DIFFRACTION OF SOUND BY NEARLY RIGID BARRIERS

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SUMMARY

An analysis is presented of the diffraction of sound by barriers whose surfaces are characterized by large, but finite, acoustic impedance. The discussion is limited to idealized source-barrier-receiver configurations in which the barriers may be considered as semi-infinite wedges. Particular attention is given to situations in which the source and receiver are at large distances from the tip of the wedge. The expression for the acoustic pressure in this limiting case is compared with the results of Pierce's analysis of diffraction by a rigid wedge. An expression for the insertion loss of a finite impedance barrier is compared with insertion loss formulas which are used extensively in selecting or designing barriers for noise control.

INTRODUCTION

The desire for effective measures to protect residential areas from noise associated with various modes of transportation has led to a resurgence of interest in the problem of sound diffraction by barriers. Wedge-shaped barriers are of particular interest because of their ubiquity both as physical entities and as subjects of scientific investigations. Little attention has been given, however, to the effect of the finite acoustic impedance of a barrier's surfaces on its performance as a noise shield. The inclusion of the effect of large, but finite, acoustic impedance in calculations of the insertion loss for a barrier is consistent with current interest in better barrier design and selection procedures. This paper describes some of the results of a theoretical study of diffraction by hard wedges and suggests a method of adapting these results to widely used formulas (refs. 1 and 2) based on rigid wedges.

SYMBOLS

Values are given in dimensionless form.

p	acoustic pressure
r,θ,z	coordinate axes in cylindrical coordinates
ω	angular frequency
c	acoustic wave speed
k	acoustic wave number
η	normalized acoustic impedance (eq. (2))
β	exterior angle of wedge
L	modified spreading distance of diffracted ray (eq. (4))
R	spherical spreading distance (eq. (14))
α	complex angle (eq. (5))
γ	angle between source-receiver path and wedge vertex (eq. (6))
IL	insertion loss (eq. (12))

PROBLEM STATEMENT

We shall restrict our attention to an idealized case in which the source may be idealized as a point source and the barrier as a wedge whose faces occupy the planes $\theta=0$ and $\theta=\beta$. The geometrical configuration of source, wedge and receiver is depicted in figure 1. The acoustic pressure field must satisfy the reduced wave equation (exp (-iwt) time dependence suppressed throughout; $k=w/c$)

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial \theta^2} + k^2 \sin^2 \gamma\right] p = 0 \qquad (1)$$

in the region $0 < \theta < \beta$ ($\pi < \beta \leq 2\pi$). The impedance boundary conditions at the surfaces $\theta=0,\beta$ may be expressed as

$$\frac{\partial p}{\partial \theta} \pm (ikr/\eta)\, p = 0, \quad \theta = 0,\beta \qquad (2)$$

where the upper sign is taken for $\theta=0$ and η is a dimensionless impedance. In addition, the pressure field must obey a radiation condition (outgoing waves from the wedge).

The solution for the point source case has been obtained from the exact solution for the diffraction of plane electromagnetic waves, particularly useful versions of which have been given by Williams (ref. 3) and Malyuzhinets (ref. 4). The details of this analysis are presented in reference 5.

PRESSURE FIELD IN THE SHADOW ZONE

The plane wave solution can be modified, following Keller's geometrical theory of diffraction (ref. 6), to yeild a solution for the pressure field due to a point source. For situations in which both the source and receiver are many wavelengths away from the tip of the wedge, with the receiver located within the acoustic shadow of the wedge, the acoustic pressure at a point (r,θ,z) due to a source at (r_0,θ_0,z_0) may be approximated as

$$p(r,\theta,z) \approx \frac{e^{ikL}\, e^{i\pi/4}}{L} \left(\frac{1}{2\, krr_0/L}\right)^{\frac{1}{2}} G(\theta,\theta_0,\alpha) \qquad (3)$$

which contains the modified spreading factor L,

$$L = \left[(z-z_0)^2 + (r+r_0)^2 \right]^{\frac{1}{2}} \qquad (4)$$

which is interpreted as the net distance a wave travels along a line from the source to the wedge tip and then along a diffracted ray to the receiver. An additional condition for the validity of equation (3) is that the quantity $(krr_0/L\pi)$ be much larger than unity. The function $G(\theta,\theta_0,\alpha)$ in equation (3) describes the variation of the strength of the diffracted pressure field, with respect to source and receiver angles θ,θ_0, and with respect to the impedance through the parameter α which is defined by

$$\cos\alpha = (\eta \sin\alpha)^{-1} \qquad (5)$$

where γ is the angle the incident ray makes with the wedge axis

$$\cos\gamma = (z-z_0)/L \; ; \; \sin\gamma=(r+r_0)/L \qquad (6)$$

The functional form of $G(\theta,\theta_0,\alpha)$ for arbitrary values of the impedance is quite complicated; it is discussed fully in reference 5. It is possible to effect a considerable simplification in the case of a hard wedge.

