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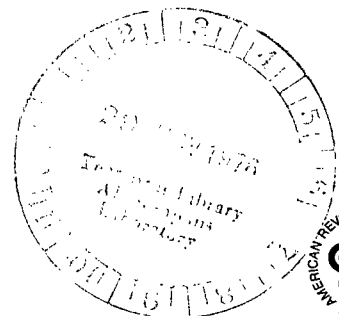


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OPTIMUM STEP DESIGN  
FOR CENTERING OF PISTONS  
MOVING IN AN INCOMPRESSIBLE FLUID

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# OPTIMUM STEP DESIGN FOR CENTERING OF PISTONS

## MOVING IN AN INCOMPRESSIBLE FLUID

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### SUMMARY

Hydrodynamic effects are analyzed for a stepped piston moving within a tight-clearance tube filled with an incompressible fluid. Together with the hydrostatic effects that were analyzed in an earlier paper, a complete solution is obtained and an optimum step design for centering of the piston is suggested. The axial speed resulting from an axial driving force is calculated, and some experimental results for pistons falling in a water-filled tube are presented.

### INTRODUCTION

The operation of hydraulic equipment often involves the sliding of a piston within a cylindrical bore. Among the applications are hydraulic valves, pumps, and actuators (refs. 1 to 3); viscometers and timing devices (refs. 4 and 5); and hydrostatic extruders (ref. 6). An important factor in all these applications is the rate of fluid leakage past the piston. It can be shown that the minimum leakage occurs when the piston is concentric within its conduit and that the leakage past the piston increases as the second power of the eccentricity. For a fully eccentric, plain cylindrical piston the leakage is  $2\frac{1}{2}$  times that when the piston is concentric (ref. 4). Hence, to control the leakage, one has to control the eccentricity. For the leakage to be minimized, the eccentricity has to be minimized as well.

Some methods have been suggested to reduce the eccentricity (refs. 7 to 9), but all these methods were analyzed or tested only with a stationary piston. This was based on the assumption that the hydrodynamic effects due to the axial motion are negligible as

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compared with the hydrostatic effects due to the pressure difference across the piston.

In reference 9 a stationary stepped piston is analyzed and an optimum step configuration that minimizes the ratio of leakage to centering force is found. It is the objective of this work to confirm experimentally the analysis of reference 9 and to find analytically the importance of hydrodynamic effects.

## HYDRODYNAMIC EFFECTS

### Step Pressure

A stepped cylindrical piston in a cylindrical tube of the type being considered is shown in figure 1. The tube is filled with a fluid, and the piston moves with a velocity  $U$  relative to the stationary tube. The leading edge of the piston has the smaller diameter so that the hydrodynamic pressure buildup around the piston produces a centering force (ref. 10). Based on this centering effect and the assumption that any other radial load acting on the piston is small enough to cause only small eccentricity, the problem can be treated as a one-dimensional case. Thus, the governing equation for a nontilted piston, where  $dh/dx = 0$ , is

$$\frac{d}{dx} \left( h^3 \frac{dp}{dx} \right) = 0$$

with the following boundary conditions (for the hydrodynamic effects):

$$p = 0 \quad \text{at } x = 0 \quad \text{and at } x = L$$

(All symbols are defined in appendix A.)

Since the film thickness is independent of  $x$ , the axial pressure variation must be linear, as shown in figure 2 where the hydrodynamic pressure on the step  $(p_s)_h$  is a function of  $\theta$ . To simplify the calculation, the coordinate system is fixed at the piston and the tube moves relative to the piston.

From the continuity of flow at the step the following can be written:

$$\frac{Uh_m}{2} - \frac{h_m^3}{12\mu} \frac{(p_s)_h}{x_s} = \frac{Uh_M}{2} + \frac{h_m^3}{12\mu} \frac{(p_s)_h}{(L - x_s)} \quad (1)$$

