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ESTIMATING NORMAL MIXTURE PARAMETERS FROM THE DISTRIBUTION OF A REDUCED FEATURE VECTOR

by

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1. INTRODUCTION

Let X be a random n-vector with density function $f(x) = \sum_{i=1}^{m} \alpha_i f_i(x)$, where $\alpha_i > 0$, $\sum_{i=1}^{m} \alpha_i = 1$ and $f_i(x) = N(x_i, \mu_i, \Sigma_i)$, the n-variate normal density with mean μ_i and covariance matrix Σ_i. Let B be a real $k \times n$ matrix of rank k and let $Y = BX$. We are interested in estimating some or all of the parameters α_i, μ_i, Σ_i from information about the distribution of Y. The density for Y is $p(y) = \sum_{i=1}^{m} \alpha_i p_i(y)$ where $p_i(y) = N(y, B\mu_i, B\Sigma_i B^T)$. We will actually estimate the non-central second moments $S_i = E\{xx^T|i\}$ rather than the covariance matrices.

Let $\{x_i\}_{i=1}^{N}$ be a sample of independent observations of X and let us assume for the moment that the α_i as well as the transformed parameters $B\mu_i$ and $B\Sigma_i B^T$ are known. Define estimates $\hat{\mu}_i$, $\hat{S}_i$ of μ_i and S_i as

$$(1a) \qquad \hat{\mu}_i = \frac{1}{N} \sum_{j=1}^{N} \frac{p_i(y_j)}{p(y_j)} x_j$$

(1b) $$\hat{S}_i = \frac{1}{N}\sum_{j=1}^{N} \frac{p_i(y_j)}{p(y_j)} x_j x_j^T$$

where $y_j = Bx_j$, $j = 1,\ldots,N$.

Then,

(2a) $$\bar{\mu}_i = E\{\hat{\mu}_i\} = E\left\{\frac{p_i(Bx)}{p(Bx)}\, x\right\}$$

(2b) $$\bar{S}_i = E\{\hat{S}_i\} = E\left\{\frac{p_i(Bx)}{p(Bx)}\, xx^T\right\} \quad .$$

Note that $B\bar{\mu}_i = B\mu_i$, $B\bar{S}_i B^T = BS_i B^T$. The biases of $\hat{\mu}_i$ and $\hat{S}_i$, as measured in arbitrary norms, satisfy

(3a) $$||\mu_i - \bar{\mu}_i||^2 \leq E\left\{\left(\frac{p_i(Bx)}{p(Bx)} - \frac{f_i(x)}{f(x)}\right)^2\right\} E\left\{||x||^2\right\}$$

$$= \left[E\left\{\frac{f_i(x)^2}{f(x)^2}\right\} - E\left\{\frac{p_i(Bx)^2}{p(Bx)^2}\right\}\right] E\left\{||x||^2\right\}$$

(3b) $$||S_i - \bar{S}_i||^2 \leq \left[E\left\{\frac{f_i(x)^2}{f(x)^2}\right\} - E\left\{\frac{p_i(Bx)^2}{p(Bx)^2}\right\}\right] E\left\{||xx^T||^2\right\} \quad .$$

If we define the overall bias by

$$\text{bias}^2 = \sum_{i=1}^{m} \alpha_i^2 \left[||\mu_i - \bar{\mu}_i||^2 + ||S_i - \bar{S}_i||^2\right]$$

then

(4) $$\text{bias}^2 \leq K\,[H - H_B]$$

where

$$K = E\{||x||^2\} + E\{||xx^T||^2\} \quad ,$$

(5) $$H = \sum_{i=1}^{m} E\left\{\frac{\alpha_i^2 f_i(x)^2}{f(x)^2}\right\} \quad , \qquad \text{and}$$

(6) $$H_B = \sum_{i=1}^{m} E\left\{\frac{\alpha_i^2 p_i(B_x)^2}{p(Bx)^2}\right\} \quad .$$

The expression H (or H_B) has been discussed by Devijver [1] as a measure of the separation of the m pattern classes. H is related to the Bayes probability of correct classification, PCC, by the inequalities

(7) $$H \leq PCC \leq \frac{1}{m} + \frac{m-1}{m}\sqrt{\frac{m\,H-1}{m-1}} \leq \sqrt{H}$$

Thus the quality of the estimate is related to the discriminatory power lost in the transformation $Y = BX$, as measured by the difference $H - H_B$. The following theorem is proved in Appendix A.

<u>Theorem:</u> If $PCC = PCC_B$ then $H = H_B$ and the estimators $\hat{\mu}_i$, $\hat{S}_i$ are unbiased.

2. APPLICATIONS

Even if the estimators $\hat{\mu}_i$ $\hat{S}_i$ are biased it is possible that they may be near enough to the true parameters that the locally contractive successive substitutions procedure UHMLE described in [4] will find the maximum likelihood estimates in very few iterations. There is some evidence that most of the information required to discriminate among agricultural classes (at least between wheat and non-wheat) lies in a fixed two dimensional subspace of 4-channel LANDSAT data space, [2], [5]. Thus there may be a good a priori choice of B, namely one whose rows span this two dimensional subspace.

