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PREFACE

The work described in this report was performed by the Telecommunications Science and Engineering Division of the Jet Propulsion Laboratory.

ACKNOWLEDGMENT

The extensive and detailed investigations in superconducting cavity technology by J. Turneaure and his group at Stanford University are gratefully acknowledged.

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ABSTRACT

The pioneering works of W. H. Hartwig; S. R. Stein and J. Turneaure; J. J. Jimenez and A. Septier; and others have shown the possibility of using a superconducting cavity to stabilize a microwave oscillator. The achievement of cavity Qs of the order of 10^{10} has made possible the realization of frequency standards with performance which could surpass that of the hydrogen maser.

The present study explores the possibility of integrating a superconducting cavity with a traveling wave maser to obtain a frequency standard with very high spectral purity.

I. INTRODUCTION

There are two basic methods for using a high-Q resonator to stabilize a microwave oscillator. In one method, a voltage-controlled oscillator (VCO) is used in an FM detector loop, as shown in Fig. 1, to stabilize the oscillator frequency. The second method, shown in Fig. 2, utilizes the high-Q resonator as a transmission filter and stabilizes a microwave oscillator to the cavity frequency. Both methods have been used by the investigators mentioned in Refs. 1-4.

The motivation for the present program was the long experience with traveling wave masers (TWMs) and closed-cycle refrigerators (CCRs), whose technologies appeared to blend in very naturally with the requirements for a good superconducting cavity-stabilized maser oscillator (SCSMO).

This report presents a discussion of those problems pertinent to the SCSMO.

II. DESIGN OF A SCSMO

The low-noise performance of a TWM would appear to make it a good device for integrating with a superconducting cavity to achieve a stable oscillator. In such an application the TWM would operate at high signal levels and in partial gain saturation in order to provide amplitude stabilization. Thus, it is apparent that the application of the TWM here is completely different from the usual linear-unsaturated mode in a receiving system.

In the usual electron tube or semiconductor oscillator, the mechanism for amplitude stabilization is contained in the Van der Pol equation (Ref. 5), which introduces damping terms proportional to the square of an exciting voltage or current. Whereas the Van der Pol equation was derived through phenomenological arguments to explain experimental observations in electronic oscillators the same saturation effects were already inherent in the equations for the solid state maser as originally proposed by Bloembergen (Ref. 6). Subsequently, it was observed that power saturation in a maser was necessarily accompanied by a rise in the spin temperature of the amplifying medium, and that the spin temperature determined the noise performance of a maser.

Siegman (Ref. 7) has postulated an equation to account for saturation in a TWM; however, for present purposes a graphical approach will be adequate. Figure 3 shows a typical response for a TWM as a function of distance along an amplifying structure of great length; typical operating points a and b for an amplifier of length L cm are shown. The small signal gain G of the TWM is given by the slope for small values of x, while the oscillator gain G_o is given by P_b/P_a and is, of course, less than G. The response shown in Fig. 3 is for fixed conditions on certain parameters, such as pump power, operating temperature, and the static magnetic field. Variations in these parameters will cause corresponding power fluctuations in the oscillator output. Moreover, to the extent that these variations also cause frequency and phase fluctuations, they will need to be examined in greater detail.

An important observation can, however, be made at this point: the low-noise performance of a TWM can be realized in an oscillator provided the input section of the maser operates in the linear-unsaturated mode. The signal-to-noise ratio then can be made sufficiently high by the time the signal suffers amplitude saturation, toward the end of the TWM, that the spectral purity is not degraded. In other words, the high spin temperature (which implies high noise temperature) toward the end of the structure, due to saturation, will not degrade a properly designed SCSMO. It is to be noted that a cavity-type maser does not provide this advantage.

The microwave design problem is then to provide the proper coupling coefficients to the superconducting cavity so as to achieve the desired gain G_o while maximizing the loaded Q.

Figure 4 shows the equivalent circuit for the SCSMO. The transmission cavity has been analyzed in great detail by Montgomery, Dicke and Purcell (Ref. 8) and only the results need to be given here.