This equation can be rewritten as

$$\frac{(p_s)_h}{12\mu} \left( \frac{h_M^3}{L - x_s} + \frac{h_m^3}{x_s} \right) = \frac{U}{2} (h_m - h_M) \quad (2)$$

where

$$h_m = C_m (1 + \epsilon \cos \theta) \quad (3)$$

$$h_M = C_m (\alpha + \epsilon \cos \theta) \quad (4)$$

and

$$\alpha = \frac{C_M}{C_m} \quad (5)$$

$$\epsilon = \frac{e}{C_m} \quad (6)$$

If the analysis is restricted to small eccentricity ratios  $\epsilon$ , the following approximate expressions can be written:

$$h_m^3 = C_m^3 (1 + 3\epsilon \cos \theta) \quad (7)$$

$$h_M^3 = C_m^3 \alpha^3 \left( 1 + \frac{3\epsilon}{\alpha} \cos \theta \right) \quad (8)$$

Substituting equations (7) and (8) into equation (2) gives

$$(p_s)_h = \frac{6\mu UL(\alpha - 1)\beta(1 - \beta)}{C_m^2 \left[ (1 - \beta)(1 + 3\epsilon \cos \theta) + \beta \alpha^3 \left( 1 + \frac{3\epsilon}{\alpha} \cos \theta \right) \right]} \quad (9)$$

where

$$\beta = \frac{x_s}{L} \quad (10)$$

After rearranging terms, the step pressure  $(p_s)_h$  becomes

$$(p_s)_h = \frac{6\mu UL(\alpha - 1)\beta(1 - \beta)}{C_m^2 [1 + \beta(\alpha^3 - 1)]} \left( \frac{1}{1 + \gamma_h \cos \theta} \right) \quad (11)$$

where

$$\gamma_h = 3\epsilon \frac{1 + \beta(\alpha^2 - 1)}{1 + \beta(\alpha^3 - 1)} \quad (12)$$

The location of the center of pressure  $(x_{cp})_h$  can be obtained directly from the location of the centroid of the typical pressure distribution shown in figure 2. Hence,

$$(x_{cp})_h = \frac{(p_\xi)_h \frac{x_s}{2} \frac{2}{3} x_s + (p_\xi)_h \frac{L - x_s}{2} \left( x_s + \frac{L - x_s}{3} \right)}{\frac{(p_\xi)_h L}{2}}$$

which yields the simple expression

$$\left( \frac{x_{cp}}{L} \right)_h = \frac{1 + \beta}{3} \quad (13)$$

The location of the center of pressure is important for eliminating undesirable tilting moments.

### Centering Force

The centering force (fig. 2) can be written as

$$W_h = -2 \int_0^\pi (p_\xi)_h \frac{L}{2} \cos \theta R d\theta$$

Substituting equation (11) into this equation gives

$$W_h = -6\mu UR \left( \frac{L}{C_m} \right)^2 \frac{(\alpha - 1)\beta(1 - \beta)}{[1 + \beta(\alpha^3 - 1)]} \int_0^\pi \frac{\cos \theta d\theta}{1 + \gamma_h \cos \theta}$$

In order that  $\gamma_h < 1$ , from equation (12) the following inequality must hold:

$$3\epsilon [1 + \beta(\alpha^2 - 1)] < 1 + \beta(\alpha^3 - 1)$$

For  $\alpha > 1$ , this is valid for any  $\epsilon \leq 1/3$ . Assuming  $\gamma_h < 1$  and using reference 11

for the evaluation of the pressure integral give

$$W_h = 6\pi\mu UR \left(\frac{L}{C_m}\right)^2 \frac{(\alpha - 1)\beta(1 - \beta)}{[1 + \beta(\alpha^3 - 1)]\gamma_h} \left[(1 - \gamma_h^2)^{-1/2} - 1\right] \quad (14)$$

The centering force diminishes to zero as the piston approaches the concentric position. Therefore, it is in that region of small eccentricity where one would like to maximize the centering force. If  $\epsilon$  is very small, it can be assumed that  $\gamma_h^2 \ll 1$  and the following approximation can be written:

$$(1 - \gamma_h^2)^{-1/2} = 1 + \frac{1}{2}\gamma_h^2 \quad (15)$$

By making use of equations (15) and (12), equation (14) can be written as

$$W_h = 9\pi\mu UR \left(\frac{L}{C_m}\right)^2 \epsilon \overline{W}_h \quad (16)$$

where

$$\overline{W}_h = \frac{\beta(\alpha - 1)(1 - \beta) [1 + \beta(\alpha^2 - 1)]}{[1 + \beta(\alpha^3 - 1)]^2} \quad (17)$$

