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TIDAL ANALYSIS OF
MET ROCKET WIND DATA

Final Report

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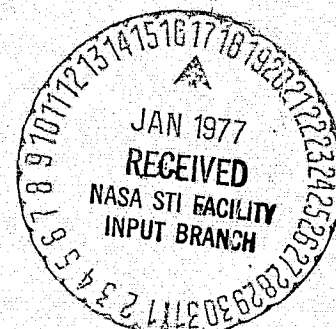
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TABLE OF CONTENTS

<u>Section</u>	<u>Title</u>	<u>Page</u>
	ABSTRACT	iv
I	INTRODUCTION	1
II	PREVIOUSLY REPORTED TIDAL ANALYSIS	2
III	ANALYTICAL METHOD	4
IV	CURRENT RESULTS	19
V	CONCLUSIONS	28
VI	REFERENCES	29
<u>Appendix</u>		
A	PREVAILING WINDS AND AMBIENT TEMPERATURE PROFILES	30
B	REMOVAL AND ASSESSMENT OF RANDOM ERRORS IN THE WIND AND TEMPERATURE DATA	39
C	COMPUTER CODES FOR DETERMINATIONS OF TIDAL MODES	56
D	HARMONIC ANALYSIS OF WIND DATA	67
<u>Figure</u>		
1	Altitude distribution of the amplitude of the solar diurnal component of u at 30° latitude for winter, equinoctial and summer conditions.	8
2	Altitude distribution of the phase of the solar diurnal component of u at 30° latitude for winter, equinoctial and summer conditions.	9
3	Smoothed northward components of the winds from June 1974.	20
4	Phase of the 7 km diurnal mode from flight 5 in Figure 3 (see text for explanation and meaning of symbols).	21
5	Phase of the 7 km diurnal mode from flight 6 in Figure 3 (see text for explanation and meaning of symbols).	22
6	Phase of the 7 km diurnal mode from flight 7 in Figure 3 (see text for explanation and meaning of symbols).	23
7	Phase of the 7 km diurnal mode from flight 8 in Figure 3 (see text for explanation and meaning of symbols).	24

TABLE OF CONTENTS

<u>Figure</u>	<u>Title</u>	<u>Page</u>
8	Derived phase of the Northward 10 km diurnal component.	25
9	Short wavelength components after removal of 7 km and 10 km components.	26
10	Long wavelength contribution to the northward wind.	27
<u>Table</u>		
1	Characteristics of some tidal modes	6

ABSTRACT

A new method of analyzing Met Rocket wind data is described. Modern tidal theory and specialized analytical techniques are used to resolve specific tidal modes and prevailing components in observed wind data. The objective is to formulate a representation of the wind which is continuous in both space and time. Such a representation allows direct comparison with theory, allows the derivation and other quantities such as temperature and pressure which in turn may be compared with observed values, and allows the formation of a wind model which extends over a broader range of space and time.

The current results for a single series of measurements indicates that significant diurnal tidal modes with wavelengths of 10 and 7 km are present in the data and are resolved by the analytical technique. The application to a wider data sample is required before the full potential of method can be determined.

SECTION I

INTRODUCTION

The primary purpose of this contract is the development of a new method for the analysis of upper atmospheric wind and temperature data, and the application of these methods to data obtained by the standard Meteorological Rocket systems. The method embraces the premise that the most useful form of the data is one which allows a continuous representation in both space and time. In order to achieve this representation, the wind data must be resolved into distinct components, each component having a distinct set of temporal and spatial characteristics. Such a resolution has predictive value, in the sense that it can predict the winds at a different place and time, and also in the sense that it can infer values of other observables, such as temperature and pressure. Furthermore, such a resolution allows direct comparison with theoretical predictions.

The analysis utilizes a data sample consisting of a series of wind measurements taken over a time interval of 24 (or more) hours, and covering a substantial portion of the altitude range from 20 to 80 kilometers. The tools of the analysis are (1) the theory of atmospheric motions in the stratosphere and mesosphere, and (2) an analytical procedure specifically suited to the Met Rocket data from Wallops Island. The method requires the resolution of the observed winds into distinct tidal modes as well as prevailing components. Each tidal mode has a characteristic period as well as spatial propagation characteristics. The number of measurements is always limited. Accordingly, harmonic analysis can resolve the observed winds according to their temporal characteristics only imperfectly. Further, there are several modes present with the same temporal period. Thus, a method to include the vertical propagation characteristics of each mode was incorporated into the analysis. The method removes modes with short wavelengths, and the remaining long-wavelength modes are then defined by use of harmonic analysis. The method relies heavily on the general results of tidal theory, and the analytical procedure is based on temporal and vertical "filtering" operations in order to resolve the observed measurements according to their temporal and spatial characteristics.

A brief survey of past observations and analysis of similar data are contained in Section II of this report. The analytical method is described in Section III and the current results are presented in Section IV.

SECTION II

PREVIOUSLY REPORTED TIDAL ANALYSIS

The presence of tidal forces in the atmosphere has long been recognized from the observations of surface pressure changes from which it was determined also that the driving force is thermal since the solar tide is much larger than the lunar. The semidiurnal component of the observed solar tide has a larger amplitude than the diurnal. For many years, this was the principal support for the "resonance theory" which assumed that the atmosphere had a natural resonance period close to 12 hours. During the past few years, however, it has been determined that the motions are actually forced oscillations in which the amplitudes are affected by varying efficiencies of the coupling to the energy source. It is now generally accepted that the thermally forced tides result from absorption of ultraviolet solar insolation by ozone in the stratosphere and mesosphere and from infrared absorption by water vapor in the troposphere. Contemporary tidal theory and limited observational data indicate that the amplitudes of tidal components increase from a few centimeters per second at the surface to several meters per second around 50 km and become the dominant component of the wind field above 100 km with amplitudes of over 100 meters per second. The development of the theory was hindered by the fact the equations are nonlinear which prohibit a complete global solution and also by the lack of observational data to furnish guidance for determination of useful approximations.

The data are difficult to analyze because the atmospheric wind system has several components that vary greatly in character and result from different sources. These differences appear in both size and time scales. Some have actually been observed and identified; others have been proposed as an explanation of observational results or on purely theoretical grounds. The shortest period, smallest size variations are in the latter group. The smallest sizes are usually categorized as turbulence and are generally attributed to nonlinearity of flow in high shear regions. The next larger scale size is often attributed to gravity waves that may result from a variety of sources including many kinds of disturbances in the atmosphere. Although these two smallest size components have not been unambiguously identified, small variations have been reported by many observers and most treatises on atmospheric dynamics include them in some form.

The next larger scale of variations is thermally driven tides with periods of 24 hours and its harmonics. Some of these motions have been identified and related through the appropriate theory to the energy source that produces them; i.e., absorption of solar insolation by ozone and water vapor. Diurnal and semi-diurnal components have been reported in the Met Rocket data and several other harmonics have been reported from other sources.

Planetary scale waves with periods of several days have been observed also. These are usually attributed to large disturbances in the atmosphere such as magnetic storms.

The largest period variations are seasonal, semiannual, annual, and quasi-biennial oscillation. The source of the 2-year oscillation is not yet known but the major seasonal changes are well described by the thermal wind equation in which the zonal wind component is defined by the meridional temperature gradient.

The observational data have been subjected to harmonic analyses in order to detect and evaluate components with periods of 24 and 12 hours. The data have been analyzed from hourly averages over varying periods of several days to months and from series of sequential measurements over a day or two. For short periods of observation, the results will be affected by the short period irregular components such as interval gravity waves or short period planetary waves. For larger observing periods, planetary waves and seasonal variations produce uncertainties. In all cases, the best fit parameters were determined by a purely mathematical process and then compared to theory. Early reports by Smith (1960) who found large diurnal and semidiurnal components from 23 observations in Nevada and Johnston Island, and by Lenhard (1963) who reported similar components with amplitudes of 10 m sec^{-1} at 60 km at Eglin, Florida were compared to the resonance theory of atmospheric tides. The observed phases were not compatible with the predictions of that theory.

Since the introduction of the forced oscillation theory and the acquisition of more data, a large number of tidal analyses of Met Rocket data have been reported.

Miers (1965) reported results from three series of rockets launched every 2 hours throughout the day from White Sands, New Mexico and from Eglin, Florida. Diurnal components with amplitudes from 10 to 20 m sec^{-1} and semidiurnal component with amplitudes of 3 to 5 m sec^{-1} were determined. Reed et al. (1966) reported diurnal amplitudes of 5 to 10 m sec^{-1} near the stratosphere from hourly values averaged over several months.

Diurnal oscillations with amplitude of 12 m sec^{-1} around 56 km were determined from 16 observations within 51 hours at White Sands by Beyers and Miers (1966). An associated temperature variation of 8°C was also reported but attempts to relate the wind and temperature variations through the generalized thermal wind equation were unsuccessful.

The presence of a semidiurnal tide with amplitude of 6 m sec^{-1} at 60 km was confirmed by Reed (1969), and Beyers and Miers (1968) confirm tidal components in the equatorial stratosphere from a series of 24 measurements within a 2-day period over Ascension Island.

In summary, it appears that tidal components with periods of 12 and 24 hours, are consistently present in the middle atmosphere but the relation of these components to the variation of temperature and pressure and to the appropriate thermal driving forces remains to be determined. The new analytical procedure which is described in Section III is specifically designed to determine these relations.

SECTION III

ANALYTICAL METHOD

A. INTRODUCTION

The primary purpose of the analysis of wind data is to obtain a representation of the winds that is continuous in space and time. In order to achieve a useful representation, the wind data must be resolved into distinct components, each component having a distinct set of temporal and spatial characteristics. Such a resolution has predictive value, in the sense that it can predict the winds at a different place and time, and also in the sense that it can infer values of other observables, such as temperature and pressure. Furthermore, such a resolution allows direct comparison with theoretical predictions.

The analysis suggested in this proposal assumes a data sample consisting of a series of wind measurements taken over a time interval of 24 (or more) hours, and covering a substantial portion of the altitude range from 20 to 80 kilometers. The tools of the analysis are (1) the theory of atmospheric motions in the stratosphere and mesosphere, and (2) an analytical procedure specifically suited to the Met Rocket data from Wallops Island. A brief account of pertinent theoretical results and of the analytical procedure is given below.

B. THEORETICAL RESULTS

In the altitude region under consideration nonlinear effects can be neglected. Accordingly the total wind is represented by a linear superposition of distinct contributions. These contributions are broken down into three general categories: (1) large-scale, long-period motions, (2) tidal oscillations, and (3) small-scale, short-period motions. The characteristics of these three types of motions are as follows:

1. Large-Scale, Long-Period Motions

Large-scale, long-period motions vary slowly or not at all during the period of observation. In other words, they are nonoscillatory with a characteristic time in excess of 24 hours. Their variation with altitude has a characteristic length in excess of 10 kilometers. The thermal wind and planetary waves are included in this category. Although these have been studied theoretically, it is not considered feasible at this time to determine their properties in detail from data confined to a single observing site.

2. Tidal Oscillations

The dominant tidal motions in this region of the atmosphere are the diurnal oscillations (with a period of 24 hours), and the semidiurnal oscillations (with a period of 12 hours). No evidence has been presented to date of the existence, to any significant extent, of tidal oscillations with

shorter periods. Pertinent features of five diurnal modes and three semidiurnal modes are listed in Table 1, which is based on the results of Lindzen (1967, 1968). All diurnal modes have the same temporal period (24 hours), but distinct variations with respect to altitude and latitude. Two types of diurnal modes are listed in Table 1: trapped and propagating. Mathematically, trapped and propagating modes are associated with negative and positive, respectively, "equivalent depths." ("Equivalent depths" are the eigenvalues of the tidal equation.) Physically, for trapped modes, the response of the atmosphere is confined to the immediate vicinity of the regions of excitation. Propagating modes are so-named because they propagate energy away from the region excitation. The term "phase" in Table 1 denotes the hour of maximum. The trapped diurnal modes, whose phase does not vary with altitude, represent motions which attain their maximum value simultaneously at all altitudes. There is evidence (Lindzen and Blabe, 1970) that the trapped diurnal modes are not significant below the mesopause.

The following characteristics are true of each mode listed in Table 1. The amplitude contains a factor which grows with altitude as $(\rho_0/\rho)^{1/2}$ where ρ is the local density and ρ_0 is the ground density. This is essentially an exponential growth. The relative amplitudes of the eastward and northward wind components are a function of latitude only, and are independent of altitude. Finally, the northward component leads the eastward component by a quarter of a period (i.e., the northward component achieves its maximum a quarter of a period before the eastward component). All of the above characteristics are independent of the nature of the exciting source. For the known sources of excitation (due to absorption by ozone and water vapor), the relative magnitudes of the different modes are known. Mathematical formulations of the stated characteristics are presented in the following subsection.

3. Small-Scale, Short-Period Motions

Small-scale, short-period oscillations may be representative of internal gravity waves or random motions. Since the continuous spectrum of allowed gravity waves is broad in terms of both periods and wavelengths, there are no general characteristics of such motions that are pertinent in the present context.

C. ANALYTICAL PROCEDURE

1. Previous Analyses

Previous analyses of Met Rocket data have treated data separately at each altitude; further, each wind component was analyzed separately. Basically, the method consisted of removing, in some fashion, a "prevailing" wind and a "trend", and then dealing with the residual wind as representing the diurnal oscillations (or the diurnal and semidiurnal oscillations together). Mathematically this procedure expresses the eastward component of the wind as

$$u(z,t) = A_1(z) \cos \omega_1 t + B_1(z) \sin \omega_1 t$$

Table 1. Characteristics of some tidal modes

Period (hours)	Name and Characteristics
24	First Trapped Diurnal. Phase does not vary with altitude. Amplitude varies smoothly with altitude. Probably not significant.
24	Second Trapped Diurnal. Phase does not vary with altitude. Amplitude varies smoothly with altitude. Probably not significant.
24	First Propagating Diurnal. Approximate wavelength: 28 km. Dominant diurnal mode.
24	Second Propagating Diurnal. Approximate wavelength: 11 km.
24	Third Propagating Diurnal. Approximate wavelength: 7 km.
12	First Semidiurnal. Approximate wavelength: 150 km.
12	Second Semidiurnal. Approximate wavelength: 54 km.
12	Third Semidiurnal. Approximate wavelength: 34 km.

$$+ A_2(z) \cos \omega_2 t + B_2(z) \cos \omega_2 t \quad (1)$$

$$+ P_0(z) + P_1(z) t,$$

where

u = eastward component of the wind

ω_1 = frequency of diurnal oscillation

ω_2 = frequency of semidiurnal oscillation

z = altitude

t = time

P_0 = prevailing wind

$P_1 t$ = "Trend"

and the amplitudes (C) and phases (δ) are determined from the equations

$$C_i^2 = A_i^2 + B_i^2, \quad \delta_i = \frac{1}{\omega_i} \arctan (B_i/A_i) \quad i = 1, 2. \quad (2)$$

A similar expression pertains to the northward component of the wind. The A 's, B 's and P 's are treated as parameters to be determined by the method of least squares from N observations

$$u(t_1), \dots, u(t_N).$$

Despite the simplicity of this approach, results fairly consistent with theory were obtained by Reed (1966, 1967), Miers (1965), and Beyers and Miers (1966, 1968). These results are shown in Figures 1 and 2 (after Lindzen, 1967).

2. Data Sample and Errors

For an adequate data sample (say nine or more observations over a 24-hour period), the analysis can be improved by taking into account general theoretical results, as well as temporal and spatial "filtering" techniques. Some of the details of those techniques depend on the temporal resolution of the data, and on the ability to ascertain the magnitude of the errors in the data. In general, it is best to acquire the data at evenly spaced intervals

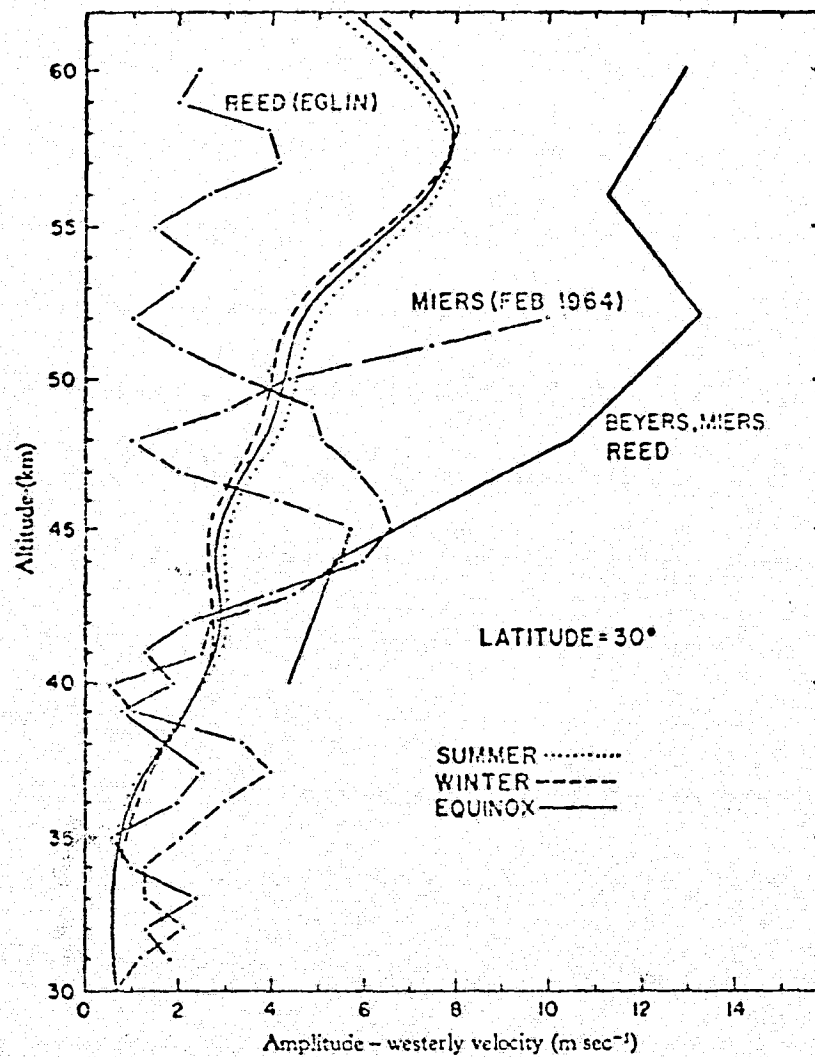


Figure 1. Altitude distribution of the amplitude of the solar diurnal component of u at 30° latitude for winter, equinoctial and summer conditions. Also shown are some distributions based on observations.

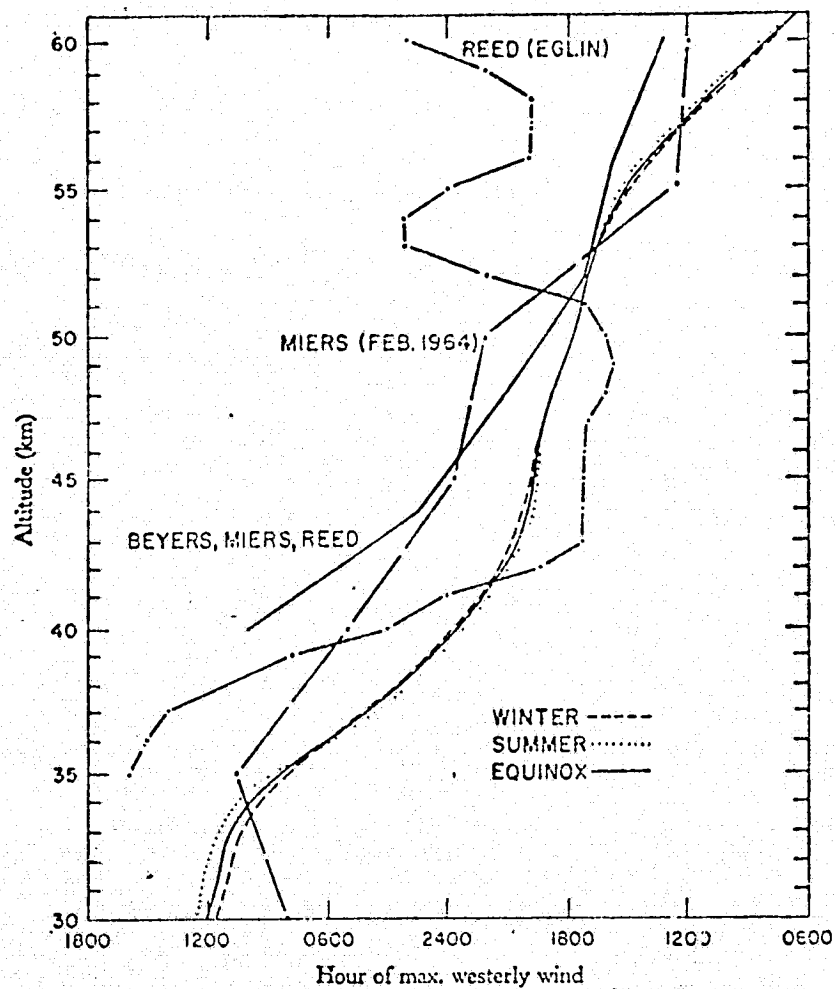


Figure 2. Altitude distribution of the phase of the solar diurnal component of u at 30° latitude for winter, equinoctial and summer conditions. Also shown are some distributions based on observations.

