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PROGRAM DOCUMENTATION
B-AVERAGE BHATTACHARYA DISTANCE

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Program Documentation

B-Average Bhattacharya Distance

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DOCUMENTATION

Computation of the Total and the B-average Bhattacharya Distance:

(Univac 1108, Univ. of Houston).

This program consists of 3 subroutines to be executed in the following sequence:

(1) Subroutine BHATT

(2) Subroutine BHATB1

(3) Subroutine BHATB2

1. SUBROUTINE BHATT

ABSTRACT

This subroutine calculates the total Bhattacharyya Distance, BDIST, using all N channels. The output of this program, BDIST, will be used in comparing the difference $\delta_H = H_B - \text{BDIST}$ where H_B is the B-average Bhattacharyya Distance computed in the subroutines BHATB1, BHATB2.

User's Information:

(Double Precision Version Only).

In order to use this subroutine the following FORTRAN calling sequence must be given:

```
CALL BHATT(COVAR, XMEAN, M,N, BDIST)
```

where:

COVAR(input) is a real 3-dimensional array (M×N×N) and contains the M N×N class covariance matrices (positive definite symmetric) used as input.

XMEAN(input)	is a real 2-dimensional array (M×N) and contains the M N-dimensional class mean vectors.
M(input)	is the no. of classes under consideration i.e. the no. of covariance matrices and mean vectors.
N(input)	is the dimension of the covariance matrices and the mean vectors.
BDIST (output)	is the value of the total Bhattacharyya Distance computed by subroutine BHATT.

SUBROUTINES USED:

Subroutine BHATT in turn calls the following subroutines

1. Subroutine MATMUL. This subroutine computes the product of 2 matrices. It calls subroutines SUPSUM and ORDER.
2. Subroutine CHLSKY. This subroutine computes the inverse of a positive definite symmetric matrix.
3. Subroutine DET. This subroutine computes the determinant of a positive definite symmetric matrix.

NOTE: (1). The format statements for input, output are dependent upon the dimensions of the input data and corresponding adjustments have to be made to formats when different sets of data are run.

(2). The variables declared in the DIMENSION statements have to similarly correspond to the dimensions of the input data.

ALGORITHM:

Subroutine BHATT computes the value of the total Bhattacharyya Distance using the covariance matrices and mean vectors as inputs.

The total Bhattacharyya Distance, BDIST, is computed by the formula

$$\text{BDIST} = \frac{1}{m} \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} H(i,j)$$

where $H(i,j)$, the interclass Bhattacharyya Distance between classes i and j is given by

$$H(i,j) = \exp\left[-\frac{1}{4}\,\delta_{ij}^{T}(\Sigma_i + \Sigma_j)^{-1}\delta_{ij} - \frac{1}{2}\,\ell n\,\frac{|\Sigma_i + \Sigma_j|}{2^N|\Sigma_i|^{1/2}|\Sigma_j|^{1/2}}\right]$$

where $\delta_{ij} = \mu_i - u_j$ and μ_i is the mean vector corresponding to class i and Σ_i is the covariance matrix corresponding to class i.

2. SUBROUTINE BHATB1:

ABSTRACT

This subroutine attempts to calculate the minimum B-average Bhattacharyya Distance using 1 Householder transformation to construct the B-matrix.

USER'S INFORMATION:

(Double Precision Version Only)

In order to use this subroutine the following FORTRAN calling sequence must be given:

```
CALL BHATB1        (COVAR, XMEAN, M,N, K, ITE, ALPHA)
```

where

COVAR(input) is a real 3-dimensional array (M N×N) containing the M N×N covariance matrices.

XMEAN(input)	is a real 2-dimensional array $(M \times N)$ and contains the M N-dimensional mean vectors used as input.
M(input)	is the number of classes under consideration (i.e. the no. of covariance matrices and mean vectors).
N(input)	is the dimension of the covariance matrices and the mean vectors.
K(input)	is the number of rows desired in the transformation matrix B (which is $K \times N$)
ITE(input)	is 1 + (the no. of iterations required)
ALPHA(input)	is a varying parameter in the iteration formula.

OUTPUT OF SUBROUTINE BHATB1

This subroutine has the following output:

1. The transformation matrix B (which has dimension $K \times N$) corresponding to a particular value of the Householder generator F.*
2. The value of the B-average interclass Bhattacharyya Distance $H_B(i,j)$, $i = 1,\ldots,M-1$; $j = i+1,\ldots,M$
3. The N-dimensional F-vector which is the generator of the Householder transformation $H = I-2FF^T$ used in constructing the B-matrix $B = (I_K | Z)H$.
4. The value of the B-average Bhattacharyya Distance, H_B corresponding to the matrix B.
5. The partial derivative vector $\frac{\partial H_B}{\partial F}$ which contains the partial derivatives of H_B with respect to the vector F.

