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16. Abstract

Determination of the thermal emission from the disk of Saturn at wavelengths in the vicinity of the thermal peak (~50 μm) is complicated by the fact that infrared telescopes currently operating at these wavelengths cannot spatially separate the rings from the disk. To account for the emission from the rings, the area of the visible disk, the area of the ansae (visible rings not overlapping the disk), and the area of overlap (visible rings overlapping the disk) must be known. The calculation presented here describes the analytical determination of these areas from parameters available in the ephemeris.

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ANALYTICAL CALCULATION OF THE AREAS

OF SATURN'S DISK AND RINGS

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SUMMARY

Determination of the thermal emission from the disk of Saturn at wavelengths in the vicinity of the thermal peak (~50 μm) is complicated by the fact that infrared telescopes currently operating at these wavelengths cannot spatially separate the rings from the disk. To account for the emission from the rings, the area of the visible disk, the area of the ansae (visible rings not overlapping the disk), and the area of overlap (visible rings overlapping the disk) must be known. The calculation presented here describes the analytical determination of these areas from parameters available in the ephemeris.

Saturn's rings and disk make comparable contributions to the thermally emitted flux from the entire system. If one assumes that the optical depth of the rings is not necessarily high, and that the only significant ring effects are due to the A and B rings, then five areas are important in estimating the relative contributions of the rings and disk. These are:

Ω_{vd} = unobscured (visible) area of the disk

Ω_A, Ω_B = visible area of the A and B rings respectively

ω_A, ω_B = area of disk obscured by the A and B rings respectively

The brightness of the entire system is described by brightness temperature T, which is related to the rings' temperatures, T_A and T_B, and the disk temperature, T_d, by

$$B_\nu(T)\Omega_t = B_\nu(T_d)(\Omega_{vd} + e^{-\tau_A}\omega_A + e^{-\tau_B}\omega_B) + B_\nu(T_A)(1 - e^{-\tau_A})\Omega_A + B_\nu(T_B)(1 - e^{-\tau_B})\Omega_B$$

Here B_ν is the Planck function,

$$\Omega_t = \Omega_{vd} + \Omega_A + \Omega_B \tag{1}$$

is the total system area, and τ_A and τ_B are the optical depths of the rings.

Both the disk and rings are elliptical in appearance to the observer. In rectangular coordinates the equation of an illipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 , \quad a \geq b \tag{2}$$

If we transform this to polar coordinates we have $x = r \cos \theta$, $y = r \sin \theta$, so

$$r^2\left(\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}\right) = 1 \quad \text{or} \quad r(\theta) = ab(a^2 \sin^2 \theta + b^2 \cos^2 \theta)^{-1/2} \tag{3}$$

The area of an ellipse is πab. Our notation for the Saturn geometry is shown in figure 1. Thus, for the problem at hand, we have

$$\Omega_{vd} = \pi a_o b_o - \omega_A - \omega_B \tag{4}$$

$$\left.\begin{aligned} \Omega_A &= \pi(a_1 b_1 - a_2 b_2) - \omega_A \\ \Omega_B &= \pi(a_3 b_3 - a_4 b_4) - \omega_B \end{aligned}\right\} \tag{5}$$

If we note that (see ref. 1)

$$\frac{a_2}{a_1} = \frac{b_2}{b_1} = 0.8801 \tag{6}$$

$$\frac{a_3}{a_1} = \frac{b_3}{b_1} = 0.8599 \tag{7}$$

and

$$\frac{a_4}{a_1} = \frac{b_4}{b_1} = 0.6650 \tag{8}$$

then the quantities a_2, b_2, a_3, b_3, a_4, and b_4 can be eliminated in (5) so that

$$\left.\begin{aligned} \Omega_A &= 0.2254\,\pi a_1 b_1 - \omega_A \\ \Omega_B &= 0.2972\,\pi a_1 b_1 - \omega_B \end{aligned}\right\} \tag{9}$$

a_o, b_o, a_1, and b_1 can be obtained from the ephemeris.

