FEATURES OF SOUND PROPAGATION THROUGH AND STABILITY OF A FINITE SHEAR LAYER*

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SUMMARY

The plane wave propagation, the stability and the rectangular duct mode problems of a compressible inviscid linearly sheared parallel, but otherwise homogeneous flow, are shown to be governed by Whittaker's equation. The exact solutions for the perturbation quantities are essentially the Whittaker M-functions $M_{i\tau,\pm 3/4}(4i\tau\eta^2)$ where the non-dimensional quantities τ, η and $4\tau\eta^2$ have precise physical meanings. A number of known results are obtained as limiting cases of our exact solutions. For the compressible finite thickness shear layer it is shown that no resonances and no critical angles exist for all Mach numbers, frequencies and shear layer velocity profile slopes except in the singular case of the vortex sheet.

INTRODUCTION AND BACKGROUND

Studies on compressible free shear layers have not been extensive. In fact with the exception of the earlier work of Graham and Graham (ref. 1) who studied sound propagation through a finite linearly sheared layer in the low-frequency limit, it has been only recently that Blumen et al (ref. 2) obtained an exact solution for the stability of the shear layer with an hyperbolic tangent profile with the significant result that this shear layer is unstable to two dimensional disturbances for all Mach numbers whereas the vortex sheet is known to be unstable only for $M<2\sqrt{2}$, a result that cautions against modeling real sheared flows with vortex sheets, as has been the practice in a number of recent noise research studies, since, as the authors point out, even the long wavelength characteristics of finite thickness shear layers may be quite different from the corresponding properties of the analogous vortex sheet. In this study we consider sound propagation and stability in linearly sheared parallel compressible inviscid homogeneous flows. Work relating to the solutions of the pressure perturbation equation has been that of Küchemann (ref. 3) who also considered the stability of a boundary layer approximated by a linear velocity profile, the study of Pridmore-Brown (ref. 4) and that of Graham and Graham (ref. 1).

Küchemann (ref. 3) obtained a formal series solution for the density perturbation equation and he also arrived at a solution supposedly valid for large values of (our) parameter $\eta = \frac{1}{K} - M$. His series solution, although it is given in a cumbersome and lengthy form, is correct but his asymptotic solution is in

*Work supported under NASA Grants NASA 2007 and NASA 676 to the Joint Institute of Aeronautics and Astronautics.

serious error. Pridmore-Brown (ref. 4) solved the pressure perturbation equation in the short wavelength approximation, i.e., for large values of (our) parameter $\tau = \frac{\omega}{4b} K$. His asymptotic solutions may also be in error. Graham and Graham (ref. 1) studied the problem of a plane wave incident on a linear velocity profile free shear compressible inviscid layer. They used entirely a series solution of the density perturbation equation which they independently rederived apparently unaware of the earlier work of Küchemann. Because they used the series solution only, they were unable to give proofs for the range of the parameters τ and η for which ordinary, total or amplified reflection occurs, although correctly identified the regions intuitively. More importantly and for the same reasons, they could neither prove the existence or non-existence of reasonances or critical angles, but for the case of "sufficiently thin -- but not zero thickness -- shear layer" nor could they draw any conclusions for either the large Mach number M or large τ cases.

SYMBOLS

a	Speed of sound of the homogeneous fluid
b	Velocity profile slope of the shear layer
c	Disturbance phase speed (in general complex)
$\vec{e}_f$	Unit vector in the direction of the mean flow
f,g	Independent solutions of equation (1)
k,k_o	Wave numbers of incident wave
m	Second index of the Whittaker M-functions
n	Index of refraction
$\vec{n}$	Wave normal unit vector
$p^{(1)}$&$p^{(2)}$	Linear combinations of f and g in equation (5)
q	Parameter of transformation in equation (1)
$w(\vec{r})$	z-component of the velocity perturbation
x,y,z	Rectanular coordinates
z_1	Shear layer thickness
A,B,C,D	Functions of f, g and their derivatives in equation (7)
$\mathcal{A},\mathcal{B}$	Functions of A,B,C and D in equation (9)
H	Heaviside function
K	Inverse of the x-component of the disturbance Mach number
$\vec{M}_d$	Disturbance vector Mach number
$\vec{M}_f$	Mean flow vector Mach number
$M(z)$	Mean flow Mach number
R.P.	"Real part of"

