INLET NOISE SUPPRESSOR DESIGN METHOD BASED UPON THE DISTRIBUTION OF ACOUSTIC POWER WITH MODE CUTOFF RATIO

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SUMMARY

Higher order spinning modes must be considered in the design of efficient noise suppressors with outer wall treatment such as in an engine inlet. These modes are difficult to measure and are in fact impossible to resolve with flush mounted wall microphones. An alternative liner design procedure is presented here which potentially circumvents the problems of resolution in modal measurement. The method is based on the fact that the modal optimum impedance and the maximum possible sound power attenuation at this optimum can be expressed as functions of cutoff ratio alone. Modes with similar cutoff ratios propagate similarly in the duct and in addition propagate similarly to the far field. Thus there is no need to determine the acoustic power carried by these modes individually, and they can be grouped together as one entity. With the optimum impedance and maximum attenuation specified as functions of cutoff ratio, the off-optimum liner performance can be estimated using a previously published approximate attenuation equation.

INTRODUCTION

The need to consider higher order spinning modes in the design of aircraft inlet suppressors with wall treatment only has been demonstrated in references 1 and 2. Using spinning modes in the propagation theory to simulate an engine inlet requires information on the modal power distribution, which is very difficult to measure. Assumptions of equal modal amplitude (ref. 3) or equal modal power (refs. 1 and 4) have been made. These assumptions may be valid for static test data (ref. 5) where the dominant source of noise may be from the interaction of the rotor with random inflow disturbances. However, in flight the character of the noise source changes considerably (ref. 6) and the modal structure giving valid liner designs has yet to be established.

Because of the difficulty of modal measurement, an alternative and more easily used method has been proposed (ref. 7). This method involves the use of the distribution of acoustic power as a function of mode cutoff ratio (hereafter called acoustic power-ξ distribution) rather than the actual modal power distribution itself. This is much simpler since many modes may have nearly the same cutoff ratio and need not be separated because they all behave the same when liner design is considered. This similar behavior is demonstrated by showing that the optimum wall impedance and the maximum possible sound power attenuation obtained at this optimum can be expressed as functions of cutoff ratio alone.

When these quantities are expressed in this way there is only a very small residual modal dependence (lobe m and radial mode μ), which can be ignored in a multimodal liner design.

It was established in reference 7 that modal optimum wall impedance was intimately related to cutoff ratio. The reference also implied that maximum attenuation and the radiation pattern were dependent upon cutoff ratio. In this paper the method will be developed into a quantitative tool useable for liner design. Approximate expressions are provided in terms of cutoff ratio for the optimum impedance and the maximum possible attenuation. Since no explicit modal identity occurs in these required inputs, modal decomposition of the noise source is replaced by the acoustic power-ξ distribution which treats all similarly propagating modes as a single entity. These equations also contain the usual input quantities such as flow Mach number, boundary layer thickness, noise frequency, and duct dimensions. Since all of the correlated quantities mentioned above involve only the optimum quantities, an off-optimum estimate procedure is also provided. This involves the approximate equation of reference 8 in which the off-optimum behavior is shown to be uniquely determined by the optimum impedance and damping along with the actual off-optimum wall impedance. The procedure outlined in this paper requires only the addition of the quantifying of the acoustic power-ξ distribution.

Methods for estimating the acoustic power-ξ distribution from the far field directivity pattern are nearing completion and should be available soon. A more desirable method using direct duct measurements with flush-mounted wall pressure transducers is currently being studied.

The problem of changes in acoustic power-ξ distribution in going from a hardwall duct to a soft wall section is discussed.