NEARLY RIGID WEDGE

For a wedge with very large impedance η, the scattering function $G(\theta,\theta_0,\alpha)$ can be simplified to the form

$$G(\theta,\theta_0,\alpha) \simeq \left[\frac{1}{M_\nu(\theta+\theta_0)} + \frac{1}{M_\nu(\theta-\theta_0)}\right]\left[1 + \frac{S_\beta(\theta,\theta_0)}{\eta \sin\gamma}\right] \qquad (7)$$

in which we have used

$$M_\nu(\theta) = \frac{\cos(\nu\pi) - \cos(\nu\theta)}{\nu \sin(\nu\pi)} \qquad (8)$$

and

$$S_\beta(\theta,\theta_0) = 2\left[M_\nu(\theta+\theta_0) + M_\nu(\theta-\theta_0)\right]^{-1} - Q_\beta(-\theta) - Q_\beta(-\theta_0) \qquad (9)$$

The function $Q_\beta(-\theta)$ takes on a rather simple form for wedge angles given by $\beta = p\pi/2q$, with p an odd integer and q and p relative primes. In such a case, we may use

$$Q_\beta(-\theta) = \sum_{n=1}^{\frac{p-1}{2}} \frac{-\nu \sin(\nu\pi)}{\sin \nu(2n\pi-\theta) \sin\left[\nu(2n-1)\pi-\theta\right]} + \sum_{m=0}^{q-1} \frac{\sin(\theta+2m\beta) + \sin\left[\theta+(2m+1)\beta\right]}{\sin(\theta+2m\beta)\sin\left[\theta+(2m+1)\beta\right]} \qquad (10)$$

If we neglect the term involving $S_\beta(\theta,\theta_0)$ in equation (7), we recover the far-field limit of Pierce's expression for the diffracted pressure field of a rigid wedge (ref. 7).

PRACTICAL APPLICATIONS

The combination of equations (3) and (7) leads to an expression for the acoustic pressure in the shadow zone at large distances from the tip of the wedge which included a first-order correction for finite wedge impedance:

$$p(r,\theta,z) \approx \frac{e^{ikL}\, e^{i\pi/4}}{L} \left(\frac{1}{2\pi k r r_0/L}\right)^{\frac{1}{2}} \left[\frac{1}{M_\nu(\theta+\theta_0)} + \frac{1}{M_\nu(\theta-\theta_0)}\right] \left[1 + \frac{S_\beta(\theta,\theta_0)}{\eta \sin\gamma}\right] \qquad (11)$$

A particularly useful measure of a barrier's shielding effect is the insertion loss, defined as

$$IL \equiv 20 \log_{10} \frac{|p_{No\ Barrier}|}{|p_{Barrier}|} \qquad (12)$$

For the present case we may take as $p_{No\ Barrier}$

$$p_{N.B.} = \frac{e^{ikR}}{R} \qquad (13)$$

with

$$R = \left[r^2 + r_0^2 - 2rr_0 \cos(\theta-\theta_0) + (z-z_0)^2\right]^{\frac{1}{2}} \qquad (14)$$

Thus we may express the insertion loss in terms of that for a rigid wedge

$$IL_{Rigid} = 20 \log_{10} (L/R) + 10 \log_{10}(2\pi k r r_0/L) - 20 \log_{10}\left[M_\nu^{-1}(\theta-\theta_0) + M_\nu^{-1}(\theta+\theta_0)\right] \qquad (15)$$

and the finite impedance correction

$$\Delta IL = -10 \log_{10} \left[\left| 1 + S_\beta(\theta,\theta_0)/(\eta \sin \gamma) \right|^2 \right] \qquad (16)$$

The restriction to barrier angles of the form $p\pi/2q$ presents no real problem: a desired barrier angle may be approximated closely enough by suitable choices of p and q, or one may interpolate for design purposes between the insertion losses for wedge angles with values of p,q which are convenient for computations. As an aid to the insertion loss computation values of the function $Q_\beta(-\theta)$, given by equation (10), may be computed for several wedge angles β and then plotted to provide the desire interpolation. A selection of the resulting curves is presented in figure 2. Numerical values for the finite-impedance correction to the insertion loss for an obtuse wedge with the interior angle 120° and surface admittance $\eta^{-1} = 0.1 - i0.05$ are presented in figure 3 for several combinations of source and receiver locations. As might be expected, the effect of the finite impedance is stronger for source and/or receiver locations nearer the surface of the wedge.

