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EVALUATION OF QUASI-SQUARE WAVE INVERTER AS A POWER SOURCE FOR INDUCTION MOTORS

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16. ABSTRACT

This study investigates the relative merits of quasi-square wave inverter-motor technology versus a sine wave inverter-motor system. The empirical results of several tests on various sizes of wye-wound induction motors are presented with mathematical analysis to support the conclusions of this study. This study concludes that, within the limitations presented in this report, the quasi-square wave inverter-motor system is superior to the more complex sine wave system for most induction motor applications in space.

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DEFINITION OF SYMBOLS

Symbol	Definition
f	frequency (hertz)
f_1	impressed-motor frequency (hertz)
I_L	true rms line or phase current in wye-wound motor (amperes)
n	speed of motor (rpm)
p	number of poles
PF	power factor of motor = $\cos\theta$ where θ is the angle between voltage and current vectors
P_i	input motor power (watts)
P_o	output motor power (watts)
Q-S	quasi-square
r_m	equivalent resistance due to core loss
r_1	primary (stator) resistance
r_2	secondary (rotor) resistance reflected to stator
s	slip
T	shaft torque of motor (kilogram-centimeters)
$T_{n\,max}$	maximum torque at the nth harmonic (kilogram-centimeter)
V_{in}	dc voltage at inverter input (volts)
V_p	peak voltage (volts)

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
V_{p-p}	peak-to-peak voltage (volts)
V_{Trms}	true root mean square voltage (volts)
V_1	fundamental frequency sine wave rms voltage (volts)
V_{1p}	peak of fundamental frequency voltage (volts)
$V_{\theta-\theta}$	phase-to-phase voltage (volts)
x_m	magnetizing reactance
x_1	primary (stator) leakage reactance
x_2	secondary (rotor) reactance reflected to stator

EVALUATION OF QUASI-SQUARE WAVE INVERTER AS A POWER SOURCE FOR INDUCTION MOTORS

SUMMARY

A group of inverter-motor tests has been analyzed in an effort to determine the relative merits of a quasi-square (Q-S) wave inverter-motor system when compared to a sine wave inverter-motor system for space applications. The results of several tests using Q-S and sine wave on various sizes of wye-wound induction motors are presented.

A Q-S waveform is inherently generated in a three-phase inverter bridge by switching the transistors on and off at the proper intervals. No heavy filters or complex timing circuits are necessary; therefore, the resulting inverter is smaller, lighter weight, and simpler, implying greater reliability and lower cost. These advantages invite a closer look at inverter-motor systems, particularly for space flight. The only apparent disadvantage is a 3 to 5 percent loss in motor efficiency and the performance degradation discussed later. Methods to eliminate the performance degradation, which depends somewhat on the switching drive to the bridge transistors, are discussed, and test results are presented.

Variations in induction motor performance may occur when a motor is driven from a Q-S wave inverter rather than a sine wave source. The performance difference may be a change in speed, maximum torque, or maximum power capability depending on the motor power factor and inverter configuration. Motor operation is normally changed very little at loads less than rated for the motors tested (most induction motors are rated at less than 70 percent peak torque). However, above rated load, motor performance may vary significantly, particularly as the power factor changes with load. Motors with power factors greater than 0.7 show a marked decrease in peak torque and power output as a direct result of fundamental voltage drops caused by waveform variations in 120 degree drive inverters.

I. INTRODUCTION

This paper presents a discussion of the variations in induction motor performance when a motor is driven from a Q-S wave inverter rather than a sine source. A series of tests was run on motors driven by a 120 degree drive

Q-S wave inverter. The performance data are analyzed and compared with data on the same motor driven with a sine wave. Data are presented on the reduction in fundamental voltage, change in waveform, power factor, and losses due to harmonics. A Fourier analysis of waveforms is presented in the Appendix.

The performance characteristics and advantages of the Q-S wave system are described relative to overall system design with methods of optimizing system design presented. The Q-S wave inverter is demonstrated adequate for virtually all space related induction motor drive applications with superiority in cost, reliability, size, and weight.

II. QUASI-SQUARE WAVE INVERTER OPERATION

A Q-S wave is inherently generated in a three-phase inverter bridge (Fig. 1) by switching the transistors on and off in the sequence shown in Table 1. Note that the transistors are on for only 120 degrees. A similar waveform is also produced if the transistors are on for 180 degrees. However, the potential problem of having two transistors in series on simultaneously is eliminated by switching for only 120 degrees. Thus, this method has been used in inverters used in this study. The disadvantage of switching for only 120 degrees is that

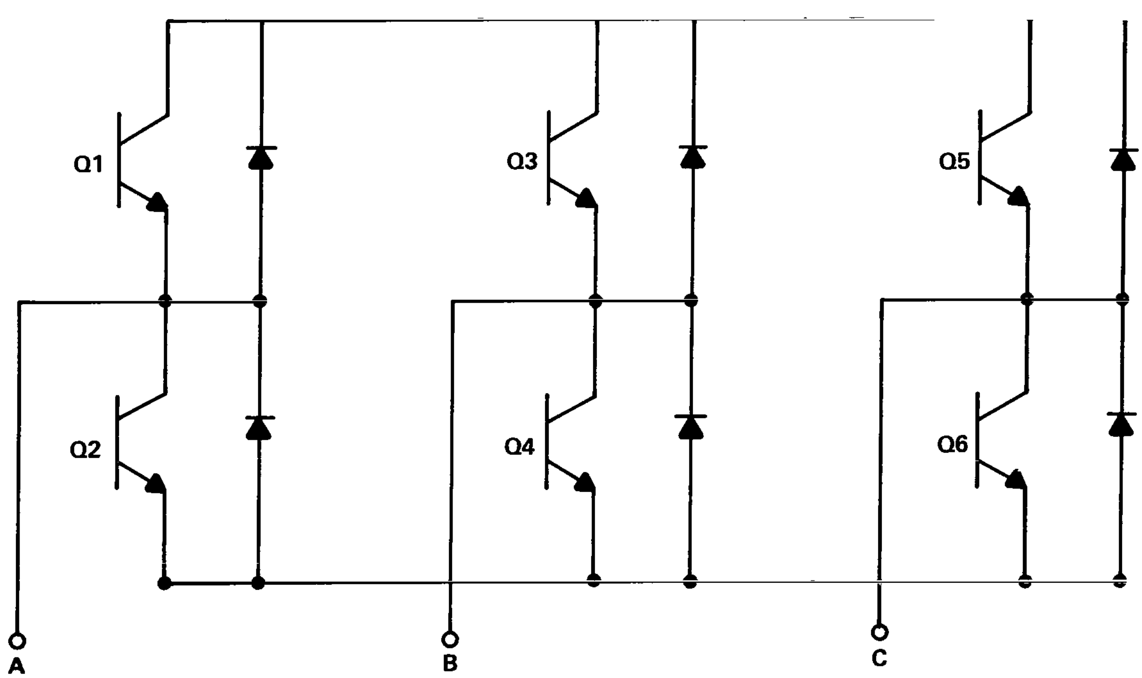

Figure 1. Three-phase Q-S inverter transistor bridge.

TABLE 1. TRANSISTOR SWITCHING SCHEMES

Q	120° DRIVE						180° DRIVE					
1	1	1	0	0	0	0	1	1	1	0	0	0
2	0	0	0	1	1	0	0	0	0	1	1	1
3	0	0	1	1	0	0	0	0	1	1	1	0
4	1	0	0	0	0	1	1	1	0	0	0	1
5	0	0	0	0	1	1	1	0	0	0	1	1
6	0	1	1	0	0	0	0	1	1	1	0	0
Degrees	0 60 120 180 240 300 360											
							0 60 120 180 240 300 360					

Note: 1 indicates transistor is driven on, 0 indicates transistor is driven off.

the waveforms and voltage regulation are dependent on the load power factor. By taking the necessary precautions to prevent short circuit current through the series transistor, this disadvantage is overcome.

