CREEP-RUPTURE DATA ANALYSIS - ENGINEERING APPLICATION OF REGRESSION TECHNIQUES

by

Donald R. Rummler

A thesis submitted to the Graduate Faculty of
North Carolina State University at Raleigh
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

DEPARTMENT OF MATERIALS ENGINEERING

RALEIGH

1 9 7 6

APPROVED BY:

Chairman of Advisory Committee

ABSTRACT

RUMMLER, DONALD ROBERT. Creep-Rupture Data Analysis - Engineering Application of Regression Techniques (Under the direction of HAYNE PALMOUR III).

The creep and rupture behavior of materials can control the design of structures which operate at elevated temperatures. In lieu of an adequate fundamental understanding, current design practice makes use of a variety of empirical techniques to predict creep behavior.

The results of investigations to apply regression techniques to the development of methodology for creep-rupture data analysis are presented. Regression analysis techniques are applied to the explicit description of the creep behavior of materials for space shuttle thermal protection systems. A regression analysis technique is then compared to five parametric methods for analyzing three simulated and twenty real data sets. Finally, a computer program for the efficient evaluation of creep- rupture data with five parametric methods is presented.

BIOGRAPHY

Donald R. Rummler was born [redacted] in [redacted], [redacted], the son of a master tailor. He received his elementary and secondary education in Cheraw, South Carolina and Belmont, North Carolina, graduating from Belmont Abbey Preparatory School in 1955.

He received his Bachelor of Science degree in Civil Engineering in 1959 and his Master of Science degree in Ceramic Engineering in 1966, both from North Carolina State University at Raleigh.

Since 1959, he has been a member of the technical staff at the National Aeronautics and Space Administration - Langley Research Center. His primary duties during this time have been concerned with the structural application of advanced materials systems to aerospace vehicles.

The author is married to the former Mary Lou [redacted]. They have three children -- Mark, Kathy, and Karen.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to the National Aeronautics and Space Administration, Langley Research Center, Hampton, Virginia, for permitting him to conduct this research as part of his work assignment. The advice and guidance of Professor H. Palmour III, Chairman of his Advisory Committee, has been invaluable in the completion of this study. A special thank you is extended to Professor Emeritus W. W. Kriegel for his continuing interest and counsel. Appreciation is also extended to the other members of the committee, including Professors R. J. Hader, H. H. Stadelmaier, and J. K. Whitfield. Gratitude is expressed to B. A. Stein for many informative discussions.

The author also expresses heartfelt thanks to his parents who stimulated and encouraged his curiosity. Finally, the author expresses his thanks to his wife, Mary, and children, Mark, Kathy, and Karen, for their patience and constant support.

TABLE OF CONTENTS

GENERAL INTRODUCTION

The creep-rupture behavior of materials can and does control the design of many structural components. Designers and analysts in the nuclear power generation, aerospace turbine, and chemical processing industries, for example, are required to design structural components which must operate reliably for periods up to forty years in complex, high temperature environments. Unfortunately, the current state of our understanding of the creep process does not allow the use of "first principles" for sizing components and predicting their service behavior. Consequently, the creep-rupture design techniques used today can at best be called "enlightened empiricism." There is no generally accepted method of analysis for the prediction of creep-rupture behavior. In fact, a method which works well for one material very often will not work well for a different material.

The purpose of the investigations reported herein was to explore the application of regression analysis techniques to the analysis of creep-rupture data of interest in aerospace applications. They constitute a part of a continuing effort, begun in 1970, to provide the materials related methodology necessary to design efficient aerospace vehicles.

The first paper deals with the application of regression analysis to the creep of space shuttle materials. Regression

techniques are used as a tool (1) to assess the effects of sheet thickness and oxygen partial pressure on the steady-state creep behavior, (2) to analytically describe the low creep strain behavior, and (3) to assess the effects of data scatter for materials where data are limited.

The third paper describes the development and use of a computer program for parametric analysis of creep rupture data. The program includes provisions for the analysis of five different parameter methods. Sample problems to aid the user in setting- up a problem are presented.

APPLICATION OF REGRESSION ANALYSIS TO CREEP OF SPACE SHUTTLE MATERIALS[1]

Donald R. Rummler

NASA Langley Research Center
Hampton, Virginia

[1]Published in proceedings of International Conference on Creep and Fatigue in Elevated Temperature Applications, Philadelphia, PA, September 1973 and Sheffield UK, April 1974; Conference publication 13, Institution of Mechanical Engineers.

APPLICATION OF REGRESSION ANALYSIS TO CREEP OF SPACE SHUTTLE MATERIALS

Donald R. Rummler

NASA Langley Research Center
Hampton, Virginia

ABSTRACT

Regression analysis techniques were used to assess the effects of sheet thickness and oxygen partial pressure and to develop constitutive creep equations. Application of prediction intervals is emphasized.

1 SYNOPSIS

Metallic heat shields for Space Shuttle thermal protection systems must operate for many flight cycles at high temperatures in low-pressure air and use thin-gage (≤ 0.65 mm) sheet. Available creep data for thin sheet under those conditions are inadequate. To assess the effects of oxygen partial pressure and sheet thickness on creep behavior and to develop constitutive creep equations for small sets of data, regression techniques are applied and discussed.

2 SYMBOLS

ϵ = creep strain

t = time, hours

th = sheet thickness, mm

T = temperature, K

σ = stress, MN/m^2

x, y, z, D, ϕ = dummy variables

3 INTRODUCTION

Recent Space Shuttle technology research and development studies ((1)* and (2)) have indicated that the creep behavior of high-temperature alloys may control the design and reusability of metallic heat shields for radiative thermal protection systems (TPS). The heat shields function as lightly loaded aerodynamic surfaces, and they must efficiently utilize thin-gage sheet to avoid weight penalties. Loads are applied at high temperature, when the local partial pressure of oxygen is low. In general, creep strains must be limited to less than 0.005 to avoid excessive panel deflections.

The creep data which exist for candidate superalloys are for steady-state creep tests run on relatively thick specimens at atmospheric pressure. These data are presented as time to a given strain level for various combinations of stress and temperature (see, for example, Refs. (3) and (4)). Attempts to use this type of data to predict the cyclic creep deformation of simple tensile specimens or for the preliminary design of heat shields underestimated the experimental creep strains by as much as a factor of 10 ((1) and (2)). These predictions typically utilized one of the parameter methods (5) combined with a life fraction approach to sum the cyclically accumulated strains. This failure to predict the experimental creep strains could be the result of one or both of the following:

(1) The data upon which calculations were based were for the creep of relatively thick specimens at atmospheric pressure, and may not be applicable to thin specimens at low pressure.

*References are given in Appendix 1.

(2) No analytic expression was available which could account for both the nonlinear primary and linear secondary creep stages.

The purpose of this paper is to present the results of an investigation to determine the applicability of regression analysis techniques to predict creep behavior when data are limited. Three applications of regression techniques which address the aforementioned shuttle TPS creep problems are discussed. Regression techniques are used as a tool (1) to assess the effects of sheet thickness and oxygen partial pressure on steady-state creep behavior, (2) to analytically describe the low creep strain behavior, and (3) to assess the effects of data scatter for materials when data are limited.

4 ANALYSIS PROCEDURES

4.1 Development

To evaluate trends in creep data and to predict creep behavior, explicit expressions for the mean and the expected upper and lower bounds for creep strain data as a function of stress, temperature, and time were desired. Little information is available about the form of these expressions for the candidate materials at low levels of creep strain. Consequently, two computer programs were written and applied to develop the desired expressions. Both programs utilize standard linear regression techniques (6). One program was of the form:

$$w = b_o + b_1 \mu \qquad (1)$$

where

$w = \log (\text{stress})$

$\mu = \log (\text{time})$

This program was used to generate coefficients, mean value estimates, and 95 percent prediction intervals* for data at specific values of strain and temperature.

The second program was used to develop models for creep strain as a function stress, temperature, and time. For this multiple regression program, the equation form assumed was:

$$f(y) = g\left\{\left(a_1x_1^2 + b_1x_1 + c_1\right)\left(a_2x_2^2 + b_2x_2 + c_2\right)\left(a_3x_3^2 + b_3x_3 + c_3\right)\right\} \qquad (2)$$

where y, x_1, x_2, and x_3 are, respectively, functions of creep strain, stress, temperature, and time.

Provision for transformation of y, x_1, x_2, and x_3 was included in the program. The transformations, which included many of those found useful for analysis of creep data (7) were as follows:

*The prediction interval (6) is used to make a statement about the anticipated value of the dependent variable (y) for a future single observation at a specific value of the independent variable (x) or variables (x_i, x_j, x_k . . .); for example, y will be between 2 and 6 for 95 percent of all future single observations taken at $x = 3$. The more familiar confidence interval, on the other hand, is used to make statements about the true mean value of y; for example, there is a 95-percent probability that the true mean value of y at $x = 3$ is between 3 and 5. The prediction interval limits are wider since these include both the sampling errors and the uncertainties in estimating the mean value of y.

Transformation Code (TC_j)	Transformation ($0 \leq i \leq 3$)
0	$x_i = z_i$
1	$x_i = \log (z_i)$
2	$x_i = 1/z_i$
3	$x_i = \log (1/z_i)$
4	$x_i = \ln (z_i)$
5	$x_i = (z_i)^{1/2}$
6	$x_i = z_i + 1.0$
7	$x_i = \log (z_i + 1.0)$
8	$x_i = (z_i)^{1/3}$

where the z_i are specific values of stress, temperature, or time. Similar functional transformations ($y = f(D)$) were used for strain. Each transformation combination was assigned a four-digit transformation number where the digits are the transformation code values for y, x_1, x_2, and x_3, respectively.

Thus transformation 1025 used the following transformations:

$$y = \log D = \log (\epsilon)$$

$$x_1 = z_1 = \sigma$$

$$x_2 = 1/z_2 = 1/T$$

$$x_3 = (z_3)^{1/2} = (t)^{1/2}$$

Creep data sets usually include a wide range of times, typically three orders of magnitude, whereas the ranges for creep strain, stress, and temperature are seldom in excess of one order of magnitude. Early analysis of

multiple regression computer runs revealed that the combination of the wide range in the variables associated with creep data sets and equation forms which include terms that can be highly colinear, such as x and x^2, led to ill-conditioned normal equations which were subject to significant round-off errors during a matrix inversion operation. In order to minimize these errors, the data were scaled from 1 to 10 after transformation of the primary variables (y, x_1, x_2, x_3) as follows:

$$y_i = 9.0 \left(y_i - y_{min}\right) \Big/ \left(y_{max} - y_{min}\right) + 1$$

$$x_{ij} = 9.0 \left(x_{ij} - x_{i\ min}\right) \Big/ \left(x_{i\ max} - x_{i\ min}\right) + 1$$

where y_{min} and y_{max} are the minimum and maximum values of the transformed strain. The $x_{i\ min}$ and $x_{i\ max}$ have similar definitions as they apply to the transformed values of stress, temperature, and time.

After transforming and scaling the primary variables, Equation (2) was expanded and new independent variables, defined as follows, were introduced:

$$y = a_1a_2a_3 \left(x_1^2x_2^2x_3^2\right) + a_1a_2b_3 \left(x_1^2x_2^2x_3\right) + \text{----} = \sum_{j=1}^{k} \phi_j z_j \qquad (3)$$

This procedure results in an equation with 27 terms having linear coefficients (ϕ_j).

Some values of ϕ_j were set equal to zero so that, in Equation (3), the order (degree of interaction) for the number of terms in the regression analysis could be reduced as follows:

k	Order	(Allowed term types)
23	4th	$\left(x_l x_m x_n^2 \text{ and } x_l^2 x_m^2\right)$
17	3rd	$\left(x_l x_m x_n \text{ and } x_l^2 x_m\right)$
10	2nd	$\left(x_l x_m\right)$
4	1st	$\left(x_l\right)$

(Note that the reduced form can no longer be factored back to Equation (2).)

4.2 Application

To perform a multiple regression analysis using Equation (3), the order of the equation (k value) was selected first. Next, the transformations to be used on the primary variables were selected. Each observation of the data set was transformed, then scaled. The transformed and scaled values for strain, stress, temperature, and time were then used to generate values for the additional variables in Equation (3). This data set was then used in the regression analysis. The mean values of creep strain were calculated from the coefficients derived during a multiple regression analysis. Explicit functions for the upper and lower bounds (95 percent prediction intervals) were calculated by treating either the upper or lower prediction limit calculated for each observed value of strain during the initial regression as another set of observed strain values; two additional regression analyses provided the desired coefficients. The residual mean square (RMS) for the prediction interval "data" sets were always extremely small ($\approx 10^{-7}$ times that of the original data set analysis). This suggests that the errors involved in these approximations for the original prediction intervals were not large.

After a regression analysis was performed, all variables and residuals were descaled and back-transformed. Several quasi-statistical parameters were then calculated to aid model development and "best-equation" selection. These parameters are described as they are introduced.

5 RESULTS AND DISCUSSION

The following examples illustrate how regression techniques were applied to three areas of creep behavior which are of interest in Space Shuttle TPS creep studies. These areas are typical of those which can occur during the preliminary design phases of any program when extensive creep data are not available.

5.1 Use of Simple Regression (Equation (1))

Haynes alloy H-188 is a cobalt base alloy which has excellent oxidation resistance and moderate elevated temperature strength. It is a candidate material for TPS application up to 1250 K. The creep data base consists primarily of the work reported in (4). This work includes creep tests on H-188 sheet from 10 production heats and for thicknesses ranging from 0.51 to 2.03 mm. All creep tests were run in air at standard pressure.

Figure 1 presents the data at 1144 K at a strain level of 0.002. A regression analysis was performed on the data set with sheet thickness ≤ 0.84 mm. These data will be defined herein as the "standard data," against which data from future observations will be compared. The regression line and the 95-percent prediction interval for the standard data are also shown on the figure. The results shown in Figure 1 allow the following statements to be made:

(1) Ninety-five (95) percent of all future observations made under the same test conditions are expected to fall within the prediction interval for sheet thicknesses between 0.51 and 0.84 mm. If creep data from tests at different test conditions generally fall outside of the prediction interval, then the new test conditions have probably changed the creep behavior of the material.

(2) Most of the data for the > 0.84 mm fall well within the prediction interval for the "standard data." Thus, the $\epsilon = 0.002$ creep strength of Haynes alloy H-188 at 1144 K is not significantly different for sheet thicknesses from 0.51 to 2.03 mm. This is in contrast to the results presented in (4) where creep rupture strengths of sheet $\leq$ 1.27 mm thick were lower than those for sheets >1.27 mm thick.

The prediction interval and mean line from Figure 1 for the "standard data" are shown in Figure 2. Also shown in Figure 2 are the results of creep tests run in another laboratory on thin-gage H-188 at both standard and reduced pressures of air. The focus provided by the prediction interval indicates that the $\epsilon = 0.002$ creep strength of H-188 for sheet thicknesses between 0.51 and 0.64 mm both at standard atmospheric and reduced pressures was not significantly different from that previously established for 0.51 to 0.84 mm sheet at standard atmospheric pressure. However, for thinner sheet (0.254 mm) at reduced pressure creep, strength was significantly higher as indicated by the many test data points (open circles) above the prediction interval. Similar results were observed for other strain levels at 1144 K.

The conclusions drawn from Figure 2 could have been reached with far fewer tests (as few as 2 or 3 for any of the test conditions shown). The use of prediction intervals data appears to be an efficient technique to explore

the effects of "nonstandard" creep conditions and to compare creep data from different sources. This is particularly useful during the preliminary design phases of a program when the consequences of "nonstandard" conditions, such as thin gage or low air pressure, must be assessed rapidly and maximum use of existing data base for thicker material at atmospheric air pressure is necessary.

5.2 Use of Multiple Regression (Equation (3))

To explore the effects of primary creep and various hardening rules, such as strain hardening, on the accumulation of cyclic creep strain, it is useful to have a constitutive relationship for steady-state creep strain. This is particularly true when the data base is limited and does not include a large number of test stresses and temperatures.

The data set (8) for René sheet (solution treated at 1450 K and aged at 1172 K) was selected to demonstrate the application of multiple-regression techniques to develop a constitutive creep equation. Creep tests were conducted at 1005, 1089, and 1172 K. Tests were not replicated. For this study, 142 strain-time data points (observations) with strain levels from 0.0005 to 0.005 were selected as input for the multiple regression analyses.

In addition to a normal regression analysis, the program numerically solved the resulting equation to estimate the time (t_ϵ) required to reach each input strain level. To assure compatibility with a strain-hardening cyclic-creep analysis, all equation forms which did not permit efficient solutions (less than 500 iterations) for all t_ϵ were rejected. The program also rejected all equation forms which calculated either a negative strain or time. Early computer runs revealed that the multiple correlation coefficient square (R^2) and the residual mean squared error (MSE), commonly used (7) to

rapidly evaluate a large number of equation alternatives were poor discriminators for this data set and these variable transformations. The following parameters were determined from the descaled and back-transformed calculated values of strain and time:

EMSE (strain mean squared error)

E/TO (maximum calculated strain at $t = 0.001$ h)

T/EO (maximum calculated time at $\epsilon = 0.000001$)

AE (average strain error)

ATP (average time error, percent)

These parameters have recognizable consequences in the preliminary design sense and were considered useful discriminators for the selection of a "best" equation. Numerous variable transformations were evaluated in a single computer run. Typically, 200 different transformations were examined in a single 600-second computer run.

Analysis of several "best" equations during early computer runs indicated that the equations were often unstable near time = zero. This unstable behavior is illustrated in Figure 3 for typical values of stress and temperature. This failure to predict $\epsilon = 0$ at $t = 0$ was eliminated by assuming an unrecorded data point ($\epsilon = 0.000001$, $t = 0.001$ h) for each creep test reported in (8). These assumed data points were added to the initial data set to yield the 167 data points and were included in all further regressions. The dashed line in Figure 3 shows that a typical predicted creep curve using the additional assumed points is reasonable, although the fit to the original data (open circular symbols) is not as good.

Even with the addition of the assumed data points, none of the variable transformations yielded a satisfactory prediction equation for the $k = 27$

version of Equation (3). The model was unstable when projected on log-stress, log-time plots. At the lowest test temperature (1005 K) and short test times ($\approx$ 10 h) these equation forms began to predict longer times for a particular level of creep strain as the stress was increased. For this particular data set, run 4124 with $k = 23$ produced the "best" model equation. This run produced the lowest values of EMSE, AE, and ATP and computed $E/TO \leq 0.000001$ and $T/EO \leq 0.01$ hr. The use of fewer terms in the model ($k < 23$) significantly increased the EMSE, AE, and ATP values calculated with the original 142 observations. This is illustrated in the following table:

	k = 23	k = 10	k = 7
EMSE ($\times 10^7$	6.71	7.25	9.14
AE ($\times 10^3$)	550	622	730
ATP (0/0)	33	37	74

Thus for this data set, the inclusion of the higher order interaction terms in the model significantly improved the model's ability to fit the data.

The degree of fit typically provided by "best" model equation is illustrated in Figure 4 for $\epsilon = 0.002$. The symbols are the data taken from (8), the solid lines are the mean stress and the 95-percent prediction interval calculated from a regression of log time on log stress using only those data points shown for each temperature. The dashed lines are the mean stress values and the 95-percent prediction intervals calculated by run 4124, $k = 23$ which included all of the 167 data points available in the data set. Agreement between the two calculated mean stress values is considered good. More importantly, however, this figure illustrates that the calculated 95-percent prediction intervals from run 4124, $k = 23$ are consistent with those obtained

from the linear regressions on the data for each temperature. This indicates that the model is probably as good as the data scatter warrant and that the consequences of this scatter can be adequately assessed in a steady-state creep analysis by utilizing the coefficients determined by run 4124 to calculate mean creep strains and the coefficients determined for the lower bounds of the prediction interval shown in Figure 4 to calculate maximum creep strains. For instance, a "best" model equation could be used to calculate creep strains at intermediate values of temperature to compare with other creep data obtained by other investigators.

Figure 5 illustrates some typical mean creep curves calculated with the coefficients determined for the "best" equation. The shapes of these curves are consistent with those obtained by fairing through the original data points. More importantly, the curvilinear nature of the creep curves demonstrate that the model equation applies even when creep strain does not accumulate linearly as a function of time. Therefore, the model is functionally capable of accounting for the effects of primary stage creep in a strain-hardening analysis of cyclic creep.

To further assess the applicability of the regression analysis, the standard deviations for the average percentage time error for strain levels 0.001, 0.0015, and 0.002 were calculated. These standard deviations were compared to similar results obtained from three optimized "C" value Larson-Miller analyses (5) of the data at these strain levels with the following results.

Comparison of Standard Deviation of Percent Time Error

ϵ	Larson-Miller	Run 4124, k = 23
0.001	44.2	20.4
0.0015	46.7	20.8
0.002	36.3	33.5

This comparison suggests that the "best" regression equation, which includes all strain levels, predicts the observed creep behavior at least as well as the family of Larson-Miller curves which would be required to cover a similar range of strain levels.

Multiple regression techniques can also be applied to fit "faired" data to estimate mean values for creep strain. This is illustrated in Figure 6. First, linear regressions of log time on log stress (Eq. (1)) were run on the original data set (8) for each level of strain and temperature. The results of several of these regressions are shown as solid lines in the figure. Next, the mean times to a given level of strain were calculated from the regression equations of the solid lines. Finally, these calculated mean times and the appropriate values of creep strain, stress, and temperature were used as input data for a multiple regression analysis (Eq. (3)). The dashed lines in Figure 6 were calculated from the results of a run 4121, k = 27, using these calculated mean times as input data. The k = 27 version of Equation (3) was not unstable with the "faired" data set, whereas, as noted before, this version was unstable with the "raw" data.

Often creep data are presented in the literature as families of faired curves for specific levels of strain and temperature. No individual creep curves are available for the material of interest.

As can be seen from this example, multiple regression techniques can be used to obtain a single equation which will coalesce families of curves. However, a prediction interval is no longer applicable because the calculations are no longer based on scattered data.

6 CONCLUDING REMARKS

Frequently, creep data are limited during the preliminary design phases of a program such as the design of Space Shuttle thermal protection systems. The examples presented herein illustrate the applicability of regression techniques for (1) evaluating the effects of "nonstandard" creep conditions such as sheet thickness or low oxygen partial pressure on creep behavior and (2) developing analytical expressions to predict creep behavior from limited data. The use of prediction intervals to evaluate the design consequences of the data scatter has been discussed.

7 ACKNOWLEDGMENTS

The author expresses his appreciation to Bland A. Stein for numerous technical discussions and encouragements and to Dick M. Royster for furnishing unpublished creep data.

APPENDIX 1

REFERENCES

(1) Harris, H. G. and Morman, K. N., Jr. 'Creep of Metallic Thermal Protection Systems,' NASA TM X-2273, Vol. II, April 1972. National Aeronautics and Space Administration, Washington, D.C.

(2) Black, W. E. et al. 'Evaluation of Coated Columbium Alloy Heat Shields for Space Shuttle Thermal Protection System Application,' NASA CR-112119, June 1972.

(3) Moon, D. P. et al. 'The Elevated-Temperature Properties of Selected Superalloys,' ASTM Data Series DS 7-S1 (American Society for Testing and Materials, 1968).

(4) Tackett, J. W. 'The Creep Rupture Properties of Haynes Alloy No. 188,' Report No. 8020, Cabot Corporation, Kokomo, Ind., November 4, 1971.

(5) Conway, J. B. 'Stress-Rupture Parameters: Origin, Calculation and Use 1969,' 1st Edition (Gordon and Breach, New York, New York).

(6) Draper, N. R. and Smith, H. 'Applied Regression Analysis 1966' (John Wiley and Sons, Inc., New York, New York).

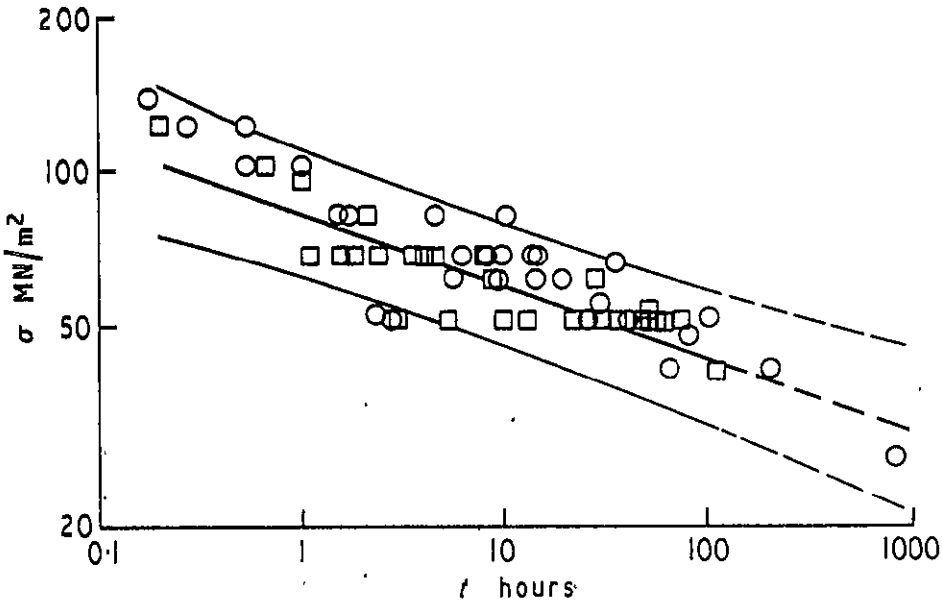
(7) Conway, J. B. 'Numerical Methods for Creep and Rupture Analysis 1967' (Gordon and Breach, New York, New York).

(8) McBride, J. G. et al. 'Creep-Rupture Properties of Six Elevated Temperature Alloys,' WADD-TR-61-99, August 1962, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio.

(9) Royster, Dick M. and Lisagor, W. Barry. 'Effect of High-Temperature Creep and Oxidation on Residual Room-Temperature Properties for Several

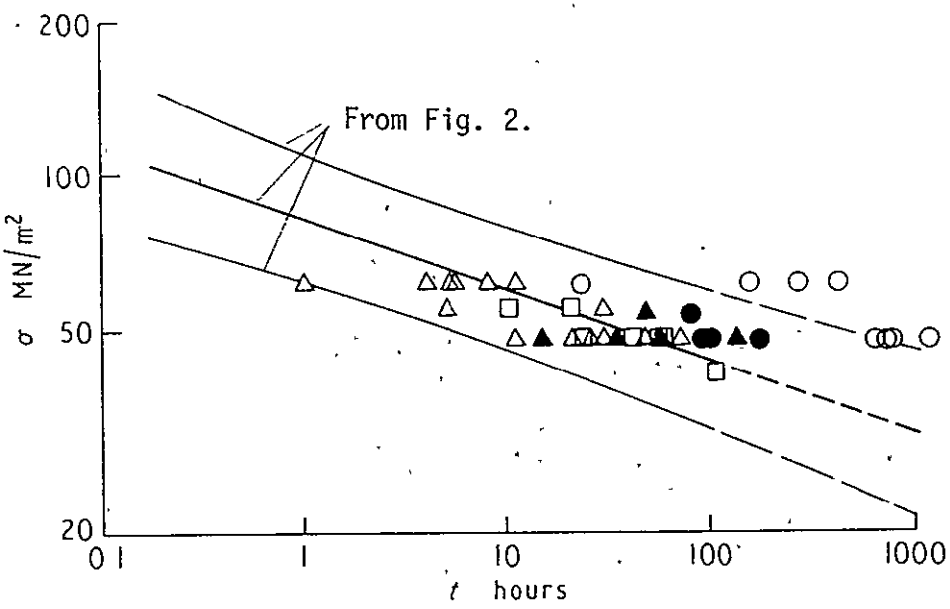
Thin-Sheet Superalloys,' NASA TN D-6893, November 1972, National Aeronautics and Space Administration, Washington, D.C.

(10) Royster, Dick M. Unpublished Data, Langley Research Center, NASA, Hampton, Virginia, December 1972.



