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DESAP 2 - A STRUCTURAL DESIGN PROGRAM WITH STRESS AND BUCKLING CONSTRAINTS

Volume II: Sample Problems

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16. ABSTRACT

DESAP 2 is a finite element program for computer-automated, minimum weight design of elastic structures with constraints on stresses (including local instability criteria) and buckling loads. No limits are placed on the number of load conditions for stress-constrained design, but only one of these load conditions can be chosen as the potential buckling load. A substantial portion of DESAP 2, particularly the analysis of the prebuckling state, is derived from the SOLID SAP finite element program developed at the University of California, Berkeley. The stress-constrained design is based on the classical stress ratio method, which drives the design toward a fully stressed state. The constraints on the buckling load are handled by solving the appropriate optimality criterion by successive iterations. During each iteration, the element sizes determined by the stress ratio method are used as the minimum size constraints. The element subroutines have been organized in a manner that permits the user to make additions and changes with a minimal programming effort. Consequently, DESAP 2 can readily be changed into a special-purpose program to handle the user's specific design requirements and failure criteria.

DESAP 2 is a companion program of DESAP 1: "A Structural Design Program with Stress and Displacement Constraints." With the exception of a few cards the same input data deck can be used for both programs.

This is Volume II of three volumes.

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VOLUME 2

K. PREAMBLE

This collection of sample problems is a supplement to Volume 1 of DESAP 2: "Theoretical and User's Manual". In making up this volume, our aim was to find examples that would best serve the following functions:

1) Illustrate and supplement the input instructions of Volume 1, and to familiarize the user with the output.

2) Explain, with examples, special problem areas and peculiarities that may arise in the use of the program.

3) Provide example problems that may be used for debugging the program during installation on a new computer system.

4) Compare the results of DESAP 2 against solutions obtained by other means, whenever possible.

Although DESAP 2 is designed primarily for the use of large structures, the stated purpose of the sample problems is clearly best fulfilled by small, simple examples that do not necessarily represent realistic design situations. Consequently, the problems appearing in this volume should be viewed strictly as tools of instruction, which in no way reflect the ultimate capabilities of the program.

Because our experience with the program is rather limited at this time, the example problems may well have overlooked some troublesome aspects of design, or even deficiencies in the program itself. The extensive computer output from each design cycle is, however, a powerful diagnostic tool that should enable the user to pinpoint the difficulty and make the appropriate correction.

An example problem is given for each element type presently used in the program. Each problem contains a complete description of the input data, including an echo of the input cards, and the computer printout of the input information. In order to reduce the bulk of the report, only a partial listing of the computer output is duplicated, containing the initial and the final designs. The complete history of a design is usually summarized by tabulating the design variables.

In compiling the sample problems, we were seriously handicapped by a lack of adequately documented optimal design problems in existing literature. For this reason, a one-to-one comparison of the results of DESAP 2 with independently obtained solutions is lacking in some of the problems.

As a final note, we would like to remind the user again that DESAP 2 is oriented towards large problems. Mainly due to an extensive use of auxiliary storage devices and other core-saving features, the program is not efficient for small structures as used for the sample problems. Consequently, the computer times for these problems are not expected to be competitive with runs obtained from programs especially designed for structures of limited size.

L. BAR ELEMENTS

L.1 Three-Panel Truss

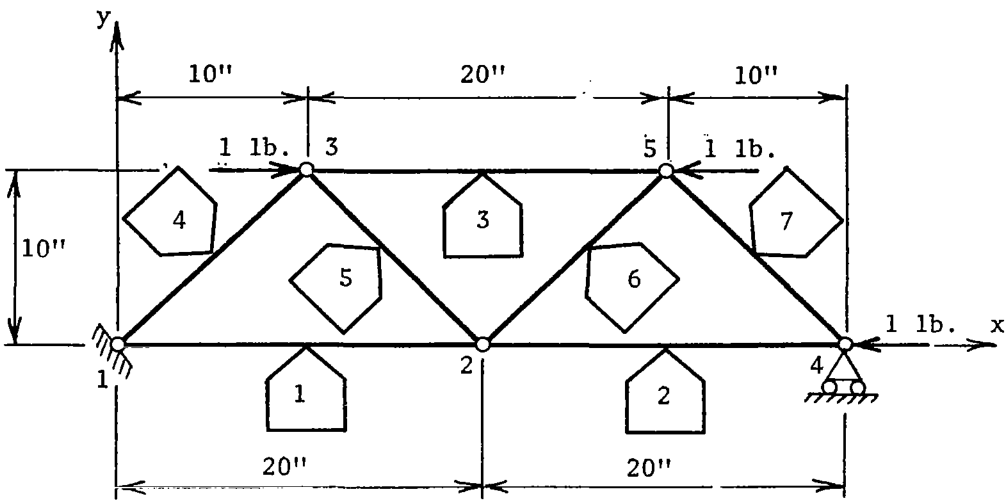


Figure L.1.1

Layout of Truss Showing Element and Node Numbers.

The truss in Fig. L.1.1 is to be optimized for the single load condition shown. The cross-sectional area of each bar is taken as an independent design variable. The data used in the design is:

E = 10 psi (Young's modulus),

$\sigma_t^* = \sigma_c^*$ = 25 psi (allowable normal stress),

A^* = 0.1 sq. in. for all elements (min. allowable cross-sectional areas),

A = 1.0 sq. in. for all elements (initial cross-sectional areas),

$p^* = 1.0$ (lower bound on the critical load parameter, i.e. the factor of safety against buckling).

Local buckling of the elements is not to be included as a design criterion. Because the two construction codes of the bar element differ only in redesign with respect to local buckling, either of the codes may be used (we chose Construction Code No. 1).

The complete design history of the problem is given on the Computer Printout sheets. The optimal design was reached in two redesigns; it is governed entirely by the buckling constraint. Because stress constraints are inactive and the prebuckling state is statically determinate, the final design is a true global optimal design.

The buckling load of the initial design (see Analysis of Design No. 0) of 2.2654 agress exactly with the analytical solution of Ref.[13], p. 148. Unfortunately, the cross-sectional areas of optimal design cannot be checked due to lack of an independently obtained solution.

<u>Special notes on input-output</u>:

1) Since the stiffness matrix of each element of the structure has the form $[K_i] = [k_i]A_i^n$, $n = 1$, uniform scaling is an exact operation. Consequently, KSCALE = 1 (=n) was specified in the Design Control Data.

2) Automatic generation of boundary condition codes was employed to suppress the z-displacement and the rotations at all the nodes (see Nodal Point Input Data and Generated Nodal Data).

3) Local buckling of elements was eliminated as a design consideration by leaving the moments of inertia blank on the geometric property card. The blanks are replaced by the computer with 10^6 in^4

(see Geometric Property Cards), so that the Euler buckling load of each element is too high to play a role in the redesign.

4) MODEIN = 1 in the Buckling Control Data means that the initial mode shape (coordinate vector) is read in with data cards.

5) NMODE = 1 in the Buckling Control Data specifies that only one buckling mode is to be considered in the design.

6) The parameter INDET = 1 in the Buckling Control Data signifies that the structure is statically determinate. Therefore, the pre-buckling stresses and the geometric stiffness matrix of the structure have to be calculated only once in the whole redesign process.

7) A "normal" value of the relaxation parameter $\alpha = n/(n+1) = 0.5$ (note that $n = 1$) was employed in the buckling-constrained redesign (see ALPA in Buckling Control Data).

8) The program was terminated when the Optimality Index of each element became sufficiently close to one (see Evaluation of Design No. 2).

```
123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
  THREEE PANEL TRUSS BUCKLING ANALYSIS
    5    1    1    7
   10         1 0.025                       1    1
    1    1    1   -1   -1   -1   -1
    2                                  20.
    3                                  10.0      10.
    4         1                        40.
    5              1    1    1    1 30.        10.
    1    7    1    1    1    1
    1    1 0.1
           10.                    25.
    1      1.
    1    1    2    1    1    1
    2    2    4    1    1    2
    3    3    5    1    1    3
    4    1    3    1    1    4
    5    2    3    1    1    5
    6    2    5    1    1    6
    7    4    5    1    1    7
 1.0          1    1    1    1  0.5       0.8
    3    1    1.
    4    1  -1.
    5    1  -1.
0.5        1.0       -1.0      1.0       -1.0      1.0       1.0
    3 1.0        0.1
    7   2.0      0.1
123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
```

Echo of Input Cards

THREEE PANEL TRUSS BUCKLING ANALYSIS

NUMBER OF NODAL POINTS = 5
NUMBER OF ELEMENT TYPES = 1
NUMBER OF LOAD CASES = 1
NUMBER OF DES. VARIABLES = 7

DESIGN CONTROL DATA

NCYCL = 10
KSCALE= 1
DELTA = 0.2500E-01
EPSIL = 0.1000E 00
LBUCK = 1

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z		T
1	1	1	-1	-1	-1	-1	0.000	0.000	0.000	0	0.000
2	0	0	0	0	0	0	20.000	0.000	0.000	0	0.000
3	0	0	0	0	0	0	10.000	10.000	0.000	0	0.000
4	0	1	0	0	0	0	40.000	0.000	0.000	0	0.000
5	0	0	1	1	1	1	30.000	10.000	0.000	0	0.000

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z	T
1	1	1	-1	-1	-1	-1	0.000	0.000	0.000	0.000
2	0	0	-1	-1	-1	-1	20.000	0.000	0.000	0.000
3	0	0	-1	-1	-1	-1	10.000	10.000	0.000	0.000
4	0	1	-1	-1	-1	-1	40.000	0.000	0.000	0.000
5	0	0	1	1	1	1	30.000	10.000	0.000	0.000

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	0	0	0	0	0	0
2	1	2	0	0	0	0
3	3	4	0	0	0	0
4	5	0	0	0	0	0
5	6	7	0	0	0	0

Computer Printout

NUMBER OF TRUSS ELEMENTS = 7
CONSTRUCTION CODE = 1
NUMBER OF MATERIALS = 1
NUMBER OF TEMPS FOR WHICH MATL PROPS GIVEN= 1
NUMBER OF DIFFERENT GEOMETRIES PROPS GIVEN= 1

MATERIAL PROPERTY CARDS

MATERIAL NUMBER	NUMBER OF TEMPS	SPECIFIC WEIGHT	TEMP	YOUNGS MODULUS	COEFFT OF THERM EXPAN	/--ALLOWABLE STRESSES--/ TENSION	COMPRESSION
1	1	0.1000E 00	0.0000E 00	0.1000E 02	0.0000E 00	0.2500E 02	0.2500E 02

GEOMETRIC PROPERTY CARDS

GEOMETRY NUMBER	X-SECT AREA	/--MOMENTS OF INERTIA--/ YY	ZZ
1	0.1000D 01	0.1000E 07	0.1000E 07

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
Y-DIR	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
Z-DIR	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
TEMP	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00

PROCESSED ELEMENT DATA

ELEMENT NUMBER	/-NODE NOS-/ I	J	/--ELEMENT ID NOS-/ MATL	GEOMY	D VAR	DESIGN VAR FRACTION	REFERENCE TEMP	END FIXITY COEFFICIENTS YY	ZZ	BAND WIDTH
1	1	2	1	1	1	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	2
2	2	4	1	1	2	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	5
3	3	5	1	1	3	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	5
4	1	3	1	1	4	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	2
5	2	3	1	1	5	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	4
6	2	5	1	1	6	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	7
7	4	5	1	1	7	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3

STRUCTURE LOAD CASE	STRUCTURE LOAD MULTIPLIERS A	B	C	D
1	0.000	0.000	0.000	0.000

BUCKLING CONTROL DATA

COEFFT = 1.00000
MODEIN = 1
NMODE = 1
INDET = 1
NVEC = 1
ALPA = 0.50000
OMEGA = 0.80000

NODAL POINT LOADS

NODE NO.	LOAD CASE	APPLIED LOADS RX	RY	RZ	MX	MY	MZ
3	1	0.100D 01	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
4	1	-0.100D 01	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
5	1	-0.100D 01	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

DESIGN VARIABLE INPUT DATA

DESIGN VARIABLE NUMBER	INITIAL VALUE	MIN ALLOWABLE VALUE
1	0.1000E 01	0.1000E 00
2	0.1000E 01	0.1000E 00
3	0.1000E 01	0.1000E 00
4	0.2000E 01	0.1000E 00
5	0.2000E 01	0.1000E 00
6	0.2000E 01	0.1000E 00
7	0.2000E 01	0.1000E 00

TOTAL NUMBER OF EQUATIONS = 7
BANDWIDTH = 7
NUMBER OF EQUATIONS IN A BLOCK = 7
NUMBER OF BLOCKS = 1

****

ANALYSIS OF DESIGN NUMBER 0

****

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
5	1	-3.000E 00	1.000E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
4	1	-4.000E 00	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	1	-1.000E 00	1.000E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	1	-2.000E 00	4.330E-15	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
1	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.1000E 01	0.1000E 01	0.1000E 01	0.2000E 01	0.2000E 01	0.2000E 01	0.2000E 01			

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FORCE
1	0.1000E 01	1	-0.1000E 01
2	0.1000E 01	1	-0.1000E 01
3	0.1000E 01	1	-0.1000E 01
4	0.2000E 01	1	0.5960E-07
5	0.2000E 01	1	0.9537E-06
6	0.2000E 01	1	-0.9537E-06
7	0.2000E 01	1	0.9537E-06

BUCKLING LOAD PARAMETERS

0.22654D 01

BUCKLING MODE SHAPES

NODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
5	1	5.895E-08	-1.939E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
4	1	1.433E 00	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	1	1.433E 00	-1.939E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	1	7.164E-01	-3.162E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01

```
   1    1   0.000E-01   0.000E-01   0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01


********************************
EVALUATION OF DESIGN NUMBER   0
********************************


          STRESS RATIO   LOAD COND   DES VARIABLE
MAX        0.1000E 00        0            1
MIN        0.5000E-01        0            4

       MAX BUCK RATIOS   LOAD COND

         0.4414E 00         1


UNIFORM SCALING OPERATION FOLLOWS


SCALE FACTOR IS  0.441AND DETERMINED BY BUCKLING     CONSTRAINTS


DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE


VALUES OF DESIGN VARIABLES

           1            2            3            4            5            6            7            8            9            10

    0  0.4414E 00  0.4414E 00  0.4414E 00  0.8828E 00  0.8828E 00  0.8828E 00  0.8828E 00


STRUCTURAL WEIGHT= 0.7643E 01


REDESIGN OPERATION FOLLOWS


OPTIMALITY INDEX OF DESIGN VARIABLES FOR BUCKLING CONSTRAINTS

    DV NO ACT/PAS    INDEX

        1      ACT     0.45474E 00
        2      ACT     0.45474E 00
        3      ACT     0.18190E 01
        4      ACT     0.22737E 00
        5      ACT     0.22737E 00
        6      ACT     0.22737E 00
        7      ACT     0.22737E 00

NO. OF ACTIVE BUCKLING CONSTRAINTS     ARE    1
```

```
********************************
ANALYSIS OF DESIGN NUMBER   1
********************************

NODAL DISPLACEMENTS AND ROTATIONS

NODE LOAD           X            Y            Z           XX           YY           ZZ
 NO. CASE

   5    1  -7.836E 00    4.622E 00    0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01

   4    1  -1.246E 01    0.000E-01    0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01

   3    1  -4.622E 00    4.622E 00    0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01

   2    1  -6.229E 00    3.015E 00    0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01

   1    1   0.000E-01    0.000E-01    0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01


VALUES OF DESIGN VARIABLES

           1           2           3           4           5           6           7           8           9          10

    0  0.3211E 00  0.3211E 00  0.6222E 00  0.5418E 00  0.5418E 00  0.5418E 00  0.5418E 00

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT  X-SECT AREA  LOAD COND   AXIAL FORCE

     1    0.3211E 00       1      -0.1000E 01
     2    0.3211E 00       1      -0.1000E 01
     3    0.6222E 00       1      -0.1000E 01
     4    0.5418E 00       1       0.2347E-06
     5    0.5418E 00       1       0.1729E-05
     6    0.5418E 00       1      -0.2603E-06
     7    0.5418E 00       1       0.9689E-06

BUCKLING LOAD PARAMETERS

         0.86583D 00

BUCKLING MODE SHAPES

NODE MODE           X            Y            Z           XX           YY           ZZ
 NO.SHAPE

   5    1  -4.127E-01    2.008E 00    0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01

   4    1  -1.706E 00    0.000E-01    0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01

   3    1  -1.293E 00    2.008E 00    0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01

   2    1  -8.528E-01    3.162E 00    0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01
```

```
   1    1   0.000E-01   0.000E-01   0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01


********************************
EVALUATION OF DESIGN NUMBER   1
********************************


          STRESS RATIO   LOAD COND   DES VARIABLE
MAX        0.3115E 00        0             2
MIN        0.1607E 00        0             3

       MAX BUCK RATIOS   LOAD COND

         0.1155E 01         1


UNIFORM SCALING OPERATION FOLLOWS


SCALE FACTOR IS  1.155AND DETERMINED BY BUCKLING     CONSTRAINTS


DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE


VALUES OF DESIGN VARIABLES

           1            2            3            4            5            6            7            8            9           10

    0  0.3708E 00  0.3708E 00  0.7186E 00  0.6257E 00  0.6257E 00  0.6257E 00  0.6257E 00


STRUCTURAL WEIGHT= 0.6460E 01


REDESIGN OPERATION FOLLOWS


OPTIMALITY INDEX OF DESIGN VARIABLES FOR BUCKLING CONSTRAINTS

    DV NO ACT/PAS    INDEX

        1       ACT     0.11317E 01
        2       ACT     0.11317E 01
        3       ACT     0.12055E 01
        4       ACT     0.79490E 00
        5       ACT     0.79490E 00
        6       ACT     0.79490E 00
        7       ACT     0.79490E 00

NO. OF ACTIVE BUCKLING CONSTRAINTS      ARE    1
```

****
ANALYSIS OF DESIGN NUMBER 2
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
5	1	-6.322E 00	3.798E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
4	1	-1.012E 01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	1	-3.798E 00	3.798E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	1	-5.060E 00	2.536E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
1	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.3952E 00	0.3952E 00	0.7924E 00	0.5616E 00	0.5616E 00	0.5616E 00	0.5616E 00			

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FORCE
1	0.3952E 00	1	-0.1000E 01
2	0.3952E 00	1	-0.1000E 01
3	0.7924E 00	1	-0.1000E 01
4	0.5616E 00	1	0.7412E-06
5	0.5616E 00	1	0.2309E-05
6	0.5616E 00	1	0.8326E-07
7	0.5616E 00	1	0.5191E-06

BUCKLING LOAD PARAMETERS

0.99102D 00

BUCKLING MODE SHAPES

NODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
5	1	3.974E-01	-1.978E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
4	1	1.586E 00	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	1	1.188E 00	-1.978E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	1	7.929E-01	-3.162E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01

```
   1    1   0.000E-01   0.000E-01   0.000E-01  0.0000E-01  0.0000E-01  0.0000E-01


********************************
EVALUATION OF DESIGN NUMBER   2
********************************


          STRESS RATIO   LOAD COND   DES VARIABLE
MAX        0.2530E 00        0             2
MIN        0.1262E 00        0             3

       MAX BUCK RATIOS   LOAD COND

         0.1009E 01        1


DESIGN IS CRITICAL



STRUCTURAL WEIGHT= 0.6343E 01


REDESIGN OPERATION FOLLOWS


OPTIMALITY INDEX OF DESIGN VARIABLES FOR BUCKLING CONSTRAINTS

    DV NO ACT/PAS    INDEX

        1      ACT     0.10241E 01
        2      ACT     0.10241E 01
        3      ACT     0.10191E 01
        4      ACT     0.10146E 01
        5      ACT     0.10146E 01
        6      ACT     0.10146E 01
        7      ACT     0.10146E 01

NO. OF ACTIVE BUCKLING CONSTRAINTS     ARE    1
BUCKLING - CRITICAL DESIGN HAS CONVERGED
```

L.2 Ten-Panel Truss

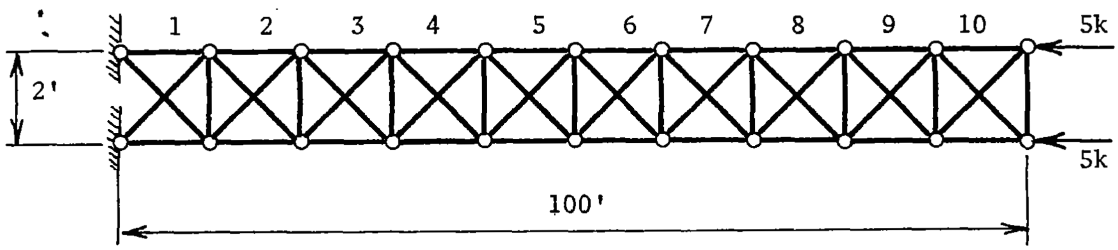


Figure L.2.1

Layout of Ten-Panel Truss Showing Panel Numbers.

The truss in Fig. L.2.1, which has been treated in existing literature [14], gives an opportunity to check the results of DESAP 2 against an independently obtained solution. Both designs use the following data:

$E = 10 \times 10^6$ psi (Young's modulus),

$\sigma_t^* = \sigma_c^* = 20{,}000$ psi (allowable stress),

$A = 2.0316$ sq. in. for all members (initial cross-sectional area),

$A^* = 0.0544$ sq. in. for all members (minimum allowable cross-sectional area).

Local buckling of the elements was not taken into consideration.

The designs obtained from DESAP 2 and Ref. [14] after five redesign cycles are listed in Table L.2.1; the correlation is excellent. It should be noted that the design is symmetric, as it should be, i.e. the two horizontal (chord) members in each panel are equal, and so are the two diagonal members.

This problem differs significantly from the three-panel truss in Sec. L.1. Firstly, the prebuckling state is not statically determinate in the current problem, which means that a prebuckling analysis must be carried out, and the geometric stiffness matrix of the structure recomputed prior to each design cycle. Secondly, the final design is governed by stress and buckling constraints simultaneously, whereas only the buckling constraint was active for the three-panel truss.

Panel number	Cross-sectional areas (sq. in.)			
	DESAP 2		Reference [14]	
	Horiz. membs.	Diag. membs.	Horiz. membs.	Diag. membs.
1	2.713	0.054	2.721	0.054
2	2.654	0.054	2.655	0.054
3	2.536	0.075	2.532	0.076
4	2.362	0.103	2.357	0.102
5	2.134	0.130	2.132	0.128
6	1.855	0.155	1.857	0.153
7	1.527	0.179	1.531	0.179
8	1.152	0.203	1.155	0.204
9	0.728	0.229	0.729	0.230
10	0.251	0.256	0.251	0.256
Weight	466.4 lb.		466.6 lb.	