Q-S inverters were used by Marshall Space Flight Center (MSFC) to drive pump and blower motors for several applications in the National Aeronautics and Space Administration (NASA) Skylab program. A block diagram of a basic Q-S inverter is shown in Figure 2. The input filter provides audio and radio frequency isolation to prevent interference between the inverter and other loads on the spacecraft bus. The dc-to-dc converter/regulator provides power and bias voltages for the oscillator, driver, and logic in the inverter. The oscillator generates the frequency necessary to drive the inverter output frequency. The logic provides the necessary countdown and phasing to operate the three-phase bridge through the driver which provides power amplification. Ancillary features of current limiting, voltage control, frequency control, slip frequency control, etc. may be added if required depending on application [1].

III. INVERTER-MOTOR TESTS

A Q-S wave inverter was tested and performance characteristics were analyzed for comparison with sine wave driven high and low power factor motors. The reduction in fundamental voltage, change in waveshape, change in power factor, and losses due to harmonics are discussed. All voltages are measured phase-to-phase unless otherwise indicated.

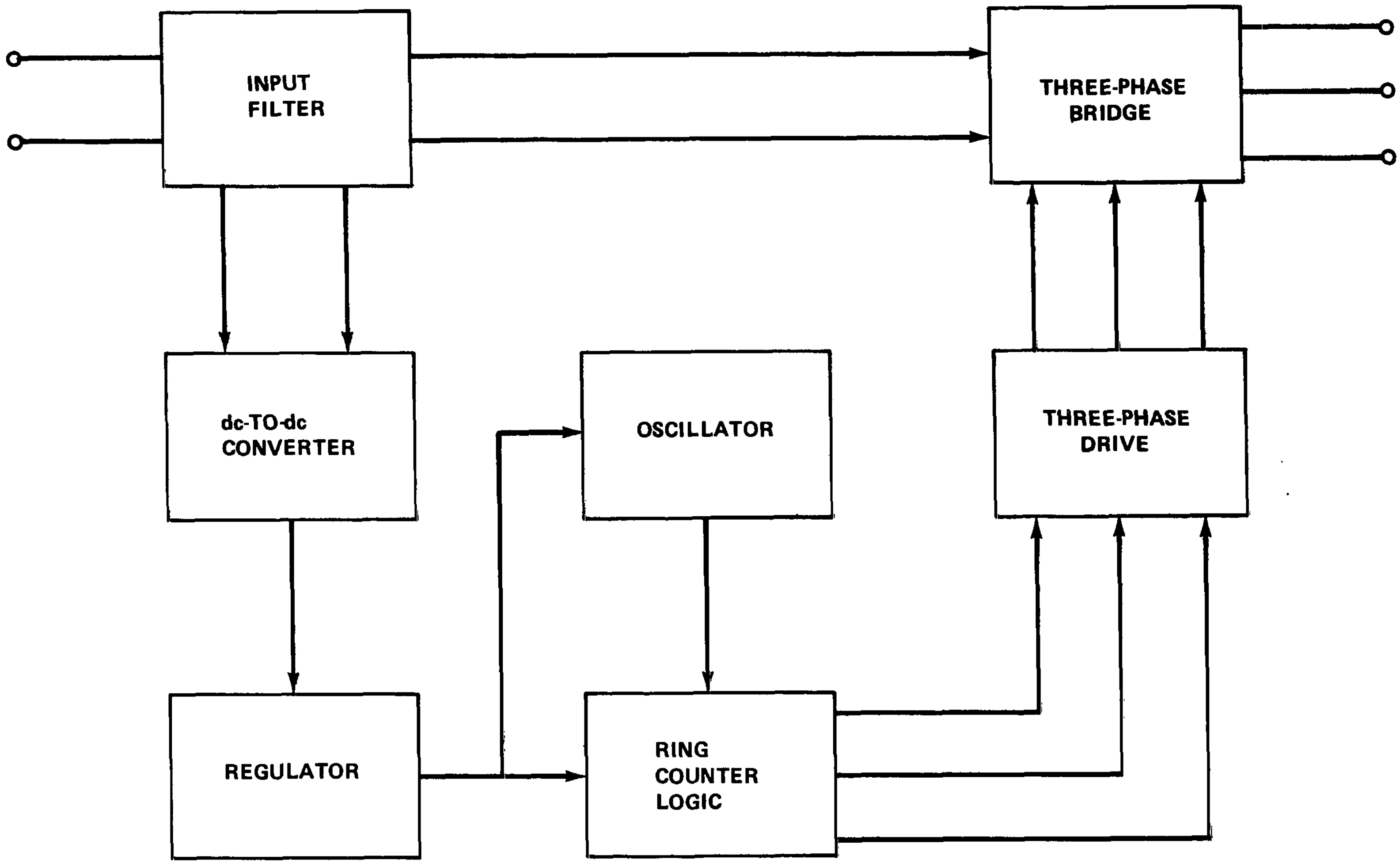

Figure 2. Block diagram of typical Q-S inverter.

The test setup used for this investigation is shown in Figure 3. The test inverter, a typical Q-S wave inverter, has the phase-to-phase voltage waveform as shown in Figure 4(a) when driving an inductive load, and the voltage waveform shown in Figure 4(b) when driving a pure resistive load. It will be shown later that, as a motor is loaded, its power factor (PF) increases and the waveform changes gradually (with 120 degree Q-S drive) from Figure 4(a) to Figure 4(b). A Fourier analysis of these waveforms is compared with the actual measured harmonic distribution in the Appendix.

Several bases are available for comparing sine wave motor operation to Q-S wave motor operation. The two waveforms could be made to have the same fundamental voltage, the same peak-to-peak voltage (V_{p-p}), or the same true rms voltage (V_{Trms}), therefore, the basis is specified for each comparison.

A. Reduction in V_1

The major effect of a Q-S wave on an induction motor with a high PF (greater than 0.7) is a decrease in fundamental frequency sine wave rms voltage (V_1) for a given rms voltage and the corresponding decrease in speed, maximum torque, and a maximum output power to the motor. The following relationships refer to the induction motor equivalent circuit shown in Figure 5:

$$T_{max} = \frac{p V_1^2}{4 \pi f_1} \quad \frac{1}{2\left(r_1 + \sqrt{r_1 + (x_1 + x_2)^2}\right)}$$

so

$$T_{max} \propto (V_1^2)$$

and

$$P_{o\ max} \propto T_{max} \quad .$$

Figures 6 and 7 are graphs of speed versus torque for two motors used in this investigation. Figure 6 shows the reduction in T_{max} of the high PF motor due

