THREE-DIMENSIONAL BOUNDARY LAYERS APPROACHING SEPARATION

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SUMMARY

The theory of semi-similar solutions of the laminar boundary layer equations is applied to several flows in which the boundary layer approaches a three-dimensional separation line. The solutions obtained are used to deduce the nature of three-dimensional separation. It is shown that in these cases separation is of the "ordinary" type. A solution is also presented for a case in which a vortex is embedded within the three-dimensional boundary layer.

INTRODUCTION

The determination of the aerodynamic forces and moments on many practical bodies requires the prediction of the location of boundary layer separation on the body. When the boundary layer is both laminar and two-dimensional this is not a very difficult problem. The phenomenon of two-dimensional separation is well understood and there are methods available which can be used to predict the flow up to separation and the location of separation with reasonable accuracy. When the boundary layer is three-dimensional the problem of predicting separation is considerably more difficult. In this case the usual methods of calculation, which involve such assumptions as similarity, small perturbations or yawed infinite cylinders, offer little aid. Furthermore, there are still pressing questions as to the nature of three-dimensional separation.

The criterion for three-dimensional laminar boundary layer separation is not necessarily the same as that for two-dimensional separation (i.e., the vanishing of the wall shear at the point of separation). In fact, both Maskell (ref. 1) and Lighthill (ref. 2) have pointed out that there are two possible modes of separation for the three-dimensional boundary layer. In one case the total wall shear may vanish at separation. This type of separation has been named by Maskell "singular" separation. In the second case the limiting streamlines, or streamlines closest to the solid wall, run close together and become tangent to the line of separation at separation. This type of separation has been named by Maskell "ordinary" separation.

The number of three-dimensional boundary layer calculations which have been carried out up to the vicinity of separation is quite limited. This is true, in part at least, because of the added mathematical difficulty arising from the addition of another independent variable (the third spatial coordinate) and the corresponding dependent variable (the third velocity component) in the three-dimensional problem. Another difficulty which has served to limit solutions in the vicinity of separation is the fact that flow reversal of one velocity component parallel to the wall often occurs near separation.

The present work presents an investigation of several three-dimensional boundary layer flows, which approach separation, with the objective of studying, in some detail, the nature of the flow in the vicinity of separation. The method employed in the present analysis is that of semi-similar solutions. Mathematically the method of semi-similar solutions is a technique by which the three independent variables are reduced to two by an appropriate scaling. In cases where separation occurs, the technique has a more important physical interpretation. It may be viewed as a scaling of the two surface coordinates in such a way that separation occurs at a constant value of the new scaled surface coordinate (although the value of the new scaled coordinate corresponding to separation is not known *a priori*). This property is extremely helpful in determining, from the solutions, the physical characteristics of separation.

Solutions are presented for two cases which lead to three-dimensional separation of the ordinary type. In one of these cases one of the velocity components parallel to the wall becomes negative prior to separation. Finally, a case is presented in which a vortex is embedded within the three-dimensional boundary layer.

SYMBOLS

A, B, C, D, E, H, I, J	coefficients of ξ in the reduced momentum equation (eq. (7))
$F(\xi,\eta)$, $G(\xi,\eta)$	dimensionless stream functions
$g(x,y)$	scaling function for the z-coordinate
ℓ	characteristic length for the flow
p	pressure
U	characteristic velocity for the flow
u, v, w	the x, y and z components of velocity, respectively
x, y	coordinate directions on the body surface (fig. 1)
z	coordinate direction normal to body surface
η	scaled z-coordinate
ν	kinematic viscosity
ξ	scaled x and y coordinate

ρ density

τ_w wall shear

Subscripts

δ conditions at the "upper" edge of the boundary layer

w condition at the body surface (wall)

ANALYSIS

The boundary layer equations for steady, incompressible motion in three-dimensions over a surface with large radii of curvature are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} \tag{2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \frac{\partial^2 v}{\partial z^2}. \tag{3}$$

The boundary conditions for this set of equations are:

$$u(x,y,0) = v(x,y,0) = w(x,y,0) = 0$$

$$\lim_{z\to\infty} u(x,y,z) = u_\delta(x,y) \qquad \lim_{z\to\infty} v(x,y,z) = v_\delta(x,y)$$