(e.g., at 3-hour intervals for nine observations). A theoretical estimate of the errors in the data is usually obtained from the characteristics of the measuring apparatus as well as from the procedure utilized to translate direct observations (e.g., radar signals) to wind values. Another estimate can be obtained by averaging the wind values over a specified height interval (e.g., half a kilometer) and treating the root-mean-square deviation from the average as the error figure. Owing to the difficulty of obtaining reliable theoretical estimates of errors, the second approach appears preferable. The error figure at each altitude determines the weight to be attached to the wind values at that altitude. For simplicity, consideration of error figures and weights are omitted from the following discussion. The methods of error assessment and data smoothing with the computer codes are contained in Appendix B of this report.

3. Analytic Representation of the Measurements

To formulate the technique of temporal and vertical filtering, it is assumed that the measured winds can be represented as a linear combination of tidal oscillations, a slowly-varying contribution, gravity waves, and noise. Analytically,

$$u(x, \theta, t) = \exp(x/2) \sum_{n=1}^2 \sum_{m=1}^{M(n)} U_{nm}(\theta) \left[A_{nm}(x) \cos \omega_n t + B_{nm}(x) \sin \omega_n t \right] \\ + P_{u0}(x) + P_{u1}(x)t + P_{u2}(x)t^2 + G_u(x, t) + N_u(x, t) \quad (3)$$

$$v(x, \theta, t) = \exp(x/2) \sum_{n=1}^2 \sum_{m=1}^{M(n)} V_{nm}(\theta) \left[B_{nm}(x) \cos \omega_n t - A_{nm}(x) \sin \omega_n t \right] \\ + P_{v0}(x) + P_{v1}(x)t + P_{v2}(x)t^2 + G_v(x, t) + N_v(x, t) \quad (4)$$

where

u = eastward component of the wind

v = northward component of the wind

θ = colatitude ($\theta = 0^\circ$ at the north pole)

t = local time

$$x = \int_0^z dz/H(z) = \text{"reduced" altitude}$$

z = altitude

$H(z)$ = scale height of altitude z

ω_n = frequency of oscillation for n th tidal harmonic, $n = 1, 2$

$U_{nm}(\theta)$ = latitudinal variation of m th mode of the n th harmonic for the eastward wind (known)

$V_{nm}(\theta)$ = latitudinal variation of m th mode of the n th harmonic for the northward wind (known)

$P_{u0} + P_{u1}t + P_{u2}t^2$ = representation of long-period eastward wind

$P_{v0} + P_{v1}t + P_{v2}t^2$ = representation of long-period northward wind

G_u = collective label for short-period contributions to eastward wind

G_v = collective label for short-period contributions to northward wind

N_u = noise in the eastward wind component

N_v = noise in the northward wind component

$M(n)$ = number of modes for n th tidal harmonic

Noise, in the present context, denotes a short-lived transient that affects only a few of the measurements. It is distinguished from the short-period contributions G , that may be present in all measurements. The coefficients A_{nm} and B_{nm} determine the amplitude (C_{nm}) and phase (δ_{nm}) of each mode as follows

$$C_{nm}^2 = A_{nm}^2 + B_{nm}^2, \quad \delta_{nm} = \frac{1}{\omega_n} \arctan (B_{nm}/A_{nm}) \quad (5)$$

Note that the growth factor $\exp(x/2)$ has been explicitly introduced as a coefficient of the tidal components in Equations (3) and (4). Note also that the general theoretical relations between the eastward and northward components pertaining to each mode have been incorporated into Equations (3) and (4) as well.

4. Temporal "Filtering"

"Filtering" usually refers to an operation on a continuous signal. In the present case the term is used to denote a linear combination of K distinct measurements conducted at the times t_k , $k = 1, \dots, K$. Let $w(k)$ be a set of coefficients and consider the combination

$$S_u(x) = \sum_{k=1}^K w(k) U(x, t_k) \quad (6)$$

where $u(x, t)$ is the eastward wind measured at time t . The sum S will be free of diurnal and semidiurnal contributions provided the coefficients w are chosen so as to satisfy the equations

$$\sum_{k=1}^K w(k) \cos \omega_n t_k = 0, \quad n = 1, 2 \quad (7)$$

$$\sum_{k=1}^K w(k) \sin \omega_n t_k = 0, \quad n = 1, 2 \quad (8)$$

Then

$$\begin{aligned} S_u = & P_{u0} \sum_{k=1}^K w(k) + P_{u1} \sum_{k=1}^K w(k) t_k + P_{u2} \sum_{k=1}^K w(k) t_k^2 \\ & + \sum_{k=1}^K w(k) G_u(t_k) + \sum_{k=1}^K w(k) N_u(t_k) \end{aligned} \quad (9)$$

Equations (7) and (8) together impose four conditions on the $w(k)$ coefficients, leaving $K-4$ degrees of freedom. (It is assumed that $K \geq 9$). Accordingly $(K-4)$ linearly independent combinations of the form of equation (9) are formed. From these combinations, the three coefficients P_{u0} , P_{u1} , P_{u2} are determined by a method of least squares, assuming the contributions from N and G cancel out. This assumption is justified provided the following conditions are met:

1. The coefficients $w(k)$ are of comparable magnitude, so that noise present in a few measurements does not lead to a spurious result.
2. The combination does not enhance excessively any short-period contribution contained in G_u . Let ω_G be the frequency of such a contribution. Forming a combination of the form of equation (9) enhances (or attenuates, as the case may be) G_u by a factor F_G , where

$$F_G^2 = \left(\sum_{k=1}^K w(k) \cos \omega_G t_k \right)^2 + \left(\sum_{k=1}^K w(k) \sin \omega_G t_k \right)^2. \quad (10)$$

These conditions are met provided there is sufficient redundancy in the data. For $K=9$ (measurements) there are $K-4=5$ equations of the form (9) from which the three P coefficients can be determined.

The first step of the temporal filtering procedure thus yields an initial estimate of the long-period wind component separately at each altitude:

$$P_{u0}(x), P_{u1}(x), P_{u2}(x) \quad \text{determined at each altitude } x.$$

The second step is the imposition of the requirement that the long-period wind component vary slowly (and smoothly) with altitude. This is equivalent to vertical filtering that establishes the smooth representations

$$P_u(x, t_k) \quad k = 1, \dots, K \quad (11)$$

These are approximations to the long-period contributions to the wind at the times of the observations.

The next step of the temporal filtering procedure is to form the quantities

$$Q_u(x, t_k) = u(x, t_k) - P_u(x, t_k) \quad (12)$$

and the combination

$$T_u(x) = \sum_{k=1}^K \alpha(k) Q_u(x, t_k). \quad (13)$$

Q_u is thus the eastward wind with the long-period contribution removed. If the requirement

$$\sum_{k=1}^K \alpha(k) \cos \omega_2 t_k = \sum_{k=1}^K \alpha(k) \sin \omega_2 t_k = 0 \quad (14)$$

is imposed on the coefficients $\alpha(k)$, then the combination (13) is free of semidiurnal contributions. Since there are K coefficients $\alpha(k)$ and two conditions imposed by equation (14), $(K-2)$ linearly independent combinations of the form of equation (13) can be obtained. If conditions (1) and (2) mentioned above are observed, then the T_u can be considered essentially free of noise and short-period contributions and thus contain the diurnal modes only.

From equation (3)

$$T_u(x, \ell) = \exp(x/2) \sum_{m=1}^{M(1)} U_{1m}(\theta) \left[A_{nm}(x) \sum_{k=1}^K \alpha(k, \ell) \cos \omega_1 t_k + B_{nm}(x) \sum_{k=1}^K \alpha(k, \ell) \sin \omega_1 t_k \right] \quad \ell = 1, \dots, K-2 \quad (15)$$

Using equation (5) this can be written as

$$\exp(-x/2) T_u(x, \ell) = \sum_{m=1}^{M(1)} U_{1m}(\theta) C_{1m}(x) \cos \omega_1 [t_1(\ell) - \delta_{1m}(x)] \quad \ell = 1, \dots, K-2 \quad (16)$$

The same steps applied to the northward wind component result in the equation

$$\exp(-x/2) T_v(x, \ell) = -F_1(\ell) \sum_{m=1}^{M(1)} V_{1m}(\theta) C_{1m}(x) \sin \omega_1 [t_1(\ell) - \delta_{1m}(x)] \quad \ell = 1, \dots, K-2 \quad (17)$$

where

$$F_1(\ell)^2 = \left(\sum_{k=1}^K \alpha(k, \ell) \cos \omega_1 t_k \right)^2 + \left(\sum_{k=1}^K \delta(k, \ell) \cos \omega_1 t_k \right)^2 \quad (18)$$

$$\bar{t}_1(\ell) = \frac{1}{\omega_1} \arctan \left[\frac{\sum_{k=1}^K \alpha(k, \ell) \sin \omega_1 t_k}{\sum_{k=1}^K \alpha(k, \ell) \cos \omega_1 t_k} \right] \quad (19)$$

Since the notation is cumbersome, it seems appropriate to review the significance of equations (16) and (17). Given k arbitrary coefficients α there are $(k-2)$ linearly independent ways of choosing these coefficients while satisfying the two conditions of equation (14). These $(k-2)$ sets of coefficients are labeled

$$\alpha(k, \ell), \quad K = 1, \dots, k, \quad \ell = 1, \dots, k-2.$$

There are, therefore $(k-2)$ linearly independent combinations T_u and T_v that can be formed:

$$T_u(x, \ell), T_v(x, \ell), \quad \ell = 1, (k-2).$$

For each ℓ , $T_u(x, \ell)$ and $T_v(x, \ell)$ represent, respectively, the total eastward and northward diurnal wind at the time $\bar{t}_1(\ell)$, enhanced by the factor $F_1(\ell)$. The unknowns in equations (16) and (17) are the amplitudes $C_{1m}(x)$ and phases $\delta_{1m}(x)$, for $m = 1, \dots, M(1)$, where $M(1)$ is the number of diurnal modes.

In an entirely analogous manner the semidiurnal winds are isolated as well.

5. Vertical "Filtering"

It has been mentioned previously that trapped diurnal modes are insignificant below the mesopause. Thus only the three propagating modes need be considered. These have wavelengths of approximately 28, 11, and 7 kilometers. Owing to the very slow variation with altitude of the amplitude factors $C_{1m}(x)$, the shorter-wavelength modes can be removed by vertical filtering. To illustrate the filtering procedure, let us consider the function

$$f(x) = \cos \left(\omega t - \frac{2\pi}{\lambda} x \right),$$

and the integral

$$\begin{aligned} I(a, x) &= \int_{x-a/2}^{x+a/2} dy f(y) \\ &= \frac{\lambda}{\pi} \sin \frac{2\pi a}{\lambda} \cos \left(\omega t - \frac{2\pi}{\lambda} x \right). \end{aligned}$$

If $a \approx \lambda/2$, then $I(a, x) \approx 0$. Thus averaging the combinations T_u and T_v of equation (16) and (17) over 7 km and 11 km successively will remove essentially all but the first propagating diurnal mode multiplied by known factors. The result can be written in the compact form

$$C_{11}(x) \cos \omega_1 \bar{t}_1(\ell) - \delta_{11}(x) = Y_u(x, \ell) \quad \ell = 1, \dots, k-2 \quad (20)$$

$$C_{11}(x) \sin \omega_1 \bar{t}_1(\ell) - \delta_{11}(x) = Y_v(x, \ell) \quad \ell = 1, \dots, k-2 \quad (21)$$

where Y_u and Y_v are known. Consider now the graphical representations of Y_u and Y_v versus x . It follows that

1. where Y_u has a maximum:

$$\delta_{11}(x) = \bar{t}_1(\ell), \quad C_{11}(x) = Y_u(x, \ell)$$

2. where Y_u has a minimum:

$$\delta_{11}(x) = \bar{t}_1(\ell) + \frac{\pi}{\omega_1}, \quad C_{11}(x) = -Y_u(x, \ell)$$

3. where Y_v has a maximum:

$$\delta_{11}(x) = \bar{t}_1(\ell) - \frac{\pi}{2\omega_1}, \quad C_{11}(x) = Y_v(x, \ell)$$

4. where $Y_v(x, \ell)$ has a minimum:

$$\delta_{11}(x) = \bar{t}_1(\ell) + \frac{\pi}{2\omega_1}, \quad C_{11}(x) = -Y_v(x, \ell).$$

From the $2(k-2)$ such graphical representations a sufficient number of points can be established in the vertical profiles of $C_{11}(x)$ and $\delta_{11}(x)$ to allow a smooth analytic representation of these functions.

After the longest-wavelength mode has been isolated in this fashion, it can be subtracted from the total diurnal contributions of equations (16) and (17) and the other modes can be investigated as well.

6. Temperature and Pressure Investigation

It is a general result of tidal theory that, for each tidal mode separately, the temperature and pressure variation can be determined from the wind components. Explicitly, the pressure variation is

$$\Delta P_{nm}(x, \theta, t) = \frac{4a\omega_e^2}{Rg\omega_n} \textcircled{H}_{nm}(\theta) \frac{1}{H} \exp(x/2) \quad (22)$$

$$x = A_{nm}(x) \cos \omega_n t + B_{nm}(x) \sin \omega_n t$$

and the temperature variation is

$$\Delta T_{nm}(x, \theta, t) = \frac{4a\omega_e^2}{Rg\omega_n} \textcircled{H}_{nm}(\theta) \frac{1}{H} \exp(x/2) \quad (23)$$

$$v \left\{ \left[H(x) A_{nm}(x) - \textcircled{H} A_{nm}(x) \right] \cos \omega_n t \right. \\ \left. - \left[H(x) B_{nm} - H B_{nm}(x) \right] \sin \omega_n t \right\}$$

where

ΔP_{nm} = pressure variation due to mth mode of nth harmonic

ΔT_{nm} = temperature variation due to mth mode of nth harmonic

a = radius of the earth

ω_e = frequency of rotation of the earth

R = gas constant

g = gravitational constant

$H_{nm}(\theta)$ = latitudinal variation (known).

The altitude-dependent functions A_{nm} and B_{nm} are the same ones involved in (and determined by) the wind components. Primes in equation (23) indicate derivatives with respect to the reduced height. In taking derivatives, it is always the case that small errors get greatly magnified. Thus, in practice one would integrate equation (23) over a small altitude interval (1/2 to 2 kilometers; depending on the accuracy of the data) and obtain a mean value for the interval.

The total tidal contribution to temperature and pressure variations is obtained by summing over all modes

$$\Delta P \text{ (tidal)} = \sum_n \sum_m P_{nm}$$

$$\Delta T \text{ (tidal)} = \sum_n \sum_m T_{nm}$$

It should be noted that the same filtering procedures developed for the winds can be applied to temperature and pressure measurements of comparable accuracy.

SECTION IV

CURRENT RESULTS

The methods described in Section III were applied to a series of measurements from Wallops Flight Center in June 1974. After application of the error assessment and smoothing procedures contained in Appendix B, the vertical profiles of the northward component of the winds for that series is shown in Figure 3. It was found that the smoothing procedure had to be applied twice in succession in order to reduce the random variations to levels which allowed the extraction of small tidal oscillations. It was also found that a broader data base was required to define the vertical profiles of prevailing zonal wind and ambient temperature which are required in the analysis. The values of these quantities which were obtained from the CIRA 1965 model are given in Appendix A of this report.

Plots such as Figure 3 are important to the analytical method especially during the initial phases. The location of maxima and minima on the vertical profiles are used to determine the relative phases of the predominant tidal modes. The values of phases which are actually utilized in the analysis are computed from tidal theory for the identified modes. Some examples of this procedure for flight nos. 5, 6, 7, and 8 of the June series are shown in Figure 4 through 7. The phase of the 7 km (diurnal) component expressed as multiples of π is shown over the observed altitude range. The solid curve was obtained from tidal theory utilizing the prevailing temperature profile. The points are from visual observation of maxima and minima on the vertical wind profiles. The points are generally alternatively the eastward maxima (●), the northward maxima (○), the eastward maxima (⊖) and the northward minima (Δ).

Verification of the amplitudes of the various superimposed oscillations in the June data was obtained by applying a standard harmonic analysis to that series of data. The results are discussed in Appendix D of this report.

The computer codes for determining the tidal modes with the filtering method are contained in Appendix C. Some examples of the results obtained from the June 1974 data are shown in Figures 8, 9, and 10. The phase of a 10 km diurnal component which could not be determined visually as was the 7 km component was, however, determined in the filtering procedure and is shown in Figure 8 for the first eight measurements in the June series.

The short wavelength residual after the removal of the 7 km and 10 km wavelength diurnal modes is shown in Figure 9. This remainder strongly resembles a 5 km wavelength semidiurnal mode.

Finally, the very long wavelength contribution to the northward wind is shown in Figure 10.

Application of the program to data from other observational series at different time has been initiated but could not be completed in time for this report.

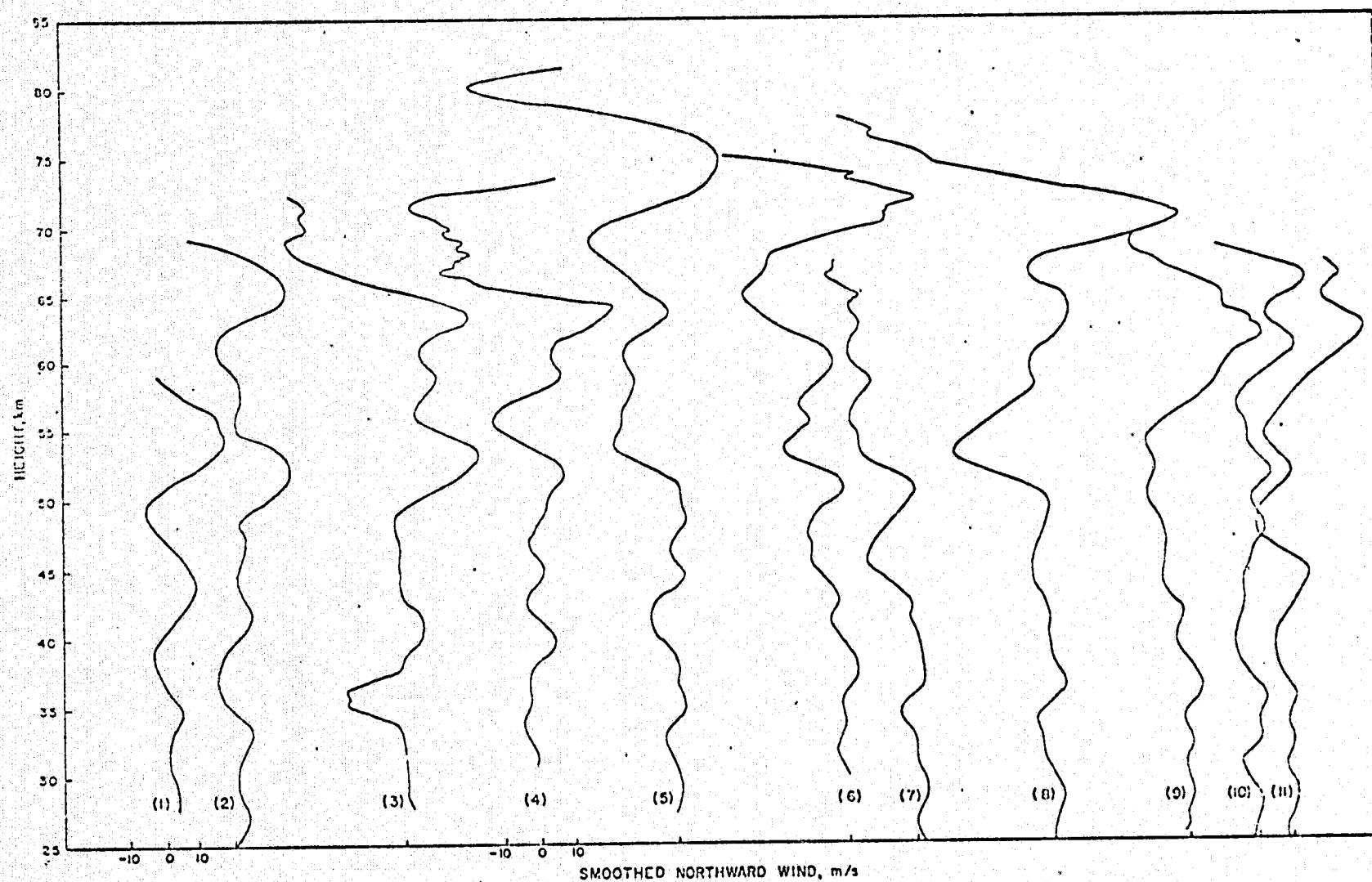


Figure 3. Smoothed northward components of the winds from June 1974.