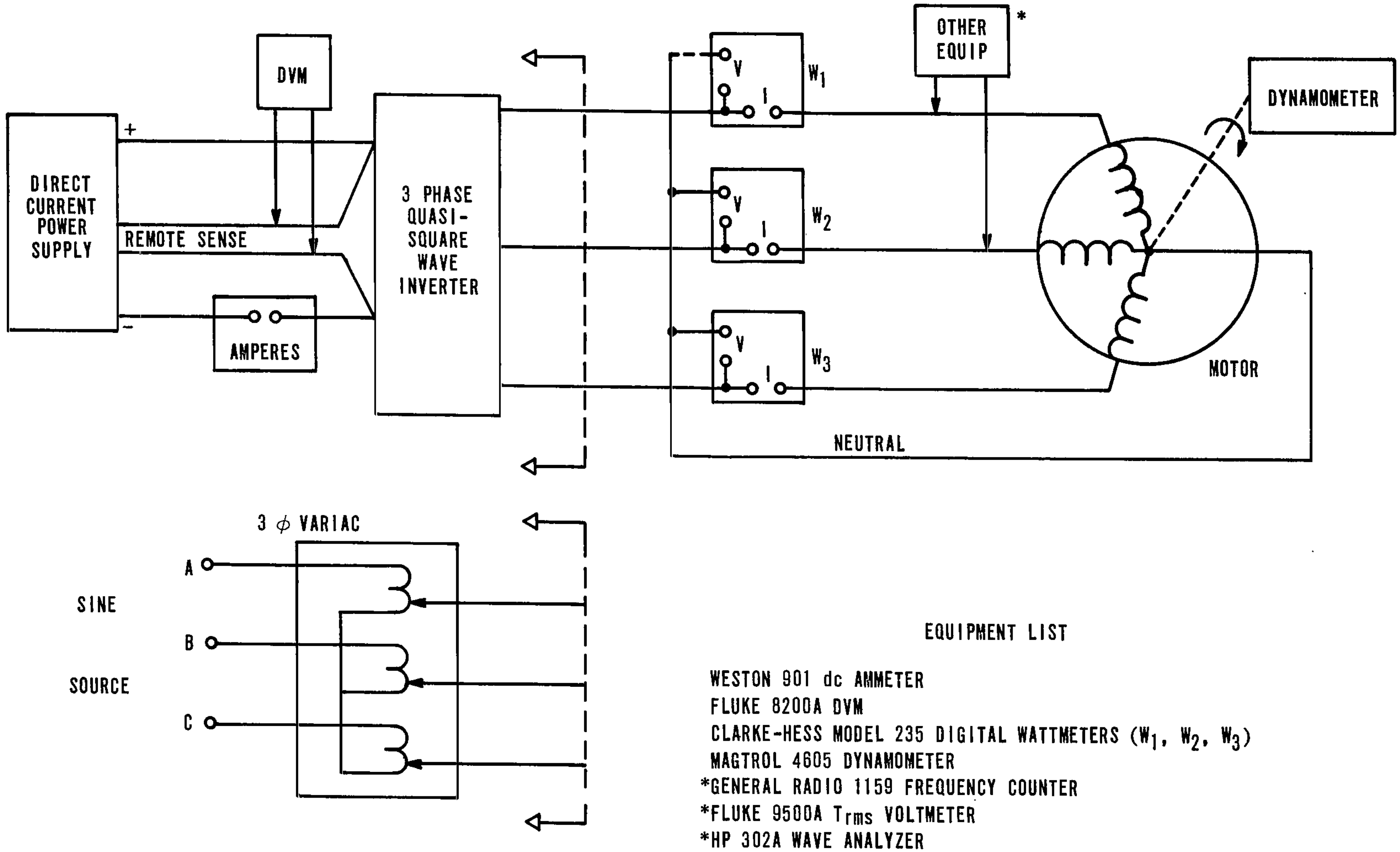

Figure 3. Test setup.

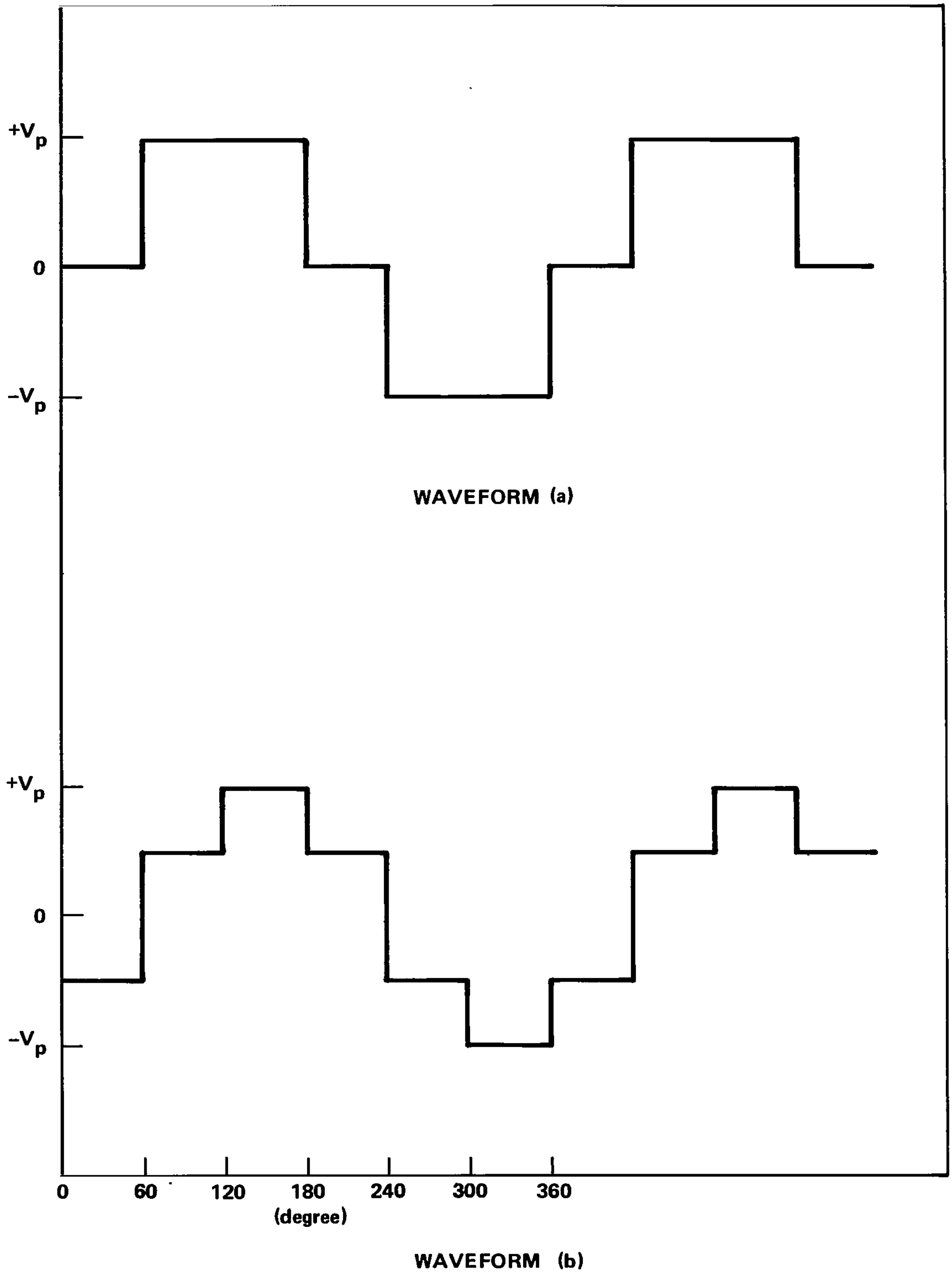

Figure 4. Q-S waveforms.

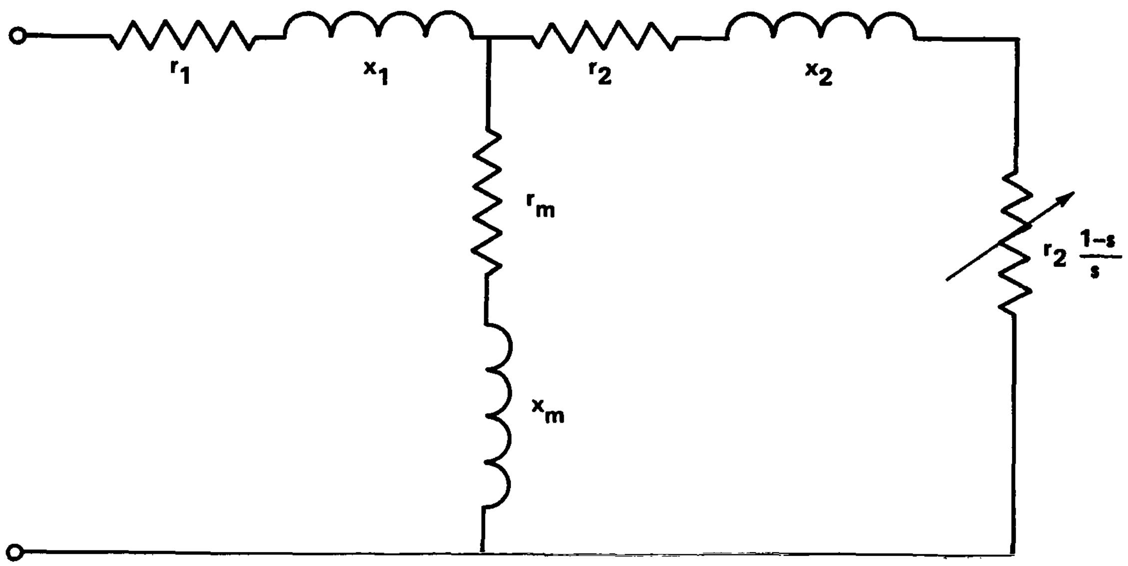

Figure 5. Induction motor equivalent circuit.

to Q-S wave, and Figure 7 (curve number 3) shows that this torque is increased in the relatively low PF motor. The reduced V_1 causes reduced speed, increased slip, increased line current, and decreased motor efficiency as shown in Figures 6 and 7.