The cross-sectional areas of all vertical members are at the minimum allowable value of 0.0544 sq. in.

Table L.2.1

Comparison Between the Results of DESAP 2 and Reference [14] After Five Design Cycles.

M. BEAM ELEMENTS

M.1 Three-Member Frame with Elastic Supports

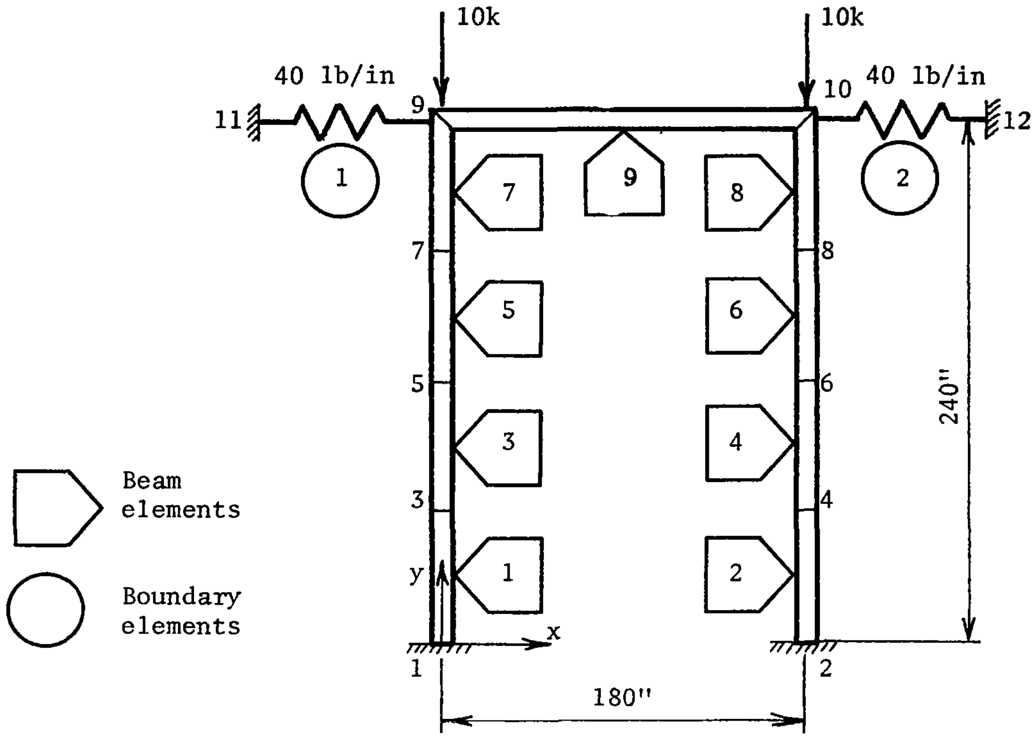


Figure M.1.1

Layout of Frame Showing Element and Node Numbers

The spring-supported frame in Fig. M.1.1, which has been extensively discussed in Ref. [4], is an example of a design problem that is governed by two buckling modes simultaneously.

The structural members consist of thin-walled beams. Only the wall thickness is to be varied during the design, which means that Construction Code No. 1 must be used. Equal size constraints are used to reduce the number of design variables to two: the cross-sectional area of the columns (A_1), and the cross-sectional area of the beam (A_2). The elastic supports are modelled by boundary elements.

The following data is used in the design:

$E = 30 \times 10^6$ psi (Young's modulus),

$\sigma_t^* = \sigma_c^* = 50{,}000$ psi (allowable stress),

$\rho = 1.0$ lb./cu. in. (specific weight),

$p^* = 1.0$ (min. allowable critical load parameter).

The geometric properties of the elements are summarized below.

Design Variable	Elements	Initial Values	
		$A(\text{in.}^2)$	$I_z(\text{in.}^4)$
1	1-8	1.50	1.50
2	9	0.15	0.15

No minimum size constraints were placed on the design variables.

The buckling of this simple frame can be treated analytically for arbitrary values of the two design variables A_1 and A_2. The results of the analysis have been plotted in the form of "design space" in Fig. M.1.2. Each point in the design space has as its coordinates A_1 and A_2, and represents, therefore, a specific design. The constraints divide the design space into two regions: the feasible region consists of designs that do not violate any constraints, whereas the infeasible region violates at least one constraint. The boundary between the two regions represents critical designs. The optimal design is that point in the feasible region that has the lowest weight. By inspection of Fig. M.1.2 it can be seen that this point lies at the intersection of the two buckling constraint lines P_s = 10k and P_a = 10k, where P_s and P_a are the buckling loads associated with the symmetric and asymmetric modes, respectively.

The designs obtained by DESAP 2 are also shown in Fig. M.1.2. As can be seen, the optimal design was reached in four redesigns, including two uniform scaling operations. The history of the design process is also given in numerical form in Table M.1.4. The stress constraints were not active at any stage of the design.

	Design Number				
	0	1	2	3	4
A_1 (in^2)	1.500	1.318	0.858	0.925	0.841
A_2 (in^2)	0.150	0.132	0.231	0.249	0.289
P_s (kips)	16.44	14.44	9.97	10.74	10.00
P_a (kips)	10.47	9.94	9.65	9.98	9.97
Wt. (lb)	N.C.*	656.2	N.C.	488.7	455.8

*Denotes that design is not critical.

Table M.1.4

Design History of Frame

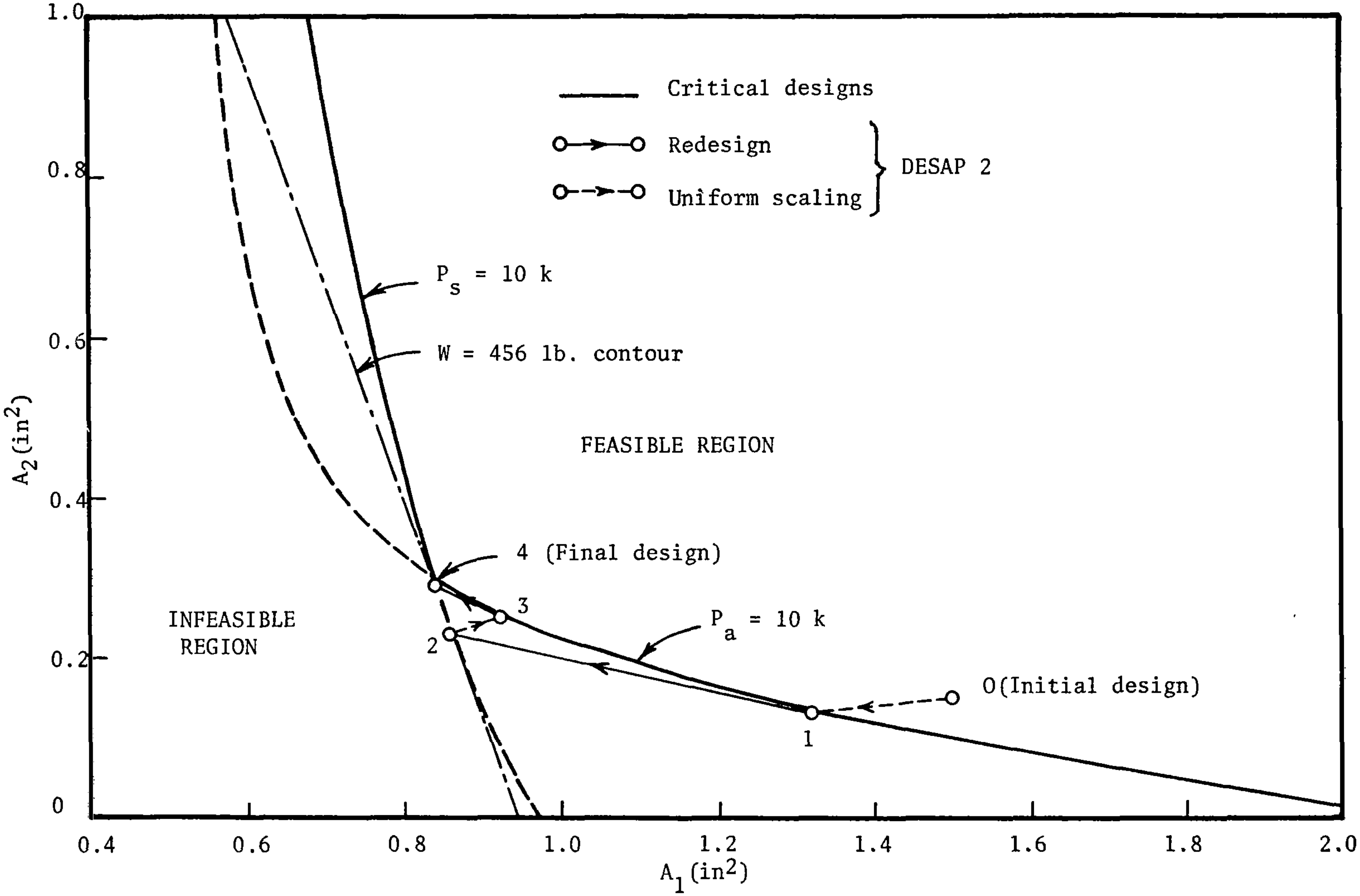


Figure M.1.2

Design Space Showing Designs Produced by DESAP 2

Special notes on input-output:

1) Although the stiffness matrix of each element has the form $[K_i] = [k_i]A_i$, the presence of the elastic supports means that uniform scaling is not an exact operation. Consequently, KSCALE = 0 is used in Design Control Data, which causes each scaled design to be analyzed and evaluated.

2) As each element bends only about the local z-axis, the properties about the two other axes are left blank on the Geometric Property Cards.

3) All section moduli have been left blank on the Geometric Property Cards. Consequently, the bending stresses are not calculated in the prebuckling state (they would be very small anyway).

4) The use of node Nos. 1 and 12 as the "third" node on the element cards (see Processed Element Data) specifies local y-axis to lie in the global x-y plane.

5) NMODE = 2 in Buckling Control Data requests that the redesign be carried out with respect to two modes simultaneously.

6) The prebuckling state is statically determinate (bending is negligible), which is specified by INDET = 1 in the Buckling Control Data. Consequently, the geometric stiffness matrix of the structure is assembled only once in the entire design process.

```
20300 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
20350   THREEE MEMBER FRAME WITH ELASTIC SUPPORTS ---- BUCKLING DESIGN
20400    12    2    1    2
20450    10           0.025     0.1          1    1    1
20500     1    1    1   -1   -1   -1    1
20550     2    1    1                   1 180.
20600     3                                            60.
20650     9                                           240.                   2
20700     4                                 180.       60.
20750    10                                 120.      240.                   2
20800    11    1    1    1    1    1    1 -100.      240.
20850    12    1    1    1    1    1    1  300.       240.
20900     2    9    1    1    1
20950     1      1.0       30000000.           50000.
21000     1    1 1.0                           1.
21050
21100
21150
21200
21250     8    8   10   12    1    1    1
21300     9    9   10    1    1    1    2
21350     7    2
21400
21450     2   10   12                   1                                   40.
21500
21550 1.0           1    2    1    2   0.5      0.8
21600     9    1               -10000.
21650    10    1               -10000.
21700
21750 0.5        -0.5                          -0.1      0.1       0.5      0.5
21800                           -0.1      -0.1      1.0     -1.0
21850 -0.2                      1.0      1.0                          -0.2
21900 1.5         -0.5                         -0.5     -0.1      1.5       0.5
21950                          -0.3      0.1       2.0
22000 -0.4        -0.1          2.0
22001                           -.4      0.1
22050     1 1.5
22100     2 .15
22150 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
22200
```

Echo of Input Cards

```
 THREEE MEMBER FRAME WITH ELASTIC SUPPORTS --- BUCKLING DESIGN

 NUMBER OF NODAL POINTS   =   12
 NUMBER OF ELEMENT TYPES  =    2
 NUMBER OF LOAD CASES     =    1
 NUMBER OF DES. VARIABLES =    2


 DESIGN CONTROL DATA

 NCYCL =    10
 KSCALE=     0
 DELTA =  0.2500E-01
 EPSIL =  0.1000E 00
 LBUCK =     1


NODAL POINT INPUT DATA

NODE    BOUNDARY CONDITION CODES   /------NODAL POINT COORDINATES--------/
NUMBER   X     Y     Z    XX    YY    ZZ          X            Y            Z            T

   1     1     1    -1    -1    -1     1       0.0          0.0          0.0     0       0.0
   2     1     1     0     0     0     1     180.000        0.0          0.0     0       0.0
   3     0     0     0     0     0     0       0.0         60.000        0.0     0       0.0
   9     0     0     0     0     0     0       0.0        240.000        0.0     2       0.0
   4     0     0     0     0     0     0     180.000       60.000        0.0     0       0.0
  10     0     0     0     0     0     0     180.000      240.000        0.0     2       0.0
  11     1     1     1     1     1     1    -100.000      240.000        0.0     0       0.0
  12     1     1     1     1     1     1     300.000      240.000        0.0     0       0.0


GENERATED NODAL DATA

NODE    BOUNDARY CONDITION CODES   /------NODAL POINT COORDINATES--------/
NUMBER   X     Y     Z    XX    YY    ZZ          X            Y            Z            T

   1     1     1    -1    -1    -1     1       0.0          0.0          0.0        0.0
   2     1     1    -1    -1    -1     1     180.000        0.0          0.0        0.0
   3     0     0    -1    -1    -1     0       0.0         60.000        0.0        0.0
   4     0     0    -1    -1    -1     0     180.000       60.000        0.0        0.0
   5     0     0    -1    -1    -1     0       0.0        120.000        0.0        0.0
   6     0     0    -1    -1    -1     0     180.000      120.000        0.0        0.0
   7     0     0    -1    -1    -1     0       0.0        180.000        0.0        0.0
   8     0     0    -1    -1    -1     0     180.000      180.000        0.0        0.0
   9     0     0    -1    -1    -1     0       0.0        240.000        0.0        0.0
  10     0     0    -1    -1    -1     0     180.000      240.000        0.0        0.0
  11     1     1     1     1     1     1    -100.000      240.000        0.0        0.0
  12     1     1     1     1     1     1     300.000      240.000        0.0        0.0


EQUATION NUMBERS

   N     X     Y     Z    XX    YY    ZZ
   1     0     0     0     0     0     0
   2     0     0     0     0     0     0
   3     1     2     0     0     0     3
   4     4     5     0     0     0     6
   5     7     8     0     0     0     9
   6    10    11     0     0     0    12
```

Computer Printout

(Input data, the initial design and the final design only are reproduced.)

7	13	14	0	0	0	15
8	16	17	0	0	0	18
9	19	20	0	0	0	21
10	22	23	0	0	0	24
11	0	0	0	0	0	0
12	0	0	0	0	0	0

THREE DIMENSIONAL BEAM ELEMENTS

```
NUMBER OF BEAM ELEMENTS        =    9
CONSTRUCTION CODE              =    1
NUMBER OF MATERIALS            =    1
NUMBER OF GEOMETRIC PROPERTIES=     1
NUMBER OF FIXED-END FORCE SETS=     0
```

MATERIAL PROPERTY CARDS

MATERIAL NUMBER	SPECIFIC WEIGHT	YOUNGS MODULUS	POISSONS RATIO	/--------ALLOWABLE STRESSES--------/ TENSION	COMPRESSION	SHEAR
1	0.1000D 01	0.3000E 08	0.0	0.5000E 05	0.5000E 05	0.2885E 05

GEOMETRIC PROPERTY CARDS

PROPERTY NUMBER	X-SECT KODE	X-SECT AREA	/------PROPERTIES OF X-SECTION-----/ X-AXIS	Y-AXIS	Z-AXIS	
1	1	0.1000D 01	0.0	0.0	0.1000E 01	MOMENTS OF INERTIA
			0.0	0.0	0.0	SECT MODULI FOR POINT A
			0.0	0.0	0.0	SECT MODULI FOR POINT B

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0	0.0	0.0	0.0
Y-DIR	0.0	0.0	0.0	0.0
Z-DIR	0.0	0.0	0.0	0.0

PROCESSED ELEMENT DATA

ELEMENT NUMBER	/---NODE NOS--/ I	J	K	/--ELEMENT ID NOS-/ MATL	GEOMY	D VAR	DESIGN VAR FRACTION	FIXED END-FORCE ID A	B	C	D	END RELEASE CODES I	J	BAND. WIDTH
1	1	3	12	1	1	1	0.1000E 01	0	0	0	0	000000	000000	3
2	2	4	12	1	1	1	0.1000E 01	0	0	0	0	000000	000000	3
3	3	5	12	1	1	1	0.1000E 01	0	0	0	0	000000	000000	9
4	4	6	12	1	1	1	0.1000E 01	0	0	0	0	000000	000000	9
5	5	7	12	1	1	1	0.1000E 01	0	0	0	0	000000	000000	9
6	6	8	12	1	1	1	0.1000E 01	0	0	0	0	000000	000000	9
7	7	9	12	1	1	1	0.1000E 01	0	0	0	0	000000	000000	9
8	8	10	12	1	1	1	0.1000E 01	0	0	0	0	000000	000000	9
9	9	10	1	1	1	2	0.1000E 01	0	0	0	0	000000	000000	6

B O U N D A R Y   E L E M E N T S

NUMBER OF ELEMENTS = 2

ELEMENT LOAD MULTIPLIERS

A	B	C	D
0.0	0.0	0.0	0.0

BOUNDARY ELEMENT DATA

CONST NUMBER	NODE N	/--NODES DEFINING CONSTRAINT DIRECTION--/ NI	NJ	NK	NL	CODES KD	KR	DISPL D	ROTATION R	STIFF S
1	9	11	0	0	0	1	0	0.0	0.0	4.00D 01
2	10	12	0	0	0	1	0	0.0	0.0	4.00D 01

STRUCTURE LOAD CASE	STRUCTURE LOAD MULTIPLIERS A	B	C	D
1	0.0	0.0	0.0	0.0

BUCKLING CONTROL DATA

COEFFT = 1.00000
MODEIN = 1
NMODE = 2
INDET = 1
NVEC = 2
ALPA = 0.50000
OMEGA = 0.80000

NODAL POINT LOADS

NODE NO.	LOAD CASE	APPLIED LOADS RX	RY	RZ	MX	MY	MZ
9	1	0.0	-0.100D 05	0.0	0.0	0.0	0.0
10	1	0.0	-0.100D 05	0.0	0.0	0.0	0.0

DESIGN VARIABLE INPUT DATA

DESIGN VARIABLE NUMBER	INITIAL VALUE	MIN ALLOWABLE VALUE
1	0.1500E 01	0.0
2	0.1500E 00	0.0

TOTAL NUMBER OF EQUATIONS = 24
BANDWIDTH = 9
NUMBER OF EQUATIONS IN A BLOCK = 22
NUMBER OF BLOCKS = 2

****
ANALYSIS OF DESIGN NUMBER 0
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
12	1	0.0	0.0	0.0	0.0	0.0	0.0
11	1	0.0	0.0	0.0	0.0	0.0	0.0
10	1	4.406E-19	-5.333E-02	0.0	0.0	0.0	-5.0772E-21
9	1	4.416E-19	-5.333E-02	0.0	0.0	0.0	-1.3254E-20
8	1	2.004E-19	-4.000E-02	0.0	0.0	0.0	-3.0171E-21
7	1	-7.467E-20	-4.000E-02	0.0	0.0	0.0	-4.5553E-21
6	1	6.796E-20	-2.667E-02	0.0	0.0	0.0	-1.4842E-21
5	1	-1.768E-19	-2.667E-02	0.0	0.0	0.0	5.5310E-22
4	1	1.172E-20	-1.333E-02	0.0	0.0	0.0	-4.7849E-22
3	1	-8.010E-20	-1.333E-02	0.0	0.0	0.0	2.0715E-21
2	1	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.1500E 01	0.1500E 00								

ANALYSIS OF BEAM ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FX	SHEAR FY	SHEAR FZ	TORQUE MX	MOMENT MY	MOMENT MZ
1	0.1500E 01	1	0.1000E 05	0.4487E-16	0.0	0.0	0.0	0.2900E-14
			-0.1000E 05	-0.4487E-16	0.0	0.0	0.0	-0.2074E-15
2	0.1500E 01	1	0.1000E 05	0.6590E-17	0.0	0.0	0.0	-0.1612E-15
			-0.1000E 05	-0.6590E-17	0.0	0.0	0.0	0.5566E-15
3	0.1500E 01	1	0.1000E 05	0.4487E-16	0.0	0.0	0.0	0.2074E-15
			-0.1000E 05	-0.4487E-16	0.0	0.0	0.0	0.2485E-14
4	0.1500E 01	1	0.1000E 05	0.6590E-17	0.0	0.0	0.0	-0.5566E-15
			-0.1000E 05	-0.6590E-17	0.0	0.0	0.0	0.9520E-15
5	0.1500E 01	1	0.1000E 05	0.4487E-16	0.0	0.0	0.0	-0.2485E-14
			-0.1000E 05	-0.4487E-16	0.0	0.0	0.0	0.5177E-14
6	0.1500E 01	1	0.1000E 05	0.6590E-17	0.0	0.0	0.0	-0.9520E-15
			-0.1000E 05	-0.6590E-17	0.0	0.0	0.0	0.1347E-14
7	0.1500E 01	1	0.1000E 05	0.4487E-16	0.0	0.0	0.0	-0.5178E-14

			-0.1000E 05	-0.4487E-16	0.0	0.0	0.0	0.7870E-14
8	0.1500E 01	1	0.1000E 05	0.6590E-17	0.0	0.0	0.0	-0.1347E-14
			-0.1000E 05	-0.6590E-17	0.0	0.0	0.0	0.1743E-14
9	0.1500E 00	1	0.2721E-16	-0.1450E-07	0.0	0.0	0.0	-0.7862E-05
			-0.2721E-16	0.1450E-07	0.0	0.0	0.0	-0.7862E-05

