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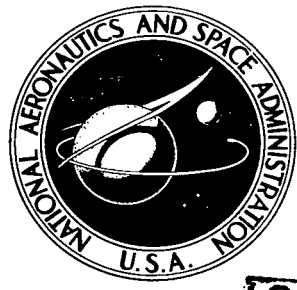

MINIMUM DISTORTION QUANTIZERS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION · WASHINGTON, D. C. • MARCH 1977

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-8384		

4. Title and Subtitle	5. Report Date
MINIMUM DISTORTION QUANTIZERS	March 1977
	6. Performing Organization Code

7. Author(s)	8. Performing Organization Report No.
Harry W. Jones, Jr.*	A-6714
	10. Work Unit No.
9. Performing Organization Name and Address	656-11-02-01
Ames Research Center Moffett Field, California 94035	11. Contract or Grant No.
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Technical Note
National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

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16. Abstract

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17. Key Words (Suggested by Author(s))	18. Distribution Statement
Quantizers, Minimum distortion Quantizers, Max Distortion measures Rate-distortion theory	Unlimited STAR Category - 32

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	123	$5.25

*For sale by the National Technical Information Service, Springfield, Virginia 22161

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SUMMARY

The well-known algorithm of Max is used to determine the minimum distortion quantizers for normal, two-sided exponential, and specialized two-sided gamma input distributions and for mean-square, magnitude, and relative magnitude error distortion criteria. The optimum equally-spaced and unequally-spaced quantizers are found, with the resulting quantizer distortion and entropy. The quantizers, and the quantizers with entropy coding, are compared to the rate distortion bounds for mean-square and magnitude error.

INTRODUCTION

The well-known optimum quantizers and optimum, equally-spaced level quantizers of Max (ref. 1) have minimum mean-square error distortion for a given number of output levels, assuming a normal or Gaussian distribution of the input parameter. Paez and Glisson (ref. 2) used the numerical algorithm of Max to find optimum quantizers and optimum, equally-spaced level quantizers for minimum mean-square error distortion, assuming either the two-sided exponential (Laplacian) distribution, or McDonald's special form of the gamma distribution. Here, the work of Max and of Paez and Glisson has been repeated and extended to higher and intermediate numbers of levels for the mean-square error distortion and for the normal, exponential, and gamma distributions. In addition, the optimum quantizers and the optimum equally-spaced level quantizers were found for these three distributions using the magnitude error distortion and the relative magnitude error distortion criteria suggested by Andrews and Pratt (ref. 3). The method of Max is reviewed, and the input distributions and distortion measures are defined. The quantizers are given and discussed, and their performance described. The new results are listed in the summary and conclusion section at the end of this report.

MAX ALGORITHM

If the input parameter distribution is $p(x)$ and the distortion for input parameters x and representative value y_i is $d(x - y_i)$, then for M representative values for the parameter, the total distortion is

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$$D = \sum_{i=1}^{M} \int_{x_i}^{x_{i+1}} d(x - y_i)p(x)dx \tag{1}$$

where x_i and x_{i+1} are the cut points determining the range of x represented by the value y_i, and x_1 and x_{M+1} are infinite. The distortion is minimized by differentiating D with respect to the x_i's and y_i's and setting the derivatives equal to zero. For $p(x) \neq 0$ for all x and $d(x)$ monotonically increasing with x, Max (ref. 1, p. 268) shows that setting the derivative with respect to x_i equal to zero requires that

$$x_i = (y_i + y_{i-1})/2 , \quad i = 2, \ldots, M \tag{2}$$

Thus, the cut point x_i is halfway between the two representative values y_i and y_{i-1}. The derivative with respect to y_i is also set to zero, giving

$$\int_{x_i}^{x_{i+1}} d'(x - y_i)p(x)dx = 0 , \quad i = 1, \ldots, M \tag{3}$$

Since $p(x)$ is symmetrical about $x = 0$, zero is a cut point for M-even and a representative value for M-odd, and the positive and negative representative levels and cut points are symmetrical, having the same magnitude and opposite signs. The indexing of the x_i and y_i is therefore changed so that x_1 and y_1 are the smallest nonnegative x_i and y_i, and $x_{\bar{M}/2}$, $y_{\bar{M}/2}$ for M-even, or $x_{(\bar{M}-1)/2}$, $y_{(\bar{M}+1)/2}$ for M-odd are the largest positive noninfinite x_i and y_i.

For unequal-level spacing and M-even, x_1 is zero and y_1 is estimated. Then, equation (3) is solved for x_2, equation (2) is solved for y_2, and so on, until equation (2) is solved for $y_{\bar{M}/2}$. If equation (3) is not satisfied by this $y_{\bar{M}/2}$, estimated y_1 is adjusted in the same direction that $y_{\bar{M}/2}$ would be adjusted to satisfy equation (3). For unequal-level spacing and M-odd, y_1 is zero and x_1 is estimated. Equation (2) is solved for y_2, equation (3) is solved for x_2, and so on, until $y_{(\bar{M}+1)/2}$ is found and tested in equation (3). If the integral is not sufficiently close to zero, the estimated x_1 is adjusted in the same direction that $y_{(\bar{M}+1)/2}$ would be adjusted to satisfy equation (3). If the output levels are equally spaced, the x_i and y_i are integral multiples of half the level spacing, and the distortion is minimized with respect to a single parameter, the level spacing. More detail is given by Max (ref. 1). Two computer programs were developed to find the equal- and unequal-level spaced quantizers. The programs use the different input distributions and distortion functions described below.

INPUT DISTRIBUTIONS

Three input distributions — the normal, the two-sided exponential, and a specialized gamma distribution — are considered. The distributions, which have zero mean and unit variance, are plotted in figure 1. The normal distribution is

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \tag{4}$$

The two-sided exponential or Laplacian distribution is (refs. 2 and 4)

$$p(x) = \frac{1}{\sqrt{2}} e^{-\sqrt{2}|x|} \tag{5}$$

McDonald's specialized gamma distribution (refs. 2 and 5) is

$$p(x) = \frac{(3/4)^{1/4}}{2\sqrt{\pi|x|}} e^{-\sqrt{3/4}|x|} \tag{6}$$

These distributions are referred to below simply as the normal, exponential, and gamma distributions.

The normal distribution is used most frequently in rate distortion theory, but Paez and Glisson (ref. 2) refer to McDonald's (ref. 5) evidence that speech amplitude variations can be modeled by the gamma distribution, which is approximated by the mathematically simple exponential density. There is also evidence that two-dimensional Hadamard transform coefficients of image data have the exponential distribution (ref. 6). Experimental Hadamard transform coefficients (ref. 7) often have a gradually decreasing exponential slope as x increases, and can be modeled by the gamma distribution, or by an even more highly peaked distribution.

DISTORTION MEASURES

Three distortion measures — the mean-square error, the magnitude error, and the relative magnitude error — are considered. The distortion for representative values of 0 and ±2.0 and of cut points of ±1.0 is plotted in figure 2. The mean-square error function, which defines distortion when substituted in equation (1), is

$$d(x - y_i) = (x - y_i)^2 \tag{7}$$

The magnitude error function is

$$d(x - y_i) = |x - y_i| \tag{8}$$

The relative magnitude error function is

$$d(x - y_i) = \frac{|x - y_i|}{|x|} \tag{9}$$

The mean-square error distortion is used most frequently in rate distortion theory and in image data compression. The magnitude distortion criterion and the relative magnitude distortion criterion were suggested for image data

by Andrews and Pratt (ref. 3). The mean-square and magnitude distortion are monotonically increasing as $|x - y_i|$ increases, but relative distortion does not have this property; the property is used in the derivation of equation (2).

For example, consider a representative value of 2, as shown in figure 2, with input values of 1 and 4. Substituting in equation (9), we have $|4 - 2| > |1 - 2|$, but $|4 - 2|/|4| < |1 - 2|/|1|$. In fact, the limit, as x becomes infinite, of $|x - 2|/|x|$ is 1. Another problem with relative distortion is that zero is a cut point for M-even, with the representative values symmetrical about zero. The distortion for small x is $|x - y_1|/|x|$ which becomes infinite as x approaches zero. It is shown in the appendix that the relative distortion criteria require one representative value to be zero, so that the only sets of cut points and representative values that are symmetrical about zero have an odd number of representative values. Only the levels for odd numbers of M are computed, but M-even can be used. For example, to use four levels, the optimum quantizer would have representative values of 0, of the one negative level for $M = 3$, and of the two positive levels for $M = 5$, or positive and negative levels could be interchanged. It is also shown in the appendix that, for M-odd, equation (2) is correct even though the monotonic distortion requirement is not satisfied.

MINIMUM DISTORTION QUANTIZERS

The results of running the Max algorithm programs for the above input distributions and distortion measures are given in tables 2 through 17; these tables are described in table 1. The number of levels increases from the lowest to highest, as indicated in parentheses, by adding 1; by multiplying by 2; or by multiplying by 2 and adding 1. The arrangement of the tables follows Max (ref. 1) directly, and differs from Paez and Glisson (ref. 2). All the numbers in the tables have been rounded to four significant digits. The final infinite cut points have not been included in tables 9 through 17. The letter a next to most of the values for $M = 1$ and $M = 2$ indicates a value derived by direct computation, rather than by the Max algorithm programs.

The equal and unequal quantizers for normal input distribution and mean-square error for $M = 1$ (1+) 36 are given by Max (ref. 1). The corresponding values given here in tables 2 and 9 are usually identical, differing at most by 5 units in the fourth place. The equal and unequal quantizers for exponential and gamma input distributions and mean-square error for $M = 2$ (2X) 32 are given by Paez and Glisson (ref. 2). The corresponding values given in tables 3, 10, and 11 differ slightly in most cases, and differ significantly for the unequal $M = 16$ and $M = 32$ gamma distribution quantizers. Since the same programs that obtained good agreement with Max were used, and since the mean-square error distortions obtained are in every case less than those given by Paez and Glisson (except for gamma, $M = 2$, where directly computed values are used), it appears that the values given here are more optimal.

The results given in the tables are partially plotted in the figures discussed below. Figure 3 shows the optimum quantizer level spacing for normal, exponential, and gamma input distribution and for mean-square error.

This figure, like many of the other figures included here, shows intermediate values not listed in any table. The level spacings for the exponential and gamma distributions are wider, reflecting their higher probabilities for large x. This also appears in figure 4, where the largest representative value is higher for the exponential and gamma distributions.

Figures 3 and 4 show also the similarity of the exponential and gamma quantizers, and indicate that the gamma quantizers have narrower spacing for small M-even than for small M-odd. This is due to the infinite value of the gamma density at $x = 0$, which requires a zero or small positive representative value. This effect, much reduced, also occurs for the exponential distribution.

Figure 5 shows mean-square distortion for the optimum equal quantizers, and figure 6 shows the distortion for odd and even M, gamma input, equal quantizers. The distortion is significantly less for the normal input distribution because of its narrower spread and lower peak.

Figure 6 shows that, for the gamma distribution and equal spacing, 2*(integer)-1 levels provide better performance than 2*(integer) levels. Figure 5 also includes the distortion for the normal input, optimum unequal spacing quantizer, which is less than the distortion for the equal level quantizer. Figure 7 shows the distortion for all the minimum mean square error unequal quantizers. As in the normal input case, the distortions for the exponential and gamma inputs are less with unequal level spacing quantizers. Although the normal input distribution again has least distortion, the differences are smaller because the optimum unequal quantizers adjust to the input distribution shape. Figure 8 gives the largest representative value for unequal-spacing, minimum mean-square error quantizers. Compared to the largest representative values for equal spacing quantizers given in figure 4, the largest representative value for unequal spacing quantizers increases more rapidly with M until a final value is approached. The largest representative values for odd and even M do not have different curves, as they did for equal spacing quantization. In all cases for unequal spacing quantizers, increasing the number of levels decreases distortion.

Figure 9 shows the optimum equal spacing for magnitude distortion and figure 11 shows the optimum equal spacing for relative distortion. The spacing is smaller for magnitude distortion, and smaller still for relative distortion, because of the reduced weighting of large magnitude errors, especially for the representative value interval that extends to infinity. The gamma distribution for magnitude distortion again has narrower spacing for an even number of levels. As discussed above and in the appendix, only odd numbers of levels are used with relative distortion. As in the mean square error case, the normal input quantizers have the smallest spacing and the gamma input quantizers have the largest spacing. Figure 10 gives the distortion for minimum magnitude error, equal-spacing quantizers. The distortion for the gamma distribution for odd and even numbers of levels is given in figure 6. As in the mean-square error case, distortion is lower for odd numbers of levels. The distortion for minimum relative error equal spacing quantizers is given in figure 12. Again, as in the case of mean-square error, the equal quantizers have least distortion for a normal input and have most distortion for a gamma

input. The optimum unequal quantizers for magnitude and relative magnitude distortion are not separately plotted, but are discussed in the next section.

Max (ref. 1) indicated that, for M-large, increasing the number of levels to 2M would, as an approximation, cause each previous representative level to be divided into two equal intervals; figures 3, 9, and 11 confirm this. For approximately constant probability density in each interval, the mean square error is reduced by one-fourth; figures 5, 6, and 7 confirm this. Similarly, doubling a high number of levels would reduce the magnitude error by one-half, as shown in figures 6 and 10. The relative distortion (fig. 12) is less well behaved, although the distributions for normal and exponential inputs nearly follow the one-half slope.

Although the minimum distortion quantizer is exactly defined, the distortion can approach the minimum for significantly different quantizers. For the $M = 38$ and $M = 40$ quantizers in table 9, distortion is 10 percent larger for $M = 38$. If an $M = 40$ quantizer is designed using the $M = 38$ quantizer with any two additional representative values, the distortion must be less than the $M = 38$ distortion, and within 10 percent of the minimum. This implies that the form of the minimum distortion quantizer is less accurately defined than its performance, and that convenient approximations to the quantizers, or even greatly differing quantizers, will often perform acceptably.

PERFORMANCE OF THE MINIMUM DISTORTION QUANTIZERS

It is well known that, for a given distortion, the normal input distribution requires a higher minimum transmission rate than any other zero mean, unit variance distribution (ref. 4, pp. 101-102). However, this rate distortion bound defines only the minimum rate for all possible transmission methods. Simply using a quantizer with an integer number of equally spaced levels for each sample is not the best transmission method, and it is relatively less efficient for the highly peaked exponential and gamma distributions. Huffman coding (ref. 8, Ch. 2) can be used to reduce the transmission rate from $\log_2 M$ to the quantizer entropy, a lower but variable rate. A plot of the equal level spacing quantizer distortion versus entropy (fig. 13) shows that the normal distribution does require higher rate (except at small M) for a transmission system consisting of an equally spaced quantizer and a Huffman coder. The optimum unequal quantizer has less distortion than the equal quantizer. Although the unequal quantizer for the normal input distribution again has lowest distortion, the difference in distortion (fig. 7) is less than for the equal quantizer. A plot of the unequal level spacing quantizer distortion versus entropy (fig. 14) shows that the normal distribution requires a transmission rate equal to or greater than the other distributions, at medium and large M.

The equally-spaced and unequally-spaced minimum distortion quantizers can each be used with entropy coding, giving four possible systems for each combination of input distribution and distortion measure. The performance of the systems listed in the tables is shown in figures 15 through 23. The rate distortion bound, or a lower bound on this bound, is given for quantizers

designed for mean-square and magnitude error (ref. 4, pp. 92-102, 141). In figure 15, the distortion is lower for entropy-coded, equal-spaced quantizers than for entropy-coded, unequally-spaced quantizers. This result was found by Wood (ref. 9), who used approximations for entropy and distortion based on the work of Max (ref. 1). Goblick and Holsinger (ref. 10) noted earlier that the entropy-coded, equal-spaced quantizers were within 0.25 (here 0.3) bits of the rate distortion bound.

Figures 16 and 17 indicate that the cases of exponential and gamma input and mean-square error are similar. Entropy-coded, equal quantizers give the best performance, and they approach the rate distortion bound. Although these highly peaked distributions give poor performance with uncoded, equal-spaced quantizers, the low probability of the higher representative levels allows larger rate reduction with entropy coding. For the gamma input and mean-square error, entropy-coded (variable rate), equal quantizers are 0.5 to 1.5 bits better than unequal quantizers. The rate reduction is comparable to the reduction obtained using variable rate adaptive Hadamard image compression (ref. 11), and these two variable rate methods can be combined.

Figures 18, 19, and 20 give the rate distortion bounds and quantizer performance, with and without entropy coding, for the magnitude distortion measure. As in the case of mean-square distortion, the entropy-coded, equal-spaced quantizers have lowest distortion, approaching the bound, and are superior to uncoded, unequal quantizers, except at some small M.

Figures 21, 22, and 23 give the quantizer performance, with and without entropy coding, for the relative distortion measure. The lower bound on the rate distortion cannot be found using the method for difference distortion measures, because the relative distortion is a function of the sample value as well as the error (ref. 4, p. 92). For the relative error, the unequally-spaced quantizers give significantly less distortion than entropy-coded, equal quantizers.

For all three distortion measures, the performance differences increase greatly for the exponential and gamma input distributions.

SUMMARY AND CONCLUSION

The results of the work include the structure of the new quantizers, the performance of the quantizers, and certain properties of the quantizers due to input distribution or distortion properties, as follows.

1. The minimum mean-square error, equal- and unequal-spaced quantizers for normal, exponential, and gamma input distributions, with the distortion and entropy, were found for new numbers of levels.

2. The magnitude error quantizers for normal, exponential, and gamma input distributions, and the distortion and entropy, were found.

3. The relative magnitude error quantizers for normal, exponential, and gamma input distributions, and the distortion and entropy, were found.

4. For exponential and gamma distributions and mean-square error, it was shown that equally-spaced quantizers with entropy coding are far superior to unequally-spaced quantizers with entropy coding, and that they approach the rate distortion bound. (These results were shown for the normal input distribution and mean-square error by Wood (ref. 9) and by Goblick and Holsinger (ref. 10), who used the work of Max (ref. 1)).

5. For magnitude error and small M, unequal-spacing quantizers with entropy coding sometimes have slightly less distortion than entropy-coded, equally-spaced quantizers. The entropy-coded, equal-spaced quantizers are superior for medium and large M, and approach the rate distortion bound.

6. For relative magnitude error, unequal quantizers have significantly less distortion than entropy-coded, equally-spaced quantizers.

7. The rate reduction for entropy-coded, equally-spaced quantizers is significantly larger for exponential or gamma input distributions than for the normal input distribution.

8. Equally-spaced quantizers with odd numbers of levels are superior to equally-spaced quantizers with even numbers of levels for the gamma input distribution. Gamma distribution quantizers usually have a representative level either equal to, or very close to, zero.

9. The exponential input distribution has a similar but much smaller superiority of the equally-spaced quantizers with odd numbers of levels.

10. The relative magnitude distortion criterion forces one representative level to be zero.

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APPENDIX

RELATIVE MAGNITUDE DISTORTION

Suppose that M is even; the cut point x_1 is zero and the distortion for the first representative value, from equation (1) is:

$$D_1 = 2 \int_{x_1}^{x_2} \frac{|x - y_1|}{|x|} p(x)dx \tag{A1}$$

x_2 is greater than y_1 and is infinite for $M = 2$. To minimize the distortion, the derivative with respect to y_1 is set to zero, as in equation (3) here, and in equations (2), (4), and (6) in reference (1). Removing the absolute value by using two integrals in the ranges 0 to y_1 and y_1 to x_2 and using Leibnitz's rule,

$$\frac{dD_1}{dy_1} = 2 \int_o^{y_1} x^{-1}p(x)dx - 2 \int_{y_1}^{x_2} x^{-1}p(x)dx \tag{A2}$$

Suppose $p(x)$ is monotonically decreasing as x increases and is continuous at $x = 0$. For small positive ε,

$$\left.\begin{aligned} \frac{dD_1}{dy_1} &= 2 \int_o^{\varepsilon} x^{-1}p(o)dx + 2 \int_{\varepsilon}^{y_1} x^{-1}p(x)dx - 2 \int_{y_1}^{x_2} x^{-1}p(x)dx \\ &= 2p(o)\ell n\ x\Big|_o^{\varepsilon} + 2 \int_{\varepsilon}^{y_1} x^{-1}p(x)dx - 2 \int_{y_1}^{x_2} x^{-1}p(x)dx \\ &= 2p(o)[\ell n\ \varepsilon + \infty] + 2 \int_{\varepsilon}^{y_1} x^{-1}p(x)dx - 2 \int_{y_1}^{x_2} x^{-1}p(x)dx \end{aligned}\right\} \tag{A3}$$

The $\ell n\ \varepsilon$ is a large finite negative number, the second integral is positive, and the third is bounded as follows:

$$2 \int_{y_1}^{x_2} x^{-1}p(x)dx < 2 \frac{1}{y_1} \int_o^{\infty} p(x)dx = \frac{1}{y_1} \tag{A4}$$

We therefore have

$$\frac{dD_1}{dy_1} > 2p(o)[\infty] + 2p(o)\ell n\ \varepsilon - \frac{1}{y_1} \tag{A5}$$

Since $\ell n\ \varepsilon$ is finite, the derivative of D_1 with respect to y_1 is infinite for all nonzero y_1. D_1 is large for y_1 not equal to zero and increases as y_1 increases, so that y_1 is forced to zero by the behavior of

dD_1/dy_1. For the case where M is odd, y_1 is zero and the relative distortion is always 1 or less, since $y_1 = 0$ may be used in equation (9). The other values of y_i are used when they are closer to x and reduce the distortion.

The assumption that p(x) is continuous at zero is not satisfied for the gamma distribution, but if we take $p(x) = p'(x)/\sqrt{|x|}$, p'(x) is continuous and, as in equation (A3)

$$\left.\begin{aligned} \frac{dD_1}{dy_1} &= 2\int_o^{\varepsilon} x^{-3/2}p'(o)dx + \ldots \\ &= 2p'(o)[-(2/5)x^{-2/5}|_o^{\varepsilon}] + \ldots \\ &= (4/5)p'(o)[-\varepsilon^{-2/5} + \infty] + \ldots \end{aligned}\right\} \quad \text{(A6)}$$

Bounding the third term of equation (A3) by $y_1^{-3/2}$, the infinite positive derivative again forces y_1 to zero.

Even for M-odd, the relative magnitude distortion is not a monotonically increasing function of $|x - y_i|$, as shown in figure 2 and mentioned in the text above. In reference 1, Max uses the monotonically increasing property to prove equation (2) above, which is used in the algorithm. Equation (2) can be shown directly, in a manner parallel to that of Max (Max eqs. (1) and (5)). From equations (1) and (9)

$$D = \sum_{i=1}^{M} \int_{x_i}^{x_{i+1}} \frac{|x - y_i|}{|x|} p(x)dx \quad \text{(A7)}$$

Setting dD/Dx_i equal to zero, by Leibnitz's rule, the i - 1 and i terms give

$$\frac{dD}{dx_i} = \frac{|x_i - y_{i-1}|}{|x_i|} p(x_i) - \frac{|x_i - y_i|}{|x_i|} p(x_i) = 0 \quad \text{(A8)}$$

For $p(x_i) \neq 0$ and $|x_i| \neq 0$, which is true for M-odd because no cut point can equal the zero representative value,

$$|x_i - y_{i-1}| = |x_i - y_i| \quad \text{(A9)}$$

Since y_{i-1}, x_i, and y_i are increasing positive values,

$$x_i = (y_i + y_{i-1})/2 \quad \text{(A10)}$$

which is equation (2).

We have shown that the relative magnitude distortion requires a representative value of zero, and that for M-odd, which implies a zero representative value, the relative magnitude distortion can be treated by the Max algorithm.

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TABLE 1.- LIST OF MINIMUM DISTORTION QUANTIZER TABLES

Table[a]	Spacing	Input distribution	Distortion criterion	Number of levels, M
2	Equal	Normal, exponential, gamma	Mean square	1(1+)40
3			Mean square	2(2x)2048
4			Magnitude	1(1+)40
5			Magnitude	2(2x)2048
6			Relative magnitude	1(1+)40
7			Relative magnitude	2(2x)2048
8		Gamma	Mean square, magnitude	1(2x,1+)511
9	Unequal	Normal	Mean square	1(1+)40, 64(2x)256
10		Exponential	Mean square	1(1+)40, 64(2x)256
11		Gamma	Mean square	1(1+)40, 64(2x)256
12		Normal	Magnitude	2(2x)16
13		Exponential	Magnitude	2(2x)16
14		Gamma	Magnitude	1(2x,1+)15
15		Normal	Relative magnitude	1(2x,1+)31
16		Exponential	Relative magnitude	1(2x,1+)31
17		Gamma	Relative magnitude	1(2x,1+)31

[a]Tables 2-8 contain the spacing, distortion, and entropy for each number of representative values. Tables 9-17 contain the cut points and representative values, distortion, and entropy for each number of representative values. The probability and distortion for each representative interval are also given.