R	Reflection coefficient
R_1, R_2	In and out of phase component of R
T	Transmission coefficient
T_1, T_2	In and out of phase component of T
$\vec{U}(z)$	Mean velocity
W	Dependent variable in Whittaker's equation
α	x-component of the wave vector of the pressure disturbance
η	Non-dimensional variable in equation (2)
θ, θ_o	Angle of incidence of plane wave
ξ	Independent variable in Whittaker's equation
τ	Non-dimensional parameter in equation (2)
Φ_u & Φ_ℓ	Perturbation velocity potentials
ω	Disturbance frequency

SOLUTION OF THE PRESSURE PERTURBATION EQUATION

In a homogeneous inviscid compressible parallel shear flow having a linear velocity profile in the z-direction only, i.e., $U = U(z) = bz$, it may be easily shown that, starting either from the linearized equations of motion or directly using the appropriate linearized form of the convective wave equation, the z-dependent part $p(z)$ of the pressure perturbation $p(\vec{r})$ is governed by the equation:

$$p_{\eta\eta} - \frac{2}{\eta} p_\eta + (4\tau)^2 (\eta^2 - 1) p = 0 \qquad (1)$$

where

$$\eta = \frac{1}{K} - M, \quad 4\tau = \frac{\omega}{b} K, \quad \text{and} \quad M = M(z) = \frac{bz}{a}. \qquad (2)$$

M is the local Mach number and K and ω acquire the following meanings depending on the problem at hand:

(i) Free Shear Layer: Propagation of a plane wave of wave vector $\vec{k}$ and frequency ω impinging on the shear layer from a half-space ($z<0$) of relative rest and at an angle θ measured from the z-axis ($-\frac{\pi}{2} \leq \theta \leq +\frac{\pi}{2}$):

$$K = \frac{1}{\sin\theta}$$

(ii) Free Shear Layer: Stability for assumed disturbances of the form $p(\vec{r}) = p(z)\, e^{i\alpha(x-ct)}$ (α and c possibly complex):

$$K = \frac{a}{c} \quad \text{and} \quad \omega = ac$$

(iii) Sound Propagation in Rectangular Ducts: Modes for assumed disturbances of the form $p(\vec{r}) = p(z)\, e^{i\alpha(kx-\omega t)}$ (k and ω real):

$$K = \frac{ak}{\omega}$$

The transformation $p = \eta^{\frac{1}{2}} W(\xi), \xi = q\eta^2$ with $q = 4i\tau$, reduces equation (1) into Whittaker's equation for W, so that the two independent solutions f and g of equation (1) are:

$$\begin{Bmatrix} f \\ g \end{Bmatrix} = (4i\tau)^{-\frac{1}{2}\pm m}\, M_{i\tau,\mp m}(4i\tau\eta^2) \quad , \quad m = \frac{3}{4} \tag{3}$$

Where M are the Whittaker M-functions and τ, η, K are defined following equation (2). It is easily shown that for τ and η real, the functions f and g are also real with f being even and g odd functions of η whereas both f and g are even functions of τ. Moreover series and asymptotic expansions for f and g are readily obtainable from the known properties of the Whittaker M-functions (ref. 5 and 6). The series expansion agrees with the series obtained by Küchemann (ref. 3) and Graham and Graham (ref. 1) although the form that results from equation (3) above is not only more compact, but also faster converging. The asymptotic forms of f and g and their derivatives with respect to η are obtained in terms of I_{2m}, I_{2m+1} with $m = \pm\frac{3}{4}$ using Oliver's method (ref. 5 and 6) and they are in disagreement with both the results of Küchemann (ref. 3) and Pridmore-Brown (ref. 4). This was expected as mentioned in the introduction since Küchemann essentially seeking an expression for large η neglected 1 compared to η^2 in the last term of our equation (1) which is tantamount to setting $\tau = 0$ in Whittaker's equation, whereas Pridmore-Brown by applying Langer's method obtained only a non-uniform leading term of an asymptotic expansion in terms of Airy functions. These and other details may be found in reference 7.