#### Flow Rate

The flow rate past the piston shown in figure 1 can be expressed as

$$Q_h = 2 \int_0^\pi \left[ U \frac{h_m}{2} + \frac{h_m^3}{12\mu} \frac{(k_s)_h}{x_s} \right] R d\theta$$

By making use of equation (7) and the tables of integrals found in reference 11, this equation can be integrated to give

$$Q_h = \pi UR C_m \overline{Q}_h \quad (18)$$

where

$$\overline{Q}_h = 1 + \frac{(1 - \beta)(\alpha - 1)}{1 + \beta(\alpha^3 - 1)} \left\{ \left(1 - \gamma_h^2\right)^{-1/2} + \frac{3\epsilon}{\gamma_h} \left[1 - \left(1 - \gamma_h^2\right)^{-1/2}\right] \right\} \quad (19)$$

If (as was done in the previous section) it is assumed that  $\gamma_h^2 \ll 1$  and that equation (15) applies, equation (19) can be rewritten as

$$\overline{Q}_h = \alpha \frac{1 + \beta(\alpha^2 - 1)}{1 + \beta(\alpha^3 - 1)} \quad (20)$$

### Friction Force

The friction force acting on the plug is

$$F_h = \int_0^{2\pi} \int_0^L \tau_h R \, d\theta \, dx \quad (21)$$

The shear stress  $\tau$  in equation (21) can be written as

$$\tau_h = \frac{\mu U}{h} + \frac{h}{2} \left( \frac{dp}{dx} \right)_h \quad (22)$$

Substituting equation (22) into (21) and integrating once over the length give the friction force as

$$F_h = \int_0^{2\pi} \left[ \mu U \left( \frac{L - x_s}{h_M} + \frac{x_s}{h_m} \right) + \frac{C_m}{2} (\alpha - 1) (p_s)_h \right] R \, d\theta$$

Substituting equation (11) and the film thickness equations (eqs. (3) and (4)) into this equation gives, after integration,

$$F_h = \frac{2\pi\mu URL}{C_m} \overline{F}_h \quad (23)$$

where

$$\overline{F}_h = \frac{\beta}{\sqrt{\alpha^2 - \epsilon^2}} + \frac{1 - \beta}{\sqrt{1 - \epsilon^2}} + \frac{3(\alpha - 1)^2 \beta (1 - \beta)}{1 + \beta(\alpha^3 - 1)} \left(1 - \gamma_h^2\right)^{-1/2} \quad (24)$$



which, for small eccentricity ratios  $\epsilon$ , becomes

$$\bar{F}_h = \frac{1 + \beta(\alpha - 1)}{\alpha} + \frac{3(\alpha - 1)^2 \beta(1 - \beta)}{1 + \beta(\alpha^3 - 1)} \quad (25)$$

### The Optimum Step

One of the main objects in piston design is to maintain the eccentricity as low as possible, thus minimizing the leakage past the piston and also diminishing the possibility of hydraulic lock (ref. 1). Minimum eccentricity can be achieved by selecting the parameters  $\alpha$  and  $\beta$  so as to maximize the nondimensional centering force  $\bar{W}_h$ .

In figure 3 the nondimensional centering force  $\bar{W}_h$  is plotted against the step length parameter  $\beta$  for various step heights. An interesting result is the existence of a maximum for  $\bar{W}_h$  at  $\alpha = 1.6$  and  $\beta = 0.26$ . This is different from the hydrostatic case of reference 9, where the centering force increases monotonically with the increase of  $\alpha$ . However, the present optimum step configuration resembles the one described in reference 12 for a flat, stepped slider, where the optimum values are  $\alpha = 1.87$  and  $\beta = 0.28$ .

Regarding the leakage, it is clear from equation (20) that the minimum for  $\bar{Q}_h$  occurs whenever  $\alpha = 1$  or  $\beta = 1$ , which corresponds to a plain cylinder as expected. However, from equation (17) it is clear that for these values the centering force vanishes. Thus, a plain cylindrical piston can assume any eccentricity up to  $\epsilon = 1$  with the result of a significant increase in the leakage as compared with the concentric position (ref. 4).