There remains simply to calculate ω_A and ω_B. To determine the angles θ_i, $i = 1$ to 4, we note that

$$r_i(\theta_i) = r_o(\theta_i) \quad \text{or} \quad r_i^2(\theta_i) = r_o^2(\theta_i) \tag{10}$$

Then

$$\frac{a_i^2 b_i^2}{a_i^2 \sin^2 \theta_i + b_i^2 \cos^2 \theta_i} = \frac{a_o^2 b_o^2}{a_o^2 \sin^2 \theta_i + b_o^2 \cos^2 \theta_i} \tag{11}$$

which can be solved for $\tan \theta_i$

$$t_i \equiv \tan \theta_i = \frac{b_o b_i}{a_o a_i} \left(\frac{a_i^2 - a_o^2}{b_o^2 - b_i^2} \right)^{1/2} \qquad \text{for} \quad b_i \leq b_o \tag{12}$$

If b_i should be greater than b_o, $t_i = +\infty$ $[\theta_i = (\pi/2)]$. Now

$$\omega_A = 2\int_{\theta_2}^{\theta_1} \int_{r_2}^{r_o} r \, dr \, d\theta + 2\int_{\theta_1}^{\pi/2} \int_{r_2}^{r_1} r \, dr \, d\theta \tag{13}$$

$$= \int_{\theta_2}^{\theta_1} (r_o^2 - r_2^2) d\theta + \int_{\theta_1}^{\pi/2} (r_1^2 - r_2^2) d\theta \tag{14}$$

$$= \int_{\theta_2}^{\theta_1} r_o^2 \, d\theta + \int_{\theta_1}^{\pi/2} r_1^2 \, d\theta - \int_{\theta_2}^{\pi/2} r_2^2 \, d\theta \tag{15}$$

$$= a_o b_o \tan^{-1}\left(\frac{a_o}{b_o} \tan \theta\right)\Bigg|_{\theta_2}^{\theta_1} + a_1 b_1 \tan^{-1}\left(\frac{a_1}{b_1} \tan \theta\right)\Bigg|_{\theta_1}^{\pi/2}$$

$$-a_2 b_2 \tan^{-1}\left(\frac{a_2}{b_2} \tan \theta\right)\Bigg|_{\theta_2}^{\pi/2} \tag{16}$$

since

$$\int r^2 \, d\theta = a^2 b^2 \int \frac{d\theta}{a^2 \sin^2 \theta + b^2 \cos^2 \theta} = ab \tan^{-1} \left(\frac{a}{b} \tan \theta\right) \tag{17}$$

Rewriting (16), and from a similar expression for ω_B, we find

$$\omega_A = a_o b_o \left[\tan^{-1}\left(\frac{a_o}{b_o} t_1\right) - \tan^{-1}\left(\frac{a_o}{b_o} t_2\right)\right] + a_1 b_1 \left[\frac{\pi}{2} - \tan^{-1}\left(\frac{a_1}{b_1} t_1\right)\right]$$

$$-a_2 b_2 \left[\frac{\pi}{2} - \tan^{-1}\left(\frac{a_2}{b_2} t_2\right)\right] \tag{18}$$

and

$$\omega_B = a_o b_o\left[\tan^{-1}\left(\frac{a_o}{b_o}\, t_3\right) - \tan^{-1}\left(\frac{a_o}{b_o}\, t_4\right)\right] + a_3 b_3\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{a_3}{b_3}\, t_3\right)\right]$$

$$-a_4 b_4\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{a_4}{b_4}\, t_4\right)\right] \tag{19}$$

From (6), (7), and (8) we obtain finally

$$\omega_A = a_o b_o\left[\tan^{-1}\left(\frac{a_o}{b_o}\, t_1\right) - \tan^{-1}\left(\frac{a_o}{b_o}\, t_2\right)\right] + a_1 b_1\left[0.1127\pi - \tan^{-1}\left(\frac{a_1}{b_1}\, t_1\right)\right.$$

$$\left. +0.7746\ \tan^{-1}\left(\frac{a_1}{b_1}\, t_2\right)\right] \tag{20}$$

and

$$\omega_B = a_o b_o\left[\tan^{-1}\left(\frac{a_o}{b_o}\, t_3\right) - \tan^{-1}\left(\frac{a_o}{b_o}\, t_4\right)\right] + a_1 b_1\left[0.1486\pi - 0.7394\ \tan^{-1}\left(\frac{a_1}{b_1}\, t_3\right)\right.$$

$$\left. +0.4422\ \tan^{-1}\left(\frac{a_1}{b_1}\, t_4\right)\right] \tag{21}$$

As an example, we calculate the areas for January 27, 1976. On this date the ephemeris parameters (in arc seconds) are

$$a_o = 10.295 \qquad b_o = 9.215$$

$$a_1 = 23.19 \qquad b_1 = 8.49$$

Then, from (1), (4), (9), (20), and (21) one obtains in square arc seconds the areas

$$\Omega_{vd} = 259.02$$
$$\Omega_A = 127.91$$
$$\Omega_B = 156.25$$
$$\omega_A = 11.52$$
$$\omega_B = 27.49$$
$$\Omega_t = 543.18$$

REFERENCE

The American Ephemeris and Nautical Almanac, issued by the Nautical Almanac Office, United States Naval Observatory. U. S. Government Printing Office, Washington, 1976, p. 416.