ANALYSIS OF BOUNDARY ELEMENTS - CONSTRAINT FORCES

CONST NUMBER	LOAD CASE	FORCE	MOMENT
1	1	-0.17666E-16	0.0
2	1	0.17622E-16	0.0

BUCKLING LOAD PARAMETERS

0.10470D 01 0.16439D 01

BUCKLING MODE SHAPES

NODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
12	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
11	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
10	1	-9.089E-02	5.825E-06	0.0	0.0	0.0	6.5071E-04
	2	4.304E-05	-6.652E-22	0.0	0.0	0.0	-7.8056E-04
9	1	-9.089E-02	-5.825E-06	0.0	0.0	0.0	6.6012E-04
	2	-4.304E-05	1.362E-21	0.0	0.0	0.0	7.8056E-04
8	1	-5.059E-02	4.369E-06	0.0	0.0	0.0	6.4761E-04
	2	-4.025E-02	-4.989E-22	0.0	0.0	0.0	-4.4184E-04
7	1	-5.010E-02	-4.369E-06	0.0	0.0	0.0	6.5293E-04
	2	4.025E-02	1.021E-21	0.0	0.0	0.0	4.4184E-04
6	1	-1.797E-02	2.913E-06	0.0	0.0	0.0	4.1155E-04
	2	-4.531E-02	-3.326E-22	0.0	0.0	0.0	2.6379E-04
5	1	-1.742E-02	-2.913E-06	0.0	0.0	0.0	4.0836E-04
	2	4.531E-02	6.808E-22	0.0	0.0	0.0	-2.6379E-04
4	1	-2.334E-03	1.456E-06	0.0	0.0	0.0	1.2060E-04
	2	-1.821E-02	-1.663E-22	0.0	0.0	0.0	5.0896E-04
3	1	-2.114E-03	-1.456E-06	0.0	0.0	0.0	1.1446E-04
	2	1.821E-02	3.404E-22	0.0	0.0	0.0	-5.0896E-04
2	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0

****
EVALUATION OF DESIGN NUMBER 0
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.1333E 00	1	1
MIN	0.3628E-20	1	2

```
MAX BUCK RATIOS   LOAD COND

  0.9551E 00          1
  0.6083E 00          1

DESIGN IS NOT CRITICAL



UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS  0.879AND DETERMINED BY BUCKLING     CONSTRAINTS
```

```
*******************************
ANALYSIS OF DESIGN NUMBER   4
*******************************
```

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
12	1	0.0	0.0	0.0	0.0	0.0	0.0
11	1	0.0	0.0	0.0	0.0	0.0	0.0
10	1	8.674E-19	-9.510E-02	0.0	0.0	0.0	0.0
9	1	8.722E-19	-9.510E-02	0.0	0.0	0.0	-1.0842E-19
8	1	7.318E-19	-7.133E-02	0.0	0.0	0.0	-4.0658E-21
7	1	-2.923E-18	-7.133E-02	0.0	0.0	0.0	-2.4417E-20
6	1	4.337E-19	-4.755E-02	0.0	0.0	0.0	-5.4210E-21
5	1	-2.817E-18	-4.755E-02	0.0	0.0	0.0	2.1654E-20
4	1	1.355E-19	-2.378E-02	0.0	0.0	0.0	-4.0658E-21
3	1	-1.083E-18	-2.378E-02	0.0	0.0	0.0	2.9793E-20
2	1	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.8412E 00	0.2892E 00								

ANALYSIS OF BEAM ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL PX	SHEAR PY	SHEAR PZ	TORQUE MX	MOMENT MY	MOMENT MZ
1	0.8412E 00	1	0.1000E 05	0.2659E-15	0.0	0.0	0.0	0.2051E-13
			-0.1000E 05	-0.2659E-15	0.0	0.0	0.0	-0.4554E-14
2	0.8412E 00	1	0.1000E 05	-0.1900E-16	0.0	0.0	0.0	-0.2280E-14
			-0.1000E 05	0.1900E-16	0.0	0.0	0.0	0.1140E-14
3	0.8412E 00	1	0.1000E 05	0.2659E-15	0.0	0.0	0.0	0.4554E-14
			-0.1000E 05	-0.2659E-15	0.0	0.0	0.0	0.1140E-13
4	0.8412E 00	1	0.1000E 05	-0.1900E-16	0.0	0.0	0.0	-0.1140E-14
			-0.1000E 05	0.1900E-16	0.0	0.0	0.0	-0.2448E-20
5	0.8412E 00	1	0.1000E 05	0.2659E-15	0.0	0.0	0.0	-0.1140E-13
			-0.1000E 05	-0.2659E-15	0.0	0.0	0.0	0.2735E-13
6	0.8412E 00	1	0.1000E 05	-0.1900E-16	0.0	0.0	0.0	-0.7438E-20
			-0.1000E 05	0.1900E-16	0.0	0.0	0.0	-0.1140E-14
7	0.8412E 00	1	0.1000E 05	0.2659E-15	0.0	0.0	0.0	-0.2735E-13

			-0.1000E 05	-0.2659E-15	0.0	0.0	0.0	0.4331E-13
8	0.8412E 00	1	0.1000E 05	-0.1900E-16	0.0	0.0	0.0	0.1140E-14
			-0.1000E 05	0.1900E-16	0.0	0.0	0.0	-0.2280E-14
9	0.2892E 00	1	0.2310E-15	-0.2828E-06	0.0	0.0	0.0	-0.2568E-05
			-0.2310E-15	0.2828E-06	0.0	0.0	0.0	-0.2568E-05

ANALYSIS OF BOUNDARY ELEMENTS - CONSTRAINT FORCES

CONST NUMBER	LOAD CASE	FORCE	MOMENT
1	1	-0.34886E-16	0.0
2	1	0.34694E-16	0.0

BUCKLING LOAD PARAMETERS

0.99673D 00 0.99951D 00

BUCKLING MODE SHAPES

NODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
12	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
11	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
10	1	-1.563E-02	3.401E-06	0.0	0.0	0.0	8.2239E-04
	2	-7.473E-02	1.627E-05	0.0	0.0	0.0	3.9239E-04
9	1	-1.561E-02	-3.401E-06	0.0	0.0	0.0	-5.9975E-04
	2	-7.474E-02	-1.627E-05	0.0	0.0	0.0	6.7293E-04
8	1	3.132E-02	2.551E-06	0.0	0.0	0.0	5.9173E-04
	2	-4.384E-02	1.221E-05	0.0	0.0	0.0	5.8019E-04
7	1	-4.644E-02	-2.551E-06	0.0	0.0	0.0	-3.1197E-04
	2	-2.850E-02	-1.221E-05	0.0	0.0	0.0	7.5846E-04
6	1	4.407E-02	1.700E-06	0.0	0.0	0.0	-1.7824E-04
	2	-1.300E-02	8.137E-06	0.0	0.0	0.0	3.9149E-04
5	1	-4.580E-02	-1.700E-06	0.0	0.0	0.0	3.2128E-04
	2	4.730E-03	-8.137E-06	0.0	0.0	0.0	2.9295E-04
4	1	1.899E-02	8.502E-07	0.0	0.0	0.0	-5.1668E-04
	2	1.831E-04	4.068E-06	0.0	0.0	0.0	6.1516E-05
3	1	-1.741E-02	-8.502E-07	0.0	0.0	0.0	5.0046E-04
	2	7.364E-03	-4.068E-06	0.0	0.0	0.0	-1.3913E-04
2	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0

****
EVALUATION OF DESIGN NUMBER 4
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.2378E 00	1	1
MIN	0.1597E-19	1	2

```
       MAX BUCK RATIOS   LOAD COND

         0.1003E 01         1
         0.1000E 01         1

DESIGN IS CRITICAL



STRUCTURAL WEIGHT= 0.4558E 03


REDESIGN OPERATION FOLLOWS


OPTIMALITY INDEX OF DESIGN VARIABLES FOR BUCKLING CONSTRAINTS

    DV NO ACT/PAS     INDEX

        1      ACT      0.10088E 01
        2      ACT      0.98958E 00

NO. OF ACTIVE BUCKLING CONSTRAINTS     ARE    2
BUCKLING - CRITICAL DESIGN HAS CONVERGED
```

N. PLANE STRESS ELEMENTS

N.1 Cantilever Beam of Variable Width

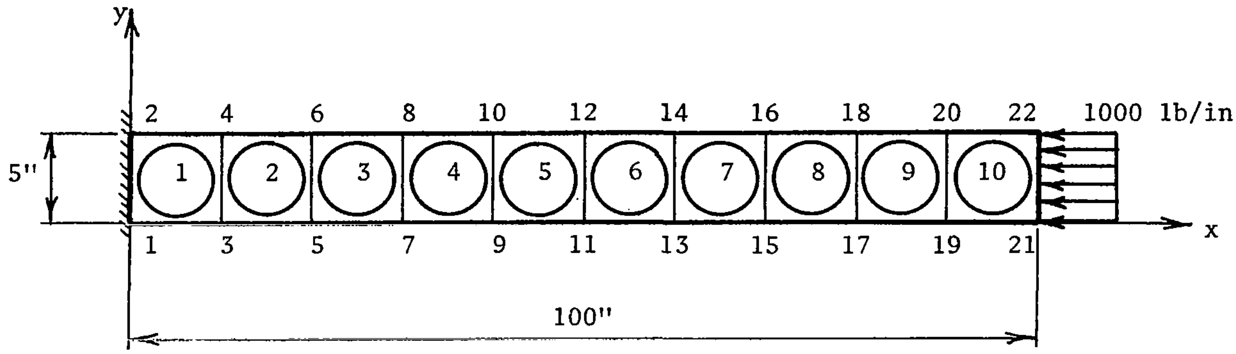


Figure N.1.1

Cantilever Beam Showing Element and Node Numbers

The cantilever beam in Fig. N.1.1 is modelled with ten membrane elements, the thickness of each element being an independent design variable. The results of DESAP 2 for this particular problem can be checked, because an exact analytical expression for the width of an optimal cantilever beam is available [15].

The material properties were taken as

$E = 10 \times 10^6$ psi (Young's modulus),

$\nu = 0$ (Poisson's ratio),

$\sigma_t^* = \sigma_c^* = 10^6$ psi (allowable stress),

$\rho = 1.0$ lb./cu. in. (specific weight).

The allowable stress was deliberately given a very high value, since we did not want the stress constraints to be active. Factor of safety against buckling is taken as one, i.e., $p^* = 1$ is used.

A uniform thickness of 0.1 in. was chosen as the initial design, and no lower limits on the thickness were used. The design history is summarized in Table N.1.1. The final design, which was reached in four iterations, can be seen to compare very well with the exact analytical solution, also given in Table N.1.1.

The solution for an optimal cantilever beam is somewhat difficult to extract from Ref. [15]. In the absence of minimum size constraints, the optimal thickness distribution can be shown to be

$$t(x) = \frac{6P\ell^2}{Eb^3}\,(1-\frac{x^2}{\ell^2}) \ , \qquad \text{(N.1.1)}$$

where P is the desired buckling load, ℓ is the length of the beam, and b represents the fixed depth of the cross section. Inserting the properties of our particular problem, the above formula reduces to

$$t(x) = 0.24\ (1-x^2/\ell^2). \qquad \text{(N.1.2)}$$

The thicknesses listed in Table N.1.1 are the values obtained from (N.1.2) at the center of each finite element.

Special notes on input-output:

1) KSCALE = 1 in Design Control Data informs the program that uniform scaling is an exact operation, i.e., the scaled structure does not have to be reanalyzed. Note that the stiffness matrix of each

element has the form $[K_i] = [k_i]t_i$ (t_i = thickness of element).

2) The layout of elements lends itself well to automatic generation of element and node data---compare computer printout of input data with the echo of the data cards.

3) The loading was prescribed on the data card of element No. 10---see Processed Element Data. Also note the proper use of Element Load Fractions (multipliers) and Structural Load Multipliers.

4) NMODE = 2 in Buckling Control Data asks for analysis of two buckling modes, which also means that both of the modes are considered in redesign. Since the design is determined by one mode only, the same results would be obtained with NMODE = 1, but the buckling analysis for the second mode would be lost.

5) INDET = 1 in Buckling Control Data declares the internal forces of the prebuckling state to be statically determinate. Consequently, the geometric stiffness matrix of the structure does not have to be updated after each redesign cycle.

6) It is important in this problem not to suppress the incompatible displacement modes of the elements. The incompatible modes enable each element to undergo pure bending deformation, thereby greatly increasing the accuracy of buckling analysis (see Element Control Card for plane stress elements in Vol. 1).

7) The design was terminated when the Optimality Index of each element was sufficiently close to one (see Evaluation of Design No. 4).

Element	Critical, Scaled Designs (thickness in inches)					Exact Solution Ref. [15]
	0	1	2	3	4	
1	.1939	.2348	.2357	.2381	.2389	.2394
2	.1939	.2283	.2309	.2333	.2341	.2346
3	.1939	.2159	.2213	.2237	.2244	.2250
4	.1939	.1989	.2067	.2092	.2100	.2106
5	.1939	.1789	.1872	.1900	.1908	.1914
6	.1939	.1578	.1625	.1660	.1668	.1674
7	.1939	.1378	.1331	.1371	.1380	.1386
8	.1939	.1207	.1010	.1032	.1043	.1050
9	.1939	.1084	.0715	.0650	.0659	.0666
10	.1939	.1019	.0531	.0313	.0242	.0234
Wt(lb)	96.94	84.17	80.14	79.84	79.86	80.00

Table N.1.1

Design History of Element Thicknesses and Total Structural Weight.

```
09200 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
09250  CANTILEVER BEAM-BUCKLING ANALYSIS AND DESIGN USING UNIFORM MEMBRANE ELEMENT
09300    22    1    1   10
09350     6         10.025                   1    1    1
09400     1    1    1   -1   -1   -1   -1
09450     2    1    1                                5.0
09500     3                                10.
09550    21                                100.                            2
09600     4                                10.       5.0
09650    22              1    1    1    1 100.       5.                    2
09700     3   10    1    1    2
09750     1    1   1.0
09800           10000000.   0.0               1000000.
09850             1.0
09900
09950
10000
10050     1    1    3    4    2    1    1
10100     2    3    5    6    4    1    2
10150     3    5    7    8    6    1    3
10200     4    7    9   10    8    1    4
10250     5    9   11   12   10    1    5
10300     6   11   13   14   12    1    6
10350     7   13   15   16   14    1    7
10400     8   15   17   18   16    1    8
10450     9   17   19   20   18    1    9
10500    10   21   22   20   19    1   10                1000.
10550 1.0
10600 1.0                2    1    2  0.5       0.8
10650
10700    10  0.1
10750 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
11000
```

Echo of Input Cards

CANTILEVER BEAM-BUCKLING ANALYSIS AND DESIGN USING UNIFORM MEMBRANE ELEMENT

NUMBER OF NODAL POINTS = 22
NUMBER OF ELEMENT TYPES = 1
NUMBER OF LOAD CASES = 1
NUMBER OF DES. VARIABLES = 10

DESIGN CONTROL DATA

NCYCL = 6
KSCALE= 1
DELTA = 0.2500E-01
EPSIL = 0.1000E 00
LBUCK = 1

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z		T
1	1	1	-1	-1	-1	-1	0.0	0.0	0.0	0	0.0
2	1	1	0	0	0	0	0.0	5.000	0.0	0	0.0
3	0	0	0	0	0	0	10.000	0.0	0.0	0	0.0
21	0	0	0	0	0	0	100.000	0.0	0.0	2	0.0
4	0	0	0	0	0	0	10.000	5.000	0.0	0	0.0
22	0	0	1	1	1	1	100.000	5.000	0.0	2	0.0

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z	T
1	1	1	-1	-1	-1	-1	0.0	0.0	0.0	0.0
2	1	1	-1	-1	-1	-1	0.0	5.000	0.0	0.0
3	0	0	-1	-1	-1	-1	10.000	0.0	0.0	0.0
4	0	0	-1	-1	-1	-1	10.000	5.000	0.0	0.0
5	0	0	-1	-1	-1	-1	20.000	0.0	0.0	0.0
6	0	0	-1	-1	-1	-1	20.000	5.000	0.0	0.0
7	0	0	-1	-1	-1	-1	30.000	0.0	0.0	0.0
8	0	0	-1	-1	-1	-1	30.000	5.000	0.0	0.0
9	0	0	-1	-1	-1	-1	40.000	0.0	0.0	0.0
10	0	0	-1	-1	-1	-1	40.000	5.000	0.0	0.0
11	0	0	-1	-1	-1	-1	50.000	0.0	0.0	0.0
12	0	0	-1	-1	-1	-1	50.000	5.000	0.0	0.0
13	0	0	-1	-1	-1	-1	60.000	0.0	0.0	0.0
14	0	0	-1	-1	-1	-1	60.000	5.000	0.0	0.0
15	0	0	-1	-1	-1	-1	70.000	0.0	0.0	0.0
16	0	0	-1	-1	-1	-1	70.000	5.000	0.0	0.0
17	0	0	-1	-1	-1	-1	80.000	0.0	0.0	0.0
18	0	0	-1	-1	-1	-1	80.000	5.000	0.0	0.0
19	0	0	-1	-1	-1	-1	90.000	0.0	0.0	0.0
20	0	0	-1	-1	-1	-1	90.000	5.000	0.0	0.0
21	0	0	-1	-1	-1	-1	100.000	0.0	0.0	0.0
22	0	0	1	1	1	1	100.000	5.000	0.0	0.0

EQUATION NUMBERS

Computer Printout

(Input data, the initial design and the final design only are reproduced)

N	X	Y	Z	XX	YY	ZZ
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	1	2	0	0	0	0
4	3	4	0	0	0	0
5	5	6	0	0	0	0
6	7	8	0	0	0	0
7	9	10	0	0	0	0
8	11	12	0	0	0	0
9	13	14	0	0	0	0
10	15	16	0	0	0	0
11	17	18	0	0	0	0
12	19	20	0	0	0	0
13	21	22	0	0	0	0
14	23	24	0	0	0	0
15	25	26	0	0	0	0
16	27	28	0	0	0	0
17	29	30	0	0	0	0
18	31	32	0	0	0	0
19	33	34	0	0	0	0
20	35	36	0	0	0	0
21	37	38	0	0	0	0
22	39	40	0	0	0	0

```
NUMBER OF MEMBRANE ELEMENTS              =   10
CONSTRUCTION MODE                        =    2
NUMBER OF MATERIALS                      =    1
NUMBER OF TEMPS FOR WHICH MATL PROPS GIVEN=   1
```

MATERIAL PROPERTY CARDS

MATL NBR	NO OF TEMP	SPECIFIC WEIGHT	TEMPERATURE	YOUNGS MODULUS	POISSONS RATIO	COEFFT OF THERM EXPN	ALLOWABLE STRESSES TENSION	COMPRESSION	SHEAR
1	1	0.1000E 01	0.0	0.1000E 08	0.0	0.0	0.1000E 07	0.1000E 07	0.0

ELEMENT LOAD FRACTIONS

LOAD CASE	TEMPERATURE	PRESSURE	X-DIRECTION	Y-DIRECTION	Z-DIRECTION
A	0.0	1.000	0.0	0.0	0.0
B	0.0	0.0	0.0	0.0	0.0
C	0.0	0.0	0.0	0.0	0.0
D	0.0	0.0	0.0	0.0	0.0

PROCESSED ELEMENT DATA

ELEMT NUMBR	NODES I	J	K	L	ID NOS MAT	DV	DES VAR FRACTION	REFERENCE TEMP	PRESSURE	BETA	PRNT CODE	BAND WDTH
1	1	3	4	2	1	1	0.1000E 01	0.0	0.0	0.0	3	4
2	3	5	6	4	1	2	0.1000E 01	0.0	0.0	0.0	3	8
3	5	7	8	6	1	3	0.1000E 01	0.0	0.0	0.0	3	8
4	7	9	10	8	1	4	0.1000E 01	0.0	0.0	0.0	3	8
5	9	11	12	10	1	5	0.1000E 01	0.0	0.0	0.0	3	8
6	11	13	14	12	1	6	0.1000E 01	0.0	0.0	0.0	3	8
7	13	15	16	14	1	7	0.1000E 01	0.0	0.0	0.0	3	8
8	15	17	18	16	1	8	0.1000E 01	0.0	0.0	0.0	3	8
9	17	19	20	18	1	9	0.1000E 01	0.0	0.0	0.0	3	8
10	21	22	20	19	1	10	0.1000E 01	0.0	0.1000D 04	0.0	3	8

STRUCTURE LOAD CASE	STRUCTURE LOAD MULTIPLIERS A	B	C	D
1	1.000	0.0	0.0	0.0

BUCKLING CONTROL DATA

```
COEFFT =   1.00000
MODEIN =    0
NMODE  =    2
INDET  =    1
NVEC   =    2
ALPA   =   0.50000
OMEGA  =   0.80000
```

NODAL POINT LOADS

NODE NO.	LOAD CASE	APPLIED LOADS PX	PY	PZ	MX	MY	MZ

DESIGN VARIABLE INPUT DATA

DESIGN VARIABLE NUMBER	INITIAL VALUE	MIN ALLOWABLE VALUE
1	0.1000E 00	0.0
2	0.1000E 00	0.0
3	0.1000E 00	0.0
4	0.1000E 00	0.0
5	0.1000E 00	0.0
6	0.1000E 00	0.0
7	0.1000E 00	0.0
8	0.1000E 00	0.0
9	0.1000E 00	0.0
10	0.1000E 00	0.0

```
TOTAL NUMBER OF EQUATIONS       =   40
BANDWIDTH                       =    8
NUMBER OF EQUATIONS IN A BLOCK =   40
NUMBER OF BLOCKS                =    1
```

****
ANALYSIS OF DESIGN NUMBER 0
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
22	1	-1.000E-01	-2.110E-13	0.0	0.0	0.0	0.0
21	1	-1.000E-01	-2.110E-13	0.0	0.0	0.0	0.0
20	1	-9.000E-02	-1.814E-13	0.0	0.0	0.0	0.0
19	1	-9.000E-02	-1.814E-13	0.0	0.0	0.0	0.0
18	1	-8.000E-02	-1.520E-13	0.0	0.0	0.0	0.0
17	1	-8.000E-02	-1.520E-13	0.0	0.0	0.0	0.0
16	1	-7.000E-02	-1.231E-13	0.0	0.0	0.0	0.0
15	1	-7.000E-02	-1.231E-13	0.0	0.0	0.0	0.0
14	1	-6.000E-02	-9.551E-14	0.0	0.0	0.0	0.0
13	1	-6.000E-02	-9.551E-14	0.0	0.0	0.0	0.0
12	1	-5.000E-02	-6.994E-14	0.0	0.0	0.0	0.0
11	1	-5.000E-02	-6.994E-14	0.0	0.0	0.0	0.0
10	1	-4.000E-02	-4.714E-14	0.0	0.0	0.0	0.0
9	1	-4.000E-02	-4.714E-14	0.0	0.0	0.0	0.0
8	1	-3.000E-02	-2.786E-14	0.0	0.0	0.0	0.0
7	1	-3.000E-02	-2.786E-14	0.0	0.0	0.0	0.0
6	1	-2.000E-02	-1.296E-14	0.0	0.0	0.0	0.0
5	1	-2.000E-02	-1.296E-14	0.0	0.0	0.0	0.0
4	1	-1.000E-02	-3.368E-15	0.0	0.0	0.0	0.0
3	1	-1.000E-02	-3.368E-15	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

