ON THE RADIATION OF SOUND FROM BAFFLED FINITE PANELS*

Patrick Leehey
Acoustics and Vibration Laboratory
Massachusetts Institute of Technology

SUMMARY

This paper is a survey of some recent theoretical and experimental research on structural-acoustic interaction carried out at the M.I.T. Acoustics and Vibration Laboratory. The emphasis is upon the radiation from and acoustic loading of baffled rectangular plates and membranes. The topics discussed include a criterion for strong radiation loading, the "mass law" for a finite panel, numerical calculation of the radiation impedance of a finite panel in the presence of a parallel mean flow, and experimental determination of the effect of vibration amplitude and Mach number upon panel radiation efficiency.

INTRODUCTION

The problem of sound radiation from a baffled finite panel is fundamental to our understanding of problems of radiation from structures and of the influences of the acoustic field upon the structural vibration itself. A rectangular panel is a reasonable representation of a structural element of an aircraft fuselage, a machine casing, or of a ship's hull or sonar dome. These are all cases where the question of acoustic radiation is of engineering significance. More generally, this structural element forms a basis for the development of techniques for dealing with multi-modal excitation of complex structures.

In the 1940's Lothar Cremer recognized that acoustic radiation from panels becomes important when the bending wave speed of the panel vibration equals or exceeds the sound speed in the adjacent medium. For an infinite plate, when the bending wave speed is less than the sound speed there is no radiation whatsoever. Modal radiation from finite panels can be classified according to the characteristics of the component traveling waves which make up the standing wave pattern of the mode. Thus, if the speed of the traveling waves is greater than the sound speed one speaks of an acoustically fast mode. It is possible for a traveling wave to have a speed less than the sound speed, but to have a trace of its wave front traveling along a panel

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*Work supported by the Sensor Technology Program, Office of Naval Research.

edge at a speed exceeding the sound speed, see figure 1. Such a mode is called an edge mode. Finally, we have the case where no traces on any edge have speeds which exceed the sound speed. Such modes are called corner modes. One might say whimsically that the trace becomes supersonic as it turns a panel corner.

The efficiency of radiation from a finite panel follows this classification. Essentially the entire surface area of the panel contributes to the radiation for an acoustically fast mode. We say that such a mode has a radiation efficiency σ_{RAD} of unity. Strips along two parallel edges of a panel, each one quarter of a bending wave length wide, contribute to the radiation from an edge mode. Here the radiation efficiency is of the order 10^{-2}. Lastly, for a corner mode, small rectangles in each panel corner about a quarter of a bending wave length on edge contribute to the radiation with an efficiency of approximately 10^{-4}.

Although the physical concepts of modal radiation resistance are straightforward to grasp, the precise calculations of radiation impedance including radiation resistance, added mass, and mode coupling terms presents a non-trivial problem in numerical analysis. The double integrals involved are improper, have integrands that are highly oscillatory, and contain lines of indeterminacy. Wallace (ref. 1) has calculated modal radiation resistance; Sandman (ref. 2) has calculated modal added mass as well. We have extended these calculations to include the capability for computing modal coupling terms for zero mean flow. In addition we can compute the effect upon modal radiation resistance and added mass of a subsonic mean flow over a panel in a direction parallel to one pair of edges. A slip flow boundary condition is imposed. Thus, the effect of the boundary layer over the panel is ignored as are also any interaction effects with flow resulting from finite amplitude displacements. A physical interpretation of the effect of mean flow upon radiation resistance is given.

It is customary to analyze the problem of panel response and radiation using *in vacuo* mode shapes. When this is done the back reaction of the acoustic field upon the panel vibration results in an infinite set of linear equations in an infinite number of unknown modal coefficients. The presence of modal coupling precludes the diagonalization of this system. For light fluid loading such as in air, the coupling terms are on one hand ignored but on the other hand are treated as the mechanism by which one obtains equipartition of vibratory energy among panel modes resonant in a narrow frequency band. This concept is fundamental to the method of statistical energy analysis of multi-modal systems as developed by Lyon and Maidanik (ref. 3) and Smith and Lyon (ref. 4). When multi-modal responses are important, statistical energy methods permit the use of average radiation resistances. Such usage appears in some of the experiments to be discussed later.

