

# EFFICIENT REANALYSIS OF STRUCTURES BY A DIRECT MODIFICATION METHOD

A. I. Raibstein, I. Kalev and A. Pipano  
Israel Aircraft Industries Ltd.

## SUMMARY

A procedure for the local stiffness modifications of large structures is described. It enables structural modifications without an a priori definition of the changes in the original structure and without loss of efficiency due to multiple loading conditions. The solution procedure, implemented in NASTRAN, involves the decomposed stiffness matrix and the displacement vectors of the original structure. It solves the modified structure exactly, irrespective of the magnitude of the stiffness changes. In order to investigate the efficiency of the present procedure and to test its applicability within a design environment, several real and large structures were solved. The results of the efficiency studies indicate that the break-even point of the procedure varies between 8% and 60% stiffness modifications, depending upon the structure's characteristics and the options employed.

## INTRODUCTION

For the efficient design of large and complex structures it is necessary to investigate several modified structural configurations. Frequently, structural modifications are made at a few discrete locations in the structure, affecting only a relatively small part of it. Consequently, it is desirable to apply a modification procedure which enables rapid re-analysis of the structure in a time step which is considerably less than that of the analysis of the regular structure. Furthermore, the procedure should be automated with minimum requirements of input data.

The various modifications procedures which have been developed so far can be subdivided into iterative and direct procedures. The iterative procedures are based either on a simple technique of successive approximations (reference 1), or on a version derived from the Gauss-Seidel technique (reference 2), or on a first order Taylor series expansion of design variables (references 3 and 4). The procedures become less efficient as more loading conditions are applied to the structure and more inaccurate as the magnitude of the stiffness changes increases. The advantage of using these procedures is that the efficiency of the procedure is not affected by the relative number of modified degrees of freedom. The direct procedures (references 1 and 5), are exact and their efficiency is mainly governed by the relative number of the modified degrees of freedom. Their efficiency is not affected by the number of loading conditions, and the accuracy of the analysis results is independent of the magnitude of the stiffness changes. Argyris and Roy (reference 6) have developed a general direct modification procedure which handles coupled combinations of three types of modifications, namely, changing stiffnesses of elements, adding degrees of freedoms and removing degrees of freedom.

To our knowledge, the direct procedures have not yet been investigated for large and complex structures and their applicability within a design environment has not been tested. In addition, operation counts alone, as it was done in references 1, 5 and 6, are not sufficient to predict the overall efficiency of the procedures. This is because, for large and real structures, the speed of in-core operations relative to the speed of out-of-core data transfer, the type of finite elements and the time required for the assembly of their stiffness matrices may have a considerable effect on the efficiency.

The present paper describes a direct procedure for the solution of structures with local stiffness modifications (reference 7). The basic concepts are similar to those of Argyris and Roy (reference 6). However, their procedure was modified and implemented in the static solution of NASTRAN (reference 8) by employing NASTRAN'S DMAP language (references 9 and 10). In order to reduce the burden of input data preparation, a NASTRAN preprocessor program, REAN, has been developed. This preprocessor, in addition to its other features, takes care of the characteristics of the modified and unmodified structure constructing the Boolean transformation matrices, which relate their corresponding degrees of freedom.

In order to investigate the efficiency of the modification procedure and to test its applicability within a design environment several real and large structures were solved. For each of the examples a number of modifications were made, including the addition or removal of elements and the investigation of regions with reduced stiffnesses in order to study the post-buckling behavior of these regions. The size of the different mathematical models varied between 60 and 1680 unconstrained degrees of freedom.

## SYMBOLS

[B]	Boolean transformation matrix
[F]	inverse of [K]
G	modulus of rigidity
[I]	unit matrix
[K]	stiffness matrix
[ $\bar{K}$ ]	modified stiffness matrix
[L]	lower triangularized stiffness matrix (figure 3)
[LLL]	lower triangularized stiffness matrix of original structure (figure 3)
{P}	load vector
{ $\bar{P}$ }	modified loading vector
{Q}	vector of reactions
{ $\bar{Q}$ }	modified vector of reactions
q	number of increments of stiffness changes (equation (39) )
[U]	upper triangularized stiffness matrix (figure 3)
{U}	displacement vector
{ $\bar{U}$ }	modified displacement vector
$a; a_1; a_2$	multiplication factors (equation (39) )
$\Delta$	increment
$\tau$	shear stress

### Subscripts

av	average
c	modified degrees of freedom
eff	effective
f	free degrees of freedom
max	maximum
r	modified constrained degrees of freedom
s	constrained degrees of freedom
u	unmodified degrees of freedom

### Superscripts

(q)	indicator of the number of stiffness increments (figure 3)
T	transpose of a matrix

## THE DIRECT MODIFICATION PROCEDURE

### The Structural Equations

The usual governing equation for the static analysis of structures by the displacement method may be written as

$$[K] \{U\} = \{P\} \quad (1)$$

where  $[K]$  denotes the structural stiffness matrix.  $\{U\}$  the unknown displacement vector and  $\{P\}$  the applied load vector.

Equation (1), after elimination of the dependent equations (MPC) and including the forces of constraints,  $Q_s$ , may be written in partitioned form as

$$\begin{bmatrix} K_{ff} & K_{fs} & 0 \\ K_{fs}^T & K_{ss} & -I \\ 0 & I & 0 \end{bmatrix} \begin{Bmatrix} U_f \\ U_s \\ Q_s \end{Bmatrix} = \begin{Bmatrix} P_f \\ P_s \\ U_s \end{Bmatrix} \quad (2)$$

where the subscripts

f denotes the unconstrained (free) set

and s denotes the constrained set.