THE FINITE THICKNESS LAYER

Plane Wave Propagation

We consider the two-dimensional finite inviscid compressible shear layer of thickness z_1 with velocity profile

$$\begin{aligned} U &= bz_1 && z \geq z_1 \\ &= bz && z_1 \geq z \geq 0 \\ &= 0 && 0 \geq z \end{aligned} \tag{4}$$

in the (x,z) plane and a time-harmonic monochromatic plane wave incident from the z<0 half-space with wave vector $\vec{k}$ and wave number $k = \frac{\omega}{a}$, in the (rest) frame of reference of the stationary fluid at z<0, in an otherwise homogeneous fluid in the entire (x,z) plane. The velocity potentials in the lower region (of relative rest) and the upper region of uniform flow are:

$$\Phi_\ell = \text{R.P.}\left\{ \mp i \left[e^{\pm ik(x\sin\theta + z\cos\theta - at)} + Re^{\pm ik(x\sin\theta - z\cos\theta - at)} \right] \right\}, z \leq 0$$

$$\Phi_u = R.P.\left\{\mp iTe^{\pm ik\left[x\sin\theta \pm (z-z_1)|\sin\theta|(\eta^2-1)^{\frac{1}{2}} - at\right]}\right\}, z \geq z_1$$

where R.P. denotes "real part of"; the first term in equation (5) represents the incident wave coming from the half-space $z \leq 0$ and R and T are respectively the complex reflection and transmission coefficients for the velocity potential, and the upper signs are taken for $\eta > 0$.* In the middle (shear layer) region with the velocity profile equation (4) the pressure perturbation $p(\vec{r})$ and the z-component $w(\vec{r})$ of the velocity perturbation are given by:

$$p(\vec{r}) = p^{(1)}(\eta)\sin\left[k(x\sin\theta - at)\right] + p^{(2)}(\eta)\cos\left[k(x\sin\theta - at)\right]$$

$$w(\vec{r}) = \frac{a}{4\tau\eta}\left\{p_\eta^{(1)}(\eta)\cos\left[k(x\sin\theta - at)\right] - p_\eta^{(2)}(\eta)\sin\left[k(x\sin\theta - at)\right]\right\}$$

where $p^{(1)}(\eta)$ and $p^{(2)}$ are linear combinations of the independent solution f and g, equation (3), of equation (1), i.e.

$$p^{(1)}(\eta) = a_{11}f + a_{12}g, \qquad p^{(2)}(\eta) = a_{21}f + a_{22}g \tag{5}$$

with a_{ij} constants. Writing also $R = R_1 + iR_2$, $T = T_1 + iT_2$ for the complex reflection and transmission coefficients R and T respectively and applying the boundary conditions (continuity of the pressure perturbation $p(\vec{r})$ and z-component $w(\vec{r})$ of the velocity perturbation) at the interfaces $z=0$ and $z=z_1$, and after separating real and imaginary parts in the resulting equations one obtains a system of eight linear algebraic equations for the determination of the eight unknowns a_{ij}, R_i, T_i. After somewhat tedious but straightforward algebra the following expressions for the reflection and transmission coefficients are obtained:

$$1 - R^2 = \pm\frac{72}{|\sin 2\theta|}\cdot\frac{\eta_1^2\sqrt{\eta_1^2 - 1}}{(A \mp B)^2 + (C \mp D)^2} = \pm|\tan\theta|\sqrt{\eta_1^2 - 1}\,T^2 \tag{6}$$

for $|\eta_1| > 1$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, R_i, T_j real, and

$|R_1|^2 = 1$, $T = T_1 = T_2 = 0$, for $|\eta_1| < 1$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, R_i, T_j complex

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*This representation used by Miles (ref. 8) and Graham and Graham (ref. 1) is consistent with the radiation condition as postulated by Miles. Actually Sommerfeld's radiation condition does not apply for plane waves and the difficulties arising in such a case have been discussed by Lighthill (ref. 9). At any rate these representations for the velocity potentials insure that the reflected and transmitted waves are outgoing in a reference frame fixed in the upper fluid and are consistent with Miles's postulate and Ribner's intuitive picture (ref. 10).