A quantitative investigation of this change in motor operation requires a knowledge of the values V_1 and V_{Trms} of the Q-S waveform. For waveform (a) of Figure 4,

$$V_{Trms} = \sqrt{\frac{1}{T}\int_o^T v^2\,dt} = \sqrt{\frac{1}{2\pi}\left[\int_{\pi/3}^{\pi} V_p^2\,dt + \int_{4\pi/3}^{2\pi} V_p^2\,dt\right]} \quad .$$

Thus,

$$V_{Trms} = 0.816\ V_p \quad .$$

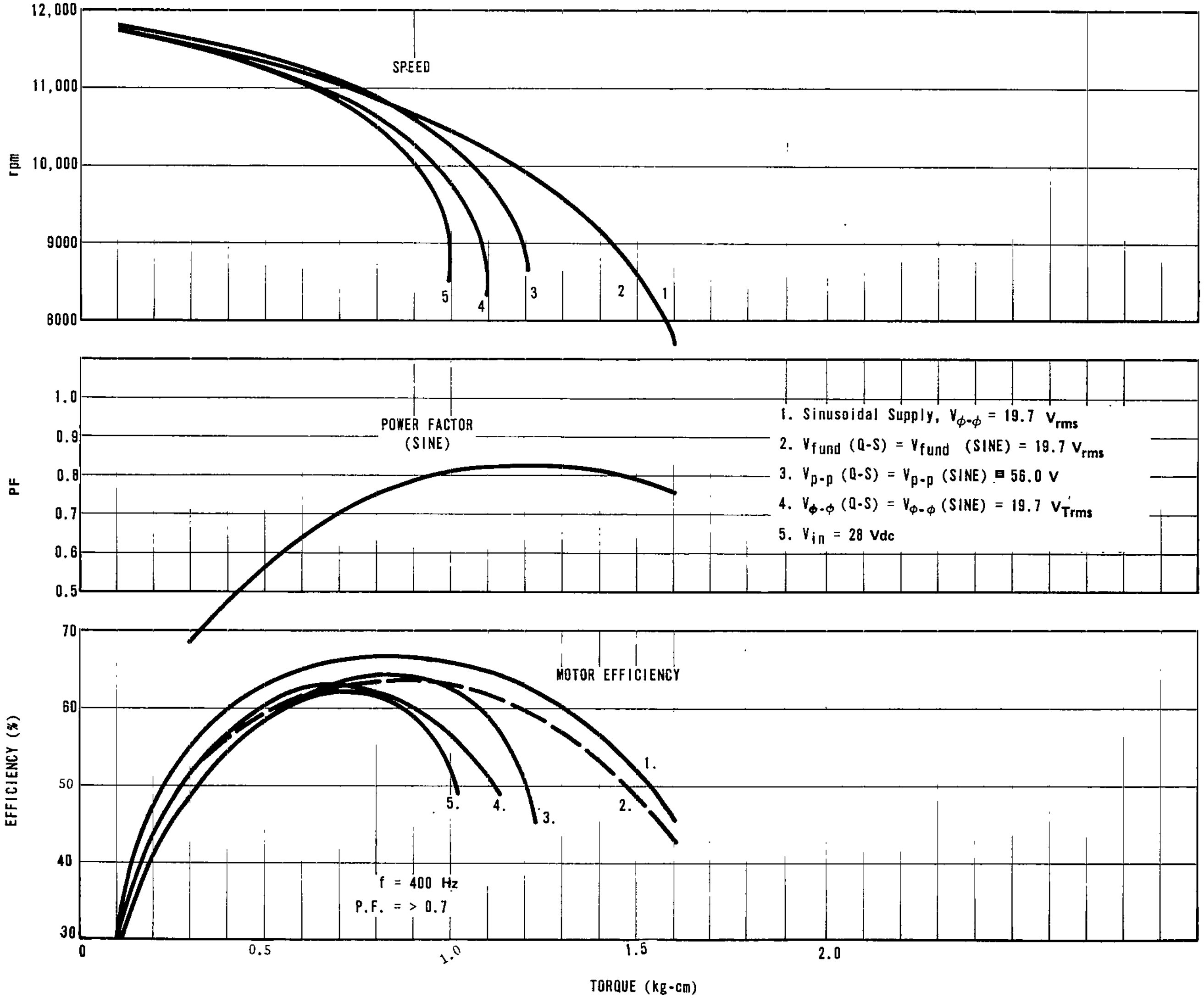

Figure 6. Motor performance Q-S versus sine wave (1/6 hp motor).

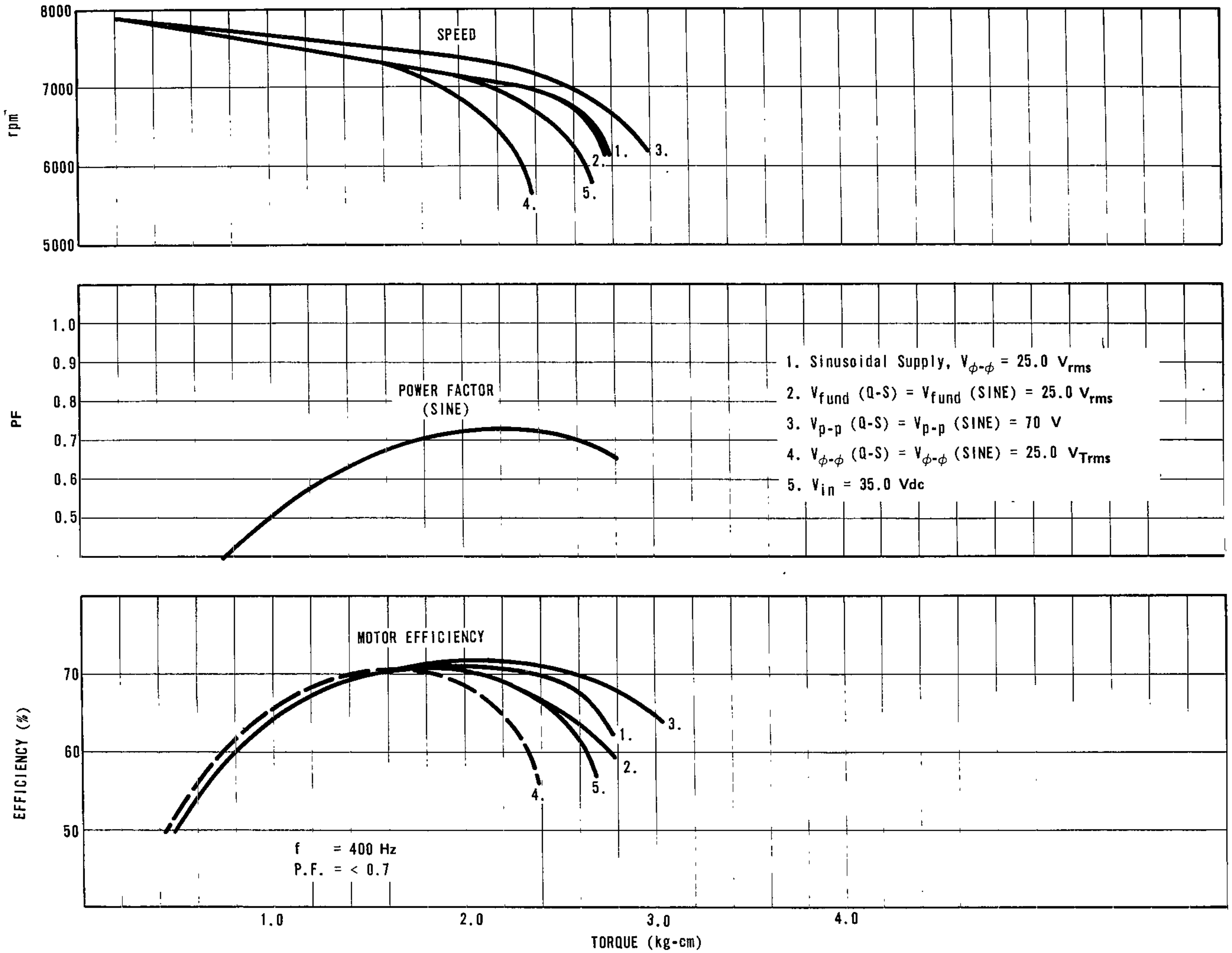

Figure 7. Motor performance Q-S versus sine wave (1/2 hp motor).