Expanding the simultaneous set of equations (2) and assuming that the enforced displacement vector  $\{U_s\} = 0$  gives us

$$[K_{ff}] \{U_f\} = \{P_f\} \quad (3)$$

$$\{Q_s\} = -\{P_s\} + [K_{fs}]^T \{U_f\} \quad (4)$$

Equation (3) is solved for the unknown displacement vector  $\{U_f\}$  and then the forces of constraints  $\{Q_s\}$ , are computed using equation (4).

### Preliminary Matrix Operations

We define that

c is the part of the f – set which is changed (modified)  
and u is the part of the f – set which remains unmodified.

Equation (3) may be written in partitioned form as

$$\begin{bmatrix} K_{uu} & K_{uc} \\ K_{uc}^T & K_{cc} \end{bmatrix} \begin{Bmatrix} U_u \\ U_c \end{Bmatrix} = \begin{Bmatrix} P_u \\ P_c \end{Bmatrix} \quad (5)$$

Now if  $[F_{ff}]$  is defined as the inverse of  $[K_{ff}]$ , then the following relation is valid

$$\begin{bmatrix} K_{uu} & K_{uc} \\ K_{uc}^T & K_{cc} \end{bmatrix} \begin{bmatrix} F_{uu} & F_{uc} \\ F_{uc}^T & F_{cc} \end{bmatrix} = \begin{bmatrix} I_{uu} & 0 \\ 0 & I_{cc} \end{bmatrix} \quad (6)$$

Expanding equation (6) gives

$$[K_{uu}] [F_{uc}] + [K_{uc}] [F_{cc}] = 0 \quad (7)$$

$$[K_{uc}]^T [F_{uc}] + [K_{cc}] [F_{cc}] = [I_{cc}] \quad (8)$$

By pre- and post-multiplying equation (7) by  $[K_{uu}]^{-1}$  and  $[F_{cc}]^{-1}$ , respectively, the following is obtained

$$[K_{uu}]^{-1} [K_{uc}] = - [F_{uc}] [F_{cc}]^{-1} \quad (9)$$

By post-multiplying equation (8) by  $[F_{cc}]^{-1}$  and substituting equation (9) gives

$$[F_{cc}]^{-1} = [K_{cc}] - [K_{uc}]^T [K_{uu}]^{-1} [K_{uc}] \quad (10)$$

The effect of element modifications will be transmitted into  $[K_{ff}]$  through a symmetric Boolean transformation matrix. The part of  $[K_{ff}]$  affected by element modifications is limited to the symmetric submatrix  $[K_{cc}]$ . Loading changes on the modified part of the structure are considered by the procedure. However, loading changes for the unmodified part of the structure should be considered through a usual restart job.

We now define the Boolean transformation matrices  $[B_{cf}]$  and  $[B_{uf}]$ . Where

$[B_{cf}]$  is the Boolean transformation matrix relating

$$[K_{ff}] \quad \text{to} \quad [K_{cc}]$$

$[B_{uf}]$  is the Boolean transformation matrix relating

$$[K_{ff}] \quad \text{to} \quad [K_{uu}]$$

Each of the rows in  $[B_{cf}]$  and  $[B_{uf}]$  contain all zeros, except for one unit value in the column relating to the position of the corresponding row in  $[K_{ff}]$  which is being changed or remains unmodified, respectively.

Due to basic algebraic considerations the following orthonormal equations are valid

$$[B_{cf}] [B_{cf}]^T = [I_{cc}] \quad (11)$$

$$[B_{uf}] [B_{uf}]^T = [I_{uu}] \quad (12)$$

$$[B_{cf}]^T [B_{cf}] + [B_{uf}]^T [B_{uf}] = [I_{ff}] \quad (13)$$

The inverse of  $[K_{ff}]$  can now be related, via the Boolean transformation matrices, to its symmetric submatrices  $[F_{uu}]$ ,  $[F_{uc}]$  and  $[F_{cc}]$  as follows

$$[F_{uu}] = [B_{uf}] [K_{ff}]^{-1} [B_{uf}]^T \quad (14)$$

$$[F_{uc}] = [B_{uf}] [K_{ff}]^{-1} [B_{cf}]^T \quad (15)$$

$$[F_{cc}] = [B_{cf}] [K_{ff}]^{-1} [B_{cf}]^T \quad (16)$$

The displacement vector  $\{U_f\}$  is related, via the Boolean transformation matrices, to its subvectors  $\{U_u\}$  and  $\{U_c\}$  as follows

$$\{U_f\} = [B_{uf}]^T \{U_u\} + [B_{cf}]^T \{U_c\} \quad (17)$$

## The Solution Procedure

The equilibrium equations of the modified structures can be written in an equivalent form to equation (5) as

$$\begin{bmatrix} K_{uu} & K_{uc} \\ K_{uc}^T & \bar{K}_{cc} \end{bmatrix} \begin{Bmatrix} \bar{U}_u \\ \bar{U}_c \end{Bmatrix} = \begin{Bmatrix} P_u \\ \bar{P}_c \end{Bmatrix} \quad (18)$$

where the element modifications in  $[K_{cc}]$  are expressed as

$$[\bar{K}_{cc}] = [K_{cc}] + [\Delta K_{cc}] \quad (19)$$

$[\Delta K_{cc}]$  is the stiffness matrix of the changes and may be singular.

