

A MICROSCOPIC DESCRIPTION OF SOUND ABSORPTION IN THE ATMOSPHERE

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SUMMARY

The various mechanisms which contribute to sound absorption in the atmosphere have been identified and a technique for computing the contribution from each is presented. The similarities between sound absorption, laser fluorescence measurements, and the opto-acoustic effect are discussed. Finally, experimental sound absorption results were compared to predictions to test the microscopic energy transfer approach.

INTRODUCTION

Perhaps the most attractive aspect of a comprehensive knowledge of basic physical phenomena is the capability of applying that knowledge to the solution of practical engineering problems. Too often, the physicist is so engrossed in his elegant esoteric theories that he neglects potential application. Too often, the engineer is so busy parameterizing his observations that he fails to identify the basic physics involved with a view towards a rigorous solution to his problems. Although these extremes are much too common, there are many cases where physicists and engineers have worked together to solve problems of mutual interest. The field of acoustics, due to the way in which it has evolved, is one of the best examples of this exchange of knowledge and interest.

This paper is going to deal with the topic of sound absorption in the atmosphere. The need for a reliable procedure to calculate sound absorption when predicting community noise around airports or when certifying aircraft is hopefully obvious to most of you. Perhaps not so familiar is the close relation between sound absorption and the design of high energy lasers or the study of the interaction potential between interacting molecules. So not only will this paper describe a procedure for computing sound absorption, it will also attempt to point out the close relation between these apparently quite different subjects.

SYMBOLS

| | |
|-----------------------------|--|
| a | absorption coefficient, db - m ⁻¹ |
| c | speed of sound, m - sec ⁻¹ |
| c _v | specific heat at constant volume, J - (kg mole) ⁻¹ - K ⁻¹ |
| c _v [∞] | specific heat at constant volume at frequencies well above the relaxation frequency, J - (kg mole) ⁻¹ - K ⁻¹ |
| c _p [∞] | specific heat at constant pressure at frequencies well above relaxation frequency, J - (kg mole) ⁻¹ - K ⁻¹ |
| c' | relaxing specific heat, J - (kg mole) ⁻¹ - K ⁻¹ |
| f | acoustic frequency, Hz |
| f _r | relaxation frequency, Hz |
| f _{r,O} | vibrational relaxation frequency of oxygen at atmospheric pressure, Hz |
| f _{r,N} | vibrational relaxation frequency of nitrogen at atmospheric pressure, Hz |
| h | absolute humidity, percent |
| P | ambient pressure, N - m ⁻² |
| P ₀ | reference pressure, 1.01325 x 10 ⁵ N - m ⁻² |
| R | universal gas constant, 8.31432 x 10 ³ J - (kg mole) ⁻¹ K ⁻¹ |
| s | relaxation strength |
| T | temperature, K |
| T ₀ | reference temperature, K |
| X | mole fraction |
| Z _{rot} | collision number |
| α | attenuation coefficient, nepers - m ⁻¹ |
| α _{cl} | attenuation coefficient for classical absorption, nepers - m ⁻¹ |
| α _{cr} | combined classical and rotational relaxation attenuation coefficient, nepers - m ⁻¹ |

- rot rotational relaxation attenuation coefficient, nepers - m⁻¹
- ratio of specific heats
- coefficient of thermal conductivity, J - (kg mole)⁻¹ - K⁻¹ - kg - m⁻¹ - sec⁻¹
- coefficient of viscosity, kg - m - sec⁻¹
- relaxation time, sec⁻¹
- angular acoustic frequency (2πf), rad - sec⁻¹