In reference 9 a design parameter for a stationary stepped piston is defined as the ratio between the nondimensional leakage and the centering force. An optimum step was found that minimized this parameter. The same procedure can be used for a moving piston. Thus, we shall look for a minimum of a parameter  $E$  given by

$$E = \left( \frac{\bar{Q}}{\bar{W}} \right)_h = \frac{1 + \beta(\alpha^3 - 1)}{\beta(\alpha - 1)(1 - \beta)} \quad (26)$$

The values of  $\alpha$  and  $\beta$  that minimize the parameter  $E$  will serve as the optimum design criteria.

Figure 4 presents the variation of the design parameter  $E$  with respect to step height  $\alpha$  and location  $\beta$ . The most interesting result is that a large margin exists for an optimum design, ranging from an  $\alpha$  of 1.3 to 1.7 and a  $\beta$  of 0.3 to 0.4; anywhere in this range,  $E$  is within 7 percent of the absolute minimum. This is the same range

as in the hydrostatic case of reference 9. Hence, the same step configuration is beneficial for both the hydrostatic and hydrodynamic effects.

### ELAPSED TIME OF MOVING PISTON

For a piston moving at a constant velocity in a tube filled with fluid, the axial driving force  $T$  is balanced by the friction force and the force due to the pressure difference across the piston. Hence,

$$T = \pi R^2 p_e + F_g + F_h \quad (27)$$

where the hydrostatic friction force  $F_g$  is given in appendix B along with the other hydrostatic results. Substituting equations (23) and (B10) into equation (27) gives

$$T = \pi R^2 p_e + \pi R C_m p_e \bar{F}_g + 2\pi \mu U L \frac{R}{C_m} \bar{F}_h \quad (28)$$

If all the fluid remains in the tube, the axial velocity of the piston is given by

$$\pi R^2 U = Q_g + Q_h$$

which, by substituting equations (18) and (B5), becomes

$$\pi R^2 U = \frac{\pi R C_m^3}{6 \mu L} p_e \bar{Q}_g + \pi U R C_m \bar{Q}_h \quad (29)$$

From equation (29) the pressure difference across the piston is

$$p_e = \frac{6 \mu R L}{C_m^3 \bar{Q}_g} U \left( 1 - \frac{C_m}{R} \bar{Q}_h \right) \quad (30)$$

and by substituting equation (30) into equation (28) the driving force becomes

$$T = 2\pi \mu U L \left( \frac{R}{C_m} \right)^3 \left[ \frac{3}{\bar{Q}_g} \left( 1 - \frac{C_m}{R} \bar{Q}_h \right) \left( 1 + \frac{C_m}{R} \bar{F}_g \right) + \left( \frac{C_m}{R} \right)^2 \bar{F}_h \right] \quad (31)$$

The dimensionless parameters  $\bar{Q}_h$  and  $\bar{F}_h$  as well as  $\bar{Q}_g$  and  $\bar{F}_g$  (see appendix B) are, for the range of optimum step design, of order of magnitude 1. Hence, the er-

ror involved in neglecting the hydrodynamic effects depends on  $C_m/R$ . For most practical applications, the value of  $C_m/R$  is less than  $10^{-2}$ , thus justifying the neglect of hydrodynamic effects.

However, in some cases where the fluid is free to flow out of the cylinder, the pressure difference across the moving piston becomes very small, and the hydrostatic effects are less significant. Such a situation occurs for instance in gun recoil bearings (ref. 13), where in spite of low radial clearance the hydrodynamic effects are more important than the hydrostatic effects.

For the piston falling in a tube, as described in the following sections, all the terms containing  $C_m/R$  in equation (31) can be neglected as compared to unity. Also the driving force  $T$  is

$$T = \pi R^2 L g (\rho_p - \rho_f) \quad (32)$$

Hence, equation (31) becomes

$$g(\rho_p - \rho_f) = \frac{6 \mu U R}{C_m^3 \bar{Q}_g} \quad (33)$$

When the velocity of the piston is constant as it falls down the tube, the time it takes the piston to fall a distance  $H$  is

$$t = \frac{H}{U}$$

which by equation (33) becomes

$$t = \frac{6 \mu R H}{C_m^3 g (\rho_p - \rho_f) \bar{Q}_g} \quad (34)$$

As a check on the validity on this equation, if one assumes a plain piston instead of a stepped piston (implying that  $\bar{Q}_g$  in eq. (34) is unity), equation (34) is in complete agreement with the equation developed in reference 4.