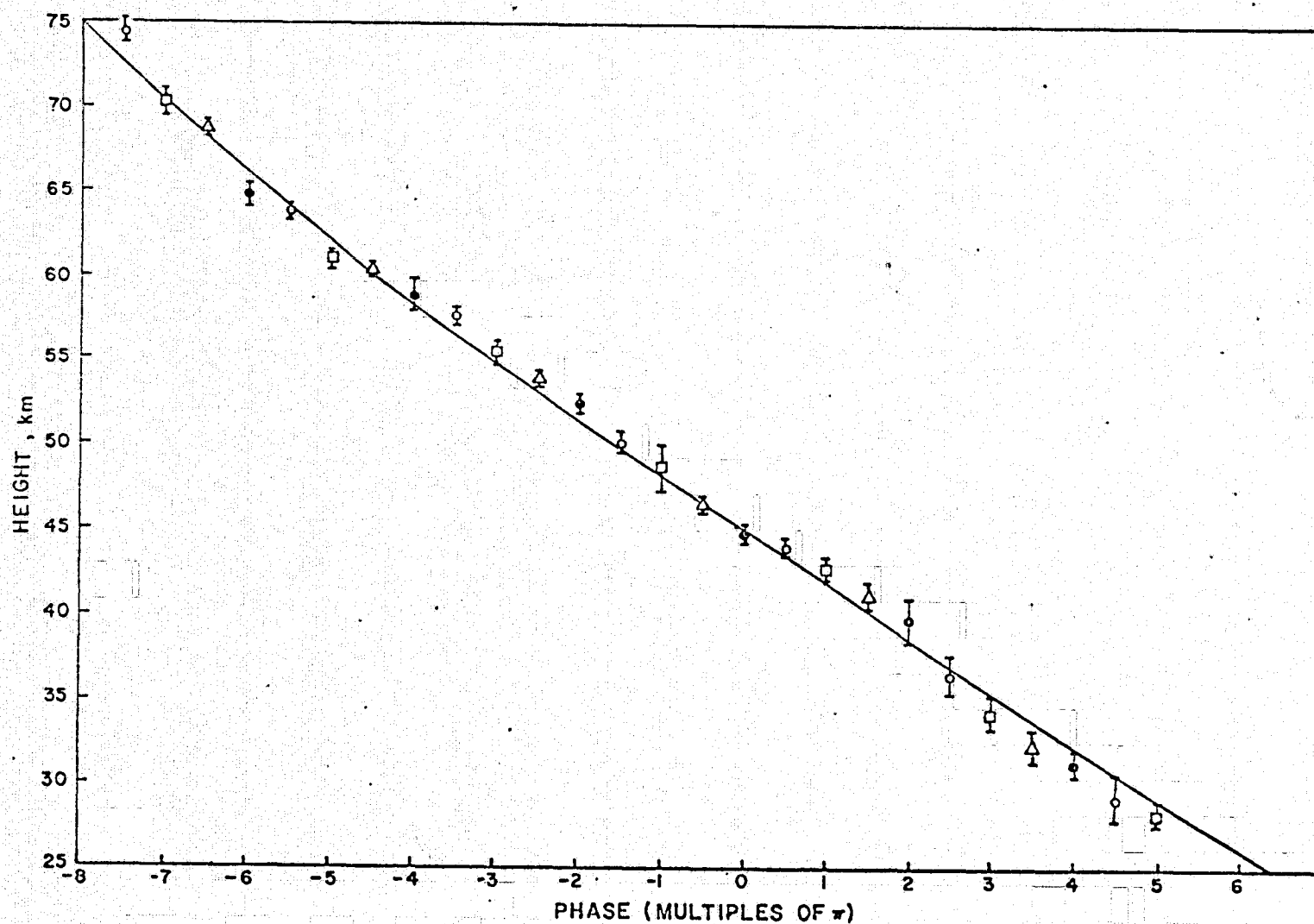


Figure 4. Phase of the 7 km diurnal mode from flight 5 in Figure 3 (see text for explanation and meaning of symbols).

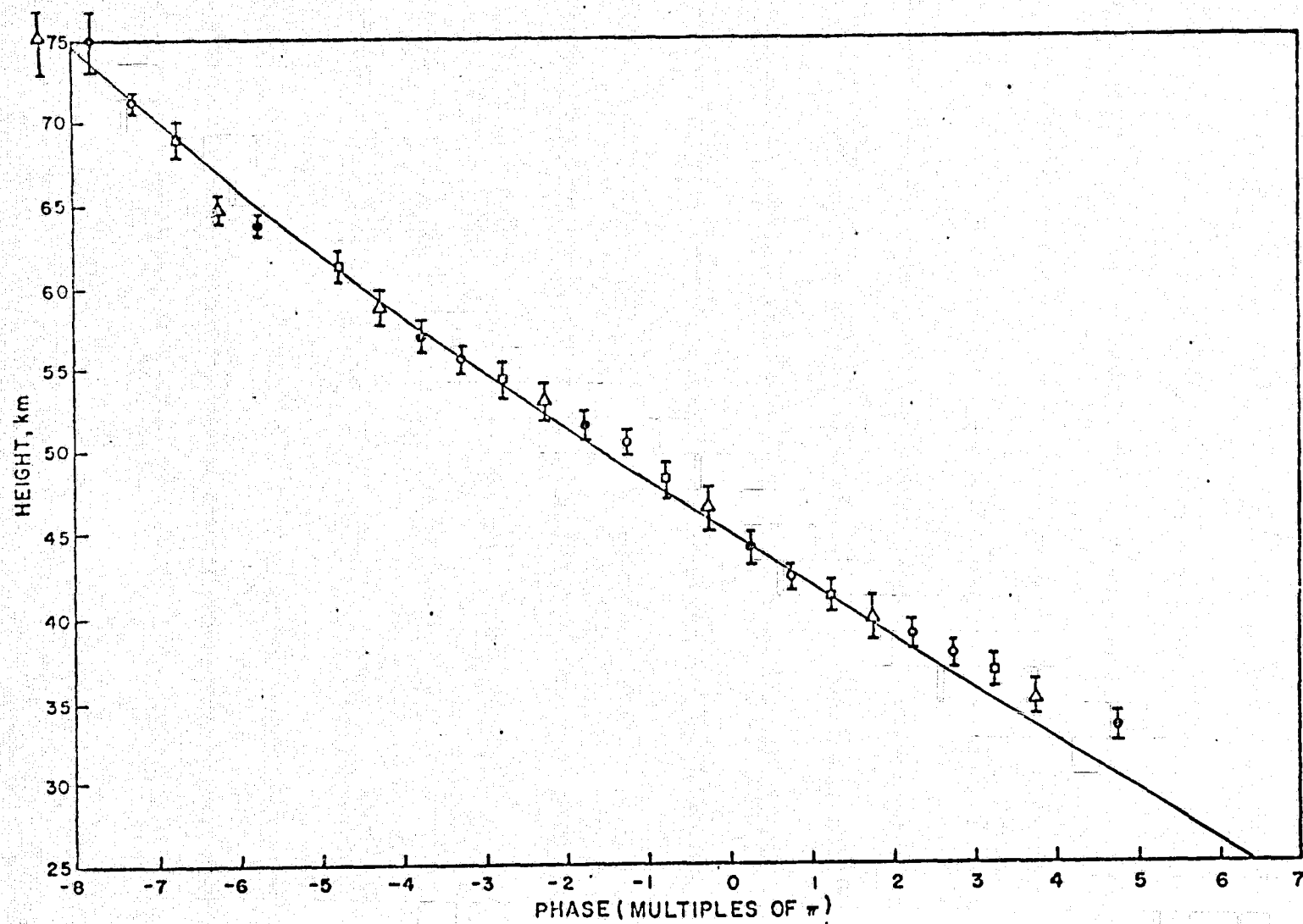


Figure 5. Phase of the 7 km diurnal mode from flight 6 in Figure 3 (see text for explanation and meaning of symbols).

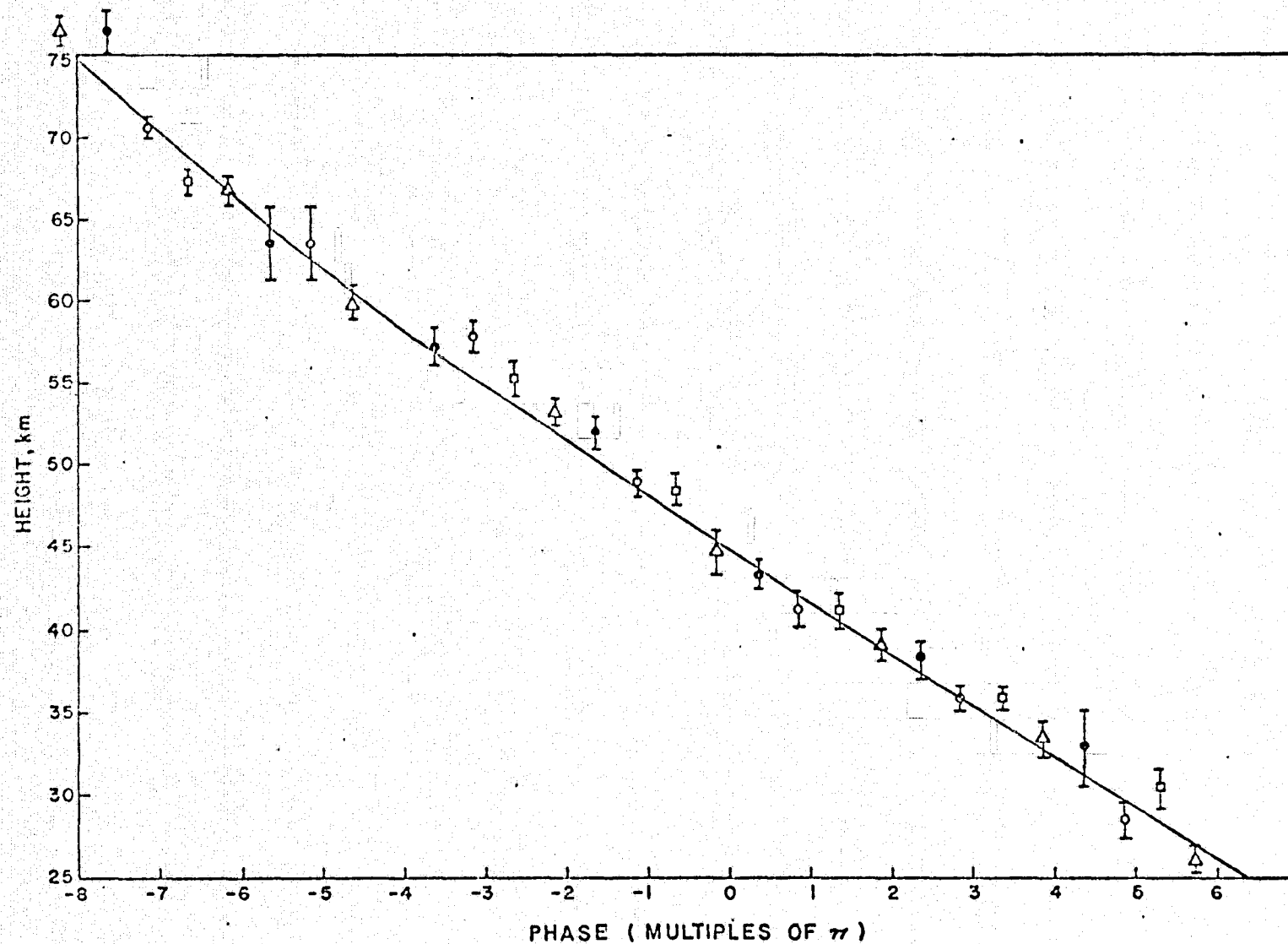


Figure 6. Phase of the 7 km diurnal mode from flight 7 in Figure 3 (see text for explanation and meaning of symbols).

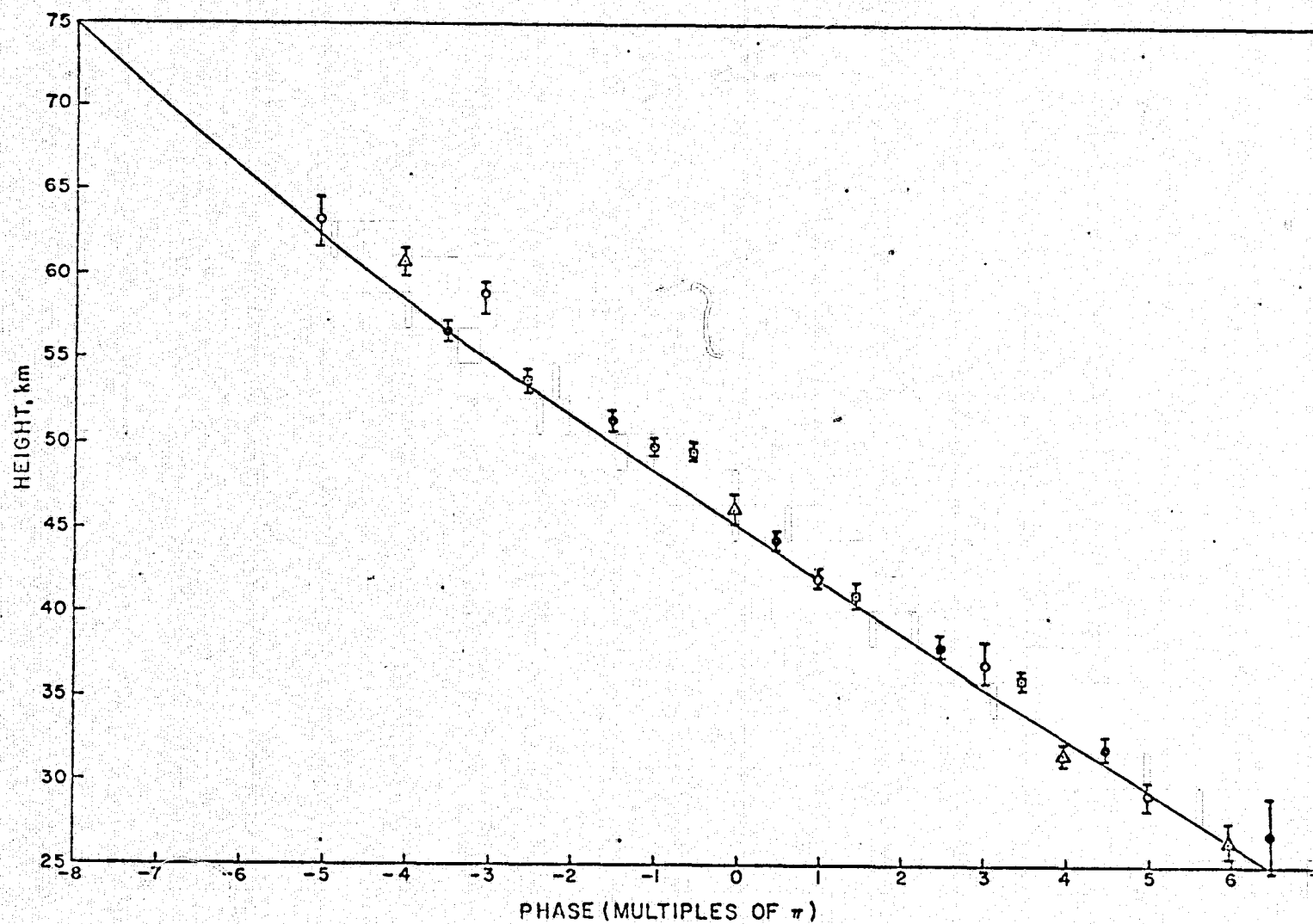


Figure 7. Phase of the 7 km diurnal mode from flight 8 in Figure 3 (see text for explanation and meaning of symbols).

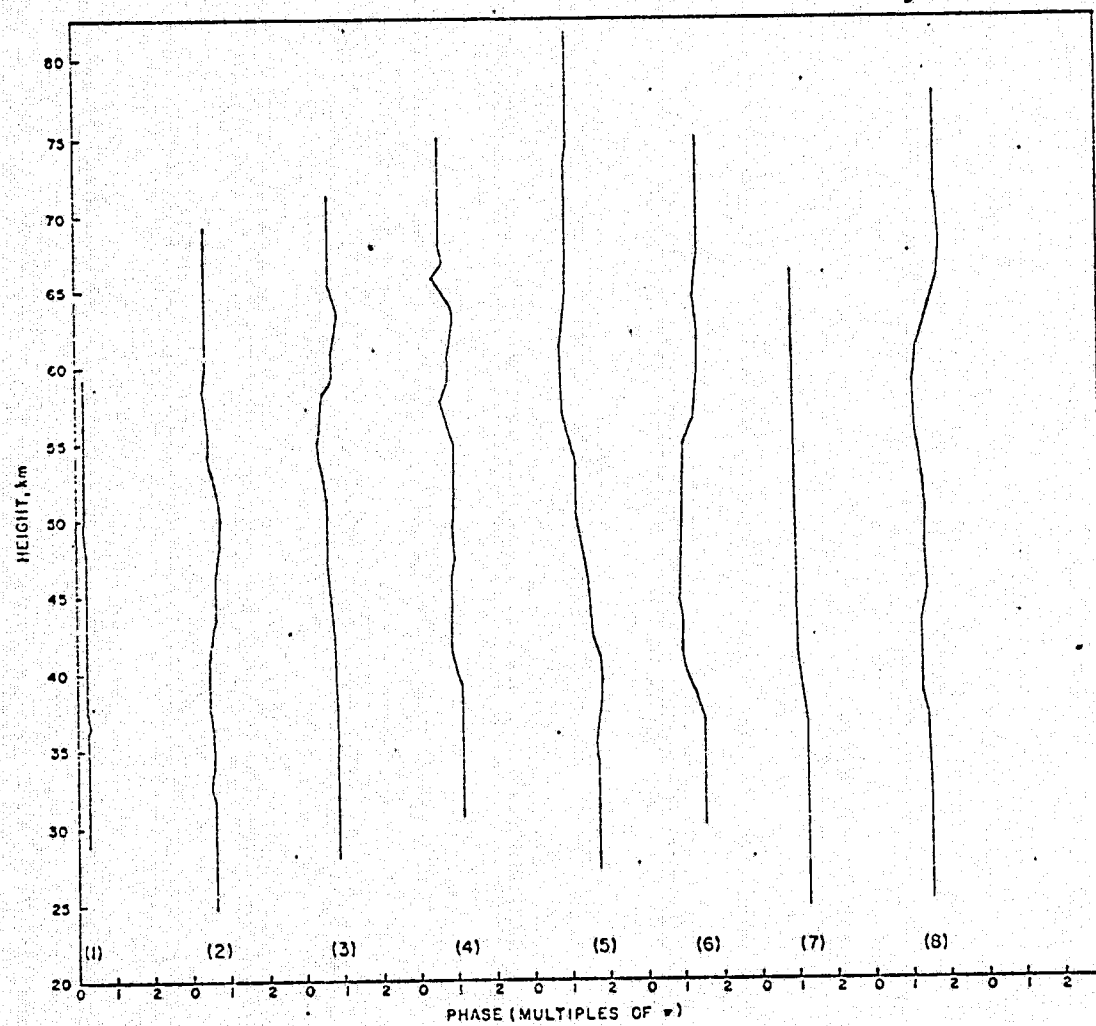


Figure 8. Derived phase of the Northward 10 km diurnal component.

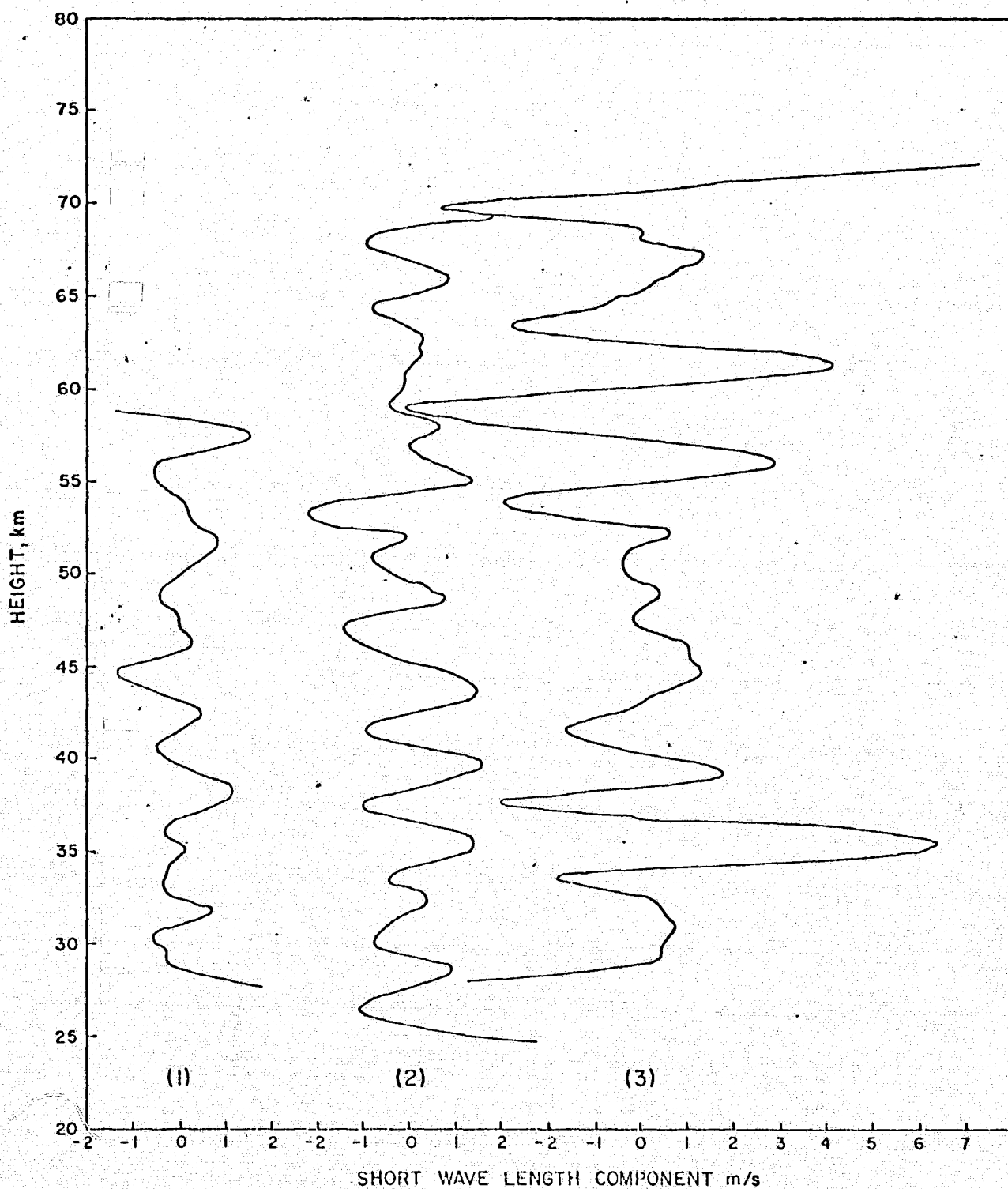


Figure 9. Short wavelength components after removal of 7 km and 10 km components.