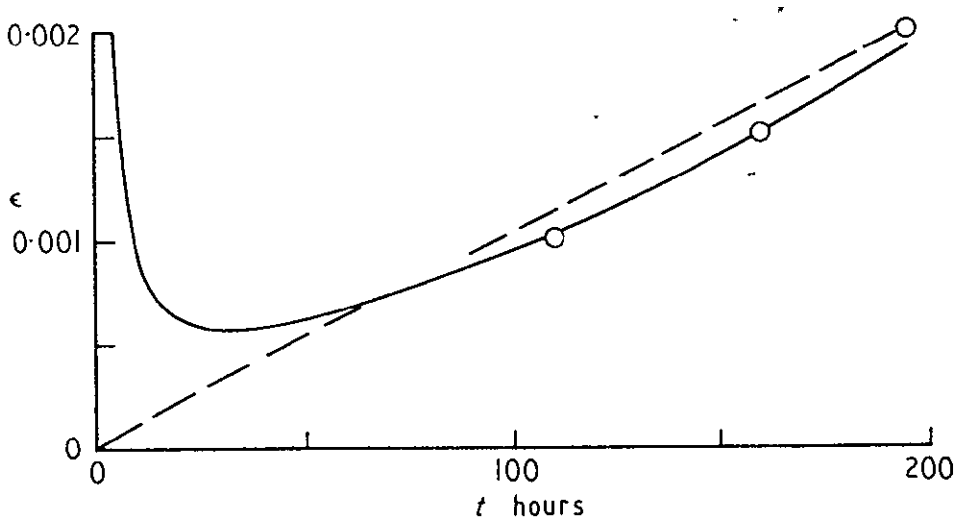
○ th > 0·84 mm (4)
□ th ⩽ 0·84 mm (4)
lines: mean and bounds of 95 per cent prediction interval from linear regression of log t on log σ for th ⩽ 0·84 mm

Fig. 1. Creep strength of Haynes alloy H-188 at 1144K, test pressure = 101 KPa,



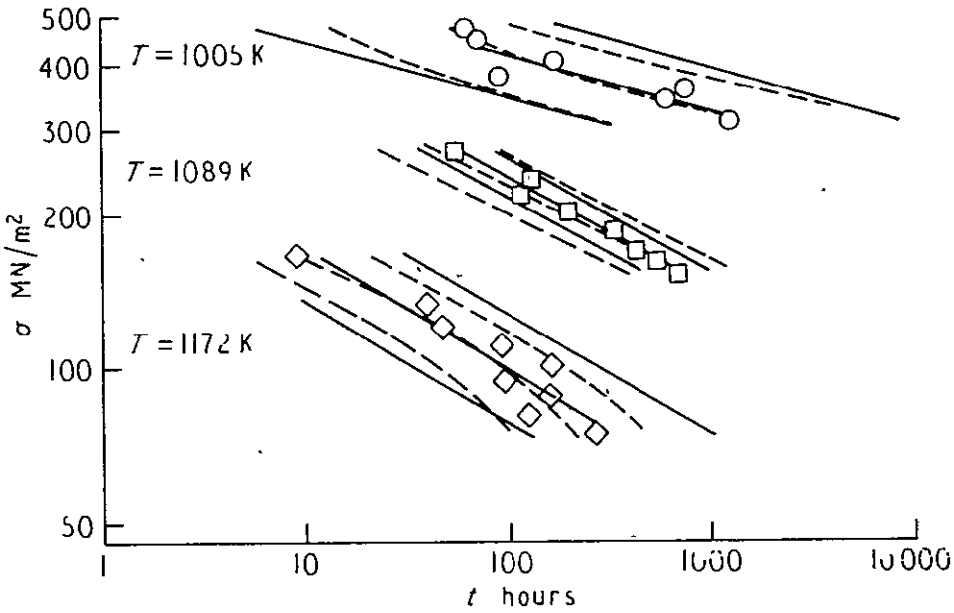
	pressure kPa	*th, mm*	*ref.*
○	0·13	0·254	10
●	101	0·254	10
□	0·13	0·510	9
△	0·13	0·640	10
▲	101	0·640	10

Fig. 2. Effect of sheet thickness and test pressure on creep strength of H-188 at 1144 K, e = 0.022



run 4024, $k = 27$: $\sigma = 207$ MN/m^2, $T = 1089$ K
o: experimental **(8)**
———: calculated, raw data, 142 observations
– – –: calculated, raw data + 'zeros', 167 observations

Fig. 3. Effect of 'zero' data points on a typical calculated creep curve for René 41



Symbols: experimental (8)

——— : linear regression at each temperature, mean and 95 per cent prediction interval

– – –: run 4124, k = 23, mean and 95 per cent prediction interval

Fig. 4. Comparison of experimental and calculated creep strength for Rene 41, e = 0.002

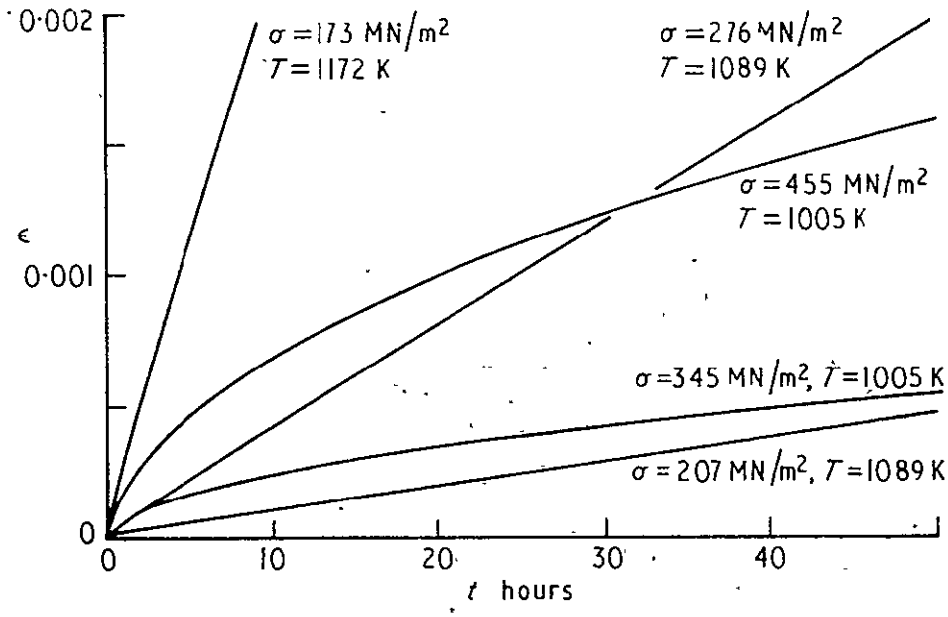
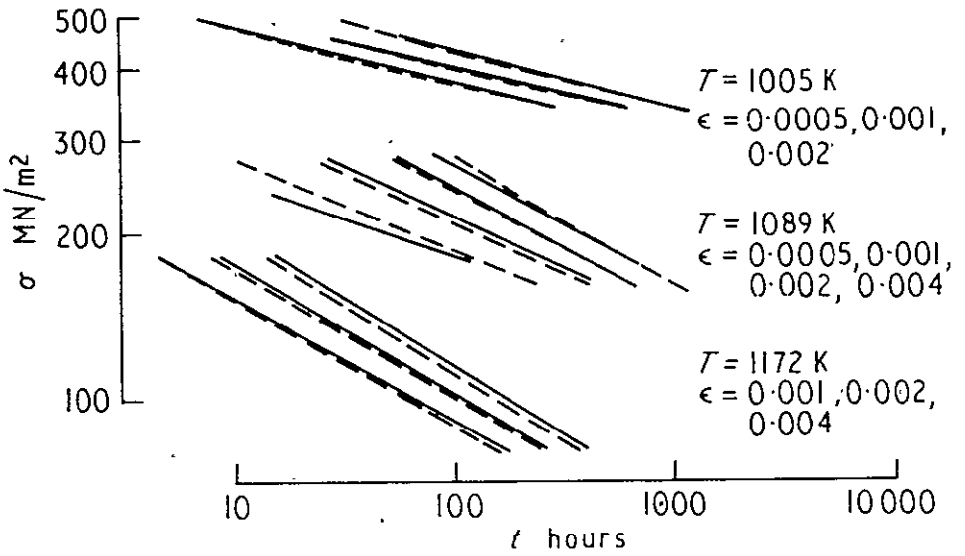


Fig. 5. Typical calculated creep curves for Rene 41, run 4124, k = 23



———: experimental means from linear regressions on data (8) for each T and ϵ shown

– – –: calculated means from run H 21, $k = 27$

Fig. 6. Comparison of experimental and calculated mean creep strengths for René 41

STRESS-RUPTURE DATA CORRELATION - GENERALIZED REGRESSION ANALYSIS AN ALTERNATIVE TO PARAMETRIC METHODS[1]

By

Donald R. Rummler

[1]This manuscript has been submitted to the ASTM for publication in the proceedings of the ASTM Symposium on Reproducibility and Accuracy of Mechanical Tests, St. Louis, Missouri, May 5, 1976.

STRESS-RUPTURE DATA CORRELATION - GENERALIZED RECESSION ANALYSIS AN ALTERNATIVE TO PARAMETRIC METHODS

By

Donald R. Rummler

ABSTRACT: The applicability of multiple regression analysis techniques to stress-rupture data correlation has been investigated. A generalized interacting variable (GIVAR) method of data correlation is proposed and evaluated. The GIVAR method is compared to six parameter methods of data correlation on three sets of simulated data and twenty sets of real data. In all cases, the GIVAR method provided the best data correlation. Application of prediction intervals and correlating variables in addition to temperature and stress is also discussed.

INTRODUCTION

Since 1952 when the first paper [1] introducing the concept of a time-temperature parameter (TTP) was published, the need to correlate and extrapolate stress-rupture data has continued unabated. The importance of stress-rupture data analysis has led to a large number of papers which either propose new parametric approaches [2-5], offer detailed comparisons of analysis techniques [5-7], and/or provide state-of-the-art surveys [8-12]. Although the development of some parametric methods can be related to creep behavior and fundamental processes, most parametric methods have been empirically derived. Most also make the assumption that there is a simple functional relationship between temperature and time-to-rupture which will yield a constant value of the parameter at a given level of applied stress. Consequently, the selection of a particular parameter to use for data analysis imposes rigid requirements on the nature of the allowable interactions between time-to-rupture, applied stress, and temperature. Methods for the selections of a particular parameter for the analysis of data sets are given in the previously cited survey papers. The application of these methods to real data sets is often difficult. Often the analyst is required to use data sets which are inadequate in terms of stress or temperature range to allow a clear selection of the parametric method best suited for data correlation. Data scatter further compounds the difficulty of selecting an analysis technique and often forces the analyst to "smooth" or approximate

his data in order to conform reasonably to the functional requirements of a particular parametric representation.

An attempt to overcome some of the difficulties has led to the concept of minimum commitment [7, 10, 13]. This method (MCM) proposes the use of a general time-temperature functional relationship. The MCM method has recently been evaluated during an investigation concerned primarily with its extrapolative characteristics [7]. Although the MCM showed promise during the evaluation, its clear superiority over other forms of parametric analysis was not demonstrated. In addition, in its present form, the MCM does not provide the analyst with an explicit form of parametric representation directly nor is it completely general in the allowed functional interactions between the primary variables of time-to-rupture, stress, and temperature.

The empirical nature of the data analysis techniques currently available is the direct result of the lack of understanding of the stress-rupture process particularly in complex engineering alloys. Until better theoretical models of creep-rupture behavior are developed, the engineer or analyst is faced with the task of establishing a functional relationship which will describe and correlate the data at hand. Regression analysis has been found to be a useful tool for the analysis of multifactor data particularly when the physical factors which control the response to be predicted are understood only in general terms. Such is currently the case in the analysis of stress-rupture data.

The purpose of this paper is to present the results of an investigation to determine the applicability of multiple regression analysis techniques to stress-rupture data correlation. The particular regression techniques developed are first compared to several parametric methods using both simulated and real stress-rupture data sets. The potential of the developed regression techniques is further explored by subjecting a large number of real data sets to a preliminary analysis designed to select the functional form of an equation to be used for detailed analysis. These results are also compared to several parametric methods.

DATA FOR ANALYSIS

Both simulated and real data sets were used to assess the capabilities of multiple regression analysis techniques for stress-rupture data correlation.

Simulated Data

Simulated data sets were derived from data for Timken 35-15 stainless steel taken from reference [8]. These data were fitted by the method of least squares to transformations of the following parametric expressions:

Larson-Miller

$$T(C + \log t_r) = b_o + b_1 \log \sigma$$

Orr-Sherby-Dorn

$$\log t_r - \Delta H/2.3RT = b_o + b_1 \log \sigma$$

Rabotnov

$$\sigma(1 + A\, t_r^b) = b_o + b_1/T + b_2/T^2$$

where

R = universal gas constant

t_r = time to rupture

T = temperature

σ = stress

C, ΔH, A, b, b_o, b_1, b_2 = constants determined by least squares

The Larson-Miller [1] and the Orr-Sherby-Dorn [2] expressions are familiar time-temperature parameters which assume that the parameter (left side of equation) is constant for a given stress. The parameter can be considered a temperature compensated time. The Rabotnov [12, 14] expression is a time-stress parameter which assumes that the value of the parameter (left side of equation) is a constant for a given temperature. The parameter represents a time compensated stress. Although the Rabotnov expression was originally developed for correlation of creep data, its use for creep-rupture correlation has been suggested [12] as an alternative to TTP methods.

The values of the constants determined by the regression analysis for each parametric expression were used with the experimental stress and temperature levels to calculate "exact" times for each simulated data set. The simulated data sets are referred to as L-M Exact, O-S-D Exact, and RAB Exact. Additional details of the fitting procedures and tabulation of the real and simulated data are presented in Appendix A.

Real Data

All real data were taken from a recent evaluation of para-

metric methods for extrapolation [7]. Careful attention was paid to the adequacy of the data in terms of range of stress and temperature exposure and long times to rupture. The data included a wide range of materials. The material types and number of observations in each data set are shown in Table 1. The data set numbering in reference [7] has been retained in this investigation. Tables 2 and 3 present the data for the two data sets (4 and 16) which are analyzed in detail. Reference [7] lists the data for the other data sets analyzed.

ANALYSIS PROCEDURES

The three types of analysis techniques used during this investigation (1) parametric, (2) minimum commitment (MCM), and (3) Generalized Iteracting Variables (GIVAR) are discussed in this section

Parametric Methods

A number of different parametric techniques have been suggested for correlating stress-rupture data. The equation forms used for multiple regression analysis of the parametric techniques selected for this investigation were as follows:

Larson-Miller (L-M)

$$Y = \log t_r = b_o + b_1/T_R + b_2 S/T_R + b_3 S^2/T_R + b_4 S^3/T_R + b_5 S^4/T_R + b_6 S^5/T_R$$

Orr-Sherby-Dorn (O-S-D)

$$Y = \log t_r = b_o + b_1/T_K + b_2 S + b_3 S^2 + b_4 S^3 + b_5 S^4 + b_6 S^5$$

Manson-Succop (M-S)

$$Y = \log t_r = b_o + b_1 T_F + b_2 S + b_3 S^2 + b_4 S^3 + b_5 S^4 + b_6 S^5$$

Manson-Haferd (M-H)

$$Y = \log t_r = b_o + b_1 T_o + b_2 T_o S + b_3 T_o S^2 + b_4 T_o S^3 + b_5 T_o S^3 + b_6 T_o S^4 + b_6 T_o S^5$$

Rabotnov (RAB)

$$Y = t_r^a = b_o + b_1/\sigma T_F + b_2/\sigma T_F^2 + b_3/\sigma T_F^3 + b_4/\sigma T_F^4 + b_5/\sigma T^5$$

where

t_r = time to rupture, hours

S = $\log \sigma$

σ = applied stress, ksi

T_F = temperature, ^oF

T_K = temperature, Kelvin

T_R = temperature, Rankin

T_o = offset temperature = $T_F - T_A$

b_i, T_A, a = constants estimated by method of least squares.

Both the M-H and RAB techniques required the use of iterative, non-linear multiple regression techniques to estimate all of the constants.

In all cases, some function of time to rupture was considered the dependent variable whose variance was minimized. High order polynomials which are functions of stress have often been used to correlate stress-rupture data [7, 8]. Although a sufficiently high order polynomial can approximate any function, it can also result in unrealistic waviness in plots of the dependent variable versus any one of the independent variables. For these reasons, the parametric model equation forms were also analyzed in functional forms which included only second or third order

polynomials in the stress function.

In addition to estimating the required constants and predicted values of log time to rupture, the parametric analysis procedures produced the following summary values to aid data correlation and parameter comparison:

$$RMS = \left(\frac{\Sigma(OTR - PTR)^2}{N} \right)^{1/2}$$

$$STD = \left(\frac{\Sigma(OTR - PTR)^2}{N - K - 1} \right)^{1/2}$$

$$DPAVG = \frac{\Sigma\ (PIMAX - PIMIN)}{N}$$

DPMAX = maximum value of PIMAX - PIMIN

where

OTR = observed log time to rupture

PTR = predicted log time to rupture

N = number of observations in data set

K = number of constants in regression model

PIMAX, PIMIN = upper and lower bounds of 95% prediction interval for each observation in a data set

The root mean square (RMS) provides an overall comparison of data correlation including both random error and functional bias. It does not, however, reflect the increases in the regression standard deviation which can occur when high order polynomial terms are included in the model equation. The added high order terms may be highly correlated with the other independent variables already in the equation and consequently may not reduce the residual sum of squares enough to account for the loss in degrees of freedom [15]. For all regressions which used log time to rupture as the dependent variable the calculated value of STD

is equivalent to the standard deviation of the regression.

The average width (DPAVG) and the maximum width (DPMAX) of the 95% prediction interval are considered useful indicators of the expected scatter for a future observation taken from the same material under the same testing conditions. The prediction interval [16, 17] is used to make a statement about the expected value of the dependent variable (log time to rupture) for a single future observation at specific values of the independent variables (functions of stress and temperature). The prediction interval is wider than the more familiar confidence interval on the mean, since it includes both sampling errors and the uncertainties in estimating the mean value of the dependent variable.

Minimum Commitment Method

The minimum commitment method (MCM) of parametric analysis [7, 10] was developed to minimize the dependence of the data analyst on the particular model equation forms of the generally used parameter methods. The MCM concept is to utilize a parameter model equation general enough to encompass most of the popular parameter methods. The parametric equation chosen has the form:

$$\log t\,(1 + AP) + P = G$$

where

t = time to rupture

A = constant

P = function of temperature

G = function of stress

The functions P and G are "station functions" which are defined by their values at selected levels of temperature and stress. Since it is not necessary for P and G to be explicitly expressed, there is no commitment on the part of the analyst to a particular parametric form. MEGA (Manson-Ensign Generalized Analysis) is the computer program developed to implement the MCM [13]. The particular version of MEGA used during this investigation utilized three stations of temperature to define P and three stations of stress to define G. In addition, the first and second derivatives of the G function at the mid station were included in the analysis. The analysis, therefore, involved the calculation of eight constants [7].

The parametric equation form which has been selected for the MCM does not readily lend itself to a least squares method of solution with log of time to rupture as the dependent variable. Consequently, the MEGA computer program in its current form does not yield least squares statistics such as the standard deviation of the solution (regression). The lack of appropriate statistics necessitated the use of RMS as the evaluator when comparing the MCM method to other methods of stress-rupture data correlation.

Generalized Interacting Variables Method

Development - The basic concept for the generalized interacting variables (GIVAR) method of data correlation was developed for the analysis and correlation of creep data [18]. Simply stated, it is assumed that the functional relationship between the dependent variable and independent variables can be

described by a low order polynomial in each independent variable. For stress-rupture data correlation, this concept leads to a model response equation of the general form:

$$f(y) = g[(a_1 + b_1X_1 + c_1X_1^2)(a_2 + b_2X_2 + c_2X_2^2 + d_2X_2^3)]$$

where y, X_1, and X_2 are respectively functions of time to rupture, temperature, and stress. Because complex interactions between time, temperature, and stress are known to occur during the creep-rupture process, the model equation is completely general and allows all interaction terms which result from the combination of the low order polynomials specified for each independent variable. Additional independent variables can be readily introduced into the general model form by the inclusion of additional low order polynomials,

$$f(y) = g[(A)\ (B)\ (C)\ (D)]$$

where A, B, C, D are low order polynomials of the independent correlating variables.

The computer program to implement the GIVAR method includes provision for transformation of y and X_i. For this investigation, the majority of data correlations were performed with the following transformations

Variable	Allowed Transformations
y	log t
X_1	T, 1/T, log T
X_2	σ, $\sigma^{1/3}$, log σ

where t, T, σ are respectively time to rupture, temperature, and stress. After transformation of the primary variables, the model

equation form is expanded and new independent variables, defined as follows, are introduced to yield a response equation for a multiple regression analysis:

$$y = a_1a_2a_3 + b_1X_1 + b_2X_2 + b_1b_2X_1X_2 - - - - = \sum_{j=1}^{k} \phi_j Z_j$$

The resulting model equation form for the multiple regression analysis is linear in the coefficients (ϕ_j) and is simply an extension of equation forms which have been used to determine optimum conditions in multifactor environments [19], for example, to determine the conditions necessary to maximize the output of a chemical process.

Application - To perform a GIVAR correlation of stress rupture data, the orders of the independent variable polynomials were selected and the general equation form expanded. A second order polynomial in temperature and a fifth order polynomial in stress were used for the majority of data correlations. When a $\sigma^{1/3}$ transformation was selected, a sixth order polynomial in stress was used. Temperature and stress interaction terms above third order ($X_1^2X_2$) were deleted from the polynomial expansions. Next, the transformations of each prime variable which would be allowed were selected. The computer program, using these control inputs plus the original data set, then analyzed all combinations of the variable transforms and printed out summary results for each analysis. The variable transforms which produced the lowest standard deviation of the regression were then resubmitted and the number of terms in the regression model was reduced using a technique known as a $t_{k,i}$-directed search [15].

When there are M potential variables in a regression model, there are 2^M possible regression equations. The $t_{k,i}$ directed search technique has been proposed as an alternative to stepwise regression techniques [16] to reduce the number of variables in a regression model. The $t_{k,i}$ directed search uses the ratio of each b_i to its standard error as follows:

$$t_{k,i} = \frac{b_i}{S(b_i)}$$

where b_i and $S(b_i)$ are the values of the coefficient and the standard error for ith variable. Following a regression on the full model equation, the variables in the full regression model are arranged in decreasing order of their $t_{k,i}$ values. Successive regressions reduce the number of variables until a "basic set" is found. The program then analyzes all model equations which can be constructed including all of the basic set of variables plus all possible combinations of the previously dropped variables. The "best" equation is selected on the basis of the lowest standard deviation of the regression.

Finally, the "best" reduced variable regression equation was analyzed in detail to verify its adequacy. If the model was to be used for significance tests or if a statistical interval such as the prediction interval were to be used, verification included careful examination of residual plots [15, 16, 20] to assess departures from the assumptions of the linear regression model.

RESULTS AND DISCUSSION

Simulated Data

The purpose of the simulated data sets was to assess the functional capability of the GIVAR method and its associated computer program without the confusing influence of the large scatter normally associated with stress-rupture data.

The results of the simulated data set analyses are summarized in Table 4 which shows the calculated values of STD for each of the six methods of data correlation for the three simulated data sets. For each data set, the generalized interacting variables method (GIVAR) produced the lowest value of STD. Of equal importance to the significantly better correlation was the fact that the GIVAR computer program selected the most correct of the prime variable transformations for the L-M and O-S-D Exact data sets. The $t_{k,i}$ search quickly reduced the original nine term model equations to the correct three term equations. The value of STD calculated for these two cases is due primarily to rounding off the calculated exact times for these data sets. For the RAB Exact data, log t, log T, and log σ were selected as the best prime variable transformations. In this case, the original eleven term model equation was reduced to nine terms during the $t_{k,i}$ search.

Table 4 also illustrates the general futility of adding higher order polynomial terms to improve correlation for the restricted models. For the four commonly used parameters, no significant improvement can be seen when expanding the model equation from four terms to seven terms (from a second order to a fifth order equation in stress). A similar lack of correlation improvement

has been reported on real data [5].

The correlations produced by the M-H and GIVAR methods for the RAB Exact data are shown in Fig. 1. The GIVAR method correlation is noticeably better than the M-H correlation. It is important to remember that in both analyses, log time to rupture was the dependent variable and consequently, minimization of differences between observed and calculated times to rupture was the regression criteria. For these data, neither of the two methods shown had model equation forms which would exactly duplicate the governing equation for the RAB Exact data generation. This is a comparable situation to most real data where correlation models seldom represent a material's behavior exactly. Since for most real data either correlation would probably be considered satisfactory, the calculation of a statistical interval such as the prediction interval to assess uncertainty about a future observation would be a natural extension of these correlations.

The residuals of the M-H and GIVAR correlations for the RAB Exact data are presented in Fig. 2. The M-H residuals clearly exhibit curvature as a function of the predicted log time to rupture. The residuals are not randomly distributed with respect to the dependent variable (predicted log time to rupture). This type of behavior indicates that the regression model is inadequate and needs additional terms. What has happened is that the M-H model equation, even with a fifth order polynomial in stress, was functionally incapable of correctly approximating the Rabotnov expression which was used to generate these data. The

random distribution of the GIVAR correlation which includes interaction terms does not suggest any functional inadequacy. An examination of the cumulative normal distribution of the residuals for the GIVAR correlation failed to indicate that the residuals were not normally distributed. Since the GIVAR correlation equation of these data does not appear to violate any of the basic regression assumptions, the calculation and use of a statistical interval would be in order [16].

Real Data

The results of the GIVAR correlation on alloy 4 (a plain carbon steel) are presented in Fig. 3. As for all GIVAR correlations, log time to rupture was the dependent variable. The prime variable transformations selected by the computer program are shown. The original eleven term model equation was reduced to seven terms during the $t_{k,i}$ search. The GIVAR mean fit seems to satisfactorily correlate this complex behavior. The STD value of the GIVAR correlation for these data was 40 percent lower (0.103 versus 0.146) than a third order M-H model which was the best of the parameter models.

To minimize the computer time, the 95% prediction interval about each observation is normally calculated during the computer run which performs the regression on the model equation. The upper and lower bounds of the 95% prediction are listed along with the calculated time to rupture. For these data, the calculated prediction interval called attention to a possible outlier, i.e., an atypical observation. This data point is shown

with the filled symbol. Examination of the residual plot with respect to predicted log time to failure (Fig. 4) suggested that the residuals were randomly distributed, had a mean of zero, and exhibited constant variance with the single exception of the residual for the possible outlier. The cumulative normal distribution plot of these residuals (Fig. 5) also appeared normal with the exception of the single suspect data point. Although there are many schemes for outlier rejection [21, 22], the present purpose is to demonstrate that the prediction interval provided a useful tool for focusing attention on a possible outlier which may have otherwise been overlooked. For other data sets, the calculated prediction interval has called attention to data transcription errors which had gone undetected because of large data scatter. It should be pointed out that the use of the prediction interval to provide a focus for possible outliers is not strictly correct in the statistical sense. Its proper use is to make estimates of the bounds which can be expected from a single future observation from the same population. Dismissing the outlier for the moment, we can say that 95% of the time a future single observation will fall within the bounds shown in Fig. 3. The implications of this kind of statement for acceptance testing, quality control, or determining the significance of a new test variable are obvious.

Temperature and stress are usually considered the prime variables for stress rupture correlation. Some authors [5], however, have been able to improve correlation by the use of an

additional variable such as elastic modulus to normalize stress. Table 5 summarizes the results of correlation analyses on alloy 16 (a nickel base alloy) to evaluate the effect of additional variables. The listing includes the analysis method, the prime variable transformations, and the calculated values for STD, DPAVG and DPMAX. The units of DPAVG and DPMAX are log (time to rupture, hours). For these data, the M-S and M-H methods were the best (lowest STD) of the parameter methods. However, the use of elastic modulus (E) to normalize stress did not significantly improve the fit in either case. Using just temperature and stress, the GIVAR method resulted in a significantly lower value of STD than the best parameter method. When second order polynomial expressions for elastic modulus and ultimate tensile strength at the test temperature were incorporated into a generalized interacting model equation, a significant further correlation improvement was achieved. The significance of the better correlation provided by the GIVAR method is more easily appreciated when it is realized, that within the average prediction interval bounds, the predicted time to rupture varies by a factor of 3 for the best parameter method and by a factor of 1.6 for the GIVAR method. For the maximum width of the prediction intervals, these values are 4.5 and 1.8, respectively. It should be pointed out that the GIVAR model equation did not allow interactions to occur between elastic modulus or ultimate tensile strength and temperature, since they are both highly correlated with temperature. In this case, the original 21 term

model equation was reduced to 13 terms during the $t_{k,i}$ search.

The best M-H and GIVAR correlations of the alloy 16 data are presented graphically in Fig. 6. The GIVAR fit is noticeably superior. Even with a fifth order polynomial in log stress, the M-H model equation appears to be functionally inadequate to correlate the complex behavior of alloy 16. This functional inadequacy is further demonstrated in Fig. 7 which presents the residuals as a function of the predicted log time to failure. The M-H residuals are not randomly distributed and definitely display a curvilinear tendency suggesting the need for interaction terms. The GIVAR residuals appear to be randomly distributed and do not suggest any inadequacies in the model equation form. The cumulative normal distribution of the residuals for the GIVAR solution (not shown) did not reveal any gross departures from normalcy. Since none of the basic assumptions of the linear regression appear to have been violated, the making of significance statements or the calculation of statistical intervals for this solution would be in order.

In order to further assess the generality of the GIVAR method, all of the data sets of reference [7] were correlated with the five parameter methods, the MCM method and the GIVAR method. The independent variables for these analyses were limited to functions of temperature and stress. For the parameter methods, second, third, and fifth order model equation forms were examined. The lowest RMS values for the five parameter methods,

MCM and GIVAR methods are tabulated in Table 6 and presented graphically in Fig. 8. RMS was selected as the basis of comparison in order to include the MCM analyses. Additional details and other summary values for these analyses are presented in Appendix B.

In Fig. 8, a range band is shown for the five parameter methods. The MCM and GIVAR method are shown with symbols. For each of the twenty data sets analyzed, the GIVAR method produced the lowest value of RMS. The GIVAR method on the average porduced a 19% lower RMS value than the MCM which was on the average the best of the other methods examined. Examination of Table 6 reveals that the GIVAR solution in several cases required less terms in the model equation than the best parameter model equation. The MEGA computer program used to implement the MCM required the determination of eight constants. Table 6 also shows that the Rabotnov method was in all cases the worst of the parametric methods. It should be pointed out, however, that a polynomial in 1/T was the only function of temperature investigated and that other functions of temperature might provide better correlations. With the exception of the GIVAR method, none of the other methods consistently produced the lowest RMS value for all twenty alloys. The failure of any single method to be consistently superior was also observed in reference [7] where the primary emphasis was on the extrapolative characteristics of the various parametric methods with these sets of data.

CONCLUSIONS

An investigation has been made to assess the applicability of a generalized interacting variable (GIVAR) multiple regression analysis method for the correlation of stress-rupture data. The GIVAR method was compared to six other methods of stress-rupture data correlation on twenty sets of data. The following conclusions are made from the analyses presented herein.

1. For all data sets examined, the GIVAR method produced the best correlation (lowest RMS value).
2. It was shown that the GIVAR method has the functional generality to satisfy criteria necessary for the calculation of statistical intervals.
3. The GIVAR method readily accepts the inclusion of correlating variables in addition to stress and temperature.
4. The prediction interval was shown to be useful for the detection of possible data outliers.

APPENDIX A

Parametric Analysis to Establish Simulated Data Sets

The purpose of simulated data sets was to evaluate the functional capabilities of the various correlation methods without the confounding influences of the large scatter normally associated with real data. Creep rupture data are seldom the result of a statistically designed experiment. The data are seldom balanced in variable space. In addition, temperature and stress are often highly correlated. Because of testing economics, low stresses are usually associated with high temperatures and high stresses are usually associated with low test temperatures. In order to include this type of inbalance in the simulated data sets, the data for Timken 35-15 stainless steel [8] were fitted to a first order Larson-Miller and Orr-Sherby-Dorn expressions and to a second order Rabotnov expression by the method of least squares. The equation forms and the fitted coefficients were as follows:

Larson-Miller

$$(T \times 10^{-4})\ (C + \log t_r) = b_o + b_1 \log \sigma$$

where T = test temperature, ^{o}R

C = iteratively determined constant = 13

t_r = time to rupture, hours

b_o = 6.39038

b_1 = -0.90584

σ = stress, psi

Orr-Sherby-Dorn

$$\log t_r - \frac{\Delta H_R}{2.3RT} = b_o + b_1 \log \sigma$$

where

t_r = time to rupture, hours

ΔH_R = apparent activation energy, iteratively calculated = 58000

R = universal gas constant = 1.986

T = temperature, K

b_o = 4.46410

b_1 = -4.60029

σ = stress, psi

Rabotnov

$$t^a = b_o + b_1/\sigma T + b_2/\sigma T^2$$

where

t = time to rupture, hours

a = constant iteratively determined = 0.3637

b_o = -1.62434

b_1 = -2.44083×10^5

σ = stress, ksi

T = temperature, ^oF

b_2 = 4.88958×10^8

The rupture times which were calculated for each of the three solution methods were substituted for the experimental times to rupture to form the "exact" simulated data sets. These calculated times and the original data for the Timken 35-15 stainless steel are presented in Table 7.

APPENDIX B

Supplementary Analysis of Correlation Methods

The purpose of this appendix is to supplement the correlation method comparison presented in the main body of the paper on the twenty real sets of data.

The results of the parametric correlations are summarized for the L-M, O-S-D, M-S, M-H and RAB in tables 8 through 12, respectively. The tables present values of RMS, STD, DPAVG and DPMAX which were calculated for each level of polynomial model equation which was evaluated. For the L-M, O-S-D, M-S, and M-H methods, second, third, and fifth order expressions in stress required 4, 5, and 7 terms, respectively. The RAB method required 3, 4, or 6 terms to develop second, third, and fifth order expressions. Table 13 presents a summary of the GIVAR method for these twenty data sets.

Parametric methods

In all cases for the L-M, O-S-D, M-S, and M-H methods, a fifth order expression produced the lowest value of RMS for a given alloy. In some cases, however, the high correlation of the power terms in stress resulted in ill-conditioned solutions which were not reliable (see Table 8, alloy 14, for example). Such was not the case for the RAB solutions (Table 12) where third order expansions (4 terms) of temperature fit better than fifth order in a number of cases (alloys 4, 6, 8, 11A, 11B, 17A).

The calculated values of STD, which for the L-M, O-S-D, M-S, and M-H methods were equivalent to the standard deviation of the

regression, did not follow the trend of better correlation with increasing degree of stress polynomial. The increased STD values reflect the fact that added variables did not reduce the residual sum of squares enough to account for the loss in degrees of freedom. These cases included the following:

Alloy	Method(s)
1	L-M
4	L-M, O-S-D, M-S, M-H
6	L-M, O-S-D, M-S, M-H
8	L-M, M-S, M-H
11A	L-M, O-S-D, M-S, M-H
11B	L-M, O-S-D, M-S
12	L-M, O-S-D, M-S, M-H

This behavior, larger values of STD with a higher order polynomial, was also exhibited for several of the alloys during the RAB method correlations (Table 12). The poorer correlation provided by the higher order polynomials can be better appreciated when we recall that the units of DPAVG and DPMAX are log time. Taking the best parametric method correlation in terms of RMS for alloy 4 (Table 11), we see that the average predicted time within the 95% prediction interval varies by a factor of 4.9 for a seven term equation and by 4.5 for a five term equation. The comparable values for the maximum width of the prediction interval are 6.4 and 5.4. In this case the use of a fifth order expression has significantly degraded the correlation. In addition to providing more sensitivity to

changes in the "goodness" of correlation, the values of DPAVG and DPMAX as preliminary evaluators of correlation have the feature of allowing all methods to be compared on an equal basis. Values of DPAVG and DPMAX can be backtransformed and averaged if necessary to accommodate different transforms of the dependent variable. They can thus provide the analyst with a "feeling" for the scatter and uncertainty in the data and its correlation.