DISCUSSION

Sound absorption in the absence of turbulence can be divided into two categories: classical absorption due to viscosity, thermal conduction, and diffusion; and relaxation absorption due to vibrational and rotational relaxation of air molecules. Classical absorption can be rigorously computed for small amplitude pressure waves provided the coefficients of viscosity, thermal conduction, and diffusion are known (ref. 1). If these three quantities are considered on a microscopic basis, it is found that each can be expressed in terms of the potential of interaction between colliding molecules (ref. 2); hence, they are not independent quantities and need not be measured separately. As an example, the unken equation (ref. 3)

$$\kappa = (15R/4)[c_v/(15R) + 3/5] \quad (1)$$

can be used to express the coefficient of thermal conductivity in terms of the coefficient of viscosity. Similar expressions exist for the diffusion coefficients. However, in air the masses of the constituent molecules do not differ appreciably; hence, diffusion need not be considered in absorption calculations (ref. 3).

By substituting numerical values into the equation for sound absorption in a viscous medium and using equation (1), the absorption due to classical mechanisms becomes

$$\alpha = [2\pi f^2 / Pc](1.88) \quad (2)$$

Equation (2) has been verified under a wide variety of pressures, frequencies, and temperatures (refs. 4 and 5) and has been found valid provided the wavelength of sound does not approach the mean free path ($f/P < 100\text{MHz/atm}$). The study of sound absorption for cases where the frequency is the same as or less than the mean free path is still an active area of study (ref. 3) but for problems in atmospheric acoustics, classical absorption can be considered known.

The second source of sound absorption, the most variable and hence most difficult to predict, is that due to relaxation. When two or more atoms join to form a molecule, that molecule is free to rotate and vibrate as well as to move translationally. The translational degrees of freedom give rise to classical absorption; the rotation and vibration give rise to relaxation absorption. For now consider only a gas composed of one specie of diatomic molecules. Since the sound wave period is generally much shorter than the time required to establish thermal equilibrium with the surroundings, the wave propagates adiabatically. Under these conditions, as the positive pressure part of the sound wave impinges on a gas segment, the local temperature rises and the molecules seek to promote a larger number to a state of excitation in order to maintain a Boltzmann distribution. But molecules can only reach a higher state during collision and even then the probability of excitation is less than unity so some time is required for the gas to react to the change in temperature. The result is a phase lag which tends to dampen the acoustic signal if the period of the sound wave is of the same order or less than the time to achieve a new distribution of excited molecules.

The time required for the molecules to reach a new equilibrium once disturbed is characterized by the relaxation time. The absorption due to a relaxation process can be written as (ref. 6)

$$\alpha = \frac{\omega S}{2c} \frac{\omega\tau}{1 + (\omega\tau)^2} \quad (3)$$

$$s = -Rc' / [c_p \infty (c_v \infty + c')]$$

The relaxation time for a specific internal mode (vibration or rotation) of a gas is proportional to the probability that a single collision will result in excitation and the frequency of collisions. Since the collision frequency is inversely proportional to the gas pressure, it is obvious that the relaxation time will vary inversely with pressure.

Each internal mode of the molecule will respond to changes in temperature at a different rate so, normally, each will have a separate relaxation time. The rotational energy levels, at reasonable temperature, are more closely spaced than vibrational levels; hence, a transition from one rotational state to another is generally more rapid than a vibrational transition. In fact, for atmospheric constituents, only about five collisions are necessary to establish rotational equilibrium (ref. 7). Hence, at atmospheric pressure, the rotational relaxation time is less than a nanosecond.

If, as is customary, we write the relaxation time as the value it has at atmospheric pressure, then the rotational relaxation time for air is on the order of a half of a nanosecond. For acoustic frequencies less than 10 MHz, equation 1 for rotational relaxation of air becomes

$$\alpha_{rot} = \alpha_{cl} \cdot .0681 Z_{rot} \quad (4)$$

where the collision frequency has been expressed in terms of the viscosity

and Z_{rot} is the number of collisions necessary to establish rotational equilibrium. Using experimental values of Z_{rot} , it can be shown that to a good approximation, over the temperature range 213 K to 373 K, the combined classical and rotational relaxation absorption is given by

$$\alpha_{cr} = 18.4 \times 10^{-12} (T/T_0)^{1/2} f^2 (P/P_0) \quad (5)$$

This equation has been thoroughly verified experimentally (ref. 4).