Once B is chosen, the transformed densities $p_i(y)$ and the α_i may be estimated by maximum likelihood. Whether this task is more or less difficult than estimating the untransformed class densities $f_i(x)$ directly depends primarily on how well B preserves the separation of the classes. Then, given the estimates of α_i and $p_i(y)$, $\hat{\mu}_i$ and $\hat{\Sigma}_i$ can be calculated and used as starting values for UHMLE in estimating α_i, μ_i, and Σ_i. Figure 1 gives a flow diagram for the procedure.

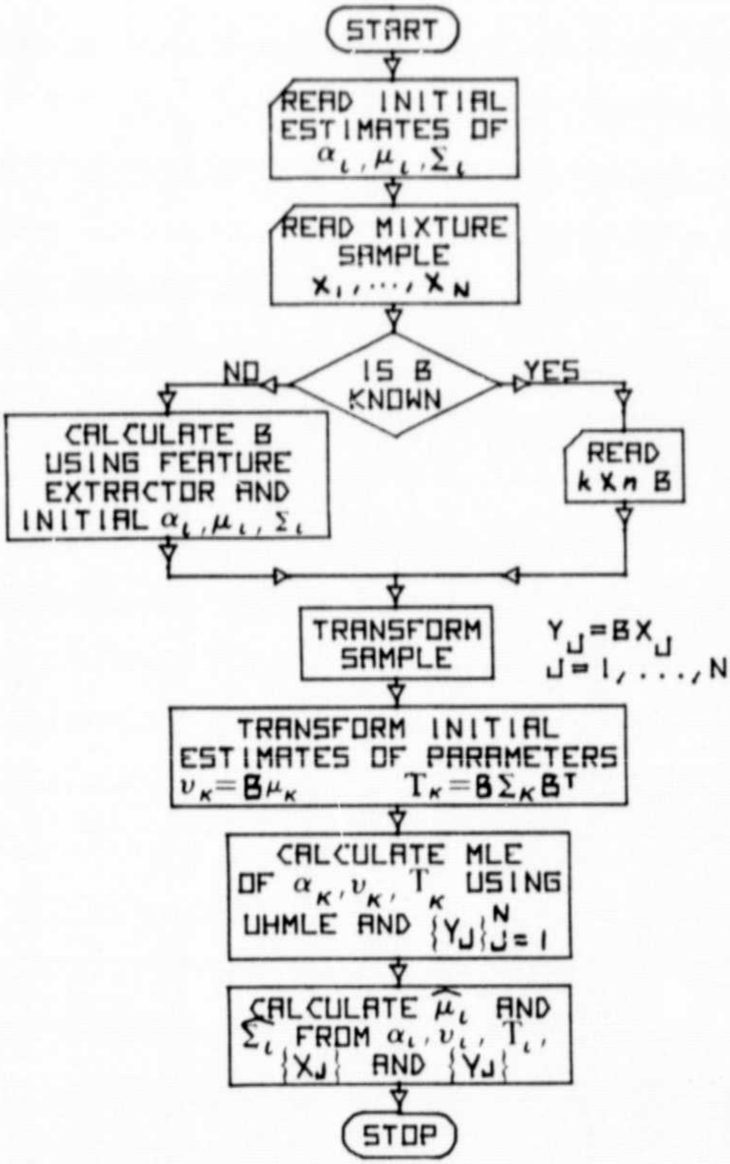

Figure 1. General Flow Diagram

3. RESULTS

A FORTRAN program (ESTMIX) for implementing the procedure outlined in Section 2 has been written and tested at Texas A&M University. Data used was obtained from the simulated LACIE segments 1851-1854 derived from Hill County statistics for ten agricultural subclasses [3], and appears in Appendix B. The measurements x_j consisted of 1000 vectors randomly chosen from segment 1851 representing subclasses 1, 3, 5, 7, and 10 (herein referred to as subclasses 1-5) in equal proportions.

In the first experiment, the vectors x_j are 4-vectors from pass 3. In each run, starting values for the means and covariances are those for the same subclasses in each of the four segments 1851-1854, pass 3. The matrix B is either a 1×4 matrix B_C computed from the input means and covariances using LFSPMC (Version 2) or the 2×4 matrix

$$B_K = \begin{pmatrix} .43258 & .63248 & .58572 & .26414 \\ -.28972 & -.56199 & .59953 & .49070 \end{pmatrix}$$

which was derived from physical considerations by R. Kauth [2,5]. The computed 1×4 B's are given in Appendix B.