From the Fourier analysis in the Appendix

$$V_{1p} = 1.1026\ V_p \quad ;$$

then the fundamental rms voltage in terms of a sine wave

$$V_1 = (0.707)\ (V_{1p}) = 0.707\ (1.1026\ V_p) \quad .$$

Thus,

$$V_1 = 0.7796\ V_p \quad .$$

and

$$\frac{V_1}{V_{Trms}} = \frac{0.7796\ V_p}{0.816\ V_p} = 0.955 \quad .$$

For waveform (b) of Figure 4,

$$V_{Trms} = \sqrt{\frac{1}{T}\int_o^T v^2\, dt}$$

$$= \sqrt{\frac{1}{\pi}\left[\int_o^{\pi/3}\left(\frac{-V_p}{2}\right)^2 dt + \int_{\pi/3}^{2\pi/3}\left(\frac{V_p}{2}\right)^2 dt + \int_{2\pi/3}^{\pi}(V_p)^2\, dt\right]} \quad .$$

Thus,

$$V_{Trms} = 0.707\ V_p \quad .$$

From the Fourier analysis in the Appendix,

$$V_{1p} = 0.9549\ V_p \qquad ;$$

then the fundamental rms voltage in terms of a sine wave

$$V_1 = (0.707)\ V_{1p}$$

$$= 0.707\ (0.9549\ V_p) \qquad ;$$

thus,

$$V_1 = 0.6752\ V_p$$

and

$$\frac{V_1}{V_{Trms}} = \frac{0.6752\ V_p}{0.707\ V_p} = 0.955 \qquad .$$

These results are compared in Table 2.

Comparing on the basis of the same peak voltage, waveform (a) has a higher fundamental voltage than a sine, while waveform (b) has a lower fundamental voltage than a sine (shown as curve 3 in Figures 6 and 7). The fundamental voltage dictates motor performance; thus performance will not be impaired unless and until the Q-S waveform degrades to waveform (b), which starts occurring at 0.9 kg-cm in Figure 6 and never occurs in Figure 7. Note that with 180 degrees drive this waveform change does not occur, and therefore, no degradation occurs (Figs. 8 and 9).

However, based on the same V_{Trms}, the fundamental voltage will be reduced 4.5 percent for either waveform. As shown in line 4 of Figures 6 and 7,

TABLE 2. COMPARISON OF CHARACTERISTICS

Parameter	Waveform		
	Sine	Type A Q-S[a]	Type B Q-S[b]
V_{Trms}	0.707 V_{peak}	0.816 V_{peak}	0.707 V_{peak}
V_1	0.707 V_{peak}	0.779 V_{peak}	0.675 V_{peak}
V_1/V_{Trms}	100%	95.5%	95.5%

a. See Figure 4(a)

b. See Figure 4(b)

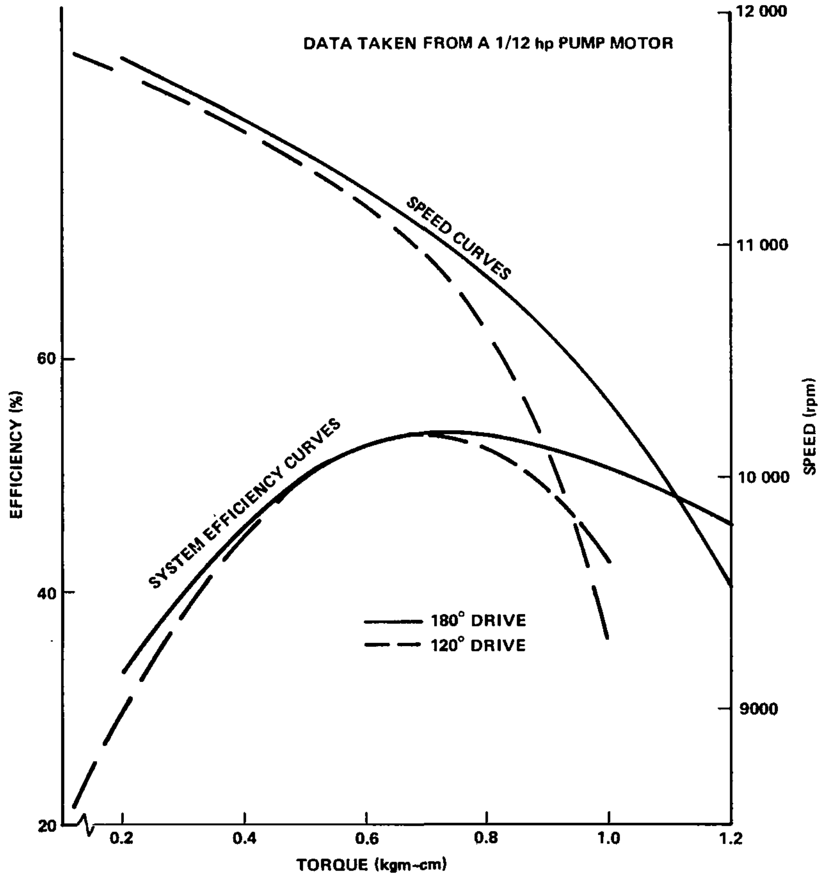

Figure 8. Motor system performance for 180 and 120 degree drive Q-S inverter.

