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16. Abstract

A revised approach to the design of transmission shafting which considers the flexure fatigue characteristics of the shaft material under combined cyclic bending and static torsion stress is presented. The proposed method is suitable to form the basis of a revised shafting design procedure as a replacement to the withdrawn ASME Code for the Design of Transmission Shafting, ASA-B17c. A fatigue failure relation, corroborated by published combined stress test data, is presented which shows an elliptical variation of reversed bending endurance strength with static torsional stress. From this elliptical failure relation, a design formula for computing the diameter of rotating solid shafts under the most common condition of loading is developed.

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INTRODUCTION

In 1927, ASME published the Code for Design of Transmission Shafting, ASA-B17c, which contained formulas and diagrams for computing shaft diameters under various conditions of loading. This standard, which was reaffirmed in 1947, became obsolete and was subsequently withdrawn in 1954. One of the principal limitations of the B17c Shafting Code was that it did not ostensibly consider fatigue as the predominate failure mode of shafting. It is now commonly accepted (e.g., refs. 1 to 3) that most rotating shafts in actual service do fail from flexure fatigue at a point where the ratio of cyclic stresses to endurance strength is a maximum. This maximum often occurs at keyways, splines, fillets and other stress concentrations.

The fatigue failure of a material is generally the complete fracture that results from the propagation of a small crack emanating from a point of high stress concentration, such as at a surface discontinuity or internal metallurgical defect, under a sufficiently large number of repeated stress cycles. The important distinction between static fracture and that resulting from fatigue is that the latter occurs even in ductile materials without plastic deformation at stresses below yield and, consequently, without prior indication of impending failure.

Shafting design formulas predicated on flexure fatigue do not exclude elastic (static) failures from consideration. In essence, a rotating shaft sized to withstand a large number of repeated stress cycles must, by necessity, be strong enough to avoid elastic failure on its first stress cycle. Thus, the avoidance of fatigue failure will be the more stringent design criterion. In view of this, the question arises why many of the shafts whose design was based upon the elastic shear failure criterion of the obsolete B17c Code provided satisfactory service life. They did so primarily because of the B17c Code's conservative working stress values and safety factors rather than the thoughtful consideration of the fatigue characteristics of the shaft material.

FATIGUE PREDICTION

Although the mechanism of fatigue failure and its statistical nature are reasonably well understood (refs. 1 to 6), prediction techniques cannot be developed to precisely pinpoint the number of stress cycles at which the onset of fatigue will begin for a given component. However, general design formulas can be used in conjunction with bench type fatigue test data and other material properties to forecast, with reasonable certainty, the likelihood that a particular component will survive a specific number of stress cycles.

The most commonly used test machine to obtain reverse bending flexure fatigue test data is the rotating beam fatigue tester. From this tester numerous stress-life or "S-N" diagrams have been published for a variety of materials (refs. 1 to 3). For several materials, most notably steel, there exists a stress, referred to as the endurance strength, at which the slope of the S-N curve approaches zero. Since most S-N diagrams, composed of numerous tests, are plotted at the middle of the test data scatter band, they represent a 50 percent probability of survival (ref. 3).

Thus, components subject to stresses at or slightly below the endurance strength will have a 50 percent probability of surviving indefinitely. This percent probability of survival can generally be increased greatly, say to 90 percent, with a relatively small decrease in operating bending stress (ref. 3).

The number of stress cycles at which the S-N curve flattens out is commonly referred to as the fatigue limit. For most plain carbon steels a distinct fatigue limit occurs between 10^5 to 10^7 stress cycles. However for alloy steels, the number of stress cycles to determine endurance strength increases with an increase in tensile strength, being $2x10^7$, $4x10^7$ and at least 10^8 cycles for specimen tensile strengths of 120,000, 160,000 and 220,000 psi, respectively, according to reference 7. It is unlikely that very high strength alloy steels, like many nonferrous metals, display a fatigue limit at all (ref. 1).

COMBINED STRESS FAILURE RELATION

In the case of simple fluctuating stresses, several failure relations have been proposed to represent the degradation of endurance strength under the presence of a static mean stress of the same kind. The most widely adopted of these for design purposes, because of its conservative nature, is the Soderberg straight line relation (ref. 8). The Soderberg line represents a linear degradation in endurance strength with static stress up to the yield strength of the material where the endurance strength becomes zero.