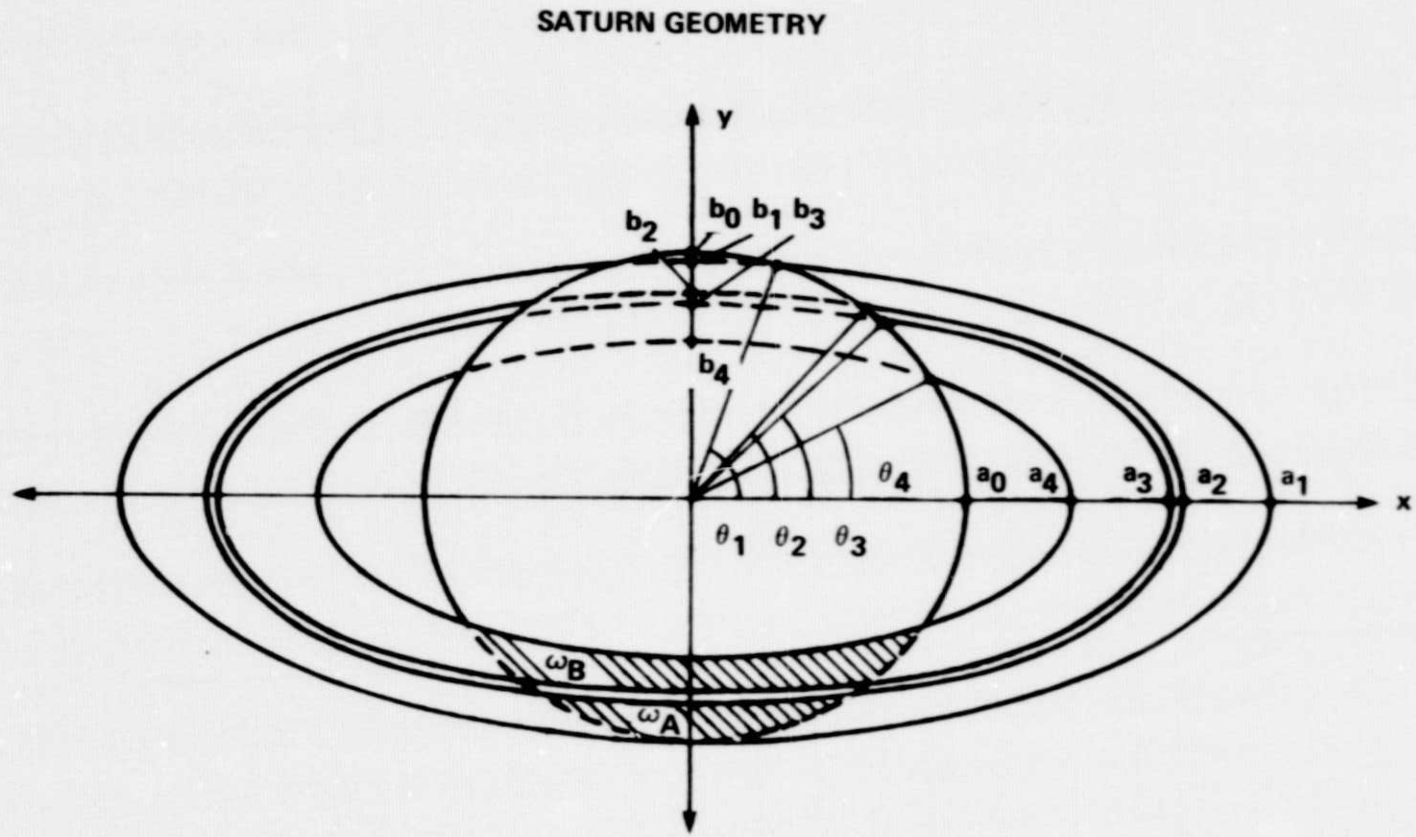

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Figure 1.- Notation for the calculation of the Saturn geometry. The quantities a_i and b_i are the semi-major and semi-minor axes of the indicated components, and the θ_i are the angles of intersection of the ring edges with the disk. The areas of overlap of the disk with ring A and ring B are ω_A and ω_B, respectively.