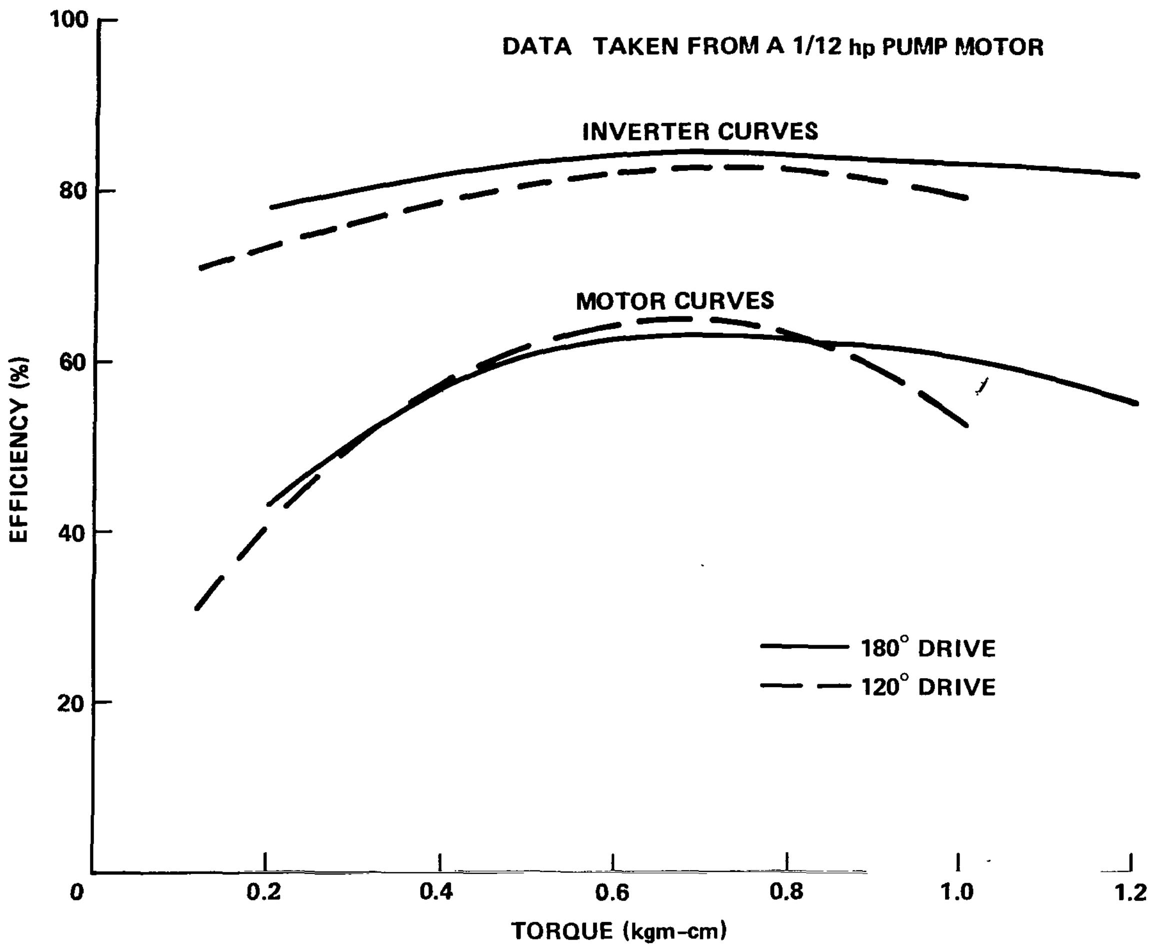

Figure 9. Inverter-motor efficiency as a function of torque for 180 and 120 degree drive Q-S inverter.

the motor operates as if on a reduced supply voltage (with resultant lower T_{max}). Therefore, performance will be degraded in either case with a corresponding increase in line current and decrease in efficiency for a given motor. However, if a motor is used which was designed to operate off this reduced fundamental, the Q-S wave inverter-motor system could be as satisfactory as the sine source.

B. Change in Waveshape

On the basis of constant peak voltage, the change in waveshape makes a large difference in motor operation because the fundamental voltage drops 13.4 percent from waveform (a) to waveform (b), (Fig. 4). Note curve 3 on

Figures 6 and 7. The waveshape changes completely from (a) to (b), with a large drop in T_{max}. However, the waveshape hardly changes in Figure 7; thus the larger V_1 is maintained with a high T_{max}. The waveshape has been found to change for some motors but not for others because of differences in motor design (specifically differences in power factor).

C. Power Factor Effect

Because the waveshape heavily depends upon the ratio of resistance to impedance, it is informative to note the power factor curves in Figures 6 and 7. Where the power factor barely exceeds 0.7, the waveform does not change greatly as shown by the high torque of line 3 in Figure 7. However, as the power factor increases above 0.7, the voltage quickly changes from waveform (a) to (b) in Figure 4 as indicated by the lower torque of line 3 in Figure 6. Although no simple formula was found to predict the change in waveform versus power factor, empirical curves are plotted in Figure 10 to indicate the rapid decrease in V_1 as the power factor exceeded 0.7. The close agreement of the curves supports the hypothesis that the change in fundamental voltage and waveshape is primarily due to the change in power factor.

D. Losses Due to Harmonics

In addition to the change in motor performance due to the reduction in V_1 there are small additional losses due to the harmonic voltages present in the Q-S waveform. These harmonic losses can be broken into two classes: reverse torques and additional I^2R losses.

The maximum torque is described by [2]:

$$T_{max} = \frac{p\,V_1^2}{4\,\pi\,f_1} \quad \frac{1}{2\,(r_1 + \sqrt{r_1 + (x_1 + x_2)^2})}$$

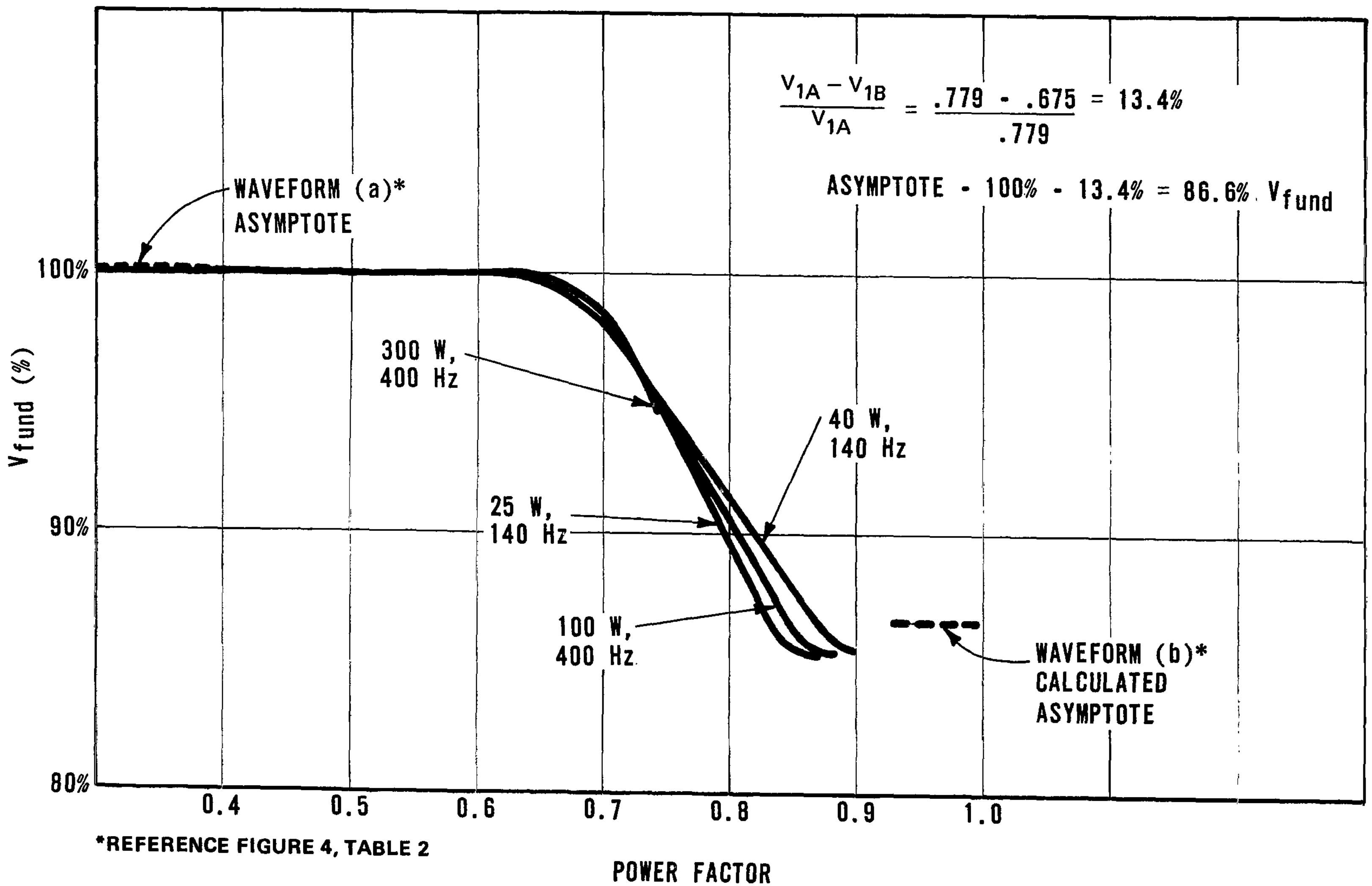

Figure 10. Fundamental voltage versus power factor for 120 degree drive.

so,

$$T_{max} = \frac{k\, V^2}{f}$$

where k is constant for a given motor and frequency. For the fundamental,

$$T_{1\ max} = \frac{k_1\, V_1^2}{f_1} \qquad .$$

For the 5th harmonic

$$T_{5\ max} = \frac{k_5\,(0.20\ V_1)^2}{5f_1} = -(0.0080)\,\frac{k_5\, V_1^2}{f_1} \qquad .$$

For the 7th harmonic

$$T_{7\ max} = \frac{k_7\,(0.14\ V)^2}{7f_1} = (0.0028)\,\frac{k_7\, V^2}{f_1} \qquad .$$

Thus,

$$T_{5\ max} = -0.0080\,\frac{k_5}{k_1}\,T_{1\ max}$$

and

$$T_{7\ max} = +\,0.0028\,\frac{k_7}{k_1}\,T_{1\ max}$$

etc.