The final loading vector,  $\{\bar{P}_c\}$ , on the modified part is expressed as

$$\{\bar{P}_c\} = \{P_c\} + \{\Delta P_c\} \quad (20)$$

where  $\{\Delta P_c\}$  defines the loading changes on the modified part of the structure.

The final displacement vectors of the modified degrees of freedom,  $\{\bar{U}_c\}$ , and the unmodified degrees of freedom,  $\{\bar{U}_u\}$ , are expressed as

$$\{\bar{U}_c\} = \{U_c\} + \{\Delta U_c\} \quad (21)$$

$$\{\bar{U}_u\} = \{U_u\} + \{\Delta U_u\} \quad (22)$$

Where  $\{\Delta U_c\}$  and  $\{\Delta U_u\}$  are the additional displacement vectors due to element modifications and loading changes.

Using equations (19) to (22), and substituting for  $[\bar{K}_{cc}]$ ,  $\{\bar{U}_c\}$ ,  $\{\bar{U}_u\}$  and  $\{\bar{P}_c\}$  in equation (18) and then subtracting equation (5) from it, yields

$$\begin{bmatrix} K_{uu} & K_{uc} \\ K_{uc}^T & K_{cc} + \Delta K_{cc} \end{bmatrix} \begin{Bmatrix} \Delta U_u \\ \Delta U_c \end{Bmatrix} = \begin{Bmatrix} 0 \\ \Delta P_c - \Delta K_{cc} U_c \end{Bmatrix} \quad (23)$$

Using the first of equation (23) we get

$$\{\Delta U_u\} = - [K_{uu}]^{-1} [K_{uc}] \{\Delta U_c\} \quad (24)$$

Using the second of equation (23) and eliminating  $\{\Delta U_u\}$  yields

$$([\Delta K_{cc}] + [K_{cc}] - [K_{uc}]^T [K_{uu}]^{-1} [K_{uc}]) \{U_c\} = \{\Delta P_c\} - [\Delta K_{cc}] \{U_c\} \quad (25)$$

By substituting equation (9) in equation (24) and equation (10) in equation (25) we obtain

$$\{\Delta U_u\} = [F_{uc}] [K_{cc}]^{-1} \{\Delta U_c\} \quad (26)$$

$$([\Delta K_{cc}] + [F_{cc}]^{-1}) \{\Delta U_c\} = \{\Delta P_c\} - [\Delta K_{cc}] \{U_c\} \quad (27)$$

Using equation (17), the equivalent relation for the modified displacement vector  $\{U_f + \Delta U_f\}$  is as follows

$$\{U_f + \Delta U_f\} = [B_{uf}]^T \{U_u + \Delta U_u\} + [B_{cf}]^T \{U_c + \Delta U_c\} \quad (28)$$

where  $\{\Delta U_f\}$  is the change of the displacement vector in the f-set.

Subtracting equation (17) from equation (28) yields

$$\{\Delta U_f\} = [B_{uf}]^T \{\Delta U_u\} + [B_{cf}]^T \{\Delta U_c\} \quad (29)$$

Substituting  $\{\Delta U_u\}$  from equation (26) in equation (29) yields

$$\{\Delta U_f\} = [B_{uf}]^T [F_{uc}] [F_{cc}]^{-1} \{\Delta U_c\} + [B_{cf}]^T \{\Delta U_c\} \quad (30)$$

Using equation (15) and substituting for  $[F_{uc}]$  in equation (30) we obtain

$$\{\Delta U_f\} = [B_{uf}]^T [B_{uf}] [K_{ff}]^{-1} [B_{cf}]^T [F_{cc}] \{\Delta U_c\} + [B_{cf}]^T \{\Delta U_c\} \quad (31)$$

Using equation (13) and substituting for  $([B_{uf}]^T [B_{uf}])$  in equation (31), yields

$$\{\Delta U_f\} = ([I_{ff}] - [B_{cf}]^T [B_{cf}]) [K_{ff}]^{-1} [B_{cf}] [F_{cc}]^{-1} \{U_c\} + [B_{cf}]^T \{\Delta U_c\} \quad (32)$$

From equation (16) we establish the identity

$$([B_{cf}] [K_{ff}]^{-1} [B_{cf}]^T) [F_{cc}]^{-1} = [I_{cc}] \quad (33)$$

Using the identity of equation (33) we reduce equation (32) to obtain

$$\{\Delta U_f\} = [K_{ff}]^{-1} [B_{cf}]^T [F_{cc}]^{-1} \{\Delta U_c\} \quad (34)$$

Using equation (27) and substituting for  $\{\Delta U_c\}$  in equation (34) we obtain the final equation for the change in the displacement vector of the f-set due to the modifications as follows

$$\{\Delta U_f\} = [K_{ff}]^{-1} [B_{cf}]^T [F_{cc}]^{-1} ([K_{cc}] + [F_{cc}]^{-1})^{-1} (\{\Delta P_c\} - [\Delta K_{cc}] \{U_c\}) \quad (35)$$

And the modified displacement vector,  $\{\bar{U}_f\}$  is expressed as

$$\{\bar{U}_f\} = \{U_f\} + \{\Delta U_f\} \quad (36)$$

Using equation (4) we define the equivalent relations for the modified vector of constraints,  $\{Q_s\}$ , as

$$\{\bar{Q}_s\} = -\{P_s\} + [\bar{K}_{fs}]^T \{\bar{U}_f\} \quad (37)$$

where the modified matrix  $[\bar{K}_{fs}]$  is expressed as

$$[\bar{K}_{fs}]^T = [K_{fs}]^T + [B_{rs}]^T [\Delta K_{cr}]^T [B_{cf}] \quad (38)$$

where

$[\Delta K_{cr}]$  is the compacted change matrix of  $[K_{fs}]$  and  $[B_{rs}]$  is the Boolean transformation matrix relating  $[K_{fs}]$  to  $[\Delta K_{cr}]$ , i.e., r denotes the part of the s-set which is changed.