The transmission loss through the cavity at resonance is given by

$$T(\omega_o) = (4\beta_1\beta_2)/(1 + \beta_1 + \beta_2)^2 \tag{1}$$

and the loaded Q is given by

$$Q_L = Q_u/(1 + \beta_1 + \beta_2) \tag{2}$$

In the above equations,

$T(\omega_o)$ = transmission loss at resonance
β_1 = coupling coefficient at input
β_2 = coupling coefficient at output
Q_L = loaded Q of cavity
Q_u = unloaded Q of cavity

An X-band TWM can easily have a gain in excess of 40 dB; hence, an oscillator gain of 30 dB is reasonable. Thus the transmission loss through the cavity should be around 10^{-3}. Assuming the input coupling can be made equal to the output coupling.

$$\beta_1 = \beta_2 = \beta \tag{3}$$

one has, finally,

$$T(\omega_0) = 4\beta^2/(1 + 2\beta)^2 \tag{4}$$

and

$$Q_L = Q_u/(1 + 2\beta) \tag{5}$$

Using the numbers quoted above,

$$Q_L \approx (1/1.03)Q_u \tag{6}$$

Thus, the high gain in a TWM makes possible a loaded Q which approaches the the unloaded Q.

One of the advantages of the servoed oscillator shown in Fig. 1 is that the time constant of the loop may be made sufficiently long to smooth out the noise in the system. With the SCSMO the noise is negligible, and the cavity performs a smoothing of any phase instabilities which may be inherent in the TWM. For Qs of the order of 10^{10} and oscillations at a frequency of 10^{10} Hz, the time constant of the cavity is around 1 second. The transit time through a TWM is something less than a microsecond, and the superconducting cavity thus averages a signal which makes over a million round trips through the TWM. Small phase fluctuations through the TWM are nullified, and a highly monochromatic signal should be realizable.

Turneaure and Stein (Ref. 2) have shown that the Q of a superconducting cavity can increase by as much as an order of magnitude per Kelvin in the region of 4 to 1.5 K. It will be shown in this report that it is essential to operate the cavity at around 1.5 K; the TWM may be operated at 4.2 K. It is apparent then that the difficult problem to be solved for a continuously operating SCSMO is that of cryogenic refrigeration. Not only is it necessary to maintain the cavity at around 1.5 K with high temperature stability but the cavity also needs to be kept free of any mechanical motion.

In the succeeding sections we discuss the frequency-pulling effects due to interconnecting transmission lines and the effects of variations of the cavity dimensions. These topics are followed by a discussion of the statistical fluctuations in high quality oscillators, and finally a brief discussion of cryogenics is given.

III. FREQUENCY-PULLING IN OSCILLATORS

It is a characteristic of all oscillators that they tend to operate in the gain-saturated mode. For the SCSMO, gain saturation is achieved when there is zero phase shift between the input and output signals. Any variations in circuit parameters which cause a phase shift of the input signal relative to the output signal result in a frequency shift which compensates for the phase shift and again establishes gain saturation. This phenomenon is called frequency-pulling and is common to most electronic oscillators. The importance of understanding this phenomenon is that it is a principal cause of instability in an oscillator.

Figure 5 shows actual experimental data for an oscillator configuration in which a phase-shifter was used to introduce large phase shifts in the signal path. The large frequency shifts produced are readily measured with a frequency counter. For a copper cavity at room temperature ($Q \approx 10^4$) the frequency dependence on phase is of the order of a few hundred kilohertz per degree of phaseshift and is virtually a vertical line in Fig. 5. It is seen that the frequency-pulling is reduced as the Q of the cavity is increased. The significance of small perturbations on a high-Q resonator is also demonstrated in Fig. 5. The mode of the superconducting cavity used was the TE_{011} mode, which is also coincident with the degenerate TM_{111} mode. Even though choke flanges were used (Fig. 6) to suppress the undesired mode, it was felt that there was still a degradation in performance. A niobium wire (0.6 mm in diameter) was used to shift the frequency of the undesired TM mode; the improved performance is shown in Fig. 5.

The Stanford group has used the TM_{010} mode in order to avoid the degeneracy problem. The TE_{111} mode should also be investigated because it is a nondegenerate mode and also the smallest resonator possible for a given frequency.