In the second experiment, the vectors x_j are 16 channel measurements, the starting means are from one of the four segments, and the starting covariances are block diagonal matrices with the 4×4 covariance matrix from each pass placed in the appropriate position on the diagonal. That is, the 16 channel starting covariances were constructed as if there were no correlation between separate passes. The matrix B is either a 1×16 matrix $\hat{B}$, computed from the input parameters using LFSPMC, or

$$\hat{B}_K = \begin{pmatrix} B_K & & & \\ & B_K & & 0 \\ & & B_K & \\ & 0 & & B_K \end{pmatrix}_{8\times16}$$

where B_K is the 2×4 matrix used in the first experiment. The computed 1×16 B's are given in Appendix B. The biases obtained for each of the above runs are listed in Figure 2.

Feature Selection Used	Data Set Used	bias^2 $\hat{\Theta}$	bias^2 Θ(MLE)
LFSPMC 1×4 B's (Pass 3)	1851	1691.46	4.70
	1852	2648.44	736.78
	1853	6779.11	12422.55
	1854	6757.28	12369.87
Kauth 2×4 (Pass 3)	1851	62.19	5.73
	1852	66.62	6.64
	1853	66.20	6.69
	1854	67.10	6.88
LFSPMC 1×16 (4 Passes)	1851	4898.90	138.33
	1852	4734.30	138.33
	1853	11195.32	13491.70
	1854	4870.29	138.33
Blocked Kauth 8 16 (4 Passes)	1851	117.92	138.33
	1852	117.92	138.33
	1853	21594.70	21387.61
	1854	117.92	138.33

Figure 2. Biases For Runs

APPENDIX A

PROOF OF THEOREM

That $H = H_B$ when $PCC = PCC_B$ follows from

<u>Theorem:</u> If $PCC = PCC_B$ then $\frac{p_i(Bx)}{p(Bx)} \equiv \frac{f_i(x)}{f(x)}$ for $i = 1,\ldots,m$.

<u>Proof:</u> Let C be an $(n-k) \times n$ matrix such that $\binom{B}{C}$ is nonsingular. Let $y = Bx$, $z = Cx$ and write f_i as a joint density $f_i(y,z)$, so that $p_i(y) = \int_{R^{n-k}} f_i(y,z)dz$.

Then,

$$PCC = \sum_{i=1}^{m} \alpha_i \iint_{R_i} f_i(y,z)dydz$$

and

$$PCC_B = \sum_{i=1}^{m} \alpha_i \iint_{R_i(B)} f_i(y,z)dzdy \ .$$

where $R_i = \{(y,z)|\alpha_i f_i(y,z) > \alpha_j f_j(y,z) \text{ for each } j \neq i\}$ and

$R_i(B) = \{(y,z)|\alpha_i p_i(y) > \alpha_j p_j(y) \text{ for each } j \neq i\}$.

Since $PCC = PCC_B$,

$\alpha_i p_i(y) > \alpha_j p_j(y)$ for each $j \neq i$ iff $\alpha_i f_i(y,z) > \alpha_j f_j(y,z)$ for each $j \neq i$

for $i = 1,\ldots,m$.

Now, $f_i(y,z)$ can be written as

$$f_i(y,z) = p_i(y)q_i(z|y) \ ,$$

where the conditional density $q_i(z|y)$ is normal with mean

$C\mu_i + C\Sigma_i B^T(B\Sigma_i B^T)^{-1}(y-B\mu_i)$ and covariance $C\Sigma_i C^T - C\Sigma_i B^T(B\Sigma_i B^T)^{-1}B\Sigma_i C^T$.

Assuming first that $\alpha_i p_i(y) > \alpha_j p_j(y)$ for each $j \neq i$, we have

$$\frac{q_i(z|y)}{q_j(z|y)} > \frac{\alpha_j p_j(y)}{\alpha_i p_i(y)} \quad \text{for each } z,\ j \neq i \ .$$

This inequality implies that

$$\mathrm{cov}(z|y,i) \geq \mathrm{cov}(z|y,j) \quad \text{for each } j \ .$$

Since $\mathrm{cov}(z|y,j)$ is independent of y , all the covariances

$$C\Sigma_j C^T - C\Sigma_j B^T(B\Sigma_j B^T)^{-1}B\Sigma_j C^T$$

are equal. This in turn implies that the means

$$C\mu_j + C\Sigma_j B^T(B\Sigma_j B^T)^{-1}(y-B\mu_j)$$

are equal. Hence, $q_i(z|y) = q(z|y)$ is independent of i . It follows that

$$\frac{p_i(y)}{p(y)} = \frac{f_i(y,z)}{f(y,z)} \quad \text{for all } y,z \ . \qquad \text{QED.}$$

<u>Remark:</u> From the proof of the theorem, it follows that

$$\Sigma_i - \Sigma_i B^T(B\Sigma_i B^T)^{-1}B\Sigma_i$$

$$\mu_i - \Sigma_i B^T(B\Sigma_i B^T)^{-1}B\mu_i$$

and

$$\Sigma_i B^T (B \Sigma_i B^T)^{-1}$$

are all independent of i. It can be shown that $PCC = PCC_B$ if and only if this condition holds.