Here x and y are orthogonal Cartesian coordinates tangent to the body surface and z is the coordinate normal to this surface (Fig. 1). As noted earlier we wish to scale the physical coordinates x, y, and z into a new set of two scaled coordinates. The appropriate scaling is: $\eta = z/g(x,y)\sqrt{\nu}$, $\xi = \xi(x,y)$, where $g(x,y)$ and $\xi(x,y)$ are at this point unknown functions. In addition, we define two dimensionless stream functions $F(\xi,\eta)$ and $G(\xi,\eta)$ constructed so that the continuity equation is identically satisfied. The velocity components written in terms of these functions become:

$$u = u_\delta \frac{\partial F}{\partial \eta} \qquad v = v_\delta \frac{\partial G}{\partial \eta} \tag{4}$$

$$w = -\sqrt{\nu}\,\left\{\frac{\partial u_\delta g}{\partial x} F + u_\delta g \frac{\partial \xi}{\partial x}\frac{\partial F}{\partial \xi} - u_\delta \frac{\partial g}{\partial x}\eta\frac{\partial F}{\partial \eta} + \frac{\partial v_\delta g}{\partial y} G + v_\delta g \frac{\partial \xi}{\partial y}\frac{\partial G}{\partial \xi} - v_\delta \frac{\partial g}{\partial y}\eta \frac{\partial G}{\partial \eta}\right\}$$

It is easily shown, by direct substitution, that this choice satisfies the continuity equation. Now if the velocity components given by equations (4) and their derivatives are introduced into the x and y momentum equations (2) and (3), one obtains the following pair of partial differential equations in the two variables η and ξ:

$$F''' + (A + B) F F'' + (C + D) G F'' + A(1 - F'^2) + E(1 - G'F') + H\left(F'' \frac{\partial F}{\partial \xi} - F' \frac{\partial F'}{\partial \xi}\right) + I\left(F'' \frac{\partial G}{\partial \xi} - G' \frac{\partial F'}{\partial \xi}\right) = 0 \quad (5)$$

$$G''' + (C + D) G G'' + (A + B) F G'' + C(1 - G'^2) + J(1 - F'G') + I\left(G'' \frac{\partial G}{\partial \xi} - G' \frac{\partial G'}{\partial \xi}\right) + H\left(G'' \frac{\partial F}{\partial \xi} - F' \frac{\partial G'}{\partial \xi}\right) = 0 \quad (6)$$

In the transformed coordinate system the boundary conditions become:

$$F(\xi,0) = F'(\xi,0) = G(\xi,0) = G'(\xi,0) = 0 \qquad \lim_{\eta\to\infty} F'(\xi,\eta) = \lim_{\eta\to\infty} G'(\xi,\eta) = 1$$

Here the primes denote differentiation with respect to η and the coefficients A, B, C, D, E, H, I, J are functions of x and y given by:

$$A = g^{*2} \frac{\partial u_\delta^*}{\partial x^*} \qquad B = u_\delta^* \frac{\partial g^{*2}}{\partial x^*} \qquad C = g^{*2} \frac{\partial v_\delta^*}{\partial y^*} \qquad D = v_\delta^* \frac{\partial g^{*2}}{\partial y^*}$$

$$E = g^{*2} \frac{v_\delta^*}{u_\delta^*} \frac{\partial u_\delta^*}{\partial y^*} \qquad H = g^{*2} u_\delta^* \frac{\partial \xi}{\partial x^*} \qquad I = g^{*2} v_\delta^* \frac{\partial \xi}{\partial y^*} \qquad J = g^{*2} \frac{u_\delta^*}{v_\delta^*} \frac{\partial v_\delta^*}{\partial x^*} \quad (7)$$

In equations (7), we have normalized u_δ, v_δ, g, x, and y by introducing the dimensionless variables:

$$u_\delta^* = \frac{u_\delta}{U} \qquad v_\delta^* = \frac{v_\delta}{U} \qquad g^* = g\sqrt{\frac{U}{\ell}} \qquad x^* = \frac{x}{\ell} \qquad y^* = \frac{y}{\ell}$$

If semi-similar solutions are to exist, the coefficients A, B, C, D, E, H, I, and J must be functions of ξ alone. There are four relations between these eight coefficients, constructed using the fact that u_δ^*, v_δ^*, g^* and ξ must be continuous functions of x^* and y^* and thus, the second derivatives of each of these functions with respect to x^* and y^* must be independent of the order of differentiation. An additional relation between u_δ^* and v_δ^* is obtained if the component of vorticity normal to the surface vanishes outside the boundary layer. These auxillary equations together with a discussion of the method of solving the total problem is presented in reference 3. Once the eight coefficients in equations (5) and (6) are defined for a given problem, the solution of equations (5) and (6) is straight forward using an implicit finite difference technique similar to that of Blottner (ref. 4). In what follows we will be interested in the angle of the streamlines relative to the x axis. In particular we will be interested in the two extremes of this angle, evaluated in the external flow and at the wall and given respectively by:

$$\tan \beta_\delta = \frac{v_\delta^*}{u_\delta^*} \qquad \tan \beta_w = \lim_{\eta \to 0} \frac{v}{u} = \frac{v_\delta^*}{u_\delta^*} \frac{G''(\xi,0)}{F''(\xi,0)}$$