SYMBOLS

A	function of eigenvalue phase angle, eq. (15)
c	speed of sound, m/sec
D	duct diameter, m
ΔdB	sound power attenuation, dB
ΔdB_m	maximum possible sound power attenuation, dB
F	boundary-layer refraction function, eq. (11)
f	frequency, Hz
G	function of maximum possible attenuation, eq. (18)
J_m	Bessel function of first kind, order m
K	wave number, $k\tau$
k	ω/c, m^{-1}
L	acoustic liner length, m
M_0	axial steady flow Mach number, free-stream uniform value
m	spinning mode lobe number (circumferential order)
N	normalized expected number of modes versus cutoff ratio
P	acoustic pressure, N/m^2
R	amplitude of eigenvalue α
R_{HW}	hardwall eigenvalue
r	radial coordinate, m
r_0	circular duct radius, m
t	time, sec
v_g	group velocity ($\partial\omega/\partial K$)
x	axial coordinate, m
α	complex radial eigenvalue ($\alpha = Re^{i\phi}$)

β	$\Delta dB_m/\Delta dB$	μ	radial mode number
δ	boundary-layer thickness, m	ξ	cutoff ratio
ε	dimensionless boundary-layer thickness, δ/r_0	ξ_{HW}	cutoff ratio in hardwall duct
ζ_m	optimum specific acoustic impedance	σ	attenuation coefficient
ζ_{m0}	optimum specific acoustic impedance for $\varepsilon = 0$	τ	propagation coefficient
η	frequency parameter, fD/c	Φ	angular coordinate, rad
θ	specific acoustic resistance	ϕ	phase angle of eigenvalue, deg
θ_m	optimum specific acoustic resistance	χ	specific acoustic reactance
		χ_m	optimum specific acoustic reactance
		ω	circular frequency, rad/sec

DEFINITION OF THE CUTOFF RATIO

Some preliminary expressions are given here to establish the notation and terminology. The modal pressure solutions are given by (in the uniform flow region)

$$P = J_m\left(\frac{\alpha r}{r_0}\right) e^{i\omega t - im\Phi - k(\sigma + i\tau)x} \tag{1}$$

where P, α, σ, and τ should actually have m, μ subscripts to associate them with the m, μ mode. For soft walled ducts the radial eigenvalue is complex and is given by

$$\alpha = Re^{i\phi} \tag{2}$$

The damping and propagation coefficients are given by

$$\sigma + i\tau = \frac{-iM_0 + i\sqrt{1 - (1 - M_0^2)\left(\frac{\alpha}{\pi\eta}\right)^2}}{1 - M_0^2} \tag{3}$$

or

$$\sigma + i\tau = \frac{-iM_0 + i\sqrt{1 - (1 - M_0^2)\left(\frac{R}{\pi\eta}\right)^2 (\cos 2\phi + i \sin 2\phi)}}{1 - M_0^2} \tag{4}$$

where η is the frequency parameter given by

$$\eta = fD/c \tag{5}$$

For hardwall duct the definition of cutoff ratio is quite direct since the eigenvalue α is real and can be given by (in a manner similar to ref. 9),

$$\xi_{HW} = \frac{\pi\eta}{R_{HW}\sqrt{1 - M_0^2}} \tag{6}$$

This definition causes the expression in the radical of equation (4) (with $\phi = 0$) to change sign at $\xi_{HW} = 1$ and causes the pressure to be damped with distance for $\xi < 1$.

For soft walled ducts the definition of cutoff ratio is not so simple. The modes possess propagating characteristics at all frequencies so there is no precise cutoff. A definition of cutoff ratio used in reference 7 was

$$\xi = \frac{\pi\eta}{R\sqrt{(1 - M_0^2) \cos 2\phi}} \tag{7}$$

which causes the real part of the radical in equation (4) to be zero at $\xi = 1$. The definition was quite arbitrary with the only advantage being that it reduced to the hardwall definition when $\phi \to 0$. A better definition might be

$$\xi = \frac{\pi\eta\sqrt{\cos\frac{2\phi}{3} - \sqrt{3}\sin\frac{2\phi}{3}}}{R\sqrt{1 - M_0^2}} \tag{8}$$

which is thought to be a new result. This was derived by insuring that the group velocity ($v_g = \partial\omega/\partial K$) be at a minimum when $\xi = 1$, which implies a minimum acoustic power propagation. Fortunately there is not much difference between the ξ definitions for the small angles ϕ encountered at the optimum impedance. For the largest difference ξ from equation (8) is about 0.87 of that from equation (7). Thus the calculated results of reference 7 are used here without modification.