Building acousticians have long utilized the so called "mass law" in calculations of transmission losses through room partitions. This law states that the acoustic power transmitted through a panel is reduced by 6 decibels

per frequency doubling or per mass doubling. The law was originally derived for the case of an infinite panel without stiffness, see London (ref. 5). Practically the mass law is found to apply for reasonably damped plates at frequencies below that for coincidence of free bending wave speed with the sound speed.

For finite panels the mass law is demonstrated here to apply to those panel modes which are driven at frequencies well above their resonant frequencies. The responses of these modes are mass-like and eventually becomes acoustically fast. These considerations have particular importance in high frequency radiation for they set a limit on the effectiveness of panel damping treatments in reducing radiated sound. One must keep in mind that the mass law applies to radiated power levels, not to sound pressure levels. For a finite panel we shall discuss the implications of the effect of directivity upon the interpretation of the mass law.

If the fluid loading is heavy it may so affect the panel vibration that the *in vacuo* modes lose their physical significance. Intuitively one would define heavy loading as that condition when a layer of fluid over a panel, an acoustic wave length in thickness, has a mass of the same order as the panel surface mass. Davies (ref. 6) has quantified this idea by analyzing the problem of a free wave on a semi-infinite membrane normally incident on a rigid baffle in the presence of an acoustic medium. He finds that there is a sharp division between heavy and light fluid loading when the parameter $\mu = \omega m/\rho c$ is equal to unity.

We conclude this review by a brief discussion of some recent experimental results of Chang (ref. 7) on the influence of mean flow Mach number and vibration amplitude upon panel radiation efficiency.

SYMBOLS

A	$= \ell_1 \ell_2$, panel area
c	sound speed
c_b	bending wave speed
c_m	membrane wave speed
c_t	trace wave speed
D	plate flexural rigidity
k	$= \omega/c$, acoustic wave number
k_1, k_2	longitudinal and transverse wave numbers
k_{mn}	$= [(m\pi/\ell_1)^2 + (n\pi/\ell_2)^2]^{1/2}$, modal wave number

k_p	resonant wave number
ℓ_1, ℓ_2	rectangular panel longitudinal and transverse lengths
M	= U/c, Mach number
m	panel mass per area
N	number of panel resonances in a frequency band
P_{mn}	modal acoustic pressure at panel
R_{mnpq}	modal coupling impedance
T	membrane tension
U	mean flow in x_1 direction
u_τ	friction velocity
$\langle V(\omega) \rangle_{PLATE}$	vibration velocity spectral density, averaged over panel
v_{pq}	modal vibration velocity of panel
x_1, x_2	longitudinal and transverse coordinates
y^+	vibration amplitude in viscous lengths ν/u_τ
λ_b	bending wave speed
μ	= $\omega m/\rho c$, ratio of membrane mass impedance to fluid characteristic impedance
μPa	micro-Pascal, 1 Pa = 1 newton/(meter)^2
ν	fluid kinematic viscosity
Π_{RAD}	radiated sound power spectral density
ρ	fluid density
σ_{mn}	modal radiation efficiency
σ_{RAD}	panel radiation efficiency
σ_{rad}	non-dimensional radiated power (Davies)
χ_{mn}	modal added mass coefficient
ω	frequency, radians/second

MODAL RADIATION IMPEDANCE

A useful classification of panel modes in terms of their radiation characteristics is given in figure 2. This graphical representation, due originally to Maidanik, shows panel modes as a lattice of points in a wave number plot. For a given frequency, wave numbers are inversely proportional to wave speeds. Hence any mode whose wave number k_{mn} is greater than the acoustic wave number k is a slow mode. The slow modes are further subdivided into edge and corner modes. The edge mode radiation shown in figure 1 is typified by the k_{23} mode in figure 2.