In addition, we will consider the total wall shear, or more specifically, the normalized form of the total shear given respectively by:

$$\tau_w = \mu \left\{\frac{\partial}{\partial z} \sqrt{u^2 + v^2}\right\}_{z = 0}$$

$$\tau_w^* = \frac{\tau_w \, g\sqrt{\nu}}{\mu \, u_\delta} = \frac{F''(\xi,0) + \tan \beta_w \tan \beta_\delta \, G''(\xi,0)}{\sqrt{1 + \tan^2 \beta_w}}$$

Finally, we will wish to consider the effects of the pressure gradient in the x and y directions given respectively by:

$$\frac{1}{\rho U^2} \frac{\partial p}{\partial x} \frac{g^{*2}}{u_\delta^*} = - A(\xi) - E(\xi) \qquad \frac{1}{\rho U^2} \frac{\partial p}{\partial y} \frac{g^{*2}}{u_\delta^*} = - C(\xi) - J(\xi)$$

SOLUTIONS FOR TWO FLOWS LEADING TO SEPARATION

In the present analysis we will assume that $H(\xi) = \xi$ and that $A(\xi) + 2B(\xi) = 1$. These assumptions are made to simplify the analysis and because they correspond to the scaling usually used in the analysis of two-dimensional non-similar boundary layers. In addition, consideration will be limited to that family of flows in which the external velocity components may be written as explicit functions of ξ. As a result of these assumptions one obtains the results $g^{*2} u_\delta^* = x^*$ and $\xi = x^*/(1 - \alpha y^*)$. Specifically we will consider the velocity distributions:

$$u_\delta^* = \xi - \xi^2 = \frac{x^*}{1 - \alpha y^*} \left\{1 - \frac{x^*}{1 - \alpha y^*}\right\}$$

$$v_\delta^* = 1 + \frac{\alpha \, x^{*2}}{(1 - \alpha y^*)^2} \left\{1 - \frac{4}{3} \frac{x^*}{(1 - \alpha y^*)}\right\}$$

It may easily be shown that these velocity distributions correspond to an irrotational outer flow (i.e. the vertical component of vorticity vanishes). Clearly the nature of the external flow field depends on the sign of the parameter α. Solutions will be presented for typical cases in which α is negative or positive.

With these external velocity distributions given, all the coefficients $A(\xi)$, $B(\xi)$, $C(\xi)$, $D(\xi)$, $E(\xi)$, $H(\xi)$, $I(\xi)$ and $J(\xi)$, may be written explicitly in terms of ξ. Equations (5) and (6) then form a pair of coupled, third order, partial differential equations which are similar in form to the transformed, two-dimensional, non-similar boundary layer equation and may be solved, as mentioned earlier, using an implicit finite difference technique.

We consider first the case in which α is negative. Solutions for this family of flows have been obtained for several values of α. The results for $\alpha = -0.5$ are typical and are presented in Figure 2. In this particular case the pressure gradient in the x direction is negative for $0 \leq \xi < 0.5$, positive for $0.5 < \xi < 0.51$ and negative for $\xi > 0.51$ while the pressure gradient in the y direction is positive for $0 \leq \xi < 0.5$, negative for $0.5 < \xi < 0.51$ and positive for $\xi > 0.51$.

In this case, as in all others presented herein, the integration of equations (5) and (6) starts at $\xi = 0$, where similar solutions are obtained, and proceeds in the ξ direction with an iteration on the velocity profile at each ξ station. At some downstream station the number of iterations required to obtain convergence starts to grow with each succeeding station until, at one station, convergence cannot be obtained in a reasonable number of iterations. This behavior is taken, by analogy with finite difference calculation of the two-dimensional boundary layer, as an indication of approaching a point of singular behavior, in the boundary layer equations, associated with separation. With $\alpha = -0.50$ a solution is obtained at 0.510 with convergence at each point in the velocity profile in 10 iterations. At $\xi = 0.511$ convergence cannot be obtained in 120 iterations. Separation is assumed to occur, then, in the vicinity of $\xi = 0.511$.