TABLE 2.- EQUIDISTANT QUANTIZATION LEVEL SPACING FOR MEAN SQUARE DISTORTION

M	Normal PDF			Exponential PDF			Gamma PDF		
	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy
1	0.0000	1.0000[a]	0.000[a]	0.0000	1.0000[a]	0.000[a]	0.0000	1.0000[a]	0.000[a]
2	1.5960[a]	.3634[a]	1.000[a]	1.4140[a]	.5000[a]	1.000[a]	1.1290[a]	.6816[a]	1.000[a]
3	1.2240	.1902	1.536	1.4140	.2634	1.324	1.8170	.2761	1.337
4	.9957	.1188	1.904	1.0850	.1931	1.752	1.0790	.2772	1.754
5	.8430	.08218	2.183	1.0240	.1322	1.872	1.3070	.1444	1.846
6	.7334	.06066	2.409	.8688	.1079	2.127	.9175	.1673	2.058
7	.6508	.04686	2.598	.8209	.08249	2.213	1.0420	.09196	2.133
8	.5860	.03744	2.761	.7295	.07075	2.394	.7926	.11160	2.268
9	.5338	.03070	2.904	.6921	.05704	2.468	.8757	.06481	2.339
10	.4908	.02569	3.032	.6309	.05020	2.608	.6990	.08033	2.433
11	.4546	.02186	3.148	.6034	.04267	2.670	.7593	.04861	2.504
12	.4238	.01885	3.253	.5594	.03834	2.783	.6245	.06022	2.571
13	.3972	.01645	3.350	.5362	.03306	2.842	.6726	.03804	2.642
14	.3739	.01450	3.440	.5028	.03007	2.937	.5649	.04687	2.697
15	.3534	.01289	3.524	.4837	.02645	2.991	.6052	.03071	2.763
16	.3352	.01154	3.602	.4573	.02428	3.074	.5318	.03855	2.779
17	.3189	.01040	3.676	.4431	.02206	3.118	.5509	.02538	2.870
18	.3042	.009434	3.746	.4218	.02044	3.191	.4837	.03127	2.890
19	.2909	.008598	3.811	.4083	.01848	3.236	.5062	.02138	2.967
20	.2788	.007873	3.874	.3906	.01723	3.301	.4565	.02662	2.964
21	.2678	.007239	3.933	.3790	.01573	3.344	.4687	.01828	3.055
22	.2576	.006682	3.990	.3640	.01474	3.402	.4192	.02315	3.063
23	.2482	.006189	4.045	.3554	.01380	3.437	.4366	.01583	3.137
24	.2396	.005751	4.097	.3423	.01278	3.491	.3939	.02002	3.139
25	.2315	.005360	4.146	.3337	.01205	3.527	.4089	.01386	3.214
26	.2241	.005008	4.194	.3226	.01140	3.576	.3716	.01749	3.211
27	.2171	.004692	4.241	.3147	.01062	3.612	.3847	.01224	3.285
28	.2105	.004406	4.285	.3050	.01007	3.657	.3519	.01542	3.278
29	.2044	.004146	4.328	.2980	.00943	3.690	.3633	.01090	3.352
30	.1987	.003909	4.370	.2892	.008969	3.733	.3342	.01370	3.341
31	.1935	.003693	4.408	.2842	.008592	3.759	.3442	.009769	3.416
32	.1881	.003495	4.449	.2751	.008040	3.805	.3182	.012260	3.402
33	.1833	.003313	4.487	.2708	.007741	3.829	.3271	.008811	3.476
34	0.1788	0.003146	4.524	0.2639	0.007405	3.866	0.3033	0.01099	3.461
35	.1744	.002991	4.559	.2586	.007012	3.895	.3118	.007991	3.533
36	.1703	.002848	4.594	.2624	.006721	3.930	.2899	.009914	3.517
37	.1664	.002715	4.628	.2476	.006382	3.958	.2978	.007282	3.588
38	.1627	.002592	4.661	.2419	.006128	3.991	.2774	.008980	3.558
39	.1591	.002477	4.693	.2374	.005834	4.018	.2851	.006666	3.640
40	.1557	.002370	4.724	.2323	.005611	4.049	.2663	.008184	3.617

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 3.- EQUIDISTANT QUANTIZATION LEVEL SPACING FOR MEAN SQUARE DISTORTION

M	Normal PDF			Exponential PDF			Gamma PDF		
	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy
2	1.5960[a]	0.3634[a]	1.000[a]	1.4140[a]	0.5000[a]	1.000[a]	1.1290[a]	0.6816[a]	1.000[a]
4	.9957	.1188	1.904	1.0850	.1931	1.752	1.0790	.2772	1.754
8	.5860	.03744	2.761	.7295	.07075	2.394	.7926	.1116	2.268
16	.3352	.01154	3.602	.4573	.02428	3.074	.5318	.03855	2.779
32	.1881	.003495	4.449	.2751	.008040	3.805	.3182	.01226	3.402
64	.1041	.001040	5.309	.1607	.002574	4.580	.1851	.003574	4.085
128	.05687	.0003043	6.182	.09018	.0007638	5.413	.09923	.0009703	4.897
256	.03076	.00008769	7.069	.04876	.0002145	6.300	.05205	.0002476	5.763
512	.01650	.00002492	7.968	.02569	.00005714	7.225	.02662	.00006318	6.682
1024	.008785	.000006997	8.878	.01323	.00001491	8.182	.01349	.00001586	7.628
2048	.004650	.000001944	9.796	.006724	.000003808	9.159	.006794	.000003952	8.593

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 4.- EQUIDISTANT QUANTIZATION LEVEL SPACING FOR MAGNITUDE DISTORTION

M	Normal PDF			Exponential PDF			Gamma PDF		
	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy
1	0.0000	0.7979[a]	0.000[a]	0.0000	0.7071[a]	0.000[a]	0.0000	0.5774[a]	0.000[a]
2	1.3490[a]	.4754	1.000[a]	.9803[a]	.4910[a]	1.000[a]	.5253[a]	.4818	1.000[a]
3	1.0270	.3406	1.574	.9776	.3540	1.503	1.1310	.3141	1.449
4	.8338	.2683	1.976	.7579	.2993	1.927	.6085	.3300	1.923
5	.7044	.2217	2.278	.7070	.2459	2.151	.8214	.2295	2.023
6	.6116	.1900	2.523	.6030	.2200	2.404	.5347	.2520	2.313
7	.5416	.1662	2.728	.5643	.1914	2.552	.6583	.1841	2.363
8	.4869	.1482	2.905	.5029	.1760	2.732	.4708	.2063	2.576
9	.4428	.1337	3.059	.4743	.1582	2.845	.5554	.1554	2.608
10	.4065	.1221	3.197	.4333	.1477	2.984	.4204	.1757	2.777
11	.3760	.1123	3.322	.4115	.1354	3.077	.4827	.1353	2.803
12	.3500	.1042	3.435	.3821	.1278	3.191	.3798	.1528	2.942
13	.3276	.09703	3.539	.3649	.1189	3.270	.4296	.1198	2.964
14	.3081	.09100	3.635	.3426	.1130	3.365	.3474	.1363	3.082
15	.2909	.08558	3.724	.3287	.1062	3.435	.3883	.1083	3.102
16	.2756	.08092	3.807	.3112	.1016	3.517	.3207	.1233	3.204
17	.2619	.07666	3.886	.2997	.09613	3.579	.3552	.09917	3.222
18	.2497	.07294	3.960	.2857	.09247	3.651	.2978	.1127	3.313
19	.2385	.06950	4.029	.2760	.08797	3.707	.3273	.09107	3.331
20	.2284	.06646	4.095	.2643	.08485	3.772	.2787	.1035	3.411
21	.2192	.06362	4.158	.2560	.08119	3.822	.3045	.08482	3.428
22	.2107	.06109	4.218	.2462	.07855	3.881	.2622	.09627	3.499
23	.2029	.05870	4.275	.2391	.07546	3.927	.2850	.07948	3.516
24	.1957	.05655	4.330	.2306	.07319	3.981	.2474	.08959	3.582
25	.1890	.05451	4.383	.2244	.07055	4.023	.2677	.07443	3.599
26	.1827	.05267	4.433	.2171	.06858	4.073	.2347	.08428	3.657
27	.1769	.05091	4.482	.2116	.06629	4.113	.2531	.07044	3.674
28	.1715	.04931	4.529	.2053	.06456	4.158	.2230	.07920	3.729
29	.1664	.04778	4.574	.2004	.06255	4.195	.2397	.06653	3.746
30	.1616	.04638	4.618	.1947	.06102	4.238	.2129	.07513	3.795
31	.1571	.04502	4.660	.1903	.05925	4.273	.2282	.06343	3.811
32	0.1529	0.04378	4.701	0.1853	0.05788	4.312	0.2035	0.07113	3.858
33	.1489	.04258	4.741	.1814	.05630	4.346	.2175	.06031	3.874
34	.1451	.04148	4.779	.1769	.05507	4.383	.1953	.06791	3.916
35	.1415	.04040	4.816	.1733	.05361	4.414	.2083	.05782	3.932
36	.1381	.03941	4.853	.1692	.05250	4.449	.1875	.06467	3.972
37	.1349	.03845	4.888	.1659	.05123	4.479	.1995	.05525	3.989
38	.1318	.03755	4.922	.1623	.05022	4.512	.1806	.06206	4.025
39	.1289	.03668	4.956	.1593	.04907	4.540	.1917	.05321	4.042
40	.1261	.03587	4.988	.1559	.04815	4.572	.1740	.05937	4.076

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 5.- EQUIDISTANT QUANTIZATION LEVEL SPACING FOR MAGNITUDE DISTORTION

M	Normal PDF			Exponential PDF			Gamma PDF		
	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy
2	1.3490[a]	0.4754	1.000[a]	0.9803[a]	0.4910[a]	1.000[a]	0.5253[a]	0.4818	1.000[a]
4	.8338	.2683	1.973	.7579	.2993	1.927	.6085	.3300	1.923
8	.4869	.1482	2.905	.5029	.1760	2.732	.4708	.2063	2.576
16	.2756	.08092	3.807	.3112	.1016	3.517	.3207	.1233	3.204
32	.1529	.04378	4.701	.1853	.05788	4.312	.2035	.07113	3.858
64	.08361	.02351	5.596	.1075	.03253	5.124	.1235	.04028	4.550
128	.04521	.01254	6.496	.06123	.01806	5.951	.0723	.02220	5.287
256	.02422	.006646	7.404	.03434	.009913	6.795	.04105	.01200	6.069
512	.01288	.003505	8.320	.01900	.005374	7.653	.02269	.006343	6.894
1024	.006810	.0018390	9.242	.010390	.002881	8.527	.012210	.003289	7.763
2048	.003581	.0009617	10.170	.005611	.001524	9.418	.006408	.001683	8.673

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 6.- EQUIDISTANT QUANTIZATION LEVEL SPACING FOR RELATIVE DISTORTION

M	Normal PDF			Exponential PDF			Gamma PDF		
	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy
1	0.0000	1.0000[a]	0.000[a]	0.0000	1.0000[a]	0.000[a]	0.0000	1.0000[a]	0.000[a]
3	.6203	.5569	1.557	.4260	.6147	1.567	.2989	.7042	1.585
5	.4338	.4128	2.282	.3103	.4791	2.273	.2182	.6009	2.206
7	.3391	.3362	2.763	.2499	.4035	2.730	.1762	.5399	2.595
9	.2809	.2873	3.121	.2118	.3536	3.070	.1497	.4976	2.883
11	.2410	.2529	3.405	.1852	.3174	3.339	.1312	.4657	3.114
13	.2118	.2270	3.641	.1653	.2896	3.563	.1174	.4405	3.306
15	.1894	.2068	3.843	.1498	.2675	3.754	.1066	.4197	3.472
17	.1717	.1904	4.019	.1374	.2493	3.922	.09790	.4022	3.618
19	.1572	.1768	4.174	.1271	.2340	4.070	.09073	.3871	3.748
21	.1452	.1653	4.314	.1185	.2209	4.203	.08469	.3739	3.866
23	.1350	.1555	4.441	.1111	.2096	4.324	.07952	.3623	3.973
25	.1262	.1469	4.557	.1047	.1997	4.435	.07504	.3518	4.072
27	.1186	.1394	4.664	.0991	.1908	4.537	.07110	.3424	4.163
29	.1119	.1328	4.763	.09414	.1829	4.631	.06762	.3339	4.248
31	.1060	.1268	4.855	.08972	.1758	4.720	.06452	.3261	4.328
33	.1008	.1215	4.942	.08575	.1694	4.802	.06172	.3189	4.403
35	.09602	.1166	5.023	.08217	.1635	4.880	.05920	.3122	4.474
37	.09174	.1122	5.099	.07891	.1581	4.954	.05690	.3061	4.541
39	.08786	.1081	5.172	.07593	.1531	5.023	.05480	.3003	4.604

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 7.- EQUIDISTANT QUANTIZATION LEVEL SPACING FOR RELATIVE DISTORTION

M	Normal PDF			Exponential PDF			Gamma PDF		
	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy
1	0.0000	1.0000[a]	0.000[a]	0.0000	1.0000[a]	0.000[a]	0.0000	1.0000[a]	0.000[a]
3	.6203	.5569	1.557	.4260	.6147	1.567	.2989	.7942	1.585
7	.3391	.3362	2.763	.2499	.4035	2.730	.1762	.5399	2.595
15	.1894	.2068	3.843	.1498	.2675	3.754	.1066	.4197	3.472
31	.1060	.1268	4.855	.08972	.1758	4.720	.06452	.3261	4.328
63	.05898	.07698	5.829	.05321	.1138	5.654	.03870	.2519	5.189
127	.03252	.04612	6.782	.03117	.07243	6.568	.02292	.1931	6.063
255	.01777	.02726	7.721	.01802	.04530	7.469	.01339	.1469	6.949
511	.009627	.01591	8.653	.01030	.02788	8.359	.00772	.1109	7.844
1023	.005175	.009160	9.581	.005817	.01690	9.243	.004400	.08321	8.745
2047	.002763	.0052390	10.510	.003254	.01011	10.120	.002480	.06205	9.650

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 8.- EQUIDISTANT QUANTIZATION LEVEL SPACING FOR GAMMA PDF

M	Mean square error			Magnitude distortion		
	Spacing	Distortion	Entropy	Spacing	Distortion	Entropy
1	0.0000	1.0000[a]	0.000	0.0000	0.5774[a]	0.000[a]
3	1.8170	.2761	1.337	1.1310	.3141	1.449
7	1.0420	.09196	2.133	.6583	.1841	2.363
15	.6052	.03071	2.763	.3883	.1083	3.102
31	.3442	.009769	3.416	.2282	.06343	3.811
63	.1882	.002955	4.147	.1322	.03636	4.535
127	.1014	.0008294	4.933	.07533	.02043	5.288
255	.05289	.0002221	5.789	.04205	.01120	6.078
511	.02678	.00005784	6.708	.02300	.006015	6.907

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 9.- OPTIMUM QUANTIZATION LEVEL SPACING FOR NORMAL PDF WITH MEAN SQUARE DISTORTION

M = 1

	X(I)	Y(I)	P(I)	D(I)
1	.0000000[a]	.0000000[a]	1.0000000[a]	.5000000

DISTORTION = 1.0000000

ENTROPY = .0000000

M = 2

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.7979000[a]	.5000000[a]	.1817000

DISTORTION = .3634000[a]

ENTROPY = 1.0000000[a]

M = 3

	X(I)	Y(I)	P(I)	D(I)
1	.6120000	.0000000	.4595000	.0545500
2	.0000000	1.2240000	.2703000	.0678100

DISTORTION = .1902000

ENTROPY = 1.5360000

M = 4

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.4528000	.3369000	.0259100

[a]Value derived by direct computation, not by the Max algorithm.

2	.9816000	1.5100000	.1631000	.0328300

DISTORTION = .1175000

ENTROPY = 1.9110000

M = 5

	X(I)	Y(I)	P(I)	D(I)
1	.3823000	.0000000	.2978000	.0142200
2	1.2440000	.7646000	.2444000	.0144300
3	.0000000	1.7240000	.1067000	.0184300

DISTORTION = .0799400

ENTROPY = 2.2030000

M = 6

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.3177000	.2450000	.0087180
2	.6589000	1.0000000	.1810000	.0088770
3	1.4470000	1.8940000	.0739600	.0113900

DISTORTION = .0579800

ENTROPY = 2.4430000

M = 7

	X(I)	Y(I)	P(I)	D(I)
1	.2803000	.0000000	.2207000	.0057200
2	.8744000	.5606000	.1987000	.0057410
3	1.6110000	1.1880000	.1373000	.0058560
4	.0000000	2.0330000	.0536200	.0075430

DISTORTION = .0440000

ENTROPY = 2.6470000

M = 8

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.2451000	.1917000	.0039650
2	.5006000	.7560000	.1615000	.0039860
3	1.0500000	1.3440000	.1066000	.0040690
4	1.7480000	2.1520000	.0402400	.0052540

DISTORTION = .0345500

ENTROPY = 2.8250000

M = 9

	X(I)	Y(I)	P(I)	D(I)
1	.2218000	.0000000	.1755000	.0028600
2	.6812000	.4436000	.1644000	.0028640
3	1.1980000	.9188000	.1323000	.0028810
4	1.8660000	1.4760000	.0844800	.0029430
5	.0000000	2.2550000	.0310500	.0038080

DISTORTION = .0278500

ENTROPY = 2.9830000

M = 10

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1996000	.1572000	.0021330
2	.4047000	.6099000	.1406000	.0021370
3	.8338000	1.0580000	.1095000	.0021510
4	1.3250000	1.5910000	.0681300	.0021980
5	1.9680000	2.3450000	.0245200	.0028490

DISTORTION = .0229400

ENTROPY = 3.1250000

M = 11

	X(I)	Y(I)	P(I)	D(I)
1	.1837000	.0000000	.1458000	.0016330
2	.5599000	.3675000	.1393000	.0016340
3	.9656000	.7524000	.1206000	.0016380
4	1.4360000	1.1790000	.0915800	.0016480
5	2.0590000	1.6930000	.0558000	.0016650
6	.0000000	2.4260000	.0197400	.0021880

DISTORTION = .0192200

ENTROPY = 3.2540000

M = 12

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1684000	.1331000	.0012780
2	.3401000	.5118000	.1231000	.0012800
3	.6943000	.8768000	.1039000	.0012830
4	1.0810000	1.2860000	.0773200	.0012920
5	1.5340000	1.7830000	.0463200	.0013210
6	2.1410000	2.4980000	.0161500	.0017160

DISTORTION = .0163400

ENTROPY = 3.3720000

M = 13

	X(I)	Y(I)	P(I)	D(I)
1	.1569000	.0000000	.1247000	.0010190
2	.4760000	.3138000	.1206000	.0010200
3	.8126000	.6383000	.1088000	.0010210
4	1.1840000	.9870000	.0900400	.0010240
5	1.6230000	1.3810000	.0658800	.0010310
6	2.2150000	1.8650000	.0389100	.0010540
7	.0000000	2.5650000	.0134000	.0013720

DISTORTION = .0140600

ENTROPY = 3.4810000

M = 14

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1457000	.1154000	.0008263
2	.2935000	.4413000	.1089000	.0008267
3	.5959000	.7505000	.0963300	.0008278
4	.9180000	1.0860000	.0784200	.0008302
5	1.2770000	1.4680000	.0566000	.0008360
6	1.7030000	1.9390000	.0330300	.0008553
7	2.2820000	2.6250000	.0112500	.0011140

DISTORTION = .0122300

ENTROPY = 3.5820000

M = 15

	X(I)	Y(I)	P(I)	D(I)
1	.1369000	.0000000	.1089000	.0006790
2	.4143000	.2739000	.1062000	.0006791
3	.7029000	.5548000	.0982800	.0006795
4	1.0130000	.8511000	.0855100	.0006805
5	1.3600000	1.1750000	.0686900	.0006825
6	1.7760000	1.5460000	.0490000	.0006873
7	2.3440000	2.0060000	.0283000	.0007033
8	.0000000	2.6810000	.0095480	.0009168

DISTORTION = .0107400

ENTROPY = 3.6770000

M = 16

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1284000	.1019000	.0005648

2	.2582000	.3880000	.0974200	.0005650
3	.5224000	.6568000	.0887100	.0005654
4	.7995000	.9423000	.0761600	.0005662
5	1.0990000	1.2560000	.0604800	.0005679
6	1.4370000	1.6180000	.0427100	.0005720
7	1.8430000	2.0690000	.0244500	.0005854
8	2.4010000	2.7320000	.0081810	.0007639

DISTORTION = .0095010

ENTROPY = 3.7650000

M = 17

	X(I)	Y(I)	P(I)	D(I)
1	.1215000	.0000000	.0967000	.0004749
2	.3669000	.2430000	.0948200	.0004750
3	.6201000	.4909000	.0892300	.0004751
4	.8874000	.7493000	.0801800	.0004755
5	1.1780000	1.0260000	.0680700	.0004762
6	1.5080000	1.3310000	.0535300	.0004777
7	1.9060000	1.6840000	.0374800	.0004811
8	2.4540000	2.1270000	.0212800	.0004925
9	.0000000	2.7810000	.0070660	.0006429

DISTORTION = .0084670

ENTROPY = 3.8490000

M = 18

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1148000	.0911700	.0004031
2	.2306000	.3463000	.0879800	.0004032
3	.4653000	.5843000	.0817100	.0004034
4	.7091000	.8339000	.0726100	.0004037
5	.9680000	1.1020000	.0610500	.0004044
6	1.2510000	1.4000000	.0476000	.0004056
7	1.5730000	1.7460000	.0330700	.0004086
8	1.9630000	2.1810000	.0186500	.0004182

9	2.5030000	2.8260000	.0061510	.0005464

DISTORTION = .0075930

ENTROPY = 3.9280000

M = 19

	X(I)	Y(I)	P(I)	D(I)
1	.1092000	.0000000	.0869600	.0003451
2	.3294000	.2184000	.0855900	.0003452
3	.5551000	.4404000	.0815100	.0003453
4	.7907000	.6698000	.0748700	.0003454
5	1.0420000	.9117000	.0659000	.0003457
6	1.3180000	1.1730000	.0549500	.0003463
7	1.6340000	1.4640000	.0425300	.0003474
8	2.0170000	1.8030000	.0293500	.0003499
9	2.5500000	2.2310000	.0164400	.0003582
10	.0000000	2.8680000	.0053890	.0004682

DISTORTION = .0068480

ENTROPY = 4.0030000

M = 20

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1038000	.0824900	.0002978
2	.2083000	.3128000	.0801300	.0002978
3	.4196000	.5265000	.0754800	.0002979
4	.6375000	.7485000	.0686800	.0002981
5	.8661000	.9837000	.0599500	.0002983
6	1.1110000	1.2380000	.0496200	.0002988
7	1.3810000	1.5230000	.0381500	.0002997
8	1.6900000	1.8570000	.0261700	.0003020
9	2.0680000	2.2790000	.0145700	.0003092
10	2.5930000	2.9080000	.0047520	.0004043

DISTORTION = .0062080

ENTROPY = 4.0740000

M = 21

	X(I)	Y(I)	P(I)	D(I)
1	.0991800	.0000000	.0790000	.0002587
2	.2989000	.1984000	.0779700	.0002587
3	.5026000	.3994000	.0749100	.0002588
4	.7137000	.6059000	.0699000	.0002588
5	.9360000	.8214000	.0630800	.0002590
6	1.1750000	1.0510000	.0546700	.0002592
7	1.4390000	1.3000000	.0449600	.0002596
8	1.7430000	1.5790000	.0343700	.0002605
9	2.1150000	1.9070000	.0234400	.0002624
10	2.6340000	2.3230000	.0129900	.0002687
11	.0000000	2.9460000	.0042130	.0003516

DISTORTION = .0056530

ENTROPY = 4.1410000

M = 22

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0946900	.0753200	.0002262
2	.1899000	.2852000	.0735300	.0002262
3	.3822000	.4792000	.0699800	.0002262
4	.5794000	.6795000	.0647600	.0002263
5	.7844000	.8893000	.0580300	.0002264
6	1.0010000	1.1130000	.0499700	.0002266
7	1.2350000	1.3570000	.0408600	.0002270
8	1.4940000	1.6320000	.0310700	.0002278
9	1.7930000	1.9550000	.0210900	.0002295
10	2.1600000	2.3650000	.0116300	.0002350
11	2.6730000	2.9810000	.0037560	.0003078

DISTORTION = .0051700

ENTROPY = 4.2060000

M = 23

	X(I)	Y(I)	P(I)	D(I)
1	.0908400	.0000000	.0723800	.0001989
2	.2735000	.1817000	.0715900	.0001989
3	.4594000	.3654000	.0692300	.0001989
4	.6507000	.5533000	.0653600	.0001990
5	.8503000	.7480000	.0600600	.0001990
6	1.0620000	.9526000	.0534800	.0001991
7	1.2910000	1.1720000	.0457900	.0001993
8	1.5460000	1.4110000	.0372500	.0001997
9	1.8400000	1.6810000	.0281900	.0002003
10	2.2020000	1.9990000	.0190500	.0002018
11	2.7100000	2.4050000	.0104600	.0002067
12	.0000000	3.0150000	.0033630	.0002708

DISTORTION = .0047460

ENTROPY = 4.2680000

M = 24

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0870700	.0693000	.0001758
2	.1746000	.2621000	.0679000	.0001758
3	.3510000	.4398000	.0651300	.0001759
4	.5311000	.6224000	.0610500	.0001759
5	.7172000	.8121000	.0557500	.0001760
6	.9121000	1.0120000	.0493700	.0001761
7	1.1190000	1.2270000	.0420600	.0001762
8	1.3440000	1.4620000	.0340500	.0001765
9	1.5950000	1.7280000	.0256600	.0001771
10	1.8850000	2.0420000	.0172700	.0001784
11	2.2420000	2.4430000	.0094420	.0001828
12	2.7450000	3.0470000	.0030250	.0002397

DISTORTION = .0043720

ENTROPY = 4.3270000

	X(I)	Y(I)	P(I)	D(I)
1	.0838100	.0000000	.0667900	.0001562
2	.2522000	.1676000	.0661700	.0001562
3	.4230000	.3368000	.0643100	.0001562
4	.5981000	.5093000	.0612500	.0001563
5	.7796000	.6870000	.0570600	.0001563
6	.9701000	.8722000	.0518200	.0001563
7	1.1730000	1.0680000	.0456500	.0001564
8	1.3940000	1.2790000	.0387200	.0001566
9	1.6410000	1.5100000	.0312200	.0001569
10	1.9270000	1.7720000	.0234200	.0001574
11	2.2810000	2.0820000	.0157000	.0001586
12	2.7790000	2.4790000	.0085550	.0001624
13	.0000000	3.0780000	.0027290	.0002129

DISTORTION = .0040410

ENTROPY = 4.3840000

M = 26

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0805900	.0641700	.0001394
2	.1615000	.2425000	.0630600	.0001394
3	.3245000	.4065000	.0608600	.0001394
4	.4904000	.5743000	.0576100	.0001394
5	.6610000	.7476000	.0533700	.0001395
6	.8382000	.9288000	.0482300	.0001395
7	1.0250000	1.1210000	.0422900	.0001396
8	1.2240000	1.3280000	.0357200	.0001398
9	1.4420000	1.5550000	.0286900	.0001400
10	1.6850000	1.8140000	.0214500	.0001405
11	1.9670000	2.1210000	.0143300	.0001415
12	2.3170000	2.5130000	.0077780	.0001450
13	2.8110000	3.1080000	.0024730	.0001901

DISTORTION = .0037460

ENTROPY = 4.4390000

M = 27

	X(I)	Y(I)	P(I)	D(I)
1	.0777800	.0000000	.0620000	.0001249
2	.2340000	.1556000	.0615000	.0001249
3	.3921000	.3124000	.0600100	.0001249
4	.5536000	.4718000	.0575600	.0001249
5	.7201000	.6354000	.0541900	.0001250
6	.8935000	.8048000	.0499500	.0001250
7	1.0770000	.9823000	.0449400	.0001251
8	1.2720000	1.1710000	.0392500	.0001251
9	1.4860000	1.3740000	.0330200	.0001253
10	1.7260000	1.5990000	.0264300	.0001255
11	2.0050000	1.8540000	.0196900	.0001259
12	2.3520000	2.1570000	.0131100	.0001268
13	2.8410000	2.5460000	.0070960	.0001300
14	.0000000	3.1360000	.0022490	.0001705

DISTORTION = .0034830

ENTROPY = 4.4920000

M = 28

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0750100	.0597400	.0001124
2	.1503000	.2256000	.0588400	.0001124
3	.3018000	.3779000	.0570700	.0001124
4	.4556000	.5332000	.0544400	.0001124
5	.6131000	.6929000	.0510000	.0001124
6	.7759000	.8588000	.0468000	.0001125
7	.9458000	1.0330000	.0419300	.0001125
8	1.1260000	1.2180000	.0364900	.0001126
9	1.3180000	1.4190000	.0305900	.0001127
10	1.5290000	1.6400000	.0244000	.0001129
11	1.7660000	1.8920000	.0181300	.0001133
12	2.0420000	2.1920000	.0120300	.0001141
13	2.3850000	2.5780000	.0064920	.0001169

14	2.8700000	3.1630000	.0020510	.0001534

DISTORTION = .0032460

ENTROPY = 4.5430000

M = 29

	X(I)	Y(I)	P(I)	D(I)
1	.0725700	.0000000	.0578500	.0001015
2	.2182000	.1451000	.0574400	.0001015
3	.3654000	.2913000	.0562300	.0001015
4	.5153000	.4396000	.0542400	.0001015
5	.6692000	.5911000	.0514900	.0001015
6	.8286000	.7474000	.0480200	.0001015
7	.9954000	.9099000	.0438900	.0001015
8	1.1720000	1.0810000	.0391800	.0001016
9	1.3620000	1.2630000	.0339800	.0001016
10	1.5700000	1.4610000	.0283900	.0001018
11	1.8040000	1.6790000	.0225800	.0001019
12	2.0770000	1.9280000	.0167300	.0001023
13	2.4170000	2.2250000	.0110700	.0001031
14	2.8980000	2.6080000	.0059570	.0001056
15	.0000000	3.1890000	.0018760	.0001386

DISTORTION = .0030320

ENTROPY = 4.5920000

M = 30

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0701600	.0558900	.0000919
2	.1405000	.2109000	.0551500	.0000919
3	.2820000	.3531000	.0537000	.0000919
4	.4254000	.4977000	.0515400	.0000919
5	.5718000	.6459000	.0487100	.0000920
6	.7224000	.7989000	.0452400	.0000920
7	.8787000	.9585000	.0412000	.0000920
8	1.0430000	1.1270000	.0366600	.0000920

9	1.2160000	1.3060000	.0316900	.0000921
10	1.4040000	1.5010000	.0264000	.0000922
11	1.6090000	1.7170000	.0209400	.0000923
12	1.8400000	1.9630000	.0154700	.0000927
13	2.1100000	2.2570000	.0102100	.0000934
14	2.4470000	2.6370000	.0054790	.0000957
15	2.9250000	3.2140000	.0017210	.0001256

DISTORTION = .0028390

ENTROPY = 4.6390000

M = 31

	X(I)	Y(I)	P(I)	D(I)
1	.0680100	.0000000	.0542200	.0000835
2	.2044000	.1360000	.0538900	.0000835
3	.3422000	.2729000	.0528900	.0000835
4	.4821000	.4115000	.0512400	.0000835
5	.6253000	.5527000	.0489700	.0000836
6	.7729000	.6978000	.0460900	.0000836
7	.9263000	.8479000	.0426600	.0000836
8	1.0870000	1.0050000	.0387200	.0000836
9	1.2590000	1.1700000	.0343400	.0000836
10	1.4430000	1.3470000	.0296000	.0000837
11	1.6460000	1.5390000	.0246000	.0000838
12	1.8750000	1.7530000	.0194600	.0000839
13	2.1420000	1.9970000	.0143400	.0000842
14	2.4760000	2.2880000	.0094420	.0000849
15	2.9510000	2.6640000	.0050540	.0000870
16	.0000000	3.2380000	.0015840	.0001142