It is beyond the scope of this paper to summarize the results of all of the analyses which were performed by the GIVAR method on the real data. Table 13 summarizes the "best" model equation results for each alloy. In most cases, the "best" equation was selected after the examination of summary computer results for nine different model equation forms. Log T and $\sigma^{1/3}$ transformations of temperature and stress were selected for several of the alloys (Table 13). These transformations are not suggested by any of the standard parametric methods. As expected, not only did the GIVAR method produce the lowest value of RMS for each of the alloys, but it also produced the lowest value of the other preliminary correlation evaluators STD, DPAVG, and DPMAX (Tables 8 through 13).

It is rare that stress-rupture data have the replicated observations that are necessary to provide an internal estimate of data scatter. The data for alloy 13 [7] was such an exception. There were seventeen experimental conditions which were replicated. These replicated observations had an average standard deviation of 0.232 with a spread of from 0.024 to 0.476,

in terms of log time. The best GIVAR correlation of these data (Table 13) had a standard deviation of 0.280 indicating that the fit was comparable to the data scatter. This value is somewhat lower than the best (M-H) parameter method STD of 0.293 (Table 11).

Summary

1. Higher order polynomial model equations do not always provide the best correlations of stress-rupture data.
2. The standard deviation of the regression (STD) is a better correlation evaluator than RMS.
3. The average and maximum width of the 95% prediction interval (DPAVG and DPMAX) are sensitive preliminary evaluators for stress-rupture data correlations.

TABLE 1--Real data sets examined.

ALLOY	MATERIAL	NUMBER OF OBSERVATIONS
1	1100-0 ALUMINUM	64
2	5454-0 ALUMINUM	75
4	PLAIN CARBON STEEL	26
5	1Cr-1Mo STEEL	33
6	1Cr-1Mo- 0.25 V STEEL	26
7	304 STAINLESS STEEL	52
8	304 STAINLESS STEEL	39
9	316 STAINLESS STEEL	38
11A, 11B	347 STAINLESS STEEL	42,44
12	A286 IRON-NICKEL	24
13	INCO 625 IRON-NICKEL	99
14	INCO 718 NICKEL-BASE	26
15	RENÉ 41 NICKEL-BASE	37
16	ASTROLOY ® NICKEL-BASE	33
17A, 17B	UDIMET 500 NICKEL-BASE	103,105
18A, 18B	L-605 COBALT-BASE	100,104
19	6061-T651 ALUMINUM	99

TOTAL = 20 DATA SETS

TABLE 2--Stress-rupture data for alloy 4.

Temperature °F	Stress ksi	Time to Rupture Hours
752	40.3	752
752	38.1	1696
752	35.8	3973
752	33.6	6134
752	31.4	10422
752	29.1	20227
842	33.6	65
842	31.4	441
842	26.9	1341
842	24.6	3023
842	22.4	3934
842	17.9	12985
842	15.7	18648
842	13.4	34753-
932	22.4	63
932	20.2	247
932	17.9	430
932	15.7	1317
932	13.5	2958
932	11.2	3202
932	9.0	7558
932	6.7	22707
1022	13.5	43
1022	11.2	142
1022	9.0	496
1022	6.9	1935

TABLE 3--Stress rupture data for alloy 16.

Temperature °F	Stress ksi	Time to Rupture Hours	Tensile Strength[a] ksi	Elastic Modulus[b] 10^{-6} psi
1400	101.0	12.8	150	25.80
1400	86.0	59.0	150	25.80
1400	80.0	176.6	150	25.80
1400	74.0	400.7	150	25.80
1400	70.0	577.0	150	25.80
1400	61.0	2279.8	150	25.80
1400	55.0	4063.2	150	25.80
1500	75.0	30.5	130	25.05
1500	64.0	142.2	130	25.05
1500	56.0	351.3	130	25.05
1500	52.0	712.0	130	25.05
1500	45.0	1228.3	130	25.05
1500	39.0	2227.4	130	25.05
1500	31.0	4393.4	130	25.05
1600	64.0	10.5	110	24.50
1600	56.5	28.8	110	24.50
1600	46.5	145.8	110	24.50
1600	41.0	253.0	110	24.50
1600	37.0	535.7	110	24.50
1600	31.0	888.0	110	24.50
1600	24.5	2899.7	110	24.50
1600	19.0	6331.0	110	24.50
1700	41.0	11.5	80	23.30
1700	33.5	44.2	80	23.30
1700	29.0	120.9	80	23.30
1700	24.0	342.7	80	23.30
1700	21.0	746.7	80	23.30
1700	17.5	1768.7	80	23.30
1700	14.5	2838.7	80	23.30
1800	29.5	6.1	40	22.15
1800	20.5	49.3	40	22.15
1800	17.0	174.0	40	22.15
1800	14.5	340.7	40	22.15

(a) Estimated from reference [23]
(b) From reference [5]

54

TABLE 4--Comparison of STD values for simulated data.

Data Set	No. of equation terms	Parameter Methods L-M 4	L-M 5	L-M 7	O-S-D 4	O-S-D 5	O-S-D 7	M-S 4	M-S 5	M-S 7	M-H 4	M-H 5	M-H 7	M-H 3	RAB 4	RAB 5	Generalized Interacting Variables GIVAR	No. of variables in "best" equation	Prime variable transformations
O-S-D Exact		.045	.045	.045	--	--	--	.092	.095	.096	.027	.023	.024	.317	.046	.003	.00003	3	log t, 1/T, log σ
L-M Exact		--	--	--	.046	.047	.047	.052	.054	.055	.027	.024	.025	.202	.052	.051	.00002	3	log t, 1/T, log σ
RAB Exact		.117	.118	.119	.145	.149	.150	.083	.084	.085	.044	.042	.044	--	--	--	.010	9	log t, log T, log σ

TABLE 5--Effect of additional variables on correlation

alloy 16 - Astroloy®

ANALYSIS METHOD	PRIME VARIABLES	STD	PREDICTION INTERVAL AVERAGE	MAXIMUM
L-M	$1/T_R$, log σ	.142	.631	.756
O-S-D	$1/T_K$, log σ	.148	.661	.824
M-S	T_F, log σ	.118	.527	.657
M-S	T_F, log σ/E	.114	.506	.648
M-H	T_F, T_A, log σ	.116	.517	.660
M-H	T_F, T_A, log σ/E	.110	.489	.652
RAB	$1/T_F$, σ	.373	1.159	4.140
GIVAR	$1/T_F$, σ	.061	.279	.353
GIVAR	log T_F, $\frac{1}{\sigma_{TU}}$, $\sigma^{1/3}$, $1/E$	.044	.213	.256

TABLE 6--Summary of RMS comparisons.

Alloy	Number of Data Points	L-M Terms	L-M RMS	O-S-D Terms	O-S-D RMS	M-S Terms	M-S RMS	M-H Terms	M-H RMS	RAB Terms	RAB RMS	MCM A	MCM RMS	GIVAR Terms	GIVAR RMS
1	64	7	.159	7	.139	7	.220	7	.153	6	.209	0	.127	9	.106
2	75	7	.082	7	.086	7	.160	7	.074	6	.245	0	.077	11	.055
4	26	7	.161	7	.149	7	.161	7	.128	4	.247	-.15	.109	7	.088
5	33	7	.063	7	.050	7	.089	7	.054	3	.304	0	.054	9	.043
6	26	7	.097	7	.057	7	.124	7	.045	4	.298	-.05	.043	6	.042
7A	52	7	.140	7	.131	7	.179	7	.121	6	.256	0	.131	10	.091
8	39	7	.178	7	.131	7	.233	7	.137	4	.291	-.05	.115	10	.074
9	38	7	.111	7	.094	7	.141	7	.111	6	.148	.15	.078	8	.068
11A	42	7	.134	7	.142	7	.122	7	.113	4	.179	-.10	.109	7	.100
11B	44	7	.132	7	.139	7	.122	7	.111	4	.218	-.05	.111	9	.099
12	24	7	.183	7	.191	7	.178	7	.178	6	.385	-.10	.175	6	.166
13	95	7	.288	7	.291	7	.291	7	.282	6	.363	-.05	.290	8	.268
14	26	5[a]	.064	5[a]	.074	5[a]	.073	5[a]	.059	6	.293	0	.056	10	.037
15	37	7	.088	7	.092	7	.100	7	.088	6	.321	0	.096	7	.068
16	33	7	.126	7	.132	7	.105	7	.103	6	.344	-.15	.072	9	.052
17A	103	7	.202	7	.232	7	.201	7	.200	4	.461	0	.198	9	.191
17B	105	7	.200	7	.228	7	.201	7	.198	4	.477	0	.201	7	.196
18A	100	7	.216	7	.251	7	.182	7	.182	6	.381	0	.186	8	.173
18B	104	7	.214	7	.252	7	.180	7	.180	6	.410	0	.187	8	.171
19	100	7	.265	7	.253	7	.308	7	.276	6	.452	0	.350	10	.225
Average			.155		.156		.169		.140		.314		.138		.116

[a]Evidence of ill-conditioned solution for seven term model.

TABLE 7--Experimental and calculated stress-rupture data for Timken 35-15 stainless steel.

Experimental			Calculated time to rupture, hours		
Temp., °F	Stress, ksi	Time to Rupture Hours	L-M Exact	O-S-D Exact	RAB Exact
1200	21.0	120	81.36	149.16	77.21
1200	19.0	170	140.47	236.38	110.82
1200	18.0	300	188.67	303.14	134.11
1200	13.0	975	1114.10	1354.55	401.32
1300	16.0	60	46.89	52.16	71.44
1300	13.0	160	136.51	135.58	150.14
1300	11.0	300	322.53	292.38	265.00
1300	7.5	1300	2315.56	1702.62	910.22
1400	8.5	120	166.00	122.73	228.84
1400	7.0	400	427.33	299.80	434.31
1400	6.0	900	905.32	609.27	711.23
1500	6.0	120	138.89	96.32	252.30
1500	4.9	300	354.14	244.54	490.24
1500	3.5	950	1676.99	1149.68	1409.44
1600	6.0	20	25.56	18.21	73.96
1600	4.0	170	152.03	117.62	302.83
1600	3.0	500	538.66	441.81	765.48
1600	2.5	1300	1200.88	1022.11	1349.54
1800	3.0	22	21.84	24.58	30.82
1800	2.0	100	110.91	158.75	139.88
1800	1.5	500	351.35	596.31	369.08
1800	1.3	1000	623.51	1151.78	585.77

TABLE 8--Summary of Larson-Miller method correlations.

Number of terms	RMS			STD			DPAVG			DPMAX		
	4	5	7	4	5	7	4	5	7	4	5	7
Alloy												
1	0.1842	0.1604	0.1587	0.1903	0.1671	0.1680	0.7937	0.7020	0.7159	0.8455	0.7960	0.9100
2	0.1473	0.0890	0.0817	0.1514	0.0921	0.0858	0.6287	0.3849	0.3632	0.6633	0.4218	0.4410
4	0.1796	0.1628	0.1606	0.1952	0.1811	0.1892	0.8497	0.8029	0.8654	0.9488	0.9214	1.0204
5	0.0823	0.0732	0.0625	0.0878	0.0795	0.0704	0.3750	0.3443	0.3131	0.4068	0.4077	0.4036
6	0.1009	0.0977	0.0970	0.1184	0.1087	0.1135	0.5153	0.4818	0.5223	0.5700	0.5662	0.6494
7	0.1769	0.1527	0.1404	0.1839	0.1606	0.1509	0.7714	0.6797	0.6497	0.8091	0.7370	0.7530
8	0.1791	0.1791	0.1777	0.1891	0.1918	0.1962	0.8008	0.8219	0.8597	0.8631	0.9337	1.0632
9	0.1367	0.1224	0.1114	0.1466	0.1313	0.1233	0.6214	0.5631	0.5410	0.7412	0.7294	1.7116
11A	0.1395	0.1358	0.1337	0.1467	0.1447	0.1464	0.6193	0.6176	0.6386	0.6664	0.6786	0.7051
11B	0.1397	0.1340	0.1317	0.1465	0.1424	0.1436	0.6174	0.6062	0.6241	0.7014	0.7087	0.7229
12	0.2356	0.1861	0.1833	0.2581	0.2091	0.2176	1.1320	0.9359	1.0142	1.4289	1.2125	1.2980
13	0.3313	0.3057	0.2681	0.3385	0.3137	0.2993	1.3935	1.2983	1.2514	1.5469	1.5613	1.6586
14	0.0677	0.0643	a	0.0736	0.0716	a	0.3201	0.3172	a	0.3578	0.3772	a
15	0.1497	0.0996	0.0870	0.1585	0.1072	0.0973	0.6729	0.4606	0.4282	0.7275	0.5297	0.5359
16	0.1663	0.1342	0.1257	0.1774	0.1457	0.1417	0.7577	0.6309	0.6303	0.8409	0.7562	0.8062
17A	0.2378	0.2064	0.2020	0.2426	0.2116	0.2092	0.9951	0.8725	0.8711	1.0364	0.9428	1.0413
17B	0.2526	0.2041	0.1999	0.2575	0.2090	0.2069	1.0554	0.8607	0.8602	1.1363	1.0042	1.1186
18A	0.2309	0.2245	0.2161	0.2356	0.2303	0.2241	0.9679	0.9510	0.9344	1.0056	1.0292	1.1236
18B	0.2283	0.2261	0.2138	0.2327	0.2338	0.2215	0.9541	0.9631	0.9212	1.0288	1.1120	1.1542
19	0.4316	0.3536	0.2654	0.4405	0.3628	0.2751	1.8096	1.4981	1.1473	1.8803	1.5931	1.2681

[a]Evidence of ill-conditioned solution.

TABLE 9—Summary of Orr-Sherby-Dorn method correlations.

Number of terms	RMS 4	RMS 5	RMS 7	STD 4	STD 5	STD 7	DPAVG 4	DPAVG 5	DPAVG 7	DPMAX 4	DPMAX 5	DPMAX 7
Alloy												
1	0.1892	0.1417	0.1387	0.1954	0.1476	0.1470	0.8149	0.6202	0.6264	0.8619	0.6899	0.7806
2	0.2025	0.1072	0.0860	0.2081	0.1109	0.0903	0.8643	0.4636	0.3823	0.9103	0.4950	0.4450
4	0.1819	0.1510	0.1493	0.1978	0.1680	0.1746	0.8608	0.7452	0.8036	0.9502	0.8133	0.9189
5	0.0818	0.0611	0.0495	0.0872	0.0663	0.0557	0.3727	0.2876	0.2482	0.3975	0.3029	0.3043
6	0.0661	0.0574	0.0566	0.0718	0.0639	0.0662	0.3128	0.2834	0.3045	0.3282	0.3110	0.3688
7	0.1582	0.1456	0.1306	0.1647	0.1531	0.1404	0.6907	0.6480	0.6048	0.7184	0.6680	0.6624
8	0.1389	0.1374	0.1308	0.1466	0.1472	0.1444	0.6209	0.6308	0.6329	0.6616	0.6944	0.7589
9	0.1264	0.1077	0.0935	0.1337	0.1156	0.1035	0.5668	0.4962	0.4541	0.5925	0.5693	0.5902
11A	0.1511	0.1437	0.1424	0.1588	0.1531	0.1560	0.6707	0.6539	0.6803	0.7124	0.6901	0.7378
11B	0.1568	0.1413	0.1393	0.1644	0.1501	0.1519	0.6930	0.6393	0.6605	0.7803	0.7149	0.7542
12	0.2689	0.1940	0.1914	0.2945	0.2180	0.2275	1.2931	0.9767	1.0607	1.4279	1.1902	1.3414
13	0.3429	0.3193	0.2910	0.3503	0.3281	0.3023	1.4425	1.3578	1.2643	1.5942	1.6033	1.5184
14	0.0873	0.0738	a	0.0949	0.0821	a	0.4131	0.3642	a	0.4261	0.3991	a
15	0.1742	0.0998	0.0923	0.1844	0.1073	0.1025	0.7830	0.4614	0.4516	0.8225	0.4857	0.5349
16	0.1797	0.1416	0.1317	0.1917	0.1537	0.1484	0.8191	0.6661	0.6605	0.8540	0.7447	0.8237
17A	0.2864	0.2370	0.2322	0.2921	0.2429	0.2405	1.1982	1.0016	1.0013	1.2249	1.0250	1.1031
17B	0.3181	0.2329	0.2284	0.3243	0.2387	0.2364	1.3290	0.9830	0.9830	1.3996	1.0309	1.1771
18A	0.2658	0.2650	0.2505	0.2713	0.2719	0.2597	1.1145	1.1226	1.0832	1.1424	1.1523	1.2127
18B	0.2702	0.2661	0.2520	0.2755	0.2727	0.2609	1.1297	1.1238	1.0855	1.1678	1.1764	1.2777
19	0.5102	0.4067	0.2533	0.5207	0.4173	0.2627	2.1390	1.7232	1.0957	2.1995	1.7714	1.1595

[a]Evidence of ill-conditioned solution.

TABLE 10--Summary of Manson-Succop method correlations.

Number of terms	RMS 4	RMS 5	RMS 7	STD 4	STD 5	STD 7	DPAVG 4	DPAVG 5	DPAVG 7	DPMAX 4	DPMAX 5	DPMAX 7
Alloy												
1	0.2417	0.2256	0.2204	0.2496	0.2350	0.2335	1.0412	0.9873	0.9951	1.1062	1.0916	1.2462
2	0.2042	0.1679	0.1602	0.2098	0.1738	0.1682	0.8714	0.7264	0.7119	0.9425	0.8035	0.8258
4	0.1776	0.1628	0.1610	0.1930	0.1812	0.1883	0.8404	0.8036	0.8666	0.8913	0.8661	0.9908
5	0.1055	0.0941	0.0886	0.1126	0.1022	0.0998	0.4809	0.4429	0.4446	0.4988	0.4680	0.5449
6	0.1264	0.1239	0.1235	0.1376	0.1378	0.1445	0.5990	0.6111	0.6647	0.6761	0.6824	0.8050
7	0.2025	0.1830	0.1788	0.2105	0.1925	0.1922	0.8830	0.8145	0.8279	0.9291	0.8500	0.9079
8	0.2339	0.2339	0.2330	0.2469	0.2505	0.2572	1.0455	1.0735	1.1276	1.1286	1.1731	1.3518
9	0.1639	0.1516	0.1409	0.1733	0.1627	0.1559	0.7348	0.6981	0.6844	0.7706	0.7975	0.8695
11A	0.1302	0.1241	0.1219	0.1369	0.1322	0.1336	0.5779	0.5646	0.5826	0.6197	0.5990	0.6316
11B	0.1317	0.1249	0.1219	0.1381	0.1327	0.1329	0.5822	0.5652	0.5778	0.6514	0.6310	0.6609
12	0.2356	0.1821	0.1779	0.2579	0.2046	0.2114	1.1325	0.9167	0.9858	1.2058	1.1163	1.2468
13	0.3492	0.3056	0.2914	0.3568	0.3140	0.3027	1.4688	1.2996	1.2661	1.6739	1.5537	1.5409
14	0.0805	0.0728	a	0.0875	0.0810	a	0.3810	0.3591	a	0.3928	0.3936	a
15	0.1673	0.1153	0.0999	0.1771	0.1240	0.1110	0.7520	0.5331	0.4889	0.7838	0.5629	0.5796
16	0.1569	0.1157	0.1051	0.1663	0.1256	0.1184	0.7106	0.5443	0.5268	0.7567	0.6055	0.6570
17A	0.2632	0.2184	0.2012	0.2736	0.2239	0.2085	1.1222	0.9231	0.8679	1.1497	0.9447	0.9559
17B	0.2795	0.2205	0.2007	0.2849	0.2260	0.2077	1.1677	0.9307	0.8637	1.2206	0.9689	1.0323
18A	0.1949	0.1942	0.1822	0.1989	0.1992	0.1889	0.8171	0.8227	0.7878	0.8378	0.8482	0.8827
18B	0.1967	0.1954	0.1805	0.2027	0.2003	0.1867	0.8309	0.8253	0.7768	0.8621	0.8620	0.9142
19	0.4161	0.3640	0.3076	0.4247	0.3735	0.3192	1.7446	1.5423	1.3313	1.8299	1.6135	1.4131

[a]Evidence of ill-conditioned solution.

TABLE 11--Summary of Manson-Haferd method correlations.

Number of terms	RMS			STD			DPAVG			DPMAX		
	4	5	7	4	5	7	4	5	7	4	5	7
Alloy												
1	0.2122	0.1653	0.1533	0.2192	0.1722	0.1624	0.9144	0.7235	0.6921	0.9716	0.7998	0.8466
2	0.1893	0.1145	0.0741	0.1946	0.1185	0.0779	0.8080	0.4954	0.3295	0.8668	0.5535	0.3652
4	0.1722	0.1312	0.1284	0.1872	0.1460	0.1502	0.8148	0.6472	0.6908	0.9018	0.7294	0.8072
5	0.0930	0.0597	0.0541	0.0992	0.0649	0.0610	0.4239	0.2809	0.2712	0.4677	0.3406	0.3510
6	0.0481	0.0455	0.0448	0.0523	0.0507	0.0525	0.2277	0.2247	0.2413	0.2468	0.2594	0.2988
7	0.1475	0.1294	0.1212	0.1535	0.1361	0.1303	0.6437	0.5759	0.5608	0.6907	0.6429	0.6534
8	0.1524	0.1390	0.1365	0.1609	0.1489	0.1507	0.6813	0.6380	0.6605	0.7561	0.7468	0.8061
9	0.1476	0.1299	0.1111	0.1560	0.1394	0.1231	0.6611	0.5975	0.5397	0.8048	0.7819	0.7105
11A	0.1248	0.1143	0.1128	0.1312	0.1217	0.1235	0.5540	0.5196	0.5388	0.5985	0.5721	0.5898
11B	0.1287	0.1144	0.1111	0.1350	0.1215	0.1212	0.5689	0.5172	0.5266	0.6492	0.6060	0.6095
12	0.2176	0.1821	0.1779	0.2384	0.2046	0.2114	1.0455	0.9157	0.9847	1.3147	1.1877	1.2502
13	0.3448	0.3041	0.2819	0.3523	0.3124	0.2929	1.4505	1.2929	1.2244	1.6561	1.5899	1.6379
14	0.0773	0.0587	a	0.0840	0.0654	a	0.3655	0.2896	a	0.4131	0.3481	a
15	0.1673	0.1015	0.0883	0.1771	0.1091	0.0980	0.7520	0.4690	0.4315	0.8203	0.5505	0.5441
16	0.1557	0.1148	0.1032	0.1660	0.1247	0.1162	0.7093	0.5398	0.5172	0.7821	0.6419	0.6600
17A	0.2677	0.2170	0.1998	0.2730	0.2224	0.2069	1.1199	0.9170	0.8614	1.1706	0.9968	1.0388
17B	0.2758	0.2179	0.1984	0.2812	0.2232	0.2054	1.1525	0.9193	0.8537	1.2466	1.0843	1.1137
18A	0.1946	0.1936	0.1822	0.1986	0.1986	0.1889	0.8158	0.8200	0.7878	0.8540	0.8953	0.9548
18B	0.1967	0.1948	0.1804	0.2006	0.1997	0.1868	0.8224	0.8226	0.7769	0.8926	0.9539	0.9775
19	0.4129	0.3619	0.2760	0.4214	0.3713	0.2862	1.7310	1.5331	1.1934	1.8026	1.6363	1.3242

[a]Evidence of ill-conditioned solution.

TABLE 12--Summary of Rabotnov method correlations.[a]

Number of terms	RMS 3	RMS 4	RMS 6	STD 3	STD 4	STD 6	DPAVG 3	DPAVG 4	DPAVG 6	DPMAX 3	DPMAX 4	DPMAX 6
Alloy												
1	0.4387	0.2185	0.2091	0.4494	0.2257	0.2178	0.0000	0.0000	0.9158	0.0000	0.0000	1.0270
2	0.3800	0.2458	0.2452	0.3879	0.2526	0.2538	1.8936	1.0257	1.0269	3.5595	1.6010	1.6469
4	0.3033	0.2473	0.2644	0.3225	0.2688	0.2942	1.4867	1.4262	1.4309	3.4359	3.8611	5.3300
5	0.3040	0.3053	0.3132	0.3188	0.3257	0.3400	1.7321	1.6602	1.7328	3.1460	2.9195	3.2385
6	0.3228	0.2975	0.3425	0.3432	0.3235	0.3811	1.6292	1.6693	1.6632	4.3534	4.2000	4.4432
7	0.4090	0.2674	0.2561	0.4214	0.2783	0.2694	1.9803	1.3364	1.3095	6.4589	4.1598	3.6349
8	0.5865	0.2912	0.3257	0.6105	0.3074	0.3489	2.9696	1.6791	1.9132	12.1315	3.4606	5.8765
9	0.2364	0.1523	0.1478	0.2464	0.1610	0.1611	1.0881	0.7106	0.7195	1.3500	1.0488	1.1497
11A	0.5037	0.1791	0.1915	0.5227	0.1882	0.2040	1.4762	0.8855	0.9193	3.7275	2.4518	2.8222
11B	0.4127	0.2180	0.2326	0.4276	0.2287	0.2471	1.8748	1.0452	1.0775	4.8993	3.7546	4.5071
12	0.4197	0.3891	0.3854	0.4487	0.4262	0.4332	2.4168	1.8572	1.9490	6.8192	3.1121	3.1838
13	0.8140	0.3982	0.3626	0.8272	0.4068	0.3746	3.7109	2.0841	1.9390	7.4721	5.0910	3.7243
14	0.3240	0.2945	0.2927	0.3445	0.3202	0.3257	0.0000	1.4057	1.4524	0.0000	1.5688	1.5985
15	0.3249	0.3306	0.3210	0.3389	0.3501	0.3451	1.6262	1.6262	1.4666	3.4998	5.7345	4.5364
16	0.5893	0.3604	0.3438	0.6181	0.3845	0.3732	1.2650	1.1264	1.1590	6.0260	3.8238	4.1400
17A	0.4785	0.4608	0.4638	0.4857	0.4700	0.4755	2.8635	2.2600	2.3512	13.3286	7.2185	8.4388
17B	0.4892	0.4770	0.4770	0.4963	0.4863	0.4887	2.8288	2.4270	2.4692	13.1340	10.3149	10.8394
18A	0.3896	0.3921	0.3807	0.3955	0.4002	0.3906	1.7884	1.8021	1.8257	5.3525	5.8298	11.8130
18B	0.4168	0.4116	0.4101	0.4229	0.4198	0.4203	1.9422	1.7542	1.8609	16.9520	5.7573	14.9883
19	0.6105	0.4668	0.4516	0.6196	0.4764	0.4634	3.4160	2.0914	2.0226	10.1529	3.4808	3.1542

[a]Based upon backtransformed log time values.

TABLE 13--Summary of GIVAR method correlations.

Alloy	No. of Observ.	Prime Variable Transformation Temp.	Stress	No. of Variables Start	"Best"	RMS	STD	DPAVG	DPMAX
1	64	1/T	$\sigma^{1/3}$	12	9	.1060	.1130	.4921	.6387
2	75	log T	$\sigma^{1/3}$	12	11	.0551	.0592	.2548	.3099
4	26	1/T	$\sigma^{1/3}$	12	7	.0883	.1033	.4754	.5273
5	33	log T	σ	11	9	.0426	.0499	.2284	.2799
6	26	1/T	log σ	9	6	.0418	.0476	.2150	.2485
7	52	log T	$\sigma^{1/3}$	12	10	.0910	.1013	.4471	.5155
8	39	1/T	σ	11	10	.0744	.0863	.3910	.4971
9	38	log T	σ	11	8	.0677	.0762	.3384	.4169
11A	42	1/T	$\sigma^{1/3}$	12	7	.0997	.1092	.4764	.5360
11B	44	1/T	σ	11	9	.0985	.1104	.4894	.5686
12	24	1/T	σ	11	6	.1664	.1921	.8784	1.0377
13	95	1/T	log σ	11	8	.2677	.2797	1.1756	1.4435
14	26	log T	$\sigma^{1/3}$	12	10	.0368	.0456	.2218	.2547
15	37	log T	σ	11	7	.0683	.0758	.3337	.4185
16	33	1/T	σ	11	9	.0520	.0610	.2788	.3529
17A	103	1/T	σ	11	9	.1913	.2002	.8417	.9649
17B	105	1/T	$\sigma^{1/3}$	12	7	.1963	.2032	.8450	.9053
18A	100	1/T	σ	11	8	.1726	.1799	.7541	.8583
18B	104	1/T	$\sigma^{1/3}$	12	8	.1712	.1782	.7451	.8174
19	100	1/T	$\sigma^{1/3}$	12	10	.2248	.2369	1.0028	1.0978

References

[1] Larson, F. R. and Miller, J., Transactions, American Society of Mechanical Engineers, Vol. 74, 1952, p. 765.

[2] Orr, R. L., Sherby, O.D., and Dorn, J.E., Transactions, American Society for Metals, Vol. 46, 1954, p. 113.

[3] Manson, S. S. and Haferd, A. M., "A Linear Time-Temperature Relation for Extrapolation of Creep and Rupture Stress Data," NACA Report TN-2890, National Advisory Committee for Aeronautics, Washington, D.C., March 1953.

[4] Manson, S. S., and Succop, G., "Stress Rupture Properties of Inconel 700 and Correlation on the Basis of Several Time-Temperature Parameters," American Society for Testing and Materials Symposium on Metallic Materials for Service at Temperatures Above 1600°F, 1955, p. 40.

[5] Goldhoff, R. M. and Hahn, G. J., "Correlation and Extrapolations of Creep-Rupture Data of Several Steels and Superalloys Using Time-Temperature Parameters," American Society for Metals Publication D8-100, American Society for Metals, Cleveland, OH, 1968, p. 199.

[6] Larke, E. C. and Inglis, N.P., Proceedings Joint International Conference on Creep, 1963, p. 6.

[7] Goldhoff, R. M., Journal of Testing and Evaluation, American Society for Testing and Materials, Vol. 2, No. 5, September 1974, pp. 387-424.

[8] Conway, J. B., "Stress-Rupture Parameters: Origin, Calculation, and Use," Gordon and Breach, New York, 1969.

[9] Van Leeuwen, H. P., "Predicting Material Behavior Under Load, Time and Temperature Conditions," NATO-AGARD Report 513, North Atlantic Treaty Organization - Advisory Group for Aeronautical Research and Development, Paris, June 1965, pp. 73-141.

[10] Manson, S.S., "Time-Temperature Parameters - A Re-evaluation and Some New Approaches," ASM publication D-8-100, American Society for Metals, Cleveland, OH, 1968, pp. 1-115.

[11] Grounes, M., J. of Basic Engineering, Series D, Transactions, American Society of Mechanical Engineers, Vol. 91, March 1969, pp. 59-62.

[12] Penny, R. K. and Marriott, D. L., "Design for Creep," McGraw-Hill Book Company (UK), Ltd, Maidenhead, Berkshire, England, 1971.

[13] Manson, S. S. and Ensign, C. R., "A Specialized Model for Analysis of Creep-Rupture Data by the Minimum Commitment Station Function Approach," NASA Report TM X-52999, National Aeronautics and Space Administration, Washington, DC, 1971.

[14] Rabotnov, Yu. N., "Creep Problems in Structural Members," North Holland Publishing Company, A Wiley Interscience Division, Amsterdam, 1969.

[15] Daniel, C. and Wood, F.S.,"Fitting Equations to Data," John Wiley and Sons, Inc., New York, 1971.

[16] Draper, N.R. and Smith, H., "Applied Regression Analysis," John Wiley and Sons, Inc., New York, 1966.