$$
\left.\begin{aligned}
\text{with,}\quad A &= \frac{1}{4\tau}\frac{K}{\sqrt{1-K^2}}\left[f_\eta(0)g_\eta(1) - f_\eta(1)g_\eta(0)\right]\\
B &= (4\tau)(\text{sgn}K)\sqrt{\eta_1^2-1}\left[f(1)g(0) - f(0)g(1)\right]\\
C &= \frac{K}{\sqrt{1-K^2}}(\text{sgn}K)\sqrt{\eta_1^2-1}\left[f(1)g_\eta(0) - f_\eta(0)g(1)\right]\\
D &= f_\eta(1)g(0) - f(0)g_\eta(1)\\
\text{with } K &= \sin\theta
\end{aligned}\right\} \quad (7)
$$

The upper signs in equation (6) hold for $\eta_1>1$ and the lower signs for $\eta_1\leq -1$, in both cases $|\eta|>1$. In equation (7) we used the notation 0 and 1 in the arguments of f and g and their derivatives with the understanding that 0 designates evaluation at $\eta=\eta_0=\eta|_{z=0}=\frac{1}{\sin\theta}$ and 1 designates evaluation at $\eta=\eta_1=\eta|_{z=z_1} = \frac{1}{\sin\theta} - M_1$, i.e., at the two edges of the shear layer.

It is clearly seen from equations (6) that the various reflection regimes are:

$$
\begin{aligned}
&\eta_1>1, && R^2<1 && : \text{Ordinary Reflection}\\
&-1<\eta_1<+1, && R^2=1 && : \text{Total Reflection}\\
&\eta_1<-1, && R^2>1 && : \text{Amplified Reflection}
\end{aligned} \quad (8)
$$

These regimes are quite analogous to the three regimes found by Miles (ref. 8) and Ribner (ref. 10) for the vortex sheet case and intutively arrived at by Graham and Graham (ref. 1). The above conditions in equation (8) imply that although the values of the reflection and transmission coefficients depend on the frequency ω and the velocity profile slope b, the dependence is only through the angle of incidence θ, and the two parameters $\tau=\frac{\omega}{b}\sin\theta$ and $\eta_1=\frac{1}{\sin\theta}-M_1$, whereas the conditions for the three reflection regimes are independent of ω or b and depend only on η_1 which is the Mach number $\frac{1}{\sin\theta}$ of the x-component of the phase velocity of the incident wave front relative to the relative Mach number M_1 of the two uniform flows confining the shear layer.

The limiting case of the vortex sheet is easily obtained in the limit $\tau\to 0$ (high-frequency or long acoustic wavelength limit) whereas the low-frequency or short wavelength limit is obtained by letting $\tau\to\infty$ in equation (6); in the former case:

$$
1-R^2 = \pm 4\,\frac{|\tan\theta|\,\eta_1^2\sqrt{\eta_1^2-1}}{(\sin^2\theta\,\eta_1^2 \pm |\tan\theta|\sqrt{\eta_1^2-1})^2}, \quad \tau\to 0
$$

which agrees with the results of Miles (ref. 8) and Ribner (ref. 10), whereas in the short wavelength case $1-R^2$ becomes the Heaviside function H:

$$1-R^2 = H\,(1-\eta_1), \qquad \tau\to\infty$$

Resonances and Critical Angles

In this section we give a formal proof that in the amplified reflection regime, $\eta_1<-1$, there are no resonances and in the ordinary reflection regime, $\eta_1>+1$, there are no cricital angles.

First it is easily deduced from equation (6) that excluding the singular cases of $\theta\to 0$ or $M\to\infty$ (in special ways) we may assume that neither A=C=O or B=D=O, nor all four A,B,C and D may be zero simultaneously. It is next seen from equation (6) that resonances exist if the denominator in the expression for R^2 is zero in the amplifying regime $\eta_1<-1$, i.e., if

$$\Delta = \mathcal{A}^2 + \mathcal{B}^2 = 0, \quad \mathcal{A} = A+B \text{ and } \mathcal{B} = C+D \tag{9}$$

with A,B,C, and D given by equations (7). But Δ in equation (9) above is just the determinant of the coefficients a_{ij} in equations (5) of the system of the four equations determining a_{ij}. Since only the first equation of that system is inhomogeneous with right hand side proportional to τ, a solution for the a_{ij} exists if and only if either $\Delta = \mathcal{A}^2 + \mathcal{B}^2 + \neq 0$ and $\tau \neq 0$ or $\Delta = \mathcal{A}^2 + \mathcal{B}^2 \to 0$ and $\tau\to 0$. It thus follows that equation (9) has solutions, only for $\tau\to 0$, and this is precisely the limiting case of the vortex sheet; i.e., resonances are possible only for the vortex sheet. One may also obtain the same result by algebraic manipulation of the general expressions for R_1 and R_2:

$$R_1 = \frac{B^2-A^2+D^2-C^2}{\Delta}, \quad R_2 = \frac{2}{\Delta}(AD-BC), \quad \Delta = (A \mp B)^2 + (C \mp D)^2 \tag{10}$$

for the case of amplified reflection (lower signs).