It may be shown that the ratio k_n/k_1 is always less than unity, further reducing the harmonic torques. Thus, the net torque due to these harmonics has little significant effect because each individual torque alternates in sign [3] and all are very small.

The other source of harmonic losses (I^2R) is also very small. The line current due to harmonics is small because the voltages are small and the motor impedance rises rapidly as the frequency increases. This variation in impedance depends upon the ratio of resistance to motor impedance, which is indicated by the power factor. A high reactance motor impedance will be higher to harmonics than a low reactance motor impedance. Thus, a high power factor motor will have higher relative harmonic currents than a low power factor motor. In all the motors used in this study, the harmonic currents amounted to less than 1.7 percent of the total line current. This is indirectly illustrated by the efficiency curves in Figures 6 and 7 where curve 2 shows a small drop in efficiency compared to curve 1, which contains no harmonics.

IV. INVERTER SELECTION

A 100 W, three-phase Q-S wave inverter typically has approximately 60 percent of the number of components in a typical 100 W sine inverter. Many of the additional components are power handling (switching and filtering) devices in the output circuit of the inverters. Therefore, the Q-S inverter is inherently smaller, simpler, and more reliable and efficient than the sine wave inverter. This advantage is somewhat offset by the 3 to 5 percent lower motor efficiency. A design possibility is to include a regulator to maintain the fundamental voltage of the Q-S wave at a level equivalent to the sine source (curve 2 of Figures 6 and 7). Thus, the same motor could be used interchangeably between the sine source and the Q-S source with this arrangement. Another possibility may be to use motors with a power factor not exceeding 0.7, thus allowing the Q-S waveform to maintain its high fundamental voltage. Perhaps the best alternative is to use a 180 degree drive Q-S inverter and take the necessary precautions to prevent the short circuit current through series transistors. In any case, the Q-S waveform tends to be better than a sine wave inverter as a result of its simplicity.

V. CONCLUSIONS

Even though the tested Q-S wave inverter-motor system has a slightly lower efficiency than a sine wave motor system when operating a high PF (greater than 0.7) motor, the Q-S system is much simpler and more reliable with fewer parts. However, if a lower PF motor is available, or if a 180 degree drive Q-S inverter is used, the Q-S system actually produces a higher fundamental voltage than the sine. Thus, in terms of overall system design, the Q-S system is adequate for virtually all applications. Though the tested inverter may not be directly substituted for a sine source with a given motor, the total inverter-motor system can be designed to be equivalent. For a given supply voltage, the high PF motor should simply be designed to produce the required speed and torque at the lower fundamental voltage supplied by the Q-S inverter. Then the only difference between the Q-S and the sine system is the 3 to 5 percent lower motor efficiency of the Q-S system which is offset by the reduced number of parts, the improved reliability of the Q-S system, and possibly the higher efficiency of the Q-S inverter.

Another possibility for Q-S system improvement is the design of a low PF motor that would allow the full fundamental voltage of the waveform to be maintained. A regulation scheme could be employed which would maintain the fundamental voltage of Q-S wave at a constant level equivalent to the sine wave.

This study, therefore, concludes that the Q-S inverter-motor system is superior to the sine wave system for most space applications in terms of reliability, reduced weight, and size.

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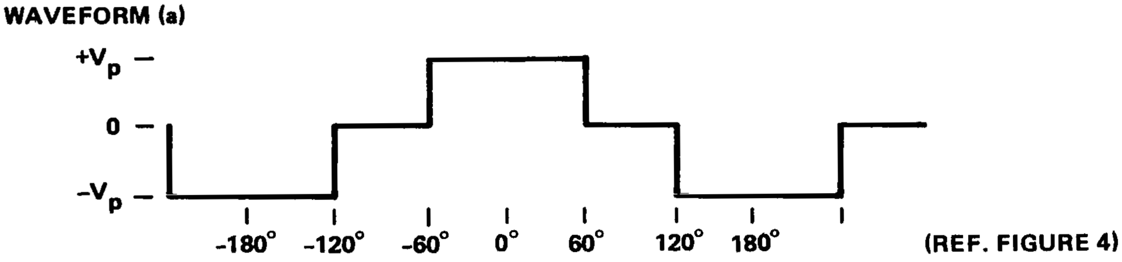
APPENDIX

FOURIER ANALYSIS OF QUASI-SQUARE WAVE



$$f(x) = \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_n = \frac{1}{\pi} \int_{-180°}^{180°} f(x) \cos nx \, dx = \frac{2}{\pi} \int_{0}^{180°} f(x) \cos nx \, dx$$

$$a_n = \frac{2}{\pi} \int_{0°}^{60°} V_p \cos nx \, dx + \frac{2}{\pi} \int_{60°}^{120°} 0 \, dx + \frac{2}{\pi} \int_{120°}^{180°} - V_p \cos nx \, dx$$

$$a_n = \frac{+2\sqrt{3}\, V_p}{n\pi} \qquad \text{for} \quad n = 1, 7, 13, 19, \ldots$$

$$a_n = \frac{-2\sqrt{3}\, V_p}{n\pi} \qquad \text{for} \quad n = 5, 11, 17, 23, \ldots$$

$$a_n = 0 \qquad \text{for all other } n.$$

Some calculated values for several of these harmonic amplitudes are presented in Table A-1.

TABLE A-1. WAVEFORM (a) ANALYSIS

Harmonic Number (n)	Relative Amplitude $V_p = 1.00$	Percentage of Fundamental
1	1.1027	100.0
5	-0.2205	20.0
7	0.1575	14.3
11	-0.1002	9.1
13	0.0848	7.7
17	-0.0649	5.9
19	0.0580	5.3
23	-0.0479	4.3
25	0.0441	4.0

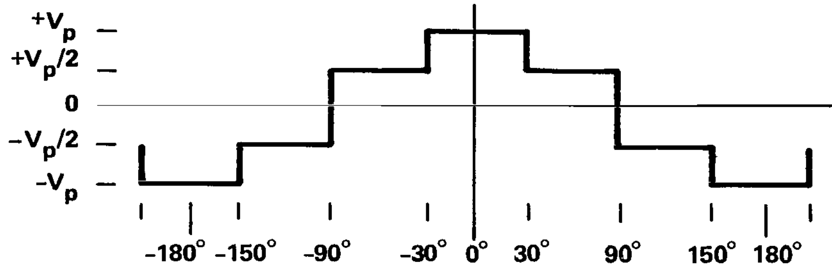
For harmonics through the 25th,

$$\text{Total Harmonic Distortion (THD)} = \frac{\sqrt{a_5^2 + a_7^2 + a_{11}^2 + a_{13} + \ldots}}{a_1}$$

$$\text{THD} = \frac{0.3201}{1.1027} = 0.290$$

$$\text{THD} = 29.0 \text{ percent}$$

WAVEFORM (b)



(REF. FIGURE 4)

$$f(x) = \sum_{n=1}^{\infty} b_n \cos nx$$

$$b_n = \frac{1}{\pi} \int_{-180°}^{180°} f(x) \cos nx \, dx = \frac{2}{\pi} \int_{0}^{180°} f(x) \cos nx \, dx$$

$$b_n = \frac{2}{\pi} \int_{0}^{30°} V_p \cos nx \, dx + \frac{2}{\pi} \int^{90°} V_p \cos nx \, dx$$

$$+ \frac{2}{\pi} \int^{150°} - V_p \cos nx \, dx + \frac{2}{\pi} \int_{150°}^{180°} - V_p \cos nx \, dx$$

where

$$b_n = \frac{3V_p}{n\pi} \qquad \text{for } n = 1, 5, 13, 17, 25, 29, \ldots$$

$$b_n = \frac{-3V_p}{n\pi} \qquad \text{for } n = 7, 11, 19, 23, 31, 35, \ldots$$

$$b_n = 0 \qquad \text{for all other } n.$$

Some calculated values for several of these harmonic amplitudes are presented in Table A-2.