Figure 2 presents the results obtained with $\alpha = -0.50$ for the angle of the streamlines in the free stream, β_δ, the angle of the limiting streamlines, β_w, and the normalized normal wall shear τ_w^*. The normalized wall shear is very large near $\xi = 0$ (in the limit as $\xi \to 0$, $\tau_w^* \to \infty$ because of the normalization) but decreases with increasing ξ. As ξ approaches 0.511, τ_w^* does not approach zero, in fact, at $\xi = 0.510$ $\tau_w^* = 0.878$. Clearly then, separation in this case is not a "singular" type separation as defined by Maskell. Now if separation occurs at a value of ξ denoted by ξ_{sep}, then the equation for the separation line is given by a rearrangement of the definition of ξ, i.e.

$$y^*_{sep} = \frac{1}{\alpha}\left(1 - \frac{x_{sep}}{\xi_{sep}}\right) \tag{8}$$

and the slope of the separation line is:

$$\beta_{sep} = \arctan\left(-\frac{1}{\alpha\xi_{sep}}\right) \tag{9}$$

Thus, if ordinary separation occurs, the angle of the limiting streamlines at the wall, β_w, should approach the angle of the separation line β_{sep}, as separation is approached. In the present case with ξ_{sep} taken to be 0.511, the value of β_w at $\xi = 0.510$ is 1.3184 which is very close to the value β_{sep} of 1.3206. This value of β_{sep} is also noted on Figure 2. Clearly β_w approaches β_{sep} as separation is approached verifying the concept of "ordinary" separation.

Next we consider a case in which α is positive. Again solutions for this family of flows have been obtained for several values of α; the results for $\alpha = 0.5$ are typical and are presented in Figure 3. In this case both the pressure gradient in the x direction and the pressure gradient in the y direction are negative for $0 \leq \xi < 0.5$ and positive for $\xi > 0.5$. The magnitude of the pressure gradient in the y direction is considerably smaller than the magnitude of the pressure gradient in the x direction.

With $\alpha = 0.5$ a solution is obtained at $\xi = 0.603$ in 39 iterations, at $\xi = 0.604$ in 43 iterations and at $\xi = 0.605$ in 68 iterations. Convergence cannot be obtained at $\xi = 0.606$ in 120 iterations. Separation is assumed to occur, then, in the vicinity of $\xi = 0.606$.

In this case the total wall shear τ_w^* decreases (from an infinite value at $\xi = 0$) with increasing ξ until it passes through zero at approximately $\xi = 0.596$. With further increase in ξ, τ_w^* becomes more negative and at $\xi = 0.605$ has the value $\tau_w^* = -0.0332$. The total wall shear is negative because the x component of velocity is reversed beyond $\xi = 0.596$. The x component of velocity is reversed because of the strong positive pressure gradient (adverse pressure gradient) which acts beyond $\xi = 0.5$. It should be noted that in this work, as in reference 3, solutions are obtained in regions where one or the other velocity components are reversed without any hint of an instability. This point will be discussed later.

The wall shear, although small, is not zero at separation. Thus, this case does not represent a "singular" type separation. As noted previously, if separation in this case is "ordinary" the angle of the limiting streamlines at the wall should approach the angle of the separation line as separation is approached. That this is the case, is shown in Figure 3. Both the angle of the limiting streamlines, β_w, and the angle of the streamlines in the freestream, β_δ, are $\pi/2$ at $\xi = 0$. As ξ increases β_δ decreases, fairly rapidly at first and then more slowly. The angle β_w decreases rapidly initially and then increases rapidly so that it approaches the value β_{sep} (noted on Figure 3) as separation is approached. Thus, the separation involved here is an "ordinary" separation.

INTEGRATION INTO REGIONS OF REVERSE FLOW

In the example just considered the x component of velocity near the wall changed directions near separation. Thus, it was necessary to integrate the boundary layer equations into a region of reverse flow to obtain the solution. Until quite recently the "conventional wisdom" was that integration of the boundary layer equations into regions of reverse flow lead to numerical instability problems since, in regions of reverse flow, the problem was ill posed. In the present case integration into regions of reverse flow apparently poses no problem. Since the next solution to be presented involves rather extensive regions of reverse flow, it is necessary to determine under what circumstances integration into regions of reverse flow is permissible.