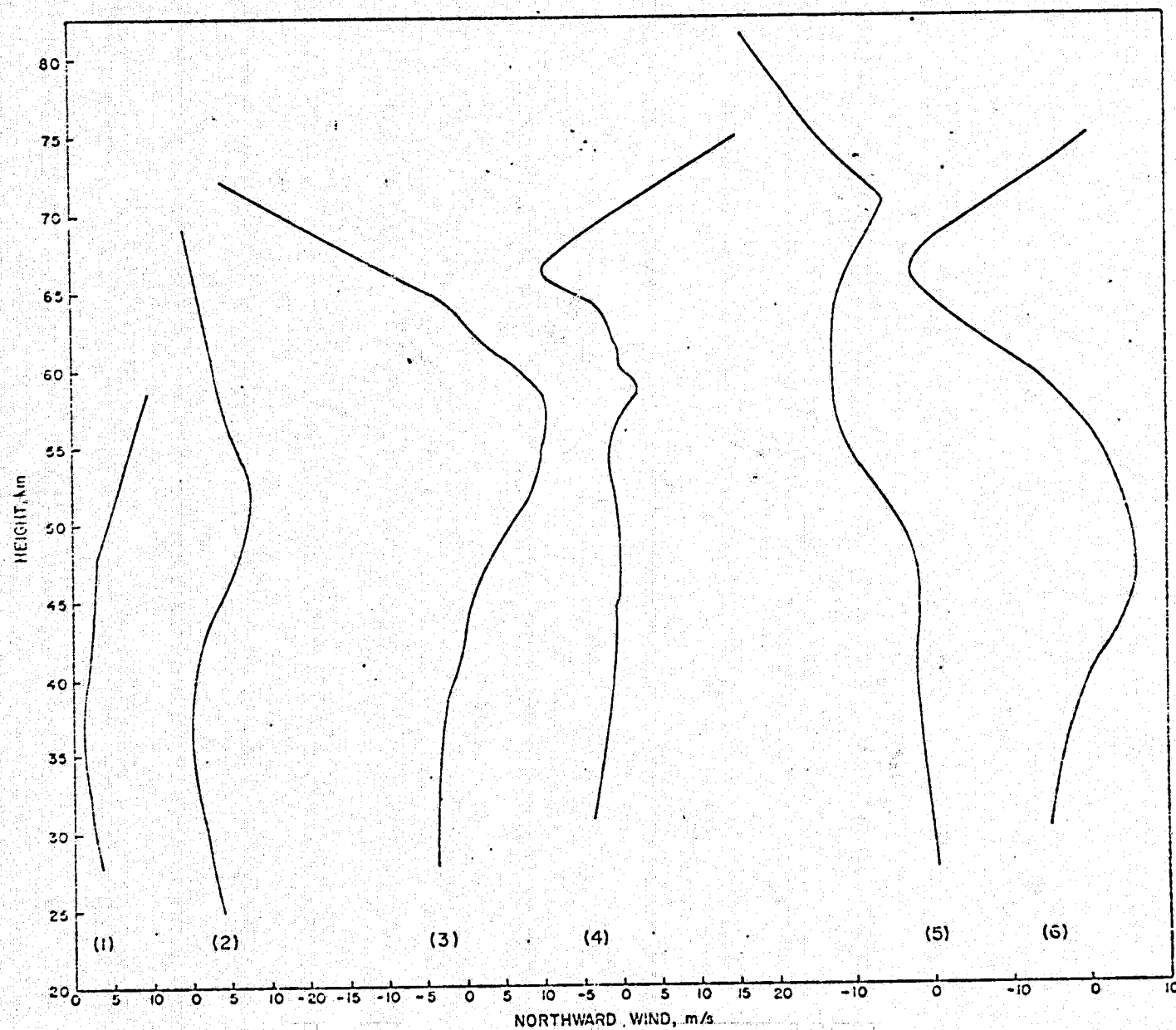


Figure 10. Long wavelength contribution to the northward wind.

SECTION V

CONCLUSIONS

Difficulties associated with the organization of the original data and the restructuring of the smoothing process were much greater and more time consuming than were initially anticipated. Thus the amount of actual processed data was less than initially expected. However, the primary objective of the contract was accomplished in that the development of the new analytical method was completed. It was demonstrated that the computer programs produce results with the vertical filtering procedure which are verified by the totally separate harmonic analysis process. Assessment of the full potential of the method must await the processing of a much longer amount of data from different times and locations.

SECTION VI

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APPENDIX A

PREVAILING WINDS AND AMBIENT TEMPERATURE PROFILES

Detailed profiles of the ambient temperature and of the prevailing zonal wind have been established by interpolation from tables found in the CIRA 1965 handbook. Values appropriate to the dates and latitudes of the measurements of interest were obtained from the tables at 5 kilometer intervals. Using interpolation techniques, we calculated the required profiles at 0.25 km intervals and smoothed the profiles using the procedures described in Appendix B. Graphical representations of the ambient temperature and prevailing zonal winds are presented in Figures A-1 through A-8.