We have tacitly assumed sinusoidal mode shapes. This assumption is quite good, even for a fully clamped plate, beyond the lowest few mode number pairs. For a plate, the resonantly responding modes are those for which $k_{mn} = k_p$ where k_p satisfies the dispersion relation

$$k_p^4 = m\omega^2/D \tag{1}$$

Obviously, the greatest radiation response will occur when a mode is both resonant and acoustically fast, i.e. when one also has $k_p \leq k$. The lowest frequency for which this can occur is the acoustical critical frequency

$$\omega = c^2 \, (m/D)^{1/2} \tag{2}$$

For a membrane, $k_p = \omega/c_{mn}$ where $c_m = (T/m)^{1/2}$ is the fixed membrane wave speed. Thus all resonant membrane modes are either fast ($c_m > c$) or slow ($c_m < c$).

By performing frequency transforms and modal expansions of the governing differential equations for the panel and the acoustic field, one obtains a relation

$$P_{mn}(\omega) = \sum_{p,q=1}^{\infty} v_{pq}(\omega) \, R_{mnpq} \qquad m,n = 1,2,3,\ldots \tag{3}$$

between the modal coefficients v_{pq} of normal panel velocity and the corresponding modal coefficients $P_{mn}(\omega)$ of the acoustic pressure field at the panel. The modal coupling impedance R_{mnpq} is a function of the acoustic wave number k and the panel geometry. The (self) radiation impedance may be written

$$R_{mnmn} = \rho c(\sigma_{mn} - i \, \chi_{mn}) \tag{4}$$

where ρc is the characteristic impedance of the field. Typical plots of the modal radiation efficiency σ_{mn} and the modal added mass coefficient χ_{mn} as functions of the ratio of acoustic wave number to modal wave number are shown in figures 3 and 4, respectively. Both peak in the neighborhood of $k/k_{mn} = 1$. At high wave numbers all modes become acoustically fast with efficiencies of unity and disappearing added mass. It is further true that modal couplings disappear at high wave numbers.

EFFECT OF MEAN FLOW UPON RADIATION IMPEDANCE

An analytical treatment of the effect of mean flow U over a vibrating plate has been given by Chang (ref. 7). The mean flow is in the positive x_1 direction at a Mach number M = U/c less than one. A linearized solution is obtained in which both the kinematic and dynamic boundary conditions are satisfied at the mean position of the panel. Viscous effects and the presence of a boundary layer are neglected. The transient version of this problem was solved earlier by Dowell (ref. 8).

The principal physical feature of the effect of mean flow upon radiation efficiency is readily grasped by reference to figure 5. The circle of radius k in figure 2 is replaced here by an ellipse centered on the negative x_1 axis. The upstream traveling longitudinal bending wave component of a mode need only exceed c - U in speed in order for that mode to become (half) fast. Except for those modes that are converted from slow to fast by the mean flow, the influence of mean flow upon radiation efficiency is quite small at moderate Mach numbers as is evident in figure 3. Much more significant increases in added mass are obtained as the Mach number is increased (see figure 4).

MASS LAW FOR A BAFFLED RECTANGULAR PANEL

Since the radiation impedances are computable, it is possible to express the modal response coefficients as a linear system of equations driven by the modal excitations. Once the modal response velocities are obtained, the radiation field can then be predicted using Rayleigh's equation. We shall not write these expressions here. It will suffice to say that the system is one of an infinite number of equations in an infinite number of unknowns. Customarily an approximate solution is obtained by truncating the system and inverting the coefficient matrix. This method works well for frequencies corresponding to the first few modal resonances. Variational techniques are available, Morse and Ingard (ref. 9), but they appear to offer no significant advantage in treating this case.

From earlier remarks, it is evident that the coefficient matrix becomes diagonal in the high frequency limit. All terms of the radiation impedance matrix vanish except for the radiation resistances, all of which represent unity efficiencies. Moreover, these modes respond as masses for they are being driven well above their resonant frequencies. Closed formed solutions for panel vibration and acoustic radiation can be obtained because the series involved may be summed explicitly.

The results of one of a series of experiments by Sledjeski (ref. 10) are shown in figure 6. A nearly plane acoustic wave was directed normally through a baffled rectangular membrane. The membrane dimensions were $\ell_1 = 0.305$ m, $\ell_2 = 0.203$ m. Its surface density was 0.36 kg/m^2, and it was tensioned uniformly to produce an *in vacuo* wave speed c_m of 100 m/sec. The measuring

microphone was placed one meter away directly over the membrane center on the side opposite the sound source. Measurements were taken of the sound transmitted through the membrane while the input to the sound source was very slowly swept in frequency with output controlled to provide an incident sound pressure level of 84 dB at the membrane.