DISTORTION = .0026640

ENTROPY = 4.6850000

M = 32

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0658900	.0525000	.0000761

2	.1320000	.1981000	.0518900	.0000761
3	.2647000	.3314000	.0506800	.0000761
4	.3990000	.4667000	.0488900	.0000761
5	.5358000	.6049000	.0465300	.0000762
6	.6760000	.7471000	.0436400	.0000762
7	.8208000	.8946000	.0402600	.0000762
8	.9717000	1.0490000	.0364300	.0000762
9	1.1300000	1.2120000	.0322200	.0000762
10	1.2990000	1.3860000	.0276900	.0000763
11	1.4810000	1.5760000	.0229600	.0000764
12	1.6820000	1.7870000	.0181100	.0000765
13	1.9080000	2.0290000	.0133200	.0000768
14	2.1730000	2.3180000	.0087490	.0000774
15	2.5040000	2.6910000	.0046720	.0000793
16	2.9760000	3.2610000	.0014610	.0001042

DISTORTION = .0025050

ENTROPY = 4.7300000

M = 33

	X(I)	Y(I)	P(I)	D(I)
1	.0639900	.0000000	.0510200	.0000696
2	.1923000	.1280000	.0507400	.0000696
3	.3217000	.2567000	.0499100	.0000696
4	.4529000	.3868000	.0485400	.0000696
5	.5868000	.5191000	.0466400	.0000696
6	.7243000	.6546000	.0442300	.0000696
7	.8666000	.7941000	.0413500	.0000696
8	1.0150000	.9390000	.0380200	.0000696
9	1.1710000	1.0910000	.0343100	.0000697
10	1.3380000	1.2520000	.0302600	.0000697
11	1.5180000	1.4240000	.0259500	.0000697
12	1.7160000	1.6120000	.0214600	.0000698
13	1.9400000	1.8200000	.0168900	.0000699
14	2.2030000	2.0600000	.0123900	.0000702
15	2.5310000	2.3460000	.0081240	.0000707
16	3.0000000	2.7170000	.0043290	.0000725
17	.0000000	3.2830000	.0013500	.0000953

DISTORTION = .0023590

ENTROPY = 4.7730000

M = 34

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0621100	.0494900	.0000638
2	.1244000	.1867000	.0489900	.0000638
3	.2494000	.3122000	.0479700	.0000638
4	.3758000	.4394000	.0464700	.0000638
5	.5042000	.5690000	.0444900	.0000638
6	.6354000	.7018000	.0420500	.0000638
7	.7704000	.8389000	.0391900	.0000638
8	.9103000	.9816000	.0359400	.0000638
9	1.0560000	1.1310000	.0323500	.0000638
10	1.2100000	1.2900000	.0284600	.0000639
11	1.3750000	1.4600000	.0243500	.0000639
12	1.5530000	1.6450000	.0200900	.0000640
13	1.7490000	1.8520000	.0157800	.0000641
14	1.9710000	2.0890000	.0115500	.0000643
15	2.2310000	2.3740000	.0075580	.0000648
16	2.5580000	2.7420000	.0040190	.0000664
17	3.0230000	3.3040000	.0012510	.0000874

DISTORTION = .0022260

ENTROPY = 4.8150000

M = 35

	X(I)	Y(I)	P(I)	D(I)
1	.0604200	.0000000	.0481800	.0000586
2	.1816000	.1208000	.0479400	.0000586
3	.3036000	.2423000	.0472400	.0000586
4	.4272000	.3649000	.0460900	.0000586
5	.5529000	.4894000	.0444800	.0000586
6	.6817000	.6165000	.0424400	.0000586
7	.8144000	.7470000	.0400000	.0000586
8	.9521000	.8818000	.0371700	.0000586

9	1.0960000	1.0220000	.0340000	.0000586
10	1.2480000	1.1700000	.0305300	.0000586
11	1.4100000	1.3260000	.0268000	.0000587
12	1.5860000	1.4950000	.0228700	.0000587
13	1.7800000	1.6780000	.0188300	.0000588
14	2.0000000	1.8830000	.0147700	.0000589
15	2.2590000	2.1180000	.0107900	.0000591
16	2.5830000	2.4000000	.0070440	.0000595
17	3.0460000	2.7660000	.0037380	.0000610
18	.0000000	3.3250000	.0011610	.0000803

DISTORTION = .0021040

ENTROPY = 4.8560000

M = 36

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0587500	.0468200	.0000540
2	.1176000	.1765000	.0463900	.0000540
3	.2358000	.2951000	.0455300	.0000540
4	.3551000	.4151000	.0442600	.0000540
5	.4761000	.5371000	.0425700	.0000540
6	.5995000	.6618000	.0405000	.0000540
7	.7260000	.7901000	.0380600	.0000540
8	.8565000	.9229000	.0352800	.0000540
9	.9921000	1.0610000	.0322000	.0000540
10	1.1340000	1.2070000	.0288400	.0000540
11	1.2840000	1.3610000	.0252600	.0000540
12	1.4450000	1.5280000	.0215200	.0000541
13	1.6190000	1.7100000	.0176800	.0000541
14	1.8110000	1.9120000	.0138400	.0000542
15	2.0290000	2.1450000	.0100900	.0000544
16	2.2860000	2.4260000	.0065770	.0000548
17	2.6070000	2.7690000	.0034840	.0000562
18	3.0670000	3.3460000	.0010800	.0000740

DISTORTION = .0019910

ENTROPY = 4.8950000

M = 37

	X(I)	Y(I)	P(I)	D(I)
1	.0572300	.0000000	.0456400	.0000498
2	.1719000	.1145000	.0454400	.0000498
3	.2874000	.2294000	.0448400	.0000498
4	.4042000	.3454000	.0438600	.0000498
5	.5228000	.4629000	.0424900	.0000498
6	.6440000	.5827000	.0407500	.0000498
7	.7684000	.7052000	.0386600	.0000498
8	.8969000	.8315000	.0362400	.0000498
9	1.0310000	.9623000	.0335100	.0000498
10	1.1710000	1.0990000	.0305100	.0000498
11	1.3190000	1.2430000	.0272700	.0000498
12	1.4780000	1.3950000	.0238400	.0000499
13	1.6500000	1.5600000	.0202700	.0000499
14	1.8400000	1.7400000	.0166300	.0000500
15	2.0570000	1.9410000	.0129900	.0000500
16	2.3110000	2.1720000	.0094550	.0000502
17	2.6310000	2.4500000	.0061510	.0000506
18	3.0880000	2.8110000	.0032520	.0000519
19	.0000000	3.3650000	.0010060	.0000682

DISTORTION = .0018880

ENTROPY = 4.9340000

M = 38

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0557300	.0444200	.0000461
2	.1116000	.1674000	.0440500	.0000461
3	.2236000	.2798000	.0433200	.0000461
4	.3366000	.3934000	.0422300	.0000461
5	.4510000	.5087000	.0407900	.0000461
6	.5675000	.6263000	.0390100	.0000461
7	.6866000	.7468000	.0369200	.0000461
8	.8090000	.8711000	.0345200	.0000461
9	.9356000	1.0000000	.0318600	.0000461
10	1.0670000	1.1350000	.0289400	.0000461

11	1.2060000	1.2770000	.0258200	.0000461
12	1.3520000	1.4280000	.0225300	.0000461
13	1.5090000	1.5910000	.0191200	.0000462
14	1.6800000	1.7690000	.0156600	.0000462
15	1.8690000	1.9690000	.0122100	.0000463
16	2.0830000	2.1980000	.0088720	.0000464
17	2.3360000	2.4740000	.0057620	.0000468
18	2.6540000	2.8330000	.0030410	.0000480
19	3.1090000	3.3840000	.0009389	.0000631

DISTORTION = .0017920

ENTROPY = 4.9720000

M = 39

	X(I)	Y(I)	P(I)	D(I)
1	.0543600	.0000000	.0433500	.0000427
2	.1633000	.1087000	.0431800	.0000427
3	.2729000	.2179000	.0426700	.0000427
4	.3836000	.3279000	.0418200	.0000427
5	.4958000	.4392000	.0406500	.0000427
6	.6103000	.5524000	.0391600	.0000427
7	.7274000	.6681000	.0373600	.0000427
8	.8480000	.7867000	.0352700	.0000427
9	.9728000	.9092000	.0329100	.0000427
10	1.1030000	1.0360000	.0303000	.0000427
11	1.2400000	1.1700000	.0274700	.0000427
12	1.3850000	1.3100000	.0244600	.0000427
13	1.5400000	1.4590000	.0213100	.0000427
14	1.7090000	1.6210000	.0180500	.0000428
15	1.8960000	1.7970000	.0147600	.0000428
16	2.1090000	1.9950000	.0114900	.0000429
17	2.3600000	2.2230000	.0083370	.0000430
18	2.6760000	2.4970000	.0054060	.0000434
19	3.1290000	2.8540000	.0028480	.0000445
20	.0000000	3.4030000	.0008778	.0000585

DISTORTION = .0017030

ENTROPY = 5.0080000

M = 40

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0530000	.0422500	.0000396
2	.1061000	.1592000	.0419300	.0000396
3	.2126000	.2660000	.0413000	.0000396
4	.3199000	.3738000	.0403700	.0000396
5	.4285000	.4831000	.0391200	.0000396
6	.5388000	.5944000	.0375900	.0000396
7	.6513000	.7082000	.0357800	.0000396
8	.7666000	.8251000	.0337000	.0000396
9	.8855000	.9459000	.0313800	.0000396
10	1.0090000	1.0710000	.0288400	.0000396
11	1.1370000	1.2030000	.0261000	.0000397
12	1.2720000	1.3420000	.0232000	.0000397
13	1.4160000	1.4900000	.0201800	.0000397
14	1.5700000	1.6500000	.0170700	.0000397
15	1.7370000	1.8250000	.0139300	.0000398
16	1.9230000	2.0210000	.0108300	.0000398
17	2.1340000	2.2470000	.0078450	.0000400
18	2.3840000	2.5200000	.0050790	.0000403
19	2.6970000	2.8750000	.0026720	.0000413
20	3.1480000	3.4210000	.0008219	.0000543

DISTORTION = .0016210

ENTROPY = 5.0440000

M = 64

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0332300	.0265000	.0000098
2	.0664800	.0997300	.0264200	.0000098
3	.1331000	.1664000	.0262700	.0000098
4	.1998000	.2333000	.0260300	.0000098
5	.2669000	.3005000	.0257200	.0000098
6	.3344000	.3682000	.0253400	.0000098
7	.4024000	.4365000	.0248800	.0000098
8	.4710000	.5054000	.0243500	.0000098
9	.5403000	.5752000	.0237400	.0000098
10	.6106000	.6459000	.0230700	.0000098
11	.6818000	.7177000	.0223300	.0000098
12	.7543000	.7908000	.0215200	.0000098
13	.8280000	.8653000	.0206500	.0000098
14	.9033000	.9414000	.0197300	.0000098
15	.9804000	1.0190000	.0187500	.0000098
16	1.0590000	1.1000000	.0177200	.0000098
17	1.1410000	1.1820000	.0166400	.0000098
18	1.2250000	1.2670000	.0155200	.0000098
19	1.3120000	1.3560000	.0143600	.0000098
20	1.4020000	1.4480000	.0131800	.0000098
21	1.4960000	1.5450000	.0119700	.0000098
22	1.5950000	1.6460000	.0107400	.0000098
23	1.7000000	1.7540000	.0095090	.0000098
24	1.8110000	1.8680000	.0082780	.0000098
25	1.9300000	1.9920000	.0070600	.0000098
26	2.0590000	2.1270000	.0058670	.0000098
27	2.2010000	2.2760000	.0047140	.0000098
28	2.3600000	2.4440000	.0036180	.0000098
29	2.5410000	2.6380000	.0026000	.0000098
30	2.7560000	2.8740000	.0016860	.0000099
31	3.0260000	3.1780000	.0009090	.0000101
32	3.4060000	3.6330000	.0003300	.0000198

DISTORTION = .0006462

ENTROPY = 5.7180000

M = 128

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0166000	.0132400	.0000012
2	.0331900	.0497900	.0132300	.0000012
3	.0664000	.0830000	.0132100	.0000012
4	.0996300	.1162000	.0131800	.0000012
5	.1329000	.1495000	.0131400	.0000012
6	.1662000	.1829000	.0130900	.0000012
7	.1996000	.2163000	.0130400	.0000012
8	.2330000	.2498000	.0129700	.0000012
9	.2666000	.2834000	.0128900	.0000012
10	.3002000	.3171000	.0128000	.0000012
11	.3340000	.3509000	.0127100	.0000012
12	.3678000	.3848000	.0126000	.0000012
13	.4019000	.4189000	.0124900	.0000012
14	.4360000	.4532000	.0123600	.0000012
15	.4704000	.4876000	.0122300	.0000012
16	.5049000	.5222000	.0120900	.0000012
17	.5397000	.5571000	.0119400	.0000012
18	.5746000	.5921000	.0117800	.0000012
19	.6098000	.6275000	.0116100	.0000012
20	.6452000	.6630000	.0114400	.0000012
21	.6810000	.6989000	.0112500	.0000012
22	.7170000	.7350000	.0110600	.0000012
23	.7533000	.7715000	.0108600	.0000012
24	.7899000	.8083000	.0106500	.0000012
25	.8269000	.8455000	.0104300	.0000012
26	.8643000	.8831000	.0102100	.0000012
27	.9021000	.9211000	.0099790	.0000012
28	.9404000	.9596000	.0097410	.0000012
29	.9791000	.9985000	.0094960	.0000012
30	1.0180000	1.0380000	.0092460	.0000012
31	1.0580000	1.0780000	.0089880	.0000012
32	1.0980000	1.1190000	.0087250	.0000012
33	1.1390000	1.1600000	.0084560	.0000012
34	1.1810000	1.2020000	.0081820	.0000012
35	1.2230000	1.2440000	.0079030	.0000012
36	1.2660000	1.2880000	.0076190	.0000012
37	1.3100000	1.3320000	.0073310	.0000012
38	1.3540000	1.3770000	.0070390	.0000012
39	1.4000000	1.4230000	.0067430	.0000012
40	1.4460000	1.4700000	.0064440	.0000012
41	1.4940000	1.5180000	.0061420	.0000012

42	1.5430000	1.5670000	.0056380	.0000012
43	1.5930000	1.6180000	.0055330	.0000012
44	1.6440000	1.6700000	.0052260	.0000012
45	1.6970000	1.7240000	.0049180	.0000012
46	1.7510000	1.7790000	.0046100	.0000012
47	1.8080000	1.8360000	.0043030	.0000012
48	1.8660000	1.8960000	.0039970	.0000012
49	1.9260000	1.9570000	.0036930	.0000012
50	1.9890000	2.0210000	.0033920	.0000012
51	2.0550000	2.0880000	.0030940	.0000012
52	2.1240000	2.1590000	.0028010	.0000012
53	2.1960000	2.2330000	.0025130	.0000012
54	2.2720000	2.3110000	.0022310	.0000012
55	2.3530000	2.3950000	.0019570	.0000012
56	2.4400000	2.4840000	.0016920	.0000012
57	2.5330000	2.5810000	.0014370	.0000012
58	2.6340000	2.6860000	.0011940	.0000012
59	2.7450000	2.8030000	.0009650	.0000012
60	2.8680000	2.9330000	.0007515	.0000012
61	3.0080000	3.0830000	.0005562	.0000012
62	3.1720000	3.2610000	.0003821	.0000012
63	3.3700000	3.4800000	.0002328	.0000012
64	3.6290000	3.7770000	.0001424	.0000090

DISTORTION = .0001713

ENTROPY = 6.7200000

M = 256

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0082950	.0066190	.0000002
2	.0165900	.0248900	.0066170	.0000002
3	.0331800	.0414800	.0066150	.0000002
4	.0497800	.0580800	.0066110	.0000002
5	.0663800	.0746800	.0066060	.0000002
6	.0829900	.0912900	.0066000	.0000002
7	.0996000	.1079000	.0065930	.0000002
8	.1162000	.1245000	.0065850	.0000002
9	.1329000	.1412000	.0065750	.0000002
10	.1495000	.1578000	.0065640	.0000002

11	.1662000	.1745000	.0065520	.0000002
12	.1828000	.1912000	.0065390	.0000002
13	.1995000	.2079000	.0065240	.0000002
14	.2162000	.2246000	.0065080	.0000002
15	.2330000	.2413000	.0064910	.0000002
16	.2497000	.2581000	.0064730	.0000002
17	.2665000	.2749000	.0064540	.0000002
18	.2833000	.2917000	.0064340	.0000002
19	.3001000	.3085000	.0064120	.0000002
20	.3170000	.3254000	.0063890	.0000002
21	.3339000	.3423000	.0063650	.0000002
22	.3508000	.3593000	.0063400	.0000002
23	.3677000	.3762000	.0063140	.0000002
24	.3847000	.3932000	.0062860	.0000002
25	.4018000	.4103000	.0062580	.0000002
26	.4188000	.4274000	.0062280	.0000002
27	.4359000	.4445000	.0061970	.0000002
28	.4531000	.4616000	.0061650	.0000002
29	.4703000	.4789000	.0061320	.0000002
30	.4875000	.4961000	.0060970	.0000002
31	.5048000	.5134000	.0060620	.0000002
32	.5221000	.5308000	.0060250	.0000002
33	.5395000	.5482000	.0059880	.0000002
34	.5569000	.5657000	.0059490	.0000002
35	.5744000	.5832000	.0059090	.0000002
36	.5920000	.6008000	.0058680	.0000002
37	.6096000	.6184000	.0058260	.0000002
38	.6273000	.6362000	.0057830	.0000002
39	.6451000	.6539000	.0057390	.0000002
40	.6629000	.6718000	.0056940	.0000002
41	.6808000	.6897000	.0056480	.0000002
42	.6987000	.7077000	.0056010	.0000002
43	.7168000	.7258000	.0055530	.0000002
44	.7349000	.7439000	.0055040	.0000002
45	.7531000	.7622000	.0054540	.0000002
46	.7713000	.7805000	.0054020	.0000002
47	.7897000	.7989000	.0053500	.0000002
48	.8082000	.8174000	.0052970	.0000002
49	.8267000	.8360000	.0052430	.0000002
50	.8453000	.8547000	.0051880	.0000002
51	.8641000	.8735000	.0051320	.0000002
52	.8829000	.8924000	.0050760	.0000002

53	.9019000	.9114000	.0050180	.0000002
54	.9209000	.9305000	.0049590	.0000002
55	.9401000	.9497000	.0049000	.0000002
56	.9594000	.9690000	.0048400	.0000002
57	.9788000	.9885000	.0047790	.0000002
58	.9983000	1.0080000	.0047170	.0000002
59	1.0180000	1.0280000	.0046540	.0000002
60	1.0380000	1.0480000	.0045910	.0000002
61	1.0580000	1.0680000	.0045260	.0000002
62	1.0780000	1.0880000	.0044610	.0000002
63	1.0980000	1.1080000	.0043960	.0000002
64	1.1180000	1.1290000	.0043290	.0000002
65	1.1390000	1.1490000	.0042620	.0000002
66	1.1590000	1.1700000	.0041940	.0000002
67	1.1800000	1.1910000	.0041260	.0000002
68	1.2010000	1.2120000	.0040570	.0000002
69	1.2230000	1.2330000	.0039870	.0000002
70	1.2440000	1.2550000	.0039160	.0000002
71	1.2650000	1.2760000	.0038460	.0000002
72	1.2870000	1.2980000	.0037740	.0000002
73	1.3090000	1.3200000	.0037020	.0000002
74	1.3310000	1.3430000	.0036290	.0000002
75	1.3540000	1.3650000	.0035560	.0000002
76	1.3760000	1.3880000	.0034830	.0000002
77	1.3990000	1.4110000	.0034090	.0000002
78	1.4220000	1.4340000	.0033350	.0000002
79	1.4460000	1.4580000	.0032600	.0000002
80	1.4690000	1.4810000	.0031850	.0000002
81	1.4930000	1.5050000	.0031100	.0000002
82	1.5180000	1.5300000	.0030340	.0000002
83	1.5420000	1.5540000	.0029580	.0000002
84	1.5670000	1.5790000	.0028820	.0000002
85	1.5920000	1.6050000	.0028050	.0000002
86	1.6180000	1.6300000	.0027290	.0000002
87	1.6430000	1.6560000	.0026520	.0000002
88	1.6700000	1.6830000	.0025750	.0000002
89	1.6960000	1.7100000	.0024980	.0000002
90	1.7230000	1.7370000	.0024220	.0000002
91	1.7510000	1.7640000	.0023450	.0000002
92	1.7780000	1.7930000	.0022680	.0000002
93	1.8070000	1.8210000	.0021910	.0000002
94	1.8360000	1.8500000	.0021140	.0000002

95	1.8650000	1.8800000	.0020380	.0000002
96	1.8950000	1.9100000	.0019620	.0000002
97	1.9250000	1.9410000	.0018860	.0000002
98	1.9560000	1.9720000	.0018100	.0000002
99	1.9880000	2.0040000	.0017350	.0000002
100	2.0210000	2.0370000	.0016600	.0000002
101	2.0540000	2.0710000	.0015850	.0000002
102	2.0880000	2.1050000	.0015120	.0000002
103	2.1220000	2.1400000	.0014380	.0000002
104	2.1580000	2.1760000	.0013650	.0000002
105	2.1950000	2.2130000	.0012930	.0000002
106	2.2320000	2.2510000	.0012220	.0000002
107	2.2710000	2.2900000	.0011520	.0000002
108	2.3100000	2.3310000	.0010820	.0000002
109	2.3520000	2.3720000	.0010140	.0000002
110	2.3940000	2.4150000	.0009465	.0000002
111	2.4380000	2.4600000	.0008802	.0000002
112	2.4830000	2.5070000	.0008151	.0000002
113	2.5310000	2.5550000	.0007514	.0000002
114	2.5800000	2.6050000	.0006891	.0000002
115	2.6310000	2.6580000	.0006284	.0000002
116	2.6850000	2.7130000	.0005693	.0000002
117	2.7420000	2.7710000	.0005119	.0000002
118	2.8020000	2.8320000	.0004565	.0000002
119	2.8650000	2.8970000	.0004031	.0000002
120	2.9320000	2.9670000	.0003518	.0000002
121	3.0040000	3.0420000	.0003030	.0000002
122	3.0820000	3.1220000	.0002566	.0000002
123	3.1660000	3.2110000	.0002130	.0000002
124	3.2590000	3.3080000	.0001724	.0000002
125	3.3630000	3.4170000	.0001349	.0000002
126	3.4790000	3.5420000	.0001010	.0000002
127	3.6150000	3.6880000	.0000709	.0000002
128	3.7770000	3.8670000	.0000792	.0000057

DISTORTION = .0000501

ENTROPY = 7.7200000

TABLE 10.- OPTIMUM QUANTIZATION LEVEL SPACING FOR EXPONENTIAL PDF WITH MEAN SQUARE ERROR

M = 1

	X(I)	Y(I)	P(I)	D(I)
1	.0000000[a]	.0000000[a]	1.0000000[a]	.5000[a]

DISTORTION = 1.000[a]

ENTROPY = .0000000[a]

M = 2

	X(I)	Y(I)	P(I)	D(I)
1	.0000000[a]	.7071[a]	.5000000[a]	.2500[a]

DISTORTION = .5000[a]

ENTROPY = 1.0000000

M = 3

	X(I)	Y(I)	P(I)	D(I)
1	.7062000	.0000000	.6279000	.0800800
2	.0000000	1.4120000	.1860000	.0910800

DISTORTION = .2622000

ENTROPY = 1.3240000

M = 4

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.4193000	.3981000	.0371700
2	1.1250000	1.8310000	.1019000	.0500800

[a]Value derived by direct computation, not by the Max algorithm.

DISTORTION = .1745000

ENTROPY = 1.7290000

M = 5

	X(I)	Y(I)	P(I)	D(I)
1	.4190000	.0000000	.4457000	.0223800
2	1.5430000	.8379000	.2200000	.0205000
3	.0000000	2.2480000	.0570800	.0274800

DISTORTION = .1184000

ENTROPY = 1.9520000

M = 6

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.2992000	.3188000	.0130100
2	.7177000	1.1360000	.1441000	.0133900
3	1.8400000	2.5430000	.0370700	.0178800

DISTORTION = .0885600

ENTROPY = 2.2100000

M = 7

	X(I)	Y(I)	P(I)	D(I)
1	.2989000	.0000000	.3441000	.0092040
2	1.0160000	.5979000	.2088000	.0085040
3	2.1360000	1.4340000	.0944700	.0087560
4	.0000000	2.8390000	.0247000	.0116000

DISTORTION = .0669200

ENTROPY = 2.3800000

M = 8

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.2326000	.2641000	.0060370
2	.5311000	.8296000	.1502000	.0060960
3	1.2470000	1.6640000	.0680700	.0062750
4	2.3640000	3.0640000	.0176700	.0082600

DISTORTION = .0533300

ENTROPY = 2.5700000

M = 9

	X(I)	Y(I)	P(I)	D(I)
1	.2325000	.0000000	.2798000	.0046380
2	.7632000	.4650000	.1900000	.0043370
3	1.4780000	1.0610000	.1081000	.0043790
4	2.5930000	1.8950000	.0490400	.0045070
5	.0000000	3.2920000	.0129700	.0058810

DISTORTION = .0428500

ENTROPY = 2.7080000

M = 10

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1903000	.2249000	.0032860
2	.4225000	.6546000	.1451000	.0033010
3	.9523000	1.2500000	.0826200	.0033320
4	1.6660000	2.0810000	.0375600	.0034290
5	2.7770000	3.4720000	.0098540	.0044550

DISTORTION = .0356100

ENTROPY = 2.8580000

M = 11

	X(I)	Y(I)	P(I)	D(I)
1	.1901000	.0000000	.2355000	.0026520
2	.6121000	.3803000	.1717000	.0025040

3	1.1410000	.8440000	.1108000	.0025150
4	1.8530000	1.4380000	.0631800	.0025390
5	2.9610000	2.2680000	.0287700	.0026120
6	.0000000	3.6530000	.0077320	.0033590

DISTORTION = .0297100

ENTROPY = 2.9740000

M = 12

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1610000	.1956000	.0019840
2	.3510000	.5409000	.1367000	.0019890
3	.7725000	1.0040000	.0882600	.0019970
4	1.3010000	1.5980000	.0503500	.0020160
5	2.0120000	2.4250000	.0229700	.0020740
6	3.1160000	3.8060000	.0061020	.0026640

DISTORTION = .0254500

ENTROPY = 3.0980000

M = 13

	X(I)	Y(I)	P(I)	D(I)
1	.1608000	.0000000	.2033000	.0016560
2	.5114000	.3217000	.1557000	.0015750
3	.9323000	.7011000	.1088000	.0015780
4	1.4600000	1.1630000	.0703200	.0015850
5	2.1680000	1.7560000	.0401700	.0016000
6	3.2670000	2.5810000	.0183700	.0016460
7	.0000000	3.9540000	.0050190	.0020860

DISTORTION = .0218000

ENTROPY = 3.1990000

M = 14

X(I)	Y(I)	P(I)	D(I)

1	.0000000	.1395000	.1729000	.0012870
2	.3001000	.4608000	.1277000	.0012890
3	.6502000	.8397000	.0893200	.0012920
4	1.0700000	1.3010000	.0577600	.0012970
5	1.5970000	1.8920000	.0330300	.0013090
6	2.3040000	2.7150000	.0151400	.0013460
7	3.3980000	4.0810000	.0040920	.0017050

DISTORTION = .019050

ENTROPY = 3.305000

M = 15

	X(I)	Y(I)	P(I)	D(I)
1	.1394000	.0000000	.1788000	.0011030
2	.4394000	.2788000	.1419000	.0010550
3	.7892000	.5999000	.1048000	.0010560
4	1.2090000	.9785000	.0733300	.0010590
5	1.7350000	1.4400000	.0474400	.0010630
6	2.4410000	2.0300000	.0271500	.0010730
7	3.5330000	2.8510000	.0124600	.0011030
8	.0000000	4.2140000	.0034560	.0013840

DISTORTION = .016690

ENTROPY = 3.393000

M = 16

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1231000	.1549000	.0008821
2	.2623000	.4015000	.1192000	.0008830
3	.5618000	.7221000	.0880500	.0008842
4	.9111000	1.1000000	.0616300	.0008863
5	1.3300000	1.5600000	.0399100	.0008901
6	1.8550000	2.1490000	.0228800	.0008984
7	2.5580000	2.9670000	.0105300	.0009231
8	3.6430000	4.3200000	.0028920	.0011590

DISTORTION = .014810

ENTROPY = 3.4850000

M = 17

	X(I)	Y(I)	P(I)	D(I)
1	.1229000	.0000000	.1594000	.0007679
2	.3847000	.2457000	.1301000	.0007380
3	.6837000	.5237000	.1001000	.0007387
4	1.0320000	.8437000	.0739800	.0007398
5	1.4500000	1.2210000	.0518300	.0007415
6	1.9730000	1.6800000	.0336100	.0007446
7	2.6730000	2.2660000	.0193000	.0007515
8	3.7510000	3.0800000	.0089240	.0007718
9	.0000000	4.4210000	.0025420	.0009507