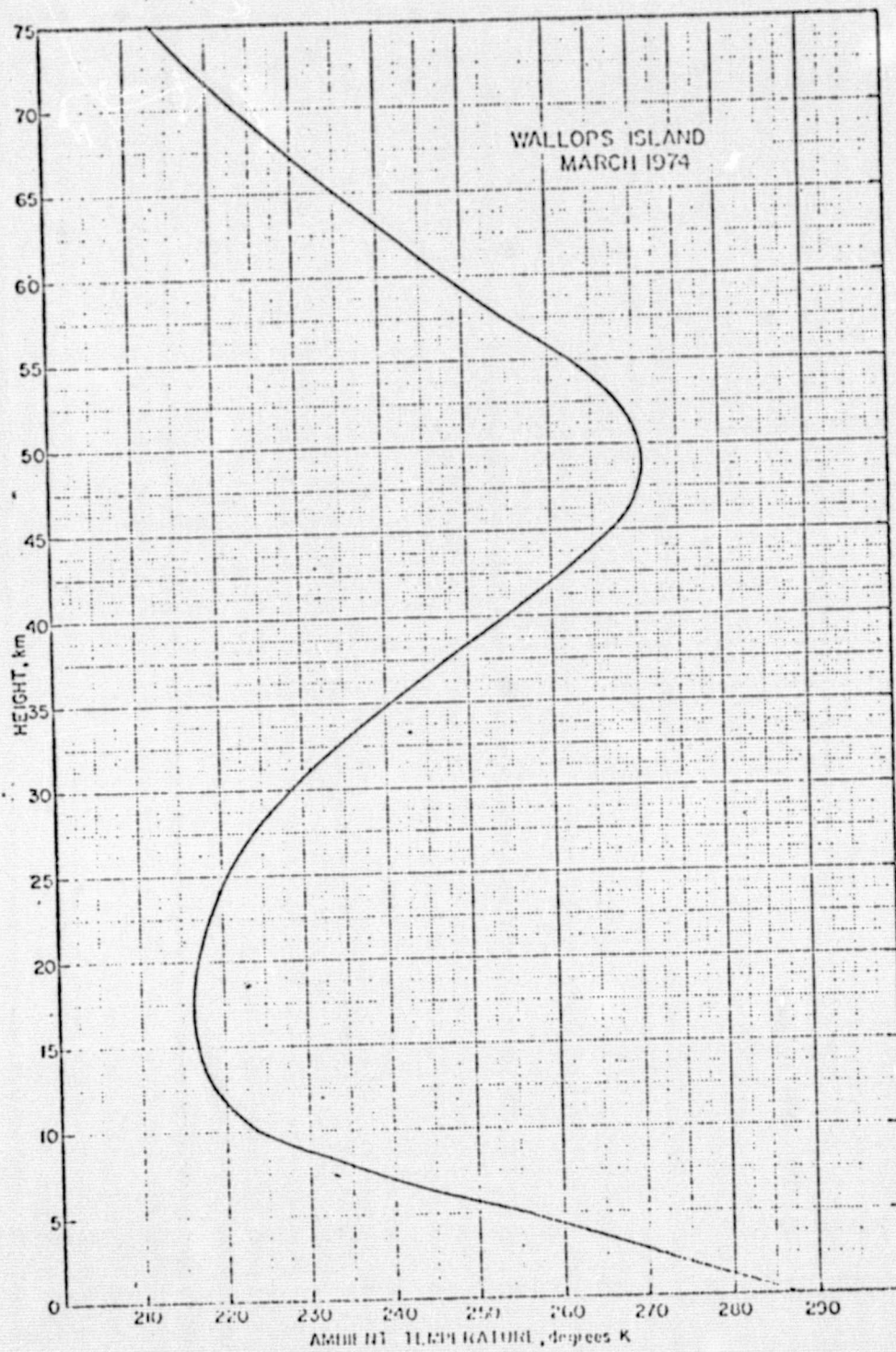


Figure A-1

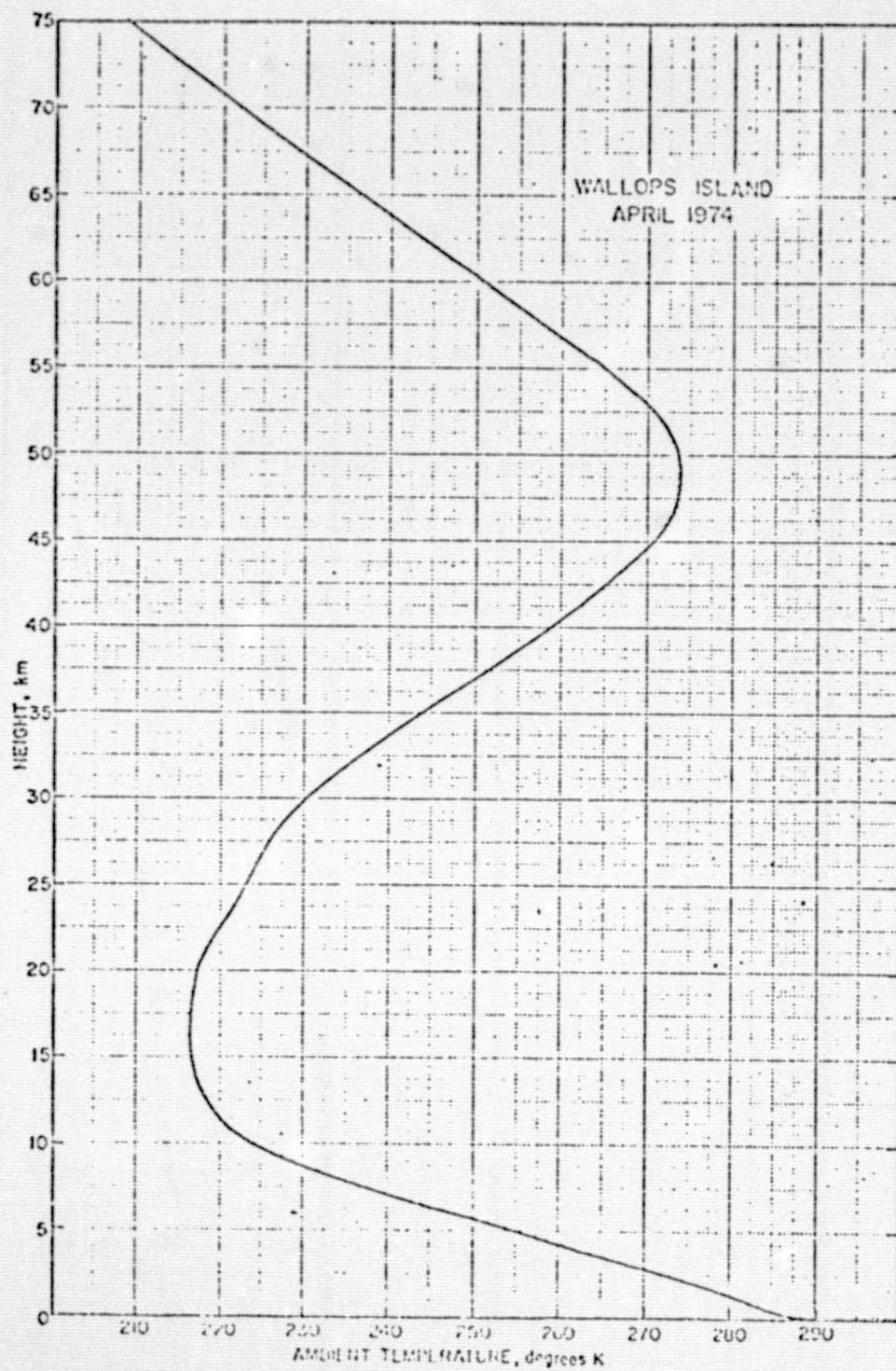


Figure A-2

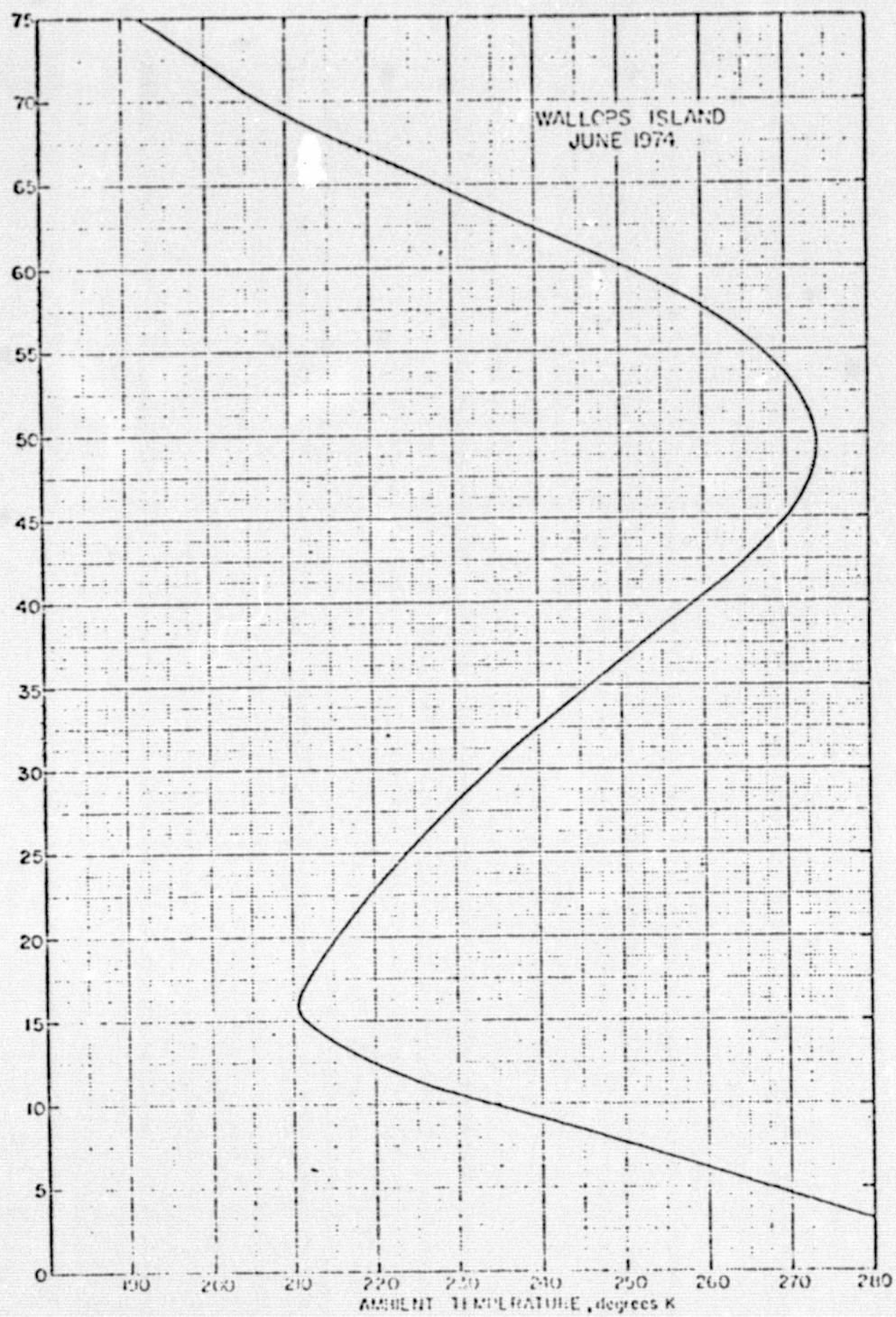


Figure A-3

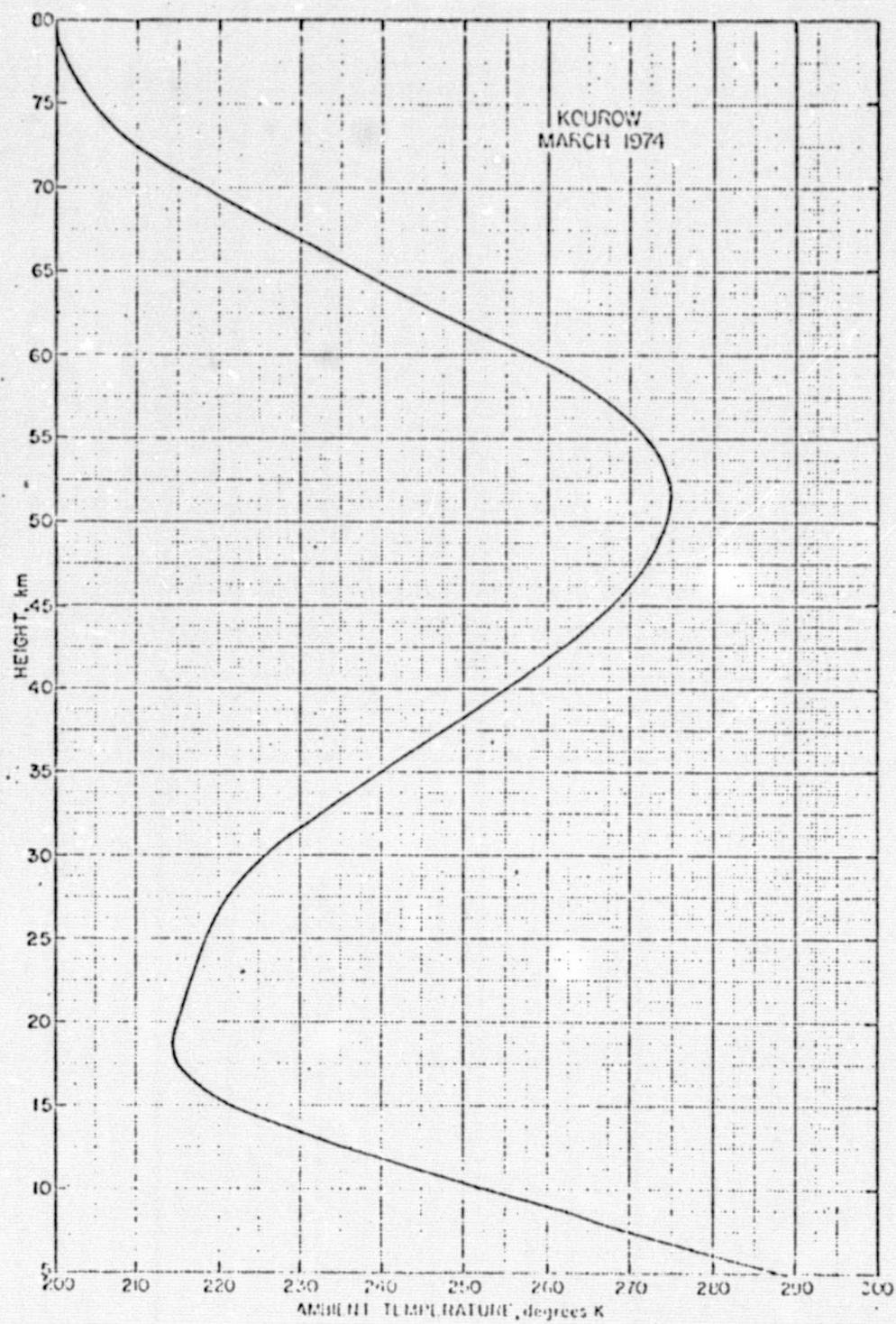


Figure A-4

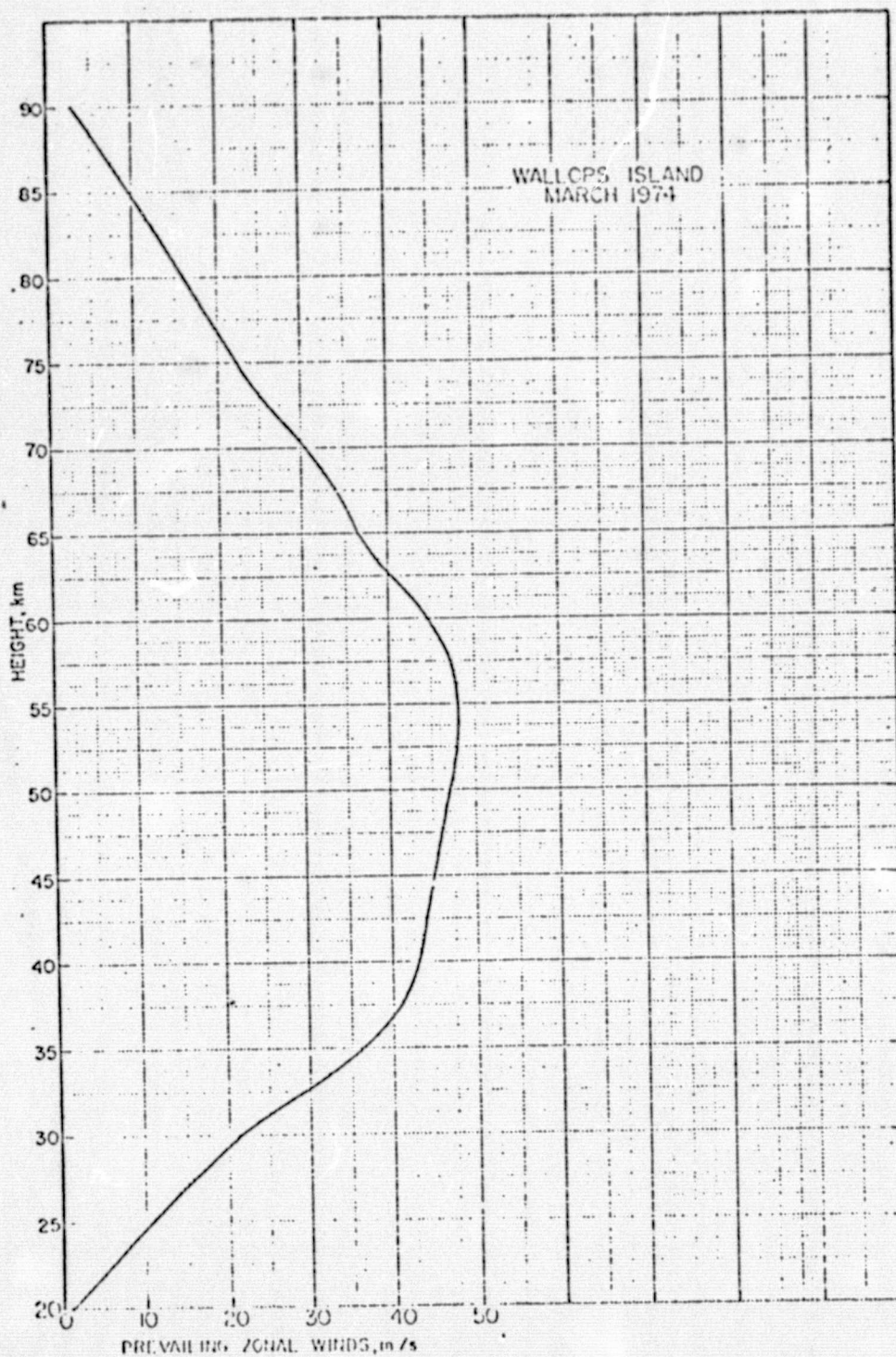


Figure A-5

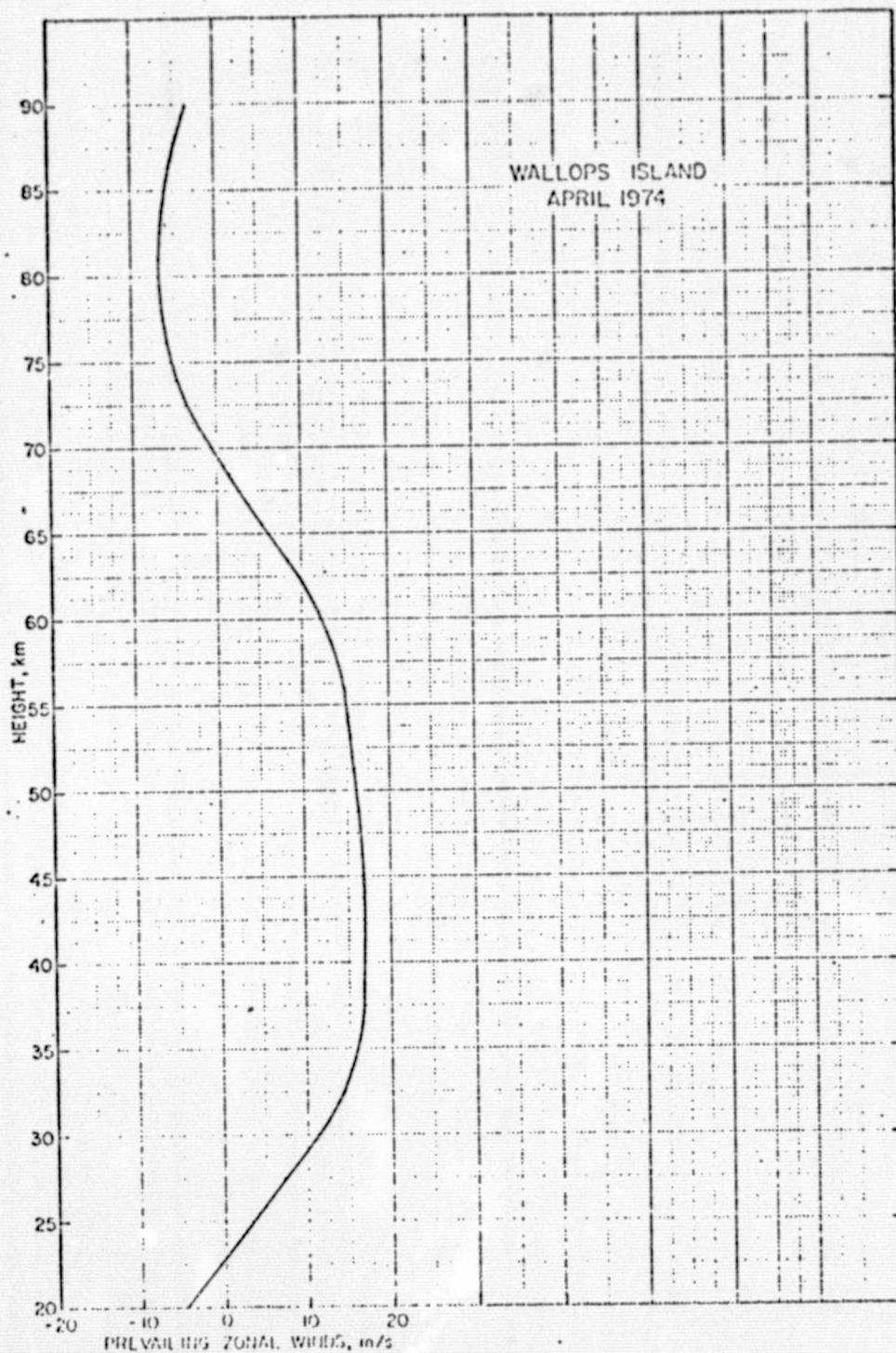


Figure A-6

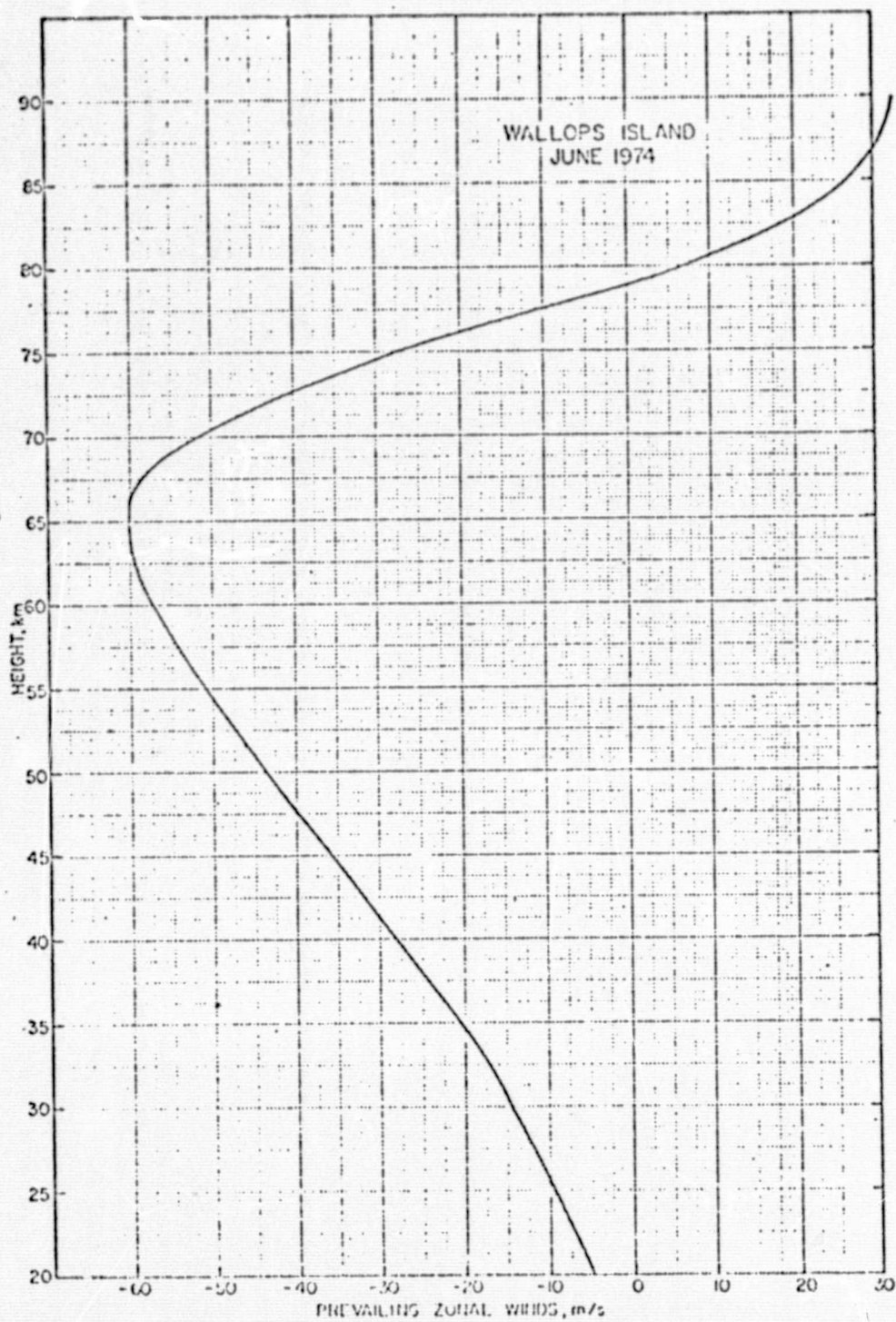


Figure A-7

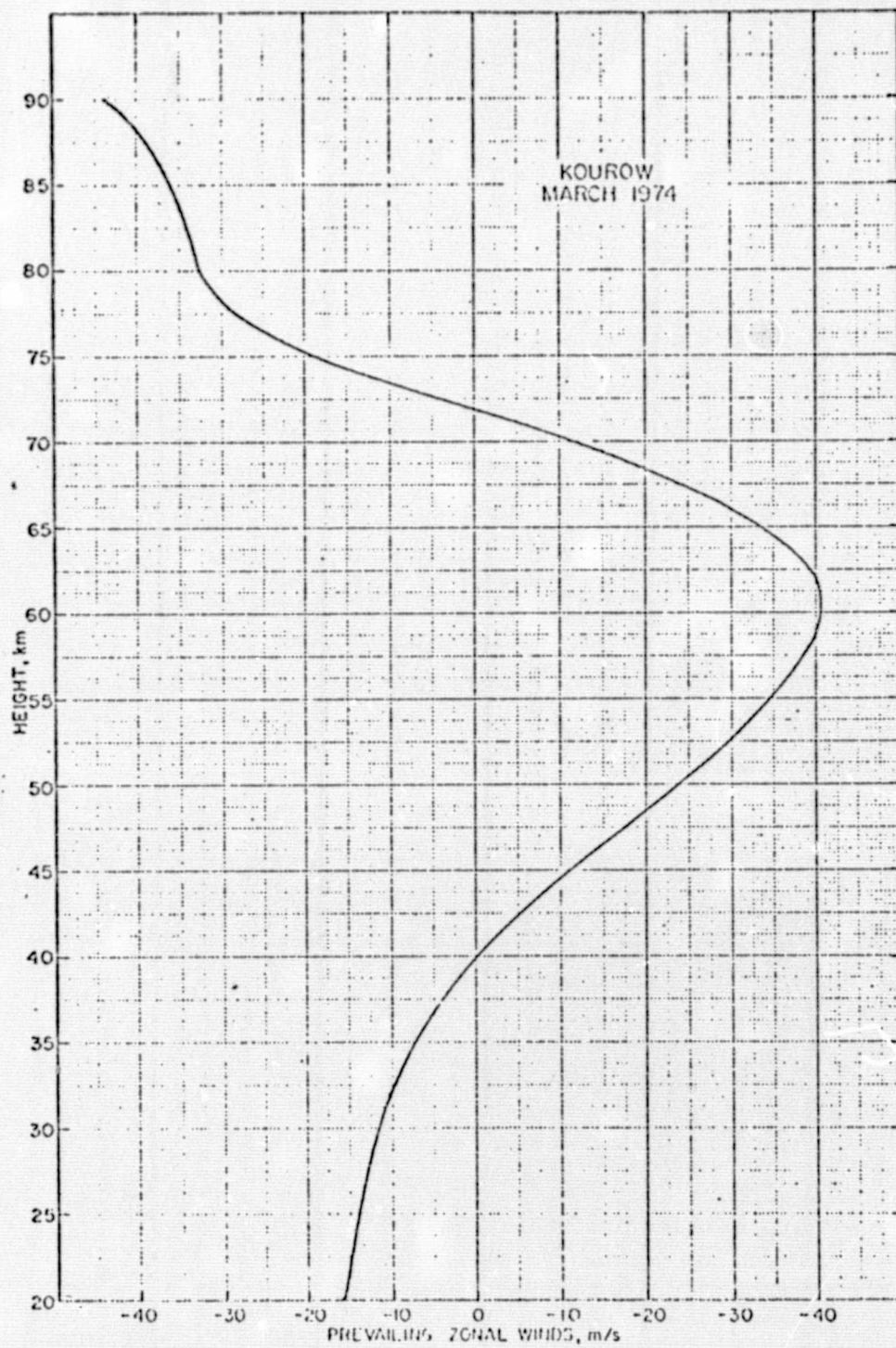


Figure A-8

APPENDIX B

REMOVAL AND ASSESSMENT OF RANDOM ERRORS IN THE WIND AND TEMPERATURE DATA

A. INTRODUCTION AND SUMMARY

A smoothing technique has been applied to the wind and temperature data supplied by the Wallops Flight Center in order to remove random errors. The technique is based on the assumption that, over an altitude interval of 2 kilometers, physically meaningful variations with respect to altitude can be represented by a three-term (quadratic) polynomial. Wavelike variations with wavelengths as short as 5 kilometers are totally unaffected by this technique. At each altitude, the technique yields an improved ("smoothed") value, the probable error associated with the smoothed value, and the random error, defined as the difference between the smoothed and original values. The random errors have been analyzed statistically. In general, the random errors associated with the wind measurements are less than 0.5 meter/second up to an altitude of about 55 kilometers. Above that altitude, the magnitude of the error tends to increase to a value between 3 and 5 meters/second at 80 kilometers. The random errors associated with temperature measurements are generally less than 0.3°C below 60 kilometers and increase to about 0.8°C at 70 kilometers.

The smoothing technique, the analysis of random errors, and estimates of errors obtained by other observers are presented in the following subsections. The computer codes utilized in the smoothing procedure and the determination of random errors is reproduced in subsection 5, along with the necessary documentation.

B. METHOD OF SMOOTHING

The smoothing procedure is first outlined in general form, and the specific form appropriate to the current data is then presented.

Let Y_k = the observed value of a physical quantity Y (e.g. temperature) corresponding to the value z_k of the independent variable z (e.g. altitude)

S_k = the probable error in Y_k (B-1)

$$X_k = \sum_{m=1}^M C_m f_{mk}$$

= a suitable analytic representation of Y ,

where $f_{mk} = f_m(z_k)$, $m = 1, M$

= the value of the expansion function f_m at z_k

C_m = expansion coefficient

M = number of terms in the analytic representation.

Define $D_k = X_k - Y_k$ (B-2)

$$\overline{S^2} = \left(\sum_{k=1}^K S_k^2 \right) / K \quad (B-3)$$

$$W_k = \left(\overline{S^2} / S_k^2 \right) \quad (B-4)$$

$$Q = \left(\sum_{k=1}^K W_k D_k^2 \right) \quad (B-5)$$

where K = number of observations of the quantity Y

The optimum value of the coefficients C_m is obtained by minimizing Q with respect to the variations of the C_m (the least squares procedure):

$$\frac{\partial Q}{\partial C_m} = 0, \quad m = 1, M.$$

The resulting equations can be formulated in terms of the matrix H and the vector U , where:

$$H_{\ell m} = \sum_{k=1}^K f_{\ell k} f_{mk} W_k, \quad U_m = \sum_{k=1}^K f_{mk} Y_k W_k, \quad m, \ell = 1, M$$

Then $U = C \cdot H$ and $C = UH^{-1}$.

$$\text{Explicitly } C_m = \sum_{k=1}^K G_{mk} Y_k W_k \quad (B-6)$$

$$\text{where } G_{mk} = \sum_{\ell=1}^M f_{\ell k} (H^{-1})_{\ell m} \quad m = 1, M, k = 1, K.$$

Corresponding to the errors S_k in the measured quantities Y_k , there are probable errors in the coefficients C_m . It can be shown that these errors are represented by the matrix:

$$\overline{\Delta C_m \Delta C_\ell} = (H^{-1})_{m\ell} \cdot \overline{S^2} \quad (B-7)$$

where the quantity on the left represents the correlation of the probable errors ΔC_m and ΔC_ℓ . Further, the most probable value of Q is given by:

$$Q^* = (K - M) \cdot \overline{S^2}. \quad (B-8)$$

Finally the most probable error in X_k is given by:

$$\overline{\Delta X_k^2} = \sum_{m=1}^M \sum_{\ell=1}^M \overline{\Delta C_m \Delta C_\ell} f_{mk} f_{\ell k} \quad (B-9)$$

$$= \sum_{m=1}^M \sum_{\ell=1}^M (H^{-1})_{m\ell} f_{mk} f_{\ell k} \cdot \overline{S^2}$$

If, as in the present instance, the uncertainties S_k in the measurements Y_k are unknown, a somewhat different procedure must be adopted. For the data under consideration, we assume that the uncertainties in the measurements are equal over the interval involved. Thus:

$$S_k^2 = \overline{S^2}, \quad W_k = 1, \quad k = 1, K \quad (B-10)$$

and using (B-5) and (B-8):

$$\overline{S^2} = \sum_{k=1}^K D_k^2 / (K - M). \quad (B-11)$$

For the present data, the analytic representation consists of a three-term polynomial with respect to the altitude. The smoothing procedure involves nine data points (or equivalently, 2 kilometers) centered at the data point in

question. This choice of smoothing interval allows most of the noise to be removed without affecting wavelike components with wavelengths as short as 5 kilometers. When applied to simulated, error-free data resulting from the superimposition of a 5-kilometer wave and an 11-kilometer wave, the method introduced less than 1 percent error. Figure B-1 illustrates the effect of smoothing on a data sample with particularly severe random errors. Note the negative correlation between the eastward and northward components.

C. ANALYSIS OF RANDOM ERRORS

The random errors established during the smoothing procedure have been analyzed statistically. They are presented graphically in Figures B-2 through B-5. Figure B-2 pertains to the random errors in the temperature data, plotted separately for each series as well as cumulatively for all the data from Wallops Island. In each instance the plotted quantity is a 2-kilometer average of the root-mean-square value of the random error D as defined by equation (B-2). Owing to the nature of the temperature measurement, the random errors are small, as expected. Below 60 kilometers the errors are generally less than 0.3°C , although the Kourou data gives errors closer to 0.5°C . Above 60 kilometers the error increases to a value of about 1°C at 70 kilometers.

Figure B-3 displays the random errors for the cumulative wind data from Wallops Island. Below 55 kilometers the errors vary between 0.5 and 1.0 meter/second for both the eastward and northward wind components. Above 55 kilometers, errors increase to a value of about 5 meters/second at 80 kilometers. The dashed curves represent the uncertainty in the smoothed wind values. These are generally about half as big as the random error at the same altitude. The apparently oscillatory nature of the random error in the eastward component above 55 kilometers is not due to any physical process. It is, rather, a fortuitous result of the superposition of all results from Wallops Island. This can be seen more clearly in Figure B-4, which displays the correlated error, as defined as:

$$D(E) \cdot D(N)$$

where $D(E)$ = Random error in eastward wind,

$D(N)$ = Random error in northward wind.

Figure B-4 reveals that above 55 km the eastward and northward errors are strongly correlated, possibly owing to the experimental difficulty of accurately establishing a direction of observation at high altitudes.

Figure B-5 is analogous to Figure B-3 but pertains to the data from Kourou.

In summary, it can be said that random temperature errors are insignificant below about 65 kilometers, whereas random wind errors are small below about 55 kilometers. It should be noted that the technique utilized in the current study does not detect systematic errors.