[17] Hahn, G. J., "Statistical Intervals for a Normal Population," General Electric Report No. 69-C-382, General Electric Research and Development Center, Schenectady, New York, November 1969.

[18] Rummler, D.R., "Application of Regression Analysis to Creep of Space Shuttle Materials," Conference Publication 13, International Conference on Creep and Fatigue in Elevated Temperature Applications, Institution of Mechanical Engineers (U.K.), 1973.

[19] Davies, O.L., "Design and Analysis of Industrial Experiments," Hafner Publishing Co., New York, 1967.

[20] Harris, R. J., "A Primer of Multivariate Statistics," Academic Press, New York, 1975.

[21] Matrella, M.G., "Experimental Statistics," National Bureau of Standards Handbook 91, August 1963.

[22] Anon., "Recommended Practice for Dealing With Outlying Observations," ASTM Standard E178-68, American Society for Testing and Materials, 1971.

[23] Lyman, Taylor; Editor, "Metals Handbook - Properties and Selection of Metals," Vol. 1, 8th Ed., American Society for Metals, 1961.

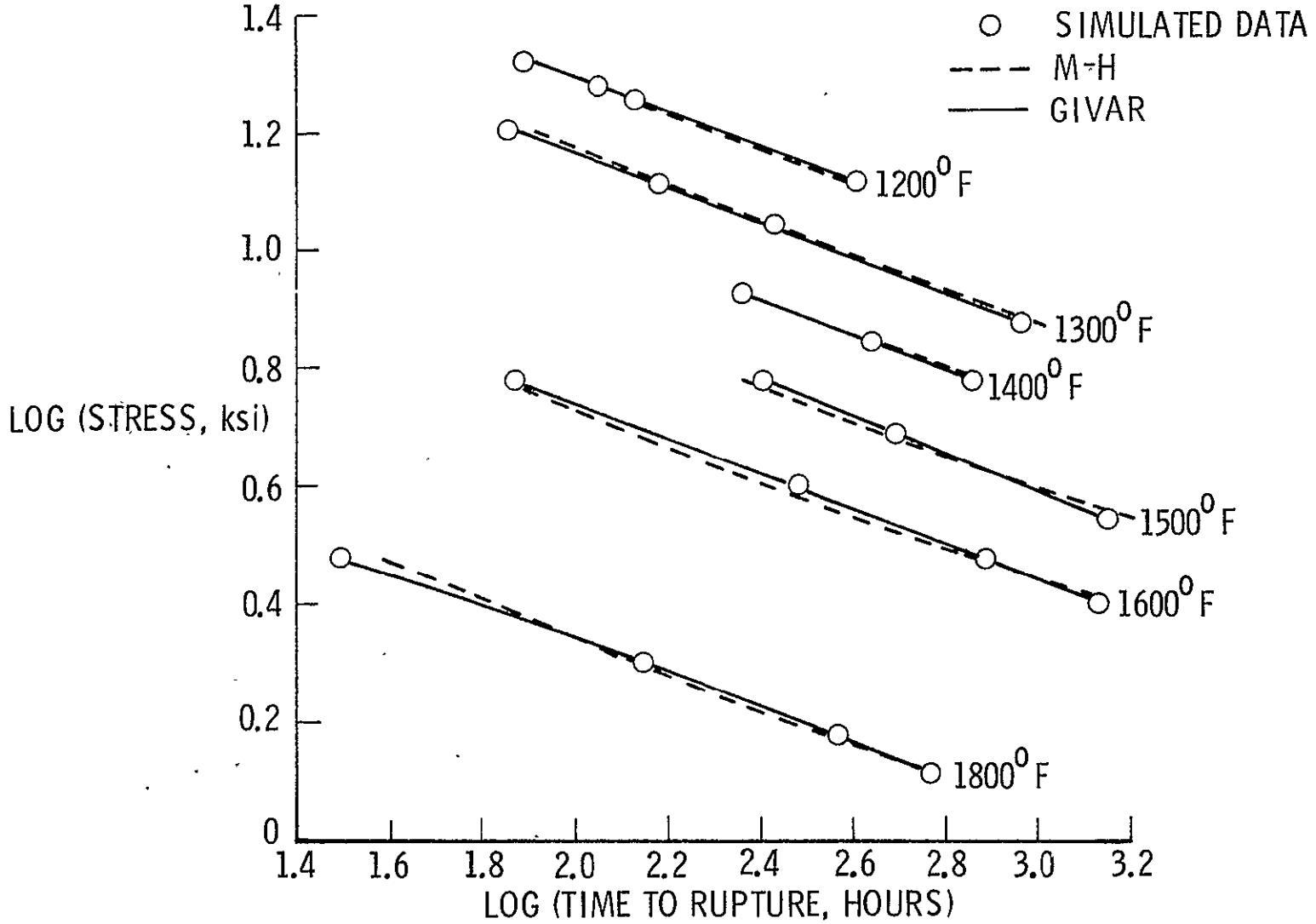


Fig. 1—Correlation of Rabotnov simulated data set.

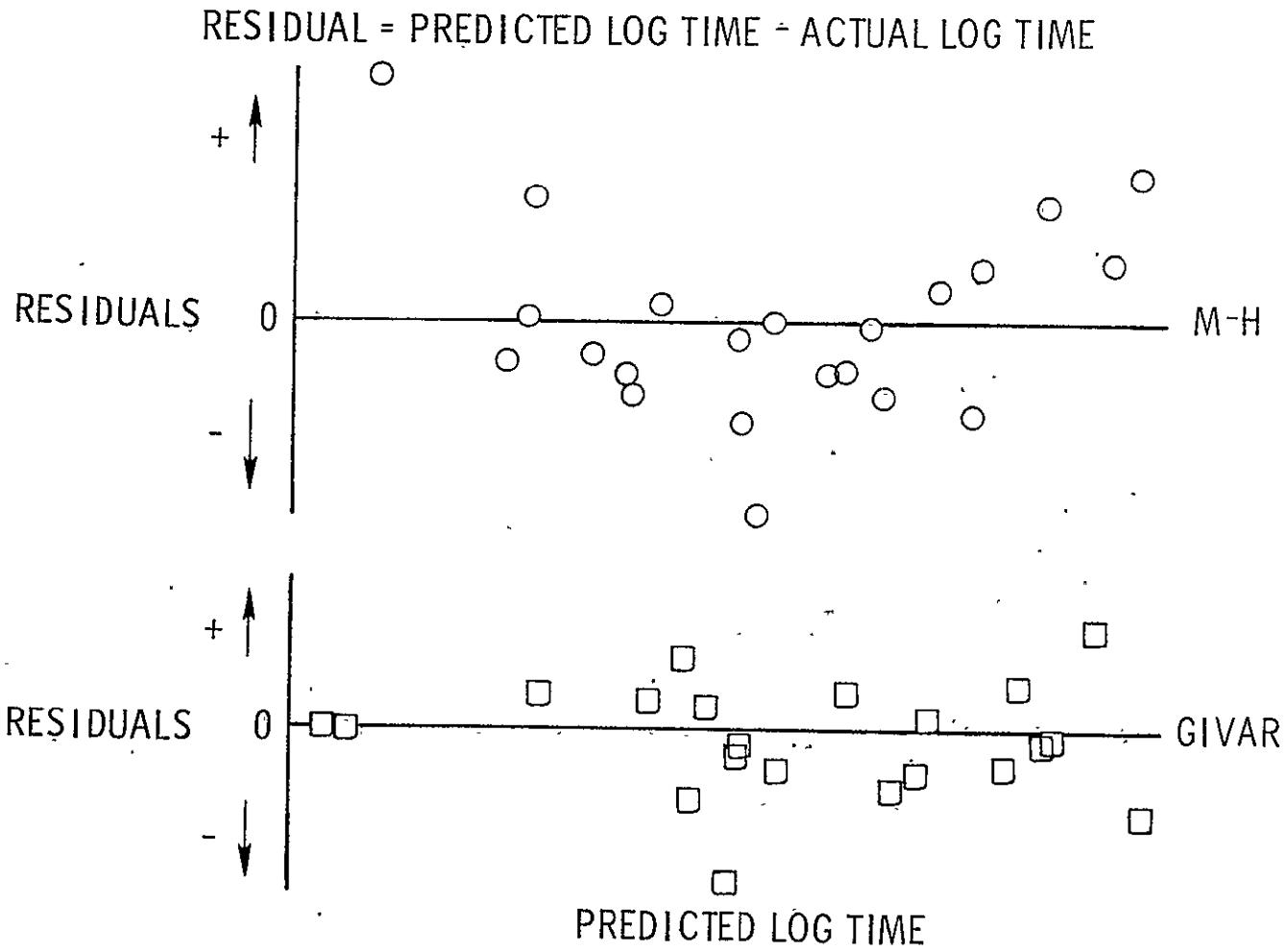


Fig. 2—Comparison of regression residuals for Rabotnov simulated data set.

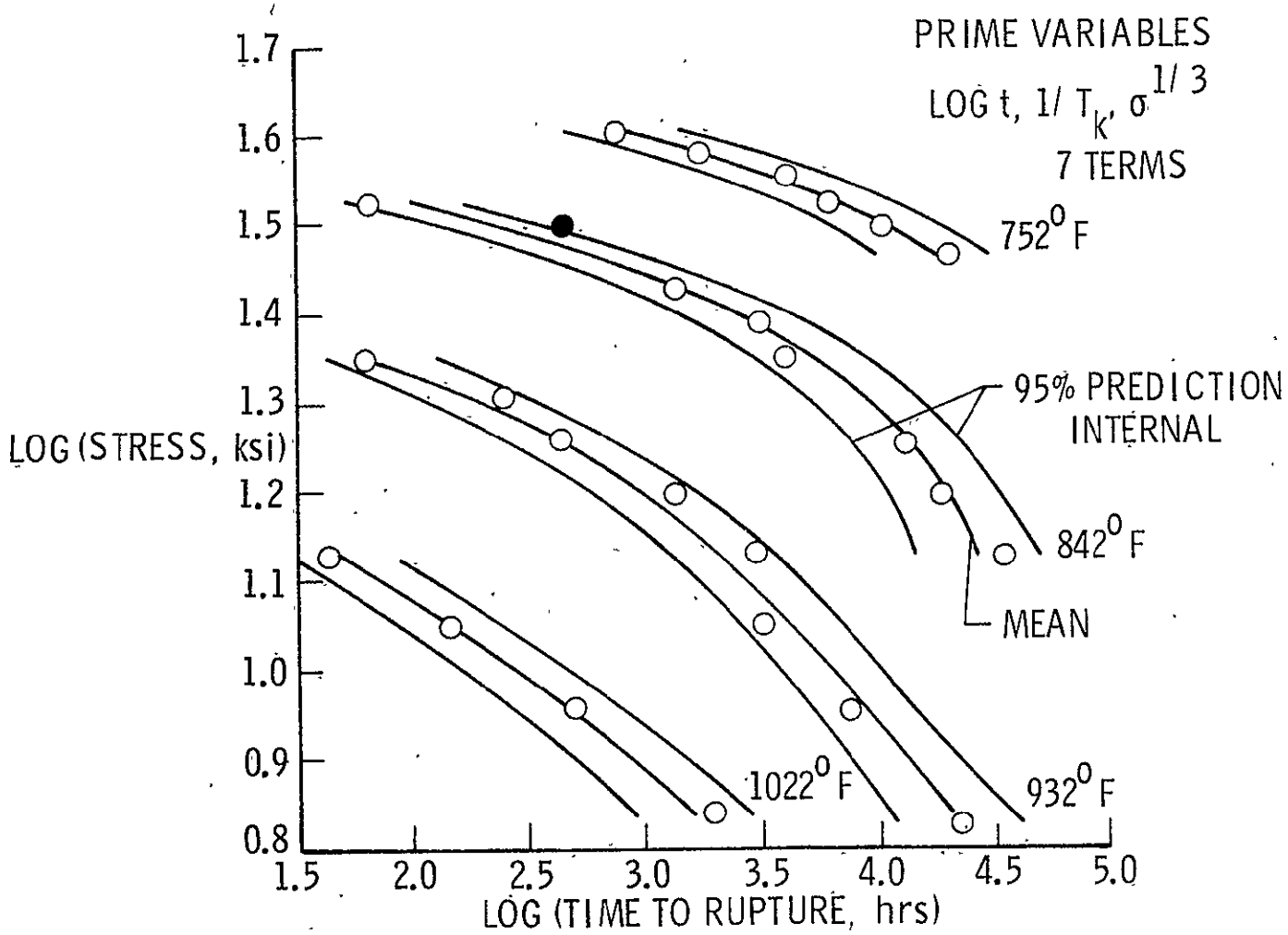


Fig. 3—GIVAR correlation for alloy 4.

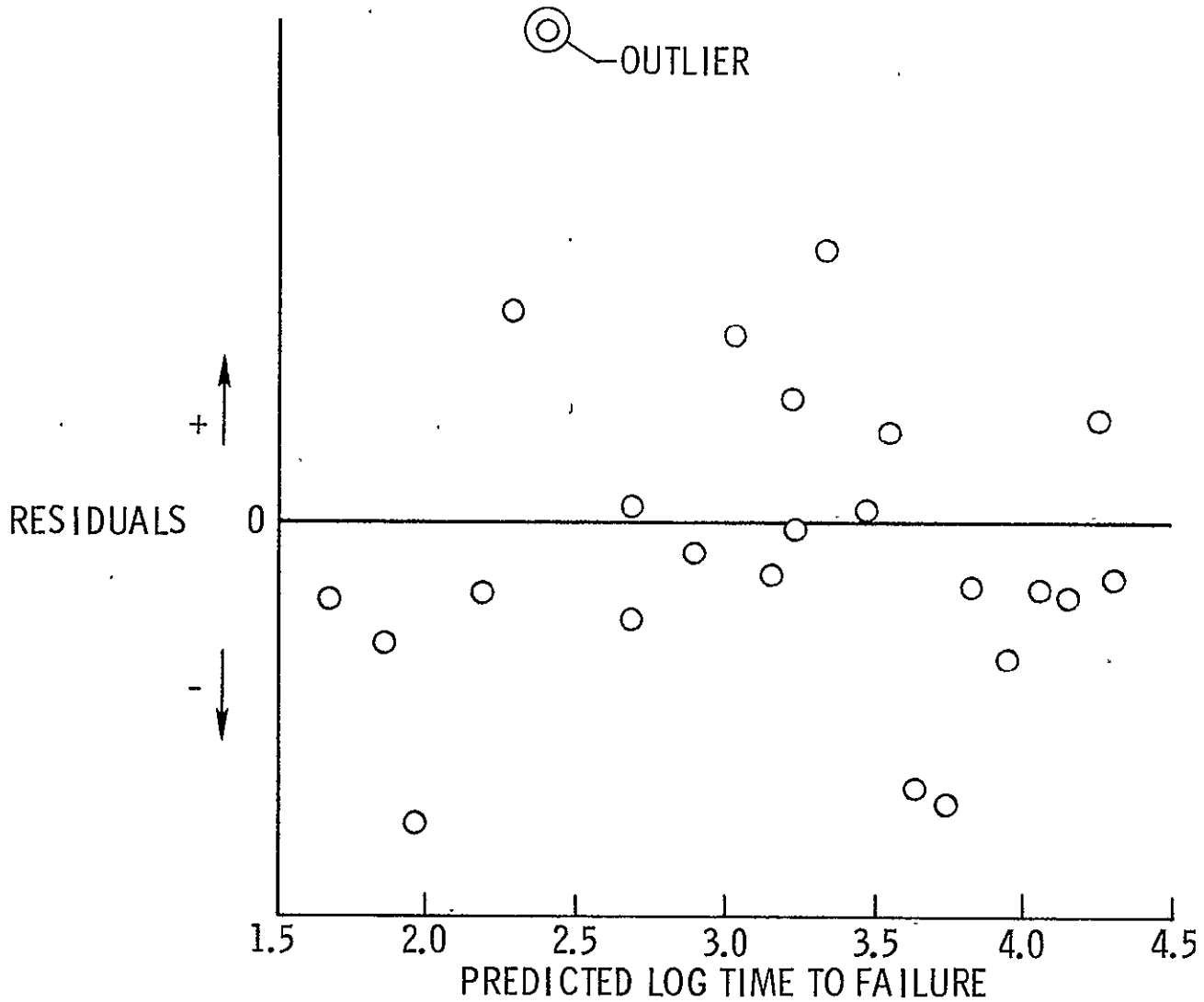


Fig. 4--Regression residuals for alloy 4, GIVAR analysis.

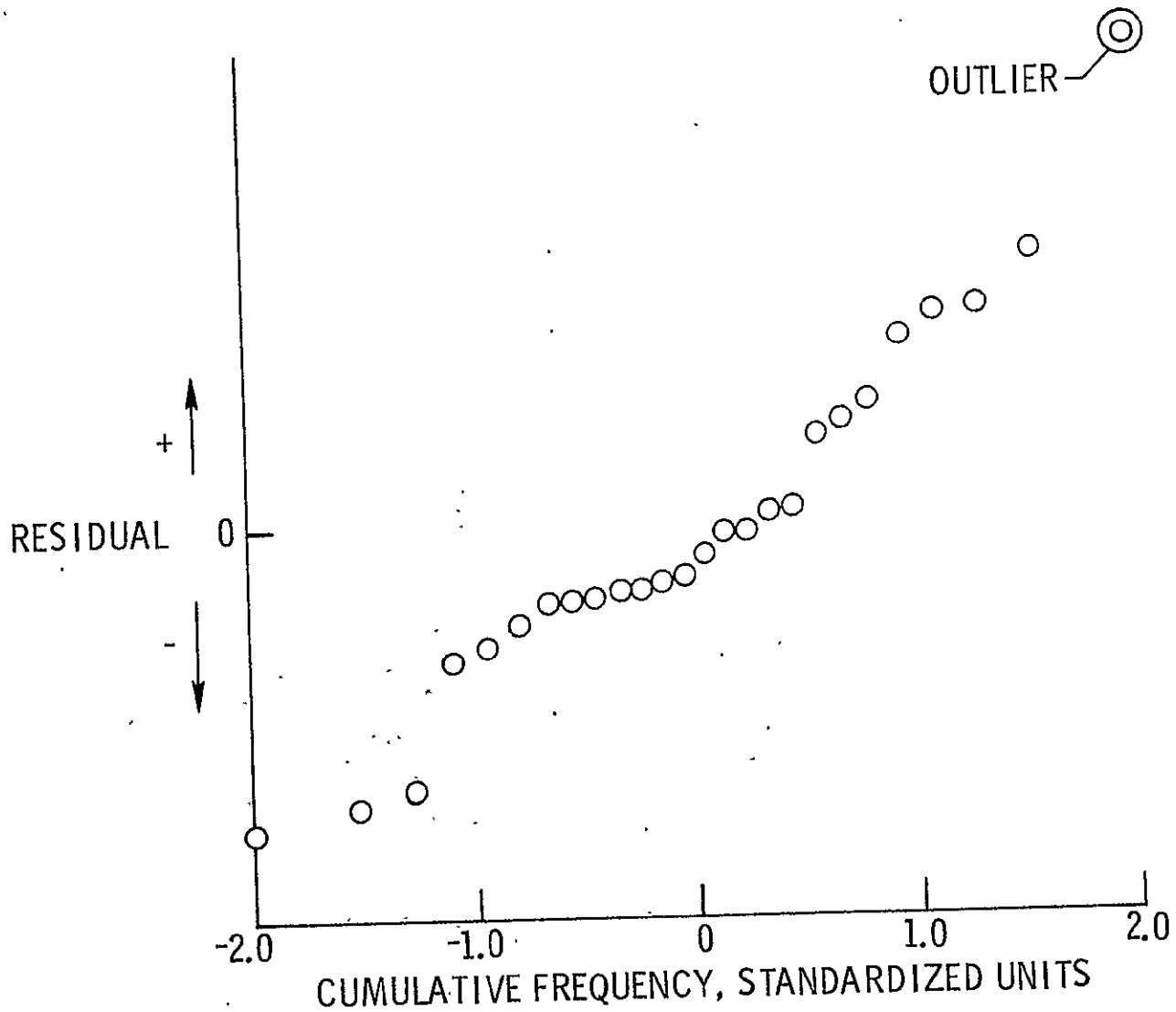


Fig. 5—Cumulative normal distribution of regression residuals for alloy 4, GIVAR analysis.

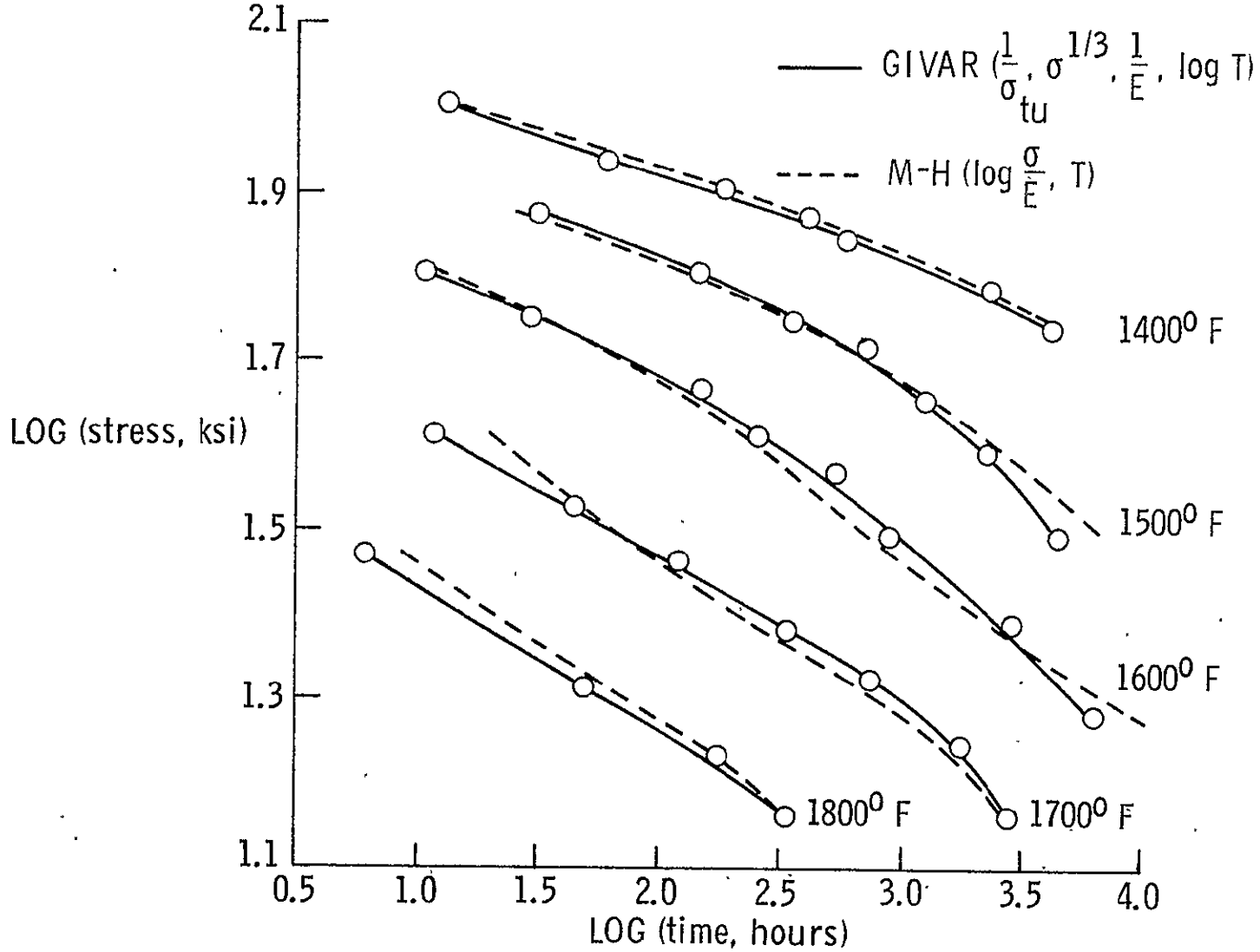


Fig. 6—Comparison of M-H and GIVAR correlations for alloy 16.

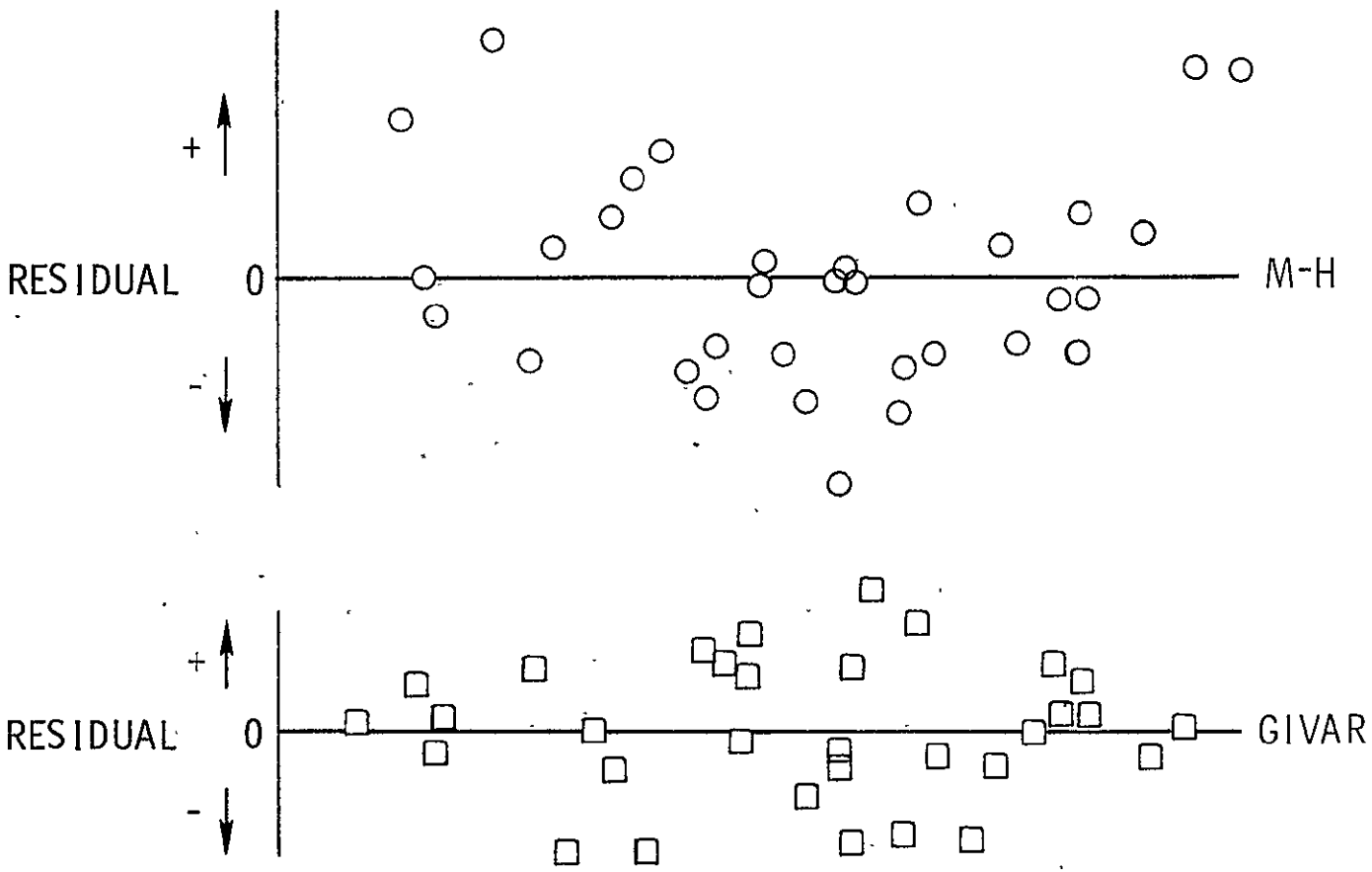


Fig. 7—Comparison of regression residuals for alloy 16.

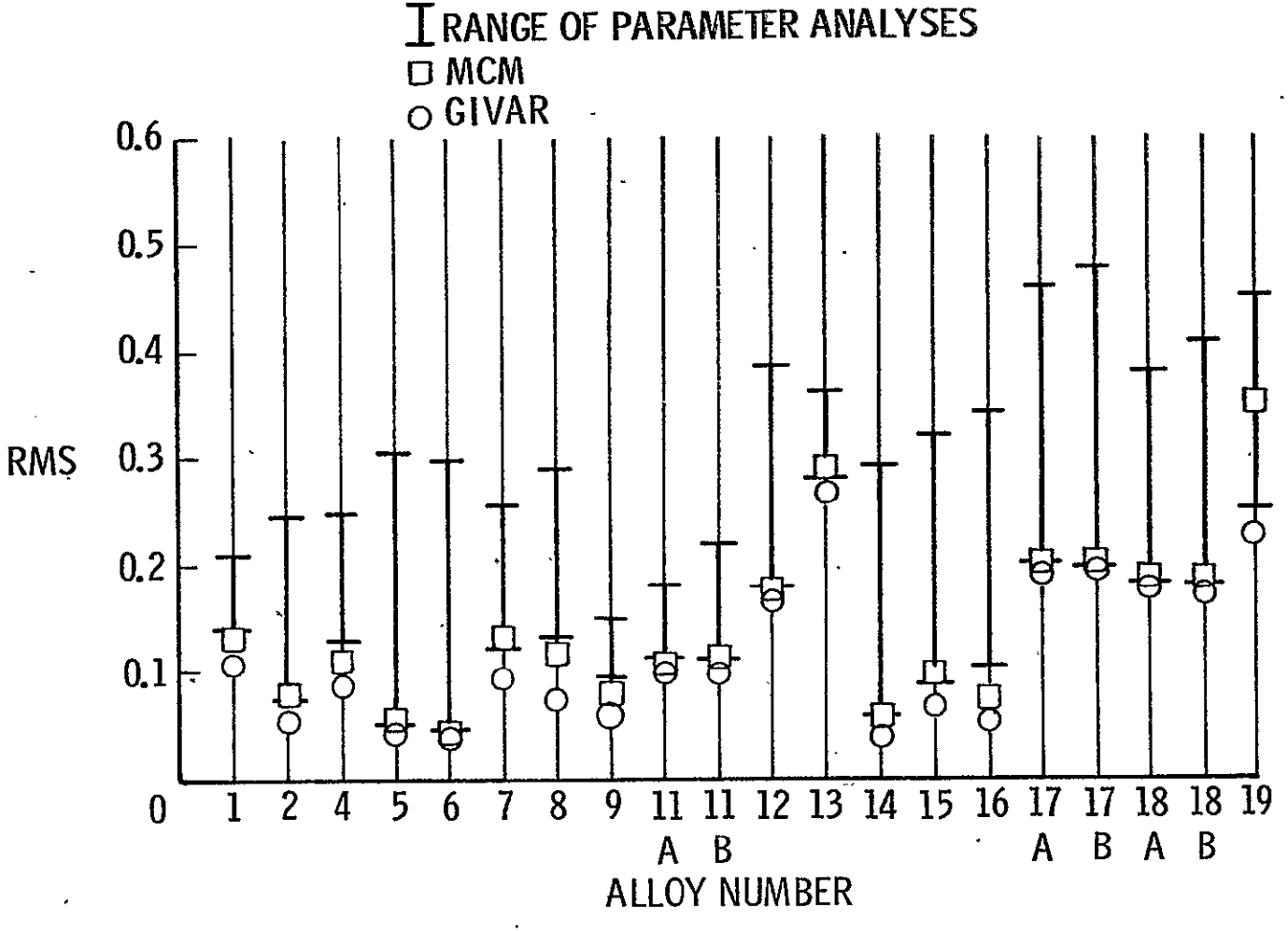


Fig. 8—RMS values for various methods of analysis.

COMPUTER PROGRAM FOR PARAMETRIC ANALYSIS OF CREEP-RUPTURE DATA[1]

By

Donald R. Rummler

[1]This manuscript will be submitted to the National Aeronautics and Space Administration for publication as a Technical Memorandum.

