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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1977

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1. Report No. NASA TN D-8412	2. Government Accession No.
4. Title and Subtitle EFFECT OF COEFFICIENT CHANGES ON STABILITY OF LINEAR RETARDED SYSTEMS WITH CONSTANT TIME DELAYS	5. Report Date March 1977
	6. Performing Organization Code
7. Author(s) L. Keith Barker	8. Performing Organization Report No. L-11298
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665	10. Work Unit No. 505-06-93-07
	11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546	13. Type of Report and Period Covered Technical Note
	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A method is developed to determine the effect of coefficient changes on the stability of a retarded system with constant time delays. The method, which uses the τ-decomposition method of stability analysis, is demonstrated by an example.

17. Key Words (Suggested by Author(s)) Differential-difference equations; Time delays; Stability; τ-decomposition method	18. Distribution Statement Unclassified - Unlimited. Subject Category 67

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	12	$3.25

* For sale by the National Technical Information Service, Springfield, Virginia 22161

EFFECT OF COEFFICIENT CHANGES ON STABILITY OF LINEAR RETARDED SYSTEMS WITH CONSTANT TIME DELAYS

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SUMMARY

In previous studies, the effects of time delays on the stability of differential-difference equations of the retarded type (retarded systems) have been examined. After the stability condition of the retarded system is determined for a fixed set of time delays, it may be of interest to determine the further change in the stability condition as a system coefficient is varied, with the time delays held fixed. This latter problem is examined in this paper by casting it in a form for which the τ-decomposition method is applicable. The approach involves a rearrangement of the characteristic equation so that the coefficient plays a role similar to a time delay in the τ-decomposition method. Then, the τ-decomposition method is applied to obtain the changes in the stability condition as the coefficient is varied.

The method is applied to a second-order differential equation with constant time delays in the velocity and displacement terms. Intervals of the coefficient over which the system is stable and unstable are computed.

INTRODUCTION

A differential-difference equation of the retarded type (retarded system) is a differential equation in which the dependent variable and all but the highest order derivative may contain time delays. (There is at least one delay.) Mathematical treatments of retarded systems can be found in references 1, 2, and 3, for example.

Some physical problems are modeled as a homogeneous retarded system, and the stability of the system is desired. Also, the stability of the homogeneous retarded system is related to the asymptotic stability of some nonlinear control problems involving time delays (ref. 4) and to the valid application of generalized harmonic analyses to retarded systems (ref. 5).

A homogeneous retarded system, which is linear and time-invariant, is asymptotically stable if and only if all the roots of the characteristic equation have negative real parts (ref. 1). The characteristic equation is transcendental and has an infinite number of roots. Therefore, it is not possible to compute all these roots to determine whether they have negative real parts. However, it is possible to compute the number of roots with zero or positive real parts.

With zero delays, the characteristic equation reduces to a polynomial, which has only a finite number of roots. With sufficiently small positive time delays, the characteristic equation has essentially these same finite roots, plus an infinite number of roots with arbitrarily large negative real parts (refs. 6 and 7). Hence, with sufficiently small positive time delays, the stability condition (stable or unstable) of the retarded system is the same as the stability condition of the system with zero delays, and the roots with nonnegative real parts are known. As the delays are increased from zero, the roots of the characteristic equation begin to move, generating root-locus curves. In order for the stability condition to change, a root-locus curve must intersect the imaginary axis. The τ-decomposition method of reference 8 has been shown to be a convenient means for determining the directions in which the root-locus curves cross the imaginary axis. Hence, the change in the stability condition can be computed as the delays are varied, one at a time, from zero to their final desired values (ref. 7).

Another stability problem occurs whenever a coefficient in the characteristic equation of a retarded system must be varied. The purpose of this study is to show how the τ-decomposition method may be used to solve this stability problem. The proposed technique involves a rearrangement of the characteristic equation so that the coefficient to be varied appears as a time delay. Then, the τ-decomposition method is applied to obtain changes in the stability condition as the coefficient is varied over a range of values.