DISTORTION = .0131200

ENTROPY = 3.5650000

M = 18

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1100000	.1402000	.0006295
2	.2327000	.3555000	.1112000	.0006299
3	.4943000	.6331000	.0856100	.0006305
4	.7929000	.9527000	.0633200	.0006313
5	1.1410000	1.3290000	.0443900	.0006328
6	1.5580000	1.7870000	.0288000	.0006355
7	2.0800000	2.3720000	.0165700	.0006413
8	2.7780000	3.1830000	.0076800	.0006584
9	3.8500000	4.5170000	.0021590	.0008041

DISTORTION = .0117900

ENTROPY = 3.6470000

M = 19

	X(I)	Y(I)	P(I)	D(I)
1	.1099000	.0000000	.1439000	.0005571
2	.3424000	.2198000	.1199000	.0005375

3	.6037000	.4650000	.0951800	.0005379
4	.9020000	.7424000	.0732700	.0005383
5	1.2490000	1.0620000	.0542200	.0005391
6	1.6660000	1.4370000	.0380200	.0005403
7	2.1860000	1.8950000	.0246900	.0005426
8	2.8830000	2.4780000	.0142200	.0005475
9	3.9500000	3.2870000	.0066080	.0005620
10	.0000000	4.6130000	.0019200	.0006777

DISTORTION = .0106000

ENTROPY = 3.7180000

M = 20

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0994800	.1281000	.0004653
2	.2093000	.3190000	.1041000	.0004655
3	.4415000	.5640000	.0826400	.0004658
4	.7025000	.8410000	.0636400	.0004663
5	1.0000000	1.1600000	.0471200	.0004669
6	1.3470000	1.5350000	.0330700	.0004680
7	1.7630000	1.9910000	.0214900	.0004699
8	2.2820000	2.5730000	.0124000	.0004741
9	2.9760000	3.3780000	.0057790	.0004865
10	4.0370000	4.6960000	.0016570	.0005994

DISTORTION = .0096550

ENTROPY = 3.7920000

M = 21

	X(I)	Y(I)	P(I)	D(I)
1	.0994100	.0000000	.1311000	.0004169
2	.3085000	.1988000	.1112000	.0004035
3	.5406000	.4182000	.0904300	.0004037
4	.8013000	.6630000	.0717900	.0004039
5	1.0990000	.9397000	.0552900	.0004043
6	1.4450000	1.2580000	.0409500	.0004049
7	1.8600000	1.6330000	.0287500	.0004058
8	2.3790000	2.0880000	.0187000	.0004075

9	3.0710000	2.6690000	.0108000	.0004111
10	4.1280000	3.4720000	.0050440	.0004217
11	.0000000	4.7840000	.0014940	.0005103

DISTORTION = .0087700

ENTROPY = 3.8560000

M = 22

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0907200	.1178000	.0003528
2	.1899000	.2891000	.0976800	.0003529
3	.3986000	.5081000	.0794500	.0003531
4	.6301000	.7522000	.0631100	.0003533
5	.8902000	1.0280000	.0486500	.0003536
6	1.1870000	1.3450000	.0360600	.0003541
7	1.5320000	1.7190000	.0253500	.0003549
8	1.9450000	2.1720000	.0165200	.0003563
9	2.4610000	2.7500000	.0095720	.0003594
10	3.1480000	3.5460000	.0044990	.0003685
11	4.1930000	4.8400000	.0013290	.0004493

DISTORTION = .0080160

ENTROPY = 3.9250000

M = 23

	X(I)	Y(I)	P(I)	D(I)
1	.0906500	.0000000	.1203000	.0003191
2	.2804000	.1813000	.1035000	.0003096
3	.4889000	.3796000	.0858800	.0003097
4	.7202000	.5983000	.0698700	.0003099
5	.9800000	.8422000	.0555100	.0003101
6	1.2760000	1.1180000	.0428000	.0003103
7	1.6210000	1.4350000	.0317400	.0003108
8	2.0340000	1.8070000	.0223300	.0003115
9	2.5480000	2.2600000	.0145600	.0003127
10	3.2330000	2.8360000	.0084470	.0003155
11	4.2740000	3.6300000	.0039800	.0003233
12	.0000000	4.9170000	.0012170	.0003859

DISTORTION = .0073380

ENTROPY = 3.9840000

M = 24

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0834400	.1091000	.0002744
2	.1740000	.2646000	.0919500	.0002745
3	.3636000	.4627000	.0762800	.0002746
4	.5719000	.6812000	.0620800	.0002747
5	.8030000	.9248000	.0493300	.0002749
6	1.0620000	1.2000000	.0380500	.0002751
7	1.3580000	1.5160000	.0282300	.0002755
8	1.7030000	1.8890000	.0198700	.0002761
9	2.1140000	2.3400000	.0129700	.0002772
10	2.6270000	2.9150000	.0075360	.0002796
11	3.3100000	3.7060000	.0035610	.0002865
12	4.3450000	4.9840000	.0010720	.0003489

DISTORTION = .0067840

ENTROPY = 4.0450000

M = 25

	X(I)	Y(I)	P(I)	D(I)
1	.0833800	.0000000	.1112000	.0002502
2	.2573000	.1668000	.0968800	.0002434
3	.4467000	.3478000	.0816800	.0002434
4	.6548000	.5457000	.0677700	.0002435
5	.8857000	.7640000	.0551600	.0002436
6	1.1450000	1.0070000	.0438500	.0002438
7	1.4400000	1.2820000	.0338300	.0002440
8	1.7840000	1.5980000	.0251100	.0002443
9	2.1950000	1.9700000	.0176800	.0002449
10	2.7070000	2.4210000	.0115500	.0002459
11	3.3880000	2.9940000	.0067210	.0002480
12	4.4180000	3.7820000	.0031840	.0002540
13	.0000000	5.0540000	.0009935	.0003017

DISTORTION = .0062510

ENTROPY = 4.1000000

M = 26

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0771300	.1014000	.0002166
2	.1603000	.2435000	.0867000	.0002167
3	.3337000	.4240000	.0731300	.0002167
4	.5227000	.6213000	.0607100	.0002168
5	.7301000	.8389000	.0494500	.0002169
6	.9602000	1.0810000	.0393400	.0002170
7	1.2180000	1.3550000	.0303800	.0002172
8	1.5130000	1.6700000	.0225800	.0002175
9	1.8550000	2.0390000	.0159300	.0002180
10	2.2630000	2.4870000	.0104400	.0002188
11	2.7710000	3.0550000	.0061010	.0002207
12	3.4450000	3.8340000	.0029160	.0002258
13	4.4580000	5.0820000	.0009139	.0002722

DISTORTION = .0057820

ENTROPY = 4.1590000

M = 27

	X(I)	Y(I)	P(I)	D(I)
1	.0771800	.0000000	.1034000	.0001998
2	.2376000	.1544000	.0910000	.0001947
3	.4112000	.3209000	.0777800	.0001947
4	.6003000	.5015000	.0655900	.0001948
5	.8080000	.6991000	.0544400	.0001948
6	1.0380000	.9169000	.0443300	.0001949
7	1.2970000	1.1600000	.0352600	.0001950
8	1.5910000	1.4340000	.0272200	.0001952
9	1.9340000	1.7490000	.0202200	.0001955
10	2.3430000	2.1190000	.0142600	.0001959
11	2.8530000	2.5680000	.0093370	.0001967
12	3.5280000	3.1370000	.0054480	.0001983
13	4.5470000	3.9190000	.0025970	.0002030
14	.0000000	5.1740000	.0008283	.0002400

DISTORTION = .0053850

ENTROPY = 4.2080000

M = 28

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0717700	.0948500	.0001745
2	.1487000	.2257000	.0820400	.0001745
3	.3087000	.3917000	.0701500	.0001746
4	.4818000	.5719000	.0591900	.0001746
5	.6703000	.7687000	.0491600	.0001747
6	.8772000	.9857000	.0400600	.0001747
7	1.1070000	1.2270000	.0318900	.0001749
8	1.3640000	1.5000000	.0246500	.0001750
9	1.6570000	1.8140000	.0183400	.0001752
10	1.9980000	2.1810000	.0129700	.0001756
11	2.4040000	2.6270000	.0085150	.0001763
12	2.9090000	3.1910000	.0049950	.0001777
13	3.5770000	3.9630000	.0024050	.0001817
14	4.5760000	5.1900000	.0007731	.0002190

DISTORTION = .0050060

ENTROPY = 4.2620000

M = 29

	X(I)	Y(I)	P(I)	D(I)
1	.0717800	.0000000	.0964900	.0001616
2	.2205000	.1436000	.0857100	.0001577
3	.3805000	.2975000	.0741200	.0001577
4	.5536000	.4636000	.0633800	.0001578
5	.7422000	.6437000	.0534800	.0001578
6	.9491000	.8406000	.0444200	.0001579
7	1.1790000	1.0580000	.0361900	.0001579
8	1.4360000	1.2990000	.0288100	.0001580
9	1.7290000	1.5720000	.0222700	.0001582
10	2.0700000	1.8860000	.0165700	.0001584
11	2.4760000	2.2540000	.0117100	.0001587
12	2.9810000	2.6990000	.0076900	.0001593

13	.6500000	3.2640000	.0045100	.0001606
14	4.6500000	4.0360000	.0021700	.0001643
15	.0000000	5.2640000	.0007160	.0001866

DISTORTION = .0046630

ENTROPY = 4.3090000

M = 30

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0670900	.0890600	.0001425
2	.1387000	.2103000	.0778100	.0001426
3	.2871000	.3640000	.0673200	.0001426
4	.4468000	.5296000	.0575800	.0001426
5	.6194000	.7093000	.0486100	.0001426
6	.8074000	.9056000	.0403900	.0001427
7	1.0140000	1.1220000	.0329400	.0001427
8	1.2420000	1.3630000	.0262400	.0001428
9	1.4990000	1.6350000	.0203100	.0001430
10	1.7910000	1.9470000	.0151300	.0001431
11	2.1300000	2.3120000	.0107100	.0001434
12	2.5340000	2.7550000	.0070540	.0001440
13	3.0350000	3.3150000	.0041560	.0001451
14	3.6960000	4.0780000	.0020170	.0001483
15	4.6810000	5.2840000	.0006669	.0001787

DISTORTION = .0043740

ENTROPY = 4.3600000

M = 31

	X(I)	Y(I)	P(I)	D(I)
1	.0671000	.0000000	.0905000	.0001326
2	.2058000	.1342000	.0810100	.0001297
3	.3543000	.2775000	.0707700	.0001297
4	.5139000	.4311000	.0612300	.0001297
5	.6866000	.5968000	.0523700	.0001297
6	.8746000	.7765000	.0442100	.0001298
7	1.0810000	.9728000	.0367400	.0001298
8	1.3100000	1.1890000	.0299500	.0001299

9	1.5660000	1.4300000	.0238600	.0001299
10	1.8580000	1.7020000	.0184600	.0001301
11	2.1970000	2.0140000	.0137600	.0001302
12	2.6010000	2.3800000	.0097390	.0001305
13	3.1030000	2.8230000	.0064120	.0001310
14	3.7650000	3.3830000	.0037770	.0001320
15	4.7500000	4.1460000	.0018320	.0001349
16	.0000000	5.3540000	.0006216	.0001524

DISTORTION = .0040910

ENTROPY = 4.4030000

M = 32

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0629700	.0839200	.0001178
2	.1299000	.1969000	.0739700	.0001179
3	.2683000	.3398000	.0646400	.0001179
4	.4164000	.4930000	.0559400	.0001179
5	.5756000	.6582000	.0478700	.0001179
6	.7478000	.8374000	.0404300	.0001179
7	.9352000	1.0330000	.0336200	.0001180
8	1.1410000	1.2490000	.0274300	.0001180
9	1.3690000	1.4890000	.0218800	.0001181
10	1.6240000	1.7590000	.0169500	.0001182
11	1.9150000	2.0700000	.0126400	.0001183
12	2.2520000	2.4330000	.0089700	.0001186
13	2.6530000	2.8730000	.0059250	.0001190
14	3.1500000	3.4280000	.0035070	.0001200
15	3.8050000	4.1810000	.0017170	.0001225
16	4.7730000	5.3640000	.0005856	.0001477

DISTORTION = .0038510

ENTROPY = 4.4510000

M = 33

	X(I)	Y(I)	P(I)	D(I)
1	.0630100	.0000000	.0852300	.0001103
2	.1930000	.1260000	.0768100	.0001080

3	.3315000	.2600000	.0677000	.0001080
4	.4797000	.4030000	.0591500	.0001080
5	.6391000	.5564000	.0511900	.0001080
6	.8114000	.7217000	.0438000	.0001081
7	.9990000	.9010000	.0369900	.0001081
8	1.2050000	1.0970000	.0307500	.0001081
9	1.4330000	1.3130000	.0250800	.0001082
10	1.6890000	1.5530000	.0200000	.0001082
11	1.9800000	1.8240000	.0154900	.0001083
12	2.3170000	2.1350000	.0115500	.0001085
13	2.7190000	2.4990000	.0081880	.0001087
14	3.2180000	2.9400000	.0054030	.0001091
15	3.8740000	3.4960000	.0031940	.0001099
16	4.8480000	4.2530000	.0015600	.0001123
17	.0000000	5.4420000	.0005417	.0001328

DISTORTION = .0036350

ENTROPY = 4.4910000

M = 34

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0593200	.0793200	.0000985
2	.1221000	.1850000	.0704600	.0000985
3	.2518000	.3185000	.0621200	.0000985
4	.3898000	.4611000	.0543100	.0000985
5	.5375000	.6139000	.0470200	.0000985
6	.6963000	.7786000	.0402500	.0000985
7	.8679000	.9572000	.0340200	.0000986
8	1.0550000	1.1520000	.0283000	.0000986
9	1.2600000	1.3670000	.0231100	.0000986
10	1.4860000	1.6060000	.0184500	.0000987
11	1.7410000	1.8750000	.0143100	.0000988
12	2.0300000	2.1840000	.0106900	.0000989
13	2.3650000	2.5450000	.0076030	.0000991
14	2.7630000	2.9810000	.0050380	.0000994
15	3.2560000	3.5310000	.0029980	.0001002
16	3.9020000	4.2740000	.0014820	.0001022
17	4.8530000	5.4310000	.0005229	.0001234

DISTORTION = .0034150

ENTROPY = 4.5370000

M = 35

	X(I)	Y(I)	P(I)	D(I)
1	.0593500	.0000000	.0804800	.0000925
2	.1816000	.1187000	.0729700	.0000907
3	.3113000	.2444000	.0648200	.0000907
4	.4494000	.3781000	.0571400	.0000907
5	.5972000	.5207000	.0499500	.0000907
6	.7561000	.6737000	.0432400	.0000907
7	.9279000	.8385000	.0370200	.0000908
8	1.1150000	1.0170000	.0312700	.0000908
9	1.3200000	1.2120000	.0260200	.0000908
10	1.5470000	1.4270000	.0212400	.0000908
11	1.8020000	1.6670000	.0169500	.0000909
12	2.0910000	1.9360000	.0131400	.0000910
13	2.4270000	2.2460000	.0098180	.0000911
14	2.8260000	2.6080000	.0069770	.0000913
15	3.3200000	3.0440000	.0046190	.0000916
16	3.9680000	3.5950000	.0027450	.0000923
17	4.9240000	4.3420000	.0013540	.0000942
18	.0000000	5.5060000	.0004864	.0001116

DISTORTION = .0032340

ENTROPY = 4.5750000

M = 36

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0561200	.0752800	.0000834
2	.1154000	.1746000	.0673200	.0000834
3	.2374000	.3002000	.0598000	.0000834
4	.3669000	.4336000	.0527300	.0000834
5	.5048000	.5760000	.0461100	.0000834
6	.6523000	.7287000	.0399300	.0000834
7	.8109000	.8932000	.0341900	.0000835
8	.9823000	1.0710000	.0289000	.0000835
9	1.1690000	1.2660000	.0240500	.0000835
10	1.3730000	1.4810000	.0196500	.0000835
11	1.6000000	1.7190000	.0156900	.0000836

12	1.8540000	1.9880000	.0121700	.0000837
13	2.1420000	2.2960000	.0091050	.0000838
14	2.4760000	2.6560000	.0064800	.0000839
15	2.8730000	3.0910000	.0043000	.0000842
16	3.3640000	3.6380000	.0025640	.0000849
17	4.0070000	4.3770000	.0012730	.0000865
18	4.9500000	5.5240000	.0004556	.0000999

DISTORTION = .0030500

ENTROPY = 4.6170000

M = 37

	X(I)	Y(I)	P(I)	D(I)
1	.0560900	.0000000	.0762300	.0000784
2	.1714000	.1122000	.0694900	.0000769
3	.2933000	.2306000	.0621500	.0000769
4	.4227000	.3560000	.0552200	.0000769
5	.5605000	.4894000	.0486900	.0000769
6	.7079000	.6316000	.0425800	.0000769
7	.8663000	.7842000	.0368800	.0000769
8	1.0380000	.9485000	.0315800	.0000769
9	1.2240000	1.1270000	.0267000	.0000770
10	1.4280000	1.3210000	.0222300	.0000770
11	1.6540000	1.5350000	.0181600	.0000770
12	1.9080000	1.7730000	.0145100	.0000771
13	2.1950000	2.0420000	.0112600	.0000771
14	2.5290000	2.3490000	.0084270	.0000772
15	2.9250000	2.7090000	.0060020	.0000774
16	3.4150000	3.1420000	.0039870	.0000776
17	4.0550000	3.6870000	.0023820	.0000782
18	4.9920000	4.4230000	.0011860	.0000797
19	.0000000	5.5620000	.0004411	.0000941

DISTORTION = .0028940

ENTROPY = 4.6550000

M = 38

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0532100	.0715700	.0000711
2	.1092000	.1652000	.0643900	.0000711
3	.2244000	.2835000	.0576000	.0000711
4	.3462000	.4088000	.0511800	.0000711
5	.4754000	.5420000	.0451400	.0000711
6	.6130000	.6840000	.0394800	.0000711
7	.7602000	.8363000	.0342000	.0000711
8	.9183000	1.0000000	.0293000	.0000711
9	1.0890000	1.1780000	.0247800	.0000711
10	1.2750000	1.3720000	.0206300	.0000712
11	1.4790000	1.5860000	.0168700	.0000712
12	1.7050000	1.8230000	.0134800	.0000712
13	1.9570000	2.0910000	.0104700	.0000713
14	2.2440000	2.3970000	.0078440	.0000714
15	2.5770000	2.7560000	.0055940	.0000715
16	2.9710000	3.1870000	.0037230	.0000718
17	3.4580000	3.7290000	.0022310	.0000723
18	4.0940000	4.4590000	.0011170	.0000736
19	5.0220000	5.5840000	.0004118	.0000904

DISTORTION = .0027520

ENTROPY = 4.6940000

M = 39

	X(I)	Y(I)	P(I)	D(I)
1	.0531700	.0000000	.0724200	.0000670
2	.1623000	.1063000	.0663400	.0000658
3	.2774000	.2183000	.0596900	.0000658
4	.3991000	.3365000	.0534000	.0000658

5	.5282000	.4617000	.0474500	.0000658
6	.6657000	.5947000	.0418600	.0000658
7	.8127000	.7366000	.0366100	.0000658
8	.9707000	.8887000	.0317200	.0000658
9	1.1410000	1.0530000	.0271800	.0000658
10	1.3270000	1.2300000	.0229900	.0000658
11	1.5310000	1.4240000	.0191500	.0000659
12	1.7560000	1.6370000	.0156600	.0000659
13	2.0080000	1.8750000	.0125200	.0000659
14	2.2950000	2.1420000	.0097300	.0000660
15	2.6260000	2.4480000	.0072920	.0000661
16	3.0200000	2.8050000	.0052040	.0000662
17	3.5050000	3.2350000	.0034680	.0000664
18	4.1390000	3.7750000	.0020810	.0000669
19	5.0600000	4.5020000	.0010460	.0000681
20	.0000000	5.6180000	.0004007	.0000789

DISTORTION = .0026040

ENTROPY = 4.730000

M = 40

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0505500	.0681600	.0000609
2	.1036000	.1567000	.0616700	.0000609
3	.2126000	.2684000	.0555000	.0000609
4	.3274000	.3864000	.0496600	.0000609
5	.4489000	.5113000	.0441400	.0000609
6	.5777000	.6441000	.0389500	.0000609
7	.7148000	.7856000	.0340800	.0000609
8	.8615000	.9374000	.0295400	.0000610
9	1.0190000	1.1010000	.0253200	.0000610
10	1.1890000	1.2780000	.0214200	.0000610
11	1.3740000	1.4710000	.0178600	.0000610
12	1.5770000	1.6840000	.0146100	.0000610
13	1.8020000	1.9200000	.0116900	.0000611
14	2.0530000	2.1860000	.0090980	.0000611
15	2.3380000	2.4900000	.0068280	.0000612
16	2.6680000	2.8450000	.0048830	.0000613
17	3.0590000	3.2720000	.0032630	.0000615
18	3.5400000	3.8080000	.0019680	.0000619
19	4.1660000	4.5250000	.0009973	.0000630
20	5.0730000	5.6210000	.0003830	.0000776

DISTORTION = .0024810

ENTROPY = 4.768000

M = 64

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0316700	.0434700	.0000150
2	.0643200	.0969700	.0408800	.0000150
3	.1306000	.1643000	.0383600	.0000150
4	.1991000	.2339000	.0359300	.0000150
5	.2699000	.3058000	.0335700	.0000150
6	.3431000	.3803000	.0312900	.0000150
7	.4189000	.4575000	.0291000	.0000150
8	.4975000	.5376000	.0269800	.0000150
9	.5792000	.6208000	.0249500	.0000150
10	.6641000	.7074000	.0229900	.0000150
11	.7526000	.7977000	.0211100	.0000150
12	.8449000	.8921000	.0193200	.0000150
13	.9414000	.9908000	.0176000	.0000150
14	1.0430000	1.0940000	.0159600	.0000150
15	1.1490000	1.2030000	.0144000	.0000150
16	1.2610000	1.3180000	.0129300	.0000150
17	1.3790000	1.4390000	.0115300	.0000150
18	1.5040000	1.5680000	.0102100	.0000150
19	1.6360000	1.7050000	.0089750	.0000150
20	1.7760000	1.8510000	.0078180	.0000150
21	1.9300000	2.0090000	.0067400	.0000150
22	2.0940000	2.1790000	.0057420	.0000150
23	2.2710000	2.3630000	.0048240	.0000150
24	2.4650000	2.5660000	.0039860	.0000150
25	2.6780000	2.7890000	.0032270	.0000150
26	2.9150000	3.0400000	.0025490	.0000150
27	3.1810000	3.3230000	.0019510	.0000150
28	3.4870000	3.6510000	.0014320	.0000151
29	3.8450000	4.0380000	.0009933	.0000151
30	4.2760000	4.5130000	.0006345	.0000152
31	4.8190000	5.1260000	.0003555	.0000153
32	5.5580000	5.9910000	.0001928	.0000230

DISTORTION = .0009768

ENTROPY = 5.4420000

M = 128

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0158100	.0220200	.0000019
2	.0318500	.0479000	.0213700	.0000019
3	.0641900	.0804800	.0207200	.0000019
4	.0970300	.1136000	.0200800	.0000019
5	.1304000	.1472000	.0194600	.0000019
6	.1643000	.1813000	.0188400	.0000019
7	.1987000	.2161000	.0182300	.0000019
8	.2337000	.2514000	.0176400	.0000019
9	.2693000	.2872000	.0170500	.0000019
10	.3055000	.3238000	.0164700	.0000019
11	.3423000	.3609000	.0159100	.0000019
12	.3798000	.3987000	.0153500	.0000019
13	.4180000	.4372000	.0148000	.0000019
14	.4568000	.4764000	.0142600	.0000019
15	.4964000	.5164000	.0137400	.0000019
16	.5367000	.5571000	.0132200	.0000019
17	.5778000	.5986000	.0127100	.0000019
18	.6198000	.6409000	.0122200	.0000019
19	.6625000	.6841000	.0117300	.0000019
20	.7062000	.7282000	.0112500	.0000019
21	.7508000	.7733000	.0107800	.0000019
22	.7963000	.8193000	.0103200	.0000019
23	.8428000	.8663000	.0098770	.0000019
24	.8904000	.9144000	.0094390	.0000019
25	.9390000	.9636000	.0090110	.0000019
26	.9888000	1.0140000	.0085930	.0000019
27	1.0400000	1.0660000	.0081850	.0000019
28	1.0920000	1.1190000	.0077870	.0000019
29	1.1460000	1.1730000	.0073980	.0000019
30	1.2010000	1.2280000	.0070200	.0000019
31	1.2570000	1.2860000	.0066520	.0000019
32	1.3150000	1.3440000	.0062930	.0000019
33	1.3750000	1.4050000	.0059450	.0000019
34	1.4360000	1.4670000	.0056060	.0000019
35	1.4990000	1.5310000	.0052770	.0000019
36	1.5640000	1.5970000	.0049590	.0000019
37	1.6310000	1.6650000	.0046500	.0000019
38	1.7010000	1.7360000	.0043510	.0000019
39	1.7720000	1.8090000	.0040620	.0000019
40	1.8460000	1.8840000	.0037830	.0000019
41	1.9230000	1.9630000	.0035140	.0000019

42	2.0030000	2.0440000	.0032550	.0000019
43	2.0860000	2.1280000	.0030050	.0000019
44	2.1720000	2.2160000	.0027660	.0000019
45	2.2620000	2.3080000	.0025370	.0000019
46	2.3560000	2.4040000	.0023170	.0000019
47	2.4540000	2.5050000	.0021070	.0000019
48	2.5570000	2.6100000	.0019080	.0000019
49	2.6660000	2.7210000	.0017180	.0000019
50	2.7800000	2.8380000	.0015380	.0000019
51	2.9000000	2.9630000	.0013690	.0000019
52	3.0280000	3.0940000	.0012090	.0000019
53	3.1650000	3.2350000	.0010590	.0000019
54	3.3100000	3.3850000	.0009187	.0000019
55	3.4660000	3.5470000	.0007885	.0000019
56	3.6350000	3.7230000	.0006684	.0000019
57	3.8180000	3.9140000	.0005581	.0000019
58	4.0190000	4.1240000	.0004578	.0000019
59	4.2410000	4.3570000	.0003674	.0000019
60	4.4880000	4.6190000	.0002869	.0000019
61	4.7690000	4.9180000	.0002163	.0000019
62	5.0930000	5.2670000	.0001557	.0000019
63	5.4750000	5.6840000	.0001050	.0000019
64	5.9440000	6.2040000	.0001118	.0000100

DISTORTION = .0002547

ENTROPY = 6.4450000

M = 256

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0079030	.0110900	.0000002
2	.0158700	.0238300	.0109300	.0000002
3	.0318500	.0398700	.0107600	.0000002
4	.0479600	.0560400	.0106000	.0000002
5	.0641900	.0723300	.0104400	.0000002
6	.0805400	.0887500	.0102800	.0000002
7	.0970300	.1053000	.0101200	.0000002
8	.1136000	.1220000	.0099630	.0000002
9	.1304000	.1388000	.0098060	.0000002
10	.1473000	.1557000	.0096510	.0000002

11	.1643000	.1728000	.0094970	.0000002
12	.1814000	.1900000	.0093440	.0000002
13	.1987000	.2074000	.0091920	.0000002
14	.2161000	.2249000	.0090410	.0000002
15	.2337000	.2425000	.0088920	.0000002
16	.2514000	.2603000	.0087440	.0000002
17	.2693000	.2783000	.0085980	.0000002
18	.2873000	.2964000	.0084520	.0000002
19	.3055000	.3146000	.0083080	.0000002
20	.3238000	.3330000	.0081650	.0000002
21	.3423000	.3516000	.0080230	.0000002
22	.3610000	.3704000	.0078830	.0000002
23	.3798000	.3893000	.0077430	.0000002
24	.3988000	.4083000	.0076050	.0000002
25	.4180000	.4276000	.0074690	.0000002
26	.4373000	.4470000	.0073330	.0000002
27	.4568000	.4666000	.0071990	.0000002
28	.4765000	.4864000	.0070660	.0000002
29	.4964000	.5064000	.0069340	.0000002
30	.5165000	.5265000	.0068030	.0000002
31	.5367000	.5469000	.0066740	.0000002
32	.5572000	.5674000	.0065460	.0000002
33	.5778000	.5882000	.0064190	.0000002
34	.5987000	.6092000	.0062940	.0000002
35	.6197000	.6303000	.0061690	.0000002
36	.6410000	.6517000	.0060460	.0000002
37	.6625000	.6733000	.0059240	.0000002
38	.6842000	.6951000	.0058030	.0000002
39	.7062000	.7172000	.0056840	.0000002
40	.7283000	.7395000	.0055660	.0000002
41	.7507000	.7620000	.0054490	.0000002
42	.7734000	.7848000	.0053330	.0000002
43	.7963000	.8078000	.0052190	.0000002
44	.8194000	.8310000	.0051060	.0000002
45	.8428000	.8545000	.0049940	.0000002
46	.8664000	.8783000	.0048830	.0000002
47	.8903000	.9024000	.0047740	.0000002
48	.9145000	.9267000	.0046650	.0000002
49	.9390000	.9513000	.0045580	.0000002
50	.9637000	.9762000	.0044530	.0000002
51	.9888000	1.0010000	.0043480	.0000002
52	1.0140000	1.0270000	.0042450	.0000002