Considering the final equation for the modified displacement vector,  $\{\bar{U}_f\}$ , (equation 36) we note that for the analysis of the modified structure, only  $[\Delta K_{cc}]$  and  $\{\Delta P_c\}$  have to be computed based on the modifications on the structure. However, the major computations are due to the following steps

- \* Performing a forward sweep and backward substitution, using the decomposed triangular matrices of  $[K_{ff}]$  on the Boolean matrix  $[B_{cf}]^T$ .
- \* Decomposition and performing a forward sweep and backward substitution using  $[F_{cc}]$  on a unit matrix  $[I_{cc}]$ . We note that  $[F_{cc}]$  is a positive definite symmetric matrix.
- \* The remaining computations are involved with smaller computing time.



The efficiency of the procedure is further improved if, after computing  $[K_{ff}]^{-1}[B_{cf}]^T$  and  $[F_{cc}]^{-1}$  (first two terms of equation(35)), changes are made only to the c degrees of freedom. This is implemented in the procedure described here by increasing or decreasing the term  $[\Delta K_{cc}]$  (last term of equation (35) ) by a constant factor as shown below:

If

$a_1$  is an initial multiplication factor for  $[\Delta K_{cc}]$  and  $[\Delta K_{cr}]$

$a_2$  is an incremental multiplication factor

q is the number of incremental stiffness changes

Then

$$a = a_1 + q \cdot a_2 \quad (39)$$

Therefore if  $[\Delta K_{cc}]$  is multiplied within the computational loop by the variable  $a$ , then only the term  $([F_{cc}]^{-1} + a [\Delta K_{cc}])^{-1}$  will have to be recomputed for each increments of stiffness change. The factors  $a_1$ ,  $a_2$  and q are defined in the NASTRAN analysis of the modified structure by standard PARAM bulk data cards.

The generation of the Boolean transformation matrices  $[B_{cf}]$  and  $[B_{rs}]$  is tedious and error-prone. Therefore, a computer program, REAN, has been developed which generates those matrices automatically in a form suitable for NASTRAN.

An overall schematic diagram of the procedure is shown in figure 1.

### DESCRIPTION OF PROGRAM REAN

The REAN (RE ANalysis) computer program is a preprocessor for NASTRAN. It performs the following main tasks

- a) Updates MPT (Material Property Table) and EST (Element Summary Table) tables
- b) Generates the Boolean transformation matrices  $[B_{cf}]$  and  $[B_{rs}]$
- c) Generates the appropriate DMAP ALTER package for Rigid Format 1.

The execution of program REAN, illustrated in figure 2, is subdivided into the following steps.

- \* The input to the program consists of
  - (a) a standard NASTRAN Data Deck defining the grid points, element connections, element properties, material properties for the structural modifications only and the values and number of incremental stiffness changes
  - (b) the NASTRAN tables CSTM, YS, GPL, USET, OQG1, MPT and EST.
- \* The program then checks
  - (a) if CBAR, CQDPLT, CQUAD2, CTRIA2 or CTRPLT elements are to be modified; if so, the EST tables are updated. This is done in order to obtain the correct stresses for the elements listed above.
  - (b) if the material properties defined for the modified part of the structure are different from those defined for the original structure; if so, the MPT tables are updated.
- \* Using the grid points defined for the modified part of the structure in conjunction with the NASTRAN tables GPL, USET and OQG1 the program first checks if any of the modified elements are connected to degrees of freedom which are constrained. If this is the case, the Boolean transformation matrices  $[B_{cf}]$  and  $[B_{rs}]$  are generated. If none of the modified elements are connected to constrained degrees of freedom, then only  $[B_{cf}]$  is generated;  $[B_{cf}]$  and  $[B_{rs}]$  are inserted via DMI card images.
- \* Using the information obtained via the NASTRAN files UT1 and UT2, the program then generates the appropriate DMAP Alter package and creates a NASTRAN input file.

After the execution of program REAN, NASTRAN is loaded and the analysis for the modified structure is performed.

### **DMAP ALTER PACKAGES**

The modification procedure is effected in NASTRAN utilizing standard DMAP statements. The following two types of DMAP Alter Packages are required:

- 1) The first DMAP Alter package, inserted in the original deck, utilizes OUTPUT1 and OUTPUT2 modules only.

OUTPUT1 is used to save all the matrices and data blocks required to perform the analysis of the modified structure and the data recovery of nodal displacements, reactions, element forces and stresses. The OUTPUT2 module is used to save data blocks required for program REAN as explained in the previous section.

- 2) The second DMAP ALTER package, generated by program REAN and which effects the solution procedure, is automatically inserted into the data deck of the modified structure and performs the actual reanalysis procedure.

A flow diagram of the DMAP functional steps and their equivalent results is shown in figure 3. The DMAP statements for the first and second packages are listed and described in the Appendix. It should be noted that, according to the problem type, program REAN automatically chooses the appropriate form of inclusion of the optional data blocks CSTM, GM, KFSO, KSS, PS and YS in the SDR1 and SDR2 modules.