1 2 3 4 5 6 7 8 9 10

0	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00

ANALYSIS OF MEMBRANE ELEMENTS, CONSTRN CODE= 2

ELEMENT	SHEET THICKNESS	LOAD COND	LOCATION	/---MEMBRANE FORCES IN LOCAL COORDS--- NXX	NYY	NXY	//--MEMBRANE FORCES IN MATERIAL COORDS-/ N11	N22	N12
1	0.1000E 00	1	CEN	-0.1000E 04	-0.1007E-13	-0.1473E-03	-0.1000E 04	-0.1007E-13	-0.1473E-03
2	0.1000E 00	1	CEN	-0.1000E 04	0.1917E-14	-0.3914E-03	-0.1000E 04	0.1917E-14	-0.3914E-03
3	0.1000E 00	1	CEN	-0.1000E 04	0.4565E-13	-0.4808E-03	-0.1000E 04	0.4565E-13	-0.4808E-03
4	0.1000E 00	1	CEN	-0.1000E 04	0.1156E-12	-0.3840E-03	-0.1000E 04	0.1156E-12	-0.3840E-03
5	0.1000E 00	1	CEN	-0.1000E 04	0.1364E-12	-0.3891E-02	-0.1000E 04	0.1364E-12	-0.3891E-02
6	0.1000E 00	1	CEN	-0.1000E 04	0.2056E-12	-0.3794E-02	-0.1000E 04	0.2056E-12	-0.3794E-02
7	0.1000E 00	1	CEN	-0.1000E 04	0.1758E-12	-0.2419E-02	-0.1000E 04	0.1758E-12	-0.2419E-02
8	0.1000E 00	1	CEN	-0.1000E 04	-0.9515E-14	-0.4006E-02	-0.1000E 04	-0.9515E-14	-0.4006E-02
9	0.1000E 00	1	CEN	-0.1000E 04	-0.5750E-13	-0.2862E-02	-0.1000E 04	-0.5750E-13	-0.2862E-02
10	0.1000E 00	1	CEN	0.4785E-13	-0.1000E 04	0.2921E-02	0.4785E-13	-0.1000E 04	0.2921E-02

BUCKLING LOAD PARAMETERS

0.51577D 00   0.47693D 01

BUCKLING MODE SHAPES

NODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
22	1	5.013E-03	-1.275E-01	0.0	0.0	0.0	0.0
	2	5.132E-03	4.278E-02	0.0	0.0	0.0	0.0
21	1	-5.013E-03	-1.275E-01	0.0	0.0	0.0	0.0
	2	-5.132E-03	4.278E-02	0.0	0.0	0.0	0.0
20	1	4.951E-03	-1.075E-01	0.0	0.0	0.0	0.0
	2	4.570E-03	6.228E-02	0.0	0.0	0.0	0.0
19	1	-4.951E-03	-1.075E-01	0.0	0.0	0.0	0.0
	2	-4.570E-03	6.228E-02	0.0	0.0	0.0	0.0
18	1	4.768E-03	-9.807E-02	0.0	0.0	0.0	0.0
	2	3.009E-03	7.751E-02	0.0	0.0	0.0	0.0
17	1	-4.768E-03	-9.807E-02	0.0	0.0	0.0	0.0
	2	-3.009E-03	7.751E-02	0.0	0.0	0.0	0.0
16	1	4.467E-03	-6.959E-02	0.0	0.0	0.0	0.0
	2	7.912E-04	8.514E-02	0.0	0.0	0.0	0.0
15	1	-4.467E-03	-6.959E-02	0.0	0.0	0.0	0.0
	2	-7.912E-04	8.514E-02	0.0	0.0	0.0	0.0
14	1	4.056E-03	-5.254E-02	0.0	0.0	0.0	0.0
	2	-1.595E-03	8.353E-02	0.0	0.0	0.0	0.0
13	1	-4.056E-03	-5.254E-02	0.0	0.0	0.0	0.0
	2	1.595E-03	8.353E-02	0.0	0.0	0.0	0.0
12	1	3.545E-03	-3.733E-02	0.0	0.0	0.0	0.0
	2	-3.629E-03	7.303E-02	0.0	0.0	0.0	0.0
11	1	-3.545E-03	-3.733E-02	0.0	0.0	0.0	0.0
	2	3.629E-03	7.303E-02	0.0	0.0	0.0	0.0
10	1	2.946E-03	-2.434E-02	0.0	0.0	0.0	0.0
	2	-4.868E-03	5.595E-02	0.0	0.0	0.0	0.0
9	1	-2.946E-03	-2.434E-02	0.0	0.0	0.0	0.0
	2	4.868E-03	5.595E-02	0.0	0.0	0.0	0.0
8	1	2.276E-03	-1.389E-02	0.0	0.0	0.0	0.0
	2	-5.046E-03	3.603E-02	0.0	0.0	0.0	0.0

7	1	-2.276E-03	-1.389E-02	0.0	0.0	0.0	0.0
	2	5.046E-03	3.603E-02	0.0	0.0	0.0	0.0
6	1	1.549E-03	-6.238E-03	0.0	0.0	0.0	0.0
	2	-4.128E-03	1.760E-02	0.0	0.0	0.0	0.0
5	1	-1.549E-03	-6.238E-03	0.0	0.0	0.0	0.0
	2	4.128E-03	1.760E-02	0.0	0.0	0.0	0.0
4	1	7.842E-04	-1.569E-03	0.0	0.0	0.0	0.0
	2	-2.314E-03	4.651E-03	0.0	0.0	0.0	0.0
3	1	-7.842E-04	-1.569E-03	0.0	0.0	0.0	0.0
	2	2.314E-03	4.651E-03	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0

****
EVALUATION OF DESIGN NUMBER 0
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.1000E-01	1	1
MIN	0.1000E-01	1	10

MAX BUCK RATIOS	LOAD COND
0.1939E 01	1
0.2097E 00	1

UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS 1.939AND DETERMINED BY BUCKLING CONSTRAINTS

DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.1939E 00	0.1939E 00	0.1939E 00	0.1939E 00	0.1939E 00	0.1939E 00	0.1939E 00	0.1939E 00	0.1939E 00	0.1939E 00

STRUCTURAL WEIGHT= 0.9694E 02

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR BUCKLING CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	0.13252E 01
2	ACT	0.12609E 01
3	ACT	0.11382E 01

4	ACT	0.96951E 00
5	ACT	0.77117E 00
6	ACT	0.56262E 00
7	ACT	0.36429E 00
8	ACT	0.19557E 00
9	ACT	0.72993E-01
10	ACT	0.85512E-02

NO. OF ACTIVE BUCKLING CONSTRAINTS ARE 1

****
ANALYSIS OF DESIGN NUMBER 4
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
22	1	-1.023E-01	-3.957E-14	0.0	0.0	0.0	0.0
21	1	-1.023E-01	-3.957E-14	0.0	0.0	0.0	0.0
20	1	-6.093E-02	-3.397E-14	0.0	0.0	0.0	0.0
19	1	-6.093E-02	-3.397E-14	0.0	0.0	0.0	0.0
18	1	-4.575E-02	-2.842E-14	0.0	0.0	0.0	0.0
17	1	-4.575E-02	-2.842E-14	0.0	0.0	0.0	0.0
16	1	-3.616E-02	-2.296E-14	0.0	0.0	0.0	0.0
15	1	-3.616E-02	-2.296E-14	0.0	0.0	0.0	0.0
14	1	-2.891E-02	-1.772E-14	0.0	0.0	0.0	0.0
13	1	-2.891E-02	-1.772E-14	0.0	0.0	0.0	0.0
12	1	-2.292E-02	-1.293E-14	0.0	0.0	0.0	0.0
11	1	-2.292E-02	-1.293E-14	0.0	0.0	0.0	0.0
10	1	-1.768E-02	-8.708E-15	0.0	0.0	0.0	0.0
9	1	-1.768E-02	-8.708E-15	0.0	0.0	0.0	0.0
8	1	-1.291E-02	-5.137E-15	0.0	0.0	0.0	0.0
7	1	-1.291E-02	-5.137E-15	0.0	0.0	0.0	0.0
6	1	-8.459E-03	-2.390E-15	0.0	0.0	0.0	0.0
5	1	-8.459E-03	-2.390E-15	0.0	0.0	0.0	0.0
4	1	-4.187E-03	-6.245E-16	0.0	0.0	0.0	0.0
3	1	-4.187E-03	-6.246E-16	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

1	2	3	4	5	6	7	8	9	10

0 0.2389E 00 0.2341E 00 0.2244E 00 0.2100E 00 0.1908E 00 0.1668E 00 0.1380E 00 0.1043E 00 0.6585E-01 0.2418E-01

ANALYSIS OF MEMBRANE ELEMENTS, CONSTRN CODE= 2

ELEMENT	SHEET THICKNESS	LOAD COND	LOCATION	/---MEMBRANE FORCES IN LOCAL COORDS---/ NXX	NYY	NXY	/--MEMBRANE FORCES IN MATERIAL COORDS-/ N11	N22	N12
1	0.2389E 00	1	CEN	-0.1000E 04	0.4035E-13	-0.1602E-03	-0.1000E 04	0.4035E-13	-0.1602E-03
2	0.2341E 00	1	CEN	-0.1000E 04	0.2325E-13	-0.2607E-03	-0.1000E 04	0.2325E-13	-0.2607E-03
3	0.2244E 00	1	CEN	-0.1000E 04	-0.1590E-13	-0.4477E-03	-0.1000E 04	-0.1590E-13	-0.4477E-03
4	0.2100E 00	1	CEN	-0.1000E 04	-0.1171E-13	-0.4211E-03	-0.1000E 04	-0.1171E-13	-0.4211E-03
5	0.1908E 00	1	CEN	-0.1000E 04	-0.2293E-13	-0.3463E-03	-0.1000E 04	-0.2293E-13	-0.3463E-03
6	0.1668E 00	1	CEN	-0.1000E 04	0.7751E-13	-0.2649E-02	-0.1000E 04	0.7751E-13	-0.2649E-02
7	0.1380E 00	1	CEN	-0.1000E 04	0.1059E-12	-0.7539E-03	-0.1000E 04	0.1059E-12	-0.7539E-03
8	0.1043E 00	1	CEN	-0.1000E 04	0.1647E-12	-0.9089E-03	-0.1000E 04	0.1647E-12	-0.9089E-03
9	0.6585E-01	1	CEN	-0.1000E 04	0.3236E-14	-0.4662E-03	-0.1000E 04	0.3236E-14	-0.4662E-03
10	0.2418E-01	1	CEN	-0.6914E-14	-0.1000E 04	-0.6347E-04	-0.6914E-14	-0.1000E 04	-0.6347E-04

BUCKLING LOAD PARAMETERS

0.99923D 00 0.62089D 01

BUCKLING MODE SHAPES

NODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
22	1	-6.097E-03	1.227E-01	0.0	0.0	0.0	0.0
	2	8.742E-03	4.743E-02	0.0	0.0	0.0	0.0
21	1	6.097E-03	1.227E-01	0.0	0.0	0.0	0.0
	2	-8.742E-03	4.743E-02	0.0	0.0	0.0	0.0
20	1	-5.518E-03	9.935E-02	0.0	0.0	0.0	0.0
	2	4.509E-03	7.462E-02	0.0	0.0	0.0	0.0
19	1	5.518E-03	9.935E-02	0.0	0.0	0.0	0.0
	2	-4.509E-03	7.462E-02	0.0	0.0	0.0	0.0
18	1	-4.903E-03	7.848E-02	0.0	0.0	0.0	0.0
	2	8.356E-04	8.541E-02	0.0	0.0	0.0	0.0
17	1	4.903E-03	7.848E-02	0.0	0.0	0.0	0.0
	2	-8.356E-04	8.541E-02	0.0	0.0	0.0	0.0
16	1	-4.289E-03	6.007E-02	0.0	0.0	0.0	0.0
	2	-1.796E-03	8.348E-02	0.0	0.0	0.0	0.0
15	1	4.289E-03	6.007E-02	0.0	0.0	0.0	0.0
	2	1.796E-03	8.348E-02	0.0	0.0	0.0	0.0
14	1	-3.676E-03	4.413E-02	0.0	0.0	0.0	0.0
	2	-3.449E-03	7.294E-02	0.0	0.0	0.0	0.0
13	1	3.676E-03	4.413E-02	0.0	0.0	0.0	0.0
	2	3.449E-03	7.294E-02	0.0	0.0	0.0	0.0
12	1	-3.063E-03	3.064E-02	0.0	0.0	0.0	0.0
	2	-4.235E-03	5.751E-02	0.0	0.0	0.0	0.0
11	1	3.063E-03	3.064E-02	0.0	0.0	0.0	0.0
	2	4.235E-03	5.751E-02	0.0	0.0	0.0	0.0
10	1	-2.450E-03	1.961E-02	0.0	0.0	0.0	0.0
	2	-4.285E-03	4.042E-02	0.0	0.0	0.0	0.0
9	1	2.450E-03	1.961E-02	0.0	0.0	0.0	0.0
	2	4.285E-03	4.042E-02	0.0	0.0	0.0	0.0
8	1	-1.838E-03	1.103E-02	0.0	0.0	0.0	0.0
	2	-3.740E-03	2.432E-02	0.0	0.0	0.0	0.0

7	1	1.838E-03	1.103E-02	0.0	0.0	0.0	0.0
	2	3.740E-03	2.432E-02	0.0	0.0	0.0	0.0
6	1	-1.225E-03	4.903E-03	0.0	0.0	0.0	0.0
	2	-2.745E-03	1.131E-02	0.0	0.0	0.0	0.0
5	1	1.225E-03	4.903E-03	0.0	0.0	0.0	0.0
	2	2.745E-03	1.131E-02	0.0	0.0	0.0	0.0
4	1	-6.126E-04	1.226E-03	0.0	0.0	0.0	0.0
	2	-1.448E-03	2.904E-03	0.0	0.0	0.0	0.0
3	1	6.126E-04	1.226E-03	0.0	0.0	0.0	0.0
	2	1.448E-03	2.904E-03	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0

****
EVALUATION OF DESIGN NUMBER 4
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.4135E-01	1	10
MIN	0.4187E-02	1	1

MAX BUCK RATIOS	LOAD COND
0.1001E 01	1
0.1611E 00	1

DESIGN IS CRITICAL

STRUCTURAL WEIGHT= 0.7986E 02

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR BUCKLING CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	0.10012E 01
2	ACT	0.10012E 01
3	ACT	0.10014E 01
4	ACT	0.10016E 01
5	ACT	0.10020E 01
6	ACT	0.10026E 01
7	ACT	0.10037E 01
8	ACT	0.10060E 01
9	ACT	0.10121E 01
10	ACT	0.93291E 00

NO. OF ACTIVE BUCKLING CONSTRAINTS ARE 1
BUCKLING - CRITICAL DESIGN HAS CONVERGED

0. SHEAR PANEL ELEMENTS

0.1 Cantilever Beam of Plate-Girder Construction

Figure 0.1.1 shows a plate girder modelled with bars and shear panels. This problem is quite similar to the cantilever beam in Sec. N.1, except that now the shear deformation of the web has a significant effect on the buckling load.

Each shear panel is sized independently, but symmetry conditions are imposed on the bar elements that make up the flanges: $A_1 = A_2$, $A_3 = A_4$, ..., $A_{19} = A_{20}$. In addition, all the transverse bars are to be of the same size, i.e., $A_{21} = A_{22} = \ldots = A_{30}$. The design variable numbers of the elements can be found in Table 0.1.1.

The design data used in the program is listed below.

Material properties:

	E(psi)	ν	$\sigma_t^* = \sigma_c^*$ (psi)	σ_s^* (psi)	ρ (lb/in^3)
Shear panels	200	0	-	3000	1.0
Bars	400	-	5000	-	1.0

Element sizes:

	A (initial)	A* (minimum)
Shear panels (in)	0.1	0.01
Flange bars (in^2)	2.0	0.1
Transv. bars (in^2)	1.0	0.1

Local buckling of all the elements is to be neglected in the design.

The design history is summarized in Table 0.1.1. Each design was governed by constraints on global buckling and the element sizes, i.e., the stress constraints never became active.

After six redesigns, the convergence criteria are not yet satisfied, although no decrease in structural weight occurred in the last three design cycles. The difficulty can be identified by inspection of the optimality indices in Evaluation of Design No. 5: the indices of all design variables, except that of No. 11 (shear panel No. 1), are within the acceptable limits, i.e., only that one element fails to satisfy the optimality criterion.

Additional design cycles would not bring an improvement, because we have encountered a fairly common convergence difficulty: oscillation of the design about the optimal point due to relaxation factor that is too large ($\alpha = 0.5$ was used). The oscillatory behavior can be observed in Table 0.1.1, where design variable No. 11 is seen to fluctuate from design No. 3 onwards; an even better indication is the optimality index of the element, shown below.

	Design Number					
	0	1	2	3	4	5
Opt. Index of Des. Var. 11	0.0301	0.1517	0.6261	1.3781	0.7076	1.2672

The design would converge readily if the procedure recommended in Sec. D.1 of Vol. 1 is followed. It was suggested that the "normal" value of the relaxation factor α (in our case $\alpha = 0.5$) is to be used for a few design cycles, after which the designs are to be evaluated so that appropriate changes to α can be made. In the current problem, the oscillatory character of the solution could be diagnosed after three redesign cycles, calling for an increase of α (under-relaxation). With $\alpha = 0.7$, the problem would probably converge after a few additional design cycles. The optimal design, however would have essentially the same weight as the final design in Table 0.1.1.

Two checks of DESAP 2 can be made using this problem. First, an analytical solution for the buckling load of the initial design is available on p. 132 of Ref. [13]:

$$P_{cr} = \frac{P_e}{1+nP_e/(AG)} \qquad (0.1.1)$$

where P_e is the critical load without shear deformation of the web (Euler buckling load), A and G are the cross-sectional area and the shear modulus of the web, respectively, and n represents a shape factor (n = 1 for our case). For the problem at hand, we get $P_e = 1.579$ lb. and $P_{cr} = 1.364$ lb., which compares favorably with $P_{cr} = 2 \times 0.6834 = 1.367$ lb. obtained from DESAP 2 (see Buckling Load Parameters of Design No. 0).

The second check is obtained by comparing the final cross-sectional areas of the flange elements with the analytical solution of Ref. [15], as was done in Sec. N.1. For this specific problem, the optimal flange area can be shown to be

$$A(x) = 3.125 \ (1-x^2/\ell^2) \ . \tag{0.1.2}$$

Since (0.1.2) does not include the weakening effect of shear deformation, the cross-sectional areas obtained from DESAP 2 should be somewhat larger, which they are indeed, as can be seen in Table 0.1.1.

Special notes on input-output:

1) NSCALE = 1 in Design Control Data informs the computer that uniform scaling is an exact operation, since the stiffness matrix of each element has the form $[K_i] = [k_i]A_i^n$, where n = 1.

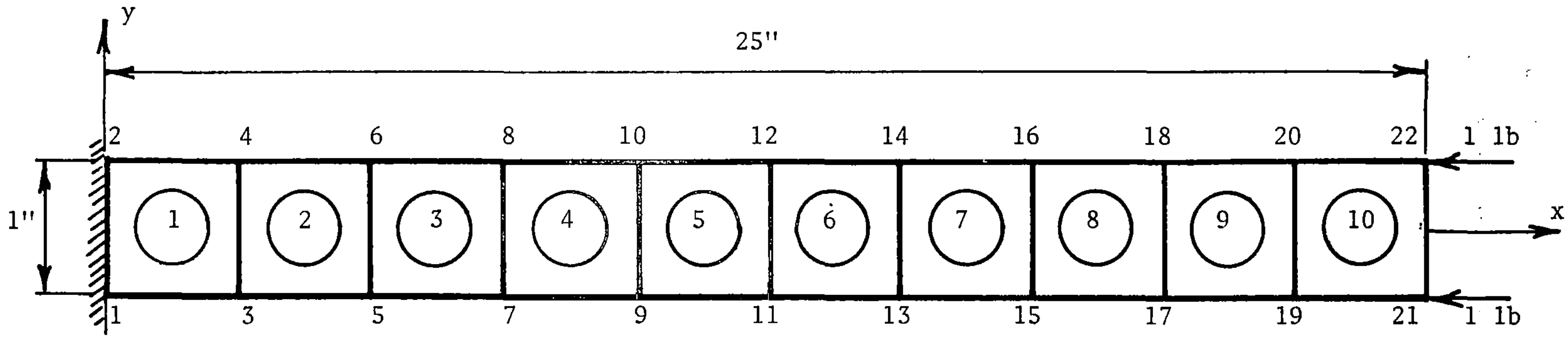
2) Extensive use was made of the automatic generation of nodal and element data.

3) Local buckling of bars was eliminated as a design consideration by leaving the moments of inertia blank on the Geometric Property Card of the truss elements. Since the computer replaces the blanks by 10^6, the Euler buckling load of each element is too high to govern the design.

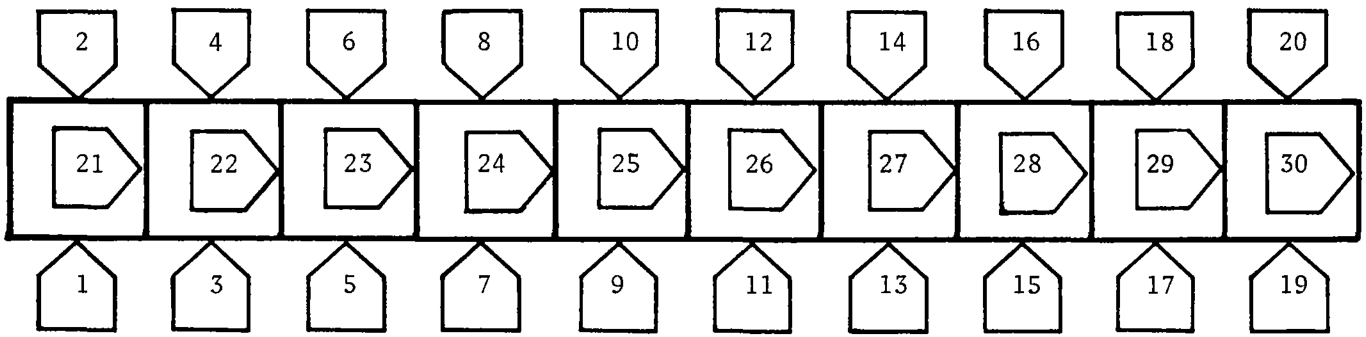
4) By setting ISU = 0 (see the Boundary Condition Code in Processed Element Data), local buckling of the shear panels was excluded from redesign equations.