APPENDIX B

1851

-0.095043	0.354845	0.169794	-0.092728
-0.130729	0.349954	0.191296	0.007083
-0.206965	0.219549	0.311299	0.382855
-0.399715	0.288661	0.178790	0.204539

1852

-0.085406	0.293773	0.142224	-0.074233
-0.108779	0.347254	0.191208	0.008994
-0.215696	0.226417	0.319528	0.394695
-0.425490	0.305334	0.187362	0.215776

1853

-0.075274	0.282277	0.135977	-0.074587
-0.138605	0.371507	0.203038	0.007567
-0.214246	0.227860	0.323095	0.397893
-0.407879	0.295570	0.183850	0.209836

1854

-0.073043	0.275001	0.132555	-0.072766
-0.125104	0.356312	0.203318	0.004111
-0.201572	0.221260	0.311475	0.394503
-0.431287	0.312728	0.192507	0.221597

B-Vectors (1×16) Obtained From Feature Selection

1851, PASS 3

0.425418	-0.177865	0.710975	0.530941

1852, PASS 3

0.460703	-0.185813	0.696812	0.517378

1853, PASS 3

0.385670	-0.161554	0.734818	0.534042

1854, PASS 3

0.423826	-0.177800	0.711105	0.532061

B-Vectors (1×4) Obtained From Feature Selection

COVARIANCE MATRIX FOR CLASS 1

PASS 1

0.600800	0.647300	0.441000	0.511400
0.647300	1.426999	0.948200	0.963600
0.441000	0.948200	1.345300	1.202100
0.511400	0.963600	1.202100	1.810900

PASS 2

0.833100	1.110100	0.311000	-0.005100
1.110100	2.978299	0.218700	-0.602200
0.311000	0.218700	1.622299	1.610399
-0.005100	-0.602200	1.610399	2.896099

PASS 3

1.996699	2.644300	0.548700	-0.895800
2.644300	4.385799	0.828300	-1.501300
0.548700	0.828300	1.248699	0.826700
-0.895800	-1.501300	0.826700	2.694300

PASS 4

3.122600	4.397499	0.967400	-1.430699
4.397499	6.988999	1.521399	-2.182500
0.967400	1.521399	1.320000	0.464100
-1.430699	-2.182500	0.464100	2.491199

MEAN VECTOR FOR CLASS 1

20.810287	22.484497	23.706894	22.901794
19.403992	19.288788	26.122986	26.973190
16.688995	15.008100	26.094193	30.928589
16.750992	15.236199	25.591492	29.169586

Segment 1851--Class Statistics

COVARIANCE MATRIX FOR CLASS 2

PASS 1

0.383500	0.176800	0.151400	0.152900
0.176800	0.838600	0.185000	0.260100
0.151400	0.185000	0.642100	0.481700
0.152900	0.260100	0.481700	0.724500

PASS 2

0.475900	0.198900	0.109200	0.053700
0.198900	0.660000	0.004600	-0.141400
0.109200	0.004600	1.639700	2.162700
0.053700	-0.141400	2.162700	3.690800

PASS 3

0.834600	0.756300	0.103000	0.139500
0.756300	1.639199	0.029400	0.224700
0.103000	0.029400	0.865300	0.518100
0.139500	0.224700	0.518100	0.774900

PASS 4

0.624200	-1.132600	-0.169900	2.476700
-1.132600	12.094800	1.815499	-21.455994
-0.169900	1.815499	1.367599	-2.237700
2.476700	-21.455994	-2.237700	41.762894

MEAN VECTOR FOR CLASS 2

19.121399	20.448792	19.709290	18.203690
21.427094	23.562988	23.272300	21.638885
22.308990	25.276199	25.340500	24.694397
21.022888	22.207993	22.212585	22.555496

Segment 1851--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 3

PASS 1

1.229699	1.424100	1.188000	1.159100
1.424100	2.821500	2.070200	2.068399
1.188000	2.070200	2.433399	2.158799
1.159100	2.068399	2.158799	2.648100

PASS 2

1.418500	1.598300	1.124200	0.0
1.598300	2.904300	1.397300	0.0
1.124200	1.397300	1.520000	0.0
0.0	0.0	0.0	0.0

PASS 3

4.774400	8.580299	0.799000	-2.181199
8.580299	16.995987	1.362599	-4.740299
0.799000	1.362599	1.202999	0.594800
-2.181199	-4.740299	0.594800	3.759299

PASS 4

3.268999	4.210899	1.372000	0.346100
4.210899	6.220400	1.674500	0.136500
1.372000	1.674500	2.851399	2.890599
0.346100	0.136500	2.890599	4.449200

MEAN VECTOR FOR CLASS 3

21.925995	24.579895	23.612595	21.899994
22.217392	24.660492	24.858887	23.187500
21.144699	21.870193	29.094086	30.987488
16.500290	13.585899	30.154388	35.137497

Segment 1851--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 4

PASS 1

0.613000	0.542600	0.421700	0.378400
0.542600	1.180400	0.565600	0.435100
0.421700	0.565600	0.765400	0.472800
0.378400	0.435100	0.472800	0.761400