The importance of reducing frequency-pulling effects becomes obvious when it is observed that variations in transmission line lengths in the cavity-stabilized oscillator are equivalent to variations in phase shift. The following results are readily obtained.

The frequency-pulling Δf is given by

$$f_0^{-1}\ \Delta f = 1/2\ Q^{-1}\ \Delta\phi \quad \text{Hz} \tag{7}$$

where

Q = the loaded cavity Q
$\Delta\phi$ = the phase shift in radians
f_0 = the nominal center frequency

If the total length of the transmission line in the oscillator loop is L, then, for gain saturation, the total phase shift ϕ is given by

$$\phi = \frac{L}{\lambda 0} = 2\pi n \tag{8}$$

where n is an integer, and

λ_0 = wavelength in transmission line at frequency f_0

By definition, the phase change resulting from a change in length ΔL is

$$\Delta\phi = \frac{\Delta L}{\lambda 0} \tag{9}$$

Therefore, substituting into Eq. (7),

$$f_0^{-1}\ \Delta f = \frac{1}{2}\ Q^{-1}\lambda_0^{-1}\Delta L \tag{10}$$

For a typical X-band oscillator,

$$f_0 \approx 10^{10}\ \text{Hz}$$

$$Q \approx 10^{10}$$

$$\frac{f}{f_0} \approx 10^{-14}$$

$$\lambda_0 \approx 3 \text{ cm}$$

Therefore,

$$\Delta f = \frac{\Delta L}{6} \qquad (11)$$

or

$$\Delta L = 6\Delta f \qquad (12)$$

Thus, it is necessary to keep $\Delta L \leq 6\Delta f$ for the desired stability. Such stability requires that the transmission lines be an integral part of the cavity and amplifier assembly. The output signal may then be extracted through a directional coupler or ferrite isolator so as to achieve maximum isolation. An added advantage of consolidating the cavity, microwave amplifier, and transmission lines is that the reduced coefficient of thermal expansion at low temperatures simplifies the problem of temperature stabilization in the oscillator.

IV. REQUIREMENTS ON DIMENSIONAL STABILITY OF THE MICROWAVE CAVITY

By far the most critical parameters to control in a SCSMO are the dimensions of the cavity. This is as it should be for, in essence, the cavity is used as an ersatz giant atom whose dimensions determine the frequency of oscillation.

In the simple case of the right circular cylinder, it is simple to derive the relationship of frequency to cavity diameter. For the TE_{011} mode,

$$f = \frac{C}{D} \qquad (13)$$

where C = a constant and D = diameter. Then,

$$\Delta f = -\frac{C}{D^2}\,\Delta. \qquad (14)$$

or

$$\frac{\Delta f}{f} = -\frac{\Delta D}{D} \qquad (15)$$

whence we obtain the not too surprising result that the fractional dimensional stability of the cavity must be the same as the fractional frequency stability desired. The coefficient of thermal expansion for niobium at 1.5 K is of the order of 10^{-10} (consistent with JPL's determination of 10^{-9} for 4.5 K) and requires a temperature stability of the order of $\pm 10^{-4}$ K for a fractional frequency stability of 10^{-14}. Intuitively it is apparent that for such dimensional stability, thermal variations are not as serious as other mechanical fluctuations. Clearly the superconducting cavity will act as a microphone, a seismometer, as well as a thermometer. Radiation pressure effects within the cavity can be made negligible by operating the oscillator in the gain-saturated mode and at low power levels. These challenging and stringent requirements still make the SCSMO an attractive frequency standard because it has great potential. An added complication is that it will be very difficult to preset the oscillator frequency to a desired value. Hopefully, it will be possible to use frequency synthesizers to resolve this difficulty.

V. STABILITY OF HIGH QUALITY FREQUENCY STANDARDS

The problem of specifying stability in frequency standards has been satisfactorily resolved in recent years (Ref. 9). The essential requirement is that definitions of stability be given only in terms of actual measurement procedures. Many investigators came to the conclusion that high-quality oscillators possessed a typical characteristic dependence in the standard deviation of fractional frequency stability on the observation time as shown in Fig. 7.