Figure 6 shows a successive pattern of resonances and "anti-resonances". The resonances occur at the loaded natural frequencies of each odd/odd mode. Both the frequency of resonance and the transmitted level can be predicted from a simple single degree of freedom analysis using the calculated radiation impedance and the measured total loss factor for the mode and frequency in question. The "anti-resonances" occur at approximately the *in vacuo* resonant frequencies of modes such as (2,2), (3,2), (2,3) (i.e. modes with at least one even mode number). Such modes cannot be excited by a normally incident plane wave. The response at these "anti-resonances" comes from the non-resonant responses of adjacent odd/odd modes and is greatly weakened by the fact that the contributions of those driven above resonance are out of phase with those driven below resonance. Here the computed contributions of only a few of these adjacent modes are required to achieve a good correlation with experiment.

At high frequencies, the "anti-resonances" become less and less pronounced and there is a tendency for the sound pressure level to asymptote to a fixed value. This is the mass law regime where the level is maintained almost entirely by the lower non-resonant acoustically fast modes.

Such behavior seems anomalous in terms of the classical mass law. However, as remarked earlier, it is the sound power, not the on-axis sound pressure, which must decrease by 6 dB per frequency doubling. In fact, our closed form solution for the sum of all non-resonant modes yields precisely the vibration of a rigid rectangular piston. At high frequencies, the directivity index for this case is well-known to be 20 log $(k\sqrt{A})$ plus a constant. The mass law is not violated, for although the on-axis pressure level approaches a constant value, the directivity increases by this rule, insuring that the radiated power decreases by 6 dB per frequency doubling. This conclusion was verified experimentally by measuring the directivity patterns of the membrane radiation.

EFFECT OF FLUID LOADING

Davies (ref. 6) has solved the problem of an acoustically slow wave on a semi-infinite membrane normally incident on the edge of a semi-infinite rigid baffle in the presence of an acoustic medium. A Wiener-Hopf technique was used to obtain the reflection coefficient at the edge and the acoustic power per span radiated from a neighborhood of the edge, both as functions of the parameter $\mu = \omega m/\rho c$, the ratio of the membrane mass impedance to the characteristic impedance of the fluid. His results for a non-dimensional radiated power σ_{rad} as a function of μ and the ratio c_m/c are shown in

figure 7. His σ_{rad} is not directly a radiation efficiency, rather σ_{rad} μ is an effective radiation efficiency for a zone of the order of an acoustic wavelength wide along the edge. Note, as figure 7 shows, that it is possible for the *in vacuo* membrane wave speed $c_m = (T/m)^{1/2}$ to be supersonic. The fluid loaded free membrane wave speed, however, must remain subsonic.

It is clear by comparison of the exact calculations with the asymptotic limits for large and small μ that there is a clear demarkation at $\mu = 1$ between the heavy and light fluid loading regimes. Davies shows further that for $c_m/c << 1$ and $\mu >> 1$, the acoustic power is radiated by a line source of a quarter membrane wave length volume velocity. The inference is strong that fluid loading effects upon mode shapes are negligible for $\mu > 1$.

EFFECT OF VIBRATION AMPLITUDE

We concluded this survey with a brief mention of a few experimental results of Chang (ref. 7) on the effects of vibration amplitude and Mach number upon panel radiation efficiency. A rectangular steel plate, 0.33 m by 0.28 m, 0.152 mm thick was mounted flush in one wall of our wind tunnel test section. The back side of the plate was enclosed in a highly absorbent and damped box which also housed a non-contact solenoid exciter and non-contact Fotonic optical displacement sensors. The opposite wall of the test section was removed in the neighborhood of the plate. When the plate was excited essentially all of its radiation was directed through the opening into a fairly large reverberant chamber enclosing the test section. The plate was excited by various 50 Hz bands of white noise at mean flow Mach numbers M = 0 and M = 0.23.