## TEST APPARATUS AND PISTONS

The test apparatus used in the experiments is shown in figure 5. The precision U-tube had a bore of  $1.5885 \pm 0.003$  centimeters with a wall thickness of 1.12 centimeters and overall length of 120 centimeters. The tube material was Pyrex glass that was op-

tically clear with graduations marked on it. Only one-half of the U-tube shown in figure 5 was used in the tests. The tube was alined in the vertical position within 0.25 degree.

Three of the pistons used in the tests are shown in figure 6. All the pistons were made of 17-4 PH stainless steel and hardened. The diameters of the pistons were machined within  $\pm 0.0025$  centimeter. Table I gives the dimensions of the pistons tested. The dimensions given in the table are shown physically in figure 1. Figure 6(a) corresponds to piston 7, figure 6(b) corresponds to piston 2, and figure 6(c) corresponds to piston 3.

## TEST PROCEDURE

Tap water was put into the U-tube, filling both ends. The top of the left half of the U-tube (fig. 5) was plugged. The first 13 cm of the tube shown in figure 5 was not used for measuring the time of the falling piston since acceleration of the piston occurs in this portion of the tube. Beyond the first 13 cm the piston falls at a constant velocity. The fall of the piston was timed with stopwatches capable of measuring to within 0.2 second. Tests were repeated to assure the accuracy of the results.

## TEST RESULTS

Table II shows the test results and how well these results agree with the theory. In this table  $\alpha$  is defined by equation (5),  $\beta$  is defined by equation (10), and  $\bar{Q}_g$  is defined by equation (B7). The theoretical elapsed time  $t$  it took the piston to fall 50.8 centimeters as described by equation (34) is also given in the table. The parameters in equation (34) and their respective values, which were constant for all the tests, are

(1) Viscosity of fluid,  $\mu$ ,  $0.96 \times 10^{-7}$  N-sec/cm<sup>2</sup> at 22° C

(2) Radius of tube,  $R$ , 0.7943 centimeter

(3) Distance piston falls,  $H$ , 50.8 centimeters

(4) Density of fluid,  $\rho_f$ , 1 g/cm<sup>3</sup>

(5) Density of piston,  $\rho_p$ , 7.85 g/cm<sup>3</sup>

The parameter  $\tilde{t}$  in table II is the experimental value of the elapsed time it took the piston to fall a distance of 50.8 centimeters.

The percentage difference between the experimental and theoretical values of the piston's elapsed falling time can be written as

$$A = \left( \frac{\tilde{t} - t}{t} \right) 100$$

In table II the value of  $A$  varies from -35 percent to +13 percent.

In reference 4 it is noted that some of the plain cylindrical pistons wobbled as they fell down the tube. The same phenomenon was observed with piston 8 of the present set (an unstepped piston). The wobble was not apparent with unstepped piston 7, which had a clearance ratio  $C/R$  of 0.0179 as compared with 0.0264 for piston 8. No wobbling occurred with any of the stepped pistons even at the large clearance ratios. This suggests a stabilizing effect of the step, as was observed in reference 4. For the stepped pistons, with the exception of piston 2, the agreement between theory and experiment was good, the deviation being 13 percent or less. The result for the nonwobbling, plain cylindrical piston also agrees with the theory.

### CONCLUDING REMARKS

Hydrodynamic effects were analyzed for a stepped piston moving within a tight-clearance tube filled with an incompressible fluid. Centering force, center of pressure, flow rate, and friction force were derived analytically. These results, together with the hydrostatic effects that were analyzed in an earlier paper constitute a complete solution of the moving stepped piston and allow an optimum step design for centering of the piston to be calculated. The elapsed time of the moving piston resulting from an axial driving force was calculated. Experimental results for pistons falling in a water-filled tube were presented. The agreement between theory and experiments was good. For a plain piston with large clearance ratio ( $C/R > 0.025$ ), a wobbling motion was observed as the piston fell down the tube. The stepped pistons always moved smoothly without any wobble.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, September 16, 1976,  
505-04.