5) NMODE = 2 in Buckling Control Data requests the use of two buckling modes in the redesign process. Because only one mode governs, an identical design would be obtained with NMODE = 1, but all information pertaining to the second mode would not be available in the computer printout.

6) INDET = 1 informs the computer that the prebuckling state is statically determinate. Consequently, the geometric stiffness matrix of the structure does not have to be updated after each redesign.



a) Node numbers and element numbers of shear panels



b) Element numbers of bar elements

Figure 0.1.1

Layout of Plate Girder Showing Node and Element Numbers

	Des. Var. Number	Element Numbers	Critical, Scaled Designs							Analytical*
			0	1	2	3	4	5	6	
Bars (sq. in.)	1	1,2	2.923	3.045	3.273	3.318	3.346	3.345	3.348	3.117
	2	3,4	2.923	2.981	3.189	3.235	3.256	3.260	3.259	3.055
	3	5,6	2.923	2.858	3.029	3.087	3.105	3.110	3.109	2.914
	4	7,8	2.923	2.690	2.801	2.877	2.892	2.897	2.896	2.742
	5	9,10	2.923	2.493	2.509	2.604	2.616	2.622	2.621	2.492
	6	11,12	2.923	2.285	2.162	2.267	2.277	2.284	2.284	2.180
	7	13,14	2.923	2.087	1.778	1.864	1.876	1.884	1.884	1.805
	8	15,16	2.923	1.192	1.395	1.396	1.412	1.421	1.422	1.336
	9	17,18	2.923	1.797	1.074	0.888	0.885	0.895	0.897	0.867
	10	19,20	2.923	1.733	0.887	0.493	0.340	0.310	0.309	0.305
	21	21-30	1.461	0.862	0.431	0.216	0.108	0.100	0.100	-
Shear Panels (in.)	11	1	1.461	.0888	.0512	.0416	.0495	.0422	.0479	-
	12	2	1.461	.1092	.0989	.0935	.0955	.0950	.0950	-
	13	3	1.461	.1479	.1545	.1434	.1456	.1451	.1450	-
	14	4	1.461	.2012	.2065	.1939	.1958	.1951	.1950	-
	15	5	1.461	.2639	.2566	.2448	.2459	.2452	.2449	-
	16	6	1.461	.3298	.3053	.2962	.2960	.2953	.2949	-
	17	7	1.461	.3925	.3507	.3480	.3461	.3454	.3449	-
	18	8	1.461	.4458	.3896	.3991	.3963	.3955	.3949	-
	19	9	1.461	.4458	.4185	.4450	.4458	.4457	.4450	-
	20	10	1.461	.4845	.4338	.4745	.4854	.4933	.4949	-
Weight (lb.)			164.4	135.5	121.5	119.0	117.8	117.9	117.9	-

*Neglects shear deformation

Table 0.1.1

Design History of Element Sizes and Total Structural Weight

```
123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
  BUCKLING DESIGN - SHEAR PANEL AND TRUSS ELEMENT
    22     2      1    21
     6     3      1 0.025                        1     1      1  0.6
     1     1      1   -1     -1    -1    -1              -0.5
     2     1      1                                    0.5
     3                                       2.5         -0.5
    21                                       25.        -0.5                           2
     4                                       2.5         0.5
    22                                        25.        0.5                           2
     1    30      1     1     1      1
     1     1   1.0
            400.                       5000.       5000.
     1        0.5




     2     2      4     1     1      1
     4     4      6     1     1      2
     6     6      8     1     1      3
     8     8     10     1     1      4
    10    10     12     1     1      5
    12    12     14     1     1      6
    14    14     16     1     1      7
    16    16     18     1     1      8
    18    18     20     1     1      9
    20    20     22     1     1     10
    30    21     22     1     1     21                                                        2
     4    10      1     1     1
     1     1  1.
              200.                     3000.



     1     1      3     4     2      1    11
     2     3      5     6     4      1    12
     3     5      7     8     6      1    13
     4     7      9    10     8      1    14
     5     9     11    12    10      1    15
     6    11     13    14    12      1    16
     7    13     15    16    14      1    17
     8    15     17    18    16      1    18
     9    17     19    20    18      1    19
    10    19     21    22    20      1    20

   1.0                  2     1      2 0.5           0.8
    21     1  -1.
    22     1  -1.

    10   2.0          0.1
    20   0.1          0.01
    21 1.0                  0.1
123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H

```

Echo of Input Cards

BUCKLING DESIGN - SHEAR PANEL AND TRUSS ELEMENT

```
NUMBER OF NODAL POINTS   =   22
NUMBER OF ELEMENT TYPES  =    2
NUMBER OF LOAD CASES     =    1
NUMBER OF DES. VARIABLES =   21
```

DESIGN CONTROL DATA

```
NCYCL =     6
KSCALE=     1
DELTA =  0.2500E-01
EPSIL =  0.1000E 00
LBUCK =     1
```

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z		T
1	1	1	-1	-1	-1	-1	0.0	-0.500	0.0	0	0.0
2	1	1	0	0	0	0	0.0	0.500	0.0	0	0.0
3	0	0	0	0	0	0	2.500	-0.500	0.0	0	0.0
21	0	0	0	0	0	0	25.000	-0.500	0.0	2	0.0
4	0	0	0	0	0	0	2.500	0.500	0.0	0	0.0
22	0	0	0	0	0	0	25.000	0.500	0.0	2	0.0

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z	T
1	1	1	-1	-1	-1	-1	0.0	-0.500	0.0	0.0
2	1	1	-1	-1	-1	-1	0.0	0.500	0.0	0.0
3	0	0	-1	-1	-1	-1	2.500	-0.500	0.0	0.0
4	0	0	-1	-1	-1	-1	2.500	0.500	0.0	0.0
5	0	0	-1	-1	-1	-1	5.000	-0.500	0.0	0.0
6	0	0	-1	-1	-1	-1	5.000	0.500	0.0	0.0
7	0	0	-1	-1	-1	-1	7.500	-0.500	0.0	0.0
8	0	0	-1	-1	-1	-1	7.500	0.500	0.0	0.0
9	0	0	-1	-1	-1	-1	10.000	-0.500	0.0	0.0
10	0	0	-1	-1	-1	-1	10.000	0.500	0.0	0.0
11	0	0	-1	-1	-1	-1	12.500	-0.500	0.0	0.0
12	0	0	-1	-1	-1	-1	12.500	0.500	0.0	0.0
13	0	0	-1	-1	-1	-1	15.000	-0.500	0.0	0.0
14	0	0	-1	-1	-1	-1	15.000	0.500	0.0	0.0
15	0	0	-1	-1	-1	-1	17.500	-0.500	0.0	0.0
16	0	0	-1	-1	-1	-1	17.500	0.500	0.0	0.0
17	0	0	-1	-1	-1	-1	20.000	-0.500	0.0	0.0
18	0	0	-1	-1	-1	-1	20.000	0.500	0.0	0.0
19	0	0	-1	-1	-1	-1	22.500	-0.500	0.0	0.0
20	0	0	-1	-1	-1	-1	22.500	0.500	0.0	0.0
21	0	0	-1	-1	-1	-1	25.000	-0.500	0.0	0.0
22	0	0	-1	-1	-1	-1	25.000	0.500	0.0	0.0

EQUATION NUMBERS

Computer Printout

(Input data, the initial design and the last two designs only are reproduced)

N	X	Y	Z	XX	YY	ZZ
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	1	2	0	0	0	0
4	3	4	0	0	0	0
5	5	6	0	0	0	0
6	7	8	0	0	0	0
7	9	10	0	0	0	0
8	11	12	0	0	0	0
9	13	14	0	0	0	0
10	15	16	0	0	0	0
11	17	18	0	0	0	0
12	19	20	0	0	0	0
13	21	22	0	0	0	0
14	23	24	0	0	0	0
15	25	26	0	0	0	0
16	27	28	0	0	0	0
17	29	30	0	0	0	0
18	31	32	0	0	0	0
19	33	34	0	0	0	0
20	35	36	0	0	0	0
21	37	38	0	0	0	0
22	39	40	0	0	0	0

```
NUMBER OF TRUSS ELEMENTS                  =   30
CONSTRUCTION CODE                         =    1
NUMBER OF MATERIALS                       =    1
NUMBER OF TEMPS FOR WHICH MATL PROPS GIVEN=    1
NUMBER OF DIFFERENT GEOMETRIES PROPS GIVEN=    1
```

MATERIAL PROPERTY CARDS

MATERIAL NUMBER	NUMBER OF TEMPS	SPECIFIC WEIGHT	TEMP	YOUNGS MODULUS	COEFFT OF THERM EXPAN	/--ALLOWABLE STRESSES--/ TENSION	COMPRESSION
1	1	0.1000E 01	0.0	0.4000E 03	0.0	0.5000E 04	0.5000E 04

GEOMETRIC PROPERTY CARDS

GEOMETRY NUMBER	X-SECT AREA	/--MOMENTS OF INERTIA--/ YY	ZZ
1	0.5000D 00	0.1000E 07	0.1000E 07

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0	0.0	0.0	0.0
Y-DIR	0.0	0.0	0.0	0.0
Z-DIR	0.0	0.0	0.0	0.0
TEMP	0.0	0.0	0.0	0.0

PROCESSED ELEMENT DATA

ELEMENT NUMBER	/-NODE NOS-/ I	J	/--ELEMENT ID NOS-/ MATL	GEOMY	D VAR	DESIGN VAR FRACTION	REFERENCE TEMP	END FIXITY COEFFICIENTS YY	ZZ	BAND WIDTH
1	1	3	1	1	1	0.1000E 01	0.0	0.1000D 01	0.1000D 01	2
2	2	4	1	1	1	0.1000E 01	0.0	0.1000D 01	0.1000D 01	2
3	3	5	1	1	2	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
4	4	6	1	1	2	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
5	5	7	1	1	3	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
6	6	8	1	1	3	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
7	7	9	1	1	4	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
8	8	10	1	1	4	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
9	9	11	1	1	5	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
10	10	12	1	1	5	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
11	11	13	1	1	6	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
12	12	14	1	1	6	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
13	13	15	1	1	7	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
14	14	16	1	1	7	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
15	15	17	1	1	8	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
16	16	18	1	1	8	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
17	17	19	1	1	9	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
18	18	20	1	1	9	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
19	19	21	1	1	10	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
20	20	22	1	1	10	0.1000E 01	0.0	0.1000D 01	0.1000D 01	6
21	3	4	1	1	21	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4
22	5	6	1	1	21	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4
23	7	8	1	1	21	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4
24	9	10	1	1	21	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4

25	11	12	1	1	21	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4
26	13	14	1	1	21	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4
27	15	16	1	1	21	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4
28	17	18	1	1	21	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4
29	19	20	1	1	21	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4
30	21	22	1	1	21	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4

NUMBER OF SHEAR PANEL ELEMENTS	=	10
CONSTRUCTION CODE	=	1
NUMBER OF MATERIALS	=	1
NUMBER OF TEMPS FOR WHICH MATL PROPS GIVEN	=	1

MATERIAL PROPERTY CARDS

MATERIAL NUMBER	NUMBER OF TEMPS	SPECIFIC WEIGHT	TEMP	YOUNGS MODULUS	POISSN RATIO	ALLOWABLE SHEAR
1	1	0.1000E 01	0.0	0.2000E 03	0.0	0.3000E 04

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0	0.0	0.0	0.0
Y-DIR	0.0	0.0	0.0	0.0
Z-DIR	0.0	0.0	0.0	0.0

PROCESSED ELEMENT DATA

ELEMENT	/-------NODE NOS			------//--EL ID NOS-/		BOUND	DES VAR	/--EFFECT PANEL DIMNS--/		BAND
NUMBER	I	J	K	L	MATL	D VAR	CODE	FRACTION	LONGER	SHORTER	WIDTH
1	1	3	4	2	1	11	0	0.1000E 01	0.0	0.0	4
2	3	5	6	4	1	12	0	0.1000E 01	0.0	0.0	8
3	5	7	8	6	1	13	0	0.1000E 01	0.0	0.0	8
4	7	9	10	8	1	14	0	0.1000E 01	0.0	0.0	8
5	9	11	12	10	1	15	0	0.1000E 01	0.0	0.0	8
6	11	13	14	12	1	16	0	0.1000E 01	0.0	0.0	8
7	13	15	16	14	1	17	0	0.1000E 01	0.0	0.0	8
8	15	17	18	16	1	18	0	0.1000E 01	0.0	0.0	8
9	17	19	20	18	1	19	0	0.1000E 01	0.0	0.0	8
10	19	21	22	20	1	20	0	0.1000E 01	0.0	0.0	8

STRUCTURE	STRUCTURE LOAD MULTIPLIERS			
LOAD CASE	A	B	C	D
1	0.0	0.0	0.0	0.0

BUCKLING CONTROL DATA

COEFFT = 1.00000
MODEIN = 0
NMODE = 2
INDET = 1
NVEC = 2
ALPA = 0.50000
OMEGA = 0.80000

NODAL POINT LOADS

NODE NO.	LOAD CASE	APPLIED LOADS PX	PY	PZ	MX	MY	MZ

21	1	-0.100D 01	0.0	0.0	0.0	0.0	0.0
22	1	-0.100D 01	0.0	0.0	0.0	0.0	0.0

DESIGN VARIABLE INPUT DATA

DESIGN VARIABLE NUMBER	INITIAL VALUE	MIN ALLOWABLE VALUE
1	0.2000E 01	0.1000E 00
2	0.2000E 01	0.1000E 00
3	0.2000E 01	0.1000E 00
4	0.2000E 01	0.1000E 00
5	0.2000E 01	0.1000E 00
6	0.2000E 01	0.1000E 00
7	0.2000E 01	0.1000E 00
8	0.2000E 01	0.1000E 00
9	0.2000E 01	0.1000E 00
10	0.2000E 01	0.1000E 00
11	0.1000E 00	0.1000E-01
12	0.1000E 00	0.1000E-01
13	0.1000E 00	0.1000E-01
14	0.1000E 00	0.1000E-01
15	0.1000E 00	0.1000E-01
16	0.1000E 00	0.1000E-01
17	0.1000E 00	0.1000E-01
18	0.1000E 00	0.1000E-01
19	0.1000E 00	0.1000E-01
20	0.1000E 00	0.1000E-01
21	0.1000E 01	0.1000E 00

```
TOTAL NUMBER OF EQUATIONS      =   40
BANDWIDTH                      =    8
NUMBER OF EQUATIONS IN A BLOCK =   40
NUMBER OF BLOCKS               =    1
```

```
********************************
ANALYSIS OF DESIGN NUMBER   0
********************************
```

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
22	1	-3.125E-02	-1.693E-15	0.0	0.0	0.0	0.0
21	1	-3.125E-02	-1.693E-15	0.0	0.0	0.0	0.0
20	1	-2.812E-02	-1.419E-15	0.0	0.0	0.0	0.0
19	1	-2.812E-02	-1.419E-15	0.0	0.0	0.0	0.0
18	1	-2.500E-02	-1.146E-15	0.0	0.0	0.0	0.0
17	1	-2.500E-02	-1.146E-15	0.0	0.0	0.0	0.0
16	1	-2.187E-02	-8.899E-16	0.0	0.0	0.0	0.0
15	1	-2.187E-02	-8.899E-16	0.0	0.0	0.0	0.0
14	1	-1.875E-02	-6.595E-16	0.0	0.0	0.0	0.0
13	1	-1.875E-02	-6.595E-16	0.0	0.0	0.0	0.0
12	1	-1.562E-02	-4.685E-16	0.0	0.0	0.0	0.0
11	1	-1.563E-02	-4.685E-16	0.0	0.0	0.0	0.0
10	1	-1.250E-02	-3.040E-16	0.0	0.0	0.0	0.0
9	1	-1.250E-02	-3.040E-16	0.0	0.0	0.0	0.0
8	1	-9.375E-03	-1.737E-16	0.0	0.0	0.0	0.0
7	1	-9.375E-03	-1.737E-16	0.0	0.0	0.0	0.0
6	1	-6.250E-03	-7.767E-17	0.0	0.0	0.0	0.0
5	1	-6.250E-03	-7.767E-17	0.0	0.0	0.0	0.0
4	1	-3.125E-03	-1.882E-17	0.0	0.0	0.0	0.0
3	1	-3.125E-03	-1.882E-17	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

1	2	3	4	5	6	7	8	9	10

0	0.2000E 01	0.2000E 01	0.2000E 01	0.2000E 01	0.2000E 01	0.2000E 01	0.2000E 01	0.2000E 01	0.2000E 01	0.2000E 01
10	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00
20	0.1000E 01									

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FORCE
1	0.2000E 01	1	-0.1000E 01
2	0.2000E 01	1	-0.1000E 01
3	0.2000E 01	1	-0.1000E 01
4	0.2000E 01	1	-0.1000E 01
5	0.2000E 01	1	-0.1000E 01
6	0.2000E 01	1	-0.1000E 01
7	0.2000E 01	1	-0.1000E 01
8	0.2000E 01	1	-0.1000E 01
9	0.2000E 01	1	-0.1000E 01
10	0.2000E 01	1	-0.1000E 01
11	0.2000E 01	1	-0.1000E 01
12	0.2000E 01	1	-0.1000E 01
13	0.2000E 01	1	-0.1000E 01
14	0.2000E 01	1	-0.1000E 01
15	0.2000E 01	1	-0.1000E 01
16	0.2000E 01	1	-0.1000E 01
17	0.2000E 01	1	-0.1000E 01
18	0.2000E 01	1	-0.1000E 01
19	0.2000E 01	1	-0.1000E 01
20	0.2000E 01	1	-0.1000E 01
21	0.1000E 01	1	0.8650E-18
22	0.1000E 01	1	-0.2580E-17
23	0.1000E 01	1	-0.6419E-17
24	0.1000E 01	1	-0.1880E-17
25	0.1000E 01	1	-0.9876E-17
26	0.1000E 01	1	-0.3930E-17
27	0.1000E 01	1	-0.1115E-16
28	0.1000E 01	1	-0.1996E-17
29	0.1000E 01	1	-0.5448E-17
30	0.1000E 01	1	-0.1077E-16

ANALYSIS OF SHEAR PANELS, CONSTRN CODE= 1

		LOAD	/--------------SHEAR FLOW AT NODES-------------/				AVERAGE
ELEMENT	THICKNESS	COND	I	J	K	L	SHEAR FLOW
1	0.1000E 00	1	-0.4657E-08	-0.4657E-08	-0.4657E-08	-0.4657E-08	-0.4657E-08
2	0.1000E 00	1	-0.1211E-07	-0.1211E-07	-0.1211E-07	-0.1211E-07	-0.1211E-07
3	0.1000E 00	1	-0.7823E-07	-0.7823E-07	-0.7823E-07	-0.7823E-07	-0.7823E-07
4	0.1000E 00	1	-0.7637E-07	-0.7637E-07	-0.7637E-07	-0.7637E-07	-0.7637E-07
5	0.1000E 00	1	-0.5215E-07	-0.5215E-07	-0.5215E-07	-0.5215E-07	-0.5215E-07
6	0.1000E 00	1	-0.1919E-06	-0.1919E-06	-0.1919E-06	-0.1919E-06	-0.1919E-06
7	0.1000E 00	1	-0.1900E-06	-0.1900E-06	-0.1900E-06	-0.1900E-06	-0.1900E-06
8	0.1000E 00	1	-0.1881E-06	-0.1881E-06	-0.1881E-06	-0.1881E-06	-0.1881E-06
9	0.1000E 00	1	-0.1863E-06	-0.1863E-06	-0.1863E-06	-0.1863E-06	-0.1863E-06
10	0.1000E 00	1	-0.1844E-06	-0.1844E-06	-0.1844E-06	-0.1844E-06	-0.1844E-06

BUCKLING LOAD PARAMETERS

0.68432D 00   0.29894D 01

BUCKLING MODE SHAPES

MODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
22	1	-8.658E-02	3.186E 00	0.0	0.0	0.0	0.0
	2	4.255E-02	1.042E 00	0.0	0.0	0.0	0.0
21	1	8.658E-02	3.186E 00	0.0	0.0	0.0	0.0
	2	-4.255E-02	1.042E 00	0.0	0.0	0.0	0.0
20	1	-8.551E-02	2.688E 00	0.0	0.0	0.0	0.0
	2	3.772E-02	1.552E 00	0.0	0.0	0.0	0.0
19	1	8.551E-02	2.688E 00	0.0	0.0	0.0	0.0
	2	-3.772E-02	1.552E 00	0.0	0.0	0.0	0.0
18	1	-8.234E-02	2.202E 00	0.0	0.0	0.0	0.0
	2	2.432E-02	1.946E 00	0.0	0.0	0.0	0.0
17	1	8.234E-02	2.202E 00	0.0	0.0	0.0	0.0
	2	-2.432E-02	1.946E 00	0.0	0.0	0.0	0.0
16	1	-7.714E-02	1.740E 00	0.0	0.0	0.0	0.0
	2	5.714E-03	2.116E 00	0.0	0.0	0.0	0.0
15	1	7.714E-02	1.740E 00	0.0	0.0	0.0	0.0
	2	-5.714E-03	2.116E 00	0.0	0.0	0.0	0.0
14	1	-7.005E-02	1.313E 00	0.0	0.0	0.0	0.0
	2	-1.359E-02	2.046E 00	0.0	0.0	0.0	0.0
13	1	7.005E-02	1.313E 00	0.0	0.0	0.0	0.0
	2	1.359E-02	2.046E 00	0.0	0.0	0.0	0.0
12	1	-6.122E-02	9.333E-01	0.0	0.0	0.0	0.0
	2	-2.985E-02	1.792E 00	0.0	0.0	0.0	0.0
11	1	6.122E-02	9.333E-01	0.0	0.0	0.0	0.0
	2	2.985E-02	1.792E 00	0.0	0.0	0.0	0.0
10	1	-5.089E-02	6.085E-01	0.0	0.0	0.0	0.0
	2	-3.954E-02	1.344E 00	0.0	0.0	0.0	0.0
9	1	5.089E-02	6.085E-01	0.0	0.0	0.0	0.0
	2	3.954E-02	1.344E 00	0.0	0.0	0.0	0.0
8	1	-3.931E-02	3.473E-01	0.0	0.0	0.0	0.0
	2	-4.052E-02	8.599E-01	0.0	0.0	0.0	0.0
7	1	3.931E-02	3.473E-01	0.0	0.0	0.0	0.0
	2	4.052E-02	8.599E-01	0.0	0.0	0.0	0.0
6	1	-2.675E-02	1.560E-01	0.0	0.0	0.0	0.0
	2	-3.286E-02	4.085E-01	0.0	0.0	0.0	0.0
5	1	2.675E-02	1.560E-01	0.0	0.0	0.0	0.0
	2	3.286E-02	4.085E-01	0.0	0.0	0.0	0.0
4	1	-1.354E-02	3.923E-02	0.0	0.0	0.0	0.0
	2	-1.827E-02	1.044E-01	0.0	0.0	0.0	0.0
3	1	1.354E-02	3.923E-02	0.0	0.0	0.0	0.0
	2	1.827E-02	1.044E-01	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0