SYMBOLS

A, A_ℓ	$N \times N$ matrices of real constants
$f(s;\tau_\ell)$	function of s in equation (3)
I	$N \times N$ identity matrix
Im( )	imaginary part of ( )
i	imaginary unit, $\sqrt{-1}$
ℓ	integer index (subscript)
K	system coefficient
K_m	final desired value of K
K^*	value of K at a point of intersection of a root-locus curve with the imaginary axis
$L(s)$	characteristic quasi-polynomial
N	dimension of $\vec{x}(t)$ in equation (1)
$N(K)$	number of roots of $W(s)$ with positive real parts

$N(\tau)$	number of roots of $W_2(s)$ with positive real parts
Re( )	real part of ( )
s	complex variable, $\sigma + i\omega$
T	number of time delays in system
t	time
$W(s)$	function of s in equation (7)
$W_2(s)$	function of s in equation (6)
x	scalar function of time
$\vec{x}$	$N \times 1$ state vector
α_1, α_2	real numbers
ε	small positive number
σ	real part of complex number s
τ, τ_ℓ	constant time delays
τ^*	value of τ at a point of intersection of a root-locus curve with the imaginary axis
ω	imaginary part of complex number s
ω_m	an upper bound on ω in $L(s) = 0$, where $s = i\omega$
ω^*	value of ω at a point of intersection of a root-locus curve with the imaginary axis

ANALYSIS

Retarded System

A class of homogeneous dynamical systems, called retarded systems, can be described by the following equation:

$$\dot{\vec{x}}(t) = A\,\vec{x}(t) + \sum_{\ell=1}^{T} A_\ell\,\vec{x}(t-\tau_\ell) \tag{1}$$

where $\vec{x}(t)$ is an $N \times 1$ vector, A and A_ℓ are $N \times N$ constant matrices, and $\tau_\ell \geq 0$ are constant time delays.

The characteristic equation associated with equation (1) is

$$L(s) = \det\left(sI - A - \sum_{\ell=1}^{T} A_\ell e^{-\tau_\ell s}\right) = 0 \tag{2}$$

where $s = \sigma + i\omega$ is a complex variable. It is known (ref. 1) that $\vec{x}(t) \to 0$ as $t \to \infty$ (asymptotically stable) if and only if all the roots of equation (2) have negative real parts ($\sigma < 0$).

The roots of equation (2) occur in complex-conjugate pairs, so that only roots in the upper half of the complex s-plane are considered herein; that is, $\omega \geq 0$.

Change in Stability With Coefficient Changes

It is assumed that the retarded system is known to be either stable or unstable for a fixed set of time delays $\tau_\ell (\ell = 1, 2, \ldots, T)$, and that the number of roots with positive or zero real parts has been determined. This information can be obtained by using the τ-decomposition method of stability analysis (ref. 7). Now, suppose it is desired to know how the stability further changes as a system coefficient K is changed. Toward this end, the characteristic equation (2) is written as

$$f(s;\tau_\ell) = Ks \tag{3}$$

where K is the system coefficient of interest, and $f(s;\tau_\ell)$ contains the remaining part of the characteristic equation. The characteristic equation can be solved for K and the resulting equation multiplied by s to obtain equation (3).

In order for the number of roots with positive real parts of equation (3) to change as K is continuously changed, a root-locus curve must intersect the imaginary axis. At an intersection point ($s = i\omega$), the following equations must be satisfied:

$$\mathrm{Re}[f(i\omega;\tau_\ell)] = 0 \tag{4}$$

and

$$K = \frac{1}{\omega}\,\mathrm{Im}[f(i\omega;\tau_\ell)] \tag{5}$$

Equations (4) and (5) are the real and imaginary parts of equation (3) with $s = i\omega$. Thus, the points where the root-locus curves of equation (3) cross the imaginary axis (eq. (4)) and the values of K (eq. (5)) for which these intersection points occur can be calculated. The behavior of the root-locus curves at the intersection points determines the change in the number of roots with positive real parts. Specifically, it is desired to know in which directions the root-locus curves cross the imaginary axis or if a root-locus curve is tangent to the imaginary axis. These are determined later by using the τ-decomposition method.

The essence of the τ-decomposition method of stability analysis is now presented. Consider the function

$$W_2(s) = e^{\tau s} \tag{6}$$

where $W_2(s)$ is a ratio of two polynomial equations in s with constant coefficients, and τ is a time delay. The following theorem is attributed to Lee and Hsu (ref. 8), where $N(\tau)$ denotes the number of roots of equation (6) with positive real parts. The theorem has been reworded for the present application.