To investigate this problem we note that equations (5) and (6) may be rewritten in the form:

$$F''' + \alpha_{11} F'' + \alpha_{21} F' + \alpha_{31} = \alpha_{41} \frac{\partial F'}{\partial \xi} \tag{10}$$

$$G''' + \alpha_{21} F'' + \alpha_{22} G' + \alpha_{32} = \alpha_{42} \frac{\partial G'}{\partial \xi} \tag{11}$$

Here again primes denote differentiation with respect to η. The exact form of the α_{ij}'s in equations (10) and (11) may be determined by comparison with equations (5) and (6); it is only important to note that $\alpha_{41} = \alpha_{42} = HF' + IG'$. If F' and G' are treated as independent variables, equations (10) and (11) closely resemble the one-dimensional heat conduction equation. As in the mathematical solution of the heat conduction equation, the problem is well posed only if the coefficient α_{41} is positive. If α_{41} is positive, equations (10) and (11) are parabolic and solutions are possible if appropriate boundary and initial conditions are prescribed. If α_{41} becomes negative for any portion of the flow field, equations (10) and (11) are parabolic equations of the mixed type and additional information is needed in order to obtain a solution to these equations. Since $\alpha_{41} = \alpha_{42} = HF' + IG'$, it is clear that this coefficient may be positive even when one of the velocities is negative. For example, if the x component of velocity is negative near the wall then in this region $F' < 0$, but α_{41} will be positive provided the product IG' is positive and greater than the absolute value of the product HF'. For this reason, solutions to equations (5) and (6) may be obtained without any numerical instability problems even when one of the velocity components is reversed.

AN EMBEDDED VORTEX

We now consider a third case in which the solution represents physically a three-dimensional boundary layer with an embedded vortex. It is assumed, as before, that $A(\xi) + 2B(\xi) = 1$, $H(\xi) = \xi$ and that the velocity components are functions of the scaled variable ξ (i.e. $u_\delta = u_\delta(\xi)$, and $v_\delta = v_\delta(\xi)$). These assumptions lead to the relations $g^{*2} u_\delta^* = x^*$ and $\xi = x^*/(1 - \alpha y^*)$. In addition we assume $I(\xi) = \xi$. This assumption yields a relation between u_δ^* and v_δ^*, namely $u_\delta^*(\xi) = \alpha\xi v_\delta^*$. It should be noted that for this flow the component of vorticity normal to the wall does not vanish outside the boundary layer. Thus, this inviscid flow will represent some type of sheared flow (rotational flow). Finally, the y component of velocity at the upper edge of the boundary layer is taken to be:

$$v_\delta^* = 1 - \gamma(\xi - 5\xi^2/3 + 8\xi^3/9)$$

This form is chosen so that the normalized y component of velocity is unity at $\xi = 0$, has a minimum at $\xi = 0.5$ and a maximum at $\xi = 0.75$. This leads to a pressure gradient in the x direction which is favorable for all ξ in the range $0 \leq \xi \leq 1$ but a pressure gradient in the y direction which is positive (adverse) for $0 \leq \xi \leq 0.5$, negative (favorable) for $0.5 < \xi < 0.75$ and positive (adverse) for $\xi > 0.75$.

Results are presented in Figures 4 and 5 for the case $\alpha = 0.5$ and for several values of γ. The variation of the limiting streamline angle, β_w, with ξ is shown in Figure 4 for $\gamma = 1.0$, 2.0 and 2.5. For $\gamma = 1.0$, β_w decreases with increasing ξ. For $\gamma = 2.0$, β_w decreases to a value of approximately zero at $\xi \cong 0.3$, increases beyond this point to a value of approximately 0.75 at $\xi \cong 0.75$ and then decreases slowly. For $\gamma = 2.5$, β_w decreases and reaches a minimum value of -0.96 and then increases again reaching a maximum at approximately $\xi = 0.8$. The variation of β_w with ξ for $\gamma = 3.0$ is similar to that for $\gamma = 2.5$ but is not shown. For $\gamma = 3.25$, 3.5 and 4 (also not shown) ordinary separation occurs. The velocity profiles for the v component of velocity are shown in Figure 5. These velocity profiles are presented for the case $\alpha = 0.5$, $\gamma = 3.0$. The y component of velocity is reversed between $\xi = 0.12$ and $\xi = 0.53$. This is also the region where the angle of the limiting streamlines is negative. Taken together, Figures 4 and 5 present a clear picture of a vortex embedded deep within the three-dimensional boundary layer. For $\xi < 0.12$ and $\xi > 0.53$ both the x and y components of velocity are positive everywhere and the flow proceeds down stream in a normal fashion. Between $\xi = 0.12$ and $\xi = 0.53$ both the x and y components of velocity are positive in the outer portion of the boundary layer but near the wall the x component of velocity is positive while the y component is reversed (negative). This results in a spiraling flow near the wall or an embedded vortex.

From the results presented for the wall shear, it is clear that for $\gamma \leq 2.0$ such a vortex does not exist (there is no flow reversal near the wall). As γ is increased beyond 2.0 a vortex is formed, a vortex which increases in size as the pressure gradient becomes more severe (γ is increased) until the pressure gradient becomes sufficiently severe that separation occurs.