****
EVALUATION OF DESIGN NUMBER 0
****

STRESS RATIO   LOAD COND   DES VARIABLE

MAX	0.1000E 00	0	11
MIN	0.5000E-01	0	1

MAX BUCK RATIOS	LOAD COND
0.1461E 01	1
0.3345E 00	1

UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS 1.461AND DETERMINED BY BUCKLING CONSTRAINTS

DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.2923E 01	0.2923E 01	0.2923E 01	0.2923E 01	0.2923E 01	0.2923E 01	0.2923E 01	0.2923E 01	0.2923E 01	0.2923E 01
10	0.1461E 00	0.1461E 00	0.1461E 00	0.1461E 00	0.1461E 00	0.1461E 00	0.1461E 00	0.1461E 00	0.1461E 00	0.1461E 00
20	0.1461E 01									

STRUCTURAL WEIGHT= 0.1644E 03

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR BUCKLING CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	0.76543E 00
2	ACT	0.72820E 00
3	ACT	0.65738E 00
4	ACT	0.55991E 00
5	ACT	0.44533E 00
6	ACT	0.32484E 00
7	ACT	0.21026E 00
8	ACT	0.11279E 00
9	ACT	0.41971E-01
10	ACT	0.47413E-02
11	ACT	0.30071E-01
12	ACT	0.26621E 00
13	ACT	0.71539E 00
14	ACT	0.13336E 01
15	ACT	0.20604E 01
16	ACT	0.28245E 01
17	ACT	0.35513E 01
18	ACT	0.41695E 01
19	ACT	0.46187E 01
20	ACT	0.48549E 01
21	ACT	0.0

NO. OF ACTIVE BUCKLING CONSTRAINTS ARE 1

****
ANALYSIS OF DESIGN NUMBER 5
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
22	1	-4.795E-02	-2.243E-15	0.0	0.0	0.0	0.0
21	1	-4.795E-02	-2.244E-15	0.0	0.0	0.0	0.0
20	1	-2.777E-02	-1.949E-15	0.0	0.0	0.0	0.0
19	1	-2.777E-02	-1.950E-15	0.0	0.0	0.0	0.0
18	1	-2.079E-02	-1.653E-15	0.0	0.0	0.0	0.0
17	1	-2.079E-02	-1.653E-15	0.0	0.0	0.0	0.0
16	1	-1.639E-02	-1.366E-15	0.0	0.0	0.0	0.0
15	1	-1.639E-02	-1.367E-15	0.0	0.0	0.0	0.0
14	1	-1.307E-02	-1.102E-15	0.0	0.0	0.0	0.0
13	1	-1.307E-02	-1.102E-15	0.0	0.0	0.0	0.0
12	1	-1.034E-02	-8.624E-16	0.0	0.0	0.0	0.0
11	1	-1.034E-02	-8.621E-16	0.0	0.0	0.0	0.0
10	1	-7.952E-03	-6.502E-16	0.0	0.0	0.0	0.0
9	1	-7.952E-03	-6.495E-16	0.0	0.0	0.0	0.0
8	1	-5.795E-03	-4.629E-16	0.0	0.0	0.0	0.0
7	1	-5.795E-03	-4.628E-16	0.0	0.0	0.0	0.0
6	1	-3.785E-03	-3.071E-16	0.0	0.0	0.0	0.0
5	1	-3.785E-03	-3.072E-16	0.0	0.0	0.0	0.0
4	1	-1.868E-03	-1.718E-16	0.0	0.0	0.0	0.0
3	1	-1.868E-03	-1.719E-16	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

1	2	3	4	5	6	7	8	9	10

0	0.3345E 01	0.3260E 01	0.3110E 01	0.2897E 01	0.2622E 01	0.2284E 01	0.1884E 01	0.1421E 01	0.8952E 00	0.3097E 00
10	0.4222E-01	0.9501E-01	0.1451E 00	0.1951E 00	0.2452E 00	0.2953E 00	0.3454E 00	0.3955E 00	0.4457E 00	0.4933E 00
20	0.1000E 00									

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FORCE
1	0.3345E 01	1	-0.1000E 01
2	0.3345E 01	1	-0.1000E 01
3	0.3260E 01	1	-0.1000E 01
4	0.3260E 01	1	-0.1000E 01
5	0.3110E 01	1	-0.1000E 01
6	0.3110E 01	1	-0.1000E 01
7	0.2897E 01	1	-0.1000E 01
8	0.2897E 01	1	-0.1000E 01
9	0.2622E 01	1	-0.1000E 01
10	0.2622E 01	1	-0.1000E 01
11	0.2284E 01	1	-0.1000E 01
12	0.2284E 01	1	-0.1000E 01
13	0.1884E 01	1	-0.1000E 01
14	0.1884E 01	1	-0.1000E 01
15	0.1421E 01	1	-0.1000E 01
16	0.1421E 01	1	-0.1000E 01
17	0.8952E 00	1	-0.1000E 01
18	0.8952E 00	1	-0.1000E 01
19	0.3097E 00	1	-0.1000E 01
20	0.3097E 00	1	-0.1000E 01
21	0.1000E 00	1	0.1348E-17
22	0.1000E 00	1	0.4055E-17
23	0.1000E 00	1	-0.1950E-17
24	0.1000E 00	1	-0.2921E-16
25	0.1000E 00	1	-0.1334E-16
26	0.1000E 00	1	0.6324E-18
27	0.1000E 00	1	0.6869E-17
28	0.1000E 00	1	0.2524E-18
29	0.1000E 00	1	0.3862E-16
30	0.1000E 00	1	0.3166E-16

ANALYSIS OF SHEAR PANELS, CONSTRN CODE= 1

ELEMENT	THICKNESS	LOAD COND	/--------------SHEAR FLOW AT NODES-------------/ I	J	K	L	AVERAGE SHEAR FLOW
1	0.4222E-01	1	-0.5318E-08	-0.5318E-08	-0.5318E-08	-0.5318E-08	-0.5318E-08
2	0.9501E-01	1	-0.1133E-07	-0.1133E-07	-0.1133E-07	-0.1133E-07	-0.1133E-07
3	0.1451E 00	1	-0.7261E-07	-0.7261E-07	-0.7261E-07	-0.7261E-07	-0.7261E-07
4	0.1951E 00	1	-0.9990E-07	-0.9990E-07	-0.9990E-07	-0.9990E-07	-0.9990E-07
5	0.2452E 00	1	-0.2238E-06	-0.2238E-06	-0.2238E-06	-0.2238E-06	-0.2238E-06
6	0.2953E 00	1	-0.1831E-06	-0.1831E-06	-0.1831E-06	-0.1831E-06	-0.1831E-06
7	0.3454E 00	1	-0.2041E-06	-0.2041E-06	-0.2041E-06	-0.2041E-06	-0.2041E-06
8	0.3955E 00	1	-0.2332E-06	-0.2332E-06	-0.2332E-06	-0.2332E-06	-0.2332E-06
9	0.4457E 00	1	-0.1366E-05	-0.1366E-05	-0.1366E-05	-0.1366E-05	-0.1366E-05
10	0.4933E 00	1	-0.1957E-05	-0.1957E-05	-0.1957E-05	-0.1957E-05	-0.1957E-05

BUCKLING LOAD PARAMETERS

0.99984D 00   0.29937D 01

BUCKLING MODE SHAPES

NODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
22	1	1.156E-01	-3.124E 00	0.0	0.0	0.0	0.0
	2	1.015E-02	1.037E 00	0.0	0.0	0.0	0.0
21	1	-1.156E-01	-3.124E 00	0.0	0.0	0.0	0.0
	2	-1.015E-02	1.037E 00	0.0	0.0	0.0	0.0
20	1	1.041E-01	-2.552E 00	0.0	0.0	0.0	0.0
	2	7.981E-03	1.086E 00	0.0	0.0	0.0	0.0
19	1	-1.041E-01	-2.552E 00	0.0	0.0	0.0	0.0
	2	-7.981E-03	1.086E 00	0.0	0.0	0.0	0.0
18	1	9.248E-02	-2.037E 00	0.0	0.0	0.0	0.0
	2	5.901E-03	1.125E 00	0.0	0.0	0.0	0.0
17	1	-9.248E-02	-2.037E 00	0.0	0.0	0.0	0.0
	2	-5.901E-03	1.125E 00	0.0	0.0	0.0	0.0
16	1	8.091E-02	-1.581E 00	0.0	0.0	0.0	0.0
	2	3.969E-03	1.152E 00	0.0	0.0	0.0	0.0
15	1	-8.091E-02	-1.581E 00	0.0	0.0	0.0	0.0
	2	-3.969E-03	1.152E 00	0.0	0.0	0.0	0.0
14	1	6.935E-02	-1.182E 00	0.0	0.0	0.0	0.0
	2	2.197E-03	1.170E 00	0.0	0.0	0.0	0.0
13	1	-6.935E-02	-1.182E 00	0.0	0.0	0.0	0.0
	2	-2.197E-03	1.170E 00	0.0	0.0	0.0	0.0
12	1	5.779E-02	-8.410E-01	0.0	0.0	0.0	0.0
	2	5.947E-04	1.178E 00	0.0	0.0	0.0	0.0
11	1	-5.779E-02	-8.410E-01	0.0	0.0	0.0	0.0
	2	-5.947E-04	1.178E 00	0.0	0.0	0.0	0.0
10	1	4.623E-02	-5.578E-01	0.0	0.0	0.0	0.0
	2	-8.270E-04	1.177E 00	0.0	0.0	0.0	0.0
9	1	-4.623E-02	-5.578E-01	0.0	0.0	0.0	0.0
	2	8.270E-04	1.177E 00	0.0	0.0	0.0	0.0
8	1	3.468E-02	-3.324E-01	0.0	0.0	0.0	0.0
	2	-2.054E-03	1.167E 00	0.0	0.0	0.0	0.0
7	1	-3.468E-02	-3.324E-01	0.0	0.0	0.0	0.0
	2	2.054E-03	1.167E 00	0.0	0.0	0.0	0.0
6	1	2.312E-02	-1.648E-01	0.0	0.0	0.0	0.0
	2	-3.061E-03	1.148E 00	0.0	0.0	0.0	0.0
5	1	-2.312E-02	-1.648E-01	0.0	0.0	0.0	0.0
	2	3.061E-03	1.148E 00	0.0	0.0	0.0	0.0
4	1	1.157E-02	-5.495E-02	0.0	0.0	0.0	0.0
	2	-3.792E-03	1.115E 00	0.0	0.0	0.0	0.0
3	1	-1.157E-02	-5.495E-02	0.0	0.0	0.0	0.0
	2	3.792E-03	1.115E 00	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0

****
EVALUATION OF DESIGN NUMBER   5
****

STRESS RATIO   LOAD COND   DES VARIABLE

MAX	0.1000E 01	0	21
MIN	0.2027E-01	0	20

MAX BUCK RATIOS	LOAD COND
0.1000E 01	1
0.4776E 00	1

DESIGN IS CRITICAL

STRUCTURAL WEIGHT= 0.1179E 03

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR BUCKLING CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	0.10016E 01
2	ACT	0.99931E 00
3	ACT	0.99931E 00
4	ACT	0.99937E 00
5	ACT	0.99950E 00
6	ACT	0.99974E 00
7	ACT	0.10002E 01
8	ACT	0.10011E 01
9	ACT	0.10035E 01
10	ACT	0.99732E 00
11	ACT	0.12672E 01
12	ACT	0.10004E 01
13	ACT	0.99867E 00
14	ACT	0.99922E 00
15	ACT	0.99796E 00
16	ACT	0.99769E 00
17	ACT	0.99753E 00
18	ACT	0.99709E 00
19	ACT	0.99675E 00
20	ACT	0.10066E 01
21	PASS	0.0

NO. OF ACTIVE BUCKLING CONSTRAINTS ARE 1

****
ANALYSIS OF DESIGN NUMBER 6
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
22	1	-4.796E-02	-1.777E-15	0.0	0.0	0.0	0.0
21	1	-4.796E-02	-1.777E-15	0.0	0.0	0.0	0.0
20	1	-2.776E-02	-1.506E-15	0.0	0.0	0.0	0.0
19	1	-2.776E-02	-1.507E-15	0.0	0.0	0.0	0.0
18	1	-2.079E-02	-1.237E-15	0.0	0.0	0.0	0.0
17	1	-2.079E-02	-1.238E-15	0.0	0.0	0.0	0.0
16	1	-1.639E-02	-9.881E-16	0.0	0.0	0.0	0.0
15	1	-1.639E-02	-9.877E-16	0.0	0.0	0.0	0.0
14	1	-1.307E-02	-7.594E-16	0.0	0.0	0.0	0.0
13	1	-1.307E-02	-7.590E-16	0.0	0.0	0.0	0.0
12	1	-1.034E-02	-5.620E-16	0.0	0.0	0.0	0.0
11	1	-1.034E-02	-5.618E-16	0.0	0.0	0.0	0.0
10	1	-7.953E-03	-3.989E-16	0.0	0.0	0.0	0.0
9	1	-7.953E-03	-3.985E-16	0.0	0.0	0.0	0.0
8	1	-5.795E-03	-2.671E-16	0.0	0.0	0.0	0.0
7	1	-5.795E-03	-2.669E-16	0.0	0.0	0.0	0.0
6	1	-3.785E-03	-1.619E-16	0.0	0.0	0.0	0.0
5	1	-3.785E-03	-1.619E-16	0.0	0.0	0.0	0.0
4	1	-1.867E-03	-7.870E-17	0.0	0.0	0.0	0.0
3	1	-1.867E-03	-7.874E-17	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

1	2	3	4	5	6	7	8	9	10

0	0.3348E 01	0.3259E 01	0.3109E 01	0.2896E 01	0.2621E 01	0.2284E 01	0.1884E 01	0.1422E 01	0.8967E 00	0.3093E 00
10	0.4786E-01	0.9503E-01	0.1450E 00	0.1950E 00	0.2449E 00	0.2949E 00	0.3449E 00	0.3949E 00	0.4450E 00	0.4949E 00
20	0.1000E 00									

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FORCE
1	0.3348E 01	1	-0.1000E 01
2	0.3348E 01	1	-0.1000E 01
3	0.3259E 01	1	-0.1000E 01
4	0.3259E 01	1	-0.1000E 01
5	0.3109E 01	1	-0.1000E 01
6	0.3109E 01	1	-0.1000E 01
7	0.2896E 01	1	-0.1000E 01
8	0.2896E 01	1	-0.1000E 01
9	0.2621E 01	1	-0.1000E 01
10	0.2621E 01	1	-0.1000E 01
11	0.2284E 01	1	-0.1000E 01
12	0.2284E 01	1	-0.1000E 01
13	0.1884E 01	1	-0.1000E 01
14	0.1884E 01	1	-0.1000E 01
15	0.1422E 01	1	-0.1000E 01
16	0.1422E 01	1	-0.1000E 01
17	0.8967E 00	1	-0.1000E 01
18	0.8967E 00	1	-0.1000E 01
19	0.3093E 00	1	-0.1000E 01
20	0.3093E 00	1	-0.1000E 01
21	0.1000E 00	1	0.1907E-17
22	0.1000E 00	1	-0.9388E-18
23	0.1000E 00	1	-0.8194E-17
24	0.1000E 00	1	-0.1692E-16
25	0.1000E 00	1	-0.8269E-17
26	0.1000E 00	1	-0.1699E-16
27	0.1000E 00	1	-0.1405E-16
28	0.1000E 00	1	0.5753E-16
29	0.1000E 00	1	0.2604E-16
30	0.1000E 00	1	-0.1050E-18

ANALYSIS OF SHEAR PANELS, CONSTRN CODE= 1

		LOAD	/--------------SHEAR FLOW AT NODES-------------/				AVERAGE
ELEMENT	THICKNESS	COND	I	J	K	L	SHEAR FLOW
1	0.4786E-01	1	-0.5388E-08	-0.5388E-08	-0.5388E-08	-0.5388E-08	-0.5388E-08
2	0.9503E-01	1	-0.1304E-07	-0.1304E-07	-0.1304E-07	-0.1304E-07	-0.1304E-07
3	0.1450E 00	1	-0.6880E-07	-0.6880E-07	-0.6880E-07	-0.6880E-07	-0.6880E-07
4	0.1950E 00	1	-0.8874E-07	-0.8874E-07	-0.8874E-07	-0.8874E-07	-0.8874E-07
5	0.2449E 00	1	-0.2242E-06	-0.2242E-06	-0.2242E-06	-0.2242E-06	-0.2242E-06
6	0.2949E 00	1	-0.2151E-06	-0.2151E-06	-0.2151E-06	-0.2151E-06	-0.2151E-06
7	0.3449E 00	1	-0.2225E-06	-0.2225E-06	-0.2225E-06	-0.2225E-06	-0.2225E-06
8	0.3949E 00	1	-0.2067E-06	-0.2067E-06	-0.2067E-06	-0.2067E-06	-0.2067E-06
9	0.4450E 00	1	-0.1911E-05	-0.1911E-05	-0.1911E-05	-0.1911E-05	-0.1911E-05
10	0.4949E 00	1	-0.1391E-05	-0.1391E-05	-0.1391E-05	-0.1391E-05	-0.1391E-05

BUCKLING LOAD PARAMETERS

0.99997D 00 0.23690D 01

BUCKLING MODE SHAPES

NODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
22	1	1.156E-01	-3.119E 00	0.0	0.0	0.0	0.0
	2	-1.019E-02	-1.061E 00	0.0	0.0	0.0	0.0
21	1	-1.156E-01	-3.119E 00	0.0	0.0	0.0	0.0
	2	1.019E-02	-1.061E 00	0.0	0.0	0.0	0.0
20	1	1.041E-01	-2.547E 00	0.0	0.0	0.0	0.0
	2	-7.857E-03	-1.110E 00	0.0	0.0	0.0	0.0
19	1	-1.041E-01	-2.547E 00	0.0	0.0	0.0	0.0
	2	7.857E-03	-1.110E 00	0.0	0.0	0.0	0.0
18	1	9.250E-02	-2.032E 00	0.0	0.0	0.0	0.0
	2	-5.637E-03	-1.148E 00	0.0	0.0	0.0	0.0
17	1	-9.250E-02	-2.032E 00	0.0	0.0	0.0	0.0
	2	5.637E-03	-1.148E 00	0.0	0.0	0.0	0.0
16	1	8.093E-02	-1.576E 00	0.0	0.0	0.0	0.0
	2	-3.581E-03	-1.174E 00	0.0	0.0	0.0	0.0
15	1	-8.093E-02	-1.576E 00	0.0	0.0	0.0	0.0
	2	3.581E-03	-1.174E 00	0.0	0.0	0.0	0.0
14	1	6.937E-02	-1.177E 00	0.0	0.0	0.0	0.0
	2	-1.705E-03	-1.189E 00	0.0	0.0	0.0	0.0
13	1	-6.937E-02	-1.177E 00	0.0	0.0	0.0	0.0
	2	1.705E-03	-1.189E 00	0.0	0.0	0.0	0.0
12	1	5.781E-02	-8.357E-01	0.0	0.0	0.0	0.0
	2	-1.931E-05	-1.195E 00	0.0	0.0	0.0	0.0
11	1	-5.781E-02	-8.357E-01	0.0	0.0	0.0	0.0
	2	1.931E-05	-1.195E 00	0.0	0.0	0.0	0.0
10	1	4.624E-02	-5.524E-01	0.0	0.0	0.0	0.0
	2	1.461E-03	-1.190E 00	0.0	0.0	0.0	0.0
9	1	-4.624E-02	-5.524E-01	0.0	0.0	0.0	0.0
	2	-1.461E-03	-1.190E 00	0.0	0.0	0.0	0.0
8	1	3.468E-02	-3.270E-01	0.0	0.0	0.0	0.0
	2	2.719E-03	-1.177E 00	0.0	0.0	0.0	0.0
7	1	-3.468E-02	-3.270E-01	0.0	0.0	0.0	0.0
	2	-2.719E-03	-1.177E 00	0.0	0.0	0.0	0.0
6	1	2.312E-02	-1.594E-01	0.0	0.0	0.0	0.0
	2	3.726E-03	-1.154E 00	0.0	0.0	0.0	0.0
5	1	-2.312E-02	-1.594E-01	0.0	0.0	0.0	0.0
	2	-3.726E-03	-1.154E 00	0.0	0.0	0.0	0.0
4	1	1.155E-02	-4.961E-02	0.0	0.0	0.0	0.0
	2	4.419E-03	-1.115E 00	0.0	0.0	0.0	0.0
3	1	-1.155E-02	-4.961E-02	0.0	0.0	0.0	0.0
	2	-4.419E-03	-1.115E 00	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0

```
********************************
EVALUATION OF DESIGN NUMBER    6
********************************
```

STRESS RATIO   LOAD COND   DES VARIABLE

0.1.25

```
MAX        0.1000E 01        0          21
MIN        0.2021E-01        0          20

      MAX BUCK RATIOS   LOAD COND

        0.1000E 01        1
        0.4221E 00        1

DESIGN IS CRITICAL


STRUCTURAL WEIGHT= 0.1179E 03

REDESIGN OPERATION FOLLOWS

TERMINAL DESIGN---NUMBER OF CRITICAL DESIGNS  =    6
```

P. PLATE ELEMENTS

P.1 Circular Plate with Simple Support

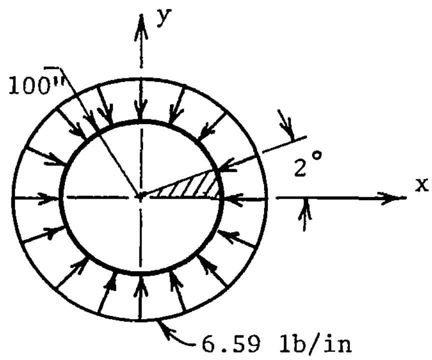


Figure P.1.1
Simply Supported Plate under Uniform Compression

The problem under consideration is to find the optimal distribution of thickness for the plate shown in Fig. P.1.1. As the design will be axisymmetric, it is sufficient to model only a sector of the plate by finite elements, as indicated by the shaded region in the figure.