PASS 2

0.892600	0.566400	0.710300	0.629600
0.566400	1.025900	0.640400	0.608900
0.710300	0.640400	1.391399	1.288600
0.629600	0.608900	1.288600	2.121599

PASS 3

0.616100	0.464300	0.375300	0.402200
0.464300	1.020700	0.350000	0.438200
0.375300	0.350000	0.957800	0.826000
0.402200	0.438200	0.826000	1.292100

PASS 4

0.949300	1.243299	-0.082000	-0.812100
1.243299	2.726500	-0.407800	-1.901500
-0.082000	-0.407800	0.871100	1.043699
-0.812100	-1.901500	1.043699	2.817599

MEAN VECTOR FOR CLASS 4

20.258856	22.083496	21.609192	20.473694
20.875992	22.596893	23.177887	22.153488
19.757599	21.715790	21.689697	21.206085
20.797592	22.788391	22.246689	20.846497

Segment 1851--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 5

PASS 1

0.745800	0.603100	0.414300	0.524400
0.603100	1.247600	0.414300	0.570600
0.414300	0.414300	0.816100	0.728900
0.524400	0.570600	0.728900	1.324100

PASS 2

0.446000	0.292400	0.393200	0.218300
0.292400	0.771500	0.398300	0.351900
0.393200	0.398300	1.057099	0.510200
0.218300	0.351900	0.510200	0.907800

PASS 3

0.514900	0.294500	0.034700	0.005300
0.294500	0.873900	0.176100	0.206100
0.034700	0.176100	0.513100	0.395600
0.005300	0.206100	0.395600	0.907800

PASS 4

1.388200	1.515800	0.162700	-0.394800
1.515800	2.308499	0.024100	-0.627700
0.162700	0.024100	0.934100	0.850200
-0.394800	-0.627700	0.850200	1.918400

MEAN VECTOR FOR CLASS 5

19.197566	18.622192	20.090500	19.319992
19.376190	18.483292	21.777100	21.479996
19.088486	17.927689	22.531189	23.519989
19.118195	17.401886	22.719695	23.000000

Segment 1851--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 1

PASS 1

1.056800	1.126800	0.763800	0.881600
1.126800	2.458699	1.625299	1.643800
0.763800	1.625299	2.294000	2.040199
0.881600	1.643800	2.040199	3.059099

PASS 2

0.908700	1.209499	0.338800	-0.005600
1.209499	3.241199	0.238000	-0.654900
0.338800	0.238000	1.765499	1.751599
-0.005600	-0.654900	1.751599	3.148100

PASS 3

2.148000	2.841399	0.589200	-0.961400
2.841399	4.707199	0.888500	-1.609500
0.589200	0.888500	1.338699	0.885700
-0.961400	-1.609500	0.885700	2.885099

PASS 4

3.296000	4.637799	1.020000	-1.507999
4.637799	7.365100	1.602900	-2.298699
1.020000	1.602900	1.390499	0.488700
-1.507999	-2.298699	0.488700	2.622199

MEAN VECTOR FOR CLASS 1

26.926697	29.083693	30.676392	29.618088
20.363892	20.191696	27.294693	28.141388
17.388397	15.605399	27.053299	32.021698
17.268585	15.683399	26.292389	29.938797

Segment 1852--Class Statistics

COVARIANCE MATRIX FOR CLASS 2

PASS 1

0.674600	0.307800	0.262200	0.263500
0.307800	1.444000	0.317100	0.443800
0.262200	0.317100	1.094899	0.817600
0.263500	0.443800	0.817600	1.223900

PASS 2

0.519100	0.216700	0.119000	0.058500
0.216700	0.718300	0.005000	-0.153800
0.119000	0.005000	1.784499	2.352300
0.058500	-0.153800	2.352300	4.011900

PASS 3

0.897900	0.812700	0.110600	0.149700
0.812700	1.759399	0.031500	0.240900
0.110600	0.031500	0.927600	0.555100
0.149700	0.240900	0.555100	0.829800

PASS 4

0.658800	-1.194500	-0.179200	2.610499
-1.194500	12.745799	1.912900	-22.597595
-0.179200	1.912900	1.440599	-2.356299
2.610499	-22.597595	-2.356299	43.959198

MEAN VECTOR FOR CLASS 2

24.586685	26.413696	25.456085	23.511887
22.476898	24.650497	24.279099	22.579788
23.217499	26.243088	26.272888	25.570496
21.557495	22.840286	22.824387	23.153091

Segment 1852--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 3

PASS 1

2.163099	2.479199	2.057500	1.998099
2.479199	4.861199	3.548300	3.528700
2.057500	3.548300	4.149400	3.664000
1.998099	3.528700	3.664000	4.473300

PASS 2

1.547199	1.741400	1.224799	0.958500
1.741400	3.160600	1.520599	1.115299
1.224799	1.520599	1.654200	0.910500
0.958500	1.115299	0.910500	1.185900