Theorem 1. Let $s = i\omega^*$, where $\omega^* > 0$, be a purely imaginary root of equation (6) with corresponding time delay $\tau^* \geqq 0$. Let ϕ be the corresponding point of intersection of the testing function $W_2(i\omega)$ with the unit circle in equation (6). Then,

(1) $N(\tau)$ increases by 1 if the testing function enters the unit circle at ϕ (which occurs when $\tau = \tau^*$ and $\omega = \omega^*$);

(2) $N(\tau)$ decreases by 1 as τ increases across τ^* if the testing function leaves the unit circle as ω increases across ω^*; and

(3) $N(\tau)$ remains the same as τ increases across τ^* if the testing function remains on the same side of the unit circle as ω increases across ω^*.

Lee and Hsu used the results of theorem 1 to examine the stability of a retarded dynamical system with one constant time delay τ. The development of the theorem is of such generality, however, that the only restriction required on the testing function $W_2(s)$ in equation (6) is that it be analytic (or regular) at the intersection points being considered. A significant feature of the theorem is that it is not necessary to be concerned with finding the first nonzero derivative of the real part of the root-locus curves with respect to K, or whether the intersection points are simple.

Now, equation (3) is expressed in a form for which the τ-decomposition method is applicable. Taking the exponential of both sides of equation (3) results in

$$W(s) = e^{Ks} \tag{7}$$

where

$$W(s) = e^{f(s;\tau_\ell)} \tag{8}$$

Equation (7) is in the standard form for application of the τ-decomposition method, where K is playing the role of a time delay; and a modified testing function has been introduced in equation (8).

The following theorem results from the general proof of theorem 1 with equation (6) replaced by equation (7).

Theorem 2. Let $s = i\omega^*$ be a purely imaginary root of equation (7) when $K = K^*$. Moreover, let $\alpha_1 < \omega^*$ and $\alpha_2 > \omega^*$ be real numbers for which $W(i\alpha_1)$ and $W(i\alpha_2)$ are defined and let ω^* be the only intersection point in the interval $[\alpha_1,\alpha_2]$. Then for ε an arbitrarily small positive number,

(1) $N(K^*+\varepsilon) = N(K^*) + 1$ if $|W(i\alpha_1)| > 1$ and $|W(i\alpha_2)| < 1$

(2) $N(K^*+\varepsilon) = N(K^*) - 1$ if $|W(i\alpha_1)| < 1$ and $|W(i\alpha_2)| > 1$

(3) $N(K^*+\varepsilon) = N(K^*)$ if both $|W(i\alpha_1)|$ and $|W(i\alpha_2)|$ are greater than 1, or if both are less than 1.

The values of K^* at the intersection points are ordered by increasing magnitudes to obtain the change in the stability as K increases to its final desired value, say K_m.

Geometrically, items (1), (2), and (3) of theorem 2 correspond, respectively, to the testing function $W(i\omega)$ entering, leaving, and being tangent to the unit circle. The testing function is evaluated along the imaginary axis to infer the directions in which the root-locus curves cross the imaginary axis.

Theorem 2 is applied as follows: Let $s = i\omega^*$ be a point of intersection of a root-locus curve of equation (3) with the imaginary axis when $K = K^*$. Thus, $\omega = \omega^*$ and $K = K^*$ satisfy equations (4) and (5). This root-locus curve is common to both equations (3) and (7). In addition, there are no extra root-locus curves of equation (7), which are not common to equation (3), that have this intersection point when $K = K^*$. If $\omega = \omega^*$ is the only value of ω in the interval $[\alpha_1,\alpha_2]$ for which the testing function $W(i\omega)$ intersects the imaginary axis, then the change in the stability at this intersection point is determined by computing $|W(i\alpha_1)|$ and $|W(i\alpha_2)|$. For example, from item (1) of theorem 2, if $|W(i\alpha_1)| > 1$ and $|W(i\alpha_2)| < 1$, as indicated in figure 1, then the system gains exactly one root with positive real part as K increases across K^*; that is, $N(K^*+\varepsilon) = N(K^*) + 1$.

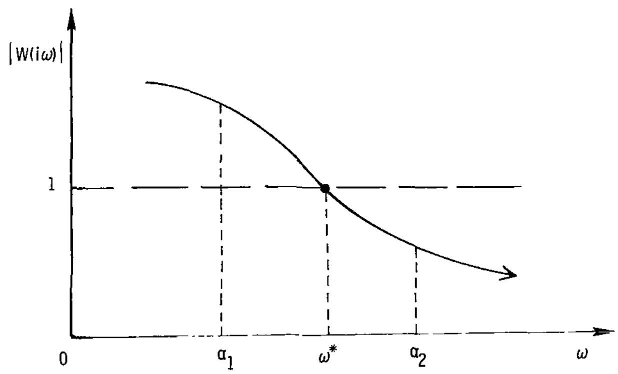

Figure 1.- Hypothetical variation of the absolute value of the testing function, evaluated along the imaginary axis.