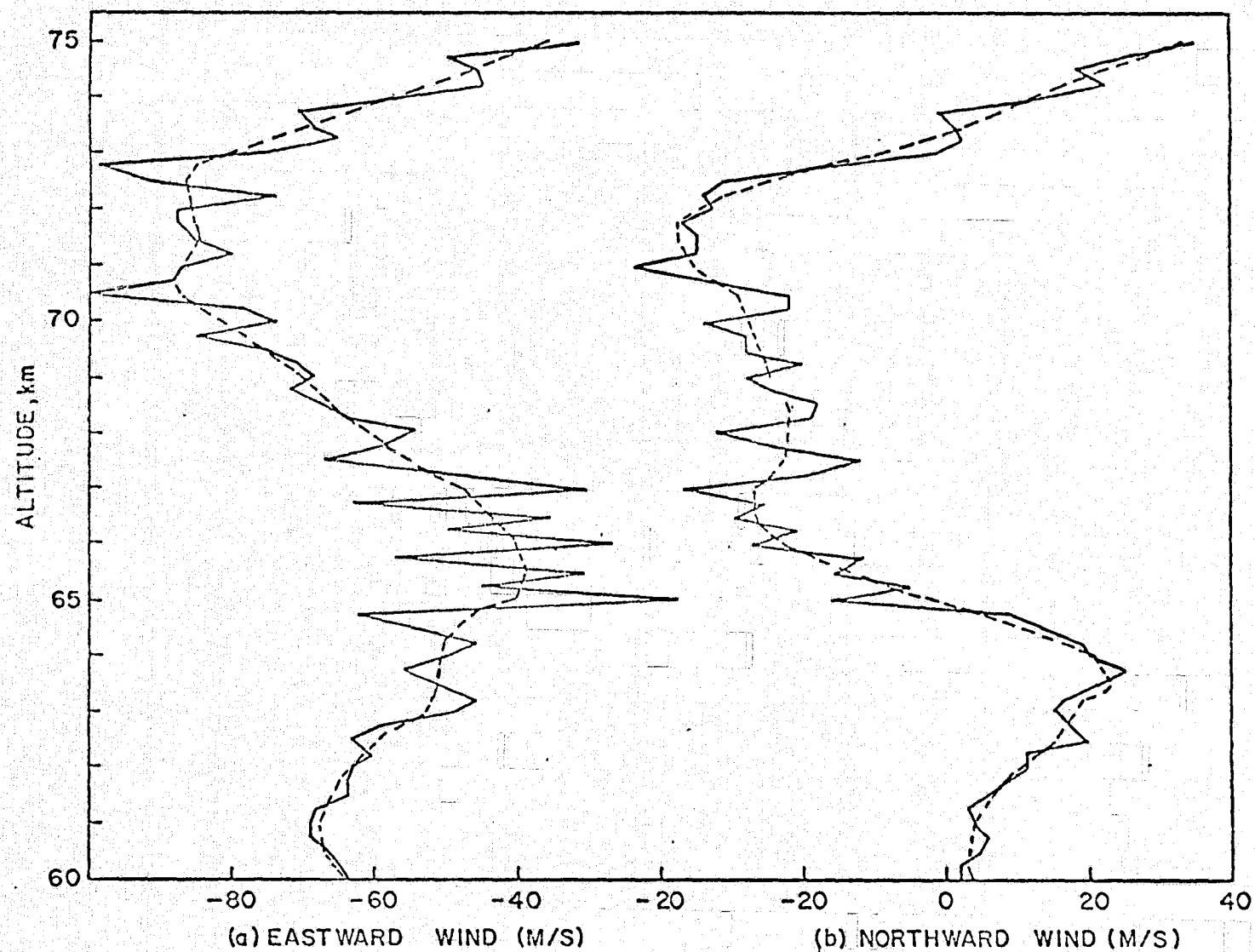


Figure B-1. Altitude Profiles of the Original and Smoothed Wind Components Measured at Wallops Island (23 20 UT, June 29, 1974). Solid Curves are Original Data. Dashed Curves are Smoothed Data.

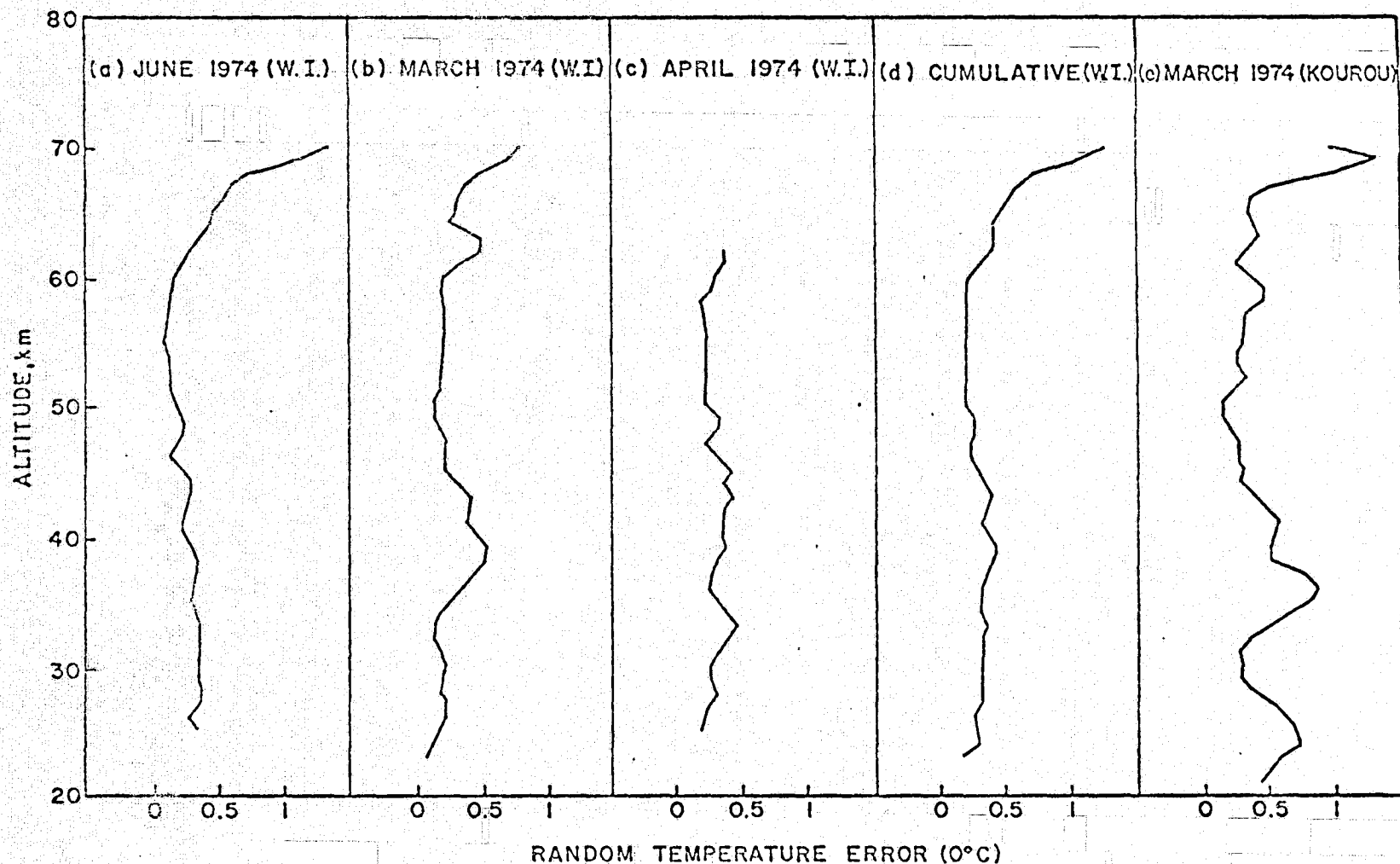


Figure B-2. Random Error in the Temperature Data as a Function of Altitude. (a) For the June 1974 Series at Wallops Island. (b) For the March 1974 Series at Wallops Island. (c) For the April 1974 Series at Wallops Island. (d) Cumulatively for all the Series at Wallops Island. (e) For the March 1974 Series at Kourou.

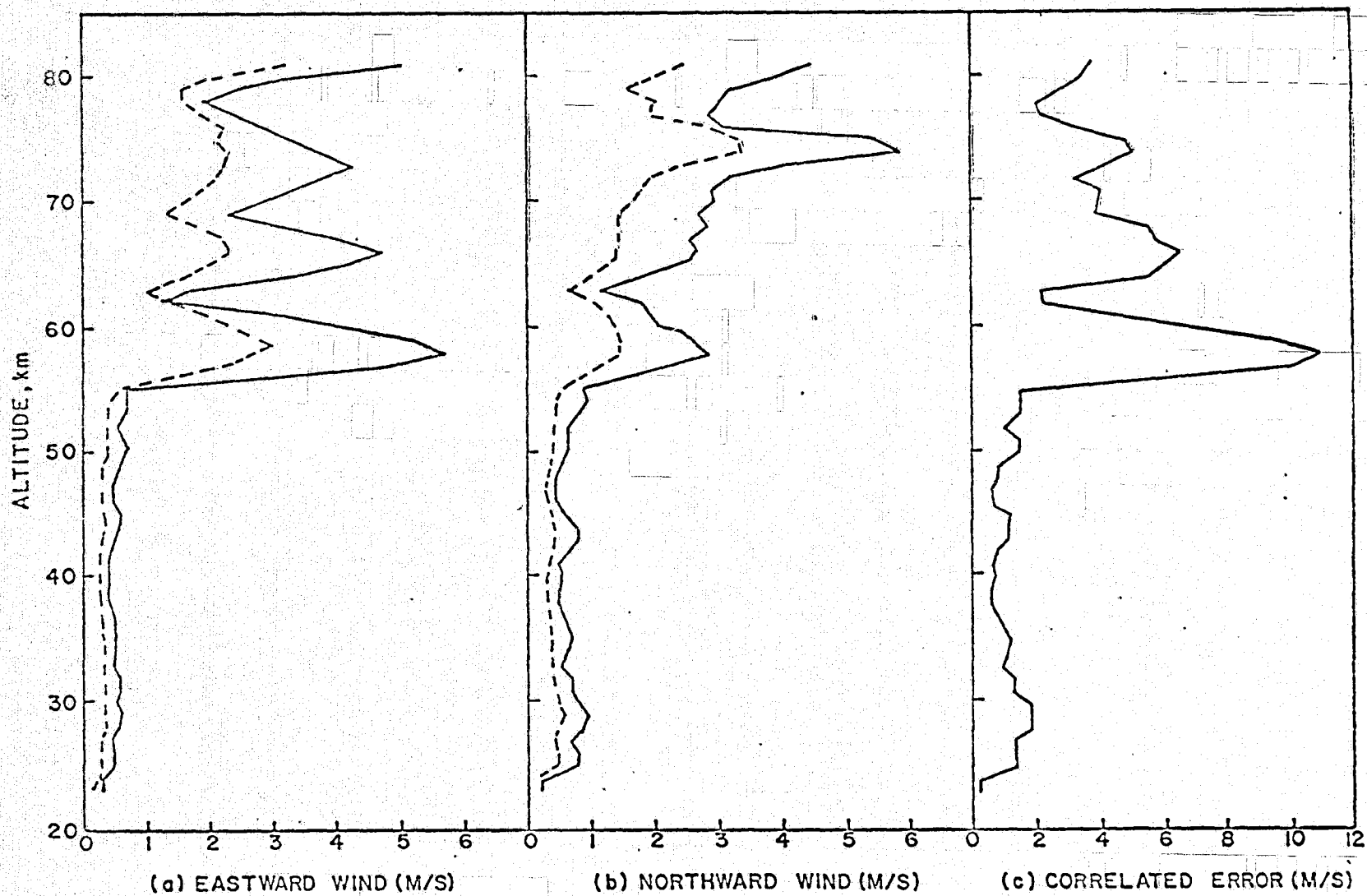


Figure B-3. Random Error in the Wind Data From Wallops Island as a Function of Altitude. (a) For the Eastward Component. (b) For the Northward Component. (c) For the Correlated Error. Solid Curves Depict Error in Original Data. Dashed Curves Depict Uncertainty Associated with Smoothed Data.

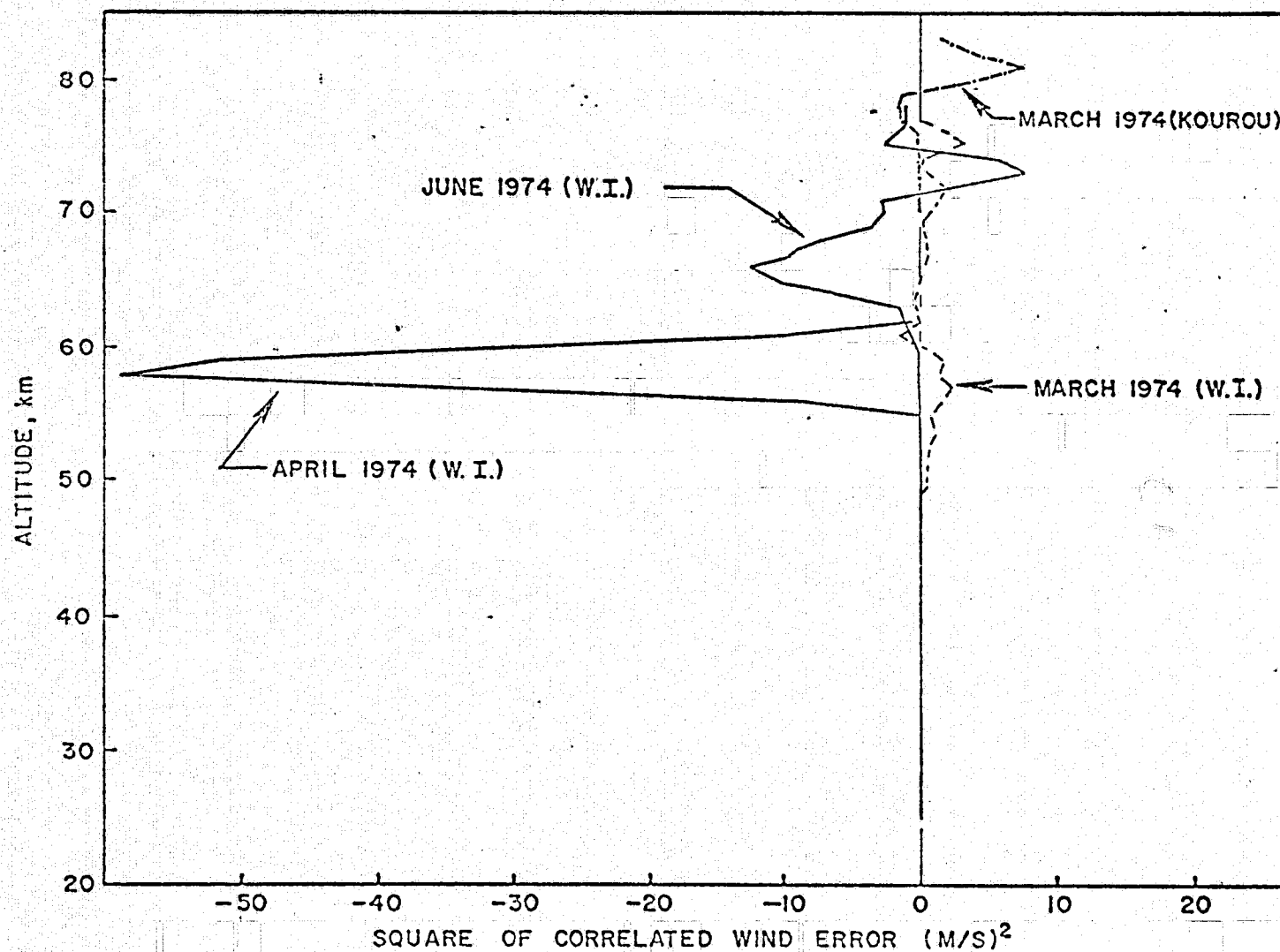


Figure B-4. The Square of the Correlated Wind Error as a Function of Altitude, Plotted Separately for Each Series (see text for definition). Curves Have Been Drawn Only Where the Error Differs Appreciably from Zero.

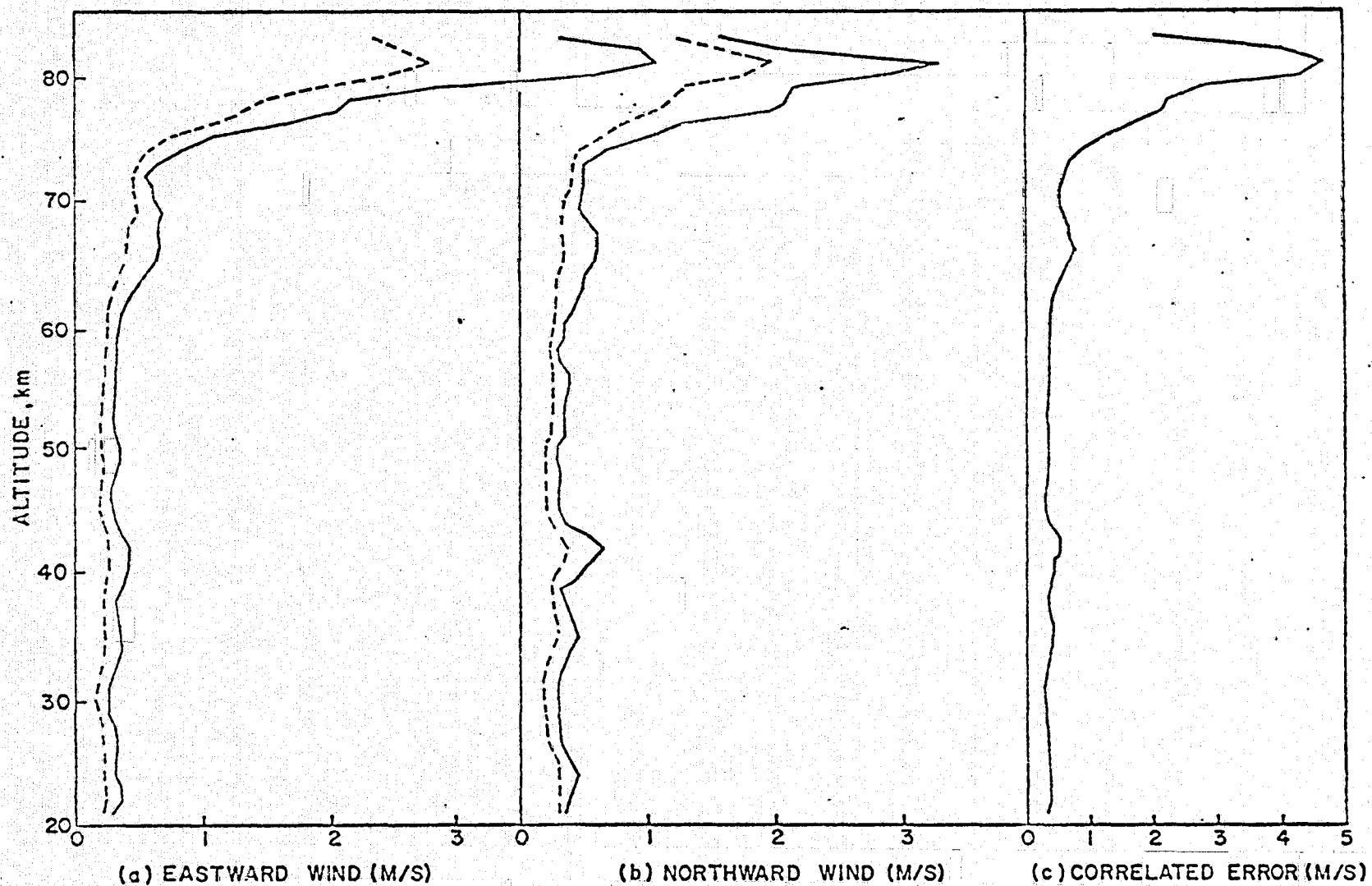


Figure B-5. Random Error in the Wind Data from Kourou as a Function of Altitude.

D. OTHER ERROR ESTIMATES

Some of the estimates of error associated with standard Met rocket measurements of wind and temperature are listed in Table B-1. The general estimates were determined by the authors from the various factors involved in the measurements. The observed differences were obtained from comparison of two or more measurements. Susko and Vaughan, tracked Jimsphere wind sensors with two different FPS-16 radars and compared the results. Miller, Wolf, and Finger, compared the results of simultaneous temperature measurements made with a U.S. and a Japanese sensor. Miller and Shmidlin compared the data obtained from pairs of identical rocket systems launched 5 minutes apart, along the same trajectory. Finally, the uncertainty produced by the angular error of the tracking radar is estimated for data taken at altitude increments of 250 meters.

E. THE COMPUTER CODES USED FOR SMOOTHING THE DATA

This subsection contains a listing of the computer codes utilized to obtain smoothed values of the wind and temperature data, as well as to calculate the random errors in the original data. The smoothing procedure utilizes two subroutines, SMOOTH and MISCELZ, together with a calling sequence. MISCELZ is called once to establish parameters common to the smoothing of all of the quantities of interest. Subroutine SMOOTH is called successively to accomplish various tasks, as indicated by the calling sequence. Each subroutine is listed below, together with explanations identifying program variables with the mathematical quantities defined in subsection 2 this appendix.