However most rotating shafts are subjected to a condition of combined stresses of different types, namely, a fully reversing bending moment in combination with a static torsion load with negligible axial loading. Although little experimental work is available for this loading condition, tests performed in references 9 and 10 with alloy steel rotating beam specimens under combined stress conditions show an elliptical variation of reversed bending endurance strength, S_b, with static torsional stress, S_s, (see figure 1) of the form:

$$\left(\frac{S_b}{S_e^*}\right)^2 + \left(\frac{S_s}{S_{sy}}\right)^2 = 1 \qquad (1)$$

where S_e^* is the reversed bending endurance strength of the test specimen at zero static torsional stress and S_{sy} is simply the static torsional yield strength of the test specimen. The combined stress fatigue data reported in references 11 and 12 for steels specimens subjected to a reversing bending stress in combination with a reversing torsional stress also show an elliptical failure relation similar to that shown in equation (1).

However, the failure relation of equation (1) has not been the only one proposed for the case of static torsion superimposed on cyclic bending. The work reported in references 13 and 14 suggest that the bending endurance strength of low carbon steel is virtually unaffected by the presence of a static torsional stress, even at torsional stresses in the plastic range. It is not clear why these latter results do not follow the aforementioned elliptical failure relation, but from a design standpoint the failure relation of equation (1) clearly provides a much safer approach to shaft design. Furthermore, the elliptical combined stress method is currently being advocated by many design specialists (refs. 3, 9, 15, and 16) and has, in fact, become part of the Australian Shaft Design Standard (ref. 17).

SHAFT DESIGN FORMULA

The following shaft design formulas are applicable to rotating solid shafts under the most common variety of loading conditions, namely, fully reversed bending in combination with static torsion, less than torsional yield, with negligible axial loading.

For design purposes, incorporating a factor of saftey, FS, into the failure relation of equation (1) the following equation can be written:

$$\left(\frac{S_b}{S_{ew}}\right)^2 + \left(\frac{S_s}{S_{syw}}\right)^2 = 1 \qquad (2)$$

where the working endurance strength factor, S_{ew}, the working torsional yield strength factor, S_{syw}, are defined as:

$$S_{ew} = \frac{S_e}{FS} \qquad (3)$$

$$S_{syw} = \frac{S_{sy}}{FS} \qquad (4)$$

and where

$$S_b = \frac{32M}{\pi d^3} \qquad (5)$$

$$S_s = \frac{16T}{\pi d^3} \qquad (6)$$

By making the appropriate substitutions into equation (2) and noting that for most steels,

$$S_{sy} = \left(\frac{S_y}{3}\right) \qquad (7)$$

we can calculate the required shaft diameter from the following expression:

$$d = \left[\frac{32\ FS}{\pi}\sqrt{\left(\frac{M}{S_e}\right)^2 + \frac{3}{4}\left(\frac{T}{S_y}\right)^2}\,\right]^{1/3} \qquad (8)$$

In equation (8), the reversed bending endurance strength of shaft to be designed, S_e, is generally different than the endurance strength found from rotating beam specimens made from the same material, S_e^*, commonly listed in design tables. A number of factors have been identified which can effect the endurance strength of a material in actual service. In accordance with reference 16, certain modifying factors may be applied to the uncorrected bending endurance strength of test specimens to account for certain application differences as follows:

$$S_e = k_a \, k_b \, k_c \, k_d \, k_e \, k_f \, k_g \; S_e^* \qquad (9)$$

where

S_e = corrected reversed bending endurance strength of shaft

S_e^* = reversed bending endurance strength of rotationg beam specimen

k_a = surface finish factor

k_b = size factor

k_c = reliability factor

k_d = temperature factor

k_e = duty cycle factor

k_f = fatigue stress concentration factor

k_g = miscellaneous effects factor

The appropriate values of these fatigue modifying factors to be used in equation (9) can be found in references 1, 2, 3, and 16.