The magnitude of $W(i\omega)$ in figure 1 intersecting the horizontal dashed line at 1 corresponds to $W(i\omega)$ intersecting the unit circle. Hence, in figure 1, $W(i\omega)$ enters the unit circle as ω increases across ω^*.

Procedure

To apply the stability procedure of this paper, ω is incremented in the following equations:

$$s = i\omega \tag{9}$$

$$\mathrm{Im}(K) = \mathrm{Im}\left[\frac{f(s;\tau_\ell)}{s}\right] \tag{10}$$

$$\mathrm{Re}(K) = \mathrm{Re}\left[\frac{f(s;\tau_\ell)}{s}\right] \tag{11}$$

$$|W(s)| = \left|e^{f(s;\tau_\ell)}\right| \tag{12}$$

The coefficient K in equation (3) is assumed to be a real number. If $\mathrm{Im}(K) = 0$ in equation (10) for some value of ω during the incrementation, say ω^*, then equation (3) is satisfied by $s = i\omega^*$ when K in equation (11) has the real value denoted by K^*. Theorem 2 is applied by examining tabulated values of equation (12) as ω increases across ω^*.

Since $\omega > 0$ is incremented in equations (9) to (12), there is the question of when to terminate the iteration process. It is always possible to compute an upper bound on ω for $K \leqq K_m$. The procedure is illustrated in a subsequent application. It is important to choose the increment on ω small enough to preclude overlooking any values of $s = i\omega$ which satisfy the characteristic equation when $K \leqq K_m$.

APPLICATION

Consider the following equation from reference 6 (in the present notation):

$$\frac{d^2x}{dt^2}(t) + 2K\frac{dx}{dt}(t-\tau_1) + x(t-\tau_2) = 0 \tag{13}$$

where τ_1 and τ_2 are fixed time delays, and the real number $K > 0$ is a system coefficient.

The characteristic equation associated with equation (13) is

$$s^2 + 2Kse^{-\tau_1 s} + e^{-\tau_2 s} = 0 \tag{14}$$

With $K = 0.5$, $\tau_1 = 0.8$, and $\tau_2 = 0.2$, the solution of equation (13) is asymptotically stable; that is, all roots of equation (14) have negative real parts (refs. 6 and 7).

Now, as K is varied, root-locus curves may possibly intersect the imaginary axis and change the stability of equation (13). To examine this possibility, write equation (14) as

$$Ks = -\frac{1}{2}\left(s^2 + e^{-\tau_2 s}\right)e^{\tau_1 s} \tag{15}$$

which is in the form of equation (3) with

$$f(s;\tau_\ell) = -\frac{1}{2}\left(s^2 + e^{-\tau_2 s}\right)e^{\tau_1 s} \tag{16}$$

Then, with equation (16), equations (9) to (12) along with the τ-decomposition theorem are subsequently used.

Iteration Bounds on ω

Maximum bound.- Setting $s = i\omega$ in equation (14) gives

$$-\omega^2 + 2Ki\omega e^{-i\omega\tau_1} + e^{-i\omega\tau_2} = 0 \tag{17}$$

Solving equation (17) for ω^2 and taking the magnitude of both sides of the resulting equation gives

$$|\omega|^2 \leqq 2|K\omega| + 1 \tag{18}$$

But, since $\omega > 0$ and $K > 0$,

$$\omega^2 \leqq 2K\omega + 1 \leqq 2K_m\omega + 1 \tag{19}$$

where $K \leqq K_m$, and K_m is a bound on K.