F. SUBROUTINE MISCELZ

Subroutine MISCELZ is listed in Table B-2. This subroutine defines the four sets of parameters required to carry out the smoothing procedure. The program parameters and their relation to symbols previously defined in subsection 2 are as follows:

- HI is the inverse of the matrix H as defined in the equation preceding (B-6)
- F(M,N) are the analytic (polynomial) expansion functions defined in (B-1)
- G(M,N) are the auxiliary functions defined by (B-6) and the equation that follows it
- ESQ(K) is the quantity on the right side of (B-9) without the S^2 .

G. SUBROUTINE SMOOTH

Subroutine SMOOTH is listed in Table B-3. This subroutine is designed to produce smoothed values of the two wind components and the temperature. It was found desirable to do a double smoothing of the original data. In

Table B-1. REPORTED RANDOM ERRORS

Windspeed m/s Temperature °C		References	
<u>General Estimates</u>			
3 - 5		Smith, J. of Meteor., 17, 296, 1960	
± 2.5		Lenhand, J.G.R. 68, 227, 1962	
± 3	±2	Beyers & Miers, JAS, 25, 155, 1968	
<u>Observed Differences</u>			
± 0.4		Susko & Vaughan, NASA TMX53752, 1968	
	± 2	Miller, Woolf, & Finger, J. Appl. Meteor., 7, 390, 1968	
± 3	± 1	Miller & Schmidlin, J. Appl. Meteor., 10, 320, 1971	
<u>Radar Precision FPS-16</u>			
Alt. km	Angular Error mils	Fallrate m/s	Wind Speed error m/s
60	0.1	100	3.4
20	0.1	10	0.1

Table B-2.

```

SUBROUTINE MISCEL2
C   CONTAINS MISCELLANEOUS DATA
COMMON/MISC2/HI(3,3),F(3,9),G(3,9),ESQ(9)
DATA HI/ 2.5541E-01, 0., -3.4632E-01,
1      0., 2.6667E-01, 0.,
2      -3.4632E-01, 0., 8.3117E-01/
C+++++
DO 140 N=1,9
  F(1,N)=1.0
  DO 110 M=2,3
110  F(M,N)=(0.25*(N-5))**(M-1)
  DO 130 N=1,3
    CC=0.0
    DO 120 L=1,3
120  CC=CC+HI(L,N)*F(L,N)
130  G(N,N)=CC
140  CONTINUE
C-----
DO 185 J=1,9
  CC=0.0
  DO 184 N=1,3
    DO 184 M=1,3
184  CC=CC+HI(M,N)*F(M,J)*F(N,J)
185  ESQ(J)=CC
  RETURN
END

```

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Table B-3.

```

SUBROUTINE SMOOTH (I,ITER,NMIN,NMAX,FA,FB,ES,DS,F1)
C
C   GIVEN FA(N), I=NMIN,NMAX AT 0.25 KM INTERVALS
C   SMOOTHS FA USING THREE-TERM POLYNOMIAL FIT OVER TWO KILOMETERS
C   (9 POINTS). F1=SMOOTHED VALUES OF FA
C   FOR ITER=2 CALCULATES PROBABLE ERRORS ES,ABSOLUTE ERRORS DS
C   FOR I=3 AND ITER=2,3 CALCULATES F1=DERIVATIVE OF FA
C.....
COMMON/DATAS/VVT(3,361),SM(3,361),ESH(3,361),DSH(3,361),DIM(361)
1      ,D2I(361)
COMMON/DISC2/F1(3,3),F(3,9),C(3,9),ESQ(9)
DIMENSION FA(361),FB(361),ES (361),DS(361),F1(361),C(3)
C
C   INITIALIZE OUTPUT QUANTITIES
LO 190 N=NMIN,NMAX
FB(N)=999.0
ES(N)=999.0
DS(N)=999.0
F1(N)=999.0
190 CONTINUE
C+++++
JMAX=9
JMIG=5
DHF=JMAX-3
JGO=1
JRMJL=1
JRM7X=JMAX/2+1
KLO=I+IN-1
C=====

```

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Table B-3 (continued).

```

C      SMC01: FIND POINTS AT A TIME
200  NLO=NLO+1
      NHI=NLO+JMAX-1
      IF (NHI.LT.JMAX) GO TO 210
      JNHI=JMAX/2+1
      JNMAX=JMAX
      JGC=3
C      FIND EXPANSION COEFFICIENTS
210  DO 230 N=1,3
      CC=0.0
      DO 220 JN=1,JMAX
        N=NLO+JN-1
220  CC=CC+G(N,JN)*FA(N)
230  C(N)=CC
      IF (ITER.NE.2) GO TO 251
C      CALCULATE ERRORS
      SUM=0.0
      DO 250 JN=1,JMAX
        N=NLO+JN-1
        CC=0.0
        DO 240 M=1,3
240  CC=CC+C(M)*F(M,JN)
        CC=CC-VVT(I,N)
250  SUM=SUM+CC**2
251  CONTINUE
      GO TO (270,260,270),JGC
C      JGC=2 (MIDDLE INTERVALS)
260  N=NLO+JMAX/2
      FE(N)=C(1)
      F1(N)=C(2)
      IF (ITER.NE.2) GO TO 200
      DS(N)=C(1)-VVT(I,N)
      ES(N)=SQRT(SUM*ESQ(JHI)/JNHI)
      GO TO 200
C      JGC=1,3 (FIRST, LAST INTERVALS)
270  DO 285 JN=JNHI,JNMAX
        N=NLO+JN-1
        CC=0.0
        DO 280 M=1,3
280  CC=CC+C(M)*F(M,JN)
        FB(N)=CC
        F1(N)=C(2)+2.0*C(3)*F(2,JN)
        IF (ITER.NE.2) GO TO 285
        DS(N)=CC-VVT(I,N)
        ES(N)=SQRT(SUM*ESQ(JN)/JN)
285  CONTINUE
      IF (JGC.EQ.3) GO TO 290
      JGC=2
      GO TO 200
290  CONTINUE
      RETURN
      END

```

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other words, the smoothed values obtained on the first iteration were used as the input data to obtain a new set of smoothed values. Errors are defined as the difference between the original data and the doubly smoothed data. For temperature data, the first and second derivatives with respect to altitude were evaluated as well. Initial values for the first derivative were obtained during the second smoothing of the temperature. These initial values were then smoothed to obtain the final set of first derivatives and the initial values of the second derivative. Finally the second derivative of the temperature was smoothed.

The subroutine arguments have the following significance:

I = index specifying quantity being smoothed

= 1 for eastward wind

= 2 for northward wind

= 3 for temperature

ITER = Index specifying iteration number

= 1 for first smoothing

= 2,3 for subsequent smoothings and derivative calculations

= Limits on height indices. These are determined by the availability of data subject to the restriction that a 0.25 kilometer grid is assumed.

FA = Vector containing values to be smoothed

FB = Vector containing smoothed values

DS = Vector containing differences between smoothed values and original data (as defined by (B-2))

ES = Vector containing probable error in smoothed data (as defined by equation (B-9))

F1 = Vector containing derivative of smoothed quantity.

Within the main body of the subroutine the index JGO serves to select the procedure to be used at the beginning of the altitude range (JGO = 1), the end (JGO = 3) and the middle (JGO = 2). This distinction is necessary because smoothed values at each altitude are obtained by considering the four altitudes that precede it and the four altitudes that follow it, except at the beginning and end of the altitude range.

1. Calling Sequence

The calling sequence used to accomplish the tasks described previously is evident from the listing provided in Table B-4. It is assumed that it is preceded by CALL MISCELZ which establishes the necessary parameters, as well as by the definition of the index I which serves to define the quantity to be smoothed. The variables that appear in the listing have the following significance.

VVT (I,N) is a matrix that contains the eastward and northward winds and the temperature.

SM (I,N) = Matrix containing final set of smoothed values for quantity I.

ESM (I,N) = Matrix containing probable error in smoothed value

DSM (I,N) = Matrix containing random error in original data

DIM (N) = Vector containing first temperature derivative

D2M (M) = Vector containing second temperature derivative

Table B-4. Calling Sequence

```

C   DEFINE QUANTITY TO BE SMOOTHED
DO 110 N=NMIN,NMAX
110 FS(N)=VVT(I,N)
C   FIRST SMOOTHING OF QUANTITY I
ITER=1
CALL SMOOTH (I,ITER,NMIN,NMAX,FS,FR,ES,DS,F1)
C   SECOND SMOOTHING OF QUANTITY I (WITH PROBABLE ERROR ES,ABSOLUTE
C   DIFFERENCE DS. FIRST DERIVATIVE F1 FOR I=3)
ITER=2
CALL SMOOTH (I,ITER,NMIN,NMAX,FR,FS,ES,DS,F1)
C   SAVE CALCULATED QUANTITIES
DO 120 N=NMIN,NMAX
    SM(I,N)=FS(N)
    ESM(I,N)=ES(N)
120 DSM(I,N)=DS(N)
    IF (I.LT.3) GO TO 140
C   FOR TEMPERATURE (I=3) ONLY
C   SMOOTH FIRST TEMPERATURE DERIVATIVE. FIND SECOND DERIVATIVE
ITER=3
CALL SMOOTH (I,ITER,NMIN,NMAX,F1,D1F,ES,DS,FR)
C   SMOOTH SECOND TEMPERATURE DERIVATIVE
ITER=4
CALL SMOOTH (I,ITER,NMIN,NMAX,FR,D2F,ES,DS,F1)
140 CONTINUE

```

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APPENDIX C

COMPUTER CODES FOR DETERMINATIONS OF TIDAL MODES

This program resolves short-wavelength modes from vertical wind profiles. The following subroutines are required:

1. RPARV - Reads parameters for vertical analysis.
2. RFILEV - Reads data from smoothed vertical profile.
3. RPREV - Reads prevailing wind, temperature and temperature derivatives.
4. ZFUNS - Defines height as a function of height index.
5. VFUNS - Calculates expansion functions for vertical analysis.
6. VFilter - Extracts short-wavelength modes.

```

      PROGRAM TMODES(INPUT,OUTPUT,TAPE1=INPUT,TAPE2=OUTPUT,TAPE11,
1          TAPE12)
C
C      RESOLVES SHORT-WAVELENGTH MODES FROM VERTICAL PROFILES
C      REQUIRES RPARV,RFILEV,VFUNS,ZFUN,ARRAY,MINV,PHASE
C
000003      COMMON/IDDEV/NIT,NOT,MTAPE,LTAPE,PI
000003      COMMON/HEADS/ISLR,L,LSMAX,TCB(7),FLAT,FLONG,TLOC(14),NHR(14),
1          MPI(14),NDAY(14),NMO(14),NVE(14),NFI(14),NTP(14),VEHICLE(14)
000003      COMMON/DATAS/VVT(3,260),SM(3,260),ESM(3,260),DSM(3,260),D:M(260)
1          ,D2M(260),NMN(3,14),NMX(3,14)
000003      COMMON/PREVS/PREV(3,260),NMIN,NMAX
000003      COMMON/PARV/KPOL,ZB,SCALE,KHR,NH(3),MH(3),HMN(3),NOPT,HINT
000003      COMMON/CODES/IFPRNT,IFTAPE
000003      COMMON/FKNS/FKN(7,260),AL(2,260),XI(260)
C
000003      NIT=1
000004      NOT=2
000005      MTAPE=11
000006      LTAPE=12
000007      PI=3.14159265
C      READ PARAMETERS FOR VERTICAL ANALYSIS
000011      CALL RPARV
000012      WRITE (NOT,100)
000016      100 FORMAT (1H1/* PARAMETERS FOR VERTICAL ANALYSIS*/)
000016      WRITE (NOT,110) KPOL,ZB,SCALE
000030      110 FORMAT (13,* POLYNOMIAL TERMS, ZB=*,F5.2,* SCALE=*,F4.2)
000030      WRITE (NOT,120) KHR
000036      120 FORMAT (7I3,* HARMONICS*/// * K N M      HMN*)
000036      DO 130 K=1,KHR
000040      130 WRITE (NOT,140) K,NH(K),MH(K),HMN(K)
000056      140 FORMAT (3I3,F8.4)
000056      WRITE (NOT,150) NOPT,HINT
000065      150 FORMAT (13,*=OPTION, HINT=*,F5.2)
C-----
C      READ PRINT/TAPE CODES
000065      READ (NIT,140) IFPRNT,IFTAPE
C-----
C      READ FILE NUMBER LIMITS
000075      READ (NIT,140) LMIN,LMAX,LSMAX
C-----
000107      CALL RPREV
000110      DO 160 N=1,260
000112      160 SM(3,N)=PREV(3,N)-273.0
000122      L=1
000123      NMN(3,L)=1
000125      NMX(3,L)=260
000127      TLOC(L)=0.0
C      DEFINE EXPANSION FUNCTIONS
000130      CALL VFUN
000131      DO 800 LL=LMIN,LMAX
C      READ DATA
000133      CALL RFILEV
000134      CALL VFILTER
000135      800 CONTINUE
000140      END

```

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SUBROUTINE RPARV
C READ PARAMETERS FOR VERTICAL ANALYSIS
C 24 MAY 75
000002 COMMON/IODEV/NIT,NOT,MTAPE,LTAPE,PI
000002 COMMON/PARV/KPOL,ZB,SCALE,KHR,NH(3),MH(3),HMN(3),NOPT,HINT
C READ NUMBER OF POLYNOMIAL COEFFICIENTS
000002 READ (NIT,100) KPOL,ZB,SCALE
000014 100 FORMAT (I3,2F8.4)
C READ HARMONIC FUNCTION DATA
000014 READ (NIT,120) KHR
000022 DO 110 K=1,KHR
000024 110 READ (NIT,120) NH(K),MH(K),HMN(K)
000043 120 FORMAT (2I3,F8.4)
C READ OPTION (NOPT=-1 FOR FIXED FITTING INTERVAL HINT)
000043 READ (NIT,100) NOPT,HINT
000052 RETURN
000053 END

```

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SUBROUTINE RFILEV
C READS DATA FROM SMOOTHED VERTICAL FILE
C 23 MAY 75
000002 COMMON/I(DEV/NIT,NOT,MTAPE,LTAPE,PI
000002 COMMON/HEADS/ISER,L,LSMAX,TCS(7),FLAT,FLONG,TLOC(14),NHR(14),
1 IMI(14),NDAY(14),NMO(14),NVE(14),NFI(14),NTP(14),VEHICLE(14)
000002 COMMON/DATAS/VVT(3,260),SM(3,260),ESM(3,260),DSM(3,260),D1M(260)
1 ,D2M(260),NMIN(3,14),NMAX(3,14)
C
000002 READ (MTAPE,210) ISER,L,LSMAX,TCS,FLAT,FLONG,TLOC(L),NHR(L),
1 IMI(L),NDAY(L),NMO(L),NVE(L),NFI(L),NTP(L),VEHICLE(L)
000055 210 FORMAT (3I3,7A10/2F11.6,F9.4,7I3,A9)
000055 DO 240 I=1,3
000057 READ (MTAPE,220) NMIN,NMAX
000066 220 FORMAT (2I3)
000066 IMIN(I,L)=NMIN
000072 IMAX(I,L)=NMAX
000075 READ (MTAPE,230) ( SM(I,N),N=NMIN,NMAX)
000111 READ (MTAPE,230) (ESM(I,N),N=NMIN,NMAX)
000125 READ (MTAPE,230) (VVT(I,N),N=NMIN,NMAX)
000141 IF (I.NE.3) GO TO 240
000143 READ (MTAPE,230) (D1M(N),N=NMIN,NMAX)
000156 READ (MTAPE,230) (D2M(N),N=NMIN,NMAX)
000171 230 FORMAT (10F8.2)
000171 240 CONTINUE
000173 READ (MTAPE,130) A
000201 130 FORMAT(A1)
000201 IF (EOF,MTAPE) 160,140
000204 140 WRITE (NOT,150) L
000212 150 FORMAT (/* EOF NOT THERE AFTER FILE*,I3/)
000212 160 RETURN
000213 END

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SUBROUTINE HPREV
C READS PREVAILING WIND, TEMPERATURE AND TEMPERATURE DERIVATIVES
000002 COMMON/IODEV/NIT,NGT,MTAPE,LTAPE,PI
000002 COMMON/HEADS/ISER,L,LSMAX,TCB(7),FLAT,FLONG,TLOC(14),NHR(14),
1 NHI(14),NDAY(14),HMO(14),NYL(14),NFI(14),NTP(14),VEHICLE(14)
000002 COMMON/DATAS/VVT(3,260),SM(3,260),ESM(3,260),DSM(3,260),D1M(260)
1 ,D2M(260),NMIN(3,14),NMAX(3,14)
000002 COMMON/PREV5/PREV(3,260),NMIN,NMAX
000002 DIMENSION DUMMY(80)
000002 NMIN=1
000003 NMAX=260
000004 READ (NIT,100) ISER
000012 100 FGRFA1 (13)
000012 DO 150 I=1,3,2
000014 READ (NIT,100)
000017 IF (I.EQ.3) GO TO 120
000021 READ (NIT,110) (PREV(I,N),N=NMIN,NMAX),BUNK
000036 110 FORMAT (10F8.2)
000036 GO TO 150
000037 120 READ (NIT,110) DUMMY,(PREV(I,N),N=NMIN,NMAX),BUNK
000056 READ (NIT,110) DUMMY,(D1M(N),N=NMIN,NMAX),BUNK
000074 READ (NIT,110) DUMMY,(D2M(N),N=NMIN,NMAX),BUNK
000112 150 CONTINUE
000114 RETURN
000115 END

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		FUNCTION ZFUN(N)
	C	DEFINES HEIGHT AS A FUNCTION OF HEIGHT-INDEX
	C	ZFUN(1)=20.00 KM, ZFUN(2)=20.25 KM, ..., ZFUN(281)=90.00 KM
000003		ZFUN=19.75+0.25*N
000007		RETURN
000007		END

ORIGINAL PAGE 4
OF 127 PAGES

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SUBROUTINE VFUNS
C   CALCULATE EXPANSION FUNCTIONS FOR VERTICAL ANALYSIS
COMMON/IODEFV/NIT,NGT,MTAPE,LTAPE,PI
COMMON/HEADS/ISER,L,LSMAX,TCB(7),FLAT,FLONG,TLOC(14),NHR(14),
1   HKI(14),HDAY(14),NMU(14),NVE(14),NFI(14),NTP(14),VEHICLE(14)
COMMON/DATA5/VVT(3,260),SM(3,260),ESM(3,260),DSM(3,260),D1M(260)
1   ,D2M(260),NMN(3,14),NMX(3,14)
COMMON/PARV/KPOL,Z5,SCALE,KHR,NH(3),MH(3),HMN(3),NOPT,HINT
COMMON/CODES/IFPRINT,IFTAPE
COMMON/FKNS/FK1(7,260),AL(2,260),XI(260)
DIMENSION ALP(3),QXN(3),QXH(3),QXR(3),WAVE(3)

C
000002   NMN=NMN(3,L)
000006   NMX=NMX(3,L)
000011   IF (NMN.EQ.NMX) GO TO 500
C   DEFINE POLYNOMIAL FUNCTIONS
000012   DO 100 N=NMN,NMX
000014   FK1(1,N)=1.0
000017   Z=SCALE*(ZFUN(N)-Z5)
000024   DO 100 K=2,KPOL
000025   100 FK1(K,N)=Z**K-1)
C=====
C   DEFINE HARMONIC FUNCTIONS
000045   CHT=2.932E-02
000046   CGAM=2.0/7.0
000050   DZ=0.25
000051   NMID=101
C   INITIALIZE
000052   DO 110 K=1,KHR
000054   110 AL(K,NMN)=0.0
000062   XI(NMN)=0.0
000064   LINES=60
000065   ITER=1
000066   120 N=NMN-1
000070   130 N=N+1
000072   IF (N.GT.NMX) GO TO 180
000075   IF (N.EQ.NMN) GO TO 150
C   FOR (N.GT.NMN) REDEFINE QUANTITIES
000076   HH=HN
000077   DO 140 K=1,KHR
000101   140 QXH(K)=QXN(K)
000106   150 CONTINUE
C   CALCULATE NEW SCALE HEIGHT AND ITS DERIVATIVES WRT HEIGHT Z
000106   HN=CHT*(SM(3,N)+273.0)
000113   H1=CHT*D1M(N)
000115   H2=CHT*D2M(N)
000117   PH=HN*(H1+CGAM)
C   CALCULATE NEW WAVENUMBERS AND WAVELENGTHS
000122   DO 160 K=1,KHR
000124   CC=-0.25+PH/HMN(K)
000130   160 QXN(K)=SORT(CC)
000136   IF (ITER.EQ.2) GO TO 200
C   FOR FIRST ITERATION, FIND UNSHIFTED XI,PHASES
000140   IF (N.EQ.NMN) GO TO 130
000142   CC=0.5*DZ
000144   XI(N)=XI(N-1)+CC*(1.0/HH+1.0/HN)
000153   DO 170 K=1,KHR

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000155      170 AL(K,N)=AL(K,N-1)+CC*(QXH(K)/HH+QXN(K)/HN)
000177      GO TO 130
000177      180 IF (ITER.EQ.2) GO TO 500
C          SHIFT XI, PHASES AT END OF FIRST ITERATION
000201      XIM=XI(NMID)
000204      DO 185 K=1,KHR
000205      185 ALP(K)=AL(K,NMID)
000215      DO 190 N=NMID,NMAX
000217      XI(N)=XI(N)-XIM
000222      DO 190 K=1,KHR
000223      190 AL(K,N)=ALP(K)-AL(K,N)
000236      ITER=2
000237      GO TO 120

=====
C          SECOND ITERATION ONLY
000237      200 Z=ZFUM(N)
000242      EX=EXP(0.5*XI(N))
000247      PH1=0.5*(PH*H1+H2*HN**2)
000254      DO 210 K=1,KHR
000255      QXR(K)=PH1/(HMN(K)*QXN(K)**2)
000262      CC=EX/SCRT(QXN(K))
000266      CC=1.0
000267      M=KPOL+2*K-1
000272      FKN(M,N)=CC*COS(AL(K,N))
000302      M=M+1
000303      FKN(M,N)=CC*SIN(AL(K,N))
000313      WAVE(K)=2.0*PI*HN/QXN(K)
000320      210 ALP(K)=AL(K,N)/PI
000326      IF (IFPRNT.EQ.1) GO TO 300
000330      IF (LINES.LT.60) GO TO 230
000332      WRITE (NOT,220) ISER,L,TLOC(L)
000345      220 FORMAT (1H1,*SERIES*,I3(* FLIGHT*,I3,* TLOC=*,F7.3/
1          3X,1HN,5X,1HZ,5X,1HH,4X,2HH1,4X,2HX1,2(5X,2HDX,5X,
2          2HQR,3X,4HWAVE,2X,5HPPHASE),2(5X,2HFC,5X,2HFS))
000345      LINES=3
000346      230 MMIN=KPOL+1
000350      MMAX=KPOL+2*KHR
000352      WRITE (NOT,240) N,Z,HN,H1,XI(N),((QXN(K),QXR(K),WAVE(K),ALP(K)),
1          K=1,KHR),(FKN(M,N),M=MMIN,MMAX)
000422      240 FORMAT (1HH,I3,4F6.2,(12F7.3))
000422      LINES=LINES+1
000424      300 CONTINUE
000424      GO TO 130
000425      500 RETURN
000426      END

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SUBROUTINE VFILTER

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C
C   EXTRACTS SHORT WAVELENGTH MODES
000002   COMMON/IODEV/HIT,NOT,NTAPE,LTAPE,PI
000002   COMMON/HEADS/ISER,L,LSMAX,TCB(7),FLAT,FLONG,TLOC(14),NHR(14),
1       NMI(14),NDAY(14),RHO(14),RYE(14),NFI(14),NTP(14),VEHICLE(14)
000002   COMMON/DATAS/VVT(3,260),SM(3,260),ESM(3,260),DSM(3,260),DIM(260)
1       ,D2M(260),HMR(3,14),HMX(3,14)
000002   COMMON/PARV/KPOL,ZB,SCALE,KHR,NH(3),MH(3),HMM(3),NOPT,HINT
000002   COMMON/CODES/IFPRNT,IFTAPE
000002   COMMON/FKNS/FKN(7,260),AL(2,260),XI(260)
000002   COMMON/PREVS/PREV(3,260),NMIN,NMAX
000002   DIMENSION H(10,10),HI(10,10),U(10),C(10),SV(100),AMP(3),PH(3)
000002   DIMENSION YC(81),ER(81)
000002   DIMENSION Y(260),W(260)
000002   DO 400 I=1,3
000004       NMIN=HMM(I,L)
000010       NMAX=HMX(I,L)
000013       IF (NMIN.EQ.NMAX) GO TO 400
000014       IF (I.EQ.2) GO TO 110
000016       CC=0.0
000017       IF (I.EQ.3) CC=273.0
000022       DO 100 N=NMIN,NMAX
000024   100   SM(I,N)=SM(I,N)-PREV(I,N)+CC
000037   110   DO 120 N=NMIN,NMAX
000041       Y(N)=SM(I,N)
000045       IF (ESM(I,N).LT.1.0) ESM(I,N)=1.0
000054       W(N)=1.0/(ESM(I,N)**2)
000062   120   CONTINUE

C
000064   WRITE (NOT,130) ISER,L,I
000076   130   FORMAT (1H1/314,* = SERIES, FLIGHT, QUANTITY*/3X,1HN,7X,1HZ,6X,
1       2HYC,7X,1HY,6X,2HER,5X,3HSUM,6X,2HYP,2(5X,3HAMP,6X,2HHP)/)
000076       JMAX=4*(HINT+0.01)+1
000104       KMAX=KPOL+2*KHR
000106       JNMIN=JMAX-(KMAX)
000110       JGO=1
000111       JNMIN=1
000112       JNMAX=JMAX/2+1
000114       NLO=NMIN-1
000116   200   NLO=NLO+1
000120       NHI=NLO+JMAX-1
000122       IF (NHI.LT.NMAX) GO TO 210
000124       JNMIN=JMAX/2+1
000125       JNMAX=JMAX
000126       JGO=3
000127   210   CONTINUE
C   DEFINE/REDEFINE MATRIX H, VECTOR U
000127   IF (NLO.GT.NMIN) GO TO 250
C   FIRST DEFINITION OF H,U
000133   DO 240 K=1,KMAX
000134       CC=0.0
000135       DO 220 JN=1,JMAX
000137       N=NLO+JN-1
000141   220   CC=CC+W(N)*FKN(K,N)*Y(N)
000153       U(K)=CC
000155       DO 240 M=1,KMAX

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000156      CC=0.0
000157      DO 230 JN=1,JMAX
000161      N=NLO+JN-1
000163      230 CC=CC+W(N)*FKN(K,N)*FKN(M,N)
000177      240 H(K,M)=CC
000210      GO TO 270
000210      250 CONTINUE
C      REDEFINE H,U
000210      NF=NLO-1
000212      NL=NH1
000214      DO 260 K=1,KMAX
000215      U(K)=U(K)+W(NL)*FKN(K,NL)*Y(NL)-W(NF)*FKN(K,NF)*Y(NF)
000235      DO 260 M=1,KMAX
000236      260 H(K,M)=H(K,M)+W(NL)*FKN(K,NL)*FKN(M,NL)-W(NF)*FKN(K,NF)*FKN(M,NF)
000270      270 CONTINUE
C      INVERT MATRIX H
000270      KK=KMAX*KMAX
000272      MODE=2
000273      CALL ARRAY(MODE,KMAX,KMAX,KK,10,10,SV,H)
000303      CALL MINV(SV,KMAX,KK,D)
000306      MODE=1
000307      CALL ARRAY(MODE,KMAX,KMAX,KK,10,10,SV,HI)
C      FIND EXPANSION COEFFICIENTS
000317      DO 290 K=1,KMAX
000321      CC=0.0
000322      DO 280 M=1,KMAX
000324      280 CC=CC+U(M)*HI(M,K)
000336      290 C(K)=CC
C      CALCULATE ERRORS
000342      SUM=0.0
000343      DO 310 JN=1,JMAX
000344      N=NLO+JN-1
000346      CC=0.0
000347      DO 300 K=1,KMAX
000351      300 CC=CC+C(K)*FKN(K,N)
000362      YC(JN)=CC
000365      ER(JN)=CC-Y(N)
000370      310 SUM=SUM+W(N)*ER(JN)**2
000376      SUM=SQRT(SUM/DNM)
000402      GO TO (330,320,330),JGO
000411      320 JNMIN=JNMAX
000413      330 DO 370 JN=JNMIN,JNMAX
000415      N=NLO+JN-1
000417      Z=ZFUI(N)
000422      YP=0.0
000423      DO 340 K=1,KPOL
000424      340 YP=YP+C(K)*FKN(K,N)
000435      DO 350 K=1,KHR
000437      KK=KPOL+2*K-1
000442      AMP(K)=SQRT(C(KK)**2+C(KK+1)**2)
000452      350 PH(K)=PHASE(C(KK+1),C(KK))/PI
000463      YY=YC(JN)+YP
000466      WRITE (NOT,360) N,Z,YC(JN),Y(N),ER(JN),SUM,YP,((AMP(K),PH(K)),K=1
1      ,KHR),YY,Z
000527      360 FORMAT (I4,12F8.2)
000527      370 CONTINUE
000532      IF (JGO.EQ.3) GO TO 400
000534      JGO=2