## NUMERICAL EXAMPLES

The direct modification procedure has been applied to several real aircraft structures. Each of the examples described below is governed by different structural characteristics influencing the efficiency of the procedure. In order to achieve the minimum decomposition time for the regular analysis, resequencing of the grid points was performed. The examples were run on a CDC 6500 computer with 50000<sub>10</sub> words of memory available for the execution of NASTRAN.

### Wing Structure (model 1)

The wing structure was idealized using shear panels to represent the shear stiffness of the skins and webs, and rod elements representing their extensional stiffnesses. The mathematical model, figure 4, consisted of 6 BAR, 501 ROD, 288 SHEAR elements and 220 GRID points representing 1320 degrees of freedom of which 648 were unconstrained. The structure was subjected to two loading conditions and its stiffness matrix had 66 RMS column lengths.

Three structural modifications were investigated; (a) the stiffness properties of 40 elements were changed affecting 39 degrees of freedom; i.e. 6% of the structure; (b) the stiffness properties of 68 elements were changed affecting 85 degrees of freedom; i.e. 13% of the structure; and (c) the stiffness properties of 130 element were changed affecting 136 degrees of freedom; i.e. 21% of the structure. The changes were referenced as Regions "A", "B" and "C", respectively, on a plan view of the wing structure shown schematically in table 1. This table also presents comparisons of the CPU execution time for a regular analysis versus analysis by the modification procedure.

### Wing Structure (model 2)

This wing structure was a highly idealized mathematical model, as shown in figure 5. The structural elements used were the same as those used in model 1. The mathematical model consisted of 26 BAR, 1200 ROD, 660 CSHEAR elements and 560 GRID points representing 3360 degrees of freedom, of which 1680 were unconstrained. The structure was subjected to two loading conditions and its stiffness matrix had 92 RMS column lengths.

Three structural modifications were investigated: (a) The stiffness properties of 60 elements were changed affecting 66 degrees of freedom, i.e. 4% of the structure; (b) the stiffness properties of 105 elements were changed affecting 132 degrees of freedom, i.e. 8% of the structure; and (c) the stiffness properties of 222 elements were changed affecting 231 degrees of freedom, i.e. 14% of the structure. The changes were referenced as regions "A", "B" and "C", respectively, on a plan view of the wing structure shown schematically in table 2. This table also presents comparison of the CPU execution time for a regular analysis versus analysis by the modification procedure.

### Wing Structure (model 3)

This model is similar in size to the one described as model 2, as shown in figure 5. However instead of shear panels, isoparametric membrane elements, QDMEM1, were utilized. The mathematical model consisted of 42 CBAR elements, 662 QDMEM1, 860 ROD, and 500 GRID points representing 3000 degrees of freedom, of which 1580 were unconstrained. The structure was subjected to three loading conditions and its stiffness matrix had 114 RMS column lengths.

Three structural modifications were investigated: (a) The stiffness property of one element was changed affecting 6 degrees of freedom, i.e. 0.4% of the structure; (b) The stiffness properties of 22 elements were changed affecting 48 degrees of freedom, i.e. 3% of the structure; and (c) the stiffness properties of 130 elements were changed affecting 225 degrees of freedom, i.e. 14% of the structure. The changes are referenced as Regions "A", "B" and "C", respectively, on a plan view of the wing structure shown schematically in table 3. This table also presents comparisons of the CPU execution time for a regular analysis versus analysis by the modification procedure.

### Fuselage Structure

The fuselage was idealized using shear panels to represent the skins, rod elements to represent the stringers and beam elements to represent the bulkheads. The mathematical model, (figure 6) consisted of 160 BAR, 590 ROD, 350 SHEAR elements and 404 GRID points representing 2424 degrees of freedom, of which 1293 were unconstrained. The structure was subjected to one loading condition and its stiffness matrix had 70 RMS column lengths.

Three structural modifications were investigated: (a) 3 elements were added to the structure affecting 28 degrees of freedom, i.e. 2.8% of the structure; (b) the properties of 12 elements were changed affecting 72 degrees of freedom, i.e. 6% of the structure; and (c) the properties of 90 elements were changed affecting 180 degrees of freedom i.e. 14% of the structure. The changes are referenced as Regions "A", "B" and "C", respectively, on an axonometric view of the fuselage structure shown schematically in table 4. This table also presents comparisons of the CPU execution time for a regular analysis versus analysis by the direct modification procedure.

#### Panel In Shear

Figure 7 shows the effect of reduced thickness at the center of a rectangular panel on the modulus of rigidity and on the maximum shear stress. The panel was idealized by 25 rectangular QDMEM elements and the thickness of the center element was reduced to zero in 10 increments.

This example demonstrates the procedure's capability of "one-shot" solution of several cases differing in stiffness (i.e. with varying  $[\Delta K_{CC}]$  in equation (35) ). In addition, the required CPU execution time utilizing the present procedure was 1/4 of the time required for 10 regular runs.

#### DISCUSSION

The efficiency studies for the solution times of the numerical examples solved, indicated the following intrinsic features of the direct modification procedure utilized.