Such a flow, with an embedded vortex, may at first appear strange. Such embedded vorticities do, however, occur in aerodynamics. The classical example occurs in the case of supersonic flow past a cone at moderate angle of attack. Moore (ref. 5) was apparently the first to recognize the nature of such an embedded vortex.

CONCLUDING REMARKS

The theory of semi-similar solutions has been used to investigate several three-dimensional laminar boundary layer flows which approach a separation line. The use of semi-similar solutions makes it possible to investigate the nature of the boundary layer as separation is approached. When separation occurred in the cases considered the three-dimensional separation was of the "ordinary" type in which the limiting or "wall" streamlines run close together and approach a tangent to the separation line. In one case considered, it is shown that as the pressure gradient becomes more severe, a vortex is formed within the boundary layer. If the pressure gradient becomes sufficiently large the boundary layer separates. The separation in this case is again an "ordinary" type separation.

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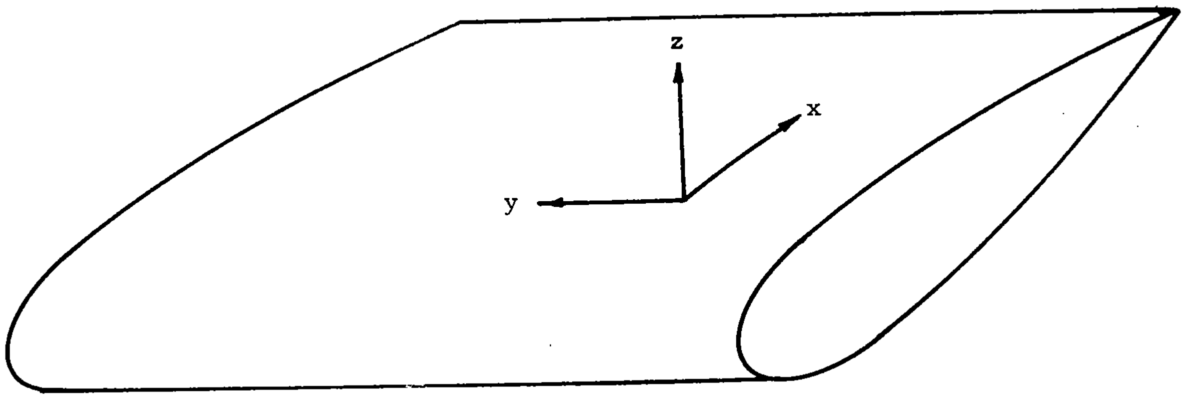

Figure 1.- Coordinate system for three-dimensional boundary layer analysis.