## APPENDIX A

### SYMBOLS

A	$100(\tilde{t} - t)/t$
$C_M$	larger radial clearance, $(D - D_2)/2$
$C_m$	smaller radial clearance, $(D - D_1)/2$
D	diameter of tube
$D_1$	major diameter of piston
$D_2$	minor diameter of piston
E	design parameter, $\bar{Q}/\bar{W}$
e	eccentricity
F	friction force
$\bar{F}_g$	$F_g/\pi R C_m p_e$
$\bar{F}_h$	$F_h C_m/2\pi\mu U R L$
g	gravitational acceleration
H	distance piston falls
h	film thickness
L	length of piston
p	pressure
$p_e$	pressure difference across piston
Q	flow rate
$\bar{Q}_g$	$Q_g/12\mu(L/D)\pi C_m^3 p_e$
$\bar{Q}_h$	$Q_h/\pi U R C_m$
R	radius of tube
T	axial driving force
t	theoretical elapsed time of falling piston
$\tilde{t}$	experimental elapsed time of falling piston
U	velocity of moving piston
W	centering force
$\bar{W}_g$	$2W_g/3\pi L R p_e \epsilon$

$\bar{W}_h$	$W_h/9\pi\mu UR(L/C_m)^2\epsilon$
$x$	axial coordinate
$\alpha$	step height parameter, $C_M/C_m$
$\beta$	step length parameter, $x_s/L$
$\gamma_g$	defined by eq. (B2)
$\gamma_h$	defined by eq. (12)
$\epsilon$	eccentricity ratio, $e/C_m$
$\theta$	angular coordinate
$\mu$	viscosity of fluid
$\rho_f$	fluid density
$\rho_p$	piston density
$\tau$	shear stress

Subscripts:

cp	center of pressure
g	hydrostatic
h	hydrodynamic
M	larger clearance
m	smaller clearance
s	step

## APPENDIX B

### HYDROSTATIC EFFECTS

The hydrostatic step pressure as given in reference 9 is

$$(p_s)_g = p_e \left[ \frac{\alpha^3 \beta}{1 + \beta(\alpha^3 - 1)} \right] \frac{1}{1 + \gamma_g \cos \theta} \quad (B1)$$

where

$$\gamma_g = 3\epsilon \left[ \frac{1 - \beta}{1 + \beta(\alpha^3 - 1)} \right] \frac{\alpha - 1}{\alpha} \quad (B2)$$

The other parameters from reference 9 are

$$\left( \frac{x_{cp}}{L} \right)_g = \frac{1 + \beta}{6} \quad (B3)$$

$$W_g = \frac{3}{2} \pi L R p_e \bar{W}_g \epsilon \quad (B4)$$

$$Q_g = \frac{\pi C_m^3}{12 \mu \left( \frac{L}{D} \right)} p_e \bar{Q}_g \quad (B5)$$

where

$$\bar{W}_g = \frac{\alpha^2 \beta (1 - \beta) (\alpha - 1)}{\left[ 1 + \beta(\alpha^3 - 1) \right]^2} \quad (B6)$$

$$\bar{Q}_g = \frac{\alpha}{1 + \beta(\alpha^3 - 1)} \quad (B7)$$

The hydrostatic friction force is found from

$$F_g = 2 \int_0^L \int_0^\pi \tau_g R \, d\theta \, dx$$

where



$$\tau_g = \frac{h}{2} \left( \frac{dp}{dx} \right)_g$$

Integrating first over the length of the two portions of the piston gives

$$F_g = \int_0^\pi \left[ h_m(p_s)_g + h_M(p_e - p_s)_g \right] R d\theta \quad (B8)$$

which can be written as

$$F_g = \int_0^\pi C_m \left[ (1 - \alpha)(p_s)_g + (\alpha + \epsilon \cos \theta)p_e \right] R d\theta \quad (B9)$$

Substituting  $(p_s)_g$  from equation (B1) and integrating gives for small eccentricity ratios

$$\overline{F}_g = \frac{\alpha^3 \beta}{1 + \beta(\alpha^3 - 1)} + \alpha \quad (B10)$$

where

$$F_g = \pi R C_m p_e \overline{F}_g \quad (B11)$$

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TABLE I. - DIMENSIONS OF CYLINDRICAL PISTONS TESTED

Piston	Major diameter of piston, $D_1$ , cm	Minor diameter of piston, $D_2$ , cm	Length of major-diameter part of piston, $x_s$ , cm	Overall length of piston, $L$ , cm
1	1.5606	1.5545	0.6299	1.5697
2	1.5603	1.5494	.3150	1.5692
3	1.5606	1.5494	.6299	1.5710
4	1.5606	1.5494	.9322	1.5697
5	1.5606	1.5438	.6299	1.5687
6	1.5469	1.5298	.6299	1.5690
7	1.5608	-----	1.5674	1.5674
8	1.5476	-----	1.5672	1.5672