ADAPTATION TO CONVENTIONAL DESIGN PROCEDURES

The most widely used barrier design charts (ref. 1,2 and 8) consider only rigid barriers and generally deal only with the effective path difference, L-R, in the form of the Fresnel number $N = 2(L-R)/\lambda$. In reference 9, Pierce has shown that in general the insertion loss formula thus obtained (ref. 1, equation 7.15),

$$IL_{Rigid} \simeq 20 \log_{10} \left[\frac{\sqrt{2\pi N}}{\tanh \sqrt{2\pi N}} \right] + 5dB \qquad (17)$$

is valid primarily near the edge of the shadow boundary, which corresponds to having one of the functions M_ν (eq. (8)) very small.

In such cases it would seem to be an acceptable practice to add the correction term, equation (16), to the rigid wedge insertion loss computed from equation (17).

CONCLUDING REMARKS

The results of a theoretical study of the diffraction of sound into the shadow of a wedge with large but finite acoustic impedance have been presented. The finite-impedance correction for the insertion loss of the wedge is cast in a form which is amenable for some wedge angles to calculations using modern desk calculators. The insertion loss correction can be used in conjunction with other calculations for rigid barriers, although the rigid wedge insertion loss formula obtained here is of greater utility and involves little additional computational effort.

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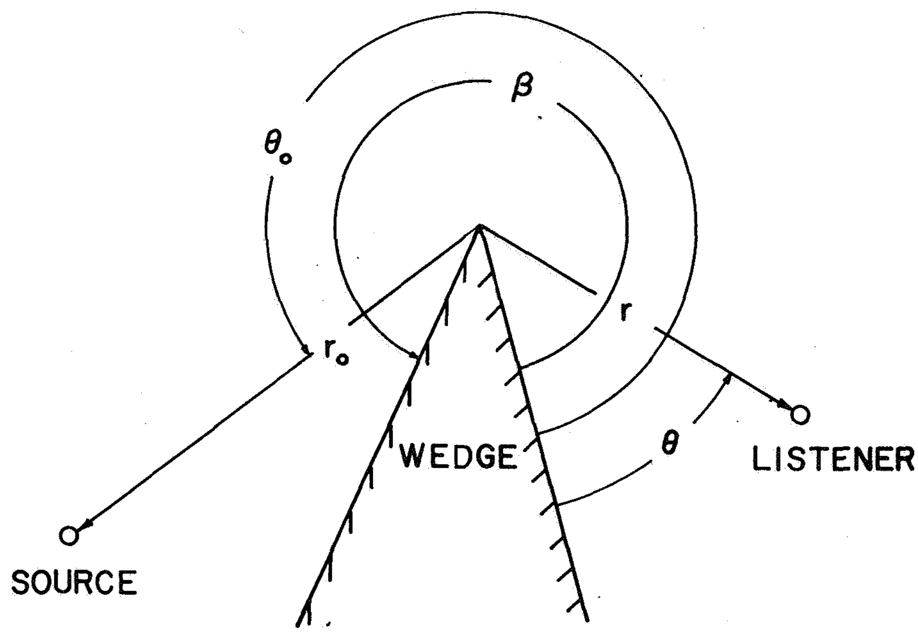


Figure 1.- Source-wedge-receiver configuration.