- (a) For large problems whose stiffness matrices have large RMS values for their semi-bandwidth, the forward sweep and back substitution using the triangularized stiffness matrix of  $[K_{ff}]$  on  $[B_{cf}]$  (First two terms of equation (35) ) is the most time-consuming. Table 3 shows that for 3% modification, for example, the solution time for the modification procedure is equal to the solution time for the regular analysis. However, for a structure with lower RMS values, the break-even point in the solution time is reached at approximately 5% modifications, as can be seen from table 1.
- (b) For modifications affecting a relatively large number of degrees of freedom, the decomposition and subsequent forward sweep and back substitution using  $[F_{CC}]$  on the unit matrix  $[I_{CC}]$  (third term of equation (35) ) may take a considerable portion of the solution time needed for the modification procedure. For example in table 1, for modification "C", the computation of  $[F_{CC}]^{-1}$  took 1/3 of the total solution time.
- (c) The types of elements used in the finite element model may have a considerable effect on the efficiency of the procedure. Table 3 shows that when changing the stiffness property of one element only 1/20 of the total CPU execution time of the regular analysis was required to obtain the new displacement vector. However the same

change for the same mesh with different types of elements would require 1/12 of the execution time of the regular analysis, as can be deduced from table 2.

- (d) When using the option of incremental stiffness changes, the efficiency of the procedure is increased substantially. The comparison between the regular analysis and the modification procedure (table 5) shows that for all the cases solved, the break-even CPU time for solution of the displacement vectors came to more than 30% modifications of degrees of freedom. For Wing model 1 the break-even point was approximately 60% modifications of degrees of freedom when including 4 additional stiffness increments.

## CONCLUSIONS

A direct modification procedure has been presented and its implementation in NASTRAN has been described. The efficiency of the procedure has been investigated by solving several real and large structures. The study of the relative CPU execution times for the various problems indicated that the efficiency of the procedure is dependent on the following main structural characteristics: (a) Size of the original structure and the number of active columns in the stiffness matrix; (b) the number of stiffness modifications; (c) the types of finite elements used in the mathematical model; and (d) the number of required additional stiffness modifications.

The break-even point of the procedure varies between 8% to 60% modifications of the degrees of freedoms. However, the higher percentage can only be achieved when using the option of incremental stiffness modifications. It is therefore concluded that the direct modification procedure is highly efficient when investigating regions with continuous varying stiffness magnitudes, such as occurs during the post-buckling phenomena. Furthermore, the procedure was found to be very useful when studying the effect of removing or adding elements between existing grid points to a structure or changing the stiffness properties of elements at different locations.

## APPENDIX

### LISTING AND DESCRIPTION OF THE DMAP STATEMENTS

#### DMAP Alter Statements For The Original Structure

1. ALTER 119
2. OUTPUT1 LLL,ULV,KFS,,//C,N,-1/C,N,8/C,N,USERTPM \$
3. OUTPUT1 USET,EQEXIN,SIL,BGPDT,//C,N,0/C,N,8 \$
4. OUTPUT1 GM,CSTM,YS,PS,KSS//C,N,0/C,N,8 \$
5. OUTPUT2 MPT,EST,,//C,N,-1/C,N,11/C,N,USERTP2 \$
6. OUTPUT2 CSTM,YS,GPL,USET,OQG1//C,N,12/C,N,USERTP3 \$
7. ENDALTER

#### Description of DMAP Alter Statements

2. The lower triangularized stiffness matrix of  $[K_{ff}]$ , the displacement vector and  $[K_{fs}]$ , which are used in the modification procedure, are written on file INP8.
- 3, 4. NASTRAN tables to be used for the recovery of the nodal displacements, nodal reactions, element forces and stresses are written on file INP8.
5. The Element Summary Table (EST) and the Material Property Table (MPT) are written on file UT1 such that, if necessary, they can be updated by Program REAN.
6. Writes NASTRAN Tables on file UT2, with the help of which Program REAN computes the Boolean transformation matrices and generates the DMAP ALTER package for the subsequent analysis of the modified structure.

#### DMAP Alter Package For The Analysis Of The Modified Structure

1. ALTER 78
2. INPUTT1 /LLLO,ULVO,KFSO,,/C,N,-1/C,N,8/C,N,USERTP1 \$
3. INPUTT1 /USET1,EQEXIN1,SIL1,BGPDT1,/C,N,0/C,N,8 \$
4. INPUTT1 /GM1,CSTM1,YS1,PS1,KSS1//C,N,0/C,N,8 \$
5. INPUTT2 /MPT1,EST1,,//C,N,-1/C,N,11/C,N,USERTP2 \$
6. FILE KFF1=SAVE/KFF2=SAVE/Q1=SAVE/XB=SAVE/V=SAVE/  
BBT=SAVE/BS=SAVE/KFSA=SAVE /KFSB=SAVE \$
7. PARAM //C,N,NOP/V,N,NC=-1 \$