CONCLUDING REMARKS

A revised approach to the design of transmission shafting under combined cyclic bending and static torsion loading is presented. The proposed method can serve as the basis of a revised shafting design procedure as a replacement to the withdrawn ASME Code for the Design of Transmission Shafting, ASA-B17C. The proposed shafting design formula embodies the following features:

1. The design formula is predicated on a fatigue failure relation which consists of an elliptical variation of reversed bending endurance strength with static torsional stress.

2. The elliptical failure relation is corroborated by the combined stress fatigue test data published by two independent investigators.

3. The design formula is of simple form and can be readily used with generally available specimen test data to compute the diameter of rotating shafts under the common shaft loading condition of cyclic bending and static torsion.

4. Fatigue modifying factors have been incorporated into the design formula to adjust endurance strength test data published for rotating beam specimens for design differences between the shaft in actual service and that experienced by the test specimen.

REFERENCES

1. Forrest, P.G., Fatigue of Metals; Pergaman Press, London, 1962.

2. Sines, G. and Waisman, J.L. (eds.), Metal Fatigue; McGraw-Hill Book Co., Inc. New York, 1959.

3. Shigley, J.E. Mechanical Engineering Design, McGraw-Hill Book, Co., Inc. New York, 1963.

4. Weibull, W., "Statistical Design of Fatigue Experiments," J. Appl. Mech. Vol. 19, 1952, Pg. 109.

5. A.S.T.M. Manual on Fatigue Testing, Publication No. 91, 1949.

6. Johnson, L.G., "The Statistical Treatment of Fatigue Experiments," GMR-202, General Motors Corp., 1956.

7. Frith, P.H., "Fatigue of Wrought High-Tensile Alloy Steels," Int. Conf. on Fatigue, Inst. Mech. Engrs., 1956.

8. Soderberg, C.R., "Factor of Safety and Working Stress," Trans. ASME, J. Appl. Mech. Vol. 52, Pt 1, APM-52-2, 1930, pp. 13-28.

9. Kececiogl., D.B. and Lalli, V.R., "Reliability Approach to Rotating Component Design," NASA TN D-7846, Feb. 1975.

10. Davies, V.C., Discussion to "The Strength of Metals Under Combined Alternating Stresses," by H.J. Gough and H.V. Pollard, Proc. Inst. Mech. Engrs., Vol. 131, No. 3, 1935, pp. 3-103.

11. Findley, W.N., Coleman J.J. and Hanley, B.C., "Theory for Combined Bending and Torsion Fatigue," Int. Conf. on Fatigue, Inst. Mech. Engrs., 1956, pg. 150.

12. Gough, H.J. and Pollard, H.V., "The Strength of Metals under Combined Alternating Stresses," Proc. Inst. Mech. Engrs., Vol. 131, No. 3, 1935, pp. 3-103.

13. Ono, A., "Fatigue of Steel under Combined Bending and Torsion," Mem. Coll. Eng., Kyushu Imp. Univ., Vol. 2, 1929, pp 117-142.

14. Lea, E.C. and Bodgen, H.P., "Combined Torsional and Repeated Bending Stresses," Engineering, Vol. 122, 1926, pp. 242-245.

15. Borchardt, H.A., "Shortcuts for Designing Shafts," Machine Design, Feb. 8, 1973. pp. 139-141.

16. Marin, J., "Design for Fatigue Loading, Part 3," Machine Design, Feb. 21, 1957, pp. 124-133.

17. Australian Standard-Recommendations for the Design of Shafts for Cranes and Hoists, AS B249-1969.

SYMBOLS

d = diameter of shaft, cm (in.)