For short observation times, denoted by Region 1 in Fig. 7, the instability is due to frequency modulation by noise in the oscillator. Generally the noise bandwidth of importance is sufficiently narrow that white noise is assumed. In this case the slope is negative and follows a cubic law. As the observation time is increased the noise bandwidth is decreased, and the stability is limited by the Q of the resonant element in the oscillator; this is Region 2 in Fig. 7. In this region noise processes cause flicker FM of the oscillator within the line-width of the resonant element. The center frequency of the resonator may be assumed to be fixed in Region 2. Nature despises stagnation, however, and the center frequency cannot remain fixed for very long. Eventually, due to physical changes in parameters in the oscillator, the center frequency does a random walk and is characterized by Region 3 in Fig. 7. It is fairly evident that Region 3 is determined by the overall time constant of the oscillator. It is also noted that for long averaging times (months or years) the usual meaning attached to the standard deviation of fractional frequency stability ceases to apply. In the first place, the measuring equipment itself would probably have fluctuations which would not be discernible from oscillator fluctuations. Furthermore, most frequency standards would not operate continuously for more than a few years. This kind of observation leads to the principle stated at the beginning of this discussion that definitions must be stated in terms of measurement procedures. Allan (Ref. 10) has suggested (as is now generally accepted) the concept of pair variance to cope with the problem of measurements over long averaging times.

The three regimes of frequency stabilities shown in Curve A of Fig. 7 may be described as short-, medium-, and long-term stabilities, and will have a character approaching the idealizations described above. An actual measurement will look more like Curve B in Fig. 7. Curve B would be representative of a frequency standard of improved stability relative to one which is represented by Curve A. To summarize the present discussion, the following remarks may be made: in Region 1 the improvement in stability results from an increased signal-to-noise ratio in the oscillator; in Region 2 the improvement results from a higher Q in the resonant element; and Region 3 is determined by the system time constant. Hence, the attractiveness of the SCSMO lies in the possibilities for improvements in Regions 1 and 2. The unknown characteristic of a SCSMO is its behavior in Region 3. No experimental data is available for the simple reason that none exists.

The discussion here on frequency stability was based on measurements in the time domain; alternatively, it is also possible to specify stability through measurements in the frequency domain. This topic is discussed in a previously cited reference (Ref. 11) and need not be repeated here.

We conclude this discussion with an order-of-magnitude comparison of the 1-second stabilitites of a hydrogen maser and a SCSMO. The standard deviation for fractional frequency fluctuations in a feedback oscillator is given by W. Edson (Ref. 12) as

$$\sigma\left(\frac{\Delta f}{f}\right) = \left(\frac{kT}{2PQ^2\tau}\right)^{1/2} \tag{16}$$

where

k = Boltzmann constant
T = effective noise temperature
P = power output of oscillator
Q = quality factor of parameter determining stability of oscillator
τ = averaging time for measurement

For the hydrogen maser, we use $T = 300$ K; $P = 10^{-10}$ watts (for an output of 10^{-12} watts), and $Q = 10^9$ (this is Q for the hydrogen atoms which determine the stability), and $\tau = 1$ second. Then

$$\sigma_1 = 4.5 \times 10^{-14} \tag{17}$$

For the SCSMO, we use

$T = 10$ K
$P = 10^{-10}$ watts (input to TWM)
$Q = 10^{10}$

and

$\tau = 1$ second

Then,

$$\sigma_2 = 10^{-16} \tag{18}$$

This result, of course, implies only that thermal noise is negligible compared with other effects we have mentioned.

VI. CRYOGENICS; A DISCUSSION OF THE PROPERTIES OF HELIUM AT VARIOUS TEMPERATURES

As shown in Fig. 8, the superconducting cavity should be operated at around 1.5 K or a lower temperature. Hence, the discussion in this concluding section is on methods for achieving continuous cryogenic refrigeration. Not too surprisingly, such a discussion turns out to be a discussion of the properties of the element helium, for no other substance could replace it as a cryogenic refrigerant. The many and varied properties of helium may best be described in terms of the working temperatures as follows:

A. HIGH TEMPERATURES

At high temperatures, helium behaves much like an ideal gas and is used as a working fluid in expansion engines. The Van der Waals forces are small in the range of 20 to 400 K (and at moderate pressures), and helium is used extensively in Gifford-McMahon and Philips cycles, which are modern variants of the Stirling cycle. These thermodynamic cycles are preferred over the Claude type of expansion engines because they have the potential for providing refrigeration with relatively small mechanical vibration. The Gifford-McMahon refrigerator, in particular, operates at low speeds and provides long engine life.