PASS 3

5.136200	9.219899	0.858000	-2.341100
9.219899	18.241699	1.461599	-5.081800
0.858000	1.461599	1.289700	0.744400
-2.341100	-5.081800	0.744400	4.025499

PASS 4

3.450399	4.441000	1.446699	0.364700
4.441000	6.555200	1.764299	0.143700
1.446699	1.764299	3.003699	3.043799
0.364700	0.143700	3.043799	4.683200

MEAN VECTOR FOR CLASS 3

28.406387	31.836197	30.553192	28.316086
23.302185	25.795395	25.975998	24.194397
22.009796	22.714493	30.159286	32.082596
17.011093	13.989200	30.975586	36.061691

Segment 1852--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 4

PASS 1

1.078300	0.944600	0.730300	0.652300
0.944600	2.033799	0.969400	0.742300
0.730300	0.969400	1.305300	0.802500
0.652300	0.742300	0.802500	1.286200

PASS 2

0.973600	0.517100	0.773900	0.685500
0.517100	1.116400	0.697000	0.662300
0.773900	0.697000	1.514299	1.401500
0.685500	0.662300	1.401500	2.306199

PASS 3

0.662800	0.498900	0.403100	0.431600
0.498900	1.095500	0.375500	0.469700
0.403100	0.375500	1.026799	0.885000
0.431600	0.469700	0.885000	1.333599

PASS 4

1.002000	1.311299	-0.086400	-0.855900
1.311299	2.873300	-0.429600	-2.002700
-0.086400	-0.429600	0.917700	1.099000
-0.855900	-2.002700	1.099000	2.965799

MEAN VECTOR FOR CLASS 4

26.208694	28.559296	27.936996	26.462296
21.901199	23.642685	24.222397	23.116394
20.571198	22.554489	22.492889	21.960785
21.425995	23.436096	22.859390	21.399689

Segment 1852--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 5

PASS 1

1.311799	1.049999	0.717600	0.903900
1.049999	2.145400	0.710200	0.973400
0.717600	0.710200	1.391600	1.237100
0.903900	0.973400	1.237100	2.236799

PASS 2

0.486500	0.318600	0.428400	0.237700
0.318600	0.839600	0.433400	0.382700
0.428400	0.433400	1.150399	0.555000
0.237700	0.382700	0.555000	0.986800

PASS 3

0.553600	0.316400	0.037300	0.005600
0.316400	0.938000	0.188900	0.221000
0.037300	0.186900	0.550100	0.423900
0.005600	0.221000	0.423900	0.972100

PASS 4

1.465199	1.598700	0.171600	-0.416100
1.598700	2.432799	0.025400	-0.661100
0.171600	0.025400	0.984000	0.874200
-0.416100	-0.661100	0.874200	2.091299

MEAN VECTOR FOR CLASS 5

24.787796	24.016098	25.953888	24.962799
20.334900	19.351288	22.761093	22.414200
19.877197	18.630096	23.364090	24.355286
19.700699	17.906494	23.344894	23.609085

Segment 1852--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 1

PASS 1

1.220699	1.300200	880000	1.014799
1.300200	2.833899	.870500	1.890100
0.880000	1.870500	2.636000	2.342199
1.014799	1.890100	2.342199	3.508900

PASS 2

0.931300	1.238999	0.347000	-0.005700
1.238999	3.319200	0.243700	-0.670500
0.347000	0.243700	1.807300	1.792700
-0.005700	-0.670500	1.792700	3.221499

PASS 3

2.341000	3.092299	0.641300	-1.045600
3.092299	5.115499	0.965600	-1.747899
0.641300	0.965600	1.454900	0.961900
-1.045600	-1.747899	0.961900	3.131000

PASS 4

3.770300	5.293900	1.163699	-1.718800
5.293900	8.389099	1.824900	-2.614799
1.163699	1.824900	1.582199	0.555500
-1.718800	-2.614799	0.555500	2.977799

MEAN VECTOR FOR CLASS 1

31.791595	33.245285	34.223495	32.107791
24.447388	21.613495	28.461594	29.316788
19.501785	17.256485	28.852386	33.674500
18.789597	16.955688	28.130588	31.962585

Segment 1853--Class Statistics

COVARIANCE MATRIX FOR CLASS 2

PASS 1

0.779300	0.355200	0.302100	0.303400
0.355200	1.665400	0.364900	0.510300
0.302100	0.364900	1.258200	0.938700
0.303400	0.510300	0.938700	1.403799

PASS 2

0.531900	0.222000	0.121900	0.059900
0.222000	0.735600	0.005100	-0.157400
0.121900	0.005100	1.826699	2.407499
0.059900	-0.157400	2.407499	4.105399

PASS 3

0.978600	0.884500	0.120400	0.162800
0.884500	1.912000	0.034300	0.261600
0.120400	0.034300	1.008100	0.602800
0.162800	0.261600	0.602800	0.900500