8. PARAM //C,N,NOP/V,N,NA=-1 \$  
 9. PARAM //C,N,SUB/V,Y,NCASES/V,Y,NCASES=1/C,N,1 \$  
 10. LABEL W3 \$  
 11. COND W7,NC \$  
 12. COND W1,NA \$  
 13. LABEL W7 \$  
 14. PARAM //C,N,ADD/V,N,NC/C,N,1 \$  
 15. FBS LLLO,,BBT/V/C,N,1/C,N,1/C,N,1/C,N,1 \$  
 16. MPYAD BB,V,/Q/C,N,0/C,N,1/C,N,0/C,N,1 \$  
 17. MPYAD BB,BBT,/DD/C,N,0/C,N,1/C,N,0/C,N,1 \$  
 18. MPYAD BB,ULVO,/XB/C,N,0/C,N,1/C,N,0/C,N,1 \$  
 19. DECOMP Q/QL,QU/C,N,1/C,N,1 \$  
 20. FBS QL,,DD/Q1/C,N,1/C,N,1/C,N,1/C,N,1 \$  
 21. ADD KFS,/KFSA/C,Y,FACA = (0.0,0.0) \$  
 22. ADD KFS,/KFSA/C,Y,FACB = (1.0,0.0) \$  
 23. ADD KFF,/KFF1/C,Y,FACA = (0.0,0.0) \$  
 24. ADD KFF,/KFF1/C,Y,FACB = (1.0,0.0) \$  
 \$ \*\*\*\* TOP OF DMAP LOOP FOR INCREMENTS OF STIFFNESS CHANGES \*\*\*\*  
 25. LABEL W1 \$  
 26. ADD KFSA,KFSB/KFSC \$  
 27. EQUIV KFSC,KFSA/NA \$  
 28. ADD KFF1,KFF2/KFF3 \$  
 29. EQUIV KFF3,KFF1/NA \$  
 30. PARAM //C,N,ADD/V,N,NC/V,N,NC/C,N,1 \$  
 31. PRTPARM //C,N,0/C,N,NC \$  
 32. MPYAD KFF3,XB,/XBB/C,N,0/C,N,1/C,N,0/C,N,1 \$  
 33. ADD Q1,KFF3/R \$  
 34. DECOMP R/RL,RU/C,N,1/C,N,1 \$  
 35. FBS RL,,XBB/U1/C,N,1/C,N,1/C,N,1/C,N,1 \$  
 36. SMPYAD V,Q1,U1,,/U2/C,N,3/C,N,-1/C,N,1 \$  
 37. ADD U2,ULVO/ULVN \$  
 38. JUMP LBL9 \$  
 39. ALTER 110,119  
 40. SMPYAD BBT,KFSC,BS,,KFSO/KFS1/C,N,3/C,N,1/C,N,1 \$



```

41.  SDR1          USET1,,ULV1,,YS1,,GM1,PS1,KFS1,KSS1,/UGV,PGG,QG/
          V,N,NSKIP/C,N,STATICS $
42.  SDR2          CASECC,CSTMM,MPT1,,EQEXIN1,SIL1,,,BGPDT1,PGG,QG,
          UGV,EST1,/OPG1,OQG1,OES1,OE1,PUGV1/C,N,STATICS $
43.  ALTER        121
44.  PARAM        //C,N,SUB/V,Y,NCASES/V,Y,NCASES/C,N,1 $
45.  COND         W3,15 $
$ **** BOTTOM OF DMAP LOOP FOR INCREMENTS OF STIFFNESS CHANGES ****
46.  ENDALTER

```

### Description Of DMAP ALTER Statements

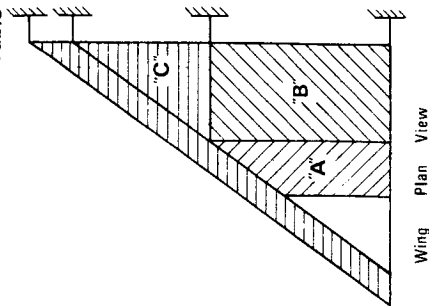
1. The procedure starts after, the stiffness matrix for the modified structure,  $[\Delta K_{CC}]$  has been assembled.
2. Retrieves the lower triangularized stiffness Matrix of  $[K_{ff}]$ , LLLO, Displacement vector, ULVO, and KFSO for the unmodified structure.
- 3, 4. Retrieves NASTRAN tables to be used for the recovery of the nodal displacements, reactions, element forces and stresses. It should be noted that not all the data block names saved via OUTPUT1 in the original analysis appear here. Program REAN checks if the files saved are empty, retrieving only those data blocks which contain information required for the analysis of the modified structure.
5. Retrieves the updated Material Property Table (MPT1) and Element Stiffness Table (EST1). The update of the tables is performed by REAN.
- 7-9. Defines the default values for  $a_1$ ,  $a_2$  and  $q$ .
15. Performs a forward sweep and backward substitution using the lower triangular matrix of  $[K_{ff}]$ , on  $[B_{cf}]^T$  this yields  $[K_{ff}]^{-1} \cdot [B_{cf}]^T$
16. Multiplies  $[B_{cf}] \cdot [K_{ff}]^{-1} \cdot [B_{cf}]^T$  which yields the matrix  $[F_{cc}]$ .
18. Multiplies  $[B_{cf}]$  by the displacement vector ULVO of the unmodified structure. This yields the displacement vector,  $\{U_c\}$ , for the part of the structure which is to be modified.
19. Decompose the matrix  $[F_{cc}]$  into an upper and lower triangular matrix.
20. Performs a forward sweep and backward substitution using the lower triangularized matrix of  $[F_{cc}]$  on a unit matrix, obtained by multiplying  $[B_{cf}] \cdot [B_{cf}]^T$ , this yields  $[F_{cc}]^{-1}$