FS = factor of safety

k = fatigue modifying factor

M = maximum bending moment, N-m (in.-l)

S_b = maximum bending stress, N/m^2 (psi)

S_e = reversed bending endurance strength of shaft, N/m^2 (psi)

S_e^* = reversed bending endurance strength of test specimen, N/m^2 (psi)

S_{ew} = working bending endurance strength of shaft, N/m^2 (psi)

S_e = maximum torsional shearing stress, N/m^2 (psi)

S_{sy} = torsional yield strength of shaft material, N/m^2 (psi)

S_{syw} = working torsion yield strength of shaft material, N/m^2 (psi)

S_y = tensile yield strength of shaft material, N/m^2 (psi)

T = mean static torsional moment, N-m (in.-lb)

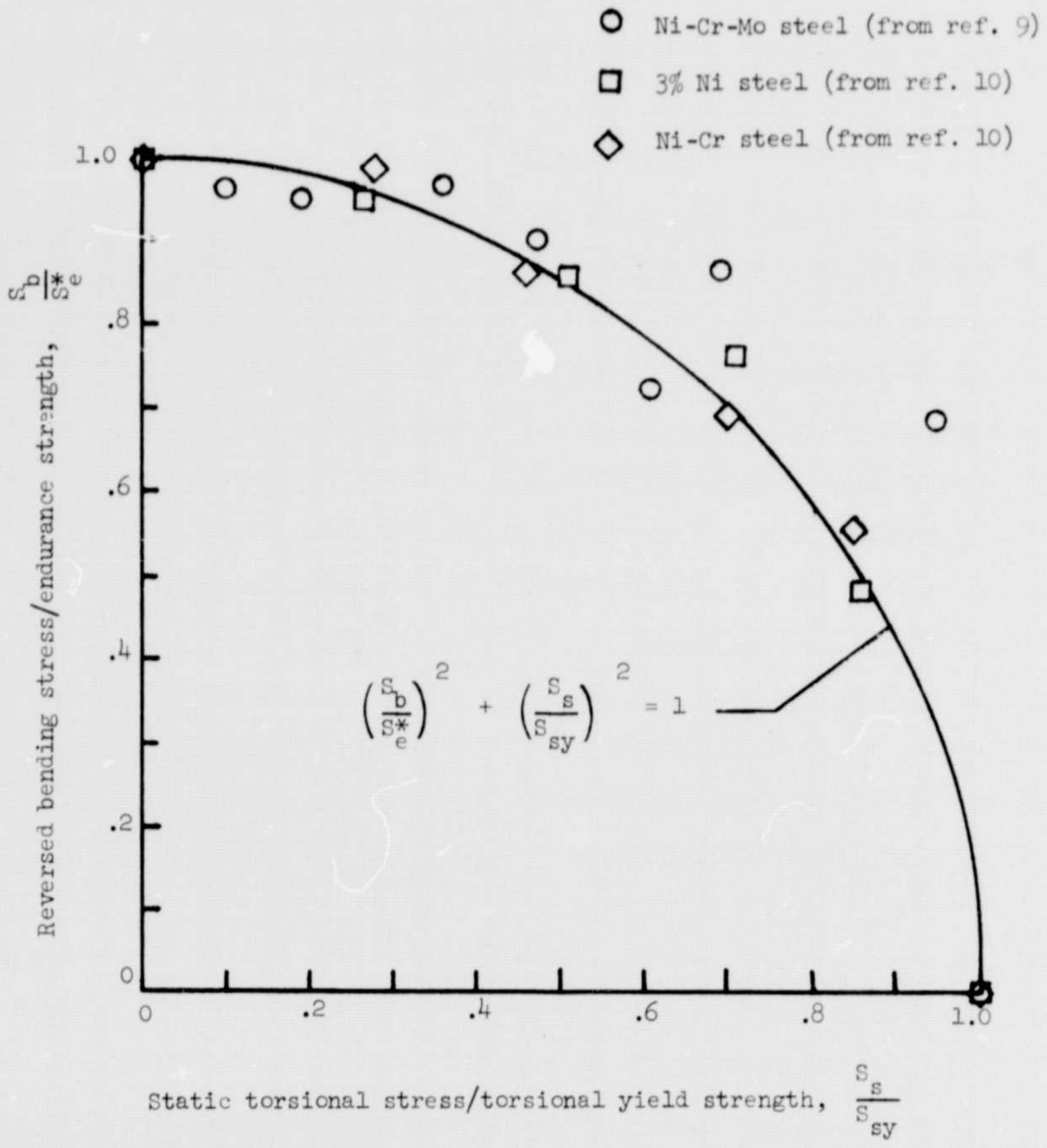

Figure 1 - Combined stress fatigue test data for reversed bending in combination with static torsion.