TABLE II. - TEST RESULTS AND COMPARISON WITH THEORY

Piston	Step height parameter, $\alpha$	Step length parameter, $\beta$	Dimensionless hydrostatic flow, $\bar{Q}_g$	Smaller radial clearance, $C_m$ , cm	Theoretical time of piston fall, $t$ , sec	Experimental time of piston fall, $\tilde{t}$ , sec	Percentage difference between experimental and theoretical times, $A = \left( \frac{\tilde{t} - t}{t} \right) 100$
1	1.22	0.4013	1.3652	0.0140	97.4	102.0	+5
2	1.39	.2007	2.0000	.0141	64.7	42.2	-35
3	1.40	.4010	1.6147	.0140	82.4	77.4	-6
4	1.40	.5939	1.3479	.0140	98.7	104.1	+5
5	1.60	.4016	1.8258	.0141	72.9	78.0	+7
6	1.41	.4015	1.6242	.0208	24.7	27.8	+13
7	1.00	1.0	1	.0138	136.7	140.9	+3
8	1.00	1.0	1	.0204	42.4	33.1	-22

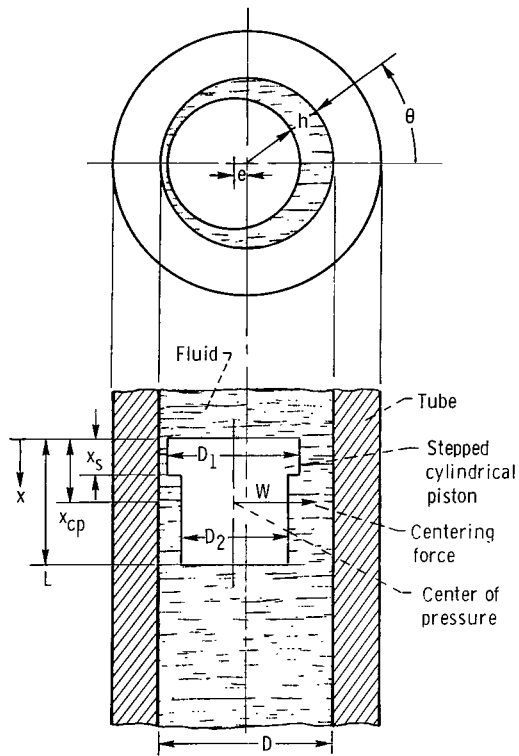


Fig. 1. - Stepped cylindrical piston in a cylindrical tube.

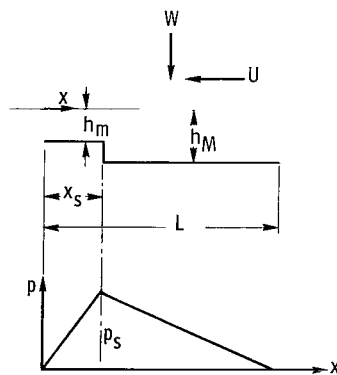


Figure 2. - Stepped piston with linear pressure distribution.

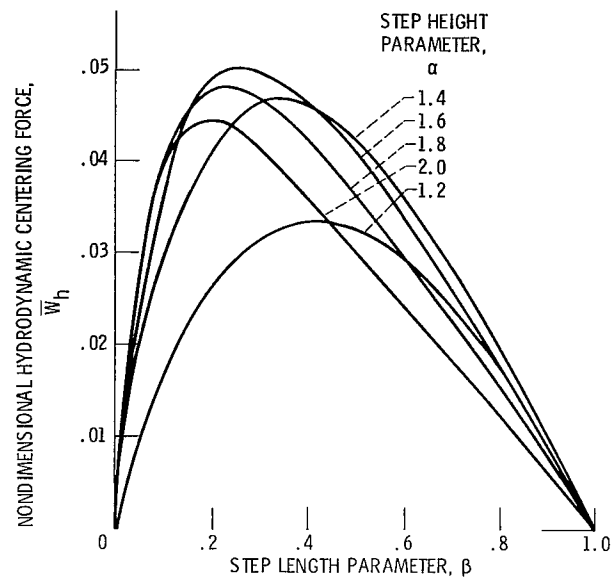


Figure 3. - Centering force for various step configurations.