TABLE A-2. WAVEFORM (b) ANALYSIS

Harmonic Number (n)	Relative Amplitude $V_p = 1.00$	Percentage of Fundamental
1	0.9549	100.0
5	0.1910	20.0
7	-0.1364	14.3
11	-0.0868	9.1
13	0.0735	7.7
17	0.0562	5.9
19	-0.0503	5.3
23	-0.0415	4.3
25	0.0382	4.0

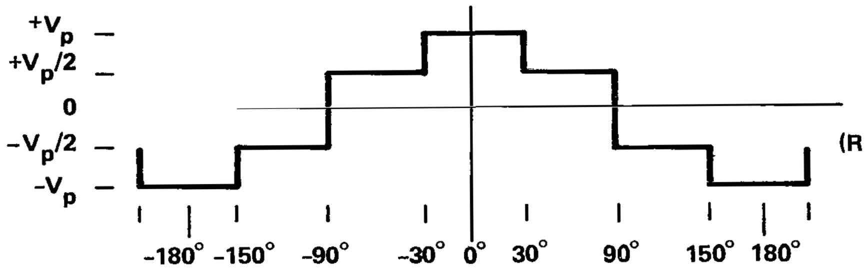
$$\text{Total Harmonic Distortion} = \frac{\sqrt{b_5^2 + b_7^2 + b_{11}^2 + b_{13}^2 + \ldots}}{b_1}$$

$$= \frac{0.2772}{0.9549} = 0.290$$

THD = 29.0 percent.

Figure A-1 depicts empirical data obtained from one wye-wound motor. These data were found to be typical for all inverter-motor combinations used in this study and agree exceptionally well with the preceding analysis.

WAVEFORM (b)



(REF. FIGURE 4)

$$f(x) = \sum_{n=1}^{\infty} b_n \cos nx$$

$$b_n = \frac{1}{\pi} \int_{-180^\circ}^{180^\circ} f(x) \cos nx \, dx = \frac{2}{\pi} \int_{0}^{180^\circ} f(x) \cos nx \, dx$$

$$b_n = \frac{2}{\pi} \int_{0}^{30^\circ} V_p \cos nx \, dx + \frac{2}{\pi} \int^{90^\circ} V_p \cos nx \, dx$$

$$+ \frac{2}{\pi} \int^{150^\circ} - V_p \cos nx \, dx + \frac{2}{\pi} \int_{150^\circ}^{180^\circ} - V_p \cos nx \, dx$$

where

$$b_n = \frac{3V_p}{n\pi} \qquad \text{for } n = 1, 5, 13, 17, 25, 29, \ldots$$

$$b_n = \frac{-3V_p}{n\pi} \qquad \text{for } n = 7, 11, 19, 23, 31, 35, \ldots$$

$$b_n = 0 \qquad \text{for all other } n.$$

Some calculated values for several of these harmonic amplitudes are presented in Table A-2.

TABLE A-2. WAVEFORM (b) ANALYSIS

Harmonic Number (n)	Relative Amplitude $V_p = 1.00$	Percentage of Fundamental
1	0.9549	100.0
5	0.1910	20.0
7	-0.1364	14.3
11	-0.0868	9.1
13	0.0735	7.7
17	0.0562	5.9
19	-0.0503	5.3
23	-0.0415	4.3
25	0.0382	4.0

$$\text{Total Harmonic Distortion} = \frac{\sqrt{b_5^2 + b_7^2 + b_{11}^2 + b_{13}^2 + \ldots}}{b_1}$$

$$= \frac{0.2772}{0.9549} = 0.290$$

THD = 29.0 percent.

Figure A-1 depicts empirical data obtained from one wye-wound motor. These data were found to be typical for all inverter-motor combinations used in this study and agree exceptionally well with the preceding analysis.

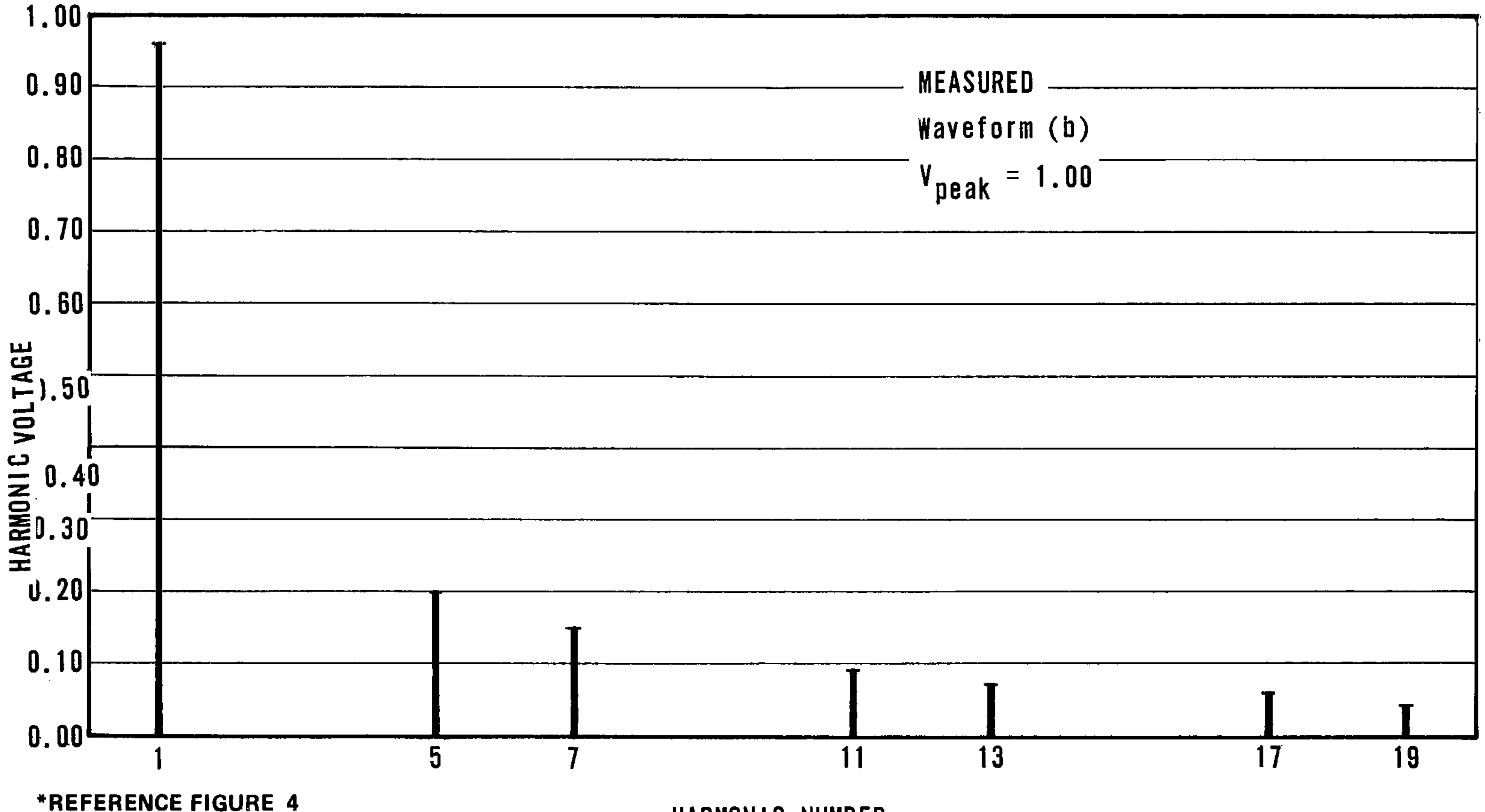


Figure A-1. Q-S voltage spectrum.

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