MODE CUTOFF RATIO AS THE BASIC PROPAGATION PARAMETER

In this section the cutoff ratio will be shown to be the basic parameter governing noise propagation in acoustically lined ducts. This will be done by showing that the optimum wall impedance for all of the modes can be accurately correlated by the cutoff ratio alone and that the maximum attenuation at this optimum can be adequately correlated by the cutoff ratio. All of the calculations presented here were obtained using the calculation procedure of reference 10. A uniform flow region was assumed in the duct interior with a boundary layer present near the wall. The classic uniform-flow sound propagation solutions were coupled to a Runge-Kutta integration solution through the boundary layer. The definition of modal optimum impedance is the same as in references

2 and 10 as well as that of reference 11 but with the additional considerations of Mach number, boundary-layer thickness, and higher order modes.

Optimum Wall Impedance

The discovery that the optimum liner wall impedance depends only on mode cutoff ratio was documented in reference 7. Figure 1 is repeated from reference 7 for completeness. Figure 1 shows sample optimum impedance calculations plotted in the wall impedance plane. The conditions used are those for a General Electric TF-34 engine. Note that a common locus of optima is evident despite the wide range of modes used. Only the first radial of each of the higher lobe number modes deviates from this common line, and this deviation is quite small. (The first radial is the furthest point toward the left side for a given lobe number.) Two coincident modes are singled out and compared in the insert table. The only thing these two modes have in common is the cutoff ratio. Additional results were shown in reference 7 which illustrated the excellent correlation of optimum impedance with cutoff ratio.

Maximum Possible Sound Power Attenuation

The maximum possible sound power attenuation is generally expressed as $\Delta dB/(L/D)$ and plotted against the frequency parameter ($\eta = fD/c$) as in figure 5 of reference 2. This type of plot has been recast in terms of the cutoff ratio (fig. 2). Several radial modes (μ = 1, 2, 5, and 10) for lobe numbers of m = 1, 7, and 20 are shown. In each case for a given m, the $\mu = 1$ curve is the lowest and the attenuation increases monotonically with increasing μ. Except for the first two radial modes of the lowest lobe number ($m = 1$), the curves cluster together. If an average curve is used through the cluster of curves, most of the modes will be adequately represented with the maximum error deviation from the average being about a factor of two for the lowest order modes. In a multimodal liner design, this error in only a few of the lower order modes is not anticipated to be significant. In those cases where a few low-order modes are known to carry the bulk of the acoustic power, the method proposed in this paper should not be used since for these conditions the direct modal approach is both simpler and more accurate.

With the exception noted previously, the maximum attenuation of the multitude of modes can be adequately represented by a single function of the cutoff ratio alone. The equations involved with the attenuation will be given in the next section where approximate correlations are discussed.

APPROXIMATE CORRELATING EQUATIONS

In this section approximate equations are developed for the optimum impedance and the maximum possible attenuation. The correlation between exactly calculated optimum impedance and cutoff ratio is considered to be firmly established, but the optimum impedance correlation equation given here must be considered preliminary. If more exact results are required at this time, the complete calculation procedure of reference 10 is suggested with a representative mode used at each cutoff ratio value under consideration.