For the critical angles we use the expressions in equation (10) with the upper signs (ordinary reflection $\eta_1>1$). If critical angles exist, then $R_1=R_2=0$ and using equation (10) we may easily deduce that for $B\neq 0$, $D\neq 0$ the ratio $\frac{A}{B} = \frac{C}{D}$ may then only attain the value -1 for zero reflection. But this implies that A+B=C+D=O which is precisely the condition for the existence of resonances equation (9) which we have just shown that do not exist for a finite thickness shear layer.

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Stability Considerations

For the layer equation (4) the boundary value problem leads to the following equation:

$$i(A + B) + (C - D) = 0 \tag{11}$$

where A,B,C, and D in general complex are given by equation (7) with $K = \frac{a}{c}$ and with (sgn K) ommitted in the expressions for B and C. The roots of the above equation give the dependence of the phase speed c, or of the frequency αc, on the wavenumber α. For temporal amplification α is real and positive, c and $\omega=\alpha c$ are real. For the neutral stability line however, in either case $c=c_r+ic_i$

with c_i=0, i.e., c is real, thus K is real and one may distinguish the cases $K \lessgtr 1$, $|\eta| \lessgtr 1$ corresponding to supersonic (upper signs) or subsonic (lower signs) disturbances and relative Mach numbers respectively.

Comparing equation (11) and equation (9) we see that the resonances, were they to exist, would obey the system of equations A+B=O, C+D=O whereas the neutral stability characteristics are determined by the system of equations A+B=O, C-D=O, with A,B,C,D real. Thus, in general one does not expect any connection between resonances and neutral stability eigenvalues except in the singular case of the compressible vortex sheet case which is discussed below.

Special Case: The Compressible Vortex Sheet

As before, we let $\tau \to 0$ in equation (11) and (7) to obtain

$$\left(1-\frac{1}{K^2}\right)^{\frac{1}{2}} K^2\eta^2 + (1-\eta^2)^{\frac{1}{2}} = 0 \qquad (12)$$

where η and K may be complex. Excluding the singular cases $K \to 1$ and $\eta \to 1$, as well as $K = \frac{a}{c} \to \infty$, we consider the case where the square root terms in equation (7) have the same signs, i.e., the cases $K<1, |\eta|>1$ or $K>1, |\eta|<1$. The formal solution of equation (12) above is

$$\frac{1}{K} = \frac{c}{a} = \frac{1}{2}\left\{ M \pm \left[M^2+4\mp 4\sqrt{1+M^2}\right]^{\frac{1}{2}}\right\},$$

which is in agreement with the results of Landau (ref. 11). It is a matter of simple algebra to show that in order to satisfy the inequalities $|\eta| \gtrless 1$ only the upper (minus) sign in the square root term in the above equation for the eigenvalues should be retained. Thus two neutral eigenvalues are not permissible which is precisely the result of Miles (ref. 12) which he arrived at in a totally different way, namely by considering the vortex sheet stability as an initial value problem. It is finally worth noting that for the vortex sheet case the stability equation (11) for neutral eigenvalues, becomes C-D=O, whereas for the plane wave propagation case as we saw previously equation (9) for the resonances becomes C+D=O, since for the vortex sheet, $\tau \to 0$, and A and B are $O(\tau)$ whereas C and D are O(1). Thus it is only for the vortex sheet that resonances and neutral stability eignevalues are given by the same equation i.e. $C^2 = D^2$. For the finite shear layer the roots of equation (9) and (11) the two equations are in general different. In fact, we have shown that although equation (9) may have real roots, there are no resonances for the finite thickness compressible shear layer.

PHYSICAL MEANING OF $\eta, \frac{\omega}{b}$, τ and $4\tau\eta^2$.