The results of these experiments are shown in figure 8. The radiation efficiencies σ_{RAD} for each of the bands were determined from the expression.

$$\Pi_{RAD}(\omega) = \rho c A \sigma_{RAD} \langle V(\omega) \rangle_{PLATE} \tag{5}$$

where Π_{RAD} is the spectral density of radiated sound power and $\langle V(\omega) \rangle_{PLATE}$ is the vibratory velocity spectral density, averaged over the plate surface. This radiation efficiency can be related to the modal radiation efficiencies σ_{mn} by

$$\sigma_{AVG} = \frac{1}{N} \sum_{m,n} \sigma_{mn} \tag{6}$$

where N is the number of resonant modes in the band and the summation extends over these modes. Equations (5) and (6) are for multi-modal resonance dominated radiation. Their validity is based on the statistical energy arguments referred to in the introduction. For this plate approximately 10 modes are resonant in a 50 Hz band.

At Mach number 0.23 we see a significant increase in radiation efficiency with increasing vibration amplitude y^+ measured in viscous lengths ν/u_τ. Both boundary layer thickening and amplitude change were used to vary y^+. The computed values are from the linear "slip flow" theory for resonantly responding modes. The no flow results corresponded closely with the $y^+ = 128$ data and with the no flow computations. Some calculations were made which indicated that changes in non-resonant mode contributions with Mach number were also too small to account for the increases in radiation efficiency. It thus appears likely that a non-linear interaction with the boundary layer is involved. Unfortunately, it was not possible to vary M independently of y^+ over a sufficient range to determine whether or not there was an independent Mach number effect.

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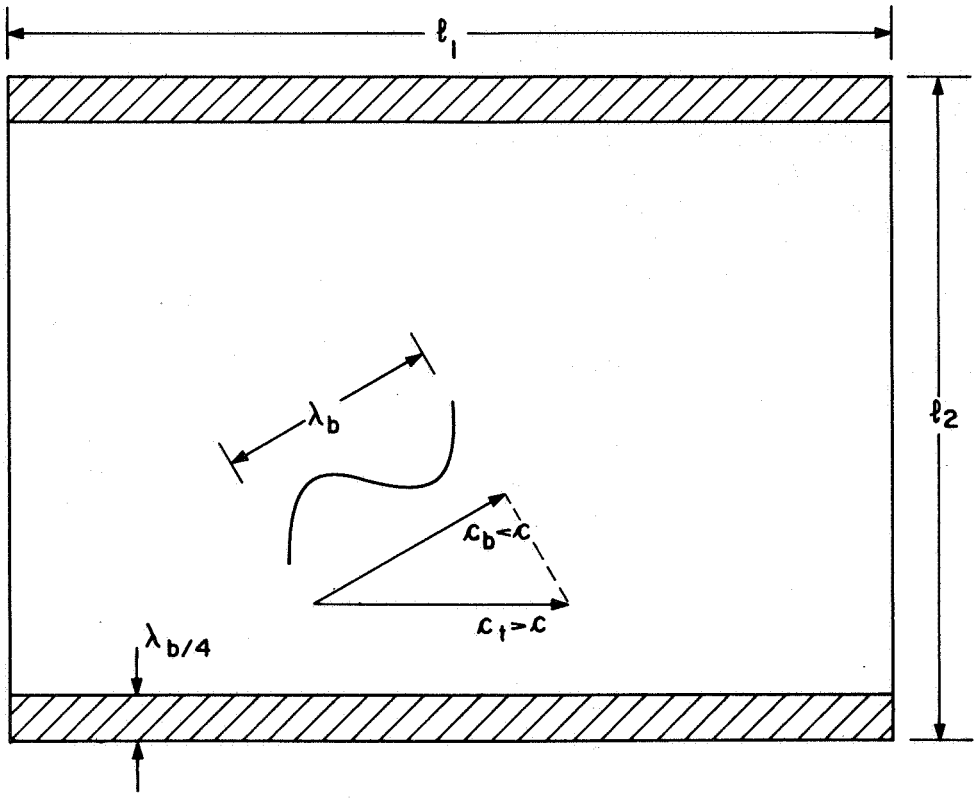

Figure 1.- Edge mode radiation.