Vibrational relaxation represents a more formidable problem. The vibrational energy levels available in air are shown in figure 1. If each of these vibrational modes exchanged energy only with translation, the problem would be relatively simple. But that's not the way nature plays the game. Instead, if there are water vapor molecules around, oxygen vibration is more likely to gain a quantum of vibrational energy during a collision with a water molecule than it is directly from translation. If there is no water vapor in the air, oxygen is most likely to become excited by energy transfer from carbon dioxide during an oxygen-carbon dioxide collision. These different energy transfer paths make the resulting mathematical equations more complex. The equations and their solutions are quite similar to the treatment of coupled springs (ref. 8). In both cases, a normal mode analysis is appropriate.

Before continuing this discussion of sound absorption, consider the similarities between the above problem and that of designing a high power carbon dioxide laser. Efficient laser operation is achieved if the upper lasing level loses little energy through collisional processes; that is, if the relaxation time for the upper lasing level is long. The rate of energy transfer between, for example, carbon dioxide and nitrogen is the same no matter where the gas is. So many of the same relaxation times important in laser design are the same as those considered in sound absorption calculations (ref. 9). It is often most convenient to measure those rates using laser excitation of a test gas and then monitoring the fluorescence decay (ref. 10) rather than trying to determine the rates directly from sound absorption measurements. Another technique, the opto-acoustic effect (ref. 11), involves the use of a pulse from a carbon dioxide laser which gives an acoustic pulse in the test gas. When determining a particular rate, the experimental approach which gives the rate most accurately should be employed without regard for what type of calculation the rate is going to be used for (e.g., sound absorption or laser design).

But now back to sound absorption. Table 1 lists the rate of energy transfer amongst various atmospheric constituents. The various rates were measured using a variety of techniques (ref. 12). It should also be noted that this table was taken from a theoretical paper on the rate at which a carbon dioxide laser beam will defocus due to atmospheric absorption and subsequent heating (ref. 13), another application of a complete kinetic description of energy transfer in the atmosphere. Once this list of rates is accurately known, one can predict sound absorption as well. When considering sound absorption, only reactions numbered 1-6 in Table 1 are important; the others affect the important relaxation times only by an immeasurably small amount.

The binary energy transfer rates from Table 1 can be substituted into a rigorous formalism for sound absorption in a multicomponent gas to give the relaxation times and strengths rigorously. This has been done (ref. 14) with the result that only two relaxation processes need be considered to calculate sound absorption in air. Further, it was found that the relaxation strengths of these two processes are given to within a fraction of a percent by the relaxing specific heats of nitrogen and oxygen. From the Planck-Einstein relation (ref. 15), the vibrational specific heat is given by

$$c' = X(\theta/T)^2 e^{-\theta/T} / (1 - e^{-\theta/T})^2 \quad (6)$$

For the atmosphere, the mole fraction of nitrogen is close to 0.78 and from spectroscopic data (ref. 16), θ for nitrogen is 3352.0 K. The mole fraction of oxygen in the atmosphere is about 0.21 and θ is 2239.1 K. The remaining one percent is argon which does not contribute to the relaxation strength.

Referring back to equation (3), it can now be seen that for vibrational relaxation there are two terms which must be added to give the relaxation absorption. The strength of each process is also given by equation (3) with the relaxing specific heat from equation (6). The only quantity yet to be determined is the relaxation time for the two vibrational relaxation processes.