B. LOW TEMPERATURES

When cooled below 20 K, helium can provide refrigeration through the Joule-Thompson (J-T) expansion process. (Although the inversion temperature for helium is around 50 K, the J-T process is seldom used above 20 K.) This is the standard method used in the liquefaction of helium. The gas is first compressed to 10 to 20 atmospheres pressure and cooled to below 20 K and then allowed to expand through a J-T valve to a return pressure of 1 or 2 atmospheres. A good heat exchanger is imperative to improve efficiency. Temperatures obtained in this way are typically in the 4 to 5 K range. Small closed-cycle refrigerators with a capacity of around 1 watt at 4.5 K consume around 6 kilowatts of electrical power and can operate continuously for several months. The working temperature is usually selected to be above 4.2 K so that the back pressure is above one atmosphere and any leaks in the system would not contaminate the helium.

C. VERY LOW TEMPERATURES

The critical temperature for helium is 5.2 K; hence it is easy to liquefy helium by cooling the gas to around 4.5 K. The liquid may then be allowed to boil to provide further refrigeration, much like a household refrigerator. Temperatures in the range of 1 to 3 K with load capacities of a few milliwatts to a few hundred milliwatts, respectively, may be obtained in this way in conjunction with the previously cited 4.5 K refrigerator. A booster pump is required for this purpose.

Some precautions are required to achieve long-term (weeks) continuous operation. The booster pump should be a high-quality unit and should be located in the proximity of the refrigerator to minimize possibilities for leaks. The working temperature should be chosen to be above or below the lambda point (2.18 K) because the viscosity of liquid helium changes abruptly at the lambda point and can cause thermal oscillations.

D. ULTRALOW TEMPERATURES

The isotopic ^3He may be used as a working fluid in an evaporation type of refrigerator to provide cooling in the range 0.01 to 1 K. The ^3He may also be mixed with ^4He to provide dilution refrigeration in the same temperature range. The heat loads are typically of the order of a microwatt or smaller in these refrigerators. At present, it is probably not necessary to cool a superconducting cavity to below around 1.5 K.

Table 1 summarizes the various properties of helium as a working fluid in various refrigeration processes. Fig. 9 is a schematic drawing of one possible configuration for a 1.5 K refrigerator to cool a SCSMO.

In order to stabilize temperature of the cavity to the order of ± 10 μK it will be necessary to regulate the pressure of the evaporating helium gas, and in addition it will be necessary to provide a servoed heater and temperature sensor at the 1.5 K station. Mechanical isolation techniques are to be used to reduce vibrations of the cavity to an absolute minimum.

VII. CONCLUSIONS

The SCSMO has a potential for providing a highly monochromatic microwave signal. The emphasis made in this investigation is that a very stable cryogenic refrigerator operating at 1.5 K is essential for the SCSMO.

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Table 1. Properties of helium as a working fluid in refrigeration processes

Working temperatures	Thermodynamic process	Useful property of helium	Typical[a] heat load
400 → 80 K	Expansion engine	Ideal gas	8 watts
80 → 15 K	Expansion engine	Ideal gas	3 watts
15 → 4.5 K	Joule-Thompson expansion	Van der Waals gas	1 watt
4.5 → 1.5 K	Evaporative cooling	Heat of vaporization	10 milliwatts

[a]For a small closed-cycle system with total electrical power input of around 6 kW.