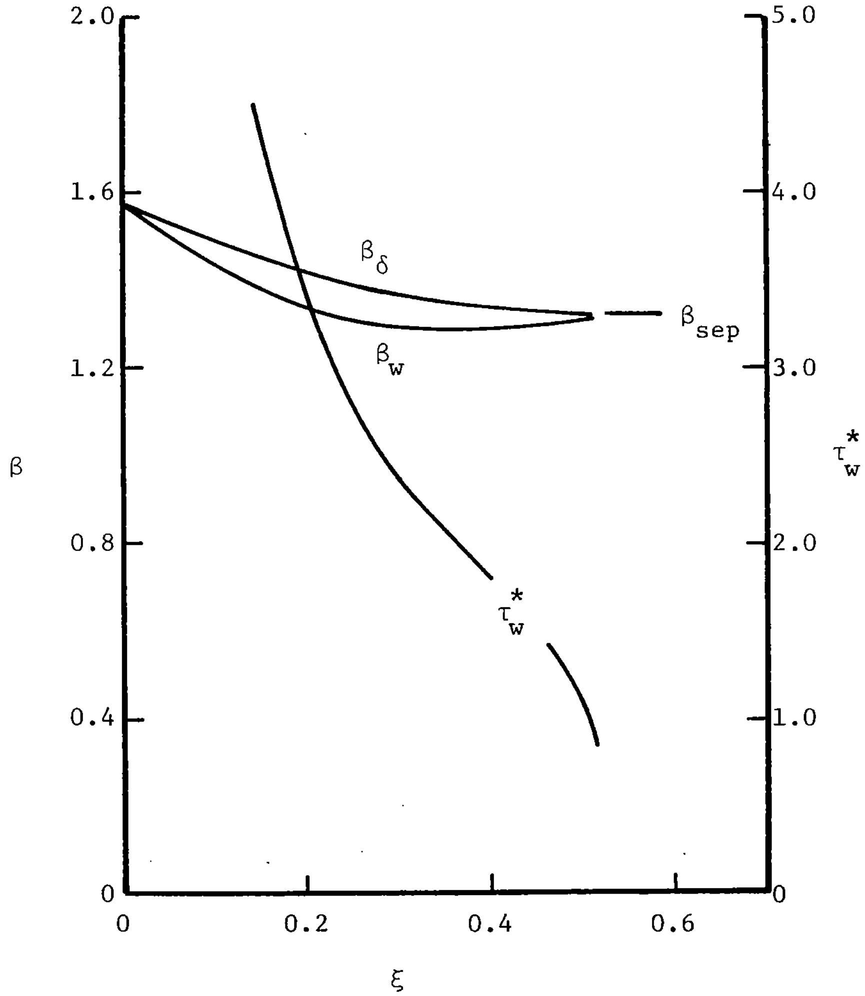

Figure 2.- Freestream streamline angle, limiting streamline angle, and total wall shear for $\alpha = -0.5$.

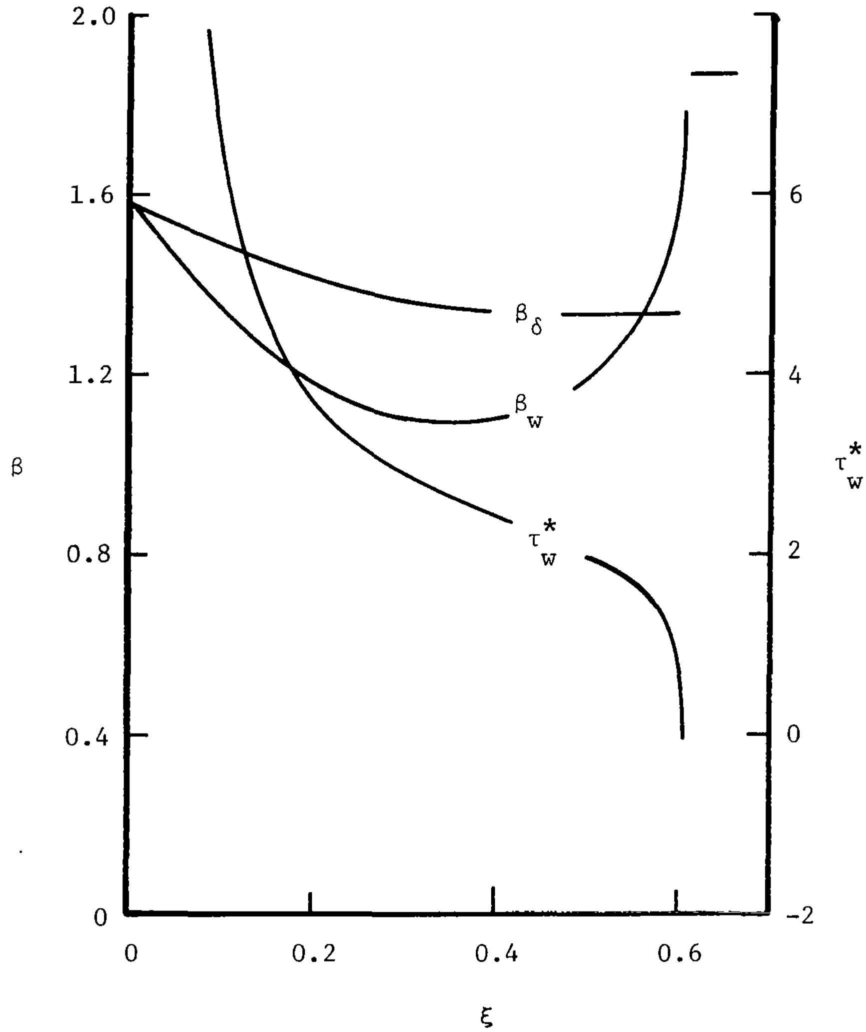

Figure 3.- Freestream streamline angle, limiting streamline angle, and total wall shear for $\alpha = 0.5$.