PASS 4

0.753700	-1.363500	-0.204400	2.975300
-1.363500	14.517799	2.177699	-25.700790
-0.204400	2.177699	1.639299	-2.678499
2.975300	-25.700790	-2.678499	49.921097

MEAN VECTOR FOR CLASS 2

29.384293	30.376587	28.627686	25.567688
26.566395	26.125595	25.410385	23.690796
25.587189	28.346100	28.038788	26.953995
23.483597	24.593887	24.481186	24.731293

Segment 1853--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 3

PASS 1

2.498599	2.860600	2.370399	2.299899
2.860600	5.603000	4.083599	4.057300
2.370399	4.083599	4.768000	4.206499
2.299899	4.057300	4.206499	5.131100

PASS 2

1.585600	1.783899	1.254499	0.981600
1.783899	3.236699	1.556899	1.141700
1.254499	1.556899	1.693399	0.931900
0.981600	1.141700	0.931900	1.213599

PASS 3

5.597699	10.033999	0.933800	-2.546100
10.033999	19.824097	1.588499	-5.518800
0.933800	1.588499	1.401600	0.808400
-2.546100	-5.518800	0.808400	4.368600

PASS 4

3.946899	5.069300	1.650499	0.415700
5.069300	7.466599	2.008599	0.163500
1.650499	2.008599	3.417800	3.459999
0.415700	0.163500	3.459999	5.318299

MEAN VECTOR FOR CLASS 3

33.331989	36.198196	34.091492	30.712891
27.421997	27.284286	27.127396	25.324097
24.326385	24.667496	32.090393	33.737991
18.514099	15.147599	33.176193	38.487396

Segment 1853--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 4

PASS 1

1.245500	1.089999	0.841300	0.750800
1.089999	2.344199	1.115700	0.853500
0.841300	1.115700	1.499900	0.921400
0.750800	0.853500	0.921400	1.475300

PASS 2

0.997800	0.632200	0.792700	0.702000
0.632200	1.143299	0.713600	0.678000
0.792700	0.713600	1.550099	1.434400
0.702000	0.678000	1.434400	2.359900

PASS 3

0.722300	0.542900	0.438700	0.469400
0.542900	1.190499	0.408100	0.510100
0.438700	0.408100	1.115999	0.961100
0.469400	0.510100	0.961100	1.501499

PASS 4

1.146199	1.496799	-0.098600	-0.975500
1.496799	3.272699	-0.499100	-2.277699
-0.098600	-0.499100	1.044200	1.249299
-0.975500	-2.277699	1.249299	3.367999

MEAN VECTOR FOR CLASS 4

31.019989	32.680191	31.287094	28.727493
26.003693	25.105789	25.353088	24.237490
22.824493	24.500793	24.098099	23.193588
23.235992	25.229797	24.518494	22.862793

Segment 1853--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 5

PASS 1

1.515200	1.211499	0.826700	1.040500
1.211499	2.477500	0.817300	1.119200
0.826700	0.817300	1.599000	1.420300
1.040500	1.119200	1.420300	2.565700

PASS 2

0.498600	0.326400	0.438800	0.243400
0.326400	0.859800	0.443800	0.391800
0.438800	0.443800	1.177699	0.568000
0.243400	0.391800	0.568000	1.009800

PASS 3

0.603400	0.344400	0.040600	0.006100
0.344400	1.019300	0.205300	0.240000
0.040600	0.205300	0.597900	0.460300
0.006100	0.240000	0.460300	1.054899

PASS 4

1.676000	1.824800	0.195700	-0.474300
1.824800	2.771000	0.028900	-0.751800
0.195700	0.028900	1.119699	0.993700
-0.474300	-0.751800	0.993700	2.293099

MEAN VECTOR FOR CLASS 5

29.492996	27.802490	29.161285	27.121597
24.417999	20.763092	23.874588	23.523193
22.399991	20.409698	25.006393	25.687988
21.390686	19.328293	25.036392	25.217285

Segment 1853--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 1

PASS 1

1.330299	1.414300	0.956800	1.102699
1.414300	3.076599	2.029900	2.049999
0.956800	2.029900	2.859599	2.539399
1.102699	2.049999	2.539399	3.801900

PASS 2

1.000600	1.329900	0.572300	-0.006100
1.329900	3.558800	0.261100	-0.718100
0.572300	0.261100	1.935699	1.919000
-0.006100	-0.718100	1.919000	3.446699

PASS 3

2.477799	3.269799	0.578400	-1.105700
3.269799	5.403700	1.020499	-1.846499
0.578400	1.020499	1.538500	1.016700
-1.105700	-1.846499	1.016700	3.307699

PASS 4

3.459100	4.863799	1.069300	-1.580299
4.863799	7.718200	1.679199	-2.407100
1.069300	1.679199	1.456100	0.511500
-1.580299	-2.407100	0.511500	2.743799