When $\omega = 0$, the right side of equation (19) exceeds the left side; however, as ω increases, the left side eventually dominates the right side. Thus, there exists a maximum value of ω, say ω_m, beyond which equation (19) is no longer satisfied. It is not difficult to see that this maximum value of ω is the largest positive real root of the equation

$$\omega^2 - 2K_m\omega - 1 = 0 \tag{20}$$

which is

$$\omega_m = K_m + \sqrt{K_m^2 + 1} \tag{21}$$

Lower bound.- One lower bound on ω is zero; however, a larger lower bound may be desired. In this case, equation (17) can be used to obtain the equation, or inequality, of magnitudes

$$|\omega^2| \geqq \left|e^{-i\omega\tau_2}\right| - \left|2Ki\omega e^{-i\omega\tau_1}\right| \tag{22}$$

or,

$$\omega^2 \geqq 1 - 2K\omega \tag{23}$$

Clearly, for $\omega = 0$, equation (23) is not satisfied. As ω increases, there is a least value of ω for which equation (23) is satisfied. This least value will be smaller for larger constant values of K. Thus, for $K \leqq K_m$, the least value of ω which satisfies

$$\omega^2 \geqq 1 - 2K_m\omega \tag{24}$$

is taken as a lower bound on ω. This lower bound, called ω_L, is the least positive real root of the equation

$$\omega^2 + 2K_m\omega - 1 = 0 \tag{25}$$

which is

$$\omega_L = -K_m + \sqrt{K_m^2 + 1} \tag{26}$$

Hence, as K is increased to K_m, all root-locus curves must intersect the imaginary axis in the interval

$$\omega_L \leqq \omega \leqq \omega_m \tag{27}$$

Specific Computations

As mentioned previously, equation (13) is asymptotically stable when $\tau_1 = 0.8$, $\tau_2 = 0.2$, and $K = 0.5$. It is desired to determine the change in stability as K is varied from 0.5 to 1 with the values of τ_1 and τ_2 held fixed. In this case, equation (16) becomes

$$f(s;\tau_\ell) = -\frac{1}{2}(s^2 + e^{-0.2s})e^{0.8s} \tag{28}$$

and equation (27) becomes

$$0.41 \leqq \omega \leqq 2.5 \tag{29}$$

With $f(s;\tau_\ell)$ given by equation (28), equations (9) to (12) were iterated simultaneously on a digital computer by incrementing ω in increments of 0.01 over the interval of values in equation (29).

Table I shows the regions of incremented values where $\mathrm{Im}(K)$ changes sign. The values of ω where $\mathrm{Im}(K) = 0$ and the corresponding values of K are given approximately in the first two columns of table II as $\omega = \omega^*$ and $K = K^*$. The results in the third column of table II were determined by examining $|W(i\omega)|$ in table I in the neighborhood of ω^* and applying the τ-decomposition theorem. As mentioned previously, the retarded system is known to be asymptotically stable for $K = 0.5$. Thus, for K between approximately $K^* = 0.16$ and 0.63, the system will remain asymptotically stable. The system is unstable in the approximate intervals $0 \leqq K < 0.16$ and $0.63 < K \leqq 1$. To examine values of K greater than $K_m = 1$, larger values of ω_m in equation (21) may be used.

CONCLUDING REMARKS

Once the stability condition (stable or unstable) of a retarded system is determined for a fixed set of time delays, it may be of interest to determine the further change in the stability condition as a system coefficient is varied, with the time delays held fixed. This latter problem is examined in this paper by casting it in a form for which the τ-decomposition method is applicable. The approach involves a rearrangement of the characteristic equation so that the coefficient to be varied plays the role of a time delay. The τ-decomposition method is applied then to obtain changes in the stability condition as the coefficient is varied over a range of values.

The method has been applied to a second-order differential equation with time delays in the velocity and displacement terms. Ranges of values of a coefficient for which the system is stable and unstable were computed.

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January 13, 1977

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TABLE I.- INCREMENTAL VALUES

ω	Re(K)	Im(K)	$\lvert W(i\omega)\rvert$
0.410	-0.2309	0.9887	0.6667
.	.	.	.
.	.	.	.
.	.	.	.
1.09	.1382	.0135	.9854
1.10	.1452	.0087	.9905
1.11	.1523	.0040	.9956
1.12	.1593	-.0005	1.0005
1.13	.1664	-.0048	1.0055
1.14	.1736	-.0090	1.0103
.	.	.	.
.	.	.	.
.	.	.	.
1.74	.6077	-.0103	1.0180
1.75	.6144	-.0066	1.0115
1.76	.6211	-.0028	1.0049
1.77	.6277	.0012	.9979
1.78	.6343	.0052	.9908
1.79	.6408	.0093	.9835
.	.	.	.
.	.	.	.
.	.	.	.
2.50	.9371	.5343	.2629

TABLE II.- VALUES OF ω^*, K^*, AND $N(K^*+\varepsilon)$

$\omega = \omega^*$	$K = K^*$	$N(K^*+\varepsilon)$
1.12	0.16	-1
1.77	.63	1

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