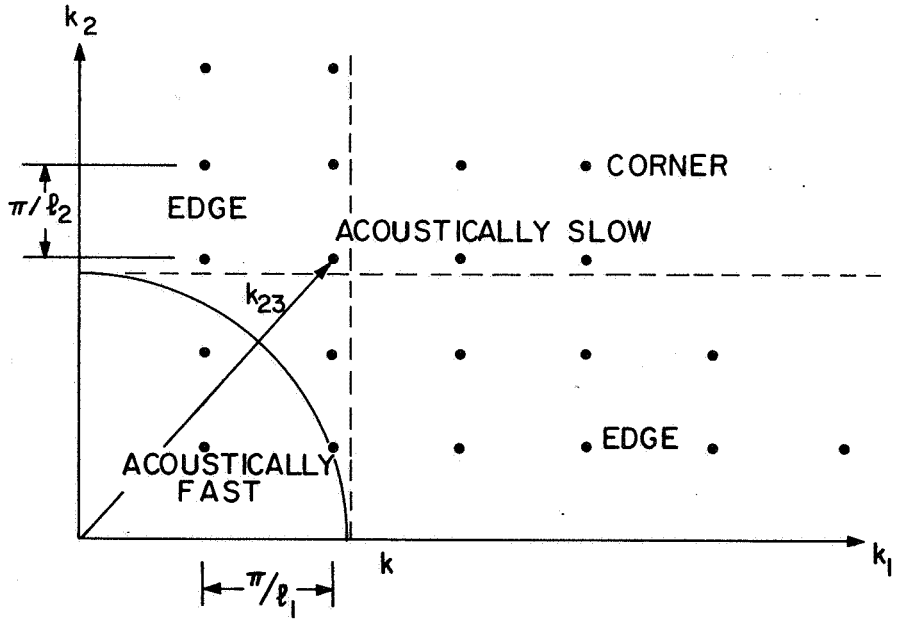

Figure 2.- Classification of panel modes in wavenumber space.

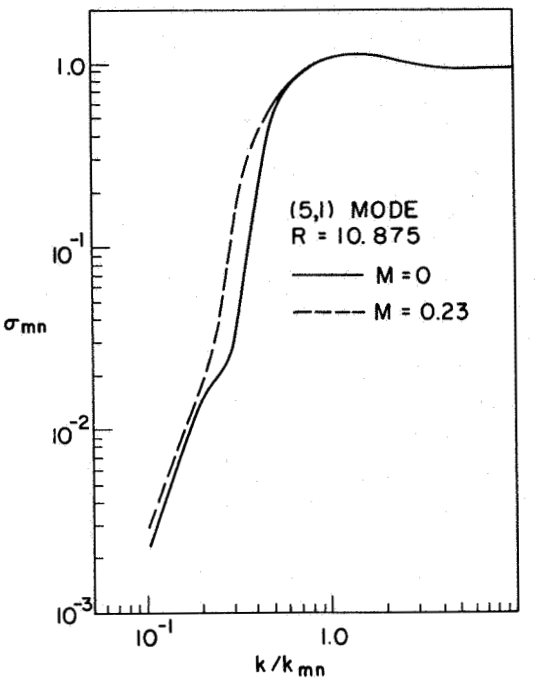

Figure 3.- Modal radiation efficiency, from Chang (ref. 7).

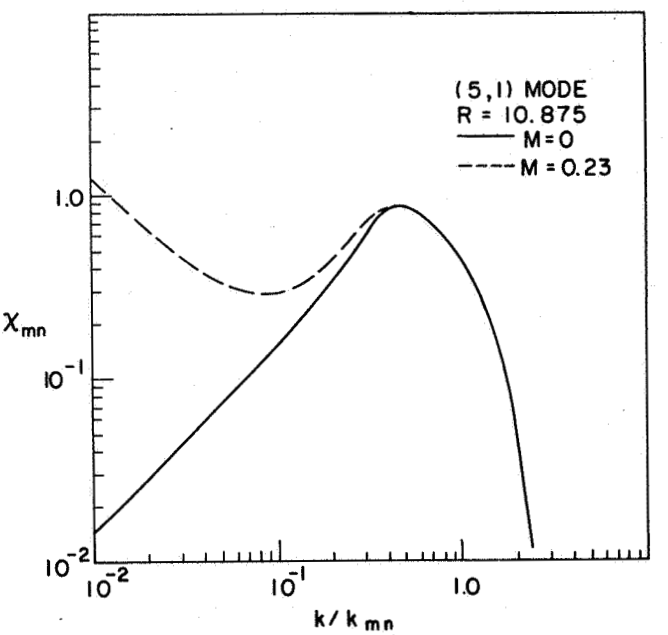

Figure 4.- Modal added mass coefficient, from Chang (ref. 7).