COMPUTER PROGRAM FOR PARAMETRIC ANALYSIS OF CREEP-RUPTURE DATA[1]

By

Donald R. Rummler

ABSTRACT

A computer program which uses several parametric model equations to analyze creep-rupture data is presented in detail. The model equations include the Larson-Miller, Orr-Sherby-Dorn, Manson-Succop, Manson-Haferd, and Rabotnov parameter methods. Standard multiple regression techniques are used to analyze data with respect to each model equation. In addition to the usual regression statistics, the program calculates statistical intervals including confidence and prediction intervals. Graphical output includes a residual plot with respect to the dependent variable and a cumulative distribution of the residuals. The computer input and output, in printed and plotted form, for sample problems are presented to aid the user in setting up and running the program.

SUMMARY

A computer program which uses several parametric model equations to analyze creep-rupture data is presented in detail. The model equations include the Larson-Miller, Orr-Sherby-Dorn, Manson-Succop, Manson-Haferd, and Rabotnov parameter methods. Standard multiple regression techniques are used to analyze data with respect to each model equation. In addition to the usual regression statistics, the program calculates statistical intervals including confidence and prediction intervals. Graphical output includes a residual plot with respect to the dependent variable and a cumulative distribution of the residuals. The program, its subroutines and their variables are listed and defined. The computer input and output, in printed and plotted form, for sample problems are presented to aid the user in setting-up and running the program. The development of the parameter model equations and the use of statistical intervals is discussed.

INTRODUCTION

The importance of creep-rupture data analysis has led to a large number of papers which either propose new parametric analysis approaches (refs. 1, 2, 3, and 4, for example) or offer detailed comparisons of different parametric methods (refs. 4, 5, and 6). Most parametric methods for creep-rupture data analysis are empirical. Consequently, it is common practice for the data analyst to fit the creep-rupture data at hand to a variety of parametric model equations to select the most appropriate analysis method.

Although several analysis methods have been presented in general terms (ref. 6, for example), there is no widely used, efficient computer program tailored specifically to the parametric analysis of creep-rupture data. In addition, most methods do not include generation of statistical intervals to aid in the selection of the "best" parametric model equation for a particular set of data.

This paper describes the development and use of a computer program for the parametric analysis of creep-rupture data. The program includes provisions for the analysis of five different parameter methods. The parametric equations used and the statistical quantities calculated are discussed. The computer program input and output, in printed and plotted form, for three sample problems are presented to aid the user in setting up and running a problem with the program.

PROGRAM DESCRIPTION

The computer program (PARAM) was developed to analyze and correlate creep-rupture data utilizing a variety of parametric method model equations. For each model equation, a function of the time to a particular creep event (such as time to 0.005 strain) is the dependent variable. Functions of stress and temperature are the only correlating independent variables. The major features of the program are as follows:

(1) The method of least squares is used to establish the coefficients for the parametric model equation selected for analysis.

(2) Provisions are made for analysis with four widely used time-temperature methods (Larson-Miller, Orr-Sherby-Dorn, Manson-Succop, and Manson-Haferd) and one time-stress (Rabotnov) method.

(3) Polynomial forms of the parametric model equations up to the fifth order are included.

(4) Multiple analyses can be accomplished during a single computer run.

(5) In addition to the usual regression statistics, the program calculates the maximum and minimum value of each independent variable, as well as its range and average value.

(6) The program also calculates the relative influence, contribution to the sums of squares, and warns of coefficient solution errors for each independent variable.

(7) Listings are made of the observed and fitted values of

the dependent variable in both regression and real variable coordinates.

(8) Two statistical intervals, the 95 percent confidence and the 95 percent prediction, are approximated and calculated for each observation.

(9) Residual plots are made to indicate how the regression residuals are distributed over all of the fitted values of the dependent variable and whether they are normally distributed.

PARAM was written in FORTRAN IV language for the Control Data 6000 series digital computer under the SCOPE 3.0 operating system. The program is dimensioned for a maximum of 5 input variables, a maximum of 10 derived independent variables and a maximum of 200 observations for each data set. It requires approximately 60,000 octal locations of core storage. A source listing of the main program and its subroutines is presented in appendix A. A detailed description of the matrix equation solution subroutine MATINV and the plotting subroutines PSEUDO, DDIPLT and CALPLT are presented in appendix B.

ANALYSIS

The analysis utilizes standard least squares multiple regression analysis techniques (refs. 7 and 8) to solve parametric equations of the following form:

$$Y = b_o + b_1X_1 + b_2X_2 + \text{---} + b_iX_i \qquad (1)$$

where Y = fitted value of dependent variable

$X_1, X_2 \ldots, X_i$ = independent variables

b_o = estimated Y intercept when all $X_i = 0$

$b_1, b_2, - - - b_i$ = estimated coefficients of independent variables

Specifically, the equation forms chosen for each of the parametric methods selected are as follows:

Larson-Miller (L-M)

$$Y = \log t = b_o + b_1/T_R + b_2 S/T_R + b_3 S^2/T_R + b_4 S^3/T_R + b_5 S^4/T_R + b_6 S^5/T_R \quad (2)$$

Orr-Sherby-Dorn (O-S-D)

$$Y = \log t = b_o + b_1/T_K + b_2 S + b_3 S^2 + b_4 S^3 + b_5 S^4 + b_6 S^5 \quad (3)$$

Manson-Succop (M-S)

$$Y = \log t = b_o + b_1 T_F + b_2 S + b_3 S^2 + b_4 S^3 + b_5 S^4 + b_6 S^5 \quad (4)$$

Manson-Haferd (M-H)

$$Y = \log t = b_o + b_1 T_o + b_2 T_o S + b_3 T_o S^2 + b_4 T_o S^3 + b_5 T_o S^3 + b_6 T_o S^4 + b_6 T_o S^5 \quad (5)$$

Rabotnov (RAB)

$$Y = t^a = b_o + b_1/\sigma T_F + b_2/\sigma T_F^2 + b_3/\sigma T_F^3 + b_4/\sigma T_F^4 + b_5/\sigma T_F^5 + b_6/\sigma T_F^6 \quad (6)$$

where

t = time to a particular creep-rupture event, rupture, for example

$S = \log \sigma$

σ = applied stress

T_F = temperature, ^oF

T_K = temperature, Kelvin

T_R = temperature, Rankine

T_o = offset temperature = $T_F - T_A$

b_1, T_A, a = constants estimated by method of least squares.

Both the M-H and RAB techniques require the use of iterative, non-linear multiple regression techniques to estimate all of the constants.

Each parametric equation can be analyzed in truncated form since the number of equation terms (LLO) is selected with input case control cards.

The development of each of the parametric method model equations is presented in appendix C.

PROGRAM USAGE

To submit a problem, information is normally entered on punched cards. Four types of information cards (option, case control, data set identification, and data) are the only input required. Output includes listings and plots.

Input

The option card controls both the printed and graphic output of the program. It also establishes the initial values to be used for the iteratively modified constants for the Manson-Haferd and Rabotnov parametric analyses. The case control cards determine the parametric equation forms to be evaluated and their

degree of truncation. A data identification card and the data cards complete the deck set up. The input card order, format, permitted values and comments follow:

Option card (215, 2F10.0)

Column	FORTRAN Variable	Value	Comments
5	INPUT	0	No listing of input cards
		1	List data set I.D., option, and case control cards
		2	List 1 + data observations
		3	List 2 + regression variables for first five observations
10	OUTPUT	0	No listing of residuals
		1	List regression residuals
		2	1 + list back transformed residuals
		3	2 + regression residual plots
11 to 20	TA		Initial value for constant in non-linear M-H equation; A value of -5000.0 is recommended
21 to 30	RA		Initial value for constant in non-linear RAB equation; A value of 0.2 is recommended.

Column	FORTRAN Variable	Value	Comments
5	NPAM(I)		Parametric expression to be evaluated
		1	Larson-Miller
		2	Orr-Sherby-Dorn
		3	Manson-Succop
		4	Manson-Haferd
		5	Rabotnov
10	LLO(I)	2to6	Number of coefficients to be determined for parametric expression selected, see ANALYSIS section of paper.

The program is dimensioned for a maximum of 20 case control cards. During a single computer run, a data set can be evaluated with 20 different parametric model equation forms. A blank card must follow the last case control card.

Blank Card

Data identification card (8A10)

Column	FORTRAN Variable	Comment
1 to 80	TYPE	Data I.D. Any characters in columns 1 to 80. This title is included in all listed output

Data cards (3F12.0)

Column	FORTRAN Variables	Comments
1-12	RS(I,1)	Time to a particular creep event
13-24	RS(I,3)	Temperature, ^{o}F
25-36	RS(I,2)	Applied stress

The program is dimensioned for a maximum of 200 observations in a data set. Round-off errors can be minimized by limiting the range of the variables. This range reduction is helpful since most creep-rupture data is ill-conditioned (see refs. 7 and 8).

Last data card must be followed by a blank card.

Blank card

More than one set of data may be analyzed with a single set up of the option and case control cards. To analyze additional data sets during a single computer run, assemble the deck as follows:

Option card

Case control cards

Blank card — First data set

Data identification card

Data cards

Blank card

Data identification card

Data cards — Second data set

Blank card

Data identification card

Data cards — Third data set

Blank card

As many data sets as desired may be analyzed during a single computer run with this type of deck setup.

Output

Examples of printed and plotted output are presented in the discussion of sample problems. Most of the output headings are self-explanatory or standard statistical terms (refs. 7 and 8). Some headings are abbreviations of standard terms and/or require additional description. These headings and brief descriptions, in the order of their appearance for the printed output are as follows:

Heading	Description
STANDARD ERROR	Standard error of estimate is square root of residual mean square, sometimes called residual root mean square
MULT. CORREL. COEFF. SQUARED	The multiple correlation coefficient squared, sometimes called coefficient of determination

MIN	The minimum value of indicated variable; independent variables are in tabular form
MAX	The maximum value of indicated variable
Y	Tabulated values of independent variable
X1-X(L2)	Tabulated values of independent variables; L2 is number of variables in case
VARIABLE	Transformation required for parametric method being evaluated
COEF. P.I.	Calculated coefficients for the fitted equation, indexed by I starting with b_o
S.E. COEF.	Estimated standard error of the coefficient
T	COEF.P(I)/S.E. COEF.
RAN X(I)	Range of independent variable
RINF(I)	Relative influence of independent variable, $\frac{(\text{COEF.P(I)(RANX(I))}}{\text{Y RANGE}}$
PSUM	The fraction of the total sums of squares explained by an independent variable; corrected for those independent variables which preceed it in the listing

CERR	The percentage difference between MATINV and Gaussian elimination solutions for coefficient; values in excess of 0.01 suggest round-off errors due to ill-conditioned normal equations
95 PERCENT PREDICTION INTERVAL STATISTICS	The 95 percent prediction interval for a single future observation is estimated for each observation in regression variable space; these values are back transformed into log time space to calculate average and maximum values; values for the t distribution are approximated with a third order polynomial in log (degrees of freedom)
REAL TIME FACTOR	$10.^{(WIDTH)}$
RESIDUALS - REGRESSION SPACE	Values listed under this heading are in terms of the regression dependent variable coordinates
RESIDUAL	Observed value of dependent variable-calculated value of dependent variable
PCTERR	$\frac{(100)(RESIDUAL)}{Y}$

ORDER	The rank order of the residual in regression coordinates; the rank order of the PCTERR in real space coordinates; ordered with respect to the largest absolute value.
CIMIN	Estimated lower limit of 95% confidence interval for the mean
CIMAX	Estimated upper limit of the 95% confidence interval for the mean
PIMIN	Estimated lower limit of 95% prediction interval for a single future observation
PIMAX	Estimated upper limit of the 95% prediction interval for a single future observation

The values of the t distribution required for the calculation of the statistical intervals are approximated with the following expression:

$$TVALUE = (10.0)^{T1}$$

where

$$T1 = 0.86186 - 0.98427\ DF + 0.58495(DF)^2 - 0.11594(DF)^3$$

DF = residual degrees of freedom for regression.

The graphical output of the program includes a plot of the residuals with respect to the calculated value of the dependent variable (FITTED Y) and a cumulative normal distribution of the residuals (ZP NORMAL). For the

ZP NORMAL plot, the plotting points for the abscissa, P, are in terms of the inverse of the standardized normal distribution and are calculated in the following manner:

for FZ = 0 → 0.5

$$ZP_1 = 1.0451 + 4.3598XP + 3.4606(XP)^2 + 1.9088(XP)^3 + 0.5446(XP)^4 + 0.0608(XP)^5$$

where XP = log FZ

FZ = (j - 3/8)/(N + 1/4)

j = 1, 2, - - - N when the residuals are arranged in order of increasing magnitude.

for FZ = 0.5 → 1.0

XP = log (1-FZ)

$ZP_2 = -ZP_1$

The ZP expression approximates the inverse of the standard normal distribution.

SAMPLE CASES

Three sample cases are presented to illustrate operation of the computer program and a method for rapidly selecting the most applicable parametric equation for a single set of creep-rupture data. The data are for a type 316 stainless steel (ref. 5). The three sample cases described in this section required a total of 10.9 seconds of CDC 6600 CPU time to compile and run.

Case 1

For this case, all five parametric methods in second degree form were used to correlate the data. The purpose of this case was to quickly scan the parametric models to select a single parameter for further study. Output was minimized by using INPUT = 1 and IOUT = 0. The program input and output for case 1 are presented in Figures 1 and 2, respectively.

When compared to the other four parameter methods, the O-S-D method had the highest MULT. CORREL. COEF. SQUARED, the lowest AVERAGE and MAXIMUM WIDTH of the 95% prediction interval. It also had the lowest STANDARD ERROR of the four time-temperature parameters.

Case 2

Based upon the results of case 1, the Orr-Sherby-Dorn parameter (NPAM = 2) was selected for further evaluation. The purpose of this case was to quickly determine the degree of the O-S-D expression which would provide the best correlation of the data. Once again, output was minimized (INPUT = 0, IOUT = 0).

The program input and output for case 2 are presented in Figures 3 and 4, respectively.

With respect to MULT. CORREL. COEF. SQUARED, there is no appreciable improvement in the correlation produced by increasing the degree of the polynomial expression. However, the STANDARD ERROR shows a steady decrease as additional variables are added up to the fifth order expression where it increases slightly. The T values for this fifth order expression clearly illustrate the inflation of the standard error of the coefficients which this high level of co-linearity produces. The CERR value for I = 2 (X(I) = LOG STRESS) suggests that the solution matrix was ill-conditioned because the two methods of solution do not agree.

The RESIDUAL SUMS OF SQUARES for the fourth order expression is approximately 30 percent lower than the third order expression. Although significant differences between the other correlation indications are not apparent, the fourth order expression is selected for further evaluation.

Case 3

Final verification of the fourth order expression selected in case 2 requires the full output capabilities of the program (INPUT = 3, IOUT = 3). The input and output for this case are presented in figures 5 and 6. The output includes a listing of the first 5 values of the regression variables, residuals and statistical intervals in regression and back transformed coordinates and plots of residuals with respect to the

calculated dependent variable (Y FITTED) and with respect to the normal cumulative distribution. The most important part of the verification of the fourth order expression is the examination of the residual plots. These plots suggest that the residuals have a zero mean and are randomly distributed with respect to the FITTED Y and that their cumulative distribution is normal. These two characteristics of the residuals are necessary for the calculation of valid statistical intervals.

The method selected for determining the "best" parametric equation for a set of data was used primarily to demonstrate the capabilities of the computer program PARAM. For other methods see references 4, 5, and 6. For a further discussion of the use of statistical intervals, the reader is referred to references 7 and 11.

CONCLUDING REMARKS

A computer program specifically developed for the parametric analysis of creep-rupture data has been discussed. The equations used for the analysis of five parametric methods and the computer program used to implement the analysis are given.

The computer program is versatile, allows rapid assessment of parametric methods for creep-rupture data, and has a relatively small core storage requirement. In addition to the statistics which are usually calculated and output by multiple regression programs, the program outputs the 95% confidence interval on the mean and the 95% prediction interval for a

future observation. Residual plots are provided to assess the validity of the calculated statistical intervals.