The relaxation times for nitrogen and oxygen in the atmosphere can be determined exactly from the binary energy transfer rates in Table 1. It turns out that due to reactions 3, 5, and 6, these relaxation times are strongly dependent on the water vapor concentration or relative humidity. As a result, the accuracy to which the relaxation times can be computed are limited by the accuracy of rate measurements for pure water vapor (reaction 6) and the rate at which nitrogen is deexcited by water vapor, (reaction 5). For this reason, the relaxation times as a function of humidity predicted from the rates in Table 1 must still be compared to values of absorption measured in moist air and the relaxation times refined to give the best agreement with laboratory air data (ref. 5). Defining the relaxation frequency, f_r , to be $1/2\pi\tau$, the best available values for the two relaxation frequencies are

$$f_{r,0} = (P/P_0) \{24 + 4.41 \times 10^4 h [(0.05 + h)/0.391 + h]\} \quad (7)$$

$$f_{r,N} = (P/P_0) (T/T_0)^{-1/2} [9 + 350h \exp\{-6.142 [(T/T_0)^{-1/3} - 1]\}].$$

$$a = 8.686 \cdot (T/T_0)^{1/2} [f^2/(P/P_0)] \{1.84 \times 10^{-11} + 2.1913 \times 10^{-4} \\ \times (T/T_0)^{-1} (P/P_0) (2239.1/T)^2 [\exp(-2239.1/T)] / [f_{r,0} + (f^2/f_{r,0})] \quad (8) \\ + 8.1619 \times 10^{-4} (T/T_0)^{-1} (P/P_0) (3352/T)^2 [\exp(-3352/T)] / [f_{r,N} + (f^2/f_{r,N})]\}$$

This expression, the procedure described previously, and supporting documentation is now being prepared for submission as a proposed American National Standards Institute Standard.

A comparison of equation (8) to experimental results is given in figures 2, 3, and 4 for different ranges of frequency and atmospheric conditions. It is obvious that the agreement is quite good.

Concluding Remarks

Sound absorption in still air is now well understood. There is still a little problem with the relaxation frequency for moist nitrogen but that problem is now being resolved by studies in our laboratory and at Langley Research Center. Once that problem is resolved, there seems to be little need for further studies in air. However, for laser development, laser isotope separation, and other applications, sound absorption measurements in gases will continue to be a fruitful field of scientific study. Studies of sound propagation through the atmosphere should now focus on effects of ground cover and turbulence. Hopefully, these studies will again bring together physicists and engineers to the solution of pressing societal problems.

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TABLE I. REACTION SCHEME

| Reactions | Forward rate constants (sec ⁻¹ atm ⁻¹) |
|---|--|
| CO ₂ *(ν_3) + O ₂ ⇌ CO ₂ ***(ν_2) + O ₂ | 6.0 × 10 ⁴ |
| CO ₂ *(ν_2) + O ₂ ⇌ CO ₂ + O ₂ | 3.0 × 10 ⁴ |
| CO ₂ + O ₂ * ⇌ CO ₂ *(ν_2) + O ₂ | 3.0 × 10 ⁵ |
| CO ₂ *(ν_1) + O ₂ ⇌ CO ₂ **(ν_2) + O ₂ | 4.5 × 10 ⁸ |
| CO ₂ *(ν_2) + CO ₂ ⇌ CO ₂ + CO ₂ | 1.8 × 10 ⁵ |
| CO ₂ *(ν_3) + CO ₂ ⇌ CO ₂ ***(ν_2) + CO ₂ | 1.5 × 10 ⁵ |
| CO ₂ *(ν_1) + CO ₂ ⇌ CO ₂ **(ν_2) + CO ₂ | 4.5 × 10 ⁸ |
| CO ₂ *(ν_2) + N ₂ ⇌ CO ₂ + N ₂ | 3.4 × 10 ⁴ |
| CO ₂ + N ₂ * ⇌ CO ₂ + N ₂ | 1.0 |
| CO ₂ *(ν_3) + N ₂ ⇌ CO ₂ + N ₂ * | 1.8 × 10 ⁷ |
| CO ₂ *(ν_3) + N ₂ ⇌ CO ₂ ***(ν_2) + N ₂ | 6.0 × 10 ⁴ |
| CO ₂ *(ν_1) + N ₂ ⇌ CO ₂ **(ν_2) + N ₂ | 4.5 × 10 ⁸ |
| CO ₂ *(ν_2) + H ₂ O ⇌ CO ₂ + H ₂ O | 4.2 × 10 ⁸ |
| N ₂ * + O ₂ ⇌ N ₂ + O ₂ * | 1.5 × 10 ² |
| O ₂ * + N ₂ ⇌ O ₂ + N ₂ | 4.0 × 10 |
| N ₂ * + O ₂ ⇌ N ₂ + O ₂ | 1.0 |
| O ₂ * + O ₂ ⇌ O ₂ + O ₂ | 6.3 × 10 |
| H ₂ O*(ν_2) + O ₂ ⇌ H ₂ O + O ₂ * | 4.6 × 10 ⁷ |
| O ₂ * + H ₂ O ⇌ O ₂ + H ₂ O | 1.1 × 10 ⁶ |
| H ₂ O*(ν_2) + O ₂ ⇌ H ₂ O + O ₂ | 6.0 × 10 ⁴ |
| N ₂ * + N ₂ ⇌ N ₂ + N ₂ | 1.0 |
| N ₂ * + H ₂ O ⇌ N ₂ + H ₂ O | 1.1 × 10 ⁵ |
| H ₂ O*(ν_2) + N ₂ ⇌ H ₂ O + N ₂ | 1.4 × 10 ⁶ |
| H ₂ O*(ν_2) + H ₂ O ⇌ H ₂ O + H ₂ O | 1.0 × 10 ⁹ |