53	1.0400000	1.0530000	.0041430	.0000002
54	1.0660000	1.0790000	.0040420	.0000002
55	1.0920000	1.1050000	.0039420	.0000002
56	1.1190000	1.1320000	.0038440	.0000002
57	1.1460000	1.1590000	.0037470	.0000002
58	1.1730000	1.1870000	.0036510	.0000002
59	1.2010000	1.2140000	.0035570	.0000002
60	1.2290000	1.2430000	.0034630	.0000002
61	1.2570000	1.2710000	.0033710	.0000002
62	1.2860000	1.3000000	.0032800	.0000002
63	1.3150000	1.3300000	.0031910	.0000002
64	1.3450000	1.3590000	.0031020	.0000002
65	1.3750000	1.3900000	.0030150	.0000002
66	1.4050000	1.4200000	.0029290	.0000002
67	1.4360000	1.4510000	.0028450	.0000002
68	1.4670000	1.4830000	.0027610	.0000002
69	1.4990000	1.5150000	.0026790	.0000002
70	1.5310000	1.5480000	.0025980	.0000002
71	1.5640000	1.5810000	.0025190	.0000002
72	1.5970000	1.6140000	.0024400	.0000002
73	1.6310000	1.6480000	.0023630	.0000002
74	1.6660000	1.6830000	.0022870	.0000002
75	1.7010000	1.7180000	.0022120	.0000002
76	1.7360000	1.7540000	.0021390	.0000002
77	1.7720000	1.7900000	.0020660	.0000002
78	1.8090000	1.8270000	.0019950	.0000002
79	1.8460000	1.8650000	.0019260	.0000002
80	1.8840000	1.9040000	.0018570	.0000002
81	1.9230000	1.9430000	.0017900	.0000002
82	1.9630000	1.9830000	.0017240	.0000002
83	2.0030000	2.0230000	.0016590	.0000002
84	2.0440000	2.0650000	.0015960	.0000002
85	2.0860000	2.1070000	.0015330	.0000002
86	2.1290000	2.1500000	.0014720	.0000002
87	2.1720000	2.1940000	.0014120	.0000002
88	2.2170000	2.2390000	.0013540	.0000002
89	2.2620000	2.2850000	.0012960	.0000002
90	2.3080000	2.3320000	.0012400	.0000002
91	2.3560000	2.3800000	.0011850	.0000002
92	2.4040000	2.4290000	.0011320	.0000002
93	2.4540000	2.4790000	.0010790	.0000002
94	2.5050000	2.5310000	.0010280	.0000002

95	2.5570000	2.5830000	.0009783	.0000002
96	2.6110000	2.6380000	.0009297	.0000002
97	2.6650000	2.6930000	.0008823	.0000002
98	2.7220000	2.7500000	.0008361	.0000002
99	2.7790000	2.8090000	.0007911	.0000002
100	2.8390000	2.8690000	.0007475	.0000002
101	2.9000000	2.9310000	.0007050	.0000002
102	2.9630000	2.9950000	.0006638	.0000002
103	3.0280000	3.0610000	.0006238	.0000002
104	3.0950000	3.1290000	.0005851	.0000002
105	3.1640000	3.1990000	.0005476	.0000002
106	3.2350000	3.2720000	.0005113	.0000002
107	3.3090000	3.3470000	.0004763	.0000002
108	3.3860000	3.4250000	.0004426	.0000002
109	3.4650000	3.5060000	.0004100	.0000002
110	3.5480000	3.5900000	.0003788	.0000002
111	3.6340000	3.6780000	.0003487	.0000002
112	3.7230000	3.7690000	.0003199	.0000002
113	3.8170000	3.8650000	.0002924	.0000002
114	3.9150000	3.9650000	.0002660	.0000002
115	4.0170000	4.0700000	.0002410	.0000002
116	4.1250000	4.1800000	.0002171	.0000002
117	4.2380000	4.2970000	.0001945	.0000002
118	4.3580000	4.4200000	.0001732	.0000002
119	4.4850000	4.5510000	.0001531	.0000002
120	4.6210000	4.6910000	.0001342	.0000002
121	4.7650000	4.8400000	.0001166	.0000002
122	4.9200000	5.0010000	.0001002	.0000002
123	5.0880000	5.1750000	.0000850	.0000002
124	5.2690000	5.3640000	.0000711	.0000002
125	5.4680000	5.5720000	.0000584	.0000002
126	5.6870000	5.8030000	.0000470	.0000002
127	5.9320000	6.0620000	.0000368	.0000002
128	6.2090000	6.3570000	.0000768	.0000043

DISTORTION = .0000677

ENTROPY = 7.4450000

TABLE 11.- OPTIMUM QUANTIZATION LEVEL SPACING FOR GAMMA PDF WITH MEAN SQUARE ERROR

M = 1

	X(I)	Y(I)	P(I)	D(I)
1	.0000000[a]	.0000000[a]	1.0000000[a]	.5000[a]

DISTORTION = 1.000[a]

ENTROPY = .0000000[a]

M = 2

	X(I)	Y(I)	P(I)	D(I)
1	.0000000[a]	.5643[a]	.5000000[a]	.3408[a]

DISTORTION = .6816[a]

ENTROPY = 1.0000000[a]

M = 3

	X(I)	Y(I)	P(I)	D(I)
1	.9088000	.0000000	.6209000	.0956300
2	.0000000	1.8180000	.1896000	.0902800

DISTORTION = .2762000

ENTROPY = 1.3370000

M = 4

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.3314000	.4060000	.0442400

[a]Value derived by direct computation, not by the Max algorithm.

2	1.2640000	2.1970000	.0939800	.0600400

DISTORTION = .2086000

ENTROPY = 1.6970000

M = 5

	X(I)	Y(I)	P(I)	D(I)
1	.4910000	.0000000	.5202000	.0262900
2	1.8940000	.9823000	.1432000	.0203800
3	.0000000	2.8050000	.0967200	.0295600

DISTORTION = .1262000

ENTROPY = 1.9450000

M = 6

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.2194000	.3531000	.0149900
2	.7444000	1.2690000	.1033000	.0164100
3	2.2260000	3.1830000	.0435800	.0202300

DISTORTION = .1032000

ENTROPY = 2.1310000

M = 7

	X(I)	Y(I)	P(I)	D(I)
1	.3426000	.0000000	.4563000	.0117000
2	1.2300000	.6853000	.1483000	.0090780
3	2.7240000	1.7750000	.0572700	.0093710
4	.0000000	3.6730000	.0662500	.0112400

DISTORTION = .0710700

ENTROPY = 2.3250000

M = 8

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1609000	.3128000	.0067510
2	.5190000	.8770000	.1135000	.0072850
3	1.4260000	1.9740000	.0457500	.0074960
4	2.9170000	3.8590000	.0279900	.0088670

DISTORTION = .0608000

ENTROPY = 2.4580000

M = 9

	X(I)	Y(I)	P(I)	D(I)
1	.2550000	.0000000	.4053000	.0058970
2	.6846000	.5100000	.1453000	.0046040
3	1.8090000	1.2590000	.0695300	.0046730
4	3.2800000	2.3590000	.0297800	.0047910
5	.0000000	4.2010000	.0527500	.0055450

DISTORTION = .0451200

ENTROPY = 2.6210000

M = 10

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1256000	.2819000	.0035900
2	.3939000	.6623000	.1148000	.0038640
3	1.0420000	1.4210000	.0573000	.0039100
4	1.9720000	2.5230000	.0249600	.0040060
5	3.4390000	4.3560000	.0210000	.0045920

DISTORTION = .0399200

ENTROPY = 2.7200000

M = 11

	X(I)	Y(I)	P(I)	D(I)
1	.2014000	.0000000	.3668000	.0033780
2	.6857000	.4028000	.1395000	.0026520
3	1.3520000	.9686000	.0749300	.0026800
4	2.2820000	1.7360000	.0395600	.0027050
5	3.7190000	2.8280000	.0178300	.0027660
6	.0000000	4.6090000	.0447800	.0031260

DISTORTION = .0312400

ENTROPY = 2.8610000

M = 12

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.1019000	.2574000	.0021140
2	.3141000	.5262000	.1127000	.0022750
3	.8127000	1.0990000	.0632300	.0022920
4	1.4830000	1.8670000	.0339700	.0023120
5	2.4090000	2.9520000	.0155300	.0023620
6	3.8300000	4.7080000	.0172100	.0026890

DISTORTION = .0280800

ENTROPY = 2.9410000

M = 13

	X(I)	Y(I)	P(I)	D(I)
1	.1649000	.0000000	.3361000	.0020950
2	.5548000	.3298000	.1330000	.0016520
3	1.0710000	.7798000	.0768600	.0016680
4	1.7450000	1.3620000	.0455700	.0016760
5	2.6640000	2.1290000	.0251900	.0016890
6	4.0500000	3.1990000	.0118200	.0017230
7	.0000000	4.9010000	.0394800	.0018770

DISTORTION = .0226600

ENTROPY = 3.0650000

M = 14

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0853500	.2377000	.0013490
2	.2598000	.4343000	.1092000	.0014520
3	.6630000	.8918000	.0659100	.0014610
4	1.1840000	1.4770000	.0397000	.0014670
5	1.8600000	2.2430000	.0221600	.0014790
6	2.7760000	3.3080000	.0105000	.0015080
7	4.1500000	4.9920000	.0147700	.0016120

DISTORTION = .0206600

ENTROPY = 3.1310000

M = 15

	X(I)	Y(I)	P(I)	D(I)
1	.1393000	.0000000	.3116000	.0013960
2	.4649000	.2786000	.1269000	.0011040
3	.8851000	.6511000	.0769500	.0011140
4	1.4140000	1.1190000	.0490000	.0011180
5	2.0900000	1.7080000	.0302700	.0011220
6	2.9970000	2.4710000	.0172000	.0011300
7	4.3440000	3.5230000	.0083050	.0011510
8	.0000000	5.1650000	.0355900	.0012610

DISTORTION = .0173900

ENTROPY = 3.2400000

M = 16

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0729200	.2213000	.0009067
2	.2200000	.3672000	.1053000	.0009766
3	.5560000	.7448000	.0668100	.0009816
4	.9792000	1.2140000	.0432900	.0009844
5	1.5070000	1.8000000	.0270500	.0009881
6	2.1780000	2.5560000	.0155400	.0009949
7	3.0730000	3.5910000	.0076110	.0010120
8	4.3900000	5.1890000	.0131300	.0011330

DISTORTION = .0159500

ENTROPY = 3.3000000

M = 17

	X(I)	Y(I)	P(I)	D(I)
1	.1196000	.0000000	.2907000	.0009657
2	.3969000	.2393000	.1210000	.0007661
3	.7482000	.5545000	.0760500	.0007727
4	1.1790000	.9420000	.0509300	.0007746
5	1.7080000	1.4150000	.0338300	.0007764
6	2.3770000	2.0010000	.0214700	.0007791
7	3.2620000	2.7520000	.0125100	.0007843
8	4.5500000	3.7720000	.0062320	.0007970
9	.0000000	5.3290000	.0326500	.0008432

DISTORTION = .0135500

ENTROPY = 3.3990000

M = 18

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0635300	.2076000	.0006406
2	.1905000	.3174000	.1014000	.0006904
3	.4779000	.6384000	.0666700	.0006937
4	.8334000	1.0280000	.0453900	.0006952
5	1.2650000	1.5020000	.0304300	.0006967
6	1.7940000	2.0860000	.0194500	.0006991
7	2.4590000	2.8320000	.0114100	.0007035
8	3.3360000	3.8410000	.0057420	.0007145

9	4.6060000	5.3700000	.0118700	.0008002

DISTORTION = .0126700

ENTROPY = 3.4500000

M = 19

	X(I)	Y(I)	P(I)	D(I)
1	.1048000	.0000000	.2735000	.0006997
2	.3461000	.2096000	.1156000	.0005562
3	.6478000	.4826000	.0746600	.0005609
4	1.0110000	.8131000	.0518300	.0005622
5	1.4460000	1.2080000	.0361400	.0005631
6	1.9750000	1.6840000	.0245800	.0005642
7	2.6370000	2.2670000	.0158900	.0005660
8	3.5050000	3.0070000	.0094220	.0005695
9	4.7500000	4.0030000	.0048090	.0005779
10	.0000000	5.4980000	.0302700	.0005925

DISTORTION = .0109200

ENTROPY = 3.5380000

M = 20

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0560200	.1958000	.0004667
2	.1671000	.2782000	.0975800	.0005032
3	.4169000	.5557000	.0659400	.0005055
4	.7217000	.8876000	.0465600	.0005064
5	1.0850000	1.2820000	.0327900	.0005072
6	1.5180000	1.7550000	.0224700	.0005081
7	2.0440000	2.3330000	.0146300	.0005097
8	2.6980000	3.0630000	.0087570	.0005127
9	3.5520000	4.0400000	.0045330	.0005197
10	4.7660000	5.4910000	.0109300	.0005841

DISTORTION = .0102500

9	4.6060000	5.3700000	.0118700	.0008002

DIST RT 0N = .0126700

ENTROPY = 3.4500000

M = 19

	X( )	Y( )	P( )	D( )
1	.1048000	.0000000	.2735000	.0006997
2	.3461000	.2096000	.1156000	.0005562
3	.6478000	.4826000	.0746600	.0005609
4	1.0110000	.8131000	.0518300	.0005622
5	1.4460000	1.2080000	.0361400	.0005631
6	1.9750000	1.6840000	.0245800	.0005642
7	2.6370000	2.2670000	.0158900	.0005660
8	3.5050000	3.0070000	.0094220	.0005695
9	4.7500000	4.0030000	.0048090	.0005779
10	.0000000	5.4980000	.0302700	.0005925

DIST RT 0N = .0119200

ENTROPY = 3.5080000

M = 20

	X( )	Y( )	P( )	D( )
1	.0000000	.0560200	.1958000	.0004667
2	.1671000	.2782000	.0975800	.0005032
3	.4169000	.5557000	.0659400	.0005055
4	.7217000	.8876000	.0465600	.0005064
5	1.0850000	1.2820000	.0327900	.0005072
6	1.5160000	1.7550000	.0224700	.0005081
7	2.0443000	2.3330000	.0146300	.0005097
8	2.6982000	3.0630000	.0087570	.0005127
9	3.5520000	4.0400000	.0045330	.0005197
10	4.7650000	5.4910000	.0109300	.0005841

DIST RT 0N = .0112500

ENT DP = 3.5860000

M = 21

	X(I)	Y(I)	P(I)	D(I)
1	.0928000	.0000000	.2584000	.0005202
2	.3054000	.1856000	.1107000	.0004143
3	.5685000	.4252000	.0730000	.0004178
4	.8805000	.7118000	.0520800	.0004187
5	1.2460000	1.0490000	.0376600	.0004192
6	1.6830000	1.4460000	.0269000	.0004197
7	2.2050000	1.9200000	.0186100	.0004205
8	2.8590000	2.4960000	.0122200	.0004217
9	3.7040000	3.2220000	.0073780	.0004241
10	4.8440000	4.1870000	.0038640	.0004297
11	.0000000	5.6100000	.0283300	.0004702

DIST NT DP = .0948320

ENT DPY = 3.5670000

M = 22

	X( )	Y( )	P( )	D( )
1	.0000050	.0501900	.1859000	.0003539
2	.1491000	.2480000	.0941200	.0003817
3	.3705000	.4930000	.0649200	.0003835
4	.6377000	.7825000	.0470700	.0003840
5	.9520000	1.1210000	.0343300	.0003845
6	1.3200000	1.5190000	.0246500	.0003849
7	1.7550000	1.9920000	.0171300	.0003856
8	2.2800000	2.5670000	.0112900	.0003867
9	2.9230000	3.2890000	.0068480	.0003889
10	3.7250000	4.2470000	.0036090	.0003938
11	4.9500000	5.6530000	.0101100	.0004061

DIST NT DP = .0954670

ENT DP = 3.7070000

ENTROPY = 3.5860000

M = 21

	X(I)	Y(I)	P(I)	D(I)
1	.0928000	.0000000	.2584000	.0005202
2	.3054000	.1856000	.1107000	.0004143
3	.5685000	.4252000	.0730000	.0004178
4	.8805000	.7118000	.0520800	.0004187
5	1.2460000	1.0490000	.0376600	.0004192
6	1.6830000	1.4460000	.0269000	.0004197
7	2.2080000	1.9200000	.0186100	.0004205
8	2.8590000	2.4960000	.0122200	.0004217
9	3.7040000	3.2220000	.0073780	.0004241
10	4.8980000	4.1870000	.0038640	.0004297
11	.0000000	5.6100000	.0283300	.0004702

DISTORTION = .0090320

ENTROPY = 3.6670000

M = 22

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0501900	.1859000	.0003539
2	.1491000	.2480000	.0941200	.0003817
3	.3705000	.4930000	.0649200	.0003835
4	.6377000	.7825000	.0470700	.0003840
5	.9520000	1.1210000	.0343300	.0003845
6	1.3200000	1.5190000	.0246500	.0003849
7	1.7560000	1.9920000	.0171300	.0003856
8	2.2800000	2.5670000	.0112900	.0003867
9	2.9280000	3.2890000	.0068480	.0003889
10	3.7680000	4.2470000	.0036090	.0003938
11	4.9500000	5.6530000	.0101100	.0004061

DISTORTION = .0084670

ENTROPY = 3.7070000

M = 23

	X(I)	Y(I)	P(I)	D(I)
1	.0832100	.0000000	.2455000	.0003964
2	.2731000	.1664000	.1063000	.0003178
3	.5062000	.3798000	.0712500	.0003205
4	.7796000	.6326000	.0519300	.0003211
5	1.0970000	.9265000	.0385800	.0003214
6	1.4660000	1.2670000	.0285400	.0003217
7	1.9010000	1.6660000	.0207000	.0003221
8	2.4220000	2.1370000	.0145100	.0003226
9	3.0630000	2.7060000	.0096480	.0003235
10	3.8890000	3.4190000	.0059090	.0003252
11	5.0420000	4.3590000	.0031610	.0003291
12	.0000000	5.7240000	.0266600	.0003727

DIST RT 0X = .0075940

ENT 0PY = 3.7640000

M = 24

	X( )	Y( )	P( )	D( )
1	.0000000	.0451500	.1768000	.0002713
2	.1337000	.2222000	.0907200	.0002927
3	.3310000	.4398000	.0636700	.0002940
4	.5672000	.6945000	.0471800	.0002944
5	.8417000	.9890000	.0353800	.0002947
6	1.1590000	1.3290000	.0263300	.0002949
7	1.5270000	1.7260000	.0192000	.0002953
8	1.9590000	2.1930000	.0135300	.0002958
9	2.4760000	2.7580000	.0090430	.0002966
10	3.1190000	3.4610000	.0055800	.0002980
11	3.9230000	4.3850000	.0030200	.0003014
12	5.0500000	5.7150000	.0095120	.0003328

DIST RT 0Y = .0071240

ENT 0P = 3.8230000

M = 25

	X( )	Y( )	P( )	D( )
1	.0751900	.0000000	.2341000	.0003108
2	.2562000	.1504000	.1022000	.0002482
3	.4545000	.3421000	.0694600	.0002503
4	.6972000	.5674000	.0515000	.0002508
5	.9756000	.8270000	.0391000	.0002510
6	1.2956000	1.1240000	.0297200	.0002512
7	1.6641000	1.4660000	.0223300	.0002514
8	2.0961000	1.8630000	.0163900	.0002516
9	2.5900000	2.3290000	.0116200	.0002520
10	3.2390000	2.8900000	.0078180	.0002527
11	4.0430000	3.5880000	.0048600	.0002539
12	5.1500000	4.4990000	.0026580	.0002566
13	.0000000	5.8020000	.0252900	.0002701

DIST AT θ^ = .0063900

ENT OPY = 3.8420000

M = 26

	X( )	Y( )	P( )	D( )
1	.0000000	.0410100	.1689000	.0002130
2	.1211000	.2011000	.0876100	.0002299
3	.2961000	.3968000	.0623400	.0002310
4	.5130000	.6240000	.0470000	.0002313
5	.7841000	.8841000	.0360100	.0002314
6	1.1330000	1.1610000	.0275400	.0002316
7	1.5520000	1.5220000	.0207900	.0002318
8	1.7190000	1.9160000	.0153300	.0002320
9	2.1470000	2.3790000	.0109200	.0002324
10	2.6560000	2.9340000	.0073840	.0002329
11	3.2760000	3.6220000	.0046250	.0002340
12	4.0090000	4.5160000	.0025590	.0002364
13	5.1510000	5.7850000	.0089860	.0002705

DIST AT θ^ = .0062760

ENT OPY = 3.9290000

M = 27

	X( )	Y( )	P( )	D( )
1	.0585000	.0000000	.2239000	.0002473
2	.2239000	.1370000	.0985300	.0001977
3	.4123000	.3108000	.0677000	.0001993
4	.6299000	.5139000	.0509000	.0001997
5	.8775000	.7459000	.0393100	.0001999
6	1.1590000	1.0090000	.0305300	.0002000
7	1.4790000	1.3080000	.0235600	.0002001
8	1.8470000	1.6500000	.0179000	.0002002
9	2.2750000	2.0440000	.0132700	.0002004
10	2.7820000	2.5060000	.0094940	.0002007
11	3.4000000	3.0590000	.0064540	.0002012
12	4.1640000	3.7410000	.0040670	.0002021
13	5.2500000	4.6260000	.0022690	.0002041
14	.0000000	5.8740000	.0240700	.0002174

DIST RT DA = .0054930

ENT OPY = 3.9930000

M = 28

	X( )	Y( )	P( )	D( )
1	.0000000	.0375500	.1619000	.0001707
2	.1106000	.1836000	.0847500	.0001843
3	.2724000	.3612000	.0509900	.0001851
4	.4837000	.5662000	.0346300	.0001853
5	.7426000	.7991000	.0363400	.0001855
6	.9507000	1.0620000	.0283800	.0001856
7	1.2120000	1.3610000	.0220000	.0001857
8	1.5310000	1.7010000	.0167800	.0001858
9	1.8970000	2.0930000	.0124600	.0001860
10	2.3220000	2.5510000	.0089690	.0001862
11	2.8250000	3.0980000	.0061280	.0001867
12	3.4350000	3.7720000	.0038870	.0001875
13	4.2070000	4.6420000	.0021920	.0001892

14	5.2500000	5.8590000	.0085240	.0002178

DIST RT ON = .0052430

ENT OP = 4.0250000

M = 29

	X( )	Y( )	P( )	D( )
1	.0522200	.0000000	.2149000	.0001998
2	.2050000	.1256000	.0951200	.0001599
3	.3766000	.2843000	.0659800	.0001612
4	.5735000	.4686000	.0501900	.0001615
5	.7961000	.6782000	.0393200	.0001616
6	1.0470000	.9140000	.0310600	.0001617
7	1.3290000	1.1790000	.0244800	.0001618
8	1.6490000	1.4790000	.0190900	.0001619
9	2.0150000	1.8190000	.0146400	.0001620
10	2.4390000	2.2110000	.0109400	.0001621
11	2.9390000	2.6670000	.0078960	.0001624
12	3.5440000	3.2110000	.0054210	.0001627
13	4.3080000	3.8780000	.0034590	.0001634
14	5.3350000	4.7370000	.0019680	.0001648
15	.0000000	5.9320000	.0230000	.0002017

DIST RT ON = .0048170

ENT OPY = 4.0870000

M = 30

	X( )	Y( )	P( )	D( )
1	.0000000	.0346100	.1557000	.0001391
2	.1017000	.1689000	.0821200	.0001502
3	.2501000	.3314000	.0596600	.0001509
4	.4247000	.5180000	.0461500	.0001511
5	.6233000	.7287000	.0364700	.0001512
6	.8469000	.9651000	.0289700	.0001513
7	1.0980000	1.2310000	.0229200	.0001513
8	1.3800000	1.5300000	.0179300	.0001514
9	1.6990000	1.8690000	.0137800	.0001515
10	2.0640000	2.2590000	.0103300	.0001516
11	2.4860000	2.7130000	.0074780	.0001518
12	2.9830000	3.2520000	.0051530	.0001522
13	3.5830000	3.9140000	.0033040	.0001528
14	4.3380000	4.7630000	.0018930	.0001541
15	5.3500000	5.9370000	.0081120	.0001734

DIST BT BN = .0045680

ENT BPY = 4.1170000

M = 31

	X( )	Y( )	P( )	D( )
1	.0579100	.0000000	.2066000	.0001635
2	.1887000	.1158000	.0919800	.0001309
3	.3460000	.2616000	.0643200	.0001321
4	.5255000	.4303000	.0494100	.0001323
5	.7273000	.6208000	.0391700	.0001324
6	.9526000	.8339000	.0313900	.0001324
7	1.2050000	1.0720000	.0251700	.0001325
8	1.4870000	1.3380000	.0200400	.0001326
9	1.8060000	1.6370000	.0157700	.0001326
10	2.1690000	1.9750000	.0121800	.0001327
11	2.5860000	2.3630000	.0091730	.0001328
12	3.0800000	2.8130000	.0066730	.0001330
13	3.6730000	3.3470000	.0046240	.0001333
14	4.4160000	3.9990000	.0029880	.0001338
15	5.4040000	4.8320000	.0017320	.0001348
16	.0000000	5.9760000	.0220500	.0001655

DISTORTION = .0042110

ENTROP = 4.1750000

M = 32

	X( )	Y( )	P( )	D( )
1	.0[illegible]	.0320900	.1502000	.0001151
2	.0941[illegible]	.1563000	.0797000	.0001243
3	.2312[illegible]	.3060000	.0583700	.0001248
4	.3915[illegible]	.4772000	.0455900	.0001250
5	.5734[illegible]	.6695000	.0364400	.0001251
6	.7767[illegible]	.8839000	.0293500	.0001251
7	1.0030000	1.1230000	.0236100	.0001252
8	1.2560000	1.3900000	.0188500	.0001252
9	1.5390000	1.6890000	.0148600	.0001253
10	1.8550000	2.0280000	.0114900	.0001254
11	2.2220000	2.4150000	.0086670	.0001255
12	2.6400000	2.8650000	.0063120	.0001256
13	3.1320000	3.3990000	.0043610	.0001259
14	3.7240000	4.0500000	.0028350	.0001264
15	4.4650000	4.8810000	.0016470	.0001274
16	5.4500000	6.0190000	.0007440	.0001363

DIS[illegible] = .0040150

E[illegible] = 4.2[illegible]

M = 33

	X( )	Y( )	P( )	D( )
1	.0536800	.0000000	.1992000	.0001356
2	.1747000	.1074000	.0891000	.0001087
3	.3197000	.2421000	.0627400	.0001096
4	.4646000	.3974000	.0486100	.0001098
5	.6690000	.5719000	.0389200	.0001099
6	.8736000	.7661000	.0315600	.0001099
7	1.1010000	.9815000	.0256600	.0001099
8	1.3540000	1.2210000	.0207800	.0001100
9	1.6360000	1.4870000	.0166900	.0001100
10	1.9530000	1.7850000	.0132200	.0001101
11	2.3130000	2.1210000	.0102800	.0001101
12	2.7270000	2.5040000	.0077910	.0001102
13	3.2110000	2.9490000	.0057080	.0001104
14	3.7930000	3.4740000	.0039900	.0001106
15	4.5160000	4.1120000	.0026080	.0001110
16	5.4700000	4.9210000	.0015380	.0001118
17	.0000000	6.0190000	.0212000	.0001344