Optimum Impedance Correlation Equation

The correlating equation was derived using the approximate equation of reference 10 as a starting point, which in turn was derived from the thin boundary-layer approximation theory of reference 12. The starting point from reference 10 is

$$\zeta_m = \frac{(1 + \varepsilon)\zeta_{m0}}{1 - iF\zeta_{m0}} = \theta_m + i\chi_m \tag{9}$$

where ζ_m is the optimum impedance with a boundary layer and where

$$\zeta_{m0} = \theta_{m0} + i\chi_{m0} \tag{10}$$

is the optimum impedance with the boundary-layer thickness $\varepsilon = \delta/r_0 = 0$. The quantity F is given by

$$F = \left(\frac{\varepsilon\pi\eta M_0}{4}\right)\left(1 + \frac{4}{\xi^4}\right) \tag{11}$$

where the first term is the simplest form of the equation in reference 10 and the second is an empirical correction needed in the vicinity of unity cutoff ratio. This correction is needed since the results of reference 10 and thus presumably reference 12 are not valid near cutoff.

The quantities in equation (10) must now be cast in terms of the cutoff ratio ξ if equation (9) is to be a function of cutoff ratio as it is known to be from the exact calculations. Because of limited space, the derivations can not be included here. Equations (21) to (23), (30) and (31) of reference 8 along with equation (31) of reference 2 were used. The variables M_0 and η on which boundary-layer refraction effects strongly depend (ref. 10) were carried intact through the derivation. Certain liberties were taken with the other variables such as replacing nearly first or second powers of the mode numbers with first and second powers of the eigenvalue. The eigenvalue had to be recovered in the equations in order to introduce the cutoff ratio from equation (8). The final equation, which must be considered as empirical, is

$$\zeta_{m0} \approx \frac{\xi}{(1 + M_0)}\left[\sqrt{\frac{1 - M_0}{1 + M_0}} - i0.84\ (1 - M_0)\ \frac{\xi}{\eta}\right] \tag{12}$$

Equation (12) is surprisingly accurate for the zero boundary-layer thickness optimum resistance, but large percent errors can occur in the reactance for very small values of reactance.

With equations (11) and (12) used in equation (9) the optimum wall impedance with a boundary layer can be calculated as a function of cutoff ratio. These approximate calculations are compared with the exact calculations (from ref. 7) in figures 3 and 4. The approximations predict the gross behavior of the exact calculations and are probably accurate enough for most liner design studies.

Maximum Attenuation Correlation

An expression for maximum possible sound power attenuation can be derived by using the real part σ of equation (4) and the cutoff ratio from equation (8) in the following expression (ref. 2)

$$\frac{\Delta dB_m}{L/D} = -17.4\pi\eta\sigma \tag{13}$$

which then yields

$$\frac{\Delta dB_m}{L/D} = \frac{-17.4R}{\sqrt{2(1 - M_0^2)}}\left[\left(\frac{\xi}{A}\right)^2\sqrt{1 - 2\left(\frac{A}{\xi}\right)^2 \cos 2\phi + \left(\frac{A}{\xi}\right)^4} + \cos 2\phi - \left(\frac{\xi}{A}\right)^2\right]^{1/2} \tag{14}$$

where A is given by

$$A \equiv \sqrt{\cos\frac{2\phi}{3} - \sqrt{3}\sin\frac{2\phi}{3}} \tag{15}$$

Equation (14) was used to generate the curves in figure 2 with the eigenvalues (R, ϕ) used for each mode and with $M_0 = 0$. An averaged equation can be obtained by using the values of R and ϕ near the center of the cluster of curves such as the 20, 1 (m, μ) mode $(R = 26.662, \phi = 5.46°)$ or the 7, 10 mode $(R = 41.881, \phi = 3.53°)$. An approximate form of equation (14) can be derived for large ξ as

$$\frac{\Delta dB_m}{L/D} \approx \frac{-8.7RA \sin 2\phi}{\xi\sqrt{1 - M_0^2}} \approx \frac{-40}{\xi\sqrt{1 - M_0^2}} \tag{16}$$

where the average-curve values of R and ϕ were used to arrive at the final expression. This compares favorably with the expression in reference 7 except that the term $\sqrt{1 - M_0^2}$ is missing in reference 7. When expressed on a modal basis, the Mach number would not appear in equation (16) as discussed in reference 2, but it is reinserted along with the cutoff ratio when equation (8) is used. Equation (16) is valid for the linear portion of the curves in figure 2 but it will underpredict the maximum attenuation near $\xi = 1$.