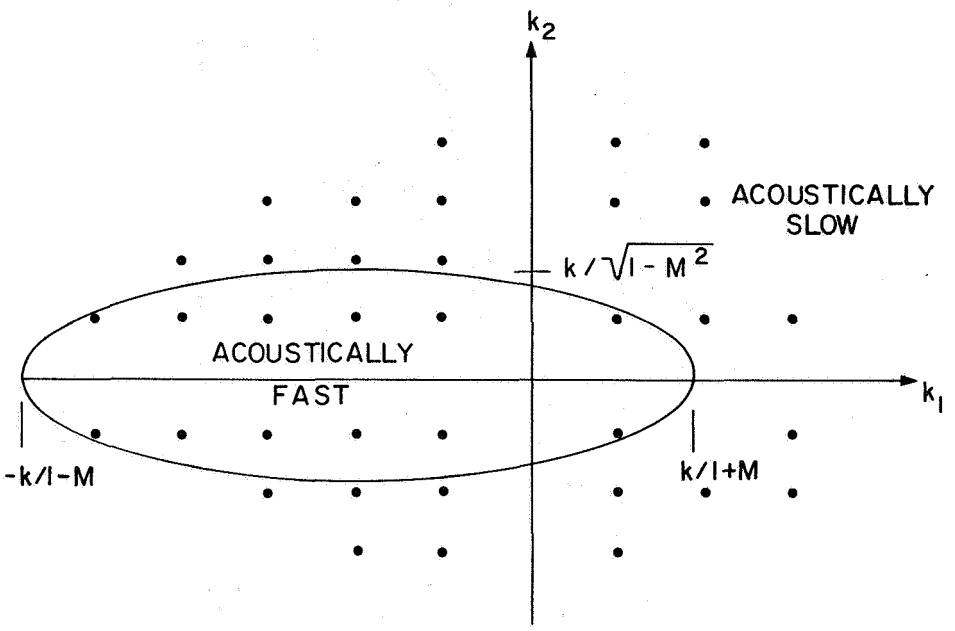

Figure 5.- Effect of mean flow Mach number upon radiation classification of modes, M = 0.6.

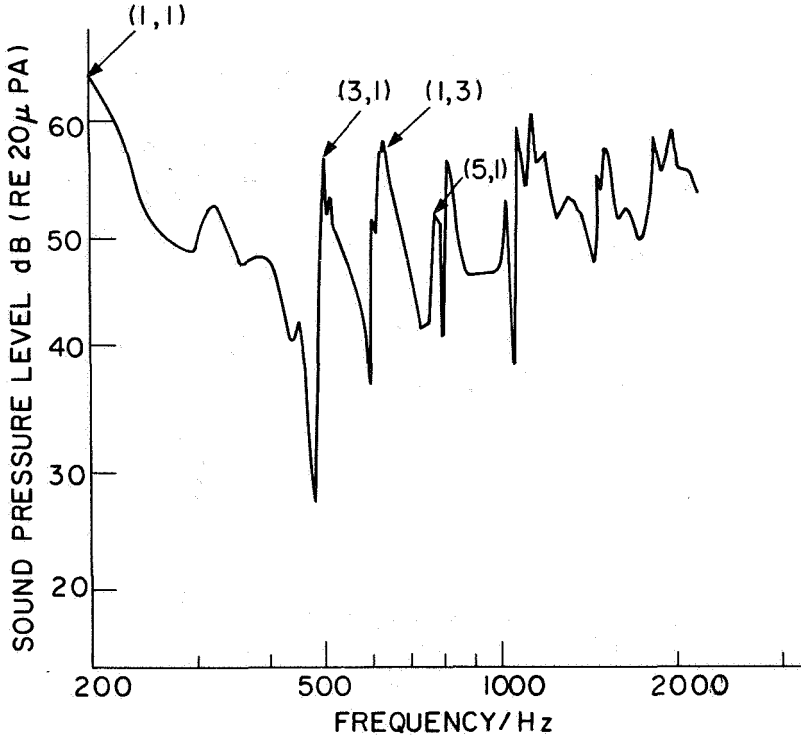

Figure 6.- Sound transmissed through a rectangular membrane, from Sledjeski (ref. 10).