* DENOTES VIBRATIONAL EXCITATION

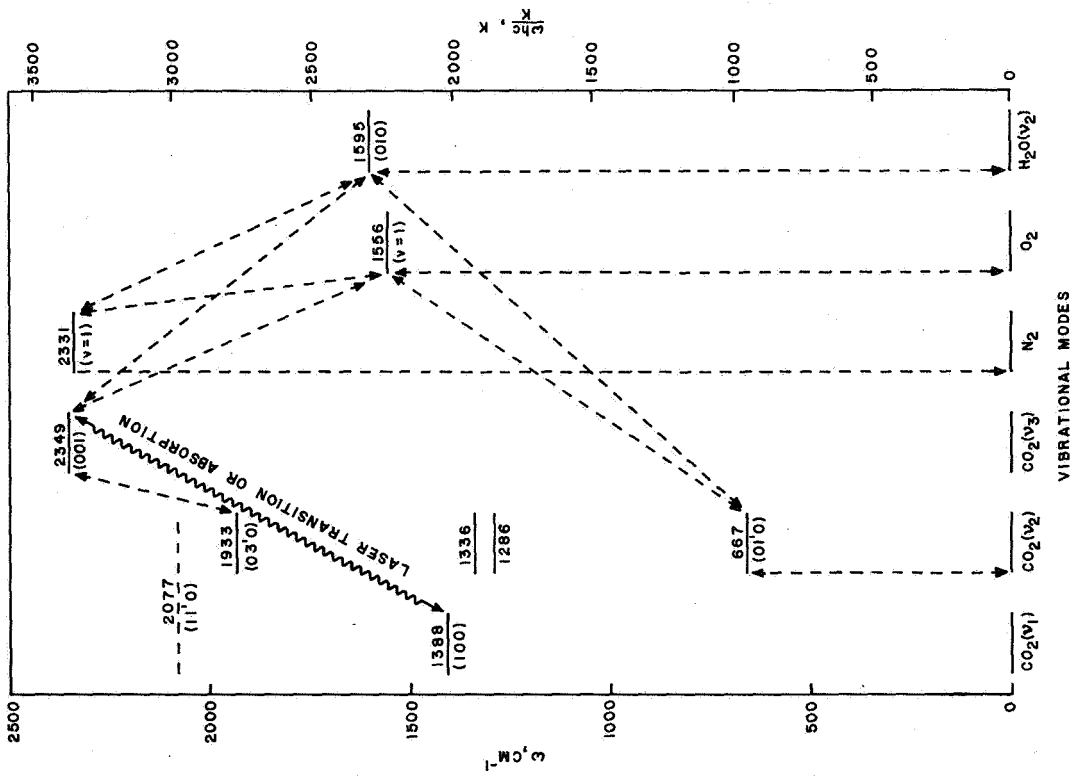


Figure 1.- Vibrational energy level diagram for air.