DIST RT BA = .0037080

ENT BPY = 4.2580000

M = 34

	X( )	Y( )	P( )	D( )
1	.0000000	.0297600	.1448000	.0000953
2	.0572200	.1447000	.0773100	.0001029
3	.2137000	.2828000	.0570400	.0001034
4	.3614000	.4401000	.0449400	.0001035
5	.5279000	.6158000	.0363000	.0001036
6	.7133000	.8108000	.0295900	.0001036
7	.9186000	1.0270000	.0241500	.0001036
8	1.1460000	1.2660000	.0196200	.0001037
9	1.3980000	1.5310000	.0157900	.0001037
10	1.6800000	1.8280000	.0125400	.0001037
11	1.9960000	2.1630000	.0097670	.0001038
12	2.3540000	2.5450000	.0074210	.0001039
13	2.7660000	2.9870000	.0054510	.0001040
14	3.2470000	3.5080000	.0038220	.0001042
15	3.8240000	4.1400000	.0025090	.0001046
16	4.5400000	4.9400000	.0014900	.0001053
17	5.4800000	6.0200000	.0074630	.0001336

DIST RT BP = .0035730

ENT BP = 4.2650000

M = 35

	X( )	Y( )	P( )	D( )
1	.0499900	.0000000	.1925000	.0001138
2	.1625000	.0999800	.0864400	.0000912
3	.2970000	.2251000	.0612300	.0000920
4	.4493000	.3689000	.0477800	.0000922
5	.6189000	.5298000	.0385900	.0000922
6	.8064000	.7081000	.0316100	.0000922
7	1.0130000	.9047000	.0260000	.0000923
8	1.2410000	1.1220000	.0213500	.0000923
9	1.4940000	1.3610000	.0174300	.0000923
10	1.7750000	1.6270000	.0140900	.0000924
11	2.0900000	1.9230000	.0112300	.0000924
12	2.4470000	2.2570000	.0087770	.0000924
13	2.8560000	2.6370000	.0066920	.0000925
14	3.3340000	3.0750000	.0049350	.0000926
15	3.9040000	3.5920000	.0034770	.0000928
16	4.6100000	4.2160000	.0022970	.0000931
17	5.5320000	5.0030000	.0013760	.0000937
18	.0000000	6.0600000	.0204400	.0001280

DIST RT BN = .0033270

ENT OPY = 4.3360000

M = 36

	X( )	Y( )	P( )	D( )
1	.0000000	.0278100	.1401000	.0000804
2	.0813900	.1350000	.0751900	.0000868
3	.1992000	.2634000	.0558100	.0000872
4	.3363000	.4092000	.0442900	.0000873
5	.4903000	.5715000	.0360800	.0000874
6	.6611000	.7506000	.0297000	.0000874
7	.8493000	.9479000	.0245200	.0000874
8	1.0560000	1.1650000	.0201900	.0000874
9	1.2850000	1.4050000	.0165200	.0000875
10	1.5370000	1.6700000	.0133800	.0000875
11	1.8180000	1.9660000	.0106800	.0000875
12	2.1330000	2.2990000	.0083620	.0000876
13	2.4880000	2.6780000	.0063860	.0000876
14	2.8960000	3.1140000	.0047190	.0000877
15	3.3710000	3.6280000	.0033330	.0000879
16	3.9380000	4.2480000	.0022090	.0000882
17	4.6380000	5.0280000	.0013300	.0000887
18	5.5510000	6.0730000	.0071770	.0001063

DIST RT ON = .0031760

ENT OPY = 4.3610000

M = 37

	X( )	Y( )	P( )	D( )
1	.0467100	.0000000	.1863000	.0000962
2	.1517000	.0934200	.0839500	.0000772
3	.2769000	.2101000	.0597900	.0000778
4	.4183000	.3437000	.0469600	.0000780
5	.5750000	.4928000	.0382000	.0000780
6	.7476000	.6573000	.0315600	.0000781
7	.9369000	.8378000	.0262200	.0000781
8	1.1450000	1.0360000	.0217900	.0000781
9	1.3730000	1.2530000	.0180400	.0000781
10	1.6250000	1.4930000	.0148200	.0000781
11	1.9050000	1.7570000	.0120500	.0000782
12	2.2170000	2.0520000	.0096490	.0000782
13	2.5710000	2.3830000	.0075830	.0000782
14	2.9740000	2.7580000	.0058150	.0000783
15	3.4440000	3.1900000	.0043150	.0000784
16	4.0020000	3.6970000	.0030650	.0000785
17	4.6900000	4.3080000	.0020470	.0000788
18	5.5790000	5.0710000	.0012460	.0000792
19	.0000000	6.0860000	.0197400	.0001042

DIST RTN = .0029630

ENT OPY = 4.4110000

M = 38

	X( )	Y( )	P( )	D( )
1	.0000000	.0260700	.1358000	.0000683
2	.0762100	.1264000	.0731900	.0000738
3	.1863000	.2463000	.0546200	.0000742
4	.3141000	.3820000	.0436200	.0000743
5	.4573000	.5325000	.0357900	.0000743
6	.6153000	.6981000	.0297200	.0000743
7	.7888000	.8795000	.0247800	.0000744
8	.9788000	1.0780000	.0206500	.0000744
9	1.1870000	1.2960000	.0171200	.0000744
10	1.4160000	1.5360000	.0140900	.0000744
11	1.6680000	1.8000000	.0114700	.0000744
12	1.9480000	2.0950000	.0091960	.0000745
13	2.2600000	2.4260000	.0072350	.0000745
14	2.6130000	2.8010000	.0055540	.0000746
15	3.0160000	3.2320000	.0041270	.0000746
16	3.4850000	3.7370000	.0029360	.0000748
17	4.0420000	4.3460000	.0019640	.0000750
18	4.7260000	5.1060000	.0011990	.0000754
19	5.6110000	6.1150000	.0069230	.0001013

DISPERSION = .0028720

ENTROPY = 4.4330000

M = 39

	X( )	Y( )	P( )	D( )
1	.0438200	.0000000	.1807000	.0000822
2	.1423000	.0876500	.0816600	.0000660
3	.2593000	.1969000	.0584200	.0000665
4	.3911000	.3217000	.0461400	.0000666
5	.5368000	.4605000	.0377800	.0000667
6	.6966000	.6131000	.0314500	.0000667
7	.8711000	.7800000	.0263500	.0000667
8	1.0620000	.9622000	.0221200	.0000667
9	1.2700000	1.1610000	.0185200	.0000668
10	1.4980000	1.3790000	.0154200	.0000668
11	1.7500000	1.6180000	.0127400	.0000668
12	2.0280000	1.8810000	.0104000	.0000668
13	2.3380000	2.1740000	.0083700	.0000668
14	2.6880000	2.5020000	.0066090	.0000669
15	3.0870000	2.8730000	.0050930	.0000669
16	3.5490000	3.3000000	.0038020	.0000670
17	4.0970000	3.7980000	.0027200	.0000671
18	4.7680000	4.3960000	.0018340	.0000673
19	5.6290000	5.1390000	.0011320	.0000677
20	.0000000	6.1180000	.0191100	.0001006

DISI RT ØN = .0026890

ENT ØPY = 4.4820000

M = 40

	X( )	Y( )	P( )	D( )
1	.0000000	.0245000	.1318000	.0000585
2	.0715400	.1186000	.0712900	.0000632
3	.1747000	.2309000	.0534600	.0000635
4	.2942000	.3576000	.0429400	.0000635
5	.4276000	.4977000	.0354600	.0000636
6	.5745000	.6513000	.0296700	.0000636
7	.7351000	.8189000	.0249500	.0000636
8	.9102000	1.0020000	.0210000	.0000636
9	1.1010000	1.2010000	.0176200	.0000636
10	1.3100000	1.4190000	.0147000	.0000636
11	1.5380000	1.6580000	.0121500	.0000637
12	1.7890000	1.9210000	.0099370	.0000637
13	2.0670000	2.2130000	.0080060	.0000637
14	2.3770000	2.5410000	.0063290	.0000637
15	2.7260000	2.9110000	.0048840	.0000638
16	3.1230000	3.3360000	.0036510	.0000639
17	3.5840000	3.8320000	.0026170	.0000640
18	4.1290000	4.4270000	.0017690	.0000641
19	4.7960000	5.1650000	.0010960	.0000645
20	5.6500000	6.1350000	.0067030	.0000817

DIST RT dN = .0025740

ENT dPY = 4.5030000

M = 64

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0139400	.0999900	.0000143
2	.0404500	.0669500	.0555500	.0000154
3	.0981300	.1293000	.0429900	.0000155
4	.1639000	.1985000	.0358000	.0000155
5	.2360000	.2736000	.0308000	.0000155
6	.3138000	.3541000	.0269800	.0000155
7	.3969000	.4398000	.0238800	.0000155
8	.4853000	.5308000	.0212900	.0000155
9	.5789000	.6270000	.0190500	.0000155
10	.6778000	.7287000	.0170800	.0000155
11	.7823000	.8360000	.0153400	.0000155
12	.8926000	.9492000	.0137700	.0000155
13	1.0090000	1.0690000	.0123600	.0000155
14	1.1320000	1.1950000	.0110700	.0000155
15	1.2620000	1.3280000	.0098910	.0000155
16	1.3990000	1.4700000	.0088150	.0000155
17	1.5450000	1.6190000	.0078270	.0000155
18	1.6990000	1.7790000	.0069210	.0000155
19	1.8630000	1.9480000	.0060880	.0000155
20	2.0380000	2.1290000	.0053240	.0000155
21	2.2260000	2.3230000	.0046250	.0000155
22	2.4270000	2.5310000	.0039850	.0000155
23	2.6430000	2.7560000	.0034020	.0000155
24	2.8780000	3.0000000	.0028720	.0000155
25	3.1330000	3.2660000	.0023930	.0000155
26	3.4130000	3.5590000	.0019640	.0000155
27	3.7220000	3.8840000	.0015810	.0000156
28	4.0670000	4.2490000	.0012430	.0000156
29	4.4560000	4.6630000	.0009489	.0000156
30	4.9020000	5.1410000	.0006968	.0000156
31	5.4230000	5.7060000	.0004854	.0000156
32	6.0510000	6.3950000	.0049820	.0000229

DISTORTION = .0010070

ENTROPY = 5.1420000

M = 128

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0060800	.0563200	.0000018
2	.0175500	.0290200	.0375600	.0000019
3	.0423700	.0557200	.0297300	.0000019
4	.0703600	.0850000	.0253900	.0000019
5	.1007000	.1164000	.0224500	.0000019
6	.1329000	.1495000	.0202600	.0000019
7	.1669000	.1843000	.0185200	.0000019
8	.2024000	.2205000	.0170800	.0000019
9	.2394000	.2582000	.0158600	.0000019
10	.2778000	.2973000	.0147900	.0000019
11	.3175000	.3377000	.0138500	.0000019
12	.3586000	.3795000	.0130000	.0000019
13	.4010000	.4225000	.0122400	.0000019
14	.4447000	.4669000	.0115400	.0000019
15	.4897000	.5125000	.0108900	.0000019
16	.5360000	.5595000	.0103000	.0000019
17	.5836000	.6078000	.0097420	.0000019
18	.6326000	.6575000	.0092220	.0000019
19	.6830000	.7085000	.0087340	.0000019
20	.7347000	.7609000	.0082740	.0000019
21	.7878000	.8148000	.0078400	.0000019
22	.8424000	.8701000	.0074290	.0000019
23	.8985000	.9270000	.0070390	.0000019
24	.9562000	.9854000	.0066680	.0000019
25	1.0150000	1.0450000	.0063160	.0000019
26	1.0760000	1.1070000	.0059790	.0000019
27	1.1390000	1.1700000	.0056580	.0000019
28	1.2030000	1.2360000	.0053520	.0000019
29	1.2690000	1.3030000	.0050590	.0000019
30	1.3370000	1.3720000	.0047790	.0000019
31	1.4070000	1.4420000	.0045110	.0000019
32	1.4790000	1.5150000	.0042540	.0000019
33	1.5530000	1.5910000	.0040080	.0000019
34	1.6290000	1.6680000	.0037720	.0000019
35	1.7080000	1.7480000	.0035460	.0000019
36	1.7890000	1.8310000	.0033300	.0000019

37	1.8730000	1.9160000	.0031220	.0000019
38	1.9600000	2.0040000	.0029240	.0000019
39	2.0490000	2.0950000	.0027330	.0000019
40	2.1420000	2.1890000	.0025510	.0000019
41	2.2370000	2.2860000	.0023770	.0000019
42	2.3370000	2.3870000	.0022100	.0000019
43	2.4390000	2.4920000	.0020510	.0000019
44	2.5460000	2.6010000	.0018990	.0000019
45	2.6570000	2.7140000	.0017540	.0000019
46	2.7730000	2.8320000	.0016150	.0000019
47	2.8930000	2.9540000	.0014840	.0000019
48	3.0180000	3.0830000	.0013580	.0000019
49	3.1500000	3.2170000	.0012390	.0000019
50	3.2870000	3.3570000	.0011270	.0000019
51	3.4310000	3.5050000	.0010200	.0000019
52	3.5830000	3.6600000	.0009189	.0000019
53	3.7420000	3.8240000	.0008238	.0000019
54	3.9110000	3.9980000	.0007345	.0000019
55	4.0890000	4.1810000	.0006507	.0000019
56	4.2790000	4.3770000	.0005724	.0000019
57	4.4810000	4.5860000	.0004995	.0000019
58	4.6980000	4.8100000	.0004319	.0000019
59	4.9310000	5.0520000	.0003695	.0000019
60	5.1830000	5.3140000	.0003123	.0000019
61	5.4570000	5.6000000	.0002601	.0000019
62	5.7580000	5.9150000	.0002129	.0000020
63	6.0900000	6.2650000	.0001707	.0000020
64	6.4620000	6.6590000	.0032950	.0000033

DISTORTION = .0002517

ENTROPY = 6.0990000

M = 256

	X(I)	Y(I)	P(I)	D(I)
1	.0000000	.0026600	.0439500	.0000002
2	.0076610	.0126600	.0251000	.0000002
3	.0184600	.0242500	.0200600	.0000002
4	.0305800	.0369000	.0173100	.0000002
5	.0436400	.0503800	.0154800	.0000002
6	.0574500	.0645300	.0141400	.0000002
7	.0719100	.0792900	.0130900	.0000002
8	.0869400	.0945900	.0122300	.0000002
9	.1025000	.1104000	.0115100	.0000002
10	.1185000	.1266000	.0108900	.0000002
11	.1350000	.1433000	.0103500	.0000002
12	.1519000	.1604000	.0098710	.0000002

13	.1692000	.1780000	.0094410	.0000002
14	.1869000	.1958000	.0090500	.0000002
15	.2050000	.2141000	.0086930	.0000002
16	.2234000	.2327000	.0083640	.0000002
17	.2422000	.2517000	.0080590	.0000002
18	.2614000	.2710000	.0077760	.0000002
19	.2809000	.2907000	.0075100	.0000002
20	.3007000	.3107000	.0072610	.0000002
21	.3209000	.3311000	.0070260	.0000002
22	.3414000	.3517000	.0068040	.0000002
23	.3623000	.3728000	.0065930	.0000002
24	.3834000	.3941000	.0063930	.0000002
25	.4049000	.4158000	.0062020	.0000002
26	.4268000	.4378000	.0060190	.0000002
27	.4489000	.4601000	.0058440	.0000002
28	.4714000	.4828000	.0056770	.0000002
29	.4942000	.5057000	.0055160	.0000002
30	.5174000	.5291000	.0053610	.0000002
31	.5409000	.5527000	.0052110	.0000002
32	.5647000	.5767000	.0050670	.0000002
33	.5889000	.6010000	.0049280	.0000002
34	.6134000	.6257000	.0047940	.0000002
35	.6382000	.6507000	.0046640	.0000002
36	.6634000	.6761000	.0045380	.0000002
37	.6889000	.7018000	.0044160	.0000002
38	.7148000	.7278000	.0042970	.0000002
39	.7410000	.7542000	.0041820	.0000002
40	.7676000	.7810000	.0040710	.0000002
41	.7945000	.8081000	.0039620	.0000002
42	.8219000	.8356000	.0038560	.0000002
43	.8496000	.8635000	.0037540	.0000002
44	.8776000	.8918000	.0036530	.0000002
45	.9061000	.9204000	.0035560	.0000002
46	.9349000	.9494000	.0034610	.0000002
47	.9641000	.9789000	.0033680	.0000002
48	.9938000	1.0090000	.0032770	.0000002
49	1.0240000	1.0390000	.0031890	.0000002
50	1.0540000	1.0700000	.0031030	.0000002
51	1.0850000	1.1010000	.0030190	.0000002
52	1.1160000	1.1320000	.0029370	.0000002
53	1.1480000	1.1640000	.0028560	.0000002
54	1.1800000	1.1970000	.0027780	.0000002

55	1.2130000	1.2290000	.0027010	.0000002
56	1.2460000	1.2630000	.0026260	.0000002
57	1.2800000	1.2960000	.0025530	.0000002
58	1.3140000	1.3310000	.0024810	.0000002
59	1.3480000	1.3660000	.0024110	.0000002
60	1.3830000	1.4010000	.0023420	.0000002
61	1.4190000	1.4370000	.0022750	.0000002
62	1.4550000	1.4730000	.0022100	.0000002
63	1.4910000	1.5100000	.0021450	.0000002
64	1.5280000	1.5470000	.0020830	.0000002
65	1.5660000	1.5850000	.0020210	.0000002
66	1.6040000	1.6240000	.0019610	.0000002
67	1.6430000	1.6630000	.0019020	.0000002
68	1.6830000	1.7030000	.0018440	.0000002
69	1.7230000	1.7430000	.0017880	.0000002
70	1.7630000	1.7840000	.0017320	.0000002
71	1.8050000	1.8260000	.0016780	.0000002
72	1.8470000	1.8680000	.0016250	.0000002
73	1.8890000	1.9110000	.0015730	.0000002
74	1.9330000	1.9550000	.0015230	.0000002
75	1.9770000	1.9990000	.0014730	.0000002
76	2.0220000	2.0440000	.0014240	.0000002
77	2.0670000	2.0900000	.0013770	.0000002
78	2.1130000	2.1370000	.0013300	.0000002
79	2.1610000	2.1840000	.0012850	.0000002
80	2.2090000	2.2330000	.0012400	.0000002
81	2.2570000	2.2820000	.0011970	.0000002
82	2.3070000	2.3320000	.0011540	.0000002
83	2.3580000	2.3830000	.0011130	.0000002
84	2.4090000	2.4350000	.0010720	.0000002
85	2.4620000	2.4880000	.0010320	.0000002
86	2.5150000	2.5420000	.0009935	.0000002
87	2.5700000	2.5970000	.0009555	.0000002
88	2.6260000	2.6540000	.0009185	.0000002
89	2.6820000	2.7110000	.0008823	.0000002
90	2.7400000	2.7690000	.0008469	.0000002
91	2.7990000	2.8290000	.0008124	.0000002
92	2.8600000	2.8900000	.0007788	.0000002
93	2.9210000	2.9520000	.0007459	.0000002
94	2.9840000	3.0160000	.0007139	.0000002
95	3.0490000	3.0810000	.0006827	.0000002
96	3.1140000	3.1480000	.0006523	.0000002

97	3.1820000	3.2160000	.0006227	.0000002
98	3.2510000	3.2850000	.0005939	.0000002
99	3.3210000	3.3570000	.0005658	.0000002
100	3.3940000	3.4300000	.0005385	.0000002
101	3.4680000	3.5050000	.0005120	.0000002
102	3.5440000	3.5820000	.0004862	.0000002
103	3.6220000	3.6610000	.0004611	.0000002
104	3.7020000	3.7420000	.0004368	.0000002
105	3.7840000	3.8260000	.0004132	.0000002
106	3.8690000	3.9120000	.0003904	.0000002
107	3.9560000	4.0000000	.0003682	.0000002
108	4.0450000	4.0910000	.0003468	.0000002
109	4.1380000	4.1850000	.0003260	.0000002
110	4.2330000	4.2810000	.0003060	.0000002
111	4.3310000	4.3810000	.0002866	.0000002
112	4.4330000	4.4850000	.0002679	.0000002
113	4.5380000	4.5910000	.0002499	.0000002
114	4.6470000	4.7020000	.0002326	.0000002
115	4.7600000	4.8170000	.0002159	.0000002
116	4.8770000	4.9360000	.0001999	.0000002
117	4.9980000	5.0600000	.0001846	.0000002
118	5.1250000	5.1900000	.0001699	.0000002
119	5.2570000	5.3240000	.0001558	.0000002
120	5.3950000	5.4650000	.0001424	.0000002
121	5.5390000	5.6130000	.0001296	.0000002
122	5.6900000	5.7680000	.0001174	.0000002
123	5.8490000	5.9300000	.0001059	.0000002
124	6.0160000	6.1020000	.0000950	.0000002
125	6.1930000	6.2840000	.0000847	.0000002
126	6.3800000	6.4760000	.0000751	.0000002
127	6.5780000	6.6810000	.0000660	.0000002
128	6.7900000	6.9000000	.0022080	.0000002

DISTORTION = .0000629

ENTROPY = 7.0650000

TABLE 12.- OPTIMUM QUANTIZATION LEVEL SPACING FOR NORMAL PDF WITH MAGNITUDE DISTORTION

M	I	X(I)	Y(I)	P(I)	D(I)
2	1	0.0000	0.6745[a]	0.5000[a]	0.2377[a]
	Distortion = 0.4754[a] Entropy = 1.0000[a]				
4	1	.0000	.3371	.2859	.05511
	2	.7921	1.247	.2141	.07733
	Distortion = .2649 Entropy = 1.9850				
8	1	.0000	.1402	.1290	.01063
	2	.3293	.5184	.1514	.01659
	3	.7736	1.0290	.1348	.01942
	4	1.3730	1.7180	.08481	.02536
	Distortion = .1440 Entropy = 2.9700				
16	1	.0000	.0460	.04099	.001062
	2	.1029	.1599	.05257	.001768
	3	.2367	.3136	.06820	.003086
	4	.4173	.5210	.08393	.005097
	5	.6610	.8010	.09321	.007563
	6	.9900	1.1790	.08531	.009181
	7	1.4340	1.6890	.05478	.007765
	8	2.0330	2.3780	.02100	.005289
	Distortion = .08162 Entropy = 3.8900				

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 13.- OPTIMUM QUANTIZATION LEVEL SPACING FOR EXPONENTIAL PDF WITH MAGNITUDE DISTORTION

M	I	X(I)	Y(I)	P(I)	D(I)
2	1	0.0000	0.4901[a]	0.5000[a]	0.2455[a]
	Distortion = 0.4910[a] Entropy = 1.0000				
4	1	.0000	.3083	.3563	.07441
	2	.8816	1.4550	.1437	.07095
	Distortion = .2907 Entropy = 1.8650				
8	1	.0000	.1701	.2159	.02096
	2	.3996	.6292	.1516	.01969
	3	.9390	1.2490	.08519	.01478
	4	1.6670	2.0850	.04732	.02322
	Distortion = .1573 Entropy = 2.8020				
16	1	.0000	.0542	.08229	.002602
	2	.1272	.2001	.08987	.003813
	3	.2984	.3968	.09139	.005201
	4	.5295	.6622	.08433	.006433
	5	.8414	1.0200	.06825	.006970
	6	1.2620	1.5040	.04633	.006326
	7	1.8310	2.1570	.02487	.004538
	8	2.5980	3.0390	.01269	.006124
	Distortion = .08401 Entropy = 3.8150				

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 14.- OPTIMUM QUANTIZATION LEVEL SPACING FOR GAMMA PDF WITH MAGNITUDE DISTORTION

M	I	X(I)	Y(I)	P(I)	D(I)
1	1	0.0000	0.0000	1.0000[a]	0.2887[a]
	Distortion = 0.5774[a] Entropy = .0000[a]				
3	1	.5716	.0000	.5470	.1134
	2		1.1430	.2265	.09965
	Distortion = .3127 Entropy = 1.4470				
7	1	.2586	.0000	.4077	.04032
	2	.8664	.5172	.1414	.02073
	3	2.0990	1.2150	.0820	.02309
	4		2.9820	.07278	.01913
	Distortion = .1662 Entropy = 2.4860				
15	1	.09313	.0000	.2589	.009482
	2	.3120	.1863	.1128	.006029
	3	.6074	.4377	.07862	.005663
	4	1.0060	.7771	.05912	.005734
	5	1.5440	1.2350	.04243	.005528
	6	2.2710	1.8540	.02730	.004770
	7	3.2520	2.6880	.01485	.003475
	8		3.8150	.03540	.005371
	Distortion = .08262 Entropy = 3.4760				

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 15.- OPTIMUM QUANTIZATION LEVEL SPACING FOR NORMAL PDF WITH RELATIVE DISTORTION

M	I	X(I)	Y(I)	P(I)	D(I)
1	1	0.0000	0.0000	1.0000[a]	0.5000[a]
			Distortion = 1.0000[a]		
			Entropy = .0000[a]		
3	1	.3160	.0000	.2480	.2480
	2		.6320	.3760	.1541
			Distortion = .5562		
			Entropy = 1.5600		
7	1	.07174	.0000	.05719	.05719
	2	.3250	.1435	.0988	.03522
	3	.7514	.5064	.1464	.03000
	4		.9964	.2262	.05763
			Distortion = .3029		
			Entropy = 2.6780		
15	1	.02397	.0000	.01912	.01912
	2	.1086	.04793	.03366	.01203
	3	.2510	.1692	.05588	.01141
	4	.4433	.3329	.07213	.01003
	5	.7029	.5538	.08771	.009887
	6	1.0530	.8520	.09495	.009369
	7	1.5260	1.2550	.08263	.007438
	8		1.7980	.06347	.008352
			Distortion = .1562		
			Entropy = 3.8110		
31	1	.00237	.0000	.001891	.001891
	2	.01065	.00474	.003302	.001176
	3	.02455	.01656	.005545	.001128
	4	.04333	.03254	.007487	.001040
	5	.06866	.05411	.01009	.001140
	6	.1028	.0832	.01358	.001349
	7	.1489	.1224	.01823	.001662
	8	.2110	.1753	.02437	.002095
	9	.2947	.2466	.03235	.002666
	10	.4077	.3428	.04235	.003388
	11	.5601	.4726	.05406	.004229
	12	.7659	.6477	.06583	.005060
	13	1.0440	.8841	.07354	.005569
	14	1.4190	1.2030	.07031	.005250
	15	1.9250	1.6340	.05089	.003753
	16		2.2160	.02713	.002761
			Distortion = .08642		
			Entropy = 4.5250		

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 16.- OPTIMUM QUANTIZATION LEVEL SPACING FOR EXPONENTIAL PDF WITH RELATIVE DISTORTION

M	I	X(I)	Y(I)	P(I)	D(I)
1	1	0.0000	0.0000[a]	1.0000[a]	0.5000[a]
			Distortion = 1.0000[a]		
			Entropy = .0000[a]		
3	1	.2160	.0000	.2629	.2632
	2		.4320	.3686	.1754
			Distortion = .6141		
			Entropy = 1.5680		
7	1	.05709	.0000	.07754	.07757
	2	.2586	.1142	.1144	.04050
	3	.7684	.4030	.1782	.04570
	4		1.1340	.1687	.04720
			Distortion = .3444		
			Entropy = 2.7550		
15	1	.0199	.0000	.02775	.02775
	2	.0901	.0398	.04594	.01636
	3	.2086	.1404	.06792	.01397
	4	.3689	.2768	.07551	.01053
	5	.5854	.4610	.07825	.008835
	6	.8775	.7097	.07396	.007317
	7	1.2720	1.0450	.06180	.005597
	8		1.4990	.08274	.01807
			Distortion = .1891		
			Entropy = 3.8640		
31	1	.00264	.0000	.003727	.003727
	2	.01194	.00528	.006507	.002324
	3	.02761	.01860	.01077	.002202
	4	.04879	.03662	.01419	.001976
	5	.07737	.06096	.01849	.002092
	6	.1159	.09379	.02379	.002364
	7	.1680	.1381	.03011	.002744
	8	.2382	.1979	.03727	.003201
	9	.3330	.2785	.04478	.003686
	10	.4609	.3874	.05167	.004123
	11	.6335	.5343	.05644	.004402
	12	.8665	.7327	.05731	.004391
	13	1.1810	1.0000	.05272	.003984
	14	1.6060	1.3620	.04248	.003176
	15	2.1790	1.8500	.02867	.002128
	16		2.5080	.02294	.003388
			Distortion = .09609		
			Entropy = 4.7300		

[a]Value derived by direct computation, not by the Max algorithm.