Variable $\eta = \frac{1}{K} - M$. 1/K is $\frac{1}{\sin\theta}$, $\frac{c}{a}$, $\frac{\omega/k}{a}$ for plane wave propagation, stability and rectangular duct mode studies respectively, and we may thus write for η:

$$\eta = (\vec{M}_d - \vec{M}_r)\cdot\vec{e}_f$$

Thus η is a relative Mach number measure i.e., it is the parallel to the mean flow component of the disturbance (phase speed based) vector Mach number $\vec{M}_d$, relative to the (relative) mean flow Mach number and thus it is a measure of the components of the relative speeds of the disturbance and the mean flow in the direction of the mean flow.

Parameter $\frac{\omega}{b}$: $\frac{\omega}{b}$ acquires the simple meaning of a characteristic Strouhal number of the flow by writing:

$$S = \frac{\text{(Shear layer thickness) x (Disturbance frequency)}}{\text{Mean Flow Speed}} = \frac{z\omega}{bz} = \frac{\omega}{b}$$

Parameters τ and $4\tau\eta^2$: For propagation of a plane wave incident from a homogeneous half-space (z<o) at an angle θ_o $(-\frac{\pi}{2}\leq\theta_o\leq+\frac{\pi}{2})$ with the z-axis, it is easy to show that the wave normals $\vec{n}$ are **independent of x, i.e., that all wave** normals of a given z-stratum are parallel. Thus defining an index of refraction $n = (1+\vec{M}_f\cdot\vec{n})^{-1} = (1+M\sin\theta)^{-1}$, it is easy to show that $n = 1-M\sin\theta_o = \sin\theta_o\eta$, $\eta = n\sin\theta\eta$. Using these relations we may write:

$$\frac{\text{Local disturbance wavelength}}{\text{Relative refraction index change}} = \frac{k_o n}{|\nabla n|/n} = \frac{\omega}{b}\sin\theta_o\eta^2 = 4\tau\eta^2$$ and $4\tau\eta^2$ is the argument of the Whittaker M-functions in our general solutions f and g of the pressure perturbation equation. τ itself also attains the simple meaning

$$\tau = \frac{\omega}{b}K = \frac{\omega/b}{1/K} \quad \frac{\text{Characteristic Strouhal Number}}{\text{Parallel component of the disturbance Mach number}}$$

CONCLUSIONS

In this paper we have examined some aspects of plane wave propagation and stability of compressible inviscid homogeneous flows characterized by a linear velocity profile. The focus of this study has been on the search for exact solutions of the perturbation equations which bring forth the salient common features of all such parallel flows. The essential conclusions of this study are: (1) The z-dependent part of the pressure (or density) disturbance of such flows is governed by Whittaker's equation with independent solutions $\eta^{-\frac{1}{2}}\mathcal{M}_{i\tau,\pm m}(4i\tau\eta^2)$, with $m = \frac{3}{4}$ where $\mathcal{M}$ are the Whittaker M-functions and η,τ, and $4\tau\eta^2$ admit the following interpretations:

$$\eta = (\vec{M}_d-\vec{M}_f)\cdot\vec{e}_f = \text{Relative Mach number parallel to mean flow}$$

$$\tau = \frac{\omega/b}{\vec{M}_d\cdot\vec{e}_f} = \text{Strouhal number} \Big/ \text{Disturbance Mach number component in the direction of the mean flow}$$

$$4\tau\eta^2 = \frac{k_o n}{|\nabla n|/n} = \text{Local disturbance wavelength} \Big/ \text{Relative refractive index change}$$

(2) Solutions to a number of other parallel flow problems may be obtained as limiting cases from our exact solutions. Such flows include the compressible vortex sheet ($\tau\to 0$), the incompressible vortex sheet ($\tau\to\infty$, $\eta\to 0$, $(\tau\eta)\to 0$), the incompressible shear layer ($\tau\to\infty,\eta\to 0$, $(\tau\eta)$-finite), and the short wavelength approximation of the compressible finite shear layer ($\tau\to\infty$). (3) The compressible finite thickness layer has <u>no resonances</u> and <u>no critical angles</u> for all

Mach numbers, frequencies, shear layer thicknesses and shear profile slopes except for combinations of the singular values of 0 or ∞ for, ω and b; two such combinations ($b \to \infty$, $z_1 \to 0$ but bz_1 finite or $\omega \to \infty$) constitute the compressible vortex sheet case.

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