The finite element model of the sector is shown in Fig. P.1.3. The conditions of symmetry on the x-axis are imposed by the boundary condition codes in the nodal point input data, but two sets of boundary elements are required on the "skewed" boundary of the segment. Boundary element Nos. 1-10 are rotational springs that suppress rotations about the boundary line, whereas element Nos. 11-20 represent extensional springs that prevent the normal, in-plane displacements. The spring constants of the boundary elements must be much larger than the corresponding stiffnesses of the elements, illustrated in Fig. P.1.2, in order to be effective in suppressing the boundary displacements and rotations. We used the ratios k_{bound}/k_{plate} = 5000

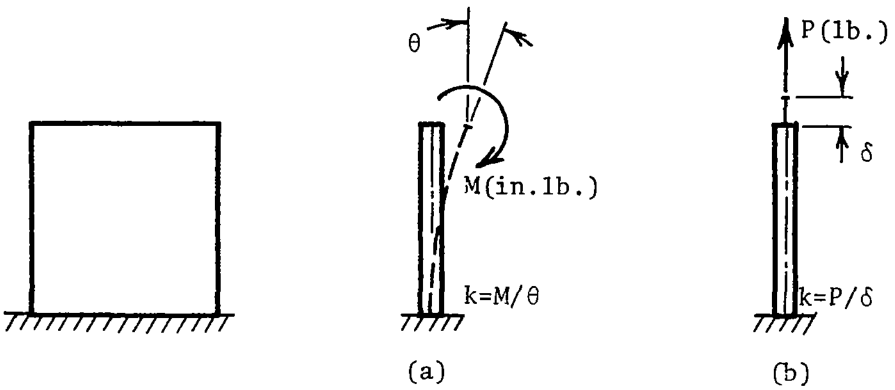


Figure P.1.2

Calculation of (a) Rotational, and (b) Extensional Stiffness of a Plate Element

for the rotational springs and 500 for the extensional springs, approximately, where k_{plate} is computed for the initial design. The material properties used in the design are:

$E = 10^6$ psi (Young's modulus),

$\nu = 0.3$ (Poisson's ratio),

$\sigma_t^* = \sigma_c^* = 50{,}000$ psi (allowable stress),

$\rho = 1.0$ lb./in^3 (specific weight),

$p^* = 1.0$ (lower bound on the critical load parameter).

The design was started with a uniform thickness of 0.5554 in., and all of the ten plate elements were sized independently. No constraint was placed on the minimum thickness. The design history of the plate is given in Table P.1.1. The optimal design, which is governed by the buckling constraint alone, was reached after five redesign cycles.

An analytical solution for the optimal plate with a continuously varying thickness has been derived in Ref. [16]. The results are plotted in Fig. P.1.4 together with the discrete element thicknesses obtained from DESAP 2. The thickness distributions correlate quite well; the small discrepancy in the two weights can be attributed to the continuous vs. discrete variation of thickness.

Special notes on input-output:

1) Buckling deformation of a flat plate does not involve stretching of the middle surface, i.e., only the bending stiffness matrix of each element: $[K_i] = [k_i]A^n$, n = 3, offers resistance to buckling. Consequently, uniform scaling is an exact operation with KSCALE = 3 (see Design Control Data).

2) The variation in the spring constants of boundary elements shown in Boundary Element Data (higher near the center than at the edge) reflects similar changes in the stiffnesses of the plate elements. The idea here is to maintain an approximately constant ratio of k_{bound}/k_{plate} throughout the finite element model.

3) NMODE = 2 in Buckling Control Data requests that two buckling modes be considered in the design. Since only one mode governs the final design, the results would be identical if NMODE = 1 were used, but the buckling load and the mode shape of the second mode would not appear in the printout.

4) INDET = 1 in Buckling Control Data informs the computer that the internal forces of the prebuckling state are statically determinate, i.e., the geometric stiffness matrix of the structure does not have to be recomputed after each design.

5) The slight variation of the membrane forces from uniform, biaxial compression (see Analysis of Plate/Shell Elements of Design No. 0) is caused by the presence of the boundary elements.

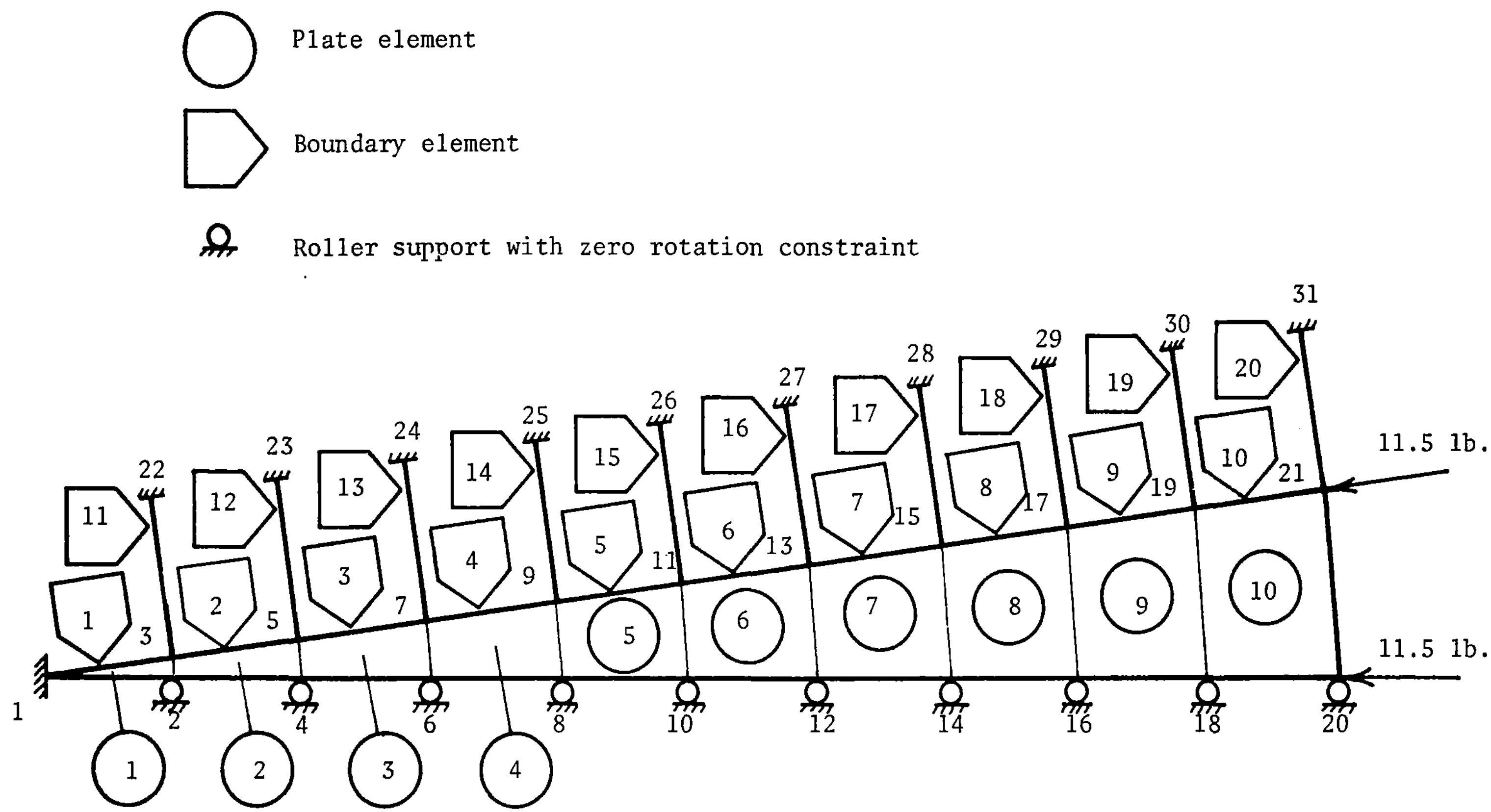


Figure P.1.3

Finite Element Model for a Two Degree Sector of Plate Showing Node and Element Numbers

Element	Critical (scaled) Designs (thickness in inches)					
	0	1	2	3	4	5
1	.5554	.6641	.7110	.7218	.7226	.7222
2	.5554	.6540	.6997	.7112	.7121	.7117
3	.5554	.6345	.6799	.6941	.6966	.6969
4	.5554	.6078	.6513	.6677	.6719	.6731
5	.5554	.5765	.6137	.6322	.6384	.6406
6	.5554	.5436	.5671	.5865	.5948	.5982
7	.5554	.5121	.5123	.5289	.5389	.5436
8	.5554	.4846	.4535	.4566	.4668	.4729
9	.5554	.4630	.3995	.3708	.3706	.3772
10	.5554	.4486	.3626	.2960	.2530	.2371
Wt(lb)*	96.92	89.94	86.68	85.14	84.54	84.59

*The weight of a 2° sector

Table P.1.1

Design History of Element Thicknesses and Structural Weight.

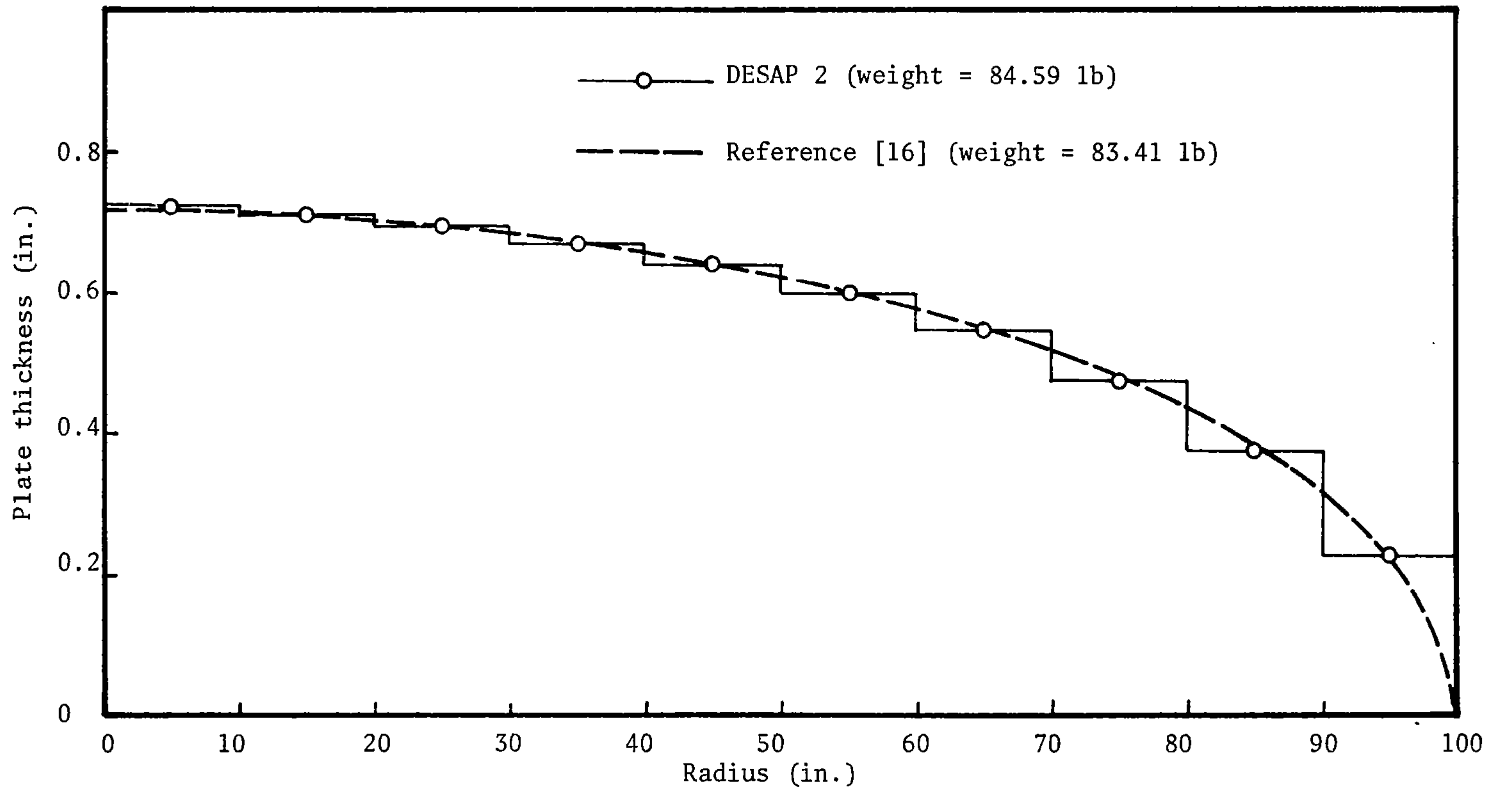


Figure P.1.4

Comparison of the Results of DESAP 2 with the Analytical Solution

```
00950 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
01000   S.S CIRCULAR PLATE - BUCKLING DESIGN UNDER UNIFORM COMPRESSIVE INPLANE LOAD
01050    31    2    1   10
01100    10         3  0.02      0.1         1    1    1
01150     1    1   -1        -1    1   -1
01200     2                               10.
01250    20         1    1    1           100.                             2
01300     3                                9.994     .349
01350    21              1              1 99.94     3.49                   2
01400    22    1    1    1    1    1    1 6.504     100.289
01450    31    1    1    1    1    1    1 96.45     103.43                 1
01500     6   10    1    1    1
01550     1    1  1.
01600            1000000.   0.3                50000.    50000.    28000.
01650
01700
01750
01800
01850
01900     1    1    2    3         1         1
01950     2    2    4    5    3    1         2
02000     3    4    6    7    5    1         3
02050     4    6    8    9    7    1         4
02100     5    8   10   11    9    1         5
02150     6   10   12   13   11    1         6
02200     7   12   14   15   13    1         7
02250     8   14   16   17   15    1         8
02300     9   16   18   19   17    1         9
02350    10   18   20   21   19    1        10
02400     7   20
02450
02500     1    3    1                        1                              1000000000
02550     2    5    3                        1                              900000000.
02600     3    7    5                        1                              800000000.
02650     4    9    7                        1                              700000000.
02700     5   11    9                        1                              600000000.
02750     6   13   11                        1                              500000000.
02800     7   15   13                        1                              400000000.
02850     8   17   15                        1                              300000000.
02900     9   19   17                        1                              200000000.
02950    10   21   19                        1                              100000000.
03000    11    3   22                   1                                   1000000000
03050    12    5   23                   1                                   900000000.
03100    13    7   24                   1                                   800000000.
03150    14    9   25                   1                                   700000000.
03200    15   11   26                   1                                   600000000.
03250    16   13   27                   1                                   500000000.
03300    17   15   28                   1                                   400000000.
03350    18   17   29                   1                                   300000000.
03400    19   19   30                   1                                   200000000.
03450    20   21   31                   1                                   100000000.
03500
03550   1.0              2    1    2 0.75       0.8
03600    20    1 -11.5
03650    21    1 -11.5     -0.4
03700
03750    10  .5554
03800 123456739A123456789B123456789C123456789D123456789E123456780F123456789G123456789H
03850
```

Echo of Input Cards

S.S CIRCULAR PLATE - BUCKLING DESIGN UNDER UNIFORM COMPRESSIVE INPLANE LOAD

```
NUMBER OF NODAL POINTS   =   31
NUMBER OF ELEMENT TYPES  =    2
NUMBER OF LOAD CASES     =    1
NUMBER OF DES. VARIABLES =   10
```

DESIGN CONTROL DATA

```
NCYCL =    10
KSCALE=     3
DELTA =  0.2000E-01
EPSIL =  0.1000E 00
LBUCK =     1
```

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z		T
1	1	-1	0	-1	1	-1	0.0	0.0	0.0	0	0.0
2	0	0	0	0	0	0	10.000	0.0	0.0	0	0.0
20	0	1	1	1	0	0	100.000	0.0	0.0	2	0.0
3	0	0	0	0	0	0	9.994	0.349	0.0	0	0.0
21	0	0	1	0	0	1	99.940	3.490	0.0	2	0.0
22	1	1	1	1	1	1	6.504	100.289	0.0	0	0.0
31	1	1	1	1	1	1	56.450	103.430	0.0	1	0.0

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z	T
1	1	-1	0	-1	1	-1	0.0	0.0	0.0	0.0
2	0	-1	0	-1	0	-1	10.000	0.0	0.0	0.0
3	0	0	0	0	0	-1	9.994	0.349	0.0	0.0
4	0	-1	0	-1	0	-1	20.000	0.0	0.0	0.0
5	0	0	0	0	0	-1	19.988	0.698	0.0	0.0
6	0	-1	0	-1	0	-1	30.000	0.0	0.0	0.0
7	0	0	0	0	0	-1	29.982	1.047	0.0	0.0
8	0	-1	0	-1	0	-1	40.000	0.0	0.0	0.0
9	0	0	0	0	0	-1	39.976	1.396	0.0	0.0
10	0	-1	0	-1	0	-1	50.000	0.0	0.0	0.0
11	0	0	0	0	0	-1	49.970	1.745	0.0	0.0
12	0	-1	0	-1	0	-1	60.000	0.0	0.0	0.0
13	0	0	0	0	0	-1	59.964	2.094	0.0	0.0
14	0	-1	0	-1	0	-1	70.000	0.0	0.0	0.0
15	0	0	0	0	0	-1	69.958	2.443	0.0	0.0
16	0	-1	0	-1	0	-1	80.000	0.0	0.0	0.0
17	0	0	0	0	0	-1	79.952	2.792	0.0	0.0
18	0	-1	0	-1	0	-1	90.000	0.0	0.0	0.0
19	0	0	0	0	0	-1	89.946	3.141	0.0	0.0
20	0	1	1	1	0	-1	100.000	0.0	0.0	0.0
21	0	0	1	0	0	1	99.940	3.490	0.0	0.0
22	1	1	1	1	1	1	6.504	100.289	0.0	0.0
23	1	1	1	1	1	1	16.498	100.638	0.0	0.0
24	1	1	1	1	1	1	26.492	100.987	0.0	0.0

Computer Printout

(Input data, the initial design and the final design only are reproduced)

25	1	1	1	1	1	1	36.486	101.336	0.0	0.0
26	1	1	1	1	1	1	46.480	101.685	0.0	0.0
27	1	1	1	1	1	1	56.474	102.034	0.0	0.0
28	1	1	1	1	1	1	66.468	102.383	0.0	0.0
29	1	1	1	1	1	1	76.462	102.732	0.0	0.0
30	1	1	1	1	1	1	86.456	103.081	0.0	0.0
31	1	1	1	1	1	1	96.450	103.430	0.0	0.0

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	0	0	1	0	0	0
2	2	0	3	0	4	0
3	5	6	7	8	9	0
4	10	0	11	0	12	0
5	13	14	15	16	17	0
6	18	0	19	0	20	0
7	21	22	23	24	25	0
8	26	0	27	0	28	0
9	29	30	31	32	33	0
10	34	0	35	0	36	0
11	37	38	39	40	41	0
12	42	0	43	0	44	0
13	45	46	47	48	49	0
14	50	0	51	0	52	0
15	53	54	55	56	57	0
16	58	0	59	0	60	0
17	61	62	63	64	65	0
18	66	0	67	0	68	0
19	69	70	71	72	73	0
20	74	0	0	0	75	0
21	76	77	0	78	79	0
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0	0	0	0	0	0
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	0	0	0	0	0	0
30	0	0	0	0	0	0
31	0	0	0	0	0	0

T H I N   P L A T E / S H E L L   E L E M E N T S

NUMBER OF ELEMENTS   =   10
NUMBER OF MATERIALS =   1
NUMBER OF TEMP CARDS=   1
CONSTRN CODE   =   1

MATERIAL PROPERTY TABLE

MATERIAL NUMBER	NUM OF TEMP	SPECIFIC WEIGHT	TEMP	YOUNGS MODULUS	POISSONS'S RATIO	COEFFT OF THERM EXPN	/------------ALLOWABLE STRESSES------------/ TENSION	COMPRESSION	SHEAR
1	1	1.00000	0.0	1.00000E 06	0.300	0.0	50000.00	50000.00	28000.00

ELEMENT LOAD CASE MULTIPLIERS

ELEMENT LOAD CASE NUMBER	PRESSURE	THERMAL EFFECTS	X- ACCELERATION	Y- ACCELERATION	Z- ACCELERATION
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0

THIN PLATE/SHELL ELEMENT DATA

ELEMENT NUMBER	NODE-I	NODE-J	NODE-K	NODE-L	MATERIAL NUMBER	DES VAR NUMBER	NORMAL PRESSURE	REFERENCE TEMPERATURE	DES VAR FRACTION	BETA	BAND WIDTH
1	1	2	3	0	1	1	0.0	0.0	1.0000	0.0	9
2	2	4	5	3	1	2	0.0	0.0	1.0000	0.0	16
3	4	6	7	5	1	3	0.0	0.0	1.0000	0.0	16
4	6	8	9	7	1	4	0.0	0.0	1.0000	0.0	16
5	8	10	11	9	1	5	0.0	0.0	1.0000	0.0	16
6	10	12	13	11	1	6	0.0	0.0	1.0000	0.0	16
7	12	14	15	13	1	7	0.0	0.0	1.0000	0.0	16
8	14	16	17	15	1	8	0.0	0.0	1.0000	0.0	16
9	16	18	19	17	1	9	0.0	0.0	1.0000	0.0	16
10	18	20	21	19	1	10	0.0	0.0	1.0000	0.0	14