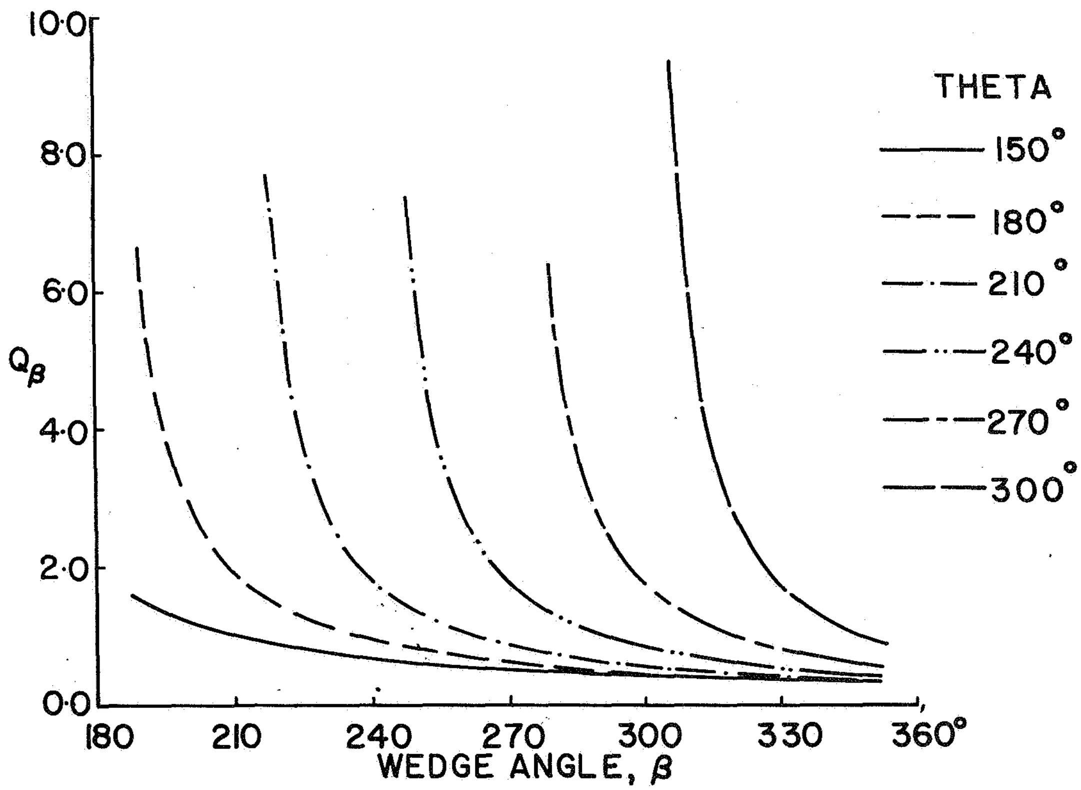
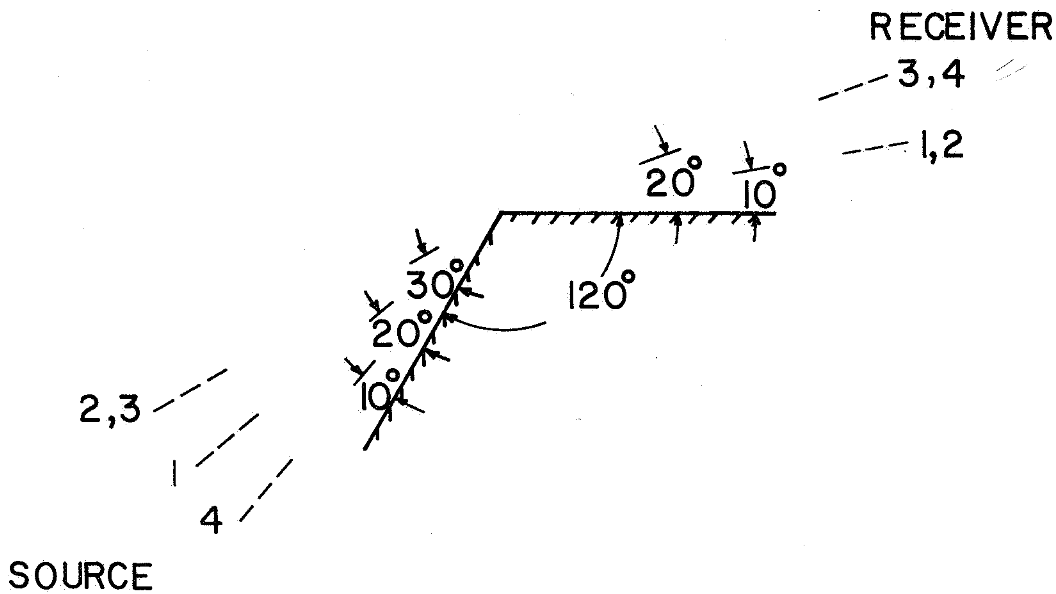


Figure 2.- Curves of Q_β (-θ).



CONFIGURATION	1	2	3	4
Δ IL (DB)	6·1	4·6	1·8	6·9

Figure 3.- Finite-impedance correction to insertion loss.
Surface admittance = 0.1 - 0.05i.