MEAN VECTOR FOR CLASS 1

30.033096	32.422195	34.177292	32.978485
21.473190	21.233887	28.626999	29.468094
19.265488	17.110992	29.242599	34.386897
17.742599	16.092392	26.928497	30.636292

Segment 1854--Class Statistics

COVARIANCE MATRIX FOR CLASS 2

PASS 1

0.849300	0.386400	0.328500	0.329600
0.386400	1.808100	0.396000	0.553400
0.328500	0.396000	1.364900	1.017699
0.329600	0.553400	1.017699	1.521099

PASS 2

0.571500	0.238300	0.130800	0.064200
0.238300	0.788700	0.005500	-0.168600
0.130800	0.005500	1.956499	2.577200
0.064200	-0.168600	2.577200	4.392400

PASS 3

1.035800	0.935200	0.127400	0.172200
0.935200	2.019699	0.036200	0.276400
0.127400	0.036200	1.066099	0.637200
0.172200	0.276400	0.637200	0.951300

PASS 4

0.691500	-1.252700	-0.187800	2.735600
-1.252700	13.356799	2.003799	-23.663193
-0.187800	2.003799	1.508599	-2.466499
2.735600	-23.663193	-2.466499	45.997787

MEAN VECTOR FOR CLASS 2

27.519897	29.433197	28.348892	26.171188
23.690399	25.906197	25.469299	23.648788
25.526199	28.508591	28.405991	27.479385
22.238785	23.418900	23.379700	23.694885

Segment 1854--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 3

PASS 1

2.723000	3.111600	2.577399	2.499200
3.111600	6.082999	4.431700	4.400499
2.577399	4.431700	5.172500	4.560499
2.499200	4.400499	4.560499	5.559600

PASS 2

1.703699	1.914700	1.345799	1.052400
1.914700	3.470400	1.668400	1.222899
1.345799	1.668400	1.813600	0.997500
1.052400	1.222899	0.997500	1.298400

PASS 3

5.924999	10.609900	0.988000	-2.692300
10.609900	20.940796	1.678900	-5.830000
0.988000	1.678900	1.482200	0.854500
-2.692300	-5.830000	0.854500	4.615100

PASS 4

3.621200	4.657399	1.516600	0.382200
4.657399	6.869499	1.848200	0.150500
1.516600	1.848200	3.145300	3.186099
0.382200	0.150500	3.186099	4.900399

MEAN VECTOR FOR CLASS 3

31.693298	35.498886	34.079688	31.526993
24.555488	27.105789	27.246185	25.338196
24.229095	24.727890	32.572388	34.452087
17.478790	14.358199	31.720795	36.899490

Segment 1854--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 4

PASS 1

1.357400	1.185599	0.914800	0.815900
1.185599	2.544999	1.210799	0.925700
0.914800	1.210799	1.627100	0.998900
0.815900	0.925700	0.998900	1.598499

PASS 2

1.072000	0.578500	0.850300	0.752700
0.578500	1.225800	0.764700	0.726100
0.850300	0.764700	1.660199	1.535500
0.752700	0.726100	1.535500	2.524899

PASS 3

0.764600	0.574100	0.464100	0.496400
0.574100	1.257600	0.431300	0.538900
0.464100	0.431300	1.180099	1.015900
0.496400	0.538900	1.015900	1.586300

PASS 4

1.051600	1.375199	-0.090600	-0.897000
1.375199	3.011000	-0.450000	-2.097099
-0.090600	-0.450000	0.960900	1.150399
-0.897000	-2.097099	1.150399	3.103299

MEAN VECTOR FOR CLASS 4

29.227493	31.833389	31.118896	29.460388
23.086288	24.850098	25.409897	24.210190
22.683990	24.556488	24.353485	23.614288
22.001587	24.026885	23.415497	21.901398

Segment 1854--Class Statistics (Cont.)

COVARIANCE MATRIX FOR CLASS 5

PASS 1

1.651299	1.317800	0.898900	1.130600
1.317800	2.689699	0.887000	1.213900
0.898900	0.887000	1.734699	1.539800
1.130600	1.213900	1.539800	2.779900

PASS 2

0.535700	0.350300	0.470700	0.261000
0.350300	0.921800	0.475500	0.419600
0.470700	0.475500	1.261299	0.608000
0.261000	0.419600	0.608000	1.080400

PASS 3

0.638700	0.364100	0.043000	0.006500
0.364100	1.076799	0.216900	0.253500
0.043000	0.216900	0.632200	0.486600
0.006500	0.253500	0.486600	1.114499

PASS 4

1.537700	1.676600	0.179900	-0.436100
1.676600	2.549399	0.026600	-0.692200
0.179900	0.026600	1.030399	0.915000
-0.436100	-0.692200	0.915000	2.112900

MEAN VECTOR FOR CLASS 5

27.633392	26.751099	28.904785	27.788696
21.442688	20.353394	23.879898	23.475494
21.938599	20.351700	25.287598	26.178085
20.234085	18.368286	23.912186	24.161392

Segment 1854--Class Statistics (Cont.)

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