```

000535
000535
000537
000540

GO TO 200
400 CONTINUE
RETURN
END

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APPENDIX D

HARMONIC ANALYSIS OF WIND DATA

Standard harmonic analysis techniques have been applied to the data from the June 1974 series at Wallops Island. The results are of a preliminary nature, in that the vertical resolution techniques have not been applied. Nevertheless, the preliminary results are sufficient to demonstrate the validity of the elements of tidal theory that have been previously mentioned. Of particular interest in the present context is the fact that the eastward (X) wind component lags the northward (Y) wind component by a quarter of a period. Let the temporal behavior of the wind components be expressed analytically as:

$$X = C_x \cos \omega t + S_x \sin \omega t \quad (D-1)$$

$$Y = C_y \cos \omega t + S_y \sin \omega t.$$

Then it follows from tidal theory that:

$$C_y = K \cdot S_x \quad S_y = -K \cdot C_x, \quad (D-2)$$

where K is a constant determined by the latitude. This result holds for each tidal mode individually.

The correlation implied by Equation (D-2) is evident in Figures D-1 and D-2. Figure 1 displays the coefficients C_x , S_x , C_y , and S_y for the diurnal wind component, as determined by harmonic analysis. It can be seen that the correlation between C_x and $-S_y$, and between C_y and S_x , is good, even though the diurnal component is made up of several modes. In Figure D-2(a), the correlation in the semidiurnal component is even more remarkable. Some degree of correlation is also evident in Figure D-2(b) for the terdiurnal component.

Figure D-3 displays the prevailing winds for the eastward (curve (a)) and northward (curve (c)) wind components. Also plotted at 5 kilometer intervals are values of the eastward prevailing wind for July 1 at the latitude of Wallops Island, as listed in the 1965 Cospar Reference Atmosphere. The general agreement between these values and the prevailing wind deduced by harmonic analysis is good. The significance of the eastward trend (curve (b)), is unclear at this time. The same remark applies to the northward prevailing wind and trend (curves (c) and (d), respectively).

In summary it can be stated that the preliminary results of the harmonic analysis are in general agreement with the results of tidal theory.

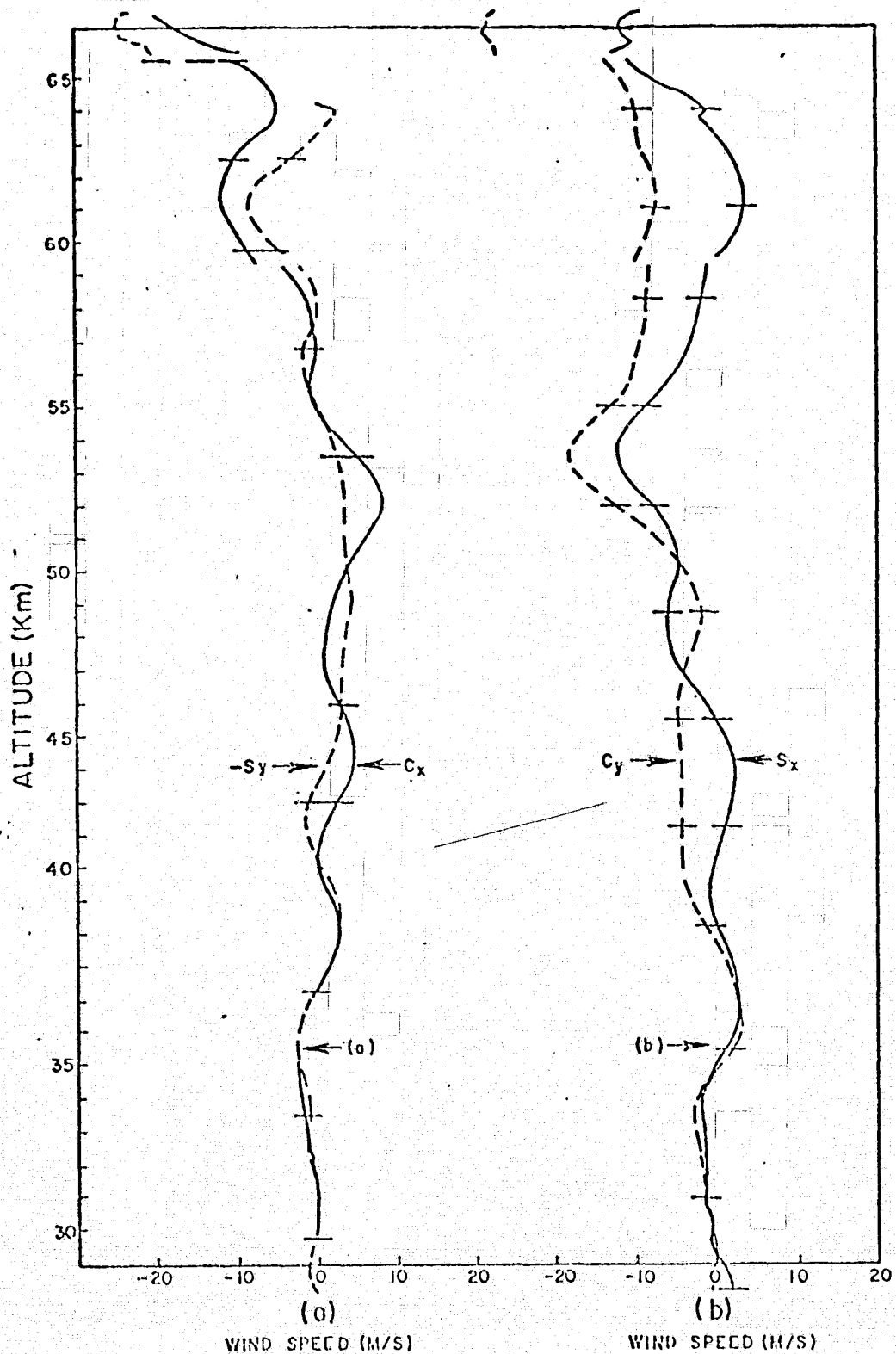


Figure D-1. Harmonic Coefficients for the Diurnal Wind Component.

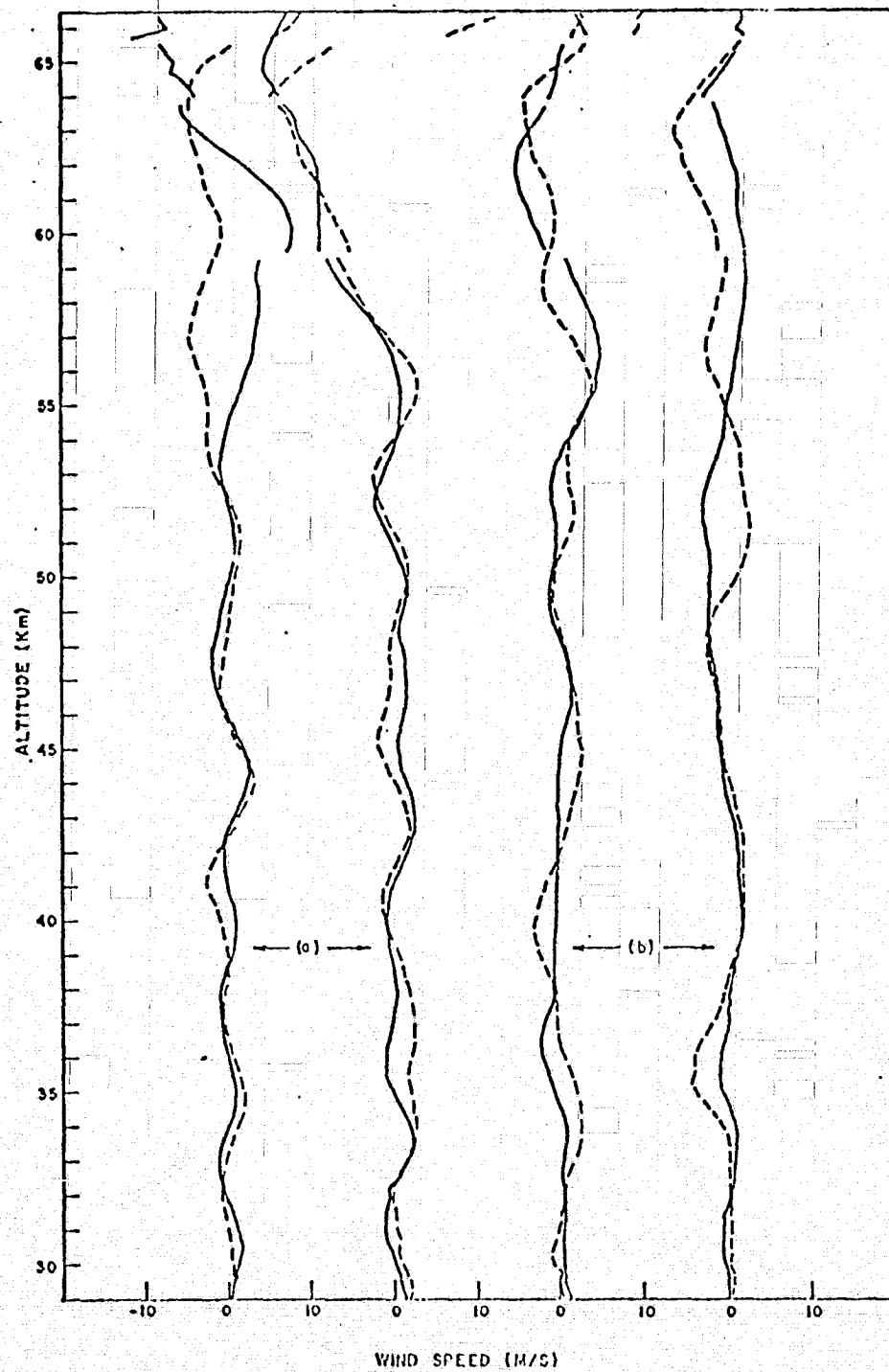


Figure D-2. Harmonic Coefficients for the Semidiurnal and Terdiurnal Wind Components.

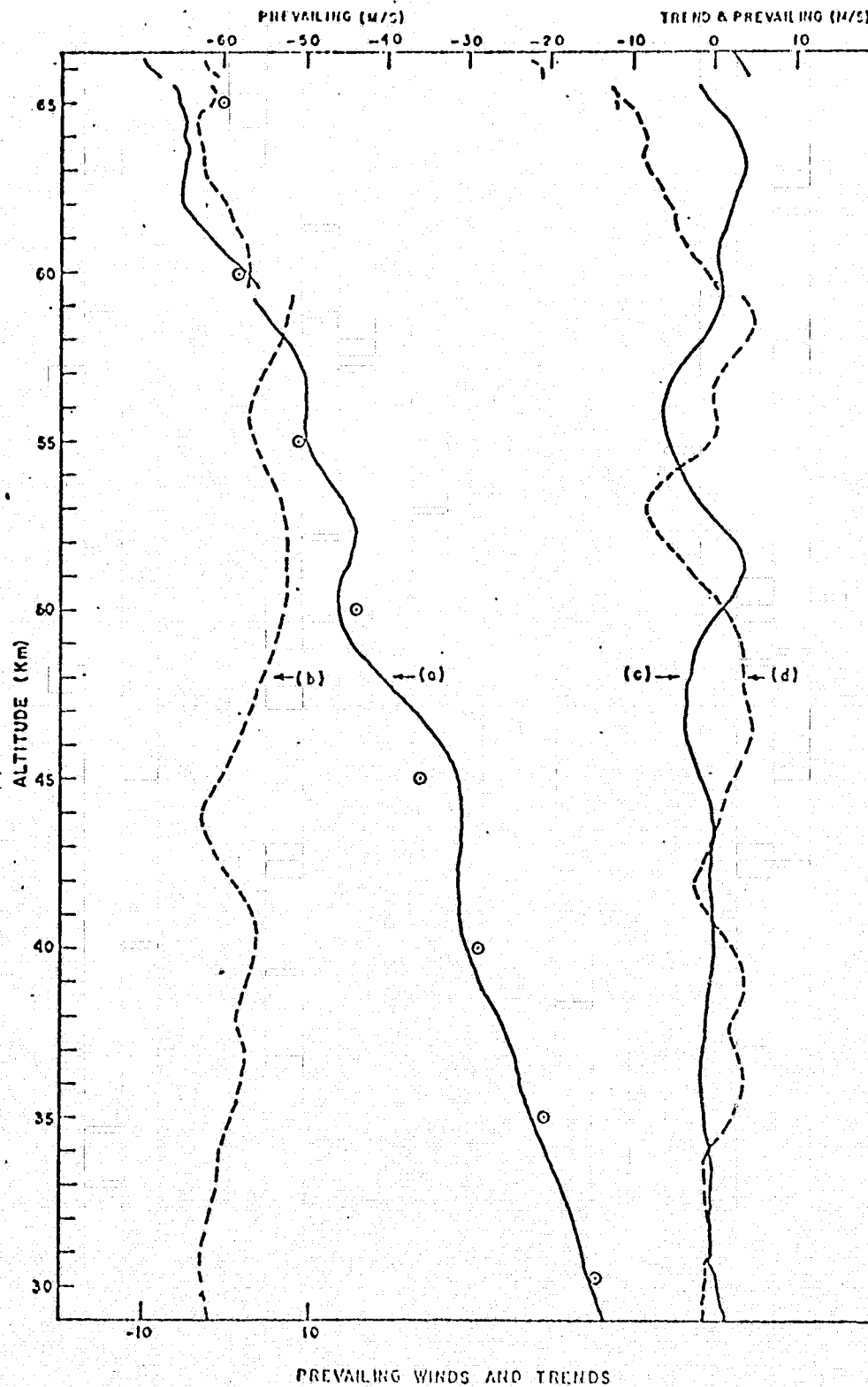


Figure D-3. Coefficients for the Prevailing and Trend Functions for the Eastward and Northward Wind Components.