B O U N D A R Y   E L E M E N T S

NUMBER OF ELEMENTS   =   20

ELEMENT LOAD MULTIPLIERS

A	B	C	D
0.0	0.0	0.0	0.0

BOUNDARY ELEMENT DATA

CONST NUMBER	NODE N	/--NODES DEFINING CONSTRAINT DIRECTION--/ NI	NJ	NK	NL	CODES KD	KR	DISPL D	ROTATION R	STIFF S
1	3	1	0	0	0	0	1	0.0	0.0	1.00D 09
2	5	3	0	0	0	0	1	0.0	0.0	9.00D 08
3	7	5	0	0	0	0	1	0.0	0.0	8.00D 08
4	9	7	0	0	0	0	1	0.0	0.0	7.00D 08
5	11	9	0	0	0	0	1	0.0	0.0	6.00D 08
6	13	11	0	0	0	0	1	0.0	0.0	5.00D 08
7	15	13	0	0	0	0	1	0.0	0.0	4.00D 08
8	17	15	0	0	0	0	1	0.0	0.0	3.00D 08
9	19	17	0	0	0	0	1	0.0	0.0	2.00D 08
10	21	19	0	0	0	0	1	0.0	0.0	1.00D 08
11	3	22	0	0	0	1	0	0.0	0.0	1.00D 09
12	5	23	0	0	0	1	0	0.0	0.0	9.00D 08
13	7	24	0	0	0	1	0	0.0	0.0	8.00D 08
14	9	25	0	0	0	1	0	0.0	0.0	7.00D 08
15	11	26	0	0	0	1	0	0.0	0.0	6.00D 08
16	13	27	0	0	0	1	0	0.0	0.0	5.00D 08
17	15	28	0	0	0	1	0	0.0	0.0	4.00D 08
18	17	29	0	0	0	1	0	0.0	0.0	3.00D 08
19	19	30	0	0	0	1	0	0.0	0.0	2.00D 08
20	21	31	0	0	0	1	0	0.0	0.0	1.00D 08

STRUCTURE LOAD CASE	STRUCTURE LOAD MULTIPLIERS A	B	C	D
1	0.0	0.0	0.0	0.0

BUCKLING CONTROL DATA

COEFFT =   1.00000
MODEIN =   0
NMODE =   2
INDET =   1
NVEC   =   2
ALPA   =   0.75000
OMEGA =   0.80000

NODAL POINT LOADS

NODE NO.	LOAD CASE	APPLIED LOADS FX	FY	FZ	MX	MY	MZ
20	1	-0.115D 02	0.0	0.0	0.0	0.0	0.0
21	1	-0.115D 02	-0.400D 00	0.0	0.0	0.0	0.0

DESIGN VARIABLE INPUT DATA

DESIGN VARIABLE NUMBER	INITIAL VALUE	MIN ALLOWABLE VALUE
1	0.5554E 00	0.0
2	0.5554E 00	0.0
3	0.5554E 00	0.0
4	0.5554E 00	0.0
5	0.5554E 00	0.0
6	0.5554E 00	0.0
7	0.5554E 00	0.0
8	0.5554E 00	0.0
9	0.5554E 00	0.0
10	0.5554E 00	0.0

```
TOTAL NUMBER OF EQUATIONS        =   79
BANDWIDTH                        =   16
NUMBER OF EQUATIONS IN A BLOCK   =   79
NUMBER OF BLOCKS                 =    1
```

****
ANALYSIS OF DESIGN NUMBER 0
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
31	1	0.0	0.0	0.0	0.0	0.0	0.0
30	1	0.0	0.0	0.0	0.0	0.0	0.0
29	1	0.0	0.0	0.0	0.0	0.0	0.0
28	1	0.0	0.0	0.0	0.0	0.0	0.0
27	1	0.0	0.0	0.0	0.0	0.0	0.0
26	1	0.0	0.0	0.0	0.0	0.0	0.0
25	1	0.0	0.0	0.0	0.0	0.0	0.0
24	1	0.0	0.0	0.0	0.0	0.0	0.0
23	1	0.0	0.0	0.0	0.0	0.0	0.0
22	1	0.0	0.0	0.0	0.0	0.0	0.0
21	1	-8.345E-04	-2.881E-05	0.0	0.0	0.0	0.0
20	1	-8.350E-04	0.0	0.0	0.0	0.0	0.0
19	1	-7.512E-04	-2.591E-05	0.0	0.0	0.0	0.0
18	1	-7.517E-04	0.0	0.0	0.0	0.0	0.0
17	1	-6.680E-04	-2.311E-05	0.0	0.0	0.0	0.0
16	1	-6.684E-04	0.0	0.0	0.0	0.0	0.0
15	1	-5.847E-04	-2.025E-05	0.0	0.0	0.0	0.0
14	1	-5.851E-04	0.0	0.0	0.0	0.0	0.0
13	1	-5.015E-04	-1.738E-05	0.0	0.0	0.0	0.0
12	1	-5.018E-04	0.0	0.0	0.0	0.0	0.0
11	1	-4.182E-04	-1.449E-05	0.0	0.0	0.0	0.0
10	1	-4.184E-04	0.0	0.0	0.0	0.0	0.0
9	1	-3.349E-04	-1.160E-05	0.0	0.0	0.0	0.0
8	1	-3.351E-04	0.0	0.0	0.0	0.0	0.0
7	1	-2.516E-04	-8.703E-06	0.0	0.0	0.0	0.0

6	1	-2.517E-04	0.0	0.0	0.0	0.0	0.0
5	1	-1.682E-04	-5.800E-06	0.0	0.0	0.0	0.0
4	1	-1.683E-04	0.0	0.0	0.0	0.0	0.0
3	1	-8.460E-05	-2.888E-06	0.0	0.0	0.0	0.0
2	1	-8.465E-05	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.5554E 00	0.5554E 00	0.5554E 00	0.5554E 00	0.5554E 00	0.5554E 00	0.5554E 00	0.5554E 00	0.5554E 00	0.5554E 00

ANALYSIS OF PLATE/SHELL ELEMENTS ,CONSTRN CODE = 1

ELEMENT NUMBER	ELEMENT THICKNESS	LOAD COND	/-------------MEMBRANE FORCES------------/ NXX	NYY	NXY	/--------BENDING/TWISTING MOMENTS--------/ MXX	MYY	MXY
1	0.5554E 00	1	-0.6682E 01	-0.6601E 01	0.2361E-03	0.0	0.0	0.0
2	0.5554E 00	1	-0.6624E 01	-0.6595E 01	-0.1694E-03	0.0	0.0	0.0
3	0.5554E 00	1	-0.6614E 01	-0.6600E 01	-0.2884E-04	0.0	0.0	0.0
4	0.5554E 00	1	-0.6610E 01	-0.6599E 01	-0.3303E-04	0.0	0.0	0.0
5	0.5554E 00	1	-0.6608E 01	-0.6597E 01	-0.3312E-04	0.0	0.0	0.0
6	0.5554E 00	1	-0.6605E 01	-0.6593E 01	-0.3758E-04	0.0	0.0	0.0
7	0.5554E 00	1	-0.6603E 01	-0.6588E 01	-0.1844E-04	0.0	0.0	0.0
8	0.5554E 00	1	-0.6601E 01	-0.6581E 01	-0.1726E-03	0.0	0.0	0.0
9	0.5554E 00	1	-0.6598E 01	-0.6568E 01	0.4057E-03	0.0	0.0	0.0
10	0.5554E 00	1	-0.6594E 01	-0.6561E 01	-0.6573E-03	0.0	0.0	0.0

ANALYSIS OF BOUNDARY ELEMENTS - CONSTRAINT FORCES

CONST NUMBER	LOAD CASE	FORCE	MOMENT
1	1	0.0	0.0
2	1	0.0	0.0
3	1	0.0	0.0
4	1	0.0	0.0
5	1	0.0	0.0
6	1	0.0	0.0
7	1	0.0	0.0
8	1	0.0	0.0
9	1	0.0	0.0
10	1	0.0	0.0
11	1	0.65954E 02	0.0
12	1	0.65998E 02	0.0
13	1	0.65996E 02	0.0
14	1	0.65975E 02	0.0
15	1	0.65946E 02	0.0
16	1	0.65908E 02	0.0
17	1	0.65846E 02	0.0

18	1	0.65749E 02	0.0
19	1	0.65614E 02	0.0
20	1	0.32613E 02	0.0

BUCKLING LOAD PARAMETERS

0.99818D 00 0.69222D 01

BUCKLING MODE SHAPES

NODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
31	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
30	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
29	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
28	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
27	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
26	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
25	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
24	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
23	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
22	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
21	1	0.0	0.0	0.0	1.1616E-03	-3.3272E-02	0.0
	2	0.0	0.0	0.0	-1.0485E-03	3.0034E-02	0.0
20	1	0.0	0.0	0.0	0.0	-3.3307E-02	0.0
	2	0.0	0.0	0.0	0.0	3.0066E-02	0.0
19	1	0.0	0.0	-3.363E-01	1.1796E-03	-3.3789E-02	0.0
	2	0.0	0.0	2.900E-01	-9.3325E-04	2.6736E-02	0.0
18	1	0.0	0.0	-3.363E-01	0.0	-3.3812E-02	0.0
	2	0.0	0.0	2.900E-01	0.0	2.6754E-02	0.0
17	1	0.0	0.0	-6.727E-01	1.1628E-03	-3.3304E-02	0.0
	2	0.0	0.0	5.056E-01	-5.3617E-04	1.5363E-02	0.0
16	1	0.0	0.0	-6.728E-01	0.0	-3.3325E-02	0.0
	2	0.0	0.0	5.056E-01	0.0	1.5372E-02	0.0
15	1	0.0	0.0	-9.993E-01	1.1110E-03	-3.1821E-02	0.0
	2	0.0	0.0	5.755E-01	-6.7685E-05	-1.9324E-03	0.0
14	1	0.0	0.0	-9.993E-01	0.0	-3.1840E-02	0.0
	2	0.0	0.0	5.755E-01	0.0	-1.9347E-03	0.0
13	1	0.0	0.0	-1.306E 00	1.0255E-03	-2.9372E-02	0.0
	2	0.0	0.0	4.584E-01	7.4580E-04	-2.1355E-02	0.0
12	1	0.0	0.0	-1.306E 00	0.0	-2.9390E-02	0.0
	2	0.0	0.0	4.583E-01	0.0	-2.1369E-02	0.0
11	1	0.0	0.0	-1.584E 00	9.0854E-04	-2.6022E-02	0.0
	2	0.0	0.0	1.556E-01	1.3390E-03	-3.8347E-02	0.0
10	1	0.0	0.0	-1.584E 00	0.0	-2.6037E-02	0.0
	2	0.0	0.0	1.556E-01	0.0	-3.8371E-02	0.0
9	1	0.0	0.0	-1.824E 00	7.6346E-04	-2.1867E-02	0.0

	2	0.0	0.0	-2.868E-01	1.7003E-03	-4.8696E-02	0.0
8	1	0.0	0.0	-1.824E 00	0.0	-2.1880E-02	0.0
	2	0.0	0.0	-2.868E-01	0.0	-4.8726E-02	0.0
7	1	0.0	0.0	-2.019E 00	5.9462E-04	-1.7032E-02	0.0
	2	0.0	0.0	-7.868E-01	1.7312E-03	-4.9585E-02	0.0
6	1	0.0	0.0	-2.019E 00	0.0	-1.7042E-02	0.0
	2	0.0	0.0	-7.868E-01	0.0	-4.9616E-02	0.0
5	1	0.0	0.0	-2.163E 00	4.0716E-04	-1.1663E-02	0.0
	2	0.0	0.0	-1.244E 00	1.4078E-03	-4.0325E-02	0.0
4	1	0.0	0.0	-2.163E 00	0.0	-1.1670E-02	0.0
	2	0.0	0.0	-1.244E 00	0.0	-4.0349E-02	0.0
3	1	0.0	0.0	-2.251E 00	2.0687E-04	-5.9273E-03	0.0
	2	0.0	0.0	-1.565E 00	7.8716E-04	-2.2554E-02	0.0
2	1	0.0	0.0	-2.251E 00	0.0	-5.9308E-03	0.0
	2	0.0	0.0	-1.565E 00	0.0	-2.2567E-02	0.0
1	1	0.0	0.0	-2.281E 00	0.0	0.0	0.0
	2	0.0	0.0	-1.673E 00	0.0	0.0	0.0

****
EVALUATION OF DESIGN NUMBER 0
****

	STRESS AREA RATIO	LOAD COND	DES VARIABLE
MAX	0.2392E-03	1	1
MIN	0.2369E-03	1	10

MAX	BUCK RATIO	LOAD COND
	0.1002E 01	1
	0.1445E 00	1

DESIGN IS CRITICAL

STRUCTURAL WEIGHT= 0.9692E 02

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR BUCKLING CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	0.17828E 01
2	ACT	0.17099E 01
3	ACT	0.15695E 01
4	ACT	0.13773E 01
5	ACT	0.11521E 01
6	ACT	0.91532E 00
7	ACT	0.68834E 00
8	ACT	0.49002E 00
9	ACT	0.33468E 00
10	ACT	0.23969E 00

NO. OF ACTIVE BUCKLING CONSTRAINTS   ARE   1

****
ANALYSIS OF DESIGN NUMBER 5
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE NO.	LOAD CASE	X	Y	Z	XX	YY	ZZ
31	1	0.0	0.0	0.0	0.0	0.0	0.0
30	1	0.0	0.0	0.0	0.0	0.0	0.0
29	1	0.0	0.0	0.0	0.0	0.0	0.0
28	1	0.0	0.0	0.0	0.0	0.0	0.0
27	1	0.0	0.0	0.0	0.0	0.0	0.0
26	1	0.0	0.0	0.0	0.0	0.0	0.0
25	1	0.0	0.0	0.0	0.0	0.0	0.0
24	1	0.0	0.0	0.0	0.0	0.0	0.0
23	1	0.0	0.0	0.0	0.0	0.0	0.0
22	1	0.0	0.0	0.0	0.0	0.0	0.0
21	1	-1.051E-03	-3.647E-05	0.0	0.0	0.0	0.0
20	1	-1.051E-03	0.0	0.0	0.0	0.0	0.0
19	1	-8.227E-04	-2.849E-05	0.0	0.0	0.0	0.0
18	1	-8.232E-04	0.0	0.0	0.0	0.0	0.0
17	1	-6.820E-04	-2.363E-05	0.0	0.0	0.0	0.0
16	1	-6.824E-04	0.0	0.0	0.0	0.0	0.0
15	1	-5.705E-04	-1.977E-05	0.0	0.0	0.0	0.0
14	1	-5.709E-04	0.0	0.0	0.0	0.0	0.0
13	1	-4.738E-04	-1.641E-05	0.0	0.0	0.0	0.0
12	1	-4.740E-04	0.0	0.0	0.0	0.0	0.0
11	1	-3.858E-04	-1.336E-05	0.0	0.0	0.0	0.0
10	1	-3.860E-04	0.0	0.0	0.0	0.0	0.0
9	1	-3.036E-04	-1.050E-05	0.0	0.0	0.0	0.0
8	1	-3.037E-04	0.0	0.0	0.0	0.0	0.0
7	1	-2.252E-04	-7.772E-06	0.0	0.0	0.0	0.0

6	1	-2.253E-04	0.0	0.0	0.0	0.0	0.0
5	1	-1.494E-04	-5.133E-06	0.0	0.0	0.0	0.0
4	1	-1.494E-04	0.0	0.0	0.0	0.0	0.0
3	1	-7.476E-05	-2.536E-06	0.0	0.0	0.0	0.0
2	1	-7.480E-05	0.0	0.0	0.0	0.0	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.7222E 00	0.7117E 00	0.6969E 00	0.6731E 00	0.6406E 00	0.5982E 00	0.5436E 00	0.4729E 00	0.3772E 00	0.2371E 00

ANALYSIS OF PLATE/SHELL ELEMENTS ,CONSTRN CODE = 1

ELEMENT NUMBER	ELEMENT THICKNESS	LOAD COND	/-------------MEMBRANE FORCES------------ NXX	NYY	NXY	//--------BENDING/TWISTING MOMENTS--------/ MXX	MYY	MXY
1	0.7222E 00	1	-0.7666E 01	-0.7546E 01	0.3476E-03	0.0	0.0	0.0
2	0.7117E 00	1	-0.7555E 01	-0.7474E 01	-0.2371E-03	0.0	0.0	0.0
3	0.6969E 00	1	-0.7509E 01	-0.7405E 01	-0.7579E-05	0.0	0.0	0.0
4	0.6731E 00	1	-0.7459E 01	-0.7270E 01	-0.4267E-04	0.0	0.0	0.0
5	0.6406E 00	1	-0.7396E 01	-0.7082E 01	-0.4418E-04	0.0	0.0	0.0
6	0.5982E 00	1	-0.7315E 01	-0.6830E 01	-0.4921E-04	0.0	0.0	0.0
7	0.5436E 00	1	-0.7214E 01	-0.6497E 01	-0.2819E-04	0.0	0.0	0.0
8	0.4729E 00	1	-0.7088E 01	-0.6043E 01	-0.5952E-04	0.0	0.0	0.0
9	0.3772E 00	1	-0.6525E 01	-0.5387E 01	0.1378E-03	0.0	0.0	0.0
10	0.2371E 00	1	-0.6706E 01	-0.4330E 01	-0.4602E-03	0.0	0.0	0.0

ANALYSIS OF BOUNDARY ELEMENTS - CONSTRAINT FORCES

CONST NUMBER	LOAD CASE	FORCE	MOMENT
1	1	0.0	0.0
2	1	0.0	0.0
3	1	0.0	0.0
4	1	0.0	0.0
5	1	0.0	0.0
6	1	0.0	0.0
7	1	0.0	0.0
8	1	0.0	0.0
9	1	0.0	0.0
10	1	0.0	0.0
11	1	0.75015E 02	0.0
12	1	0.74411E 02	0.0
13	1	0.73347E 02	0.0
14	1	0.71722E 02	0.0
15	1	0.69526E 02	0.0
16	1	0.66557E 02	0.0
17	1	0.62653E 02	0.0

18	1	0.57083E 02	0.0
19	1	0.48457E 02	0.0
20	1	0.21871E 02	0.0

BUCKLING LOAD PARAMETERS

0.99759D 00   0.429560 01

BUCKLING MODE SHAPES

NODE NO.	MODE SHAPE	X	Y	Z	XX	YY	ZZ
31	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
30	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
29	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
28	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
27	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
26	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
25	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
24	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
23	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
22	1	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0
21	1	0.0	0.0	0.0	-1.7326E-03	4.9618E-02	0.0
	2	0.0	0.0	0.0	2.1999E-03	-6.3002E-02	0.0
20	1	0.0	0.0	0.0	0.0	4.9670E-02	0.0
	2	0.0	0.0	0.0	0.0	-6.3068E-02	0.0
19	1	0.0	0.0	4.574E-01	-1.3360E-03	3.8262E-02	0.0
	2	0.0	0.0	-4.110E-01	1.6639E-04	-4.7690E-03	0.0
18	1	0.0	0.0	4.574E-01	0.0	3.8287E-02	0.0
	2	0.0	0.0	-4.110E-01	0.0	-4.7712E-03	0.0
17	1	0.0	0.0	8.062E-01	-1.0737E-03	3.0751E-02	0.0
	2	0.0	0.0	-3.326E-01	-7.0464E-04	2.0177E-02	0.0
16	1	0.0	0.0	8.062E-01	0.0	3.0769E-02	0.0
	2	0.0	0.0	-3.326E-01	0.0	2.0190E-02	0.0
15	1	0.0	0.0	1.087E 00	-8.7798E-04	2.5146E-02	0.0
	2	0.0	0.0	-7.857E-02	-1.0383E-03	2.9735E-02	0.0
14	1	0.0	0.0	1.087E 00	0.0	2.5161E-02	0.0
	2	0.0	0.0	-7.856E-02	0.0	2.9754E-02	0.0
13	1	0.0	0.0	1.317E 00	-7.1632E-04	2.0517E-02	0.0
	2	0.0	0.0	2.342E-01	-1.1183E-03	3.2028E-02	0.0
12	1	0.0	0.0	1.317E 00	0.0	2.0529E-02	0.0
	2	0.0	0.0	2.342E-01	0.0	3.2047E-02	0.0
11	1	0.0	0.0	1.502E 00	-5.7516E-04	1.6475E-02	0.0
	2	0.0	0.0	5.490E-01	-1.0587E-03	3.0325E-02	0.0
10	1	0.0	0.0	1.502E 00	0.0	1.6485E-02	0.0
	2	0.0	0.0	5.490E-01	0.0	3.0343E-02	0.0
9	1	0.0	0.0	1.649E 00	-4.4740E-04	1.2816E-02	0.0

	2	0.0	0.0	8.342E-01	-9.1607E-04	2.6241E-02	0.0
8	1	0.0	0.0	1.649E 00	0.0	1.2824E-02	0.0
	2	0.0	0.0	8.342E-01	0.0	2.6257E-02	0.0
7	1	0.0	0.0	1.761E 00	-3.2870E-04	9.4171E-03	0.0
	2	0.0	0.0	1.071E 00	-7.2214E-04	2.0688E-02	0.0
6	1	0.0	0.0	1.761E 00	0.0	9.4228E-03	0.0
	2	0.0	0.0	1.071E 00	0.0	2.0701E-02	0.0
5	1	0.0	0.0	1.839E 00	-2.1613E-04	6.1933E-03	0.0
	2	0.0	0.0	1.246E 00	-4.9672E-04	1.4233E-02	0.0
4	1	0.0	0.0	1.839E 00	0.0	6.1971E-03	0.0
	2	0.0	0.0	1.246E 00	0.0	1.4242E-02	0.0
3	1	0.0	0.0	1.885E 00	-1.0663E-04	3.0573E-03	0.0
	2	0.0	0.0	1.354E 00	-2.5139E-04	7.2080E-03	0.0
2	1	0.0	0.0	1.885E 00	0.0	3.0592E-03	0.0
	2	0.0	0.0	1.354E 00	0.0	7.2124E-03	0.0
1	1	0.0	0.0	1.901E 00	0.0	0.0	0.0
	2	0.0	0.0	1.391E 00	0.0	0.0	0.0

****
EVALUATION OF DESIGN NUMBER 5
****

	STRESS AREA RATIO	LOAD COND	DES VARIABLE
MAX	0.4969E-03	1	10
MIN	0.2107E-03	1	1

MAX BUCK RATIO	LOAD COND
0.1002E 01	1
0.2328E 00	1

DESIGN IS CRITICAL

STRUCTURAL WEIGHT= 0.8459E 02

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR BUCKLING CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	0.98764E 00
2	ACT	0.98898E 00
3	ACT	0.99151E 00
4	ACT	0.99373E 00
5	ACT	0.99652E 00
6	ACT	0.10001E 01
7	ACT	0.10051E 01
8	ACT	0.10132E 01
9	ACT	0.10295E 01
10	ACT	0.98673E 00

```
NO. OF ACTIVE BUCKLING CONSTRAINTS      ARE     1
BUCKLING - CRITICAL DESIGN HAS CONVERGED
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