APPENDIX A

SOURCE LISTING OF PROGRAM PARAM

```
      PROGRAM PARAM(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,          00000001
     1TAPE7=PUNCH)                                                        00000002
C     PARAM                                                              00000003
C     PROGRAM FOR PARAMETRIC ANALYSIS OF CREEP-RUPTURE DATA              00000004
C     COEFFICIENTS FOR PARAMETRIC MODEL EQUATIONS ARE DETERMINED BY      00000005
C     METHOD OF LEAST SQUARES                                            00000006
C     Y= B0+B1X1+B2X2 ------                                             00000007
C     PARAMETRIC METHODS INCLUDE                                         00000008
C       LARSON-MILLER(L-M)                                               00000009
C       ORR-SHERBY-DORN(O-S-D)                                           00000010
C       MANSON-SUCCOP(M-S)                                               00000011
C       MANSON-HAFERD(M-H)                                               00000012
C       RABOTNOV(RAB)                                                    00000013
C     DONALD R. RUMMLER
C     NASA-LANGLEY RESEARCH CENTER, HAMPTON, VA. , 1976
C     ARRAYS WHICH DEPEND ON NUMBER OF OBSERVATIONS IN DATA SET (L1)     00000016
      DIMENSION  AA( 200),CY( 200), CIMAX( 200), CIMIN( 200)             00000017
      DIMENSION  ERRPER( 200), F( 200,10), IPERM( 200), PYMAX( 200)      00000018
      DIMENSION  PYMIN( 200), RIS( 200), RS( 200,5), TEMP( 200), Y( 200)00000019
      DIMENSION ZP(200)                                                  00000020
C     ARRAYS WHICH DEPEND ON NUMBER OF VARIABLES IN REGRESSION MODEL     00000021
C     NUMBER OF INDEPENDENT VARIABLES (L2)                               00000022
      DIMENSION  CERR(10), PAR(10)                                       00000023
      DIMENSION  PAR1(10), SB(10), SSR(10), SUMA(10), SUMB(10)           00000024
      DIMENSION  SUMP2(10,10), SUMX(10), SUMXY(10,10), SUMX1(10,10)      00000025
      DIMENSION  SUMX2(10), T(10), XMAX(10), XMIN(10), XRAN(10)          00000026
      DIMENSION  XMEAN(10)                                               00000027
C     NUMBER OF COEFFICIENTS DETERMINED (L3)                             00000028
      DIMENSION D(11,11), DD(11,11), E(11,1), G(11,1), INDEX(11,2)       00000029
      DIMENSION IPIVOT(11), X(11)                                        00000030
C     NUMBER OF COEFFICIENTS +1 (N3)                                     00000031
      DIMENSION  A(12,12), B(12,12)                                      00000032
C     ARRAYS WHICH DEPEND UPON OTHER FACTORS                             00000033
                                                                         +
```

```
C      NUMBER OF CASES                                                    00000034
       DIMENSION  LL0(20) , NPAM(20)                                      00000035
C      MISC                                                               00000036
       DIMENSION  TYPE(8),  IN(2), VAR(30), PAM(5)
       DATA(PAM(I),I=1,5)/ 3HL-M, 5HO-S-D, 3HM-S, 3HM-H, 3HRAB/           00000038
       DATA(VAR(I),I=1,30)/3H1/T,3HS/T,6HS**2/T,6HS**3/T,6HS**4/T,        00000039
      16HS**5/T ,                                                         00000040
      23H1/T,1HS,4HS**2,4HS**3,4HS**4,4HS**5,                             00000041
      31HT,1HS,4HS**2,4HS**3,4HS**4,4HS**5,                               00000042
      42HDT,4HDT*S,7HDT*S**2,7HDT*S**3,7HDT*S**4,7HDT*S**5,               00000043
      55H1/L*T,8H1/L*T**2,8H1/L*T**3,8H1/L*T**4,8H1/L*T**5,8H1/L*T**6/    00000044
C      L1 = NUMBER OF OBSERVATIONS IN DATA SET                            00000045
C      L1 IS DETERMINED BY PROGRAM                                        00000046
C      L2 = NUMBER OF VARIABLES INPARAMETRIC EQUATION SELECTED            00000047
C      L3 = NUMBER OF COEFFICIENTS TO BE DETERMINED, INCLUDES B0          00000048
C      L3 = L2+1                                                          00000049
C      CALL PLOT VECTOR FILE ONLY WHEN OUTPUT INCLUDES PLOTTING           00000065
       CALL PSEUDO
       CALL LEROY
C      COMPLETE DATA DECK SETUP INCLUDING OPTION AND CASE CONTROL CARDS
C      FOR EACH DATA SET ARE REQUIRED IF 1 - CONTINUE CARD IS HERE.
     1 CONTINUE                                                           00000081
C      READ INPUT AND OUTPUT OPTIONS AND
C      INITIAL VALUES OF M-H AND RAB CONSTANTS
C      IPUT = INPUT LISTING OPTIONS                                       00000053
C             0 - NO INPUT LISTING
C             1 - CASE CONTROL VARIABLES
C             2 - + DATA SET OBSERVATIONS
C             3   + TRANSFORMED REGRESSION VARIABLES FOR FIRST
C                 FIVE OBSERVATIONS
C      OUTPUT = OUTPUT OPTIONS                                            00000058
C               0 - NO RESIDUALS
C               1 - RESIDUALS REGRESSED SPACE                            00000060
                                                                          +
```

```
C                2 - 1 + REAL SPACE RESIDUALS                                   00000061
C                3 - 2 + RESIDUAL PLOT IN REGRESSED SPACE                       00000062
      READ(5,4) INPUT,IOUT,    TA,RA                                           00000051
    4 FORMAT(2I5,2F10.0)                                                       00000052
      IF(EOF,5)900,9
    9 CONTINUE
C     READ CASE CONTROL CARDS                                                  00000063
C     PUT BLANK CARD AFTER LAST CASE CARD                                      00000064
C     LL0  = TOTAL NUMBER OF VARIABLES FOR CASE                                00000069
C     NPAM = PARAMETRIC EXPRESSION TO BE EVALUATED                             00000070
C              1 - LARSON-MILLER  (LM)                                         00000071
C              2 - ORR-SHERBY-DORN  (OSD)                                      00000072
C              3 - MANSON-SUCCOP  (MS)                                         00000073
C              4 - MANSON-HAFERD  (MH)                                         00000074
C              5 - RABOTNOV  (RAB)                                             00000075
      I3=1                                                                     00000067
    3 READ(5,2) NPAM(I3),LL0(I3)                                               00000068
      IF(LL0(I3)) 7,8,7                                                        00000076
    7 I3=I3+1                                                                  00000077
      GO TO 3                                                                  00000078
    2 FORMAT (2I5)                                                             00000079
    8 I3=I3-1                                                                  00000080
C     ONLY ONE SETUP OF OPTION AND CASE CONTROL CARDS ARE REQUIRED
C     FOR MANY DATA SETS IF 1 - CONTINUE CARD IS HERE
C   1 CONTINUE
C     READ DATA SET IDENIFICATION (TYPE)                                       00000082
      READ(5,777)(TYPE(I),I=1,8)                                               00000083
  777 FORMAT(8A10)                                                             00000084
      IF(EOF,5) 900,6                                                          00000085
    6  I=1                                                                     00000086
C     READ IN OBSERVATIONS                                                     00000088
C     IF NUMBER OF CORRELATING VARIABLES CHANGES,
C     CHANGE STATEMENTS 5 AND 10
C     RS(I,1)= RUPTURE TIME                                                    00000091
C     RS(I,2)= APPLIED STRESS,PSI                                              00000092
C     RS(I,3)= TEST TEMPERATURE, DEGREES F                                     00000093
```

```
C       PUT BLANK CARD BEHIND LAST DATA CARD                                    00000089
   10   READ(5,5) RS(I,1), RS(I,3), RS(I,2)                                     00000090
        IF(EOF,5) 900,901                                                       00000094
  901   CONTINUE                                                                00000095
        IF (RS(I,1)-0.) 11,12,11                                                00000096
   11   I=I+1                                                                   00000097
        GO TO 10                                                                00000098
    5   FORMAT(3F12.0)                                                          00000099
   12   L1=I-1                                                                  00000100
        IF(INPUT-1)301,300,300                                                  00000101
C       INPUT = 1 LISTING                                                       00000102
  300   WRITE(6,414)                                                            00000103
        WRITE(6,220)                                                            00000104
        WRITE(6,221)                                                            00000105
        WRITE(6,502)(TYPE(I),I=1,8)                                             00000106
  302   FORMAT(     10X,*DATA SET*/10X, 8A10/)                                  00000107
        WRITE(6,299)                                                            00000108
  299   FORMAT(* OPTION CARD*)                                                  00000109
        WRITE(6,303)INPUT,IOUT,  TA,RA                                          00000110
  303   FORMAT(* INPUT= *,I1/* IOUT= *  ,I1/* TA= *,F10.0/* RA= *,F10.4/)      00000111
        WRITE(6,304)                                                            00000112
  304   FORMAT(* CASE CONTROL CARDS*/5X,* PARAMETER CODE*,5X,                   00000113
       1*NO. COEFFICIENTS*/)                                                    00000114
        WRITE(6,305)(NPAM(I),LL0(I),I=1,13 )                                    00000115
  305   FORMAT(10X,I5,15X,I5)                                                   00000116
  301   CONTINUE                                                                00000117
C       INPUT = 2 LISTING                                                       00000118
        IF(INPUT-2) 309,308,308                                                 00000119
  308   WRITE(6,414)                                                            00000120
        WRITE(6,306)                                                            00000121
                                                                                +
```

```
  306 FORMAT(5X,*INPUT DATA OBSERVATIONS*/ 3X,*NO.*,14X,* TIME*, 5X,     00000122
     1 *STRESS*, 5X,*TEMPERATURE*)                                       00000123
      WRITE(6,307)(I,RS(I,1),RS(I,2),RS(I,3) , I=1,L1)                   00000124
  307 FORMAT(I5,10X,F10.2,    F8.0 , 4X,F10.0)                           00000125
  309 CONTINUE                                                           00000126
C     START  CASE LOOP (I3)                                              00000127
C     I3 = NUMBER OF CASES (PARAMETRIC EQUATIONS) TO BE EXAMINED
C          FOR EACH DATA SET
      DO 350 KK= 1,I3                                                    00000130
      NEGSB=0                                                            00000131
      L3=LL0(KK)                                                         00000132
      L2=L3-1                                                            00000133
      LAST=0                                                             00000134
      BMSE= 1000000.                                                     00000135
      XN=L1                                                              00000136
      LIM=0                                                              00000137
      IFG=0                                                              00000138
      L=NPAM(KK)                                                         00000139
      IF(L-4)22,21,20                                                    00000140
   20 CONTINUE                                                           00000141
C     RABOTNOV CONSTANTS                                                 00000142
      C=RA                                                               00000143
      DEL =0.1                                                           00000144
      DELMIN=0.001                                                       00000145
      GO TO 23                                                           00000146
C     MANSON-HAFERD CONSTANTS                                            00000147
   21 CONTINUE                                                           00000148
      C=TA                                                               00000149
       DEL = 1000.0                                                      00000150
      DELMIN=10.                                                         00000151
      GO TO 23                                                           00000152
   22 LAST=2                                                             00000153
   23 CONTINUE                                                           00000154
                                                                          +
```

```
   57 CONTINUE                                                          00000155
C     SELECT PARAMETRIC FORM FOR REGRESSION                             00000156
      L=NPAM(KK)                                                        00000157
      GO TO (61,62,63,64,65),L                                          00000158
   61 CALL LM(Y,RS,F,L1)                                                00000159
      GO TO 66                                                          00000160
   62 CALL OSD(Y,RS,F,L1)                                               00000161
      GO TO 66                                                          00000162
   63 CALL MS(Y,RS,F,L1)                                                00000163
      GO TO 66                                                          00000164
   64 CALL MH( Y,RS,F,L1,C )                                            00000165
      GO TO 66                                                          00000166
   65 CALL RAB (Y,RS,F,L1,C)                                            00000167
   66 CONTINUE                                                          00000168
      SSER=0.0                                                          00000169
C     ZERO A,B,SUMX1 ARRAYS                                             00000170
      DO 473 M=1,12                                                     00000171
      DO 473 J=1,12                                                     00000172
      A(M,J)=0.0                                                        00000173
      B(M,J)=0.0                                                        00000174
  473 SUMX1(M,J)=0.0                                                    00000175
      DO 105 M=1,L2                                                     00000176
      SUMX(M)=0.0                                                       00000177
      DO105I=1,L1                                                       00000178
  105 SUMX(M)=SUMX(M)+F(I,M)                                            00000179
      DO 106 M=1,L2                                                     00000180
      DO 106 J=1,L2                                                     00000181
      SUMP2(M,J)=0.0                                                    00000182
      DO106I=1,L1                                                       00000183
  106 SUMP2(M,J)=SUMP2(M,J)+F(I,M)*F(I,J)                               00000184
      SUMY=0.0                                                          00000185
      SUMY2=0.0                                                         00000186
      DO107I=1,L1                                                       00000187
                                                                        +
```

```
      SUMY=SUMY+Y(I)                                                    00000188
  107 SUMY2=SUMY2+Y(I)**2                                               00000189
      DO 108 M=1,L2                                                     00000190
      SUMXY(M)=0.0                                                      00000191
      DO108I=1,L1                                                       00000192
  108 SUMXY(M)=SUMXY(M)+F(I,M)*Y(I)                                     00000193
      DO 109 M=1,L2                                                     00000194
      DO 109 J=1,L2                                                     00000195
  109 SUMX1(M,J)=SUMP2(M,J)-(SUMX(M)*SUMX(J))/XN                        00000196
      DO 110 M=1,L2                                                     00000197
  110 SUMX1(M,L3)=SUMXY(M)-(SUMX(M)*SUMY)/XN                            00000198
      SUMX1(L3,L3)=SUMY2-(SUMY**2)/XN                                   00000199
      DO42M=1,L3                                                        00000200
   42 SUMX1(L3,M)=SUMX1(M,L3)                                           00000201
      N3=L3+1                                                           00000202
      DO16I=1,L3                                                        00000203
      SUMX1(I,N3)=0.0                                                   00000204
      DO16M=1,L3                                                        00000205
   16 SUMX1(I,N3)=SUMX1(I,N3)+SUMX1(I,M)                                00000206
      DO17J=1,N3                                                        00000207
   17 A(1,J)=SUMX1(1,J)                                                 00000208
      SUMB(1)=0.0                                                       00000209
      DO18J=1,N3                                                        00000210
      B(1,J)=A(1,J)/A(1,1)                                              00000211
   18 SUMB(1)=SUMB(1)+B(1,J)                                            00000212
      SUMB(1)=SUMB(1)-B(1,N3)                                           00000213
      DO115I=2,L3                                                       00000214
      DO115J=I,N3                                                       00000215
      NIX=I-1                                                           00000216
      TEMP=0.0                                                          00000217
      DO116I1=1,NIX                                                     00000218
  116 TEMP=-A(I1,I)*B(I1,J)+TEMP                                        00000219
      A(I,J)=TEMP+SUMX1(I,J)                                            00000220
                                                                        +
```

```
  115 B(I,J)=A(I,J)/A(I,I)                                                  00000221
      DO29I=1,L2                                                            00000222
   29 SSR(I)=A(I,L3)*B(I,L3)                                                00000223
      REGSS=SUMX1(L3,L3)-A(L3,L3)                                           00000224
      SSER=A(L3,L3)                                                         00000225
      CORC=REGSS/SUMX1(L3,L3)                                               00000226
      XN1=L1-L2-1                                                           00000227
      XMSER=SSER/XN1                                                        00000228
      ZIP=XMSER                                                             00000229
      STD=SQRT(XMSER)                                                       00000230
      XMRSS=REGSS/L2                                                        00000231
      FTRSS=XMRSS/XMSER                                                     00000232
      TOTSUM=SUMX1(L3,L3)                                                   00000233
C     ITERATE ON LOWEST 1-CORC FOR RABOTNOV SOLUTION                        00000234
      IF(NPAM(KK).EQ.5) ZIP=1.0-CORC                                        00000235
C     LOOP AROUND ITERATION FOR L-M,O-S-D, AND M-S SOLUTIONS                00000236
      L=NPAM(KK)                                                            00000237
      IF(L.LT.4) GO TO 1439                                                 00000238
      IF(LAST-1) 51,52,1439                                                 00000239
   51 CONTINUE                                                              00000240
      CALL ITER(C    ,CBEST,ZIP   ,BMSE,IFG,ICT, DEL,DELMIN,LIM,LAST)      00000241
      GO TO 57                                                              00000242
   52 CONTINUE                                                              00000243
      C=CBEST                                                               00000244
      LAST=2                                                                00000245
      GO TO 57                                                              00000246
 1439 CONTINUE                                                              00000247
      DO117I=2,L2                                                           00000248
      SUMA(I)=0.0                                                           00000249
      SUMB(I)=0.0                                                           00000250
      DO117J=1,L3                                                           00000251
      SUMA(I)=SUMA(I)+A(I,J)                                                00000252
  117 SUMB(I)=SUMB(I)+B(I,J)                                                00000253
                                                                            +
```

```
      DO80 I=1,L2                                                          00000254
      DO80 J=1,L2                                                          00000255
   80 D(I,J)=SUMX1(I,J)                                                    00000256
      DD(1,1)=L1                                                           00000257
       DO72 M=1,L2                                                         00000258
        I=M+1                                                              00000259
   72   DD(I,1)=SUMX(M)                                                    00000260
        DO73 K=1,L2                                                        00000261
       J=K+1                                                               00000262
   73   DD(1,J)=SUMX(K)                                                    00000263
      DO74 M=1,L2                                                          00000264
      I=M+1                                                                00000265
      DO74 K=1,L2                                                          00000266
      J=K+1                                                                00000267
   74 DD(I,J)= D(M,K)                                                      00000268
C  CHANGE L9 WHEN YOU REDIMENSION PROGRAM                                  00000269
      L9=11                                                                00000270
      CALL MATINV(L9,L3,DD,0,G,1,DETERM,ISCALE,IPIVOT,INDEX)               00000271
      DO81 I=1,L2                                                          00000272
   81 E(I,1)=SUMX1(I,L3)                                                   00000273
      CALL MATINV(L9,L2,D ,1,E,1,DETERM,ISCALE,IPIVOT,INDEX)               00000274
      PAR(1)=B(L2,L3)                                                      00000275
      M3=L2                                                                00000276
      K3=L2                                                                00000277
      DO113I=2,L2                                                          00000278
      MIX=I-1                                                              00000279
      M3=M3-1                                                              00000280
      TEMP1=0.0                                                            00000281
      DO114I1=1,MIX                                                        00000282
      TEMP1=-PAR(I1)*B(M3,K3)+TEMP1                                        00000283
  114 K3=K3-1                                                              00000284
      PAR(I)=TEMP1+B(K3,L3)                                                00000285
  113 K3=L2                                                                00000286
                                                                             +
```

```
      DO47M=1,L2                                                        00000287
   47 SUMX2(M)=SUMX(M)/XN                                               00000288
      SUMY3=SUMY/XN                                                     00000289
      K4=L2                                                             00000290
      DO216I=1,L2                                                       00000291
      PAR1(K4)=PAR(I)                                                   00000292
  216 K4=K4-1                                                           00000293
      PARO=0.0                                                          00000294
      DO217I=1,L2                                                       00000295
  217 PARO=PARO-PAR1(I)*SUMX2(I)                                        00000296
      PARO=PARO+SUMY3                                                   00000297
      N5=L1-L2-1                                                        00000298
      XN1=N5                                                            00000299
      XMSER=SSER/XN1                                                    00000300
      IF(XMSER .GT. 9.0E+100) GO TO 350                                 00000301
      DO49I=1,L2                                                        00000302
      IF(D(I,I).LT.0.0)WRITE(6,1100)(I,D(I,I))                          00000303
      IF(D(I,I).LT.0.0) NEGSB=NEGSB+1                                   00000304
C     AVOID MODE 2 DUMP ABORT CASE 3/3/76                               00000305
      IF(D(I,I).LT. 0.0) GO TO 350                                      00000306
 1100 FORMAT(//,5X,*****NEGATIVE SB(I),I=*,I3,*DI=*,E20.8)              00000307
   49 SB(I)=SQRT(ABS(D(I,I) *XMSER))                                    00000308
      DO118I=1,L2                                                       00000309
      T(I)=PAR1(I)/SB(I)                                                00000310
  118 T(I)= ABS(T(I))                                                   00000311
C     CY(I)= SOLUTION IN RFGRESSION SPACE                               00000312
      DO122I=1,L1                                                       00000313
      SUMCY =0.0                                                        00000314
      TEM=0.0                                                           00000315
      DO123M=1,L2                                                       00000316
  123 TEM=TEM+PAR1(M)*F(I,M)                                            00000317
      SUMCY =SUMCY+TEM                                                  00000318
  122 CY(I)=SUMCY    +PARO                                              00000319
                                                                        +
```

```
C       CALCULATE MIN, MAX, RANGE, MEAN                                  00000320
        CALL MINMAX(YMIN,YMAX,YRAN,YMEAN,Y,L1)                           00000321
        DO 95 I=1,L2                                                     00000322
        DO 96 K=1,L1                                                     00000323
   96   X(K)=F(K,I)                                                      00000324
        CALL MINMAX(XMIN(I), XMAX(I), XRAN(I),XMEAN(I),X,L1)             00000325
   95   CONTINUE                                                         00000326
C       PRINT REGRESSION STATISTICS                                      00000327
        WRITE(6,414)                                                     00000328
        WRITE(6,220)                                                     00000329
        WRITE(6,221)                                                     00000330
  220   FORMAT( 5X,* LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC*)   00000331
  221   FORMAT( 5X,* ANALYSIS OF CREEP-RUPTURE DATA*)                    00000332
        WRITE(6,526)                                                     00000333
  526   FORMAT( /*   -------  REGRESSION VALUES  --------*)              00000334
        WRITE(6,502)(TYPE(I),I=1,8)                                      00000335
  502   FORMAT(*  DATA SET                *,1X,8A10)                     00000336
        MM=NPAM(KK)                                                      00000337
        WRITE (6,506)PAM(MM)                                             00000338
  506   FORMAT(*  PARAMETER SELECTED                *, 7X, A8)          00000339
        WRITE(6,507)( L1)                                                00000340
  507   FORMAT(*  NO. OF OBSERVATIONS               *, 8X, I4)          00000341
        WRITE(6,508)( L2)                                                00000342
  508   FORMAT(*  NO. OF INDEPENDENT VARIABLES      *, 8X, I4)          00000343
        WRITE(6,509)(XN1)                                                00000344
  509   FORMAT(*  RESIDUAL DEGREES OF FREEDOM       *, 8X, F4)          00000345
        WRITE(6,514)( FTRSS )                                            00000346
  514   FORMAT(*  F - VALUE                         *, F12.1)           00000347
        WRITE(6,545)(XMSER)                                              00000348
  545   FORMAT(*  RESIDUAL MEAN SQUARE              *, E12.4)           00000349
        WRITE(6,546)(  STD)                                              00000350
  546   FORMAT(*  STANDARD ERROR                    *, E12.4)           00000351
        WRITE(6,547)( SSER)                                              00000352
                                                                          +
```

```
  547 FORMAT(*  RESIDUAL  SUM OF SQUARES          *, F12.4)                00000353
      WRITE(6, 548)(TOTSUM)                                                00000354
  548 FORMAT(*  TOTAL SUMS OF SQUARES             *, E12.4)                00000355
      WRITE(6, 549)(CORC)                                                  00000356
  549 FORMAT(*  MULT. CORREL. COEF. SQUARED        *, F12.4/)               00000357
C     WRITE(6,320)                                                         00000358
  320 FORMAT(/)                                                            00000359
      IF(MM.EQ.4) WRITE(6,432) CBEST                                       00000360
      IF(MM.EQ.5) WRITE(6,433) CBEST                                       00000361
  432 FORMAT(* MANSON - HAFERD CONSTANT(TA) =*,F10.1/)                     00000362
  433 FORMAT(* RABOTNOV CONSTANT (RA) =*,F10.5/)                           00000363
      WRITE(6,492)(YMIN,YMAX,YRAN,YMEAN)                                   00000364
  492 FORMAT(5 X,* MIN Y =*,E11.2,3X,* MAX Y =*,E11.2,3X,* Y RANGE =*,     00000365
     1    E11.2,3X* MEAN Y =*,E11.2/)                                       00000366
C     INPUT= 3 LISTING                                                     00000367
      IF(INPUT-3)311,310,311                                               00000368
  310 WRITE(6,312)                                                         00000369
  312 FORMAT(/ 5X,*FIRST 5 OBSERVATIONS - TRANSFORMED VARIABLES*/          00000370
     15X,*Y*,18X,* X1 - X(L2) *)                                           00000371
      DO 313 I=1,5                                                         00000372
      DO 315 J=1,L2                                                        00000373
  315 TEMP(J)=F(I,J)                                                       00000374
      WRITE(6,314)(Y(I),(TEMP(J),J=1,L2))                                  00000375
  313 CONTINUE                                                             00000376
  314 FORMAT(8E15.5)                                                       00000377
      WRITE(6,320)                                                         00000378
  311 CONTINUE                                                             00000379
      WRITE(6,422)                                                         00000380
  422 FORMAT(3X,*    I   *,2X,*VARIABLE*, 4X,*COEF.P(I)*,3X, *S.E.COEF.*,  00000381
     1 4X, *T*,              5X, *MEAN X(I)*, 3X, *MIN X(I)*, 3X,*MAX X(I)00000382
     2*,3X, *RAN X(I)*, 4X,*RINF*, 3X, *PSUM*, 3X,*CERR*)                  00000383
      WRITE(6,535)(PARO)                                                   00000384
  535 FORMAT(6X,*0*,11X,E14.4)                                             00000385
                                                                           +
```

```
      M=NPAM(KK)*6-6                                                    00000386
      DO 420 I=1,L2                                                     00000387
      CERR(I)=100.0*((PAR1(I)-E(I))/PAR1(I))                            00000388
      RINF=(XRAN(I)*PAR1(I)+1.0E-30) /YRAN                              00000389
      SSRR=SSR(I)/REGSS                                                 00000390
      WRITE(6,421)(I,VAR(I+M),PAR1(I), SB(I),T(I),XMEAN(I),             00000391
     1 XMIN(I),XMAX(I),XRAN(I), RINF,SSRR,CERR(I))                      00000392
  420 CONTINUE                                                          00000393
  421 FORMAT(      I7, 4X,A8,1X,  E12.4, E11.2,  F7.2,                  00000394
     1        E13.3, 3E11.2,  F8.2, F7.3, F7.2)                       00000395
      WRITE(6,424)                                                      00000396
  424 FORMAT( /* VARIABLE CODE*/10X,*S=LOG STRESS*/10X,*T=TEMPERATURE*  00000397
     1/10X,*DT=T-TA*/10X,*L=STRESS*/)                                   00000398
      CUMERR=0.0                                                        00000399
      RRIS=0.0                                                          00000400
      SRIS2=0.0                                                         00000401
      EMAX=0.0                                                          00000402
      EMAXP=0.0                                                         00000403
      NZERO=0                                                           00000404
      SDP=0.0                                                           00000405
      SSDP=0.0                                                          00000406
      DPMAX = -10.0                                                     00000407
       XDF=ALOG10(XN1)                                                  00000408
       T6=0.8618559 -0.9842715*XDF+0.5849466*XDF**2-.1159365*XDF**3     00000409
      T6=10.**T6                                                        00000410
CCCC **** START 333 LOOP*****                                           00000411
      DO333M=1,L1                                                       00000412
      X(1)=1.0                                                          00000413
      DO 92 K=1,L2                                                      00000414
      I=K+1                                                             00000415
   92  X(I)=F(M,K)                                                      00000416
      DO 100 J =1,L3                                                    00000417
        TEMP(J) =0.0                                                    00000418
                                                                        +
```

```
      DO 100 I =1,L3                                                    00000419
      TEMP(J)= TEMP(J)+ X(I)* DD(I,J)                                   00000420
  100 CONTINUE                                                          00000421
      ANS =0.0                                                          00000422
      DO 200  J=1,L3                                                    00000423
      ANS= ANS +TEMP(J)*X(J)                                            00000424
  200 CONTINUE                                                          00000425
       XMER = XMSER                                                     00000426
       XMER= ABS(XMER)                                                  00000427
       ANS = ABS(ANS)                                                   00000428
C     CALCULATE 95 PERCENT STATISTICAL INTERVALS                        00000429
      DELTA=T6*SQRT(XMER*ANS)                                           00000430
      CIMAX(M)=CY(M)+DELTA                                              00000431
      CIMIN(M)=CY(M)-DELTA                                              00000432
      DELTA=T6*SQRT(XMER*(1+ANS))                                       00000433
      PYMAX(M)=CY(M)+DELTA                                              00000434
      PYMIN(M)=CY(M)-DELTA                                              00000435
      RIS(M)= CY(M)- Y(M)                                               00000437
C     AVOID DUMP WHEN Y=0  2/25/76                                      00000438
      IF(Y(M).EQ.0.0) Y(M)=0.000001                                     00000439
      ERRPER(M)=RIS(M)/Y(M)*100.                                        00000440
      IF( ABS(RIS(M)).GT. ABS(EMAX))EMAX = RIS(M)                       00000441
      RIS2   =RIS(M)**2                                                 00000442
      SRIS2=SRIS2+RIS2                                                  00000443
      RRIS=RRIS+ABS(RIS(M))                                             00000444
      IF(ABS(ERRPER(M)).GT. ABS(EMAXP)) EMAXP = ERRPER(M)               00000445
      CUMERR=CUMERR+ABS(ERRPER(M))                                      00000446
  333 CONTINUE                                                          00000447
CCCC ****** END 333 LOOOP $$$$**                                        00000448
C     FIND OBSERVATIONS OUTSIDE OF 95 PERCENT PREDICTION INTERVAL       00000449
  401 FORMAT(/* OBSERVATIONS OUTSIDE OF 95 PERCENT PREDICTION INTERVAL*/00000450
     15X,*OBS.*, 5X,*CALC Y*, 5X,*PYMIN*, 5X,*PYMAX* /)                 00000451
      IBAD=0                                                            00000452
                                                                        +
```

```
      DO 400 I=1,L1                                                          00000453
      IF(CY(I)-PYMIN(I)) 402,399,399                                         00000454
  399 CONTINUE                                                               00000455
      IF(CY(I)-PYMAX(I)) 400,400,402                                         00000456
  402 IBAD=IBAD+1                                                            00000457
      IF(IBAD.EQ.1) WRITE(6,401)                                             00000458
      WRITE(6,403)( I, CY(I),PYMIN(I),PYMAX(I) )                             00000459
  400 CONTINUE                                                               00000460
  403 FORMAT ( I5,3E16.6)                                                    00000461
C     DETERMINE DP STATISTICS IN TERMS OF LOG TIME TO RUPTURE                00000462
      DPSUM=0.0                                                              00000463
      DPMAX= -100.0                                                          00000464
      IF(NPAM(KK)-5) 404,406,404                                             00000465
  404 DO 405 I=1,L1                                                          00000466
      DP=PYMAX(I)-PYMIN(I)                                                   00000467
      IF(DP.GT.DPMAX) DPMAX=DP                                               00000468
      DPSUM= DPSUM+DP                                                        00000469
  405 CONTINUE                                                               00000470
      GO TO 408                                                              00000471
C     RABOTNOV  DP                                                           00000472
  406 DO 407 I=1,L1                                                          00000473
C     AVOID NEGATIVE PY DUMP                                                 00000474
      IF(PYMIN(I).LT.0.0) PYMIN(I)=1.0                                       00000475
      IF(PYMAX(I).LT.0.0) PYMAX(I)=1.0                                       00000476
      RP1=PYMAX(I)**(1.0/CRFST)                                              00000477
      RP2=PYMIN(I)**(1.0/CBEST)                                              00000478
      DP= ALOG10(RP1)-ALOG10(RP2)                                            00000479
      IF(DP.GT.DPMAX)DPMAX=DP                                                00000480
      DPSUM=DPSUM+DP                                                         00000481
  407 CONTINUE                                                               00000482
  408 DPAVE = DPSUM/L1                                                       00000483
      RP1=10.**DPAVE                                                         00000484
      RP2=10.**DPMAX                                                         00000485
                                                                             +
```

```
      WRITE(6,410)                                                      00000486
  410 FORMAT( 5X,* 95 PERCENT PREDICTION INTERVAL STATISTICS*/25X,       00000487
     1*LOG TIME*,10X,*REAL TIME FACTOR (ANTILOG WIDTH)*/)               00000488
      WRITE(6,409)(DPAVE,RP1,DPMAX,RP2)                                 00000489
  409 FORMAT(* AVERAGE WIDTH *, 5X, F10.3,19X, F10.1/* MAXIMUM WIDTH*,   00000490
     1 6X, F10.3,19X,F10.1 )                                            00000491
C     ORDER RESIDUALS - LARGEST TO SMALLEST                             00000492
      L1NEG =-L1                                                        00000493
      DO 2100 I=1,L1                                                    00000494
      TEMP(I) =0.0                                                      00000495
 2100 TEMP(I)= ABS(RIS(I))                                              00000496
      CALL AORDER(TEMP , L1NEG,IPERM)                                   00000497
      DO 1202 I=1,L1                                                    00000498
      J=IPERM(I)                                                        00000499
      TEMP(J)=I                                                         00000500
 1202 CONTINUE                                                          00000501
C     OUTPUT = 1 OR GREATER                                             00000502
C     RESIDUALS IN REGRESSED SPACE                                      00000503
      IF(IOUT  -1) 413,412,412                                          00000504
  412 CONTINUE                                                          00000505
      WRITE(6,414)                                                      00000506
      WRITE(6,415)                                                      00000507
  414 FORMAT(1H1)                                                       00000508
  415 FORMAT(*  RESIDUALS - REGRESSION SPACE*/)                         00000509
      WRITE(6,416)                                                      00000510
      WRITE(6,417)(I,Y(I),CY(I),RIS(I),ERRPER(I), TEMP(I),              00000511
     1 CIMAX(I),CIMIN(I),PYMIN(I),PYMAX(I),I=1,L1)                      00000512
  417 FORMAT(I5, 1X,3E12.3,    ,F10.1, 8X,F5, 4X, 4E12.3)               00000513
  416 FORMAT( 2X,*OBS*, 7X,*Y OBS*, 6X,*Y CALC*,5X,  *RESIDUAL*,6X,     00000514
     1 *PCTERR       *, 1X,*ORDER*,7X ,*CIMIN*, 7X,*CIMAX*,7X,          00000515
     2 *PIMIN*,7X ,*PIMAX*/)                                            00000516
  413 CONTINUE                                                          00000517
      XMRSS=REGSS/L2                                                    00000518
                                                                        +
```

```
      FTRSS=XMRSS/XMSER                                                  00000519
      DPAVE= SDP/L1                                                      00000520
      DPSIG=(L1*SSDP-SDP**2)/(L1*(L1-1.0))                               00000521
      DPSIG=SQRT(DPSIG)                                                  00000522
      STD=SQRT(XMSER)                                                    00000523
C     PLOTTING ROUTINE                                                   00000524
C     PLOT RESIDUALS WITH VARIAN ON LINE PLOTTER                         00000525
      IF(IOUT  -3)445,440,440                                            00000526
  440 CONTINUE                                                           00000527
      IN(1)= 5HPARAM                                                     00000528
      IN(2)= 4HPLOT                                                      00000529
      N=L1                                                               00000530
      ISYMD=12                                                           00000531
      IEC=1                                                              00000532
      CALL MINMAX(YL,YH,YRAN,YMEAN,RIS,L1)                               00000533
      YL=1.8*YL                                                          00000534
      YH=1.8*YH                                                          00000535
      XL=0.0                                                             00000536
      XH=0.0                                                             00000537
      NXM=1                                                              00000538
      NYM=1                                                              00000539
      YNOTE= 10H  RESIDUAL                                               00000540
      XNOTE5 = 10HZP  NORMAL                                             00000541
      XNOTE6= 10H  FITTED Y                                              00000542
      CALL VDIPLT(IEC, IN, N,  CY(1), RIS(1),  XL,XH,YL,YH,NXM,          00000543
     1  XNOTE6, NYM, YNOTE, ISYMD)                                       00000544
      CALL AORDER(RIS,L1,IPERM)                                          00000545
      DO 430 I=1,L1                                                      00000546
      J=IPERM(I)                                                         00000547
      TEMP(I)= RIS(J)                                                    00000548
       XI =I                                                             00000549
      FZ=(XI-.375)/(L1+.25)                                              00000550
      IF(FZ-0.5)570,570,571                                              00000551
                                                                        +
```

```
  570 XX=ALOG10(FZ)                                                      00000552
         ZP(I)= 1.04505 + 4.35979*XX + 3.46057*XX**2+ 1.90878*XX**3      00000553
     1 + 0.54456*XX**4+ 0.0608*XX**5                                     00000554
      GO TO 572                                                          00000555
  571 XX=ALOG10(1.0-FZ)                                                  00000556
         ZP(I)= 1.04505 + 4.35979*XX + 3.46057*XX**2+ 1.90878*XX**3      00000557
     1 + 0.54456*XX**4+ 0.0608*XX**5                                     00000558
      ZP(I)=-ZP(I)                                                       00000559
  572 CONTINUE                                                           00000560
  430 CONTINUE                                                           00000561
      YL=0.0                                                             00000562
      YH=0.0                                                             00000563
      CALL VDIPLT(IEC, IN, N,   ZP(I),TEMP(I), XL,XH,YL,YH,NXM,          00000564
     1  XNOTES, NYM, YNOTE, ISYMD)                                       00000565
  445 CONTINUE                                                           00000566
C     OUTPUT. = 2 OR GREATER                                             00000567
C     REAL SPACE RESIDUAL OUTPUT                                         00000568
C     BACKTRANSFORM SOLUTION AND PREDICTION INTERVALS                    00000569
      MX=NPAM(KK)                                                        00000571
      DO 441 M=1,L1                                                      00000572
      GO TO(201,201,201,201,203),MX                                      00000573
  201   CY(M)= 10.0**CY(M)                                               00000574
      PYMAX(M)=10.0**PYMAX(M)                                            00000575
      PYMIN(M)=10.0**PYMIN(M)                                            00000576
      CIMAX(M)=10.0**CIMAX(M)                                            00000577
      CIMIN(M)=10.0**CIMIN(M)                                            00000578
      GO TO 205                                                          00000579
  203 CONTINUE                                                           00000580
C     AVOID NEGATIVE TO A POWER DUMP                                     00000581
       IF(PYMIN(M).LE. 0.0) PYMIN(M)=1.0                                 00000582
       IF(PYMAX(M).LE. 0.0) PYMAX(M)=1.0                                 00000583
      IF(CIMAX(M).LE.0.0) CIMAX(M)=1.0                                   00000584
      IF(CIMIN(M).LE.0.0) CIMIN(M)=1.0                                   00000585
                                                                          +
```

```
      IF(CY(M).LE.0.0) CY(M)=1.0                                        00000586
        CY(M)=     CY(M)**(1.0/CBEST)                                   00000587
      PYMAX(M)= PYMAX(M)**(1.0/CBEST)                                   00000588
      PYMIN(M)= PYMIN(M)**(1.0/CBEST)                                   00000589
      CIMAX(M)=CIMAX(M)**(1.0/CBEST)                                    00000590
      CIMIN(M)=CIMIN(M)**(1.0/CBEST)                                    00000591
  205 CONTINUE                                                          00000592
      RIS(M)= RS(M,1)-CY(M)                                             00000593
      ERRPER(M)=(RIS(M)/RS(M,1))*100.                                   00000594
  441 CONTINUE                                                          00000595
C     ORDER REAL SPACE RESIDUALS                                        00000596
      DO 425 I=1,L1                                                     00000597
      TEMP(I)=0.0                                                       00000598
  425 TEMP(I)=ABS(ERRPER(I))                                            00000599
      CALL AORDER(TEMP,L1NEG,IPERM)                                     00000600
      DO 1203 I=1,L1                                                    00000601
      J=IPERM(I)                                                        00000602
      TEMP(J)=I                                                         00000603
 1203 CONTINUE                                                          00000604
      IF(IOUT.LT.2) GO TO 350                                           00000605
      WRITE(6,414)                                                      00000606
      WRITE(6,431)                                                      00000607
  431 FORMAT(*  BACKTRANSFORMED RESIDUALS - REAL SPACE*/)               00000608
      WRITE(6,416)                                                      00000609
      WRITE(6,417)(I,RS(I,1),CY(I),RIS(I),ERRPER(I),TEMP(I),            00000610
     1 CIMIN(I),CIMAX(I), PYMIN(I),PYMAX(I),I=1,L1)                     00000611
  350 CONTINUE                                                          00000612
      GO TO 1                                                           00000613
  900 CONTINUE                                                          00000614
C     CALL CALPLT ROUTINE ONLY WHEN PLOTTING                            00000615
      IF(IOUT  .GE.3) CALL CALPLT(0,0,999)                              00000616
      STOP
      END                                                               00000617
      SUBROUTINE AORDER (AA,N,IPERM)                                    00000618
                                                                        +
```

```
C       THIS SUBROUTINE ORDERS VALUES IN AA AND STORES ORDER IN IPERM      00000619
C       N IS NUMBER OF VALUES IN AA                                        00000620
C       IPERM IS ORDERED WITH RESPECT TO LOCATION OF VALUES IN AA          00000621
C       IF N IS POSITIVE IPERM(1) HAS LOCATION IN AA OF SMALLEST VALUE
C       IPERM(N) HAS LOCATION OF LARGEST VALUE IN AA                       00000623
C       IF N IS NEGATIVE IPERM IS ORDERED BY LOCATION OF LARGEST TO        00000624
C       SMALLEST VALUES IN AA                                              00000625
C       ARRAY AA IS NOT CHANGED                                            00000626
        DIMENSION AA(1), IPERM(1)                                          00000627
        LOGICAL  SWITCH                                                    00000628
        NABS = IABS(N)                                                     00000629
        DO 100    I=1,NABS                                                 00000630
  100   IPERM(I) = I                                                       00000631
        IF( NABS .LT.2) RETURN                                             00000632
  200   SWITCH =  .FALSE.                                                  00000633
        DO500 I= 2,NABS                                                    00000634
        II= IPERM(I-1)                                                     00000635
        JJ= IPERM(I)                                                       00000636
        IF( N.LT. 0) GO TO 400                                             00000637
        IF(AA(II).LE.AA(JJ)) GO TO 500                                     00000638
  300   ITEMP= IPERM(I-1)                                                  00000639
        IPERM(I-1) = IPERM (I)                                             00000640
        IPERM(I)=ITEMP                                                     00000641
        SWITCH = .TRUE.                                                    00000642
        GO TO 500                                                          00000643
  400   IF(AA(II).LT.AA(JJ)) GO TO 300                                     00000644
  500   CONTINUE                                                           00000645
        IF( SWITCH) GO TO  200                                             00000646
  900   RETURN                                                             00000647
        END                                                                00000648
        SUBROUTINE LM (Y,RS,F,L1)                                          00000649
C       CONVERTS TIME,STRESS,AND TEMPERATURE TO FORMAT REQUIRED            00000650
C       FOR LINEAR SOLUTION OF LARSON-MILLER EXPRESSION                    00000651
                                                                           +
```


9/1

```
C     SOLUTION ALLOWS FIFTH ORDER EXPANSION OF LOG STRESS                00000652
C     Y= B0 + B1(X1)+B2(X2)--- B6(X6)                                    00000653
C     WHERE  Y = LOG(RUPTURE TIME)                                       00000654
C            S = APPLIED STRESS IN PSI                                   00000655
C            T = TEST TEMPERATURE IN DEGREES F                           00000656
C            X1= 1/(T+460)                                               00000657
C            X2= LOG(S)/(T+460) = S/TK                                   00000658
C            X3= S**2/TK                                                 00000659
C            X4= S**3/TK                                                 00000660
C            X5= S**4/TK                                                 00000661
C            X6= S**5/TK                                                 00000662
C     C,B0-B6  = CONSTANTS DETERMINED BY LINEAR LEAST SQUARES METHOD     00000663
C            B0= OPTIMUM L-M CONSTANT (C)                                00000664
      DIMENSION Y(200), RS(200,5), F(200,10)                             00000665
      DO 10 I=1,L1                                                       00000666
      Y(I)= ALOG10(RS(I,1))                                              00000667
      S= ALOG10(RS(I,2))                                                 00000668
      T=     (RS(I,3)+460.0)                                             00000669
      F(I,1) = 1.0/T                                                     00000670
      F(I,2) = S/T                                                       00000671
      F(I,3) = S**2/T                                                    00000672
      F(I,4) = S**3/T                                                    00000673
      F(I,5) = S**4/T                                                    00000674
      F(I,6) = S**5/T                                                    00000675
   10 CONTINUE                                                           00000676
      RETURN                                                             00000677
      END                                                                00000678
      SUBROUTINE OSD(Y,RS,F,L1)                                          00000679
C     CONVERTS TIME,STRESS,AND TEMPERATURE TO FORMAT REQUIRED            00000680
C     FOR LINEAR SOLUTION OF ORR-SHERBY-DORN EXPRESSION                  00000681
C     SOLUTION ALLOWS FIFTH ORDER EXPANSION OF LN STRESS                 00000682
C     WHERE  Y = LOG(TIME TO CREEP EVENT)
C            S = APPLIED STRESS IN PSI                                   00000684
                                                                        +
```

```
C             T = TEST TEMP IN DEGREES F                                        00000685
C             X1= 1/TK                                                          00000686
C             X2= LN(S) = SL                                                    00000687
C             X3= SL**2                                                         00000688
C             X4= SL**3                                                         00000689
C             X5= SL**4                                                         00000690
C             X6= SL**5                                                         00000691
C          B0-B6= CONSTANTS, DETERMINED BY LINEAR LEAST SQUARES METHOD          00000692
C             B1= DELH/R                                                        00000693
C           DELH= APPARENT ACTIVATION ENERGY                                    00000694
C              R= UNIVERSAL GAS CONSTANT                                        00000695
      DIMENSION Y(200), RS(200,5), F(200,10)                                    00000696
      DO 10 I=1,L1                                                              00000697
      Y(I)= ALOG10(RS(I,1))                                                     00000698
      S= ALOG10(RS(I,2))                                                        00000699
      T=(5./9.)*(RS(I,3)-32.) +273.                                             00000700
      F(I,1)=  1.0/ T                                                           00000701
      F(I,2)=  S                                                                00000702
      F(I,3)=  S**2                                                             00000703
      F(I,4)=  S**3                                                             00000704
      F(I,5)=  S**4                                                             00000705
      F(I,6)=  S**5                                                             00000706
   10 CONTINUE                                                                  00000707
      RETURN                                                                    00000708
      END                                                                       00000709
      SUBROUTINE MS(Y,RS,F,L1)                                                  00000710
C     CONVERTS TIME,STRESS,AND TEMPERATURE TO FORMAT REQUIRED                   00000711
C     FOR LINEAR SOLUTION OF MANSON-SUCCOP EXPRESSION                           00000712
C     SOLUTION ALLOWS FIFTH ORDER EXPANSION OF LOG STRESS                       00000713
C             B1 = OPTIMUM M-S CONSTANT (C)                                     00000714
      DIMENSION Y(200), RS(200,5), F(200,10)                                    00000715
      DO 10 I=1,L1                                                              00000716
      Y(I)= ALOG10(RS(I,1))                                                     00000717
```