OFF-OPTIMUM ATTENUATION

As seen in figure 1 for any chosen impedance, at best only one value of cutoff ratio would be at an optimum. Since a distribution of modes and their associated cutoff ratios would be the usual case encountered, off-optimum damping must be considered. Since the optimum impedance and maximum damping are now known for any cutoff ratio value, the approximate attenuation equation of reference 8 is ideally suited for use here. This equation is expressed as

$$\theta^2 - 2\theta[\theta_m + G(\beta - 1)] + \left(\chi - \frac{\chi_m}{\beta}\right)^2 + \frac{\theta_m(3\beta - 1)}{\beta(\beta + 1)}\left[\frac{\theta_m(2\beta^2 - \beta + 1)}{\beta(\beta + 1)} + 2G(\beta - 1)\right] = 0 \qquad (17)$$

where

$$G = \frac{8.7\ L/D}{\Delta dB_m(1 + M_0)^2} \qquad (18)$$

and

$$\beta = \frac{\Delta dB_m}{\Delta dB} \qquad (19)$$

The off-optimum attenuation to be solved for is ΔdB occuring at an impedance given by θ and χ with optimum input values θ_m, χ_m, and ΔdB_m and with the usual design inputs of L, D, and M_0. When operating off-optimum ΔdB cannot exceed ΔdB_m, and β is always greater than unity. Thus a possible procedure for solving equation (17) is to increment β upward from unity until the equation is satisfied and then solve for ΔdB from equation (19).

OUTLINE OF USE OF THE ACOUSTIC LINER DESIGN PROCEDURE

In the example that follows the use of the equations presented in the preceding sections will be illustrated. The final element of the technique that is needed is the distribution of acoustic power as a function of cutoff ratio. Because of space limitation as well as the preliminary state of the development of this subject, the equations will be presented without proof and are intended for illustrative purposes only.

The modal population density as a function of cutoff ratio is expressed as

$$\frac{dN}{d\xi} = \frac{2}{\xi^3} \qquad (20)$$

If equation (20) is integrated between ξ_1 and ξ_2, the normalized number of modes between these limits is

$$N = \frac{1}{\xi_1^2} - \frac{1}{\xi_2^2} \tag{21}$$

Note that N is normalized since, if $\xi_1 = 1$ and $\xi_2 = \infty$, then $N = 1$. For simplicity equal energy per mode will be used, which may be a reasonable assumption for static test data (ref. 5). Then the acoustic power between ξ_1 and ξ_2 is also given by equation (21). Next, choose 10 increments so that 0.1 of the power falls in each interval with 0.05 of the power on each side of the interval center. The 10 ξ intervals thus have centers located at $\xi = 1.026$, 1.085, 1.155, 1.24, 1.35, 1.49, 1.69, 2, 2.58, and 4.47.

Now the attenuation calculation can be made for each value of cutoff ratio ξ at the desired values of resistance θ and reactance χ. Assume that boundary-layer thickness ε, Mach number M_0, frequency parameter η, and the allowable duct diameter D and length L are known from other considerations. For each center value of ξ, then, calculate the optimum impedance components θ_m and χ_m from equation (9) using equations (11) and (12). Next, calculate the maximum possible attenuation ΔdB_m for each ξ from equation (14) using equation (15) and the R and ϕ values given just after equation (15). Now all the inputs are available (θ, χ, θ_m, χ_m, ΔdB_m) to calculate β from equation (17) and then ΔdB from equation (19), again, for each of the 10 values of ξ. The estimated liner output acoustic power can then be calculated, and an overall ΔdB calculated by applying the ΔdB for each ξ catagory to its respective input power, summing the output powers, and comparing this sum with the total input power. The calculation is now complete at the selected value of θ and χ. If a multimodal optimization study is being made, new values of θ and χ would be selected, and the calculations repeated until the total attenuation is maximized.