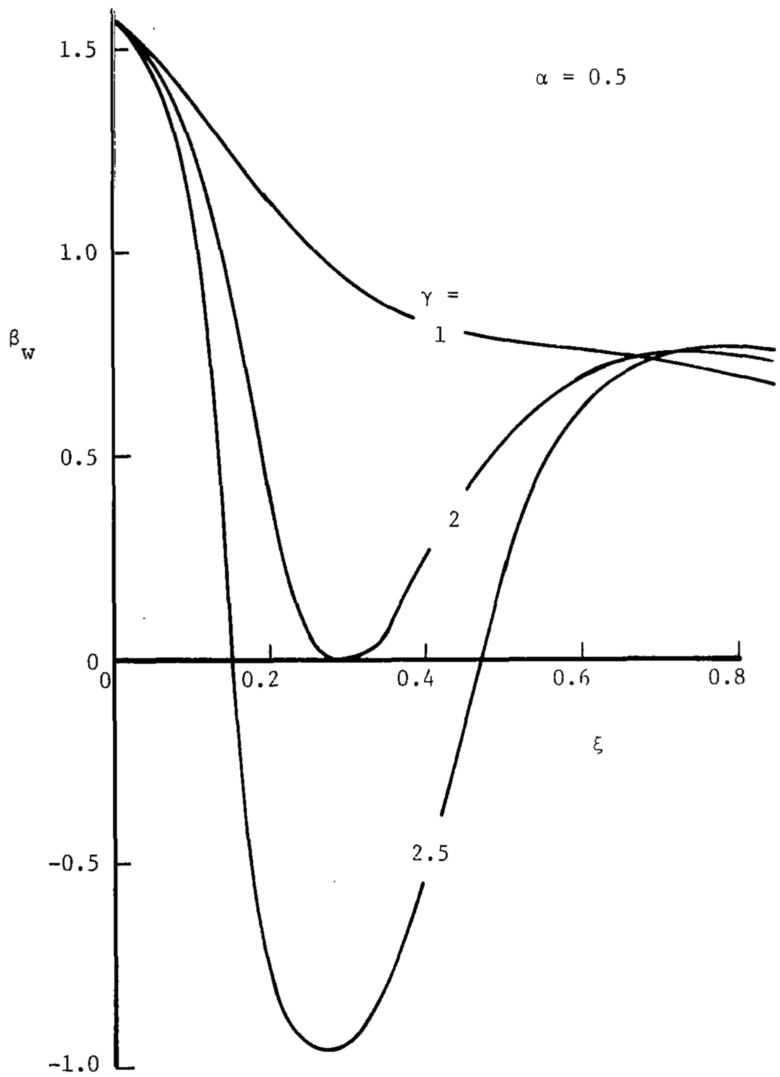

Figure 4.- Limiting streamline angles for embedded vortex flow.

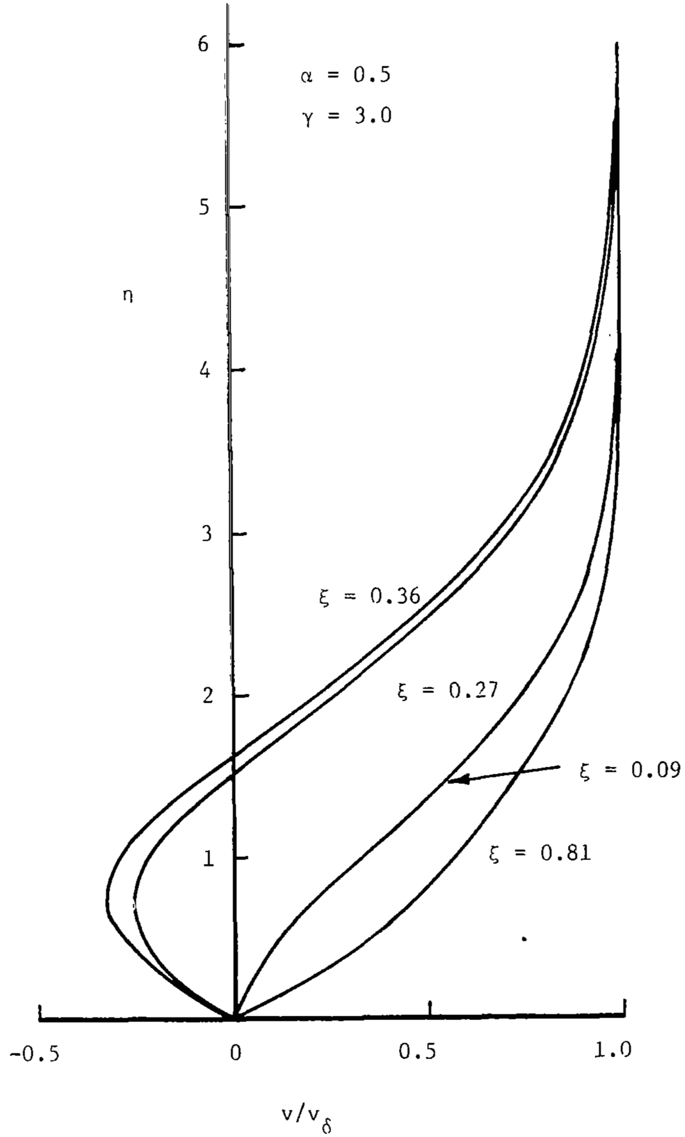

Figure 5.- y velocity profiles, embedded vortex.