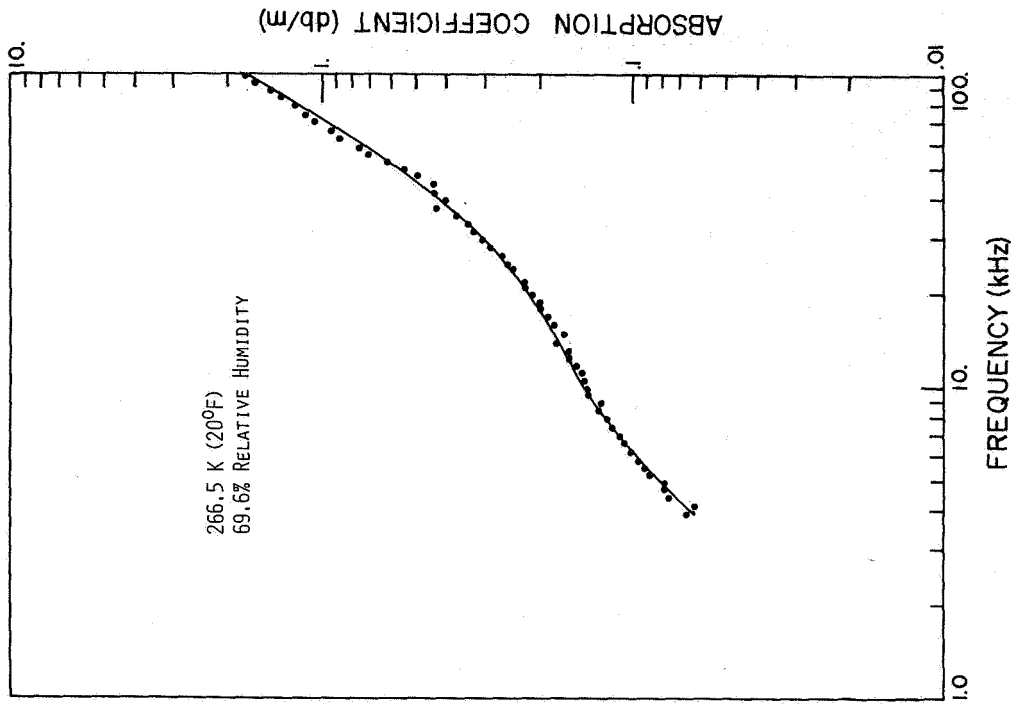


Figure 2.- Total absorption of sound in air at 266.5 K and 69.6% relative humidity.