CONCLUDING REMARKS

The acoustic liner evaluation method presented in this paper should provide a useful alternative to the more usual modal analysis approach. Some of the problems in the modal approach, which are not problems in the present approach, are as follows: The phase speed of a mode is inversely proportional to the propagation coefficient τ given by equation (3). If the cutoff ratio from equation (6) is used in equation (3), the term $\sqrt{1 - 1/\xi^2}$ will be found to contain all of the modal information. For well propagating modes ($\xi >> 1$) this radical is essentially equal to one and the axial phase speed of all these modes is nearly identical. Also in a multimodal situation several modes may have almost equal cutoff ratios even though $\xi \approx 1$. These modes would also be indistinguishable in an axial direction since they have the same axial phase velocity. Thus the modal acoustic power can not be uniquely determined by using axial microphone traverses, and radial traverses (which are undesirable) must be used. Modes with nearly coincident cutoff ratios do not present a problem to the method of this paper since they all behave similarly in the acoustic liner (with respect to optimum impedance and maximum attenuation) and are thus lumped together.

Some simplifying conditions have been tacitly assumed to hold in the development of the technique presented here. It has been assumed that modal cross coupling is not important in the attenuation calculations in the lined duct section; that is, the reduction of acoustic power of each mode can be calculated independently of all the other modes. Theoretically, the cross coupling should be considered (refs. 13 and 14), but for practical purposes this coupling might be negligible (0.5-dB error in the results of ref. 15). Although not directly affecting the results presented here, the problem of a possible change in acoustic power-ξ distribution in going from a hard walled section to a soft walled section must be recognized. Could a power density distribution determined in a hard duct be used for attenuation calculations in the lined duct section or what modifications must be made? This can not be answered definitely at this time, but some insight can be offered. The cutoff ratio should be related to the angle of incidence with the duct wall. If modes are available in the soft duct with angles similar to those in the hard duct, then these modes should be excited. Thus angle of incidence is preserved in much the same way as the lobe numbers are preserved. This analogy is not exact since modes with exactly the same lobe number are available in both duct sections while angles of incidence can only be approximately the same. Thus some scattering of acoustic power among the various cutoff ratios should occur, but in a multimodal situation this is not suspected to be an extremely important effect. A notable singular exception occurs when a plane wave in the hard duct reaches the soft walled section. The plane wave with $\xi = \infty$ is scattered into several soft wall modes with finite and possibly even small cutoff ratios (depending on frequency parameter). This situation does not comply with the conditions assumed earlier, since there is no mode in a very soft duct that matches the angle of incidence of the plane wave. Perhaps the most serious assumption of all is that an acoustic power-ξ distribution is more available than an acoustic power-modal distribution. In reference 7 it was implied that the far-field directivity pattern is intimately related to the acoustic power-ξ distribution. This approach, which has been pursued and is nearing completion, should allow at least a crude approximation to the power distribution. Also, the present method offers the potential for avoiding some of the problems associated with modal measurement. Measurements of the acoustic power-ξ distribution using wall mounted microphones in the duct should be developable and ultimately available.

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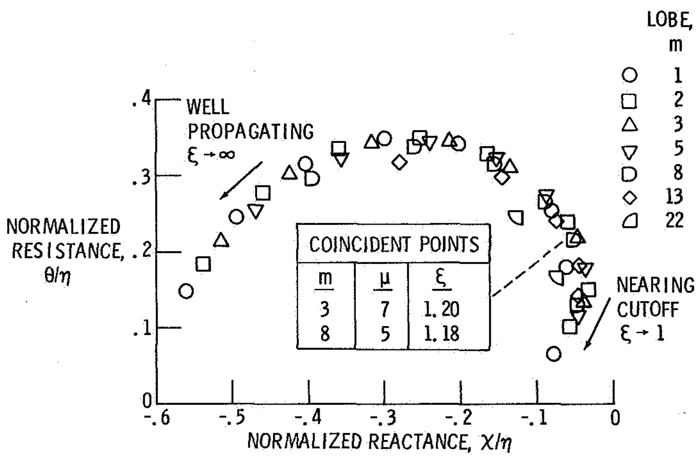

Figure 1. - Example higher order spinning mode optimum impedance locus. Frequency, 2890 hertz; frequency parameter, 9.47; Mach number, -0.36; boundary layer, δ/r_0 = 0.059.