- 21-24. Initializes  $[\Delta K_{cc}]$  and  $[\Delta K_{cr}]$  of the modified structure.
25. Start of the DMAP loop for the specified increments of stiffness changes.
32. Multiplies  $[\Delta K_{cc}] \cdot \{U_c\}$
33. Adds (  $[\Delta K_{cc}] + [F_{cc}]^{-1}$  )
34. Decomposes the matrix (  $[\Delta K_{cc}] + [F_{cc}]^{-1}$  ) into upper and lower triangular matrices.
35. Performs a forward sweep and backward substitution using the lower triangular matrix of (  $[\Delta K_{cc}] + [F_{cc}]^{-1}$  ) on (  $[\Delta K_{cc}] \cdot \{U_c\}$  )  
This yields (  $[\Delta K_{cc}] + [F_{cc}]^{-1}$  )<sup>-1</sup> ·  $[\Delta K_{cc}] \cdot \{U_c\}$
36. Multiplies the results obtained in 15, 20 and 35. This yields the displacement vector,  $\{\Delta U_f\}$ , which represent the influence of the structural modification on the original structure.
37. Adds ULVO and  $\{\Delta U_f\}$  and yields the final displacement vector ULVN of the modified structure.
40. Computes the nodal reaction (See equation (37)). The Boolean transformation matrix  $[B_{rs}]$  is computed by REAN.  
DMAP Statement 40 is not included by Program REAN if none of the modified elements are incident at degrees of freedom which are constrained.
- 41, 42. Recovers via SDR1 and SDR2 the nodal displacements, reactions element stresses and forces for the modified structure.
45. Bottom of DMAP loop.

## REFERENCES

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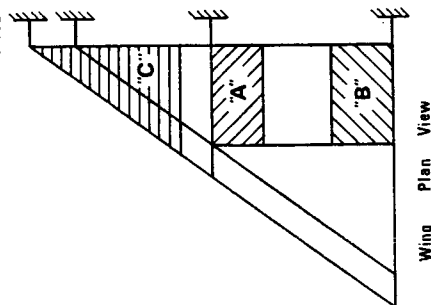
Table 1 — Comparison of CPU Execution Times for Wing Model 1



Model characteristics		Modification characteristics (Upper Skin)	
650	D.O.F.	Region "A" — 39	D.O.F.: 40 Elements
220	Grid Points	Region "B" — 85	D.O.F.: 70 Elements
800	Elements	Region "C" — 136	D.O.F.: 130 Elements

	Regular Analysis (sec.)	Analysis using the Direct Modification Procedure (sec.)					
		Region "A" — 6% Modifications		Region "B" — 13% Modifications		Region "C" — 21% Modifications	
		First Stiffness Modification	Each Additional Modification	First Stiffness Modification	Each Additional Modification	First Stiffness Modification	Each Additional Modification
Input Data Processing, Generation and Partitioning of Stiffness Matrix	164	20	0	26	0	36	0
Solution	41	51	5	100	13	177	49
Data Recovery and Output of Results	27	27	27	27	27	27	27
Total	232	98	32	153	40	240	76

Table 2 - Comparison of CPU Execution Times for Wing Model 2



Model characteristics

1650 D.O.F.

580 Grid Points

2150 Elements

Modification characteristics (Upper Skin)

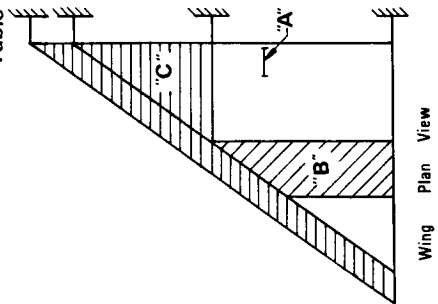
Region "A" - 66 D.O.F.: 60 Elements

Region "B" - 132 D.O.F.: 105 Elements

Region "C" - 231 D.O.F.: 222 Elements

	Regular Analysis (sec.)	Analysis using the Direct Modification Procedure (sec.)					
		Region "A" - 4% Modifications		Region "B" - 8% Modifications		Region "C" - 14% Modifications	
		First Stiffness Modification	Each Additional Modification	First Stiffness Modification	Each Additional Modification	First Stiffness Modification	Each Additional Modification
Input Data Processing, Generation and Partitioning of Stiffness Matrix	478	35	0	39	0	51	0
Solution	133	180	21	386	34	870	121
Data Recovery and Output of Results	79	79	79	79	79	79	79
Total	690	294	100	504	113	1000	200

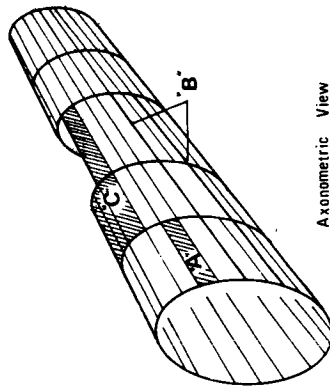
Table 3 — Comparison of CPU Execution Times for Wing Model 3



Model characteristics	Modification characteristics (Upper Skin)
1580 D.O.F.	Region "A" — 6 D.O.F.: 1 Element
500 Grid Points	Region "B" — 48 D.O.F.: 22 Elements
1570 Elements	Region "C" — 225 D.O.F.: 130 Elements

	Regular Analysis (sec.)	Analysis using the Direct Modification Procedure (sec.)					
		Region "A" — 0.4% Modifications		Region "B" — 3% Modifications		Region "C" — 14% Modifications	
		First Stiffness Modification	Each Additional Modification	First Stiffness Modification	Each Additional Modification	First Stiffness Modification	Each Additional Modification
Input Data Processing, Generation and Partitioning of Stiffness Matrix	1190	15	0	17	0	170	0
Solution	230	58	5	233	13	900	121
Data Recovery and Output of Results	140	140	140	140	140	140	140
Total	1560	213	145	390	153	1210	261

Table 4 — Comparison of CPU Execution Times for Fuselage



Axonometric View

Model characteristics		Modification characteristics	
1293	D.O.F.	Region "A" — 28	D.O.F.: 13 Elements
404	Grid Points	Region "B" — 72	D.O.F.: 12 Elements
1100	Elements	Region "C" — 180	D.O.F.: 90 Elements