8/11

```
      S= ALOG10(RS(I,2))                                                    00000718
      T= RS(I,3)                                                            00000719
      F(I,1)=T                                                              00000720
      F(I,2)=  S                                                            00000721
      F(I,3)=  S**2                                                         00000722
      F(I,4)=  S**3                                                         00000723
      F(I,5)=  S**4                                                         00000724
      F(I,6)=  S**5                                                         00000725
   10 CONTINUE                                                              00000726
      RETURN                                                                00000727
      END                                                                   00000728
      SUBROUTINE MH(Y,RS,F,L1,CMH)                                          00000729
C     FOR NONLINEAR SOLUTION OF MANSON-HAFERD EXPRESSION                    00000730
C           CMH = TEMPERATURE OFFSET (TA)                                   00000731
C             X1= T-CMH =DT                                                 00000732
C             X2= DT*S                                                      00000733
C             X3= DT*S**2                                                   00000734
C             X4= DT*S**3                                                   00000735
C             X5= DT*S**4                                                   00000736
C             X6= DT*S**5                                                   00000737
C     SOLUTION IS ITERATED TO FIND CMH WHICH PRODUCES BEST FIT              00000738
      DIMENSION Y(200), RS(200,5), F(200,10)                                00000739
      DO 10 I=1,L1                                                          00000740
      Y(I)= ALOG10(RS(I,1))                                                 00000741
      S= ALOG10(RS(I,2))                                                    00000742
      DT= RS(I,3)-CMH                                                       00000743
      F(I,1)= DT                                                            00000744
      F(I,2)= DT*S                                                          00000745
      F(I,3)= DT*S**2                                                       00000746
      F(I,4)= DT*S**3                                                       00000747
      F(I,5)= DT*S**4                                                       00000748
      F(I,6)= DT*S**5                                                       00000749
   10 CONTINUE                                                              00000750
                                                                            +
```

```
      RETURN                                                                00000751
      END                                                                   00000752
      SUBROUTINE ITER(C,   BC,X,PBX,IFG,ICT, DEL, DELMIN, LIM,LAST)         00000753
C     ITERATES CONSTANT (C) TO MINIMIZE VALUE (X)                           00000754
C     BC = VALUE OF CONSTANT ASSOCIATED WITH LOWEST(BEST) X VALUE           00000755
C     PBX= BEST PREVIOUS VALUE OF X                                         00000756
C     IFG =FLAG TO CONTROL INCREASING OR DECREASING C FOR NEXT ITERATION    00000757
C     DEC =CONTROLS SIZE OF C INCREMENT                                     00000758
C     ICT =ALLOWS C TO INCREMENT BEYOND BC BFFORF CHANGING
C          INCREMENT SIZE, ITERATION STOPS WHEN DEL .LE. DELMIN
C     LIM = COUNTER FOR ITERATIONS                                          00000761
C     LAST=END ITERATION FLAG                                               00000762
      IF( IFG) 5,5,30                                                       00000763
C     INCREASING C .                                                        00000764
    5 LIM = LIM+1                                                           00000765
C     NEXT CARD PREVENTS NEGATIVE X FROM BEING BEST X VALUE                 00000766
      IF(X.LE. 0.0) GO TO 10                                                00000767
      IF(PBX .GT. X)  BC=C                                                  00000768
      IF(PBX .GT. X)  PBX=X                                                 00000769
      IF(PBX .EQ. X)  ICT=0                                                 00000770
      IF(X .GT. PBX)  ICT=ICT+1                                             00000771
      IF(DEL .LE.DELMIN) GO TO   40                                         00000772
C     IF( ICT.LT. 2) GO TO 10                                               00000773
      IF( ICT.LT.10) GO TO 10                                               00000774
      DEL = 0.3*DEL                                                         00000775
      ICT=0                                                                 00000776
C     C= C-DEL                                                              00000777
      C=BC+5.0*DEL                                                          00000778
      IFG=1                                                                 00000779
      GO TO 50                                                              00000780
   10 C= C+DEL                                                              00000781
      GO TO 50                                                              00000782
C     DECREASING CONSTANT                                                   00000783
                                                                            +
```

```
   30 LIM =LIM+1                                                          00000784
C     NEXT CARD PREVENTS NEGATIVE X FROM BEING BEST X VALUE              00000785
      IF(X.LE. 0.0) GO TO 35                                             00000786
      IF(PBX .GT. X)  BC=C                                               00000787
      IF(PBX .GT. X)  PBX=X                                              00000788
      IF(PBX .EQ. X)  ICT=0                                              00000789
      IF(X .GT. PBX)  ICT=ICT+1                                          00000790
      IF(DEL.LE.DELMIN) GO TO 40                                         00000791
      IF( ICT.LT.10) GO TO 35                                            00000792
      DEL=0.3*DEL                                                        00000793
      ICT=0                                                              00000794
      C=BC-5.0*DEL                                                       00000795
      IFG=0                                                              00000796
   35 C=C-DEL                                                            00000797
      GO TO 50                                                           00000798
   40 LAST=1                                                             00000799
   50 CONTINUE                                                           00000800
      RETURN                                                             00000801
      END                                                                00000802
      SUBROUTINE RAB(Y,RS,F,L1,A)                                        00000803
C     FOR NONLINEAR SOLUTION OF RABOTNOV EXPRESSION                      00000804
C     SOLUTION ALLOWS FIFTH  ORDER EXPANSION OF TEMPERATURE FUNCTION     00000805
C     WHERE Y=  RUPTURE TIME **A                                         00000806
C           T=  TEST TEMPERATURE IN DEGREES F                            00000807
C          X1=  1/ST                                                     00000808
C           A=  ITERATED CONSTANT                                        00000809
C           S=  STRESS IN PSI                                            00000810
      DIMENSION Y(200), RS(200,5), F(200,10)                             00000811
      DO 10 I=1,L1                                                       00000812
      Y(I)= (RS(I,1))**A                                                 00000813
      S   = RS(I,2)                                                      00000814
      T   = RS(I,3)                                                      00000815
      F(I,1)=  1.0/(S*T)                                                 00000816
                                                                          +
```

```
      F(I,2)=   1.0/(S*T**2)                                            00000817
      F(I,3)=   1./(S*T**3)                                             00000818
      F(I,4)=   1./(S*T**4)                                             00000819
      F(I,5)=   1./(S*T**5)                                             00000820
      F(I,6)=   1./(S*T**6)                                             00000821
   10 CONTINUE                                                          00000822
      RETURN                                                            00000823
      END                                                               00000824
      SUBROUTINE MINMAX(CMIN,CMAX,CRAN,CMEAN,C,N)                       00000825
C     CALCULATES MINIMUM, MAXIMUM,RANGE, AND MEAN OF C(I)               00000826
C     WHERE N= NUMBER OF OBSERVATIONS                                   00000827
      DIMENSION C(1)                                                    00000828
      CMAX=-1.0E+100                                                    00000829
      CMIN=1.0E+100                                                     00000830
      CSUM=0.0                                                          00000831
      DO 5, I=1,N                                                       00000832
      CSUM=CSUM+C(I)                                                    00000833
      IF( C(I)-CMIN) 2,3,3                                              00000834
    2 CMIN=C(I)                                                         00000835
    3 IF( C(I)-CMAX)5,5,4                                               00000836
    4 CMAX=C(I)                                                         00000837
    5 CONTINUE                                                          00000838
      CMEAN=CSUM/N                                                      00000839
      CRAN=CMAX-CMIN                                                    00000840
    7 CONTINUE                                                          00000841
      RETURN                                                            00000842
      END                                                               00000843
                                                                        +
```

APPENDIX B

LANGLEY RESEARCH CENTER SYSTEM SUBROUTINES

SUBROUTINE MATINV

LANGUAGE: FORTRAN

PURPOSE: To invert a real square matrix A, solve the matrix equation AX = B, where B is a matrix of constant vectors, and by an option evaluate the determinant.

USE: CALL MATINV(MAX,N,A,M,B,IOP,DETERM,ISCALE,IPIVOT,IWK)

MAX — An input integer specifying the maximum order of A as stated in the dimension statement of the calling program.

N — An input integer specifying the order of A; $1 \leq N \leq MAX$.

A — An input/output two-dimensional array of the coefficients. On return to the calling program, A^{-1} is stored in A. A must be dimensioned in the calling program with first dimension MAX and second dimension at least N. The original A matrix is destroyed.

M — An input integer specifying the number of column vectors in B. M = 0 signals that the subroutine is used solely for inversion; however, in the call statement an entry corresponding to B must be present.

B — An input/output two-dimensional array of the constant vectors. On return to the calling program, the solution X is stored in B. B should have its first dimension MAX and its second dimension at least M. The original B matrix is destroyed.

IOP — Compute the determinant option.
IOP = 0, Compute the determinant.
IOP = 1, Do not compute the determinant.

DETERM — For IOP = 0, in conjunction with ISCALE, represents the value of the determinant of A as follows:

$$DET(A) = (DETERM)10^{100(ISCALE)}$$

For IOP=1, the determinant is set to 1. The determinant is set to zero for a singular matrix, for both IOP = 0 or 1 option. Upon return from MATINV, DETERM should be tested or written out in the calling program.

(See Other Coding Information)

ISCALE — For IOP = 0, the scale factor is computed by the subroutine to avoid overflow or underflow in the computation of the quantity, DETERM. For IOP = 1, ISCALE may be a dummy argument.

IPIVOT — A one-dimensional array used by the subprogram to store pivotal information. It should be dimensioned at least N. In general the user does not need to make use of this array.

IWK — An integer array of temporary storage, dimensioned at least 2 x N.

METHOD: Jordan's method is used to reduce a matrix A to the identity matrix I through a succession of elementary transformations: $\ell_n, \ell_{n-1}, \ldots, \ell_1$. $A = I$. If these transformations are simultaneously applied to I and to a matrix B of constant vectors, the results are A^{-1} and X where $AX = B$. Each transformation is selected so that the largest element is used in the pivotal position.

ACCURACY: Total pivotal strategy is used to minimize the rounding errors; however, the accuracy of the final results depends upon how well-conditioned the original matrix is. A return with DETERM $\neq 0$ does not guarantee accuracy in the solutions or inverse.

REFERENCE: Fox, L., An Introduction to Numerical Linear Algebra. Oxford University Press, New York, 1965.

STORAGE: 516_8 locations

SUBROUTINE DDIPLT

LANGUAGE: FORTRAN

PURPOSE: To provide a one-call method of preparing plotting. This routine was originally designed for recording plots on the DD80 plotter only; however, it has been redesigned to use on any plotter. This one-call routine should not be used on any new jobs; new jobs requiring one-call displays should use INFOPLT.

These displays will not meet specifications for final figures.

USE:

```
CALL DDIPLT(IEC,IN,N,XDATA,YDATA,XMIN,XMAX,YMIN,YMAX,
            NXM,XM,NYM,YM,ISYMD)
```

where

IEC is the code for terminating the frame

- 0 frame incomplete
- 1 frame complete with this data. The frame change is built in and the plotter will be spaced for the next frame.

IN is a two-word array. Each word contains 10 Hollerith characters for plot identification.

N is the number of points to be plotted.

XDATA is the name of the array containing the floating point values of X to be plotted.

YDATA is the name of the array containing the floating point values of Y to be plotted.

XMIN is the minimum value for X.

XMAX is the maximum value for X.

YMIN is the minimum value for Y.

YMAX is the maximum for Y.

The routine checks for the first call only to determine if either (XMAX-XMIN) or (YMAX-YMIN) is equal to zero. When either is zero, the routine will scan the X and/or Y array to determine the limits. For multiple curves per display, the limits must be specified on the first call to include all curves since the limits from the first call will be used for all curves.

If any data falls outside the limits, it will be eliminated; but a count will be kept of all points dropped and written at top of the plot.

Minimum/maximum values are next checked to see that the range is not zero. When it is, the specified values are adjusted by 10 percent of the minimum or set equal to ±1.0 in cases where minimum and maximum are equal to zero.

NXM is the number of central memory words in the message for the horizontal annotation. Maximum number of words is 13; each word contains 10 characters. If NXM and NYM are both negative, tic marks will be generated instead of grid.

XM is the name of array containing the label for the horizontal annotation.

NYM is the number of words in the message for the vertical annotation. Maximum number of words is 13.

YM is the name of array containing the label for the vertical annotation.

ISYMD is the integer code specifying the symbol or mode to be used for plotting the data values.

1	Circle	8	Fan	
2	Square	9	Long diamond	
3	Diamond	10	House	
4	Triangle	11	Circled dot	
5	Right Triangle	12	X	
6	Quadrant	13	Dot	
7	Dog House	14	Vectors	

RESTRICTIONS: The following arrays must be specified in a DIMENSION statement of the calling program: IN(2), XDATA(N), YDATA(N), XM(NXM), YM(NYM).

Each curve on a display requires a separate entry to the routine. X and Y coordinates for plotting must be in separate arrays of single precision, floating point data. Frame control is specified by the IEC code in the calling sequence for the routine.

METHOD: Data are scaled and plotted; axes are drawn and annotated, and grid lines or tic marks are generated.

Minimum/maximum values are adjusted to provide a range when all values of an array are equal. Adjustment is also made where needed to improve the appearance of the plot.

ACCURACY: Approximately three significant figures may be read in either direction.

REFERENCES:

STORAGE: 3021_8 locations

SUBPROGRAMS USED: CALPLT, NOTATE, NUMBER, PNTPLT, NFRAME

OTHER CODING INFORMATION: A call to PSEUDO (1.4.1) must precede the first call to DDIPLT. An entry called VDIPLT with the same parameters as DDIPLT is available which packs 8 6" x 6" plots per frame for the Varian postprocessor.

SUBROUTINE PSEUDO

LANGUAGE: COMPASS

PURPOSE: To create and write an appropriately named Plot Vector File. Through linkages set up by an initial call to PSEUDO, all subsequent graphics data generated by the user will be routed through one of the PSEUDO entry points and written on the Plot Vector File. The PSEUDO processor is designed for use with the frame dependent post-processors described in Section 1.3, Volume IV, of the Computer Programing Manual.

USE:

```
CALL PSEUDO
```

or

```
CALL PSEUDO(FN)
```

FN — file name left-justified with zero fill. Default file name is SAVPLT.

Example:

```
CALL PSEUDO
```

This will establish a Plot Vector File named SAVPLT.

```
CALL PSEUDO(6LMYFILE)
```

This will establish a Plot Vector File named MYFILE.

NOTE: The Plot Vector File (or Files) will usually be written to disk (as opposed to tape) and may be postprocessed following user program termination via appropriate specification of one or more PLOT control cards (see Section 1.3, Volume IV, Computer Programing Manual).

RESTRICTIONS:

(1) An initializing call to PSEUDO (with or without a file name argument) must be made prior to any calls to CALPLT or any other graphics output routine.

(2) Every Plot Vector File should be terminated with a 999 pen code, CALL CALPLT(0.0,0.0,999). The transmission of the 999 code will cause an EOF write on the Plot Vector File, and the file will temporarily be closed. Thus, any given Plot Vector File will contain only one 999 pen code and/or one EOF.

(3) To continue plotting execution following transmission of a 999 code to a current Plot Vector File, the user program must call the PSEUDO processor to create new Plot Vector File (i.e., CALL PSEUDO(6LMYFIL2)).

METHOD: In addition to entry PSEUDO, this processor contains two other entry points, namely PLT9999 and PLT9998. An initializing call to PSEUDO will set PLT9999 into the processor switching mechanism (PLOTSW). Subsequent plot data generation will then be routed via CALPLT, PLOTSW, and PLT9999 and written on the Plot Vector File. The entry PLT9998 is used to record special purpose data from routines NFRAME and PLTSTOP.

ACCURACY:

REFERENCES: See Section 1.3, Volume IV, Computer Programing Manual.

STORAGE: 2155_8 locations total for direct subprograms

SUBPROGRAMS USED: NUMARG, PLOTSW

APPENDIX C

DEVELOPMENT OF PARAMETRIC MODEL EQUATIONS

This appendix presents the development of the parametric model equations used in the computer program PARAM. The Larson-Miller, Orr-Sherby-Dorn, Manson-Succop, and Manson-Haferd expressions are familiar time-temperature parameters. These parameters assume that the value of the parameter (a function of stress) is a constant for each value of the temperature compensated time parameter. The Rabotnov parameter (refs. 9 and 10) is a time-stress parameter which assumes that the value of the parameter (a function of temperature) is a constant for each value of the time compensated stress parameter. Time to a given creep event and a polynomial in the parameter function (stress or temperature) were respectively the dependent and independent variables all regression model equation forms used in PARAM. The following presents the development of these five equation forms:

Larson-Miller Parameter

$$P = T_R (\log t + C) = f(\sigma)$$

$$T_R(\log t + C) = b_1 + b_2 \log \sigma + b_3(\log \sigma)^2 + b_4(\log \sigma)^3 + b_5(\log \sigma)^4 + b_6(\log \sigma)^5$$

assuming $b_o = -C$

$$\log t = b_o + b_1/T_R + b_2 \log \sigma/T_R + b_3 (\log \sigma)^2/T_R + b_4 (\log \sigma)^3/T_R + b_5 (\log \sigma)^4/T_R + b_6 (\log \sigma)^5/T_R$$

where P = the Larson-Miller parameter

T_R = temperature, ^oR

t = time to a particular creep event

C = Larson-Miller constant

σ = applied stress

b_o - - b_6 = coefficients estimated by method of least squares.

Orr-Sherby-Dorn Parameter

$$P = t \exp(-\Delta H/RT_K) = g(\sigma)$$

$$\log t - K(\Delta H/RT_K) = f(\log \sigma)$$

assuming $b_1 = K\Delta H/R$

$$\log t = b_o + b_1/T_K + b_2 \log \sigma + b_3 (\log \sigma)^2 + b_4 (\log \sigma)^3 + b_5 (\log \sigma)^4 + b_6 (\log \sigma)^5$$

where P = Orr-Sherby-Dorn parameter

t = time to a particular creep event

ΔH = apparent activation energy

R = universal gas constant

T_K = temperature, Kelvin

σ = applied stress

b_o-- b_6 = coefficients estimated by method of least squares.

Manson-Succop Parameter

$$P = \log t + CT_F = f(\sigma)$$

$$\log t = -C\,T_F + f(\sigma)$$

assuming $b_1 = -C$

$$\log t = b_o + b_1 T_F + b_2 \log \sigma + b_3 (\log \sigma)^2 + b_4 (\log \sigma)^3 + b_5 (\log \sigma)^4 + b_6 (\log \sigma)^5$$

where P = Manson-Succop parameter

t = time to a particular creep event

C = Manson-Succop constant

T_F = temperature, ^oF

σ = applied stress

b_o --b_6 = coefficients estimated by method of least squares.

Manson-Haferd Parameter

$$P = (\log t - \log t_a)/(T_F - T_A) = f(\sigma)$$

$$\log t = \log t_a + (T_F - T_A) f(\sigma)$$

assuming $b_o = \log t_a$

$D = T_F - T_A$

$$\log t = b_o + b_1 D + b_2 D \log \sigma + b_3 D (\log \sigma)^2 + b_4 D (\log \sigma)^3 + b_5 D (\log \sigma)^4 + b_6 D (\log \sigma)^5$$

where P = Manson-Haferd parameter

t = time to a particular creep event

t_a = offset time

T_F temperature, ^oF

T_A = offset temperature, ^oF

σ = applied stress

b_o -- b_6 = coefficients estimated by method of least squares which iteratively searched values of T_A to determine best fit.

Rabotnov Parameter

$$P = \sigma(1 + At^{\alpha}) = f(T)$$

$$t^{\alpha} = -1/A + 1/A\sigma\,[C_1 + C_2/T + C_3/T^2 + C_4/T^3 + C_5/T^4 + C_5/T^5 + C_6/T^6]$$

assuming $b_o = -1/A$

$b_i = C_i/A$

$$t^{\alpha} = b_o + b_1/\sigma T + b_2/\sigma T^2 + b_3/\sigma T^3 + b_4/\sigma T^4 + b_5/\sigma T^5 + b_6/\sigma T^6$$

where P = Rabotnov parameter

σ = applied stress

A, α = constants

t = time to a particular creep event

T = temperature, ^{o}F

b_o--b_6 = coefficients estimated by method of least squares which iteratively searched values of α to determine best fit.

REFERENCES

1. Larson, F. R.; and Miller, J.: A Time-Temperature Relationship for Rupture and Creep Stresses. Trans. ASME, vol. 74, 1952, pg. 765.
2. Orr, R. L.; Sherby, O.D.; and Dorn, J.E.: Correlations of Rutpure Data for Metals at Elevated Temperatures. Trans. ASM, vol. 46, 1954, pg. 113.
3. Manson, S.S.; and Haferd, A.M.: A Linear Time-Temperature Relation for Extrapolation of Creep and Rupture Stress Data. NACA TN-2890, 1953.
4. Goldhoff, R. M.; and Hahn, G. J.: Correlation and Extrapolation of Creep-Rupture Data of Several Steels and Superalloys Using Time-Temperature Parameters. ASM publication D8-100, American Society for Metals, 1968, pg. 199.
5. Goldhoff, R. M.: Towards the Standardization of Time-Temperature Parameter Usage in Elevated Temperature Data Analysis. J. Testing and Evaluation, ASTM, vol. 2, no. 5, Sept. 1974, pp. 387-424.
6. Conway, J. B.: Stress-Rupture Parameters: Origin, Calculation, and Use. Gordon and Breck, 1969.
7. Draper, N.R.; and Smith, H.: Applied Regression Analysis. John Wiley and Sons, Inc., 1966.
8. Daniel, C.; and Wood, F. S.: Fitting Equations to Data. John Wiley and Sons, Inc., 1971.

9. Penny, R.K.; and Marriott, D. L.: Design for Creep, McGraw-Hill Book Co. (UK), Ltd. (Maidenhead, Berkshire, England), 1971.

10. Rabotnov, Yu. N.: Creep Problems in Structural Members. North Holland Publishing Co. (Amsterdam), 1969.

11. Hahn, G. J.: Statistical Intervals for a Normal Population. GE Report No. 69-C-382, General Electric Research and Development Center, Schenectady, New York, Nov. 1969.

```
1     0    -5000.0         0.2
1     4
2     4
3     4
4     4
5     3

ALLOY   9    316 STAINLESS STEEL
  3142.90       1175.00          25.00
    74.60       1200.00          30.00
   213.00       1200.00          28.00
   656.20       1200.00          25.00
  3476.10       1200.00          22.00
  6825.30       1200.00          20.00
 10076.50       1200.00          18.50
 15790.80       1200.00          17.00
   290.90       1225.00          25.00
   186.50       1250.00          25.00
    81.50       1275.00          25.00
    36.50       1300.00          25.00
   104.10       1300.00          22.00
   228.20       1300.00          20.00
   258.10       1300.00          19.00
   319.00       1300.00          18.00
   377.50       1300.00          17.00
   753.70       1300.00          16.00
   785.30       1300.00          16.50
  1232.50       1300.00          15.00
  1854.60       1300.00          13.60
  2421.00       1300.00          13.00
  4078.30       1300.00          12.00
  6258.10       1300.00          11.00
    21.50       1325.00          25.00
     9.90       1350.00          25.00
     2.70       1400.00          25.00
    83.30       1400.00          15.00
   251.20       1400.00          12.50
   921.00       1400.00          10.00
    27.90       1450.00          15.00
    75.20       1450.00          12.50
     5.00       1500.00          16.40
    40.60       1500.00          12.50
    87.90       1500.00          10.00
   170.40       1500.00           9.00
   614.90       1500.00           7.00
    28.70       1550.00          10.00
```

Figure 1.-Input data for Case 1.

```
     LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
     ANALYSIS OF CREEP-RUPTURE DATA
 DATA SET                          ALLOY   9   316 STAINLESS STEEL
OPTION CARD
INPUT= 1
IOUT= 0
TA=       -5000
RA=       .2000

CASE CONTROL CARDS
     PARAMETER CODE      NO. COEFFICIENTS

          1                    4
          2                    4
          3                    4
          4                    4
          5                    3
```

Figure 2.-Output for Case 1.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

------- REGRESSION VALUES --------

DATA SET	ALLOY 9	316 STAINLESS STEEL
PARAMETER SELECTED		L-M
NO. OF OBSERVATIONS		38
NO. OF INDEPENDENT VARIABLES		3
RESIDUAL DEGREES OF FREEDOM		34
F - VALUE		476.8
RESIDUAL MEAN SQUARE		2.1495E-02
STANDARD ERROR		1.4661E-01
RESIDUAL SUM OF SQUARES		7.3083E-01
TOTAL SUMS OF SQUARES		3.1476E+01
MULT. CORREL. COEF. SQUARED		.9768

MIN Y = 4.31E-01 MAX Y = 4.20E+00 Y RANGE = 3.77E+00 MEAN Y = 2.45E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	CERR
0		-1.8792E+01									
1	1/T	4.7642E+04	2.98E+03	16.00	5.613E-04	4.98E-04	6.12E-04	1.14E-04	1.44	.335	.00
2	S/T	-3.7957E+03	4.14E+03	.92	6.929E-04	4.31E-04	8.90E-04	4.59E-04	-.46	.663	.00
3	S**2/T	-3.2979E+03	1.69E+03	1.95	8.689E-04	3.64E-04	1.31E-03	9.50E-04	-.83	.003	-.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	.621	4.2
MAXIMUM WIDTH	.741	5.5

Figure 2.-Continued.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

-------- REGRESSION VALUES --------

DATA SET	ALLOY. 9	316 STAINLESS STEEL
PARAMETER SELECTED		O-S-D
NO. OF OBSERVATIONS		38
NO. OF INDEPENDENT VARIABLES		3
RESIDUAL DEGREES OF FREEDOM		34
F - VALUE		575.9
RESIDUAL MEAN SQUARE		1.7867E-02
STANDARD ERROR		1.3367E-01
RESIDUAL SUM OF SQUARES		6.0748E-01
TOTAL SUMS OF SQUARES		3.1476E+01
MULT. CORREL. COEF. SQUARED		.9807

MIN Y = 4.31E-01 MAX Y = 4.20E+00 Y RANGE = 3.77E+00 MEAN Y = 2.45E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	CERR
0		-1.5630E+01									
1	1/T	2.1117E+04	5.27E+02	40.07	1.011E-03	8.96E-04	1.10E-03	2.06E-04	1.15	.333	-.00
2	S	1.5077E+00	2.02E+00	.75	1.228E+00	8.45E-01	1.48E+00	6.32E-01	.25	.658	.00
3	S**2	-3.3333E+00	8.36E-01	3.99	1.533E+00	7.14E-01	2.18E+00	1.47E+00	-1.30	.009	.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	.567	3.7
MAXIMUM WIDTH	.593	3.9

Figure 2.-Continued.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

-------- REGRESSION VALUES --------

DATA SET	ALLOY 9	316 STAINLESS STEEL
PARAMETER SELECTED		M-S
NO. OF OBSERVATIONS		38
NO. OF INDEPENDENT VARIABLES		3
RESIDUAL DEGREES OF FREEDOM		34
F - VALUE		338.2
RESIDUAL MEAN SQUARE		3.0019E-02
STANDARD ERROR		1.7326E-01
RESIDUAL SUM OF SQUARES		1.0206E+00
TOTAL SUMS OF SQUARES		3.1476E+01
MULT. CORREL. COEF. SQUARED		.9676

MIN Y = 4.31E-01 MAX Y = 4.20E+00 Y RANGE = 3.77E+00 MEAN Y = 2.45E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	CERR
0		2.2556E+01									
1	T	-1.1658E-02	3.80E-04	30.69	1.328E+03	1.17E+03	1.55E+03	3.75E+02	-1.16	.330	.00
2	S	-7.9643E-01	2.64E+00	.30	1.228E+00	8.45E-01	1.48E+00	6.32E-01	-.13	.665	.00
3	S**2	-2.3794E+00	1.09E+00	2.19	1.533E+00	7.14E-01	2.18E+00	1.47E+00	-.93	.005	-.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	.735	5.4
MAXIMUM WIDTH	.771	5.9

Figure 2.-Continued.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

------- REGRESSION VALUES --------

DATA SET	ALLOY	9	316 STAINLESS STEEL
PARAMETER SELECTED			M-H
NO. OF OBSERVATIONS			38
NO. OF INDEPENDENT VARIABLES			3
RESIDUAL DEGREES OF FREEDOM			34
F - VALUE			419.8
RESIDUAL MEAN SQUARE			2.4334E-02
STANDARD ERROR			1.5599E-01
RESIDUAL SUM OF SQUARES			8.2737E-01
TOTAL SUMS OF SQUARES			3.1476E+01
MULT. CORREL. COEF. SQUARED			.9737

MANSON - HAFERD CONSTANT(TA) = 300.0

MIN Y = 4.31E-01 MAX Y = 4.20E+00 Y RANGE = 3.77E+00 MEAN Y = 2.45E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	CERR
0		1.4509E+01									
1	DT	-1.0686E-02	1.26E-03	8.48	1.028E+03	8.75E+02	1.25E+03	3.75E+02	-1.06	.328	-.00
2	DT*S	4.8648E-03	2.15E-03	2.26	1.251E+03	1.01E+03	1.54E+03	5.24E+02	.68	.651	-.00
3	DT*S**2	-4.6243E-03	9.02E-04	5.13	1.548E+03	8.57E+02	2.15E+03	1.29E+03	-1.59	.021	-.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	.661	4.6
MAXIMUM WIDTH	.805	6.4

Figure 2.-Continued.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

-------- REGRESSION VALUES --------

DATA SET	ALLOY 9	316 STAINLESS STEEL
PARAMETER SELECTED		RAB
NO. OF OBSERVATIONS		38
NO. OF INDEPENDENT VARIABLES		2
RESIDUAL DEGREES OF FREEDOM		35
F - VALUE		220.4
RESIDUAL MEAN SQUARE		2.1137E-03
STANDARD ERROR		4.5975E-02
RESIDUAL SUM OF SQUARES		7.3979E-02
TOTAL SUMS OF SQUARES		1.0058E+00
MULT. CORREL. COEF. SQUARED		.9264

RABOTNOV CONSTANT (RA) = .05630

MIN Y = 1.06E+00 MAX Y = 1.72E+00 Y RANGE = 6.66E-01 MEAN Y = 1.38E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	CERR
0		8.7594E-01									
1	1/L*T	-3.6886E+04	2.06E+03	17.94	4.693E-05	2.78E-05	9.52E-05	6.75E-05	-3.74	.092	0.00
2	1/L*T**2	6.3869E+07	3.19E+06	20.01	3.505E-08	2.04E-08	6.35E-08	4.31E-08	4.13	.908	0.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	1.088	12.3
MAXIMUM WIDTH	1.350	22.4

Figure 2.-Concluded.

```
0     0    -5000.0          0.2
2     3
2     4
2     5
2     6
2     7

ALLOY   9   316 STAINLESS STEEL
 3142.90       1175.00          25.00
   74.60       1200.00          30.00
  213.00       1200.00          28.00
  656.20       1200.00          25.00
 3476.10       1200.00          22.00
 6825.30       1200.00          20.00
10076.50       1200.00          18.50
15790.80       1200.00          17.00
  290.90       1225.00          25.00
  186.50       1250.00          25.00
   81.50       1275.00          25.00
   36.50       1300.00          25.00
  104.10       1300.00          22.00
  228.20       1300.00          20.00
  258.10       1300.00          19.00
  319.00       1300.00          18.00
  377.50       1300.00          17.00
  753.70       1300.00          16.00
  785.30       1300.00          16.50
 1232.50       1300.00          15.00
 1854.60       1300.00          13.60
 2421.00       1300.00          13.00
 4078.30       1300.00          12.00
 6258.10       1300.00          11.00
   21.50       1325.00          25.00
    9.90       1350.00          25.00
    2.70       1400.00          25.00
   83.30       1400.00          15.00
  251.20       1400.00          12.50
  921.00       1400.00          10.00
   27.90       1450.00          15.00
   75.20       1450.00          12.50
    5.00       1500.00          16.40
   40.60       1500.00          12.50
   87.90       1500.00          10.00
  170.40       1500.00           9.00
  614.90       1500.00           7.00
   28.70       1550.00          10.00
```

Figure 3.-Input data for Case 2.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