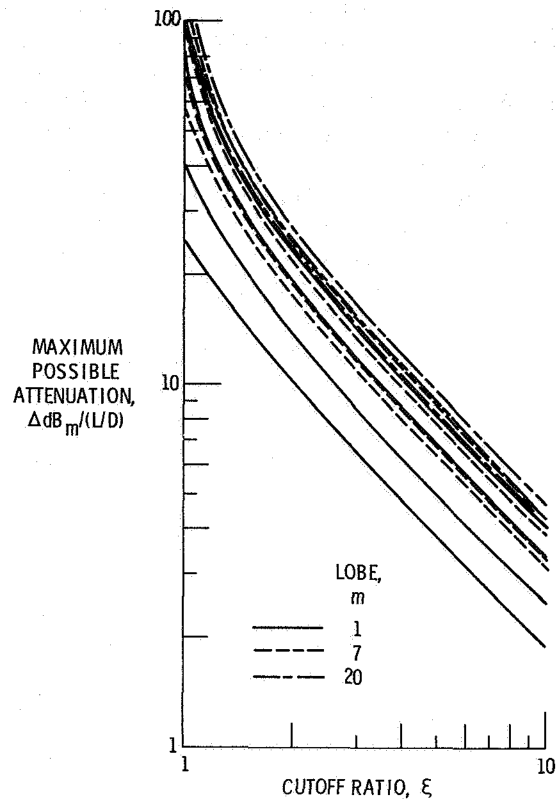

Figure 2. - Maximum possible attenuation as function of mode cutoff ratio. Zero Mach number.

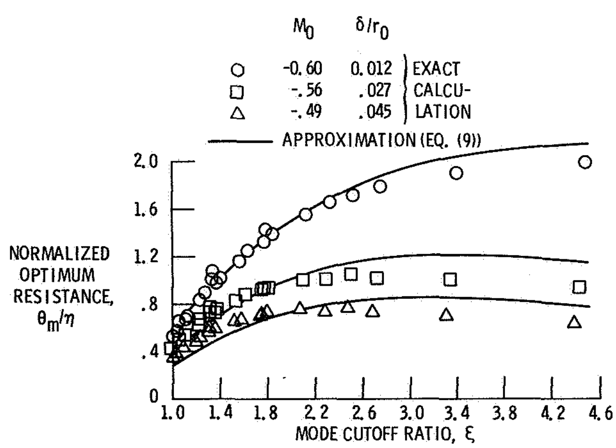

Figure 3. - Resistance correlation compared with exact calculations.

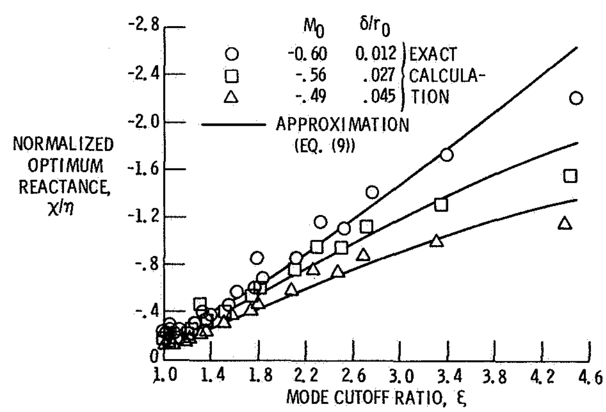

Figure 4. - Reactance correlation compared with exact calculations.