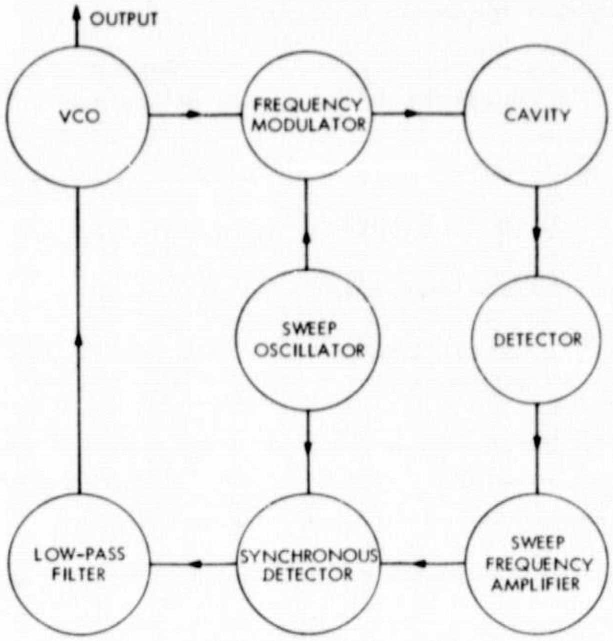

Fig. 1. Frequency modulation servoed oscillator (The superconducting cavity is used as a frequency discriminator.)

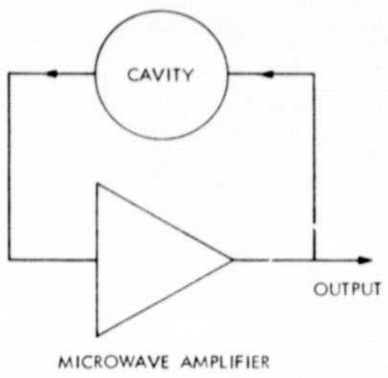

Fig. 2. Regenerative microwave oscillator (The cavity is used as a transmission filter.)

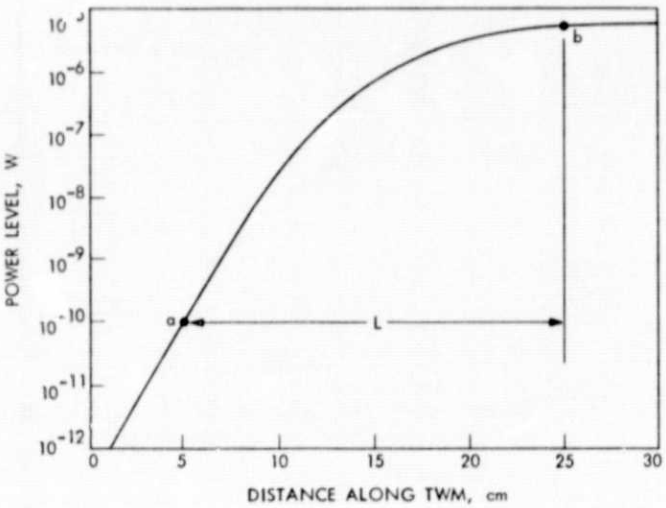

Fig. 3. Typical power saturation curve
for a TWM (see text)

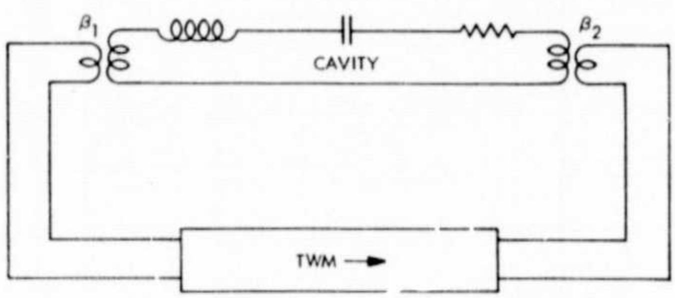

Fig. 4. Equivalent circuit for a SCSMO.
(A small part of the output signal
would be used as the stable
oscillator output.)