TABLE 17.- OPTIMUM QUANTIZATION LEVEL SPACING FOR GAMMA PDF WITH RELATIVE DISTORTION

M	I	X(I)	Y(I)	P(I)	D(I)
	1	0.0000	0.0000[a]	1.0000[a]	0.5000[a]
1			Distortion = 1.0000[a]		
			Entropy = .0000[a]		
	1	.1574	.0000	.3292	.3812
	2		.3149	.3354	.1611
3			Distortion = .7033		
			Entropy = 1.5850		
	1	.0220	.0000	.1290	.1483
	2	.1592	.0440	.1229	.0549
	3	.5660	.2744	.1387	.04134
7	4		.8575	.1740	.05744
			Distortion = .4557		
			Entropy = 2.7930		
	1	.0020	.0000	.03916	.04499
	2	.00906	.0040	.02638	.009456
	3	.02692	.01412	.03564	.009333
	4	.0721	.03972	.05262	.01256
	5	.1864	.1045	.07695	.01769
15	6	.4756	.2683	.1029	.02307
	7	1.2070	.6828	.1080	.02342
	8		1.7310	.07798	.01910
			Distortion = .2743		
			Entropy = 3.7620		
	1	.000522	.0000	.02002	.02299
	2	.002640	.001044	.01496	.00579
	3	.008263	.004236	.02066	.005652
	4	.02241	.01229	.03048	.007378
	5	.04648	.03253	.03360	.006070
	6	.07929	.06044	.03283	.004304
	7	.1236	.09815	.03368	.003666
	8	.1834	.1491	.03529	.003416
31	9	.2642	.2178	.03711	.003320
	10	.3732	.3105	.03867	.003275
	11	.5203	.4358	.03948	.003216
	12	.7189	.6048	.03899	.003087
	13	.9870	.8330	.03671	.002846
	14	1.3490	1.1410	.03233	.002466
	15	1.8370	1.5570	.02598	.001958
	16		2.1180	.03920	.007932
			Distortion = .1517		
			Entropy = 4.9140		

[a]Value derived by direct computation, not by the Max algorithm.

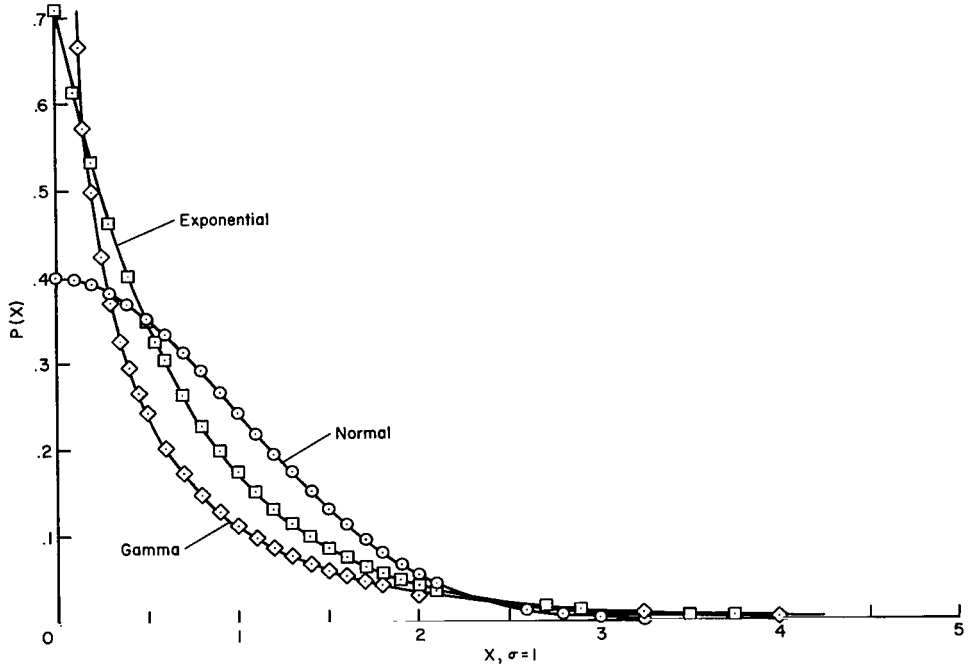

Figure 1.- Normal, exponential, and gamma probability distributions.

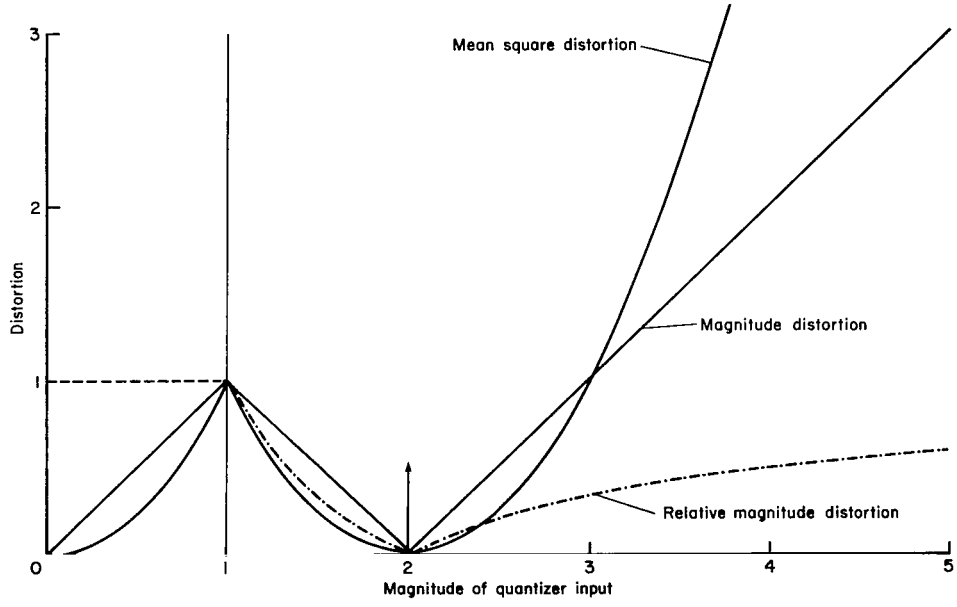

Figure 2.- The mean square, magnitude, and relative magnitude distortion measures.

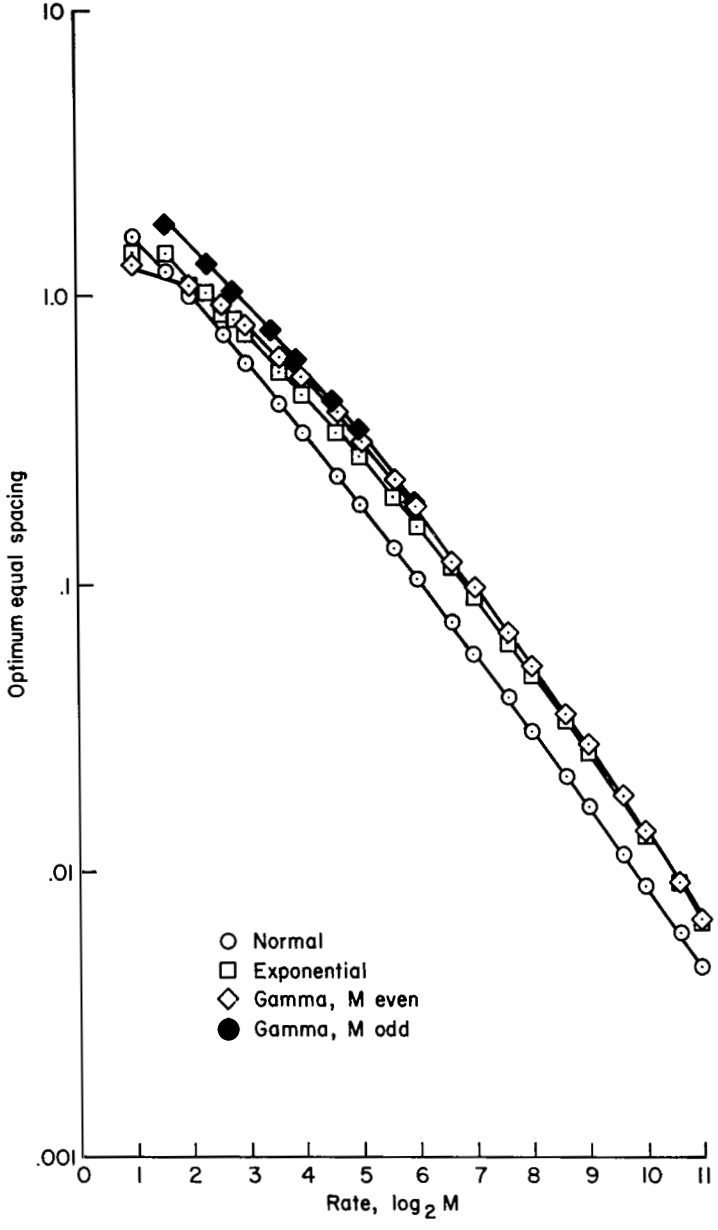

Figure 3.- Optimum equal spacing for the mean-square error quantizers.

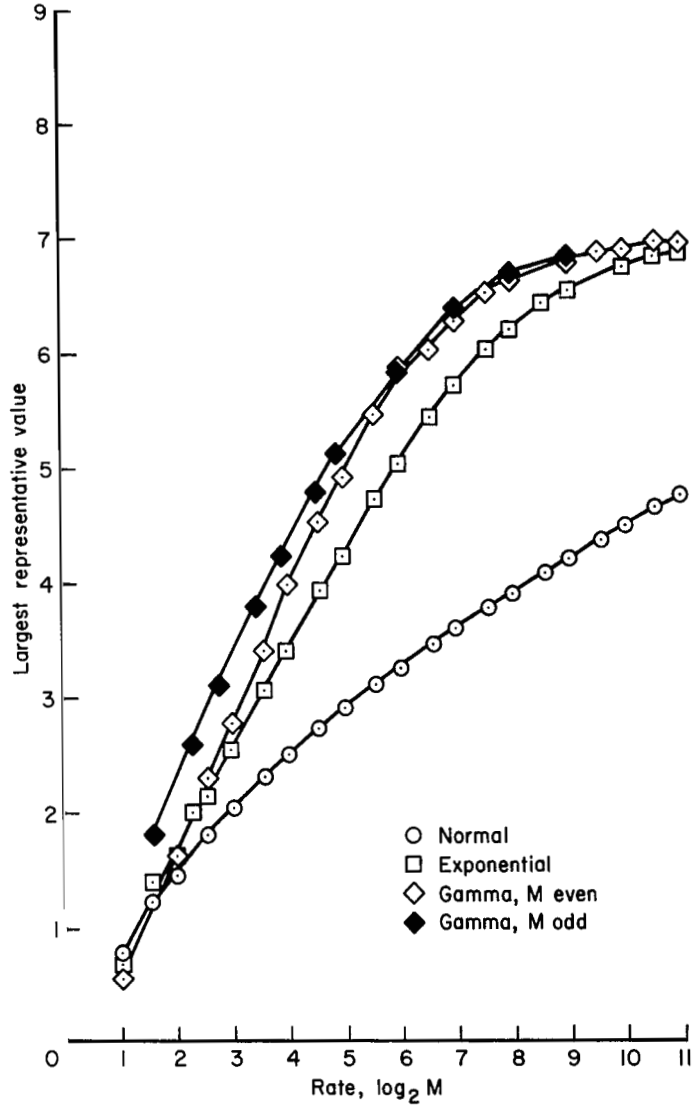

Figure 4.- Largest representative values for the equal spacing, minimum mean-square error quantizers.

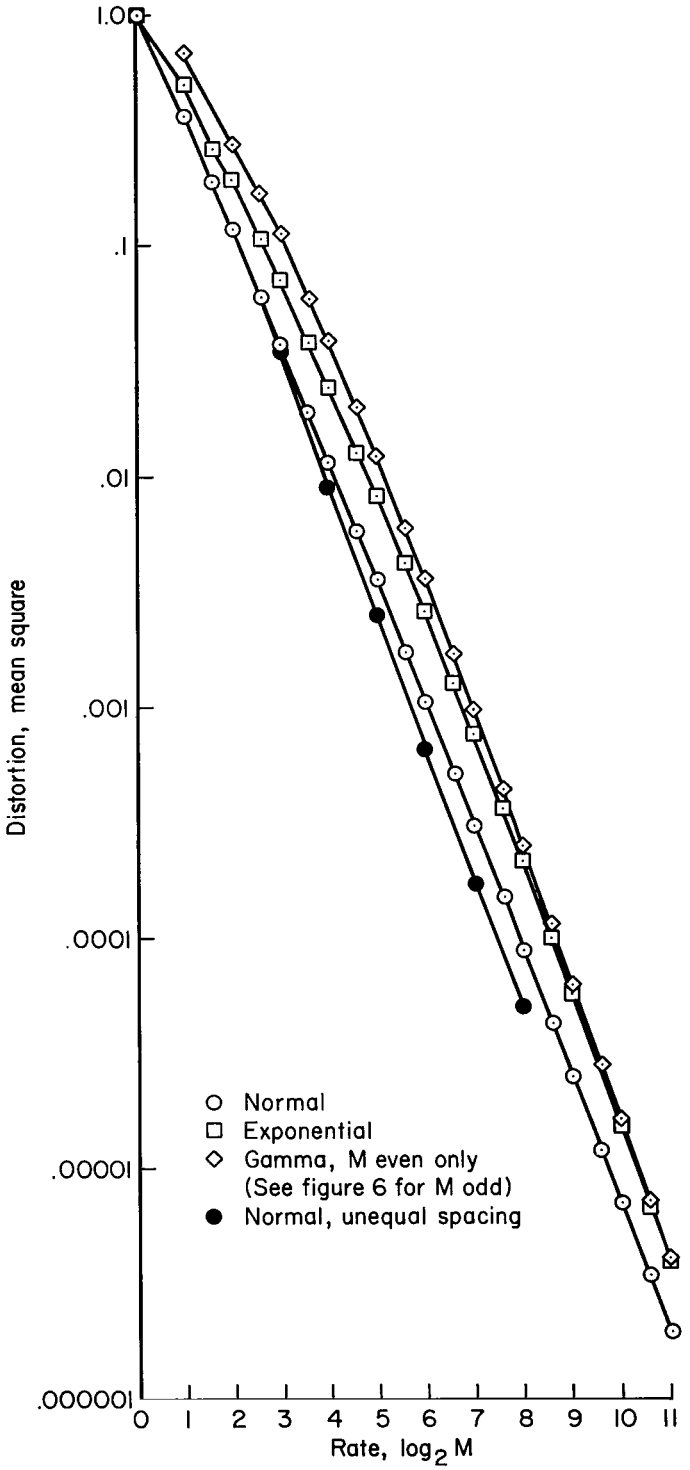

Figure 5.- Distortion for the equal spacing, minimum mean-square error quantizers.

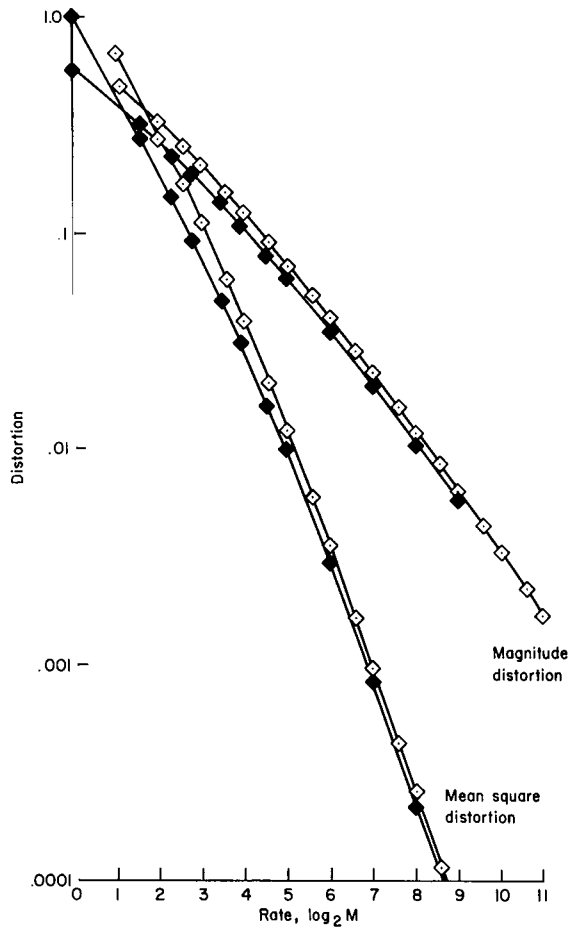

Figure 6.- Mean square and magnitude distortions for the gamma input equal-spacing quantizers.

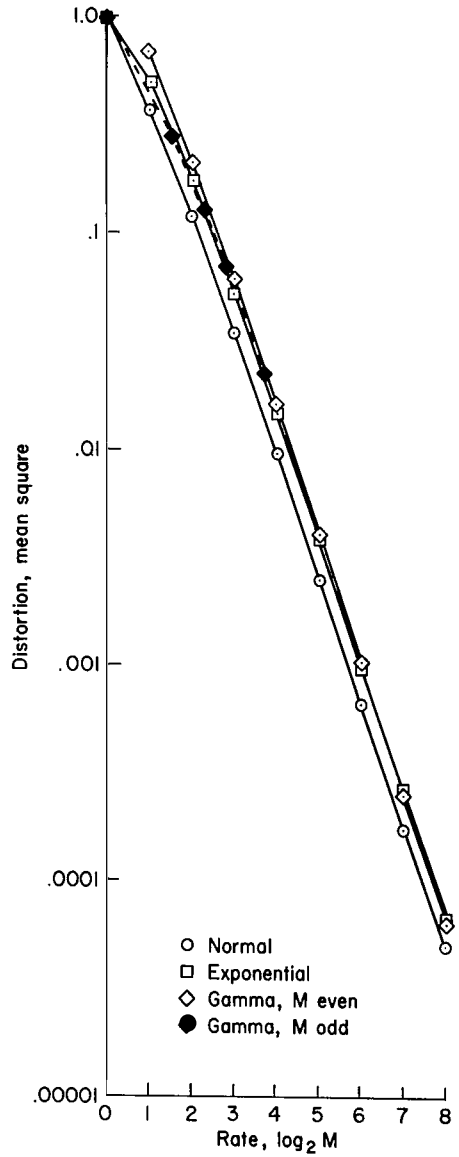

Figure 7.- Distortion for the unequal spacing, minimum mean-square error quantizers.

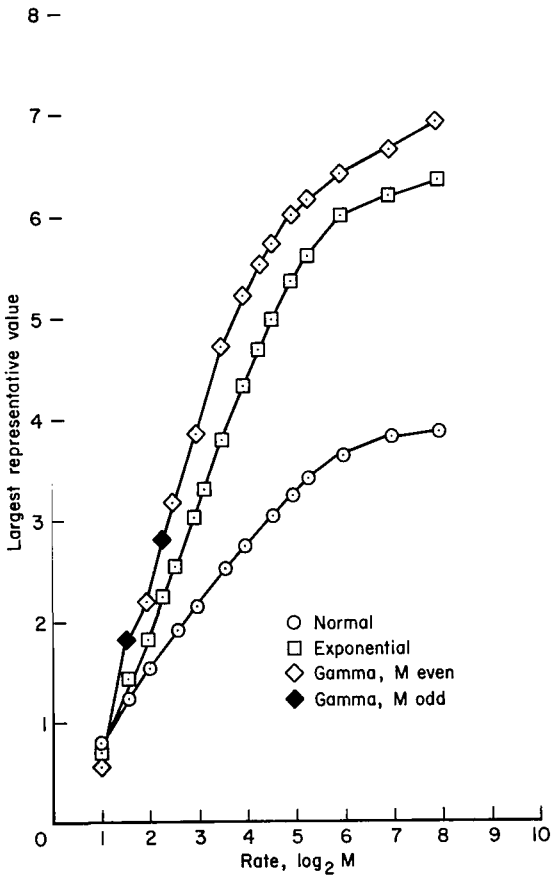

Figure 8.- Largest representative values for the unequal-spacing, minimum mean square error quantizers.

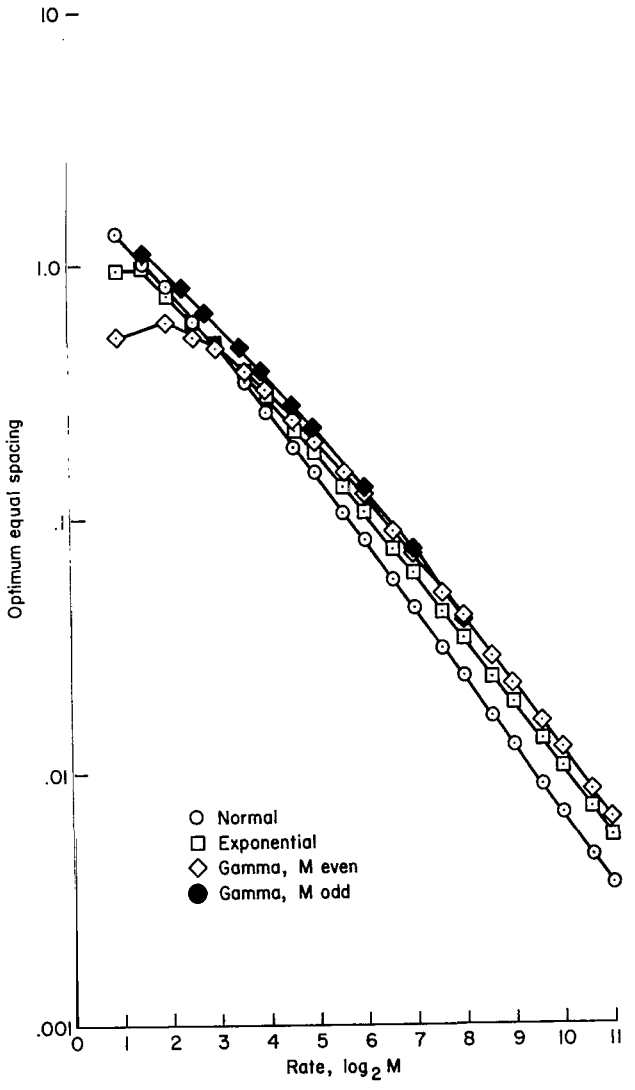

Figure 9.- Optimum equal-spacing for the magnitude error quantizers.

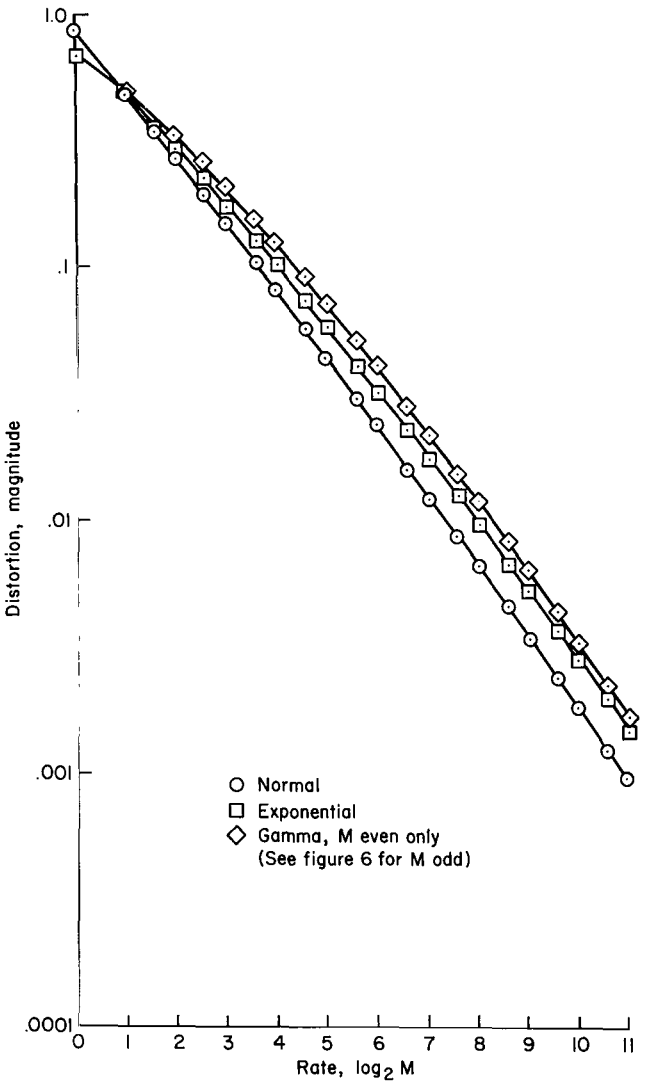

Figure 10.- Distortion for the equal spacing, minimum magnitude error quantizers.

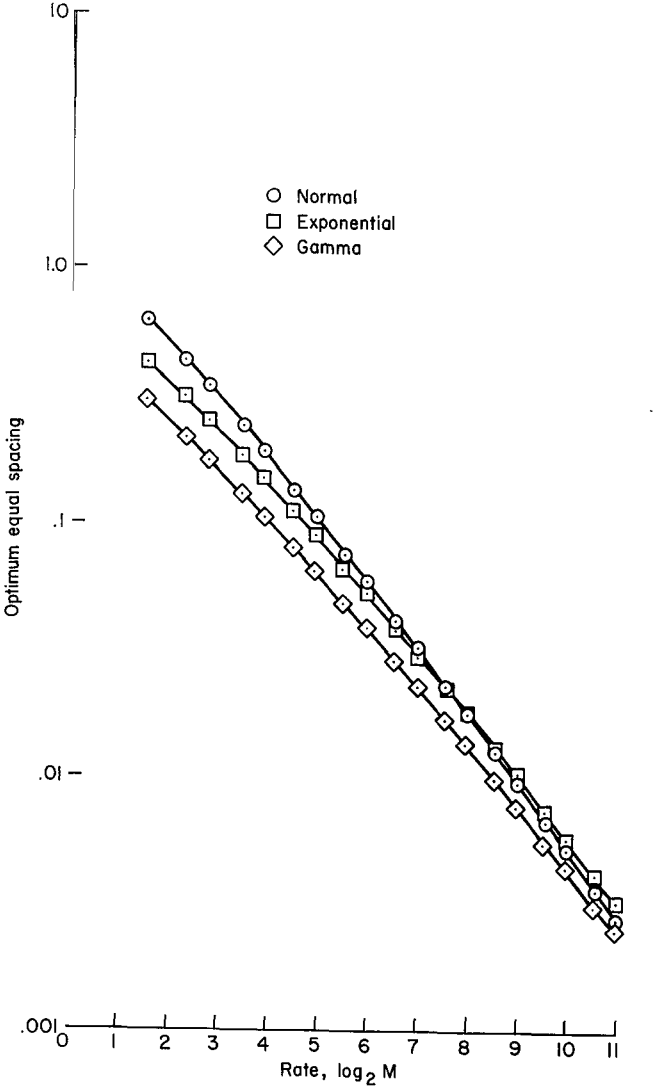

Figure 11.- Optimum equal spacing for the minimum relative error quantizers for M odd only.

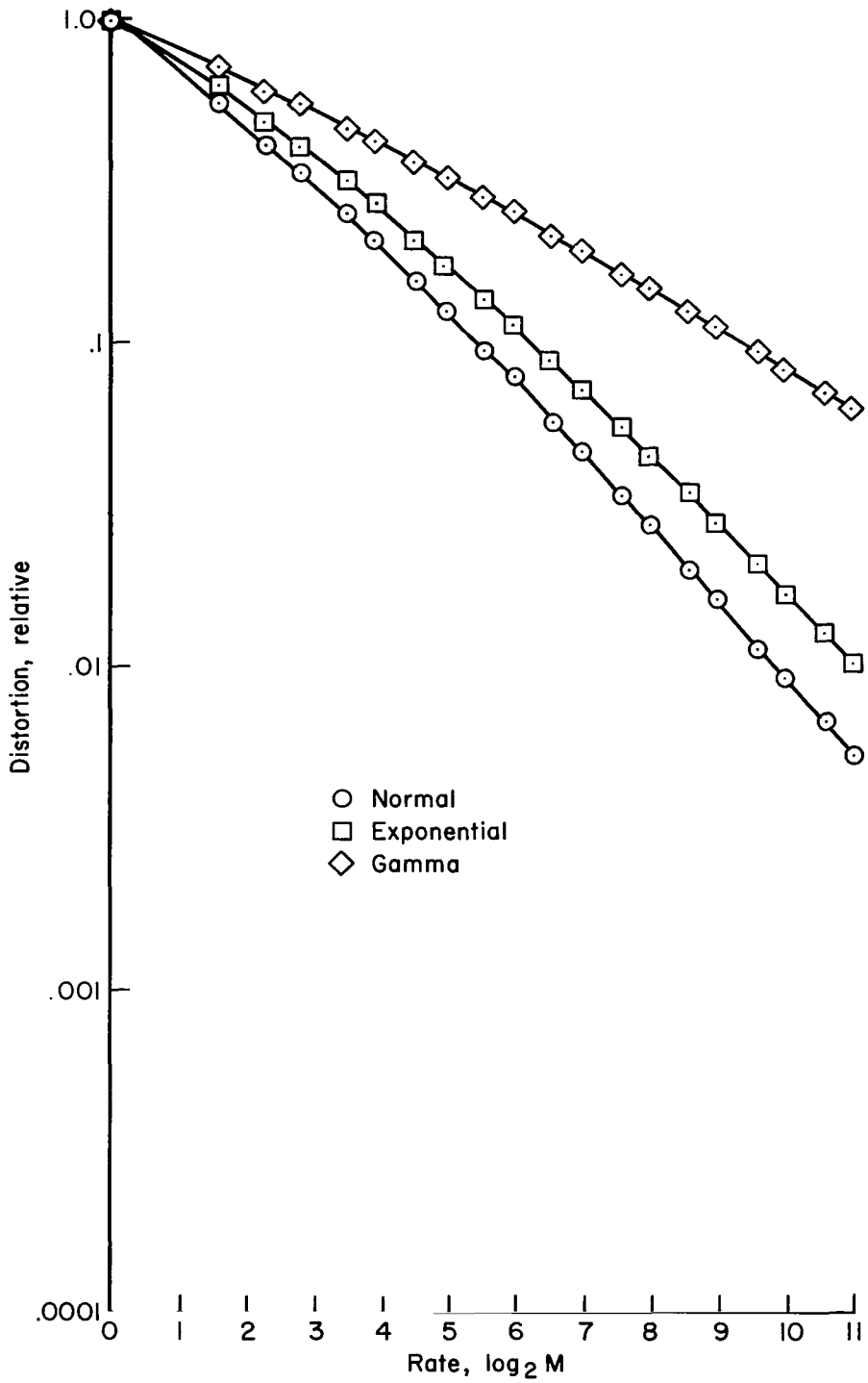

Figure 12.- Distortion for the equal spacing, minimum relative error quantizers for M odd only.