-------- REGRESSION VALUES ----------

DATA SET	ALLOY 9 316 STAINLESS STEEL
PARAMETER SELECTED	O-S-D
NO. OF OBSERVATIONS	38
NO. OF INDEPENDENT VARIABLES	2
RESIDUAL DEGREES OF FREDOM	35
F - VALUE	600.1
RESIDUAL MEAN SQUARE	2.5481E-02
STANDARD ERROR	1.5963E-01
RESIDUAL SUM OF SQUARES	8.9183E-01
TOTAL SUMS OF SQUARES	3.1476E+01
MULT. CORREL. COEF. SQUARED	.9717

MIN Y = 4.31E-01 MAX Y = 4.20E+00 Y RANGE = 3.77E+00 MEAN Y = 2.45E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	CERR
0		-1.1091E+01									
1	1/T	2.1335E+04	6.26E+02	34.09	1.011E-03	8.96E-04	1.10E-03	2.06E-04	1.16	.336	0.00
2	S	-6.5279E+00	2.31E-01	28.22	1.228E+00	8.45E-01	1.48E+00	6.32E-01	-1.10	.664	.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	.668	4.7
MAXIMUM WIDTH	.703	5.0

Figure 4.-Continued.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

-------- REGRESSION VALUES --------

DATA SET	ALLOY 9	316 STAINLESS STEEL
PARAMETER SELECTED		O-S-D
NO. OF OBSERVATIONS		38
NO. OF INDEPENDENT VARIABLES		3
RESIDUAL DEGREES OF FREEDOM		34
F - VALUE		575.9
RESIDUAL MEAN SQUARE		1.7867E-02
STANDARD ERROR		1.3367E-01
RESIDUAL SUM OF SQUARES		6.0748E-01
TOTAL SUMS OF SQUARES		3.1476E+01
MULT. CORREL. COEFF. SQUARED		.9807

MIN Y = 4.31E-01 MAX Y = 4.20E+00 Y RANGE = 3.77E+00 MEAN Y = 2.45E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	GERR
0		-1.5630E+01									
1	1/T	2.1117E+04	5.27E+02	40.07	1.011E-03	8.96E-04	1.10E-03	2.06E-04	1.15	.333	-.00
2	S	1.5077E+00	2.02E+00	.75	1.228E+00	8.45E-01	1.48E+00	6.32E-01	.25	.658	.00
3	S**2	-3.3333E+00	8.36E-01	3.99	1.533E+00	7.14E-01	2.18E+00	1.47E+00	-1.30	.009	.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	.567	3.7
MAXIMUM WIDTH	.593	3.9

Figure 4.-Continued.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

-------- REGRESSION VALUES --------

DATA SET	ALLOY 9	316 STAINLESS STEEL
PARAMETER SELECTED		O-S-O
NO. OF OBSERVATIONS		38
NO. OF INDEPENDENT VARIABLES		4
RESIDUAL DEGREES OF FREEDOM		33
F - VALUE		580.4
RESIDUAL MEAN SQUARE		1.3367E-02
STANDARD ERROR		1.1562E-01
RESIDUAL SUM OF SQUARES		4.4113E-01
TOTAL SUMS OF SQUARES		3.1476E+01
MULT. CORREL. COEF. SQUARED		.9860

MIN Y = 4.31E-01 MAX Y = 4.20E+00 Y RANGE = 3.77E+00 MEAN Y = 2.45E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	CERR
0		6.7051E+00									
1	1/T	2.0982E+04	4.57E+02	45.87	1.011E-03	8.96E-04	1.10E-03	2.06E-04	1.14	.331	.00
2	S	-5.7228E+01	1.67E+01	3.42	1.228E+00	8.45E-01	1.48E+00	6.32E-01	-9.60	.654	-.00
3	S**2	4.7687E+01	1.45E+01	3.29	1.533E+00	7.14E-01	2.18E+00	1.47E+00	18.58	.009	-.00
4	S**3	-1.4563E+01	4.13E+00	3.53	1.942E+00	6.04E-01	3.22E+00	2.62E+00	-10.13	.005	-.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	.496	3.1
MAXIMUM WIDTH	.569	3.7

Figure 4.-Continued.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

-------- REGRESSION VALUES ---------

DATA SET	ALLOY 9	316 STAINLESS STEEL
PARAMETER SELECTED		D-S-D
NO. OF OBSERVATIONS		38
NO. OF INDEPENDENT VARIABLES		5
RESIDUAL DEGREES OF FREEDOM		32
F - VALUE		594.8
RESIDUAL MEAN SQUARE		1.0472E-02
STANDARD ERROR		1.0233E-01
RESIDUAL SUM OF SQUARES		3.3509E-01
TOTAL SUMS OF SQUARES		3.1476E+01
MULT. CORREL. COEF. SQUARED		.9894

MIN Y = 4.31E-01 MAX Y = 4.20E+00 Y RANGE = 3.77E+00 MEAN Y = 2.45E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	CERR
0		-1.0736E+02									
1	1/T	2.1161E+04	4.09E+02	51.77	1.011E-03	8.96E-04	1.10E-03	2.06E-04	1.15	.330	-.00
2	S	3.4852E+02	1.28E+02	2.72	1.228E+00	8.45E-01	1.48E+00	6.32E-01	58.47	.652	-.00
3	S**2	-4.8780E+02	1.69E+02	2.89	1.533E+00	7.14E-01	2.18E+00	1.47E+00	-190.05	.009	-.00
4	S**3	2.9592E+02	9.76E+01	3.03	1.942E+00	6.04E-01	3.22E+00	2.62E+00	205.77	.005	-.00
5	S**4	-6.6787E+01	2.10E+01	3.18	2.492E+00	5.10E-01	4.76E+00	4.25E+00	-75.36	.003	-.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	.444	2.8
MAXIMUM WIDTH	.560	3.6

Figure 4.-Continued.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

------- REGRESSION VALUES --------

DATA SET	ALLOY 9	316 STAINLESS STEEL
PARAMETER SELECTED		O-S-D
NO. OF OBSERVATIONS		38
NO. OF INDEPENDENT VARIABLES		6
RESIDUAL DEGREES OF FREEDOM		31
F - VALUE		484.8
RESIDUAL MEAN SQUARE		1.0708E-02
STANDARD ERROR		1.0348E-01
RESIDUAL SUM OF SQUARES		3.3194E-01
TOTAL SUMS OF SQUARES		3.1476E+01
MULT. CORR'L. COEF. SQUARED		.9895

MIN Y = 4.31E-01 MAX Y = 4.20E+00 Y RANGE = 3.77E+00 MEAN Y = 2.45E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	CERR
0		4.7690E+01									
1	1/T	2.1181E+04	4.15E+02	51.03	1.011E-03	8.96E-04	1.10E-03	2.06E-04	1.16	.330	.00
2	S	-3.3766E+02	1.27E+03	.27	1.228E+00	8.45E-01	1.48E+00	6.32E-01	-56.65	.652	.01
3	S**2	7.1482E+02	2.22E+03	.32	1.533E+00	7.14E-01	2.18E+00	1.47E+00	278.50	.009	.00
4	S**3	-7.4798E+02	1.93E+03	.39	1.942E+00	6.04E-01	3.22E+00	2.62E+00	-520.10	.005	.00
5	S**4	3.8218E+02	8.28E+02	.46	2.492E+00	5.10E-01	4.76E+00	4.25E+00	431.24	.003	.00
6	S**5	-7.6572E+01	1.41E+02	.54	3.235E+00	4.31E-01	7.03E+00	6.60E+00	-134.18	.000	.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	.454	2.8
MAXIMUM WIDTH	.590	3.9

Figure 4.-Concluded.

```
3     3    -5000.0          0.2
2     6

ALLOY   9    316 STAINLESS STEEL
 3142.90        1175.00          25.00
   74.60        1200.00          30.00
  213.00        1200.00          28.00
  656.20        1200.00          25.00
 3476.10        1200.00          22.00
 6825.30        1200.00          20.00
10076.50        1200.00          18.50
15790.80        1200.00          17.00
  290.90        1225.00          25.00
  186.50        1250.00          25.00
   81.50        1275.00          25.00
   36.50        1300.00          25.00
  104.10        1300.00          22.00
  228.20        1300.00          20.00
  258.10        1300.00          19.00
  319.00        1300.00          18.00
  377.50        1300.00          17.00
  753.70        1300.00          16.00
  785.30        1300.00          16.50
 1232.50        1300.00          15.00
 1854.60        1300.00          13.60
 2421.00        1300.00          13.00
 4078.30        1300.00          12.00
 6258.10        1300.00          11.00
   21.50        1325.00          25.00
    9.90        1350.00          25.00
    2.70        1400.00          25.00
   83.30        1400.00          15.00
  251.20        1400.00          12.50
  921.00        1400.00          10.00
   27.90        1450.00          15.00
   75.20        1450.00          12.50
    5.00        1500.00          16.40
   40.60        1500.00          12.50
   87.90        1500.00          10.00
  170.40        1500.00           9.00
  614.90        1500.00           7.00
   28.70        1550.00          10.00
```

Figure 5.-Input data for Case 3.

```
     LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
     ANALYSIS OF CREEP-RUPTURE DATA
 DATA SET                        ALLOY   9   316 STAINLESS STEEL
OPTION CARD
INPUT= 3
IOUT= 3
TA=       -5000
RA=        .2000

CASE CONTROL CARDS
     PARAMETER CODE      NO. COEFFICIENTS

               2                    6
```

Figure 6.-Output for Case 3.

INPUT DATA OBSERVATIONS

NO.	TIME	STRESS	TEMPERATURE
1	3142.90	25	1175
2	74.60	30	1200
3	213.00	28	1200
4	656.20	25	1200
5	3476.10	22	1200
6	6825.30	20	1200
7	10076.50	18	1200
8	15790.80	17	1200
9	290.90	25	1225
10	186.50	25	1250
11	81.50	25	1275
12	36.50	25	1300
13	104.10	22	1300
14	228.20	20	1300
15	258.10	19	1300
16	319.00	18	1300
17	377.50	17	1300
18	753.70	16	1300
19	785.30	16	1300
20	1232.50	15	1300
21	1854.60	14	1300
22	2421.00	13	1300
23	4078.30	12	1300
24	6258.10	11	1300
25	21.50	25	1325
26	9.90	25	1350
27	2.70	25	1400
28	83.30	15	1400
29	251.20	13	1400
30	921.00	10	1400
31	27.90	15	1450
32	75.20	13	1450
33	5.00	16	1500
34	40.60	13	1500
35	87.90	10	1500
36	170.40	9	1500
37	614.90	7	1500
38	28.70	10	1550

Figure 6.-Continued.

LEAST-SQUARES REGRESSION PROGRAM FOR PARAMETRIC
ANALYSIS OF CREEP-RUPTURE DATA

------- REGRESSION VALUES -------

DATA SET	ALLOY 9	316 STAINLESS STEEL
PARAMETER SELECTED		0-S-0
NO. OF OBSERVATIONS		38
NO. OF INDEPENDENT VARIABLES		5
RESIDUAL DEGREES OF FREEDOM		32
F - VALUE		594.8
RESIDUAL MEAN SQUARE		1.0472E-02
STANDARD ERROR		1.0233E-01
RESIDUAL SUM OF SQUARES		3.3509E-01
TOTAL SUMS OF SQUARES		3.1476E+01
MULT. CORREL. COEF. SQUARED		.9894

MIN Y = 4.31E-01 MAX Y = 4.20E+00 Y RANGE = 3.77E+00 MEAN Y = 2.45E+00

FIRST 5 OBSERVATIONS - TRANSFORMED VARIABLES

Y	X1 - X(L2)				
3.49733E+00	1.10132E-03	1.39794E+00	1.95424E+00	2.73191E+00	3.81904E+00
1.87274E+00	1.08473E-03	1.47712E+00	2.18189E+00	3.22291E+00	4.76063E+00
2.32838E+00	1.08473E-03	1.44716E+00	2.09427E+00	3.03073E+00	4.38595E+00
2.81704E+00	1.08473E-03	1.39794E+00	1.95424E+00	2.73191E+00	3.81904E+00
3.54109E+00	1.08473E-03	1.34242E+00	1.80210E+00	2.41918E+00	3.24756E+00

I	VARIABLE	COEF.P(I)	S.E.COEF.	T	MEAN X(I)	MIN X(I)	MAX X(I)	RAN X(I)	RINF	PSUM	CERR
0		-1.0736E+02									
1	1/T	2.1161E+04	4.09E+02	51.77	1.011E-03	8.96E-04	1.10E-03	2.06E-04	1.15	.330	-.00
2	S	3.4852E+02	1.28E+02	2.72	1.228E+00	8.45E-01	1.48E+00	6.32E-01	58.47	.652	-.00
3	S**2	-4.8780E+02	1.69E+02	2.89	1.533E+00	7.14E-01	2.18E+00	1.47E+00	-190.05	.009	-.00
4	S**3	2.9592E+02	9.76E+01	3.03	1.942E+00	6.04E-01	3.22E+00	2.62E+00	205.77	.005	-.00
5	S**4	-6.6787E+01	2.10E+01	3.18	2.492E+00	5.10E-01	4.76E+00	4.25E+00	-75.36	.003	-.00

VARIABLE CODE
S=LOG STRESS
T=TEMPERATURE
DT=T-TA
L=STRESS

95 PERCENT PREDICTION INTERVAL STATISTICS

	LOG TIME	REAL TIME FACTOR (ANTILOG WIDTH)
AVERAGE WIDTH	.444	2.8
MAXIMUM WIDTH	.560	3.6

Figure 6.-Continued.

RESIDUALS - REGRESSION SPACE

OBS	Y OBS	Y CALC	RESIDUAL	PCTERR	ORDER	CIMIN	CIMAX	PIMIN	PIMAX
1	3.497E+00	3.266E+00	-2.312E-01	-6.6	2	3.334E+00	3.199E+00	3.046E+00	3.486E+00
2	1.873E+00	1.879E+00	6.405E-03	.3	32	2.024E+00	1.735E+00	1.625E+00	2.133E+00
3	2.328E+00	2.331E+00	2.687E-03	.1	36	2.426E+00	2.236E+00	2.102E+00	2.561E+00
4	2.817E+00	2.915E+00	9.799E-02	3.5	11	2.975E+00	2.855E+00	2.697E+00	3.133E+00
5	3.541E+00	3.402E+00	-1.391E-01	-3.9	6	3.467E+00	3.337E+00	3.183E+00	3.621E+00
6	3.834E+00	3.688E+00	-1.462E-01	-3.8	5	3.758E+00	3.618E+00	3.468E+00	3.908E+00
7	4.003E+00	3.892E+00	-1.109E-01	-2.8	7	3.965E+00	3.820E+00	3.671E+00	4.114E+00
8	4.198E+00	4.098E+00	-1.004E-01	-2.4	10	4.173E+00	4.023E+00	3.876E+00	4.320E+00
9	2.464E+00	2.574E+00	1.106E-01	4.5	8	2.630E+00	2.518E+00	2.358E+00	2.791E+00
10	2.271E+00	2.244E+00	-2.704E-02	-1.2	23	2.298E+00	2.189E+00	2.028E+00	2.460E+00
11	1.911E+00	1.922E+00	1.130E-02	.6	30	1.978E+00	1.867E+00	1.706E+00	2.139E+00
12	1.562E+00	1.610E+00	4.811E-02	3.1	18	1.670E+00	1.551E+00	1.393E+00	1.828E+00
13	2.017E+00	2.097E+00	7.990E-02	4.0	13	2.155E+00	2.040E+00	1.881E+00	2.314E+00
14	2.358E+00	2.383E+00	2.501E-02	1.1	26	2.440E+00	2.326E+00	2.167E+00	2.600E+00
15	2.412E+00	2.520E+00	1.082E-01	4.5	9	2.575E+00	2.465E+00	2.304E+00	2.736E+00
16	2.504E+00	2.656E+00	1.519E-01	6.1	4	2.708E+00	2.603E+00	2.440E+00	2.871E+00
17	2.577E+00	2.793E+00	2.165E-01	8.4	3	2.844E+00	2.743E+00	2.578E+00	3.009E+00
18	2.877E+00	2.937E+00	5.943E-02	2.1	15	2.987E+00	2.887E+00	2.722E+00	3.152E+00
19	2.895E+00	2.864E+00	-3.093E-02	-1.1	21	2.914E+00	2.814E+00	2.649E+00	3.079E+00
20	3.091E+00	3.089E+00	-1.443E-03	-.0	37	3.142E+00	3.037E+00	2.874E+00	3.305E+00
21	3.268E+00	3.328E+00	5.957E-02	1.8	14	3.388E+00	3.268E+00	3.110E+00	3.545E+00
22	3.384E+00	3.442E+00	5.778E-02	1.7	16	3.506E+00	3.377E+00	3.223E+00	3.661E+00
23	3.610E+00	3.651E+00	4.073E-02	1.1	19	3.724E+00	3.578E+00	3.430E+00	3.873E+00
24	3.796E+00	3.888E+00	9.195E-02	2.4	12	3.971E+00	3.806E+00	3.664E+00	4.113E+00
25	1.332E+00	1.307E+00	-2.534E-02	-1.9	25	1.372E+00	1.242E+00	1.088E+00	1.526E+00
26	9.956E-01	1.012E+00	1.654E-02	1.7	28	1.084E+00	9.400E-01	7.910E-01	1.233E+00
27	4.314E-01	4.461E-01	1.475E-02	3.4	29	5.344E-01	3.578E-01	2.192E-01	6.730E-01
28	1.921E+00	1.925E+00	4.402E-03	.2	33	1.981E+00	1.869E+00	1.709E+00	2.141E+00
29	2.400E+00	2.379E+00	-2.108E-02	-.9	27	2.437E+00	2.321E+00	2.162E+00	2.596E+00
30	2.964E+00	2.990E+00	2.611E-02	.9	24	3.062E+00	2.919E+00	2.770E+00	3.211E+00
31	1.446E+00	1.389E+00	-5.697E-02	-3.9	17	1.457E+00	1.320E+00	1.169E+00	1.609E+00
32	1.876E+00	1.843E+00	-3.369E-02	-1.8	20	1.907E+00	1.778E+00	1.624E+00	2.061E+00
33	6.990E-01	6.687E-01	-3.028E-02	-4.3	22	7.602E-01	5.772E-01	4.405E-01	8.969E-01
34	1.609E+00	1.333E+00	-2.750E-01	-17.1	1	1.408E+00	1.259E+00	1.111E+00	1.556E+00
35	1.944E+00	1.945E+00	9.271E-04	.0	38	2.018E+00	1.872E+00	1.723E+00	2.166E+00
36	2.231E+00	2.235E+00	3.831E-03	.2	34	2.319E+00	2.152E+00	2.010E+00	2.460E+00
37	2.789E+00	2.780E+00	-8.500E-03	-.3	31	2.966E+00	2.594E+00	2.500E+00	3.060E+00
38	1.458E+00	1.461E+00	3.328E-03	.2	35	1.543E+00	1.380E+00	1.237E+00	1.686E+00

Figure 6.-Continued.

BACKTRANSFORMED RESIDUALS — REAL SPACE

OBS	Y OBS	Y CALC	RESIDUAL	PCTERR	ORDER	CIMIN	CIMAX	PIMIN	PIMAX
1	3.143E+03	1.846E+03	1.297E+03	41.3	4	1.580E+03	2.156E+03	1.113E+03	3.061E+03
2	7.460E+01	7.571E+01	-1.108E+00	-1.5	32	5.428E+01	1.056E+02	4.217E+01	1.359E+02
3	2.130E+02	2.143E+02	-1.322E+00	-.6	36	1.723E+02	2.665E+02	1.264E+02	3.635E+02
4	6.562E+02	8.223E+02	-1.661E+02	-25.3	9	7.155E+02	9.451E+02	4.982E+02	1.357E+03
5	3.476E+03	2.523E+03	9.527E+02	27.4	8	2.174E+03	2.928E+03	1.525E+03	4.176E+03
6	6.825E+03	4.875E+03	1.951E+03	28.6	6	4.153E+03	5.722E+03	2.935E+03	8.096E+03
7	1.008E+04	7.806E+03	2.270E+03	22.5	11	6.611E+03	9.218E+03	4.691E+03	1.299E+04
8	1.579E+04	1.253E+04	3.258E+03	20.6	12	1.053E+04	1.491E+04	7.513E+03	2.091E+04
9	2.909E+02	3.753E+02	-8.438E+01	-29.0	5	3.300E+02	4.268E+02	2.280E+02	6.176E+02
10	1.865E+02	1.752E+02	1.126E+01	6.0	24	1.547E+02	1.986E+02	1.066E+02	2.881E+02
11	8.150E+01	8.365E+01	-2.149E+00	-2.6	30	7.359E+01	9.508E+01	5.083E+01	1.376E+02
12	3.650E+01	4.078E+01	-4.276E+00	-11.7	18	3.556E+01	4.676E+01	2.472E+01	6.726E+01
13	1.041E+02	1.251E+02	-2.103E+01	-20.2	13	1.096E+02	1.428E+02	7.595E+01	2.061E+02
14	2.282E+02	2.417E+02	-1.353E+01	-5.9	25	2.121E+02	2.755E+02	1.468E+02	3.981E+02
15	2.581E+02	3.311E+02	-7.304E+01	-28.3	7	2.917E+02	3.759E+02	2.013E+02	5.447E+02
16	3.190E+02	4.526E+02	-1.336E+02	-41.9	3	4.009E+02	5.109E+02	2.755E+02	7.435E+02
17	3.775E+02	6.215E+02	-2.440E+02	-64.6	1	5.532E+02	6.982E+02	3.787E+02	1.020E+03
18	7.537E+02	8.642E+02	-1.105E+02	-14.7	15	7.701E+02	9.699E+02	5.268E+02	1.418E+03
19	7.853E+02	7.313E+02	5.399E+01	6.9	21	6.517E+02	8.207E+02	4.458E+02	1.200E+03
20	1.232E+03	1.228E+03	4.087E+00	.3	37	1.089E+03	1.385E+03	7.479E+02	2.018E+03
21	1.855E+03	2.127E+03	-2.727E+02	-14.7	14	1.852E+03	2.443E+03	1.289E+03	3.510E+03
22	2.421E+03	2.766E+03	-3.445E+02	-14.2	16	2.383E+03	3.210E+03	1.671E+03	4.577E+03
23	4.078E+03	4.479E+03	-4.010E+02	-9.8	19	3.784E+03	5.302E+03	2.690E+03	7.459E+03
24	6.258E+03	7.734E+03	-1.476E+03	-23.6	10	6.399E+03	9.347E+03	4.610E+03	1.297E+04
25	2.150E+01	2.028E+01	1.219E+00	5.7	26	1.745E+01	2.357E+01	1.225E+01	3.358E+01
26	9.900E+00	1.028E+01	-3.842E-01	-3.9	28	8.709E+00	1.214E+01	6.180E+00	1.711E+01
27	2.700E+00	2.793E+00	-9.325E-02	-3.5	29	2.279E+00	3.423E+00	1.656E+00	4.710E+00
28	8.330E+01	8.415E+01	-8.487E-01	-1.0	33	7.397E+01	9.572E+01	5.113E+01	1.385E+02
29	2.512E+02	2.393E+02	1.190E+01	4.7	27	2.094E+02	2.734E+02	1.452E+02	3.943E+02
30	9.210E+02	9.781E+02	-5.707E+01	-6.2	23	8.302E+02	1.152E+03	5.882E+02	1.626E+03
31	2.790E+01	2.447E+01	3.430E+00	12.3	17	2.089E+01	2.866E+01	1.474E+01	4.061E+01
32	7.520E+01	6.959E+01	5.614E+00	7.5	20	6.004E+01	8.064E+01	4.206E+01	1.151E+02
33	5.000E+00	4.663E+00	3.367E-01	6.7	22	3.777E+00	5.757E+00	2.757E+00	7.887E+00
34	4.060E+01	2.155E+01	1.905E+01	46.9	2	1.813E+01	2.561E+01	1.292E+01	3.594E+01
35	8.790E+01	8.809E+01	-1.878E-01	-.2	38	7.442E+01	1.043E+02	5.290E+01	1.467E+02
36	1.794E+02	1.719E+02	-1.510E+00	-.9	34	1.418E+02	2.083E+02	1.024E+02	2.887E+02
37	6.149E+02	6.030E+02	1.192E+01	1.9	31	3.928E+02	9.257E+02	3.165E+02	1.149E+03
38	2.870E+01	2.892E+01	-2.208E-01	-.8	35	2.398E+01	3.488E+01	1.725E+01	4.848E+01

Figure 6.-Continued.

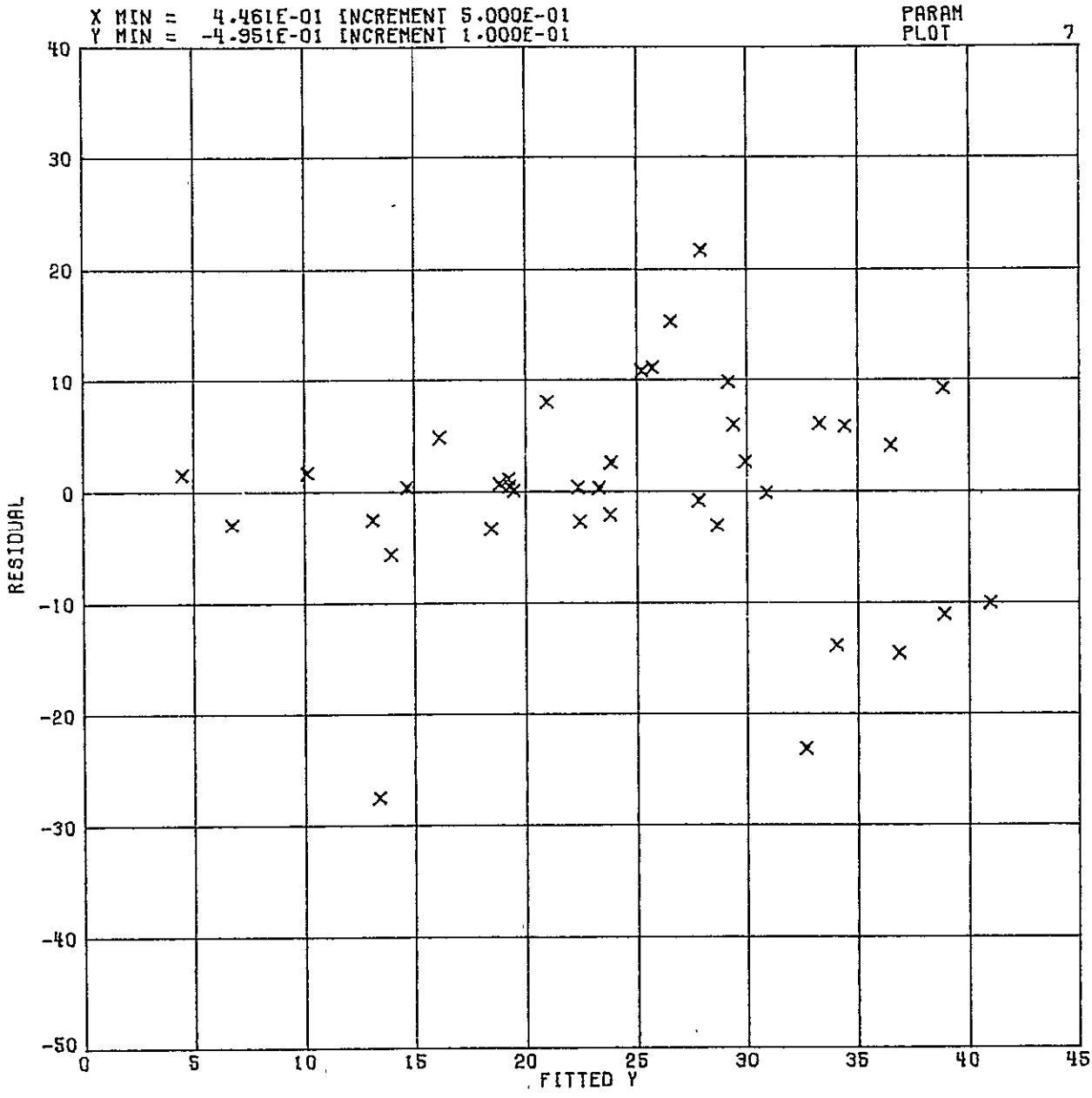


Figure 6.-Continued.

156

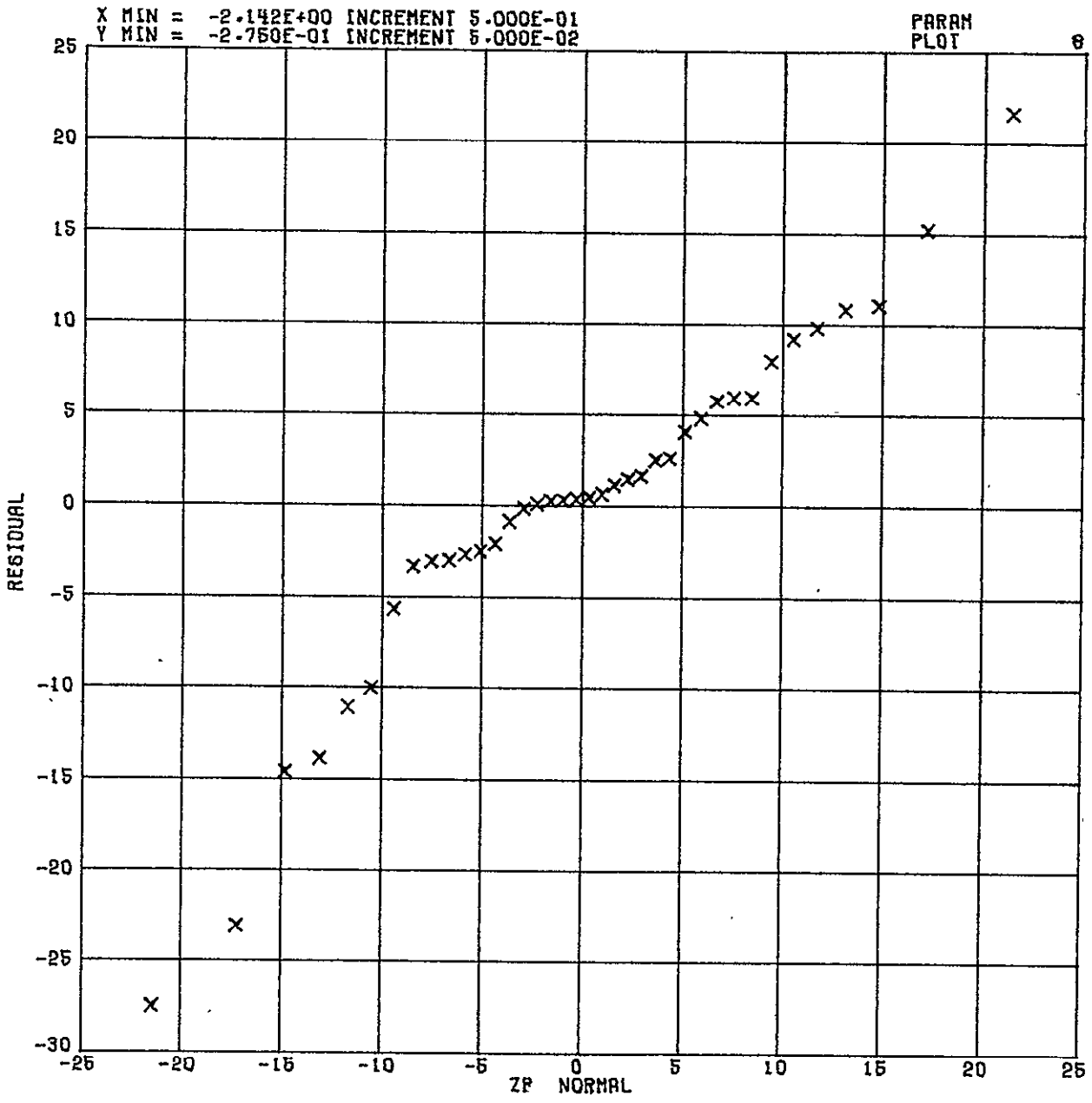


Figure 6.-Concluded.