*See 'ALGORITHM'

Subroutines Used

The following subroutines are in turn called by subroutine BHATB1:

1. Subroutine MATMUL - calls SUPSUM and ORDER.

2. Subroutine CHLSKY.

3. Subroutine DET.

ALGORITHM

Subroutine BHATB1 attempts to compute the minimum B-average Bhattacharyya Distance using one Householder transformation to compute the B-matrix. The B-average Bhattacharyya Distance is given by the formula

$$H_B = \frac{1}{m} \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} H_B(i,j)$$

where

$$H_B(i,j) = \exp[-\tfrac{1}{4}\hat{\delta}_{ij}^T(\hat{\Sigma}_i + \hat{\Sigma}_j)^{-1}\hat{\delta}_{ij} - \tfrac{1}{2}\ell n(|\hat{\Sigma}_i + \hat{\Sigma}_j|/2^k|\hat{\Sigma}_i|^{1/2}|\hat{\Sigma}_j|^{1/2}]$$

where $\delta_{ij} = B(\mu_i - u_j)$ and $\hat{\Sigma}_i = B\Sigma_i B^T$ and B is a $K \times N$ matrix of rank K of the form $B = (I_K | Z)H$ where $H = I - 2FF^T$, $||F|| = 1$. An initial guess for F is taken to be $F_o^T = [\frac{1}{\sqrt{N}}, \ldots, \frac{1}{\sqrt{N}}]^T$ and the corresponding matrix $B = (I_K | Z)(I - 2F_oF_o^T)$ is computed. The corresponding value of

$$H_B = \frac{1}{m} \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} H_B(i,j)$$

is also computed.

The steepest descent iterator is then applied to alter the value of F

$$\text{i.e.} \qquad F_{p+1} = F_p - \alpha \frac{\partial H_B}{\partial F_p} \cdot H_B$$

where α is a varying parameter and is one of the inputs to the program.

$\frac{\partial H_B}{\partial F_p}$ is the partial derivative vector (derived analytically). The value of F_{p+1} is then normalized so that $\|F_{p+1}\| = 1$. The B-matrix is recomputed with the new value of F. The corresponding value of H_B is computed. This procedure is repeated (ITE - 1) number of times (8 seems to be a good value for ITE). Two points should be noted:

(1). Whether $\frac{\partial H_B}{\partial F} \approx \Theta$.

(2). Whether $\delta_H = H_B$ - BDIST (the total Bhattacharyya Distance) is sufficiently small.

The values of α and ITE (which are both inputs to this subroutine) should be altered accordingly in order to achieve the above 2 objectives.

The value of F at which the minimum value of H_B occurs is saved. Call it F1.

3. Subroutine BHATB2

This subroutine attempts to compute the minimum B-average Bhattacharyya Distance using 2 Householder transformations.

USER'S INFORMATION:

(Double Precision Version)

(1) In order to use this subroutine the following FORTRAN calling sequence must be given:

CALL BHATB2(COVAR, XMEAN, M, N, K, ITE, ALPHA)

where

COVAR, XMEAN, M,N,K,ITE,ALPHA

have the same meanings as in SUBROUTINE BHATB1.

(2) This subroutine reads in the value of F1 computed in the previous program (subroutine BHATB1). The data cards for F1 should have the format 5F16.8 (e.g. if F1 is 12-dimensional then F1 is punched on 3 data cards; the first 2 cards contain 5 components of F1 and the last card contains 2 components of F1).

These data cards for F1 are placed following the data cards for the covariance matrices and the mean vectors.

(3) The value of F1 that is read in is then used to compute the Householder transformation $H_1 = I - 2F1F1^T$. The covariance matrices Σ_i and the mean vectors μ_i $i = 1,\ldots,m$ are transformed into $H_1\Sigma_iH_1$ and $H_1\mu_i$.

The number of Householder transformations by which the covariance matrices Σ_i and the mean vectors μ_i have to be transformed is denoted by the variable IJ.

For subroutine BHATB2 we require one Householder transformation to obtain $H_1\Sigma_iH_1$ and $H_1\mu_i$.

The FORTRAN statements "IJ = 1" appears after the comment:

"C-----------IJ Eq. No. of Householder Transformations Required---".

OUTPUT OF SUBROUTINE BHATB2

1. The vector F1 which is the generator of the Householder transformation $H_1 = I - 2E1F1^T$.
2. Same as subroutine BHATB1.

ALGORITHM:

Here each Σ_i is replaced by $H_1 \Sigma_i H_1$ and each μ_i is replaced by $H_1 u_i$. The B matrix is then taken to be $B = (I_K | Z)(I-2FF^T)$, F = 1. An initial guess for F, $F_o^T = [\frac{1}{\sqrt{N}}, \ldots, \frac{1}{\sqrt{N}}]$ is made and the same procedure as in subroutine BHATB1 is applied. The value of F = F2 at which the minimum value of H_B occurs is saved.

USING MORE THAN 2 HOUSEHOLDER TRANSFORMATIONS TO CONSTRUCT THE B-MATRIX:

If more than 2 Householder transformations are required to compute the transformation matrix B i.e. if $\delta_H = H_B$ - BDIST is not small enough, then subroutine BHATB2 can be modified in the following way. For the B-matrix requiring 3 Householder transformations do the following:

(1) Place the data cards containing the vector F2 (computed in the previous program) following the data cards containing F1.

(2) The statement following the comment "C... Ij Eq. NO. OF HOUSEHOLDER TRANSFORMATIONS REQUIRED ..." should be "IJ = 2"

For J ≥ 4 Householder transformations required in computing the B-matrix:

(1) the data cards for F1,...,F(J-1) should be placed after the data cards for the covariance matrices and mean vectors;

(2) the statement "IJ = 2" should be changed to "IJ = (J-1)".

References

1. H.P. Decell, Jr. and W.G. Smiley, III, "Householder Transformations and Optimal Linear Combinations", Dept. of Mathematics, University of Houston.

2. Salma K. Marani, Masters Thesis, "Bhattacharya Distance, Householder Transformations and Dimension Reduction in Pattern Recognition".