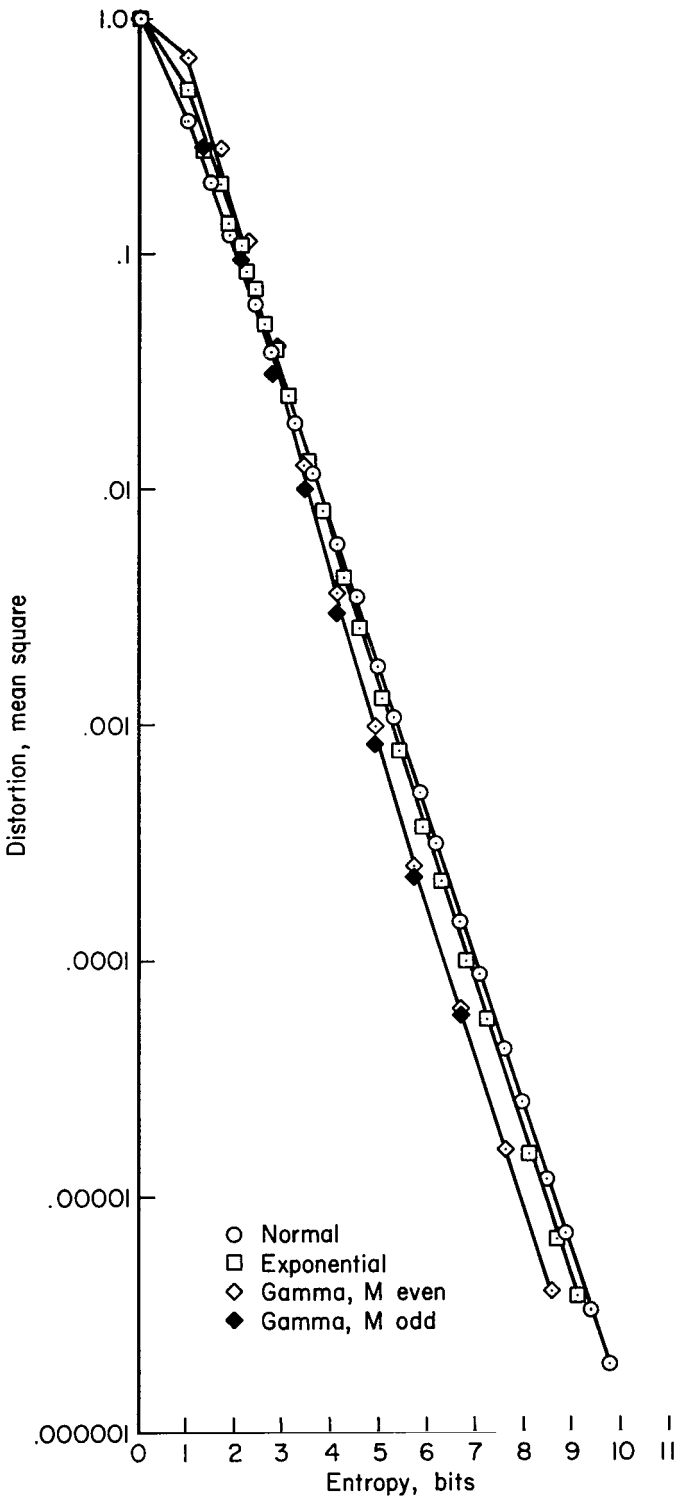

Figure 13.- Mean-square distortion vs. entropy for equal-spacing quantizers.

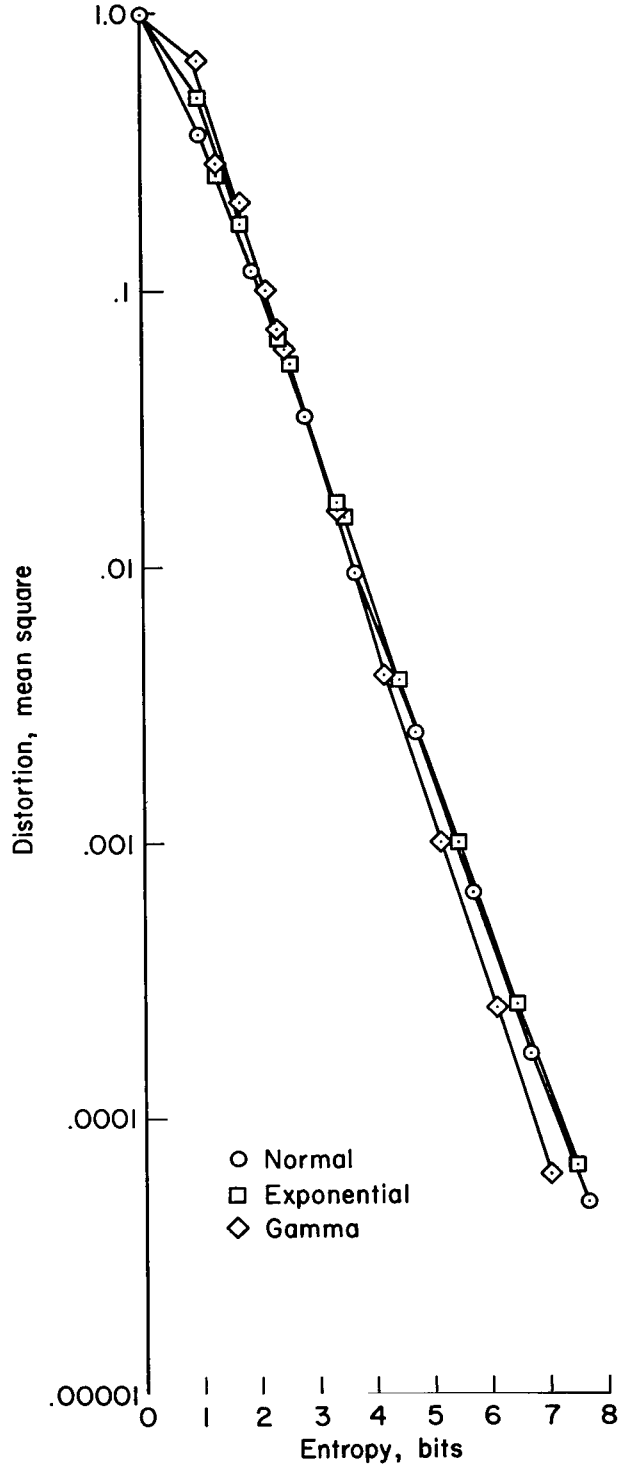

Figure 14.- Mean-square distortion vs. entropy for unequal-spacing quantizers.

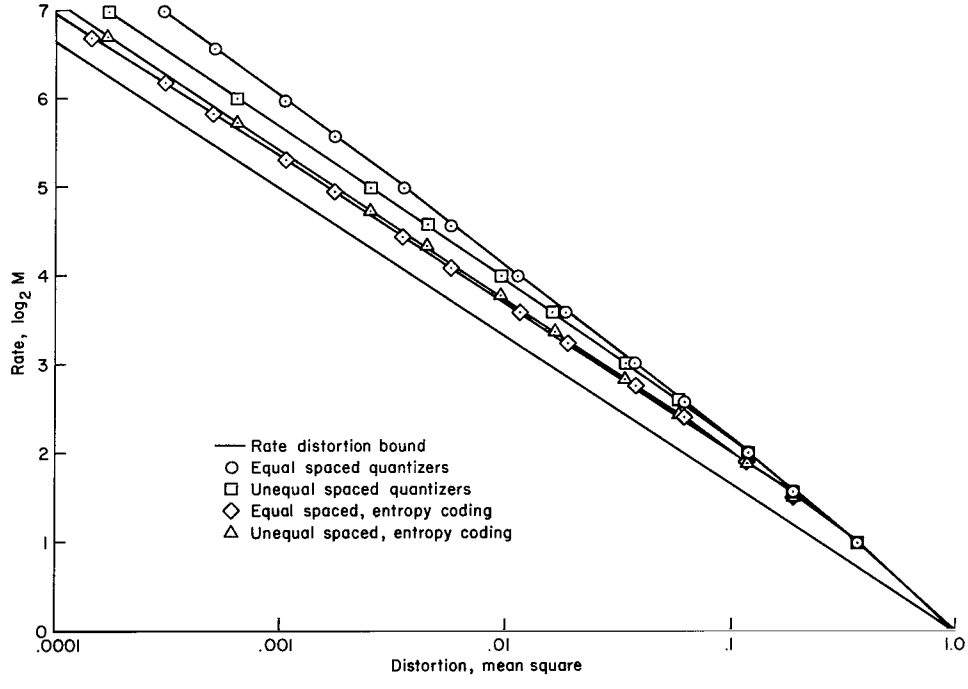

Figure 15.- Rate-distortion curves for normal input and mean-square distortion.

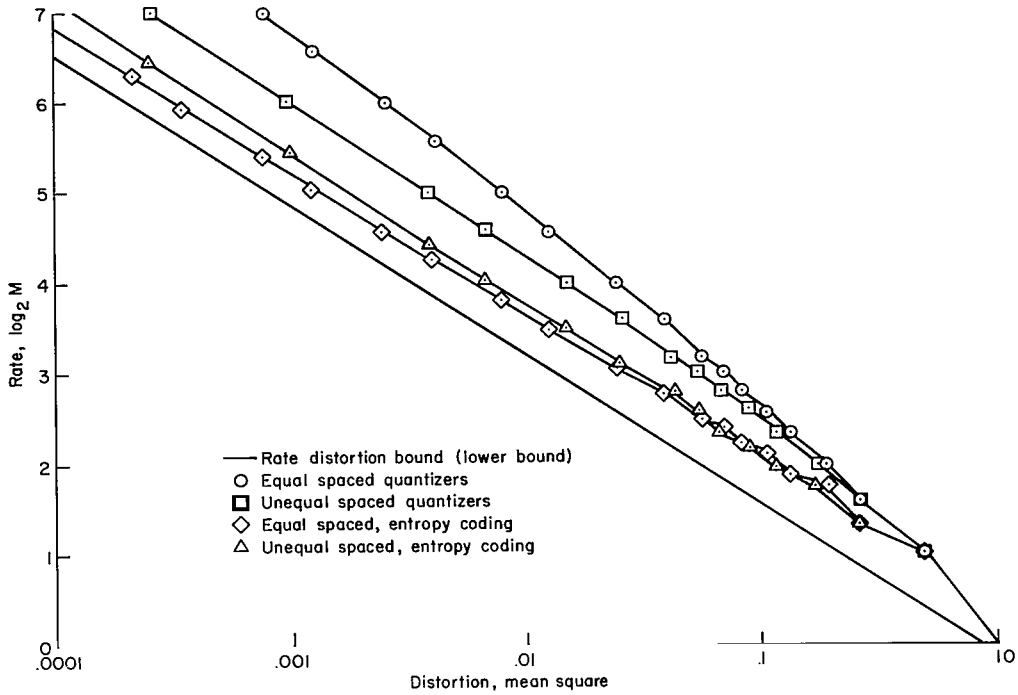

Figure 16.- Rate-distortion curves for exponential input and mean-square distortion.

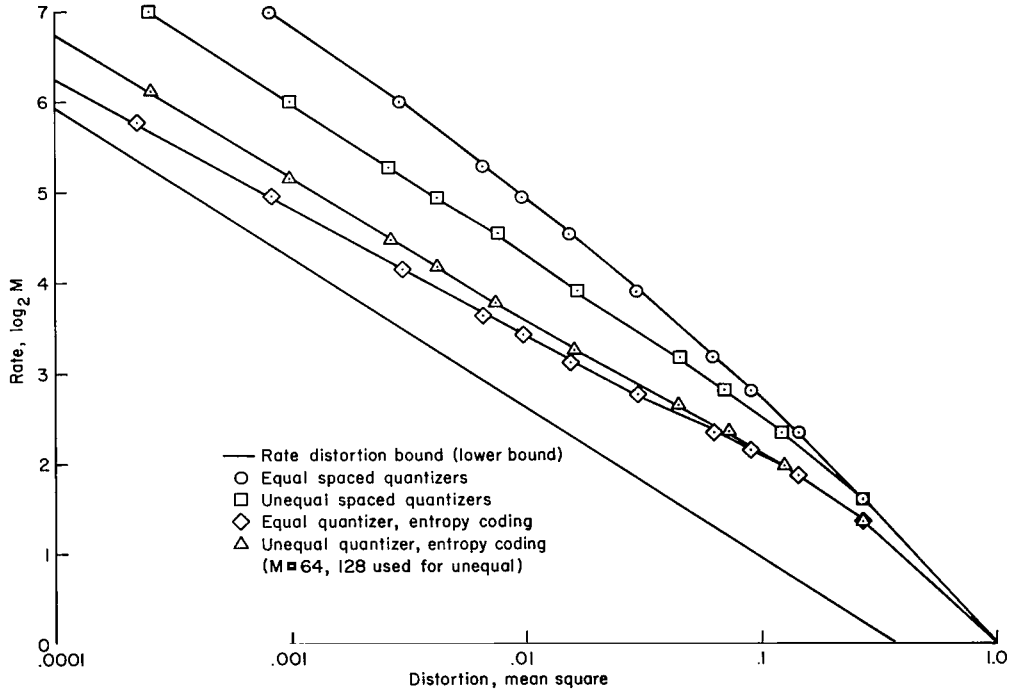

Figure 17.- Rate-distortion curves for gamma input and mean-square distortion, M odd.

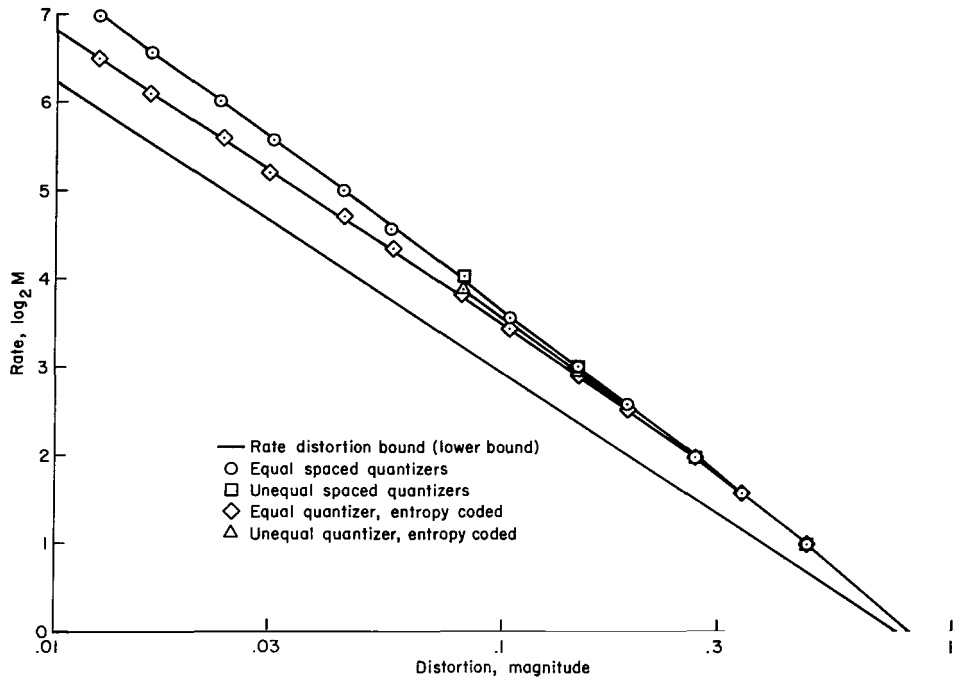

Figure 18.- Rate-distortion curves for normal input and magnitude distortion.

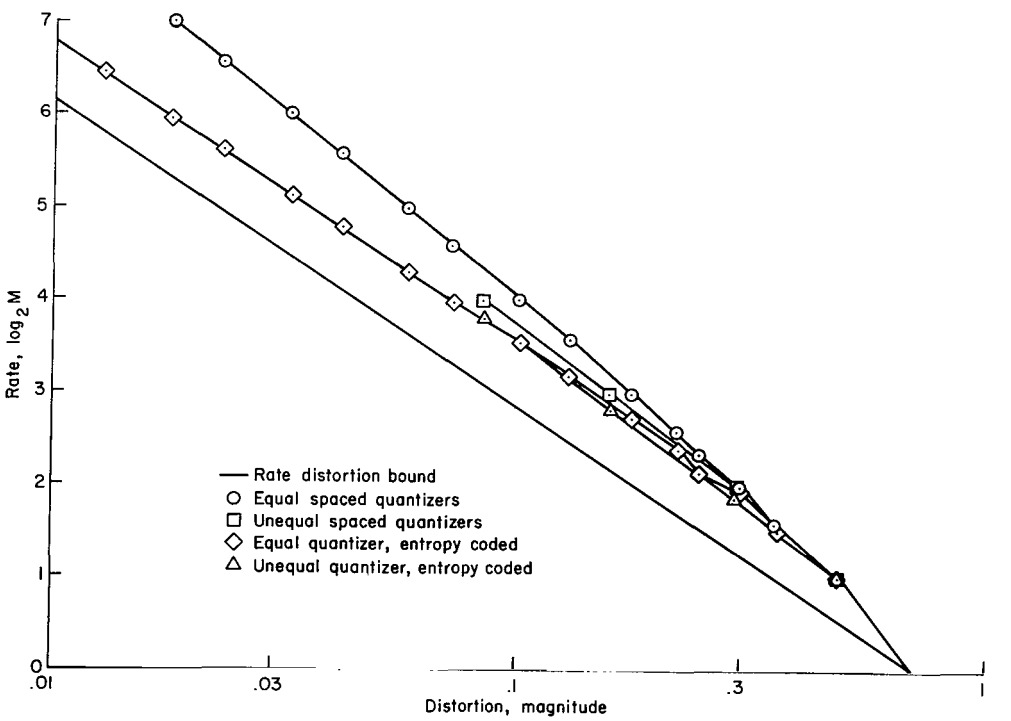

Figure 19.- Rate-distortion curves for exponential input and magnitude distortion.

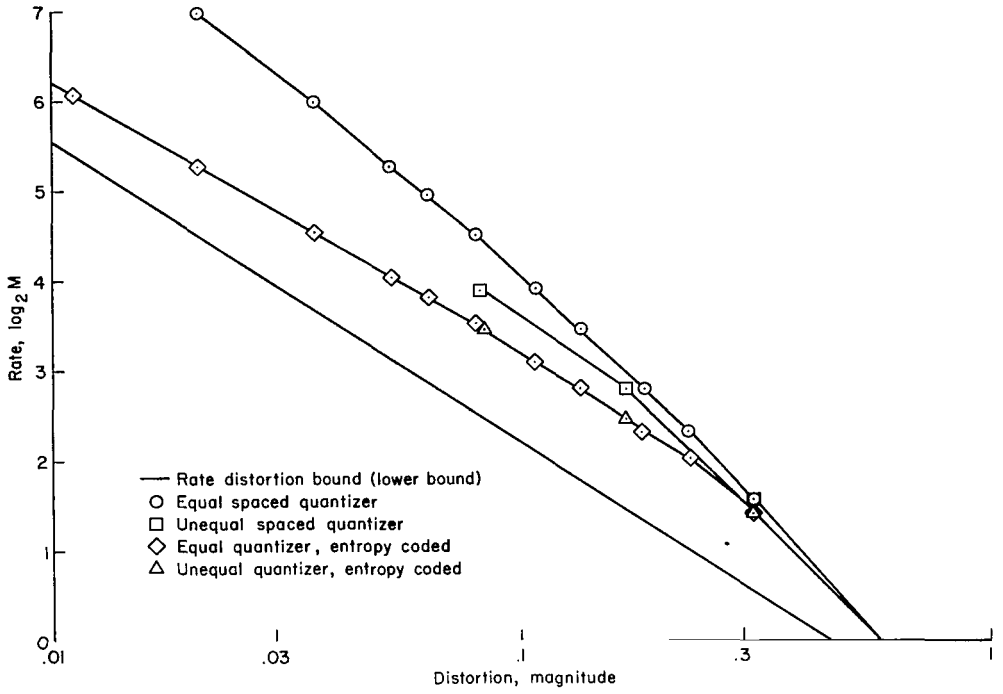

Figure 20.- Rate-distortion curves for gamma input and magnitude distortion, M odd.

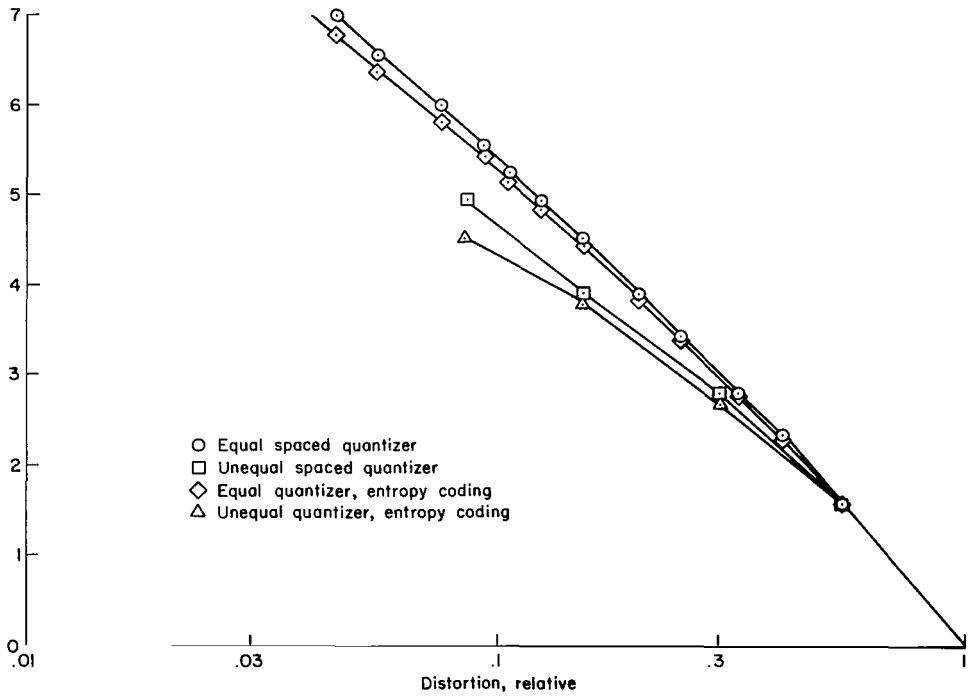

Figure 21.- Rate-distortion curves for normal input and relative distortion, M odd.

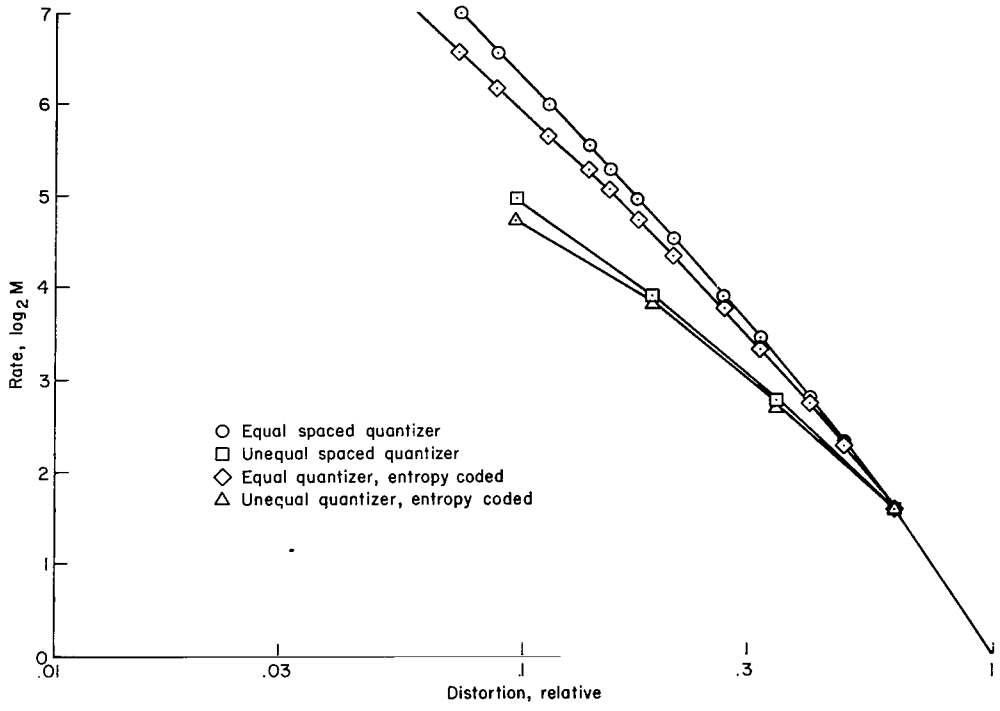

Figure 22.- Rate-distortion curves for exponential input and relative distortion, M odd.

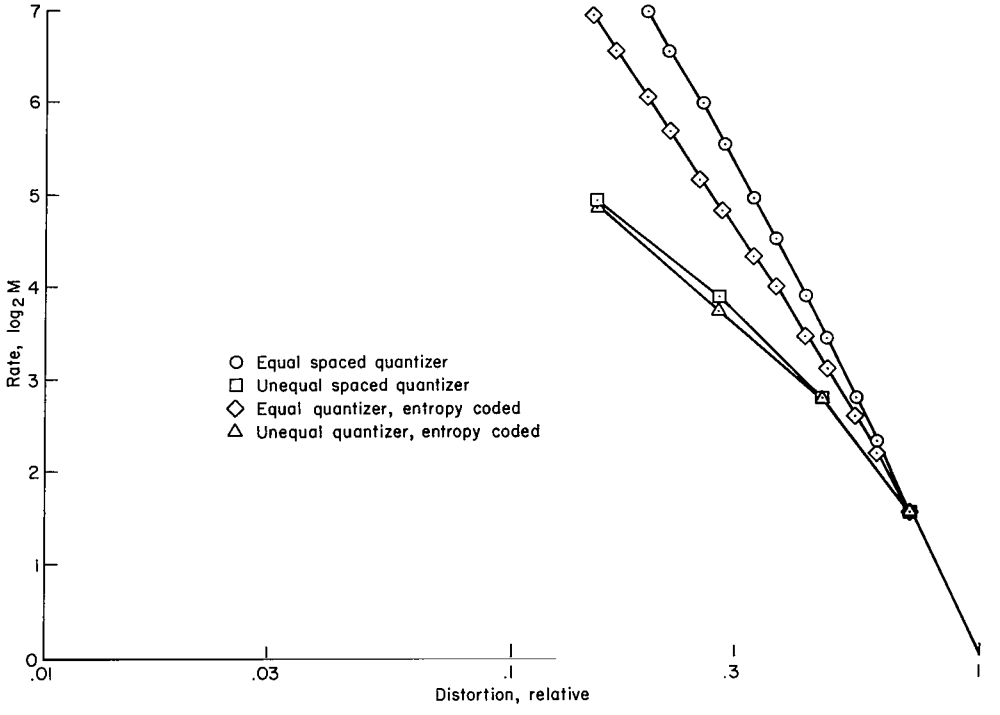

Figure 23.- Distortion curves for gamma input and relative distortion, M odd.

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