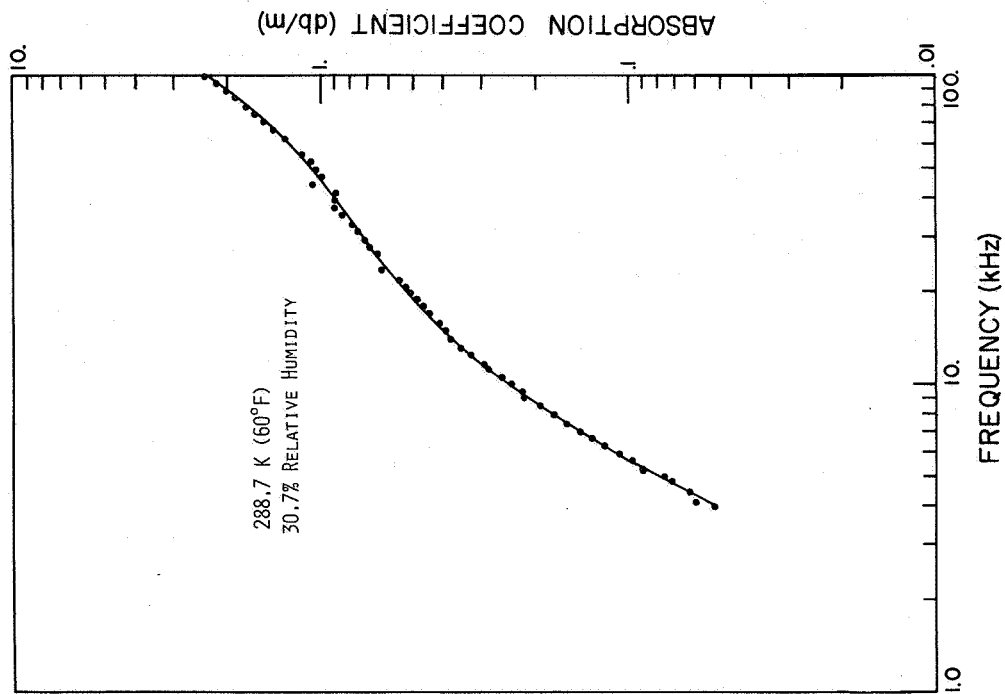


Figure 3.- Total absorption of sound in air at 288.7 K and 30.7% relative humidity.

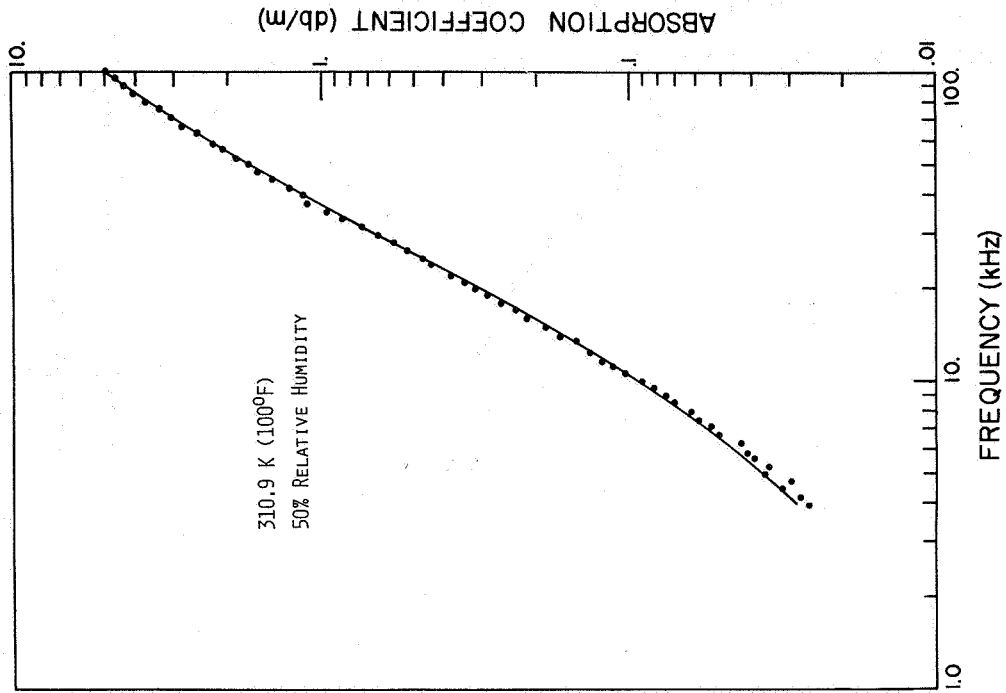


Figure 4.- Total absorption of sound in air at 310.9 K and 50% relative humidity.