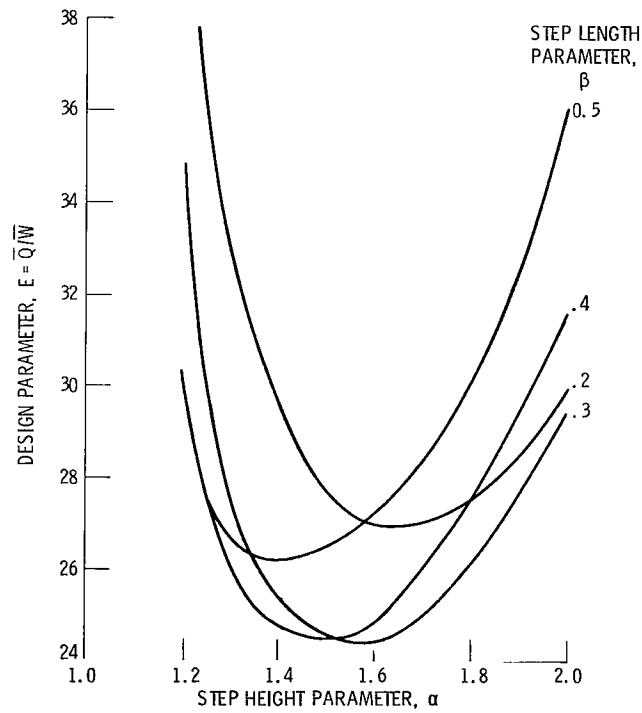


Figure 4. - Design parameters for various step configurations.

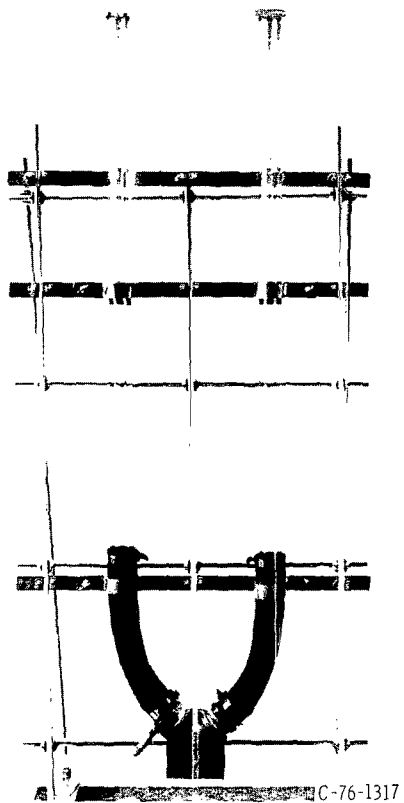


Figure 5. - Test apparatus.

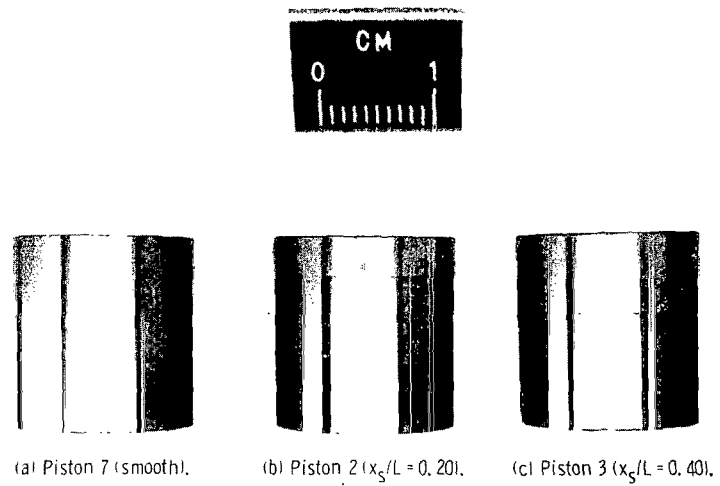


Figure 6. - Examples of pistons used in experiments.

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