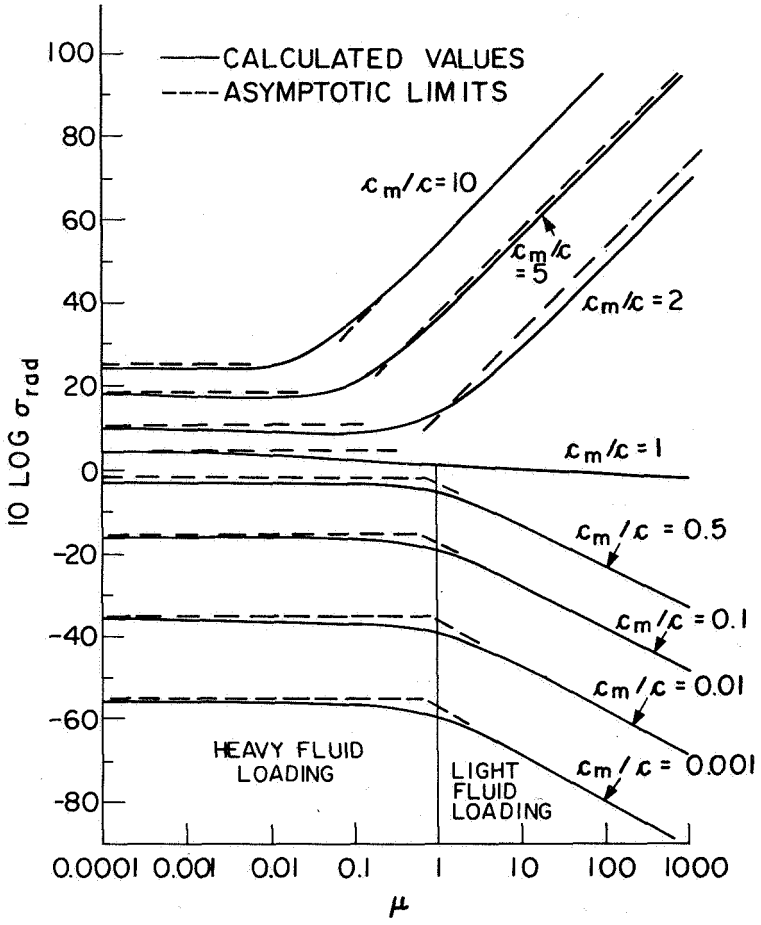

Figure 7.- Radiated power for a semi-infinite membrane, from Davies (ref. 6).

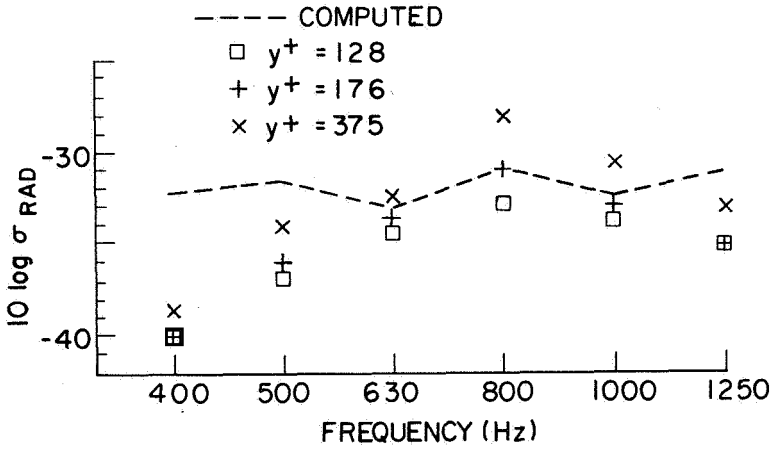

Figure 8.- Effect of vibration amplitude upon radiation efficiency, M = 0.23, from Chang (ref. 7).