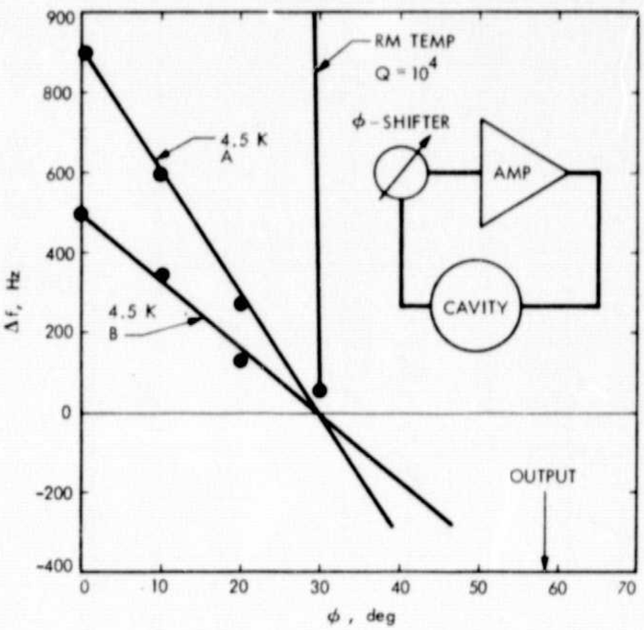

Fig. 5. Frequency deviation as a function of phase shift in a regenerative oscillator

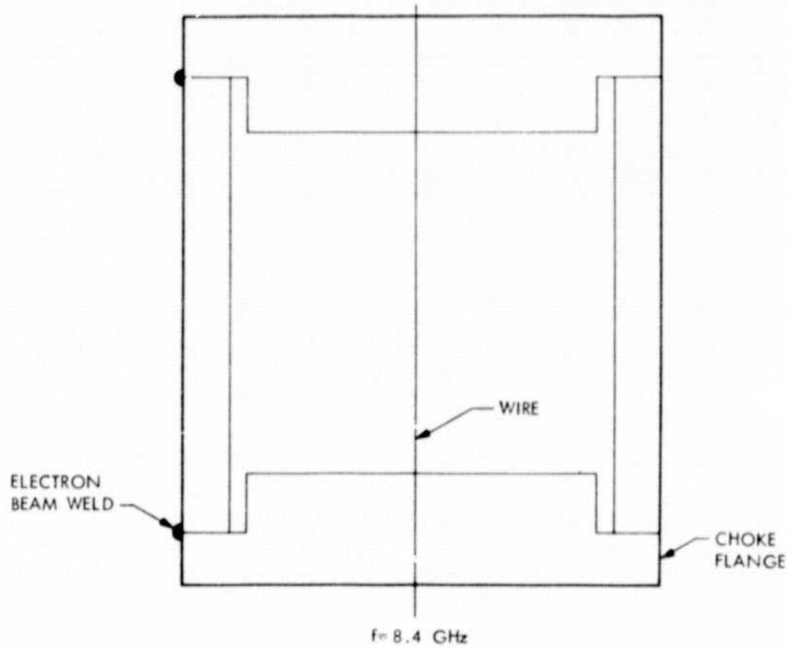

Fig. 6. Cross-section of TE_{011} mode niobium cavity

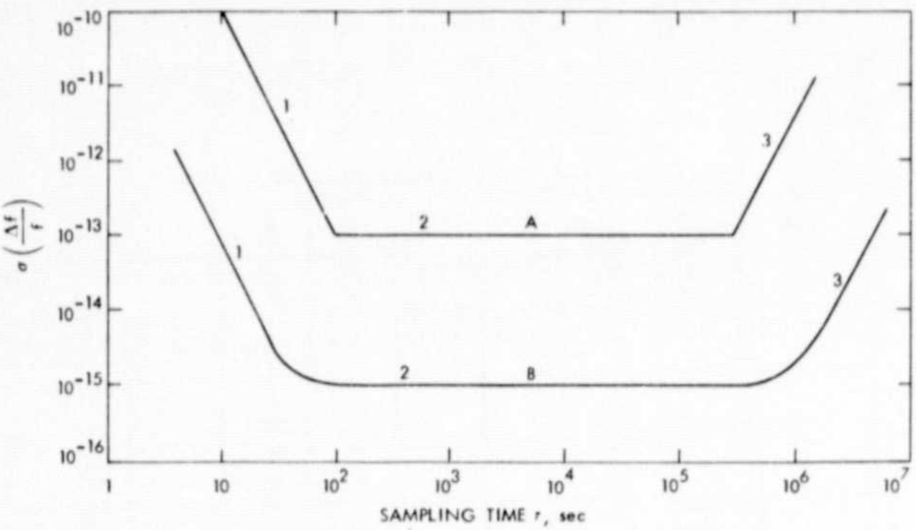

Fig. 7. Typical performance curves for high-quality oscillators (see text)

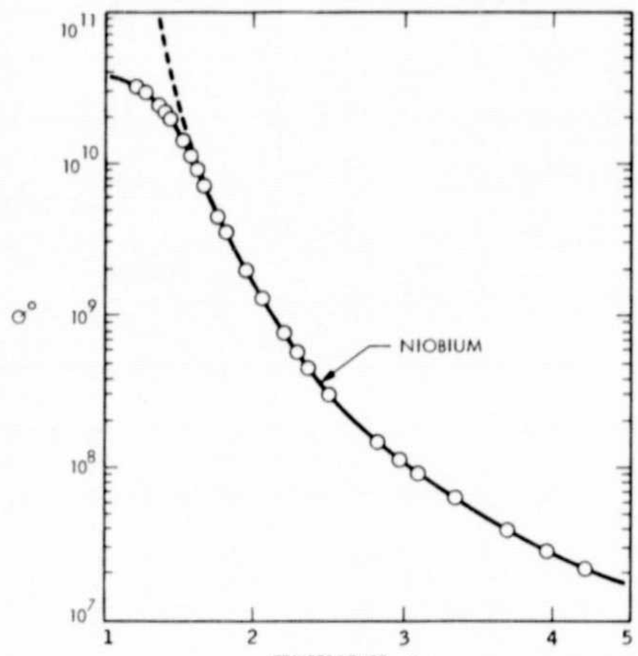

Fig. 8. The unloaded Q of an X-band TM_{010} mode niobium cavity (from Ref. 13)

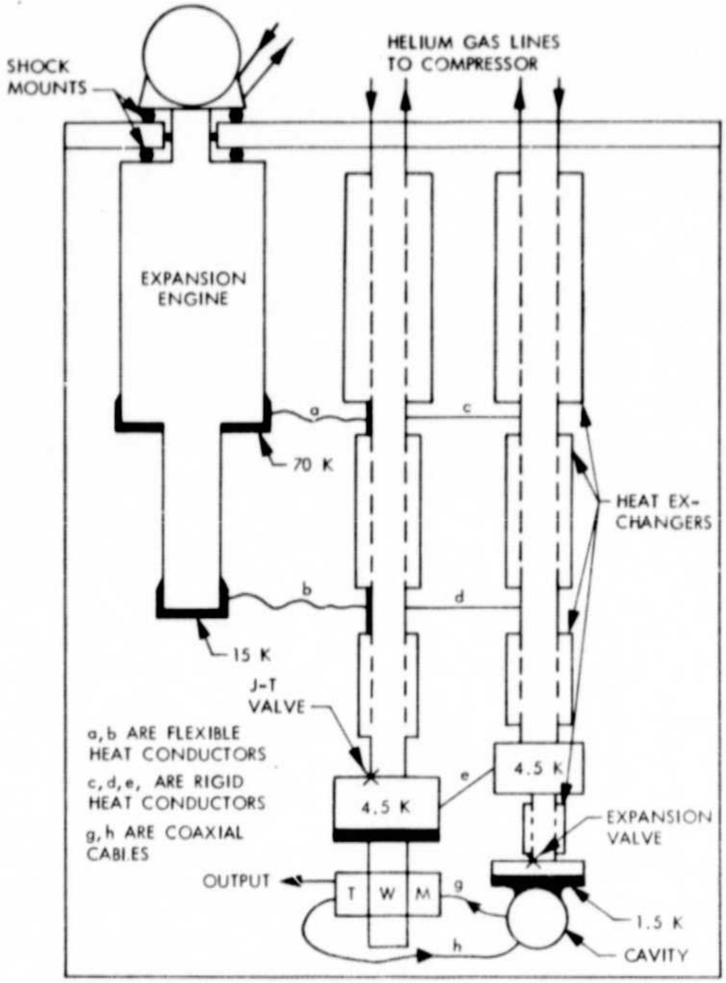

Fig. 9. Schematic diagram for 1.5 K cryogenic refrigerator with SCSMO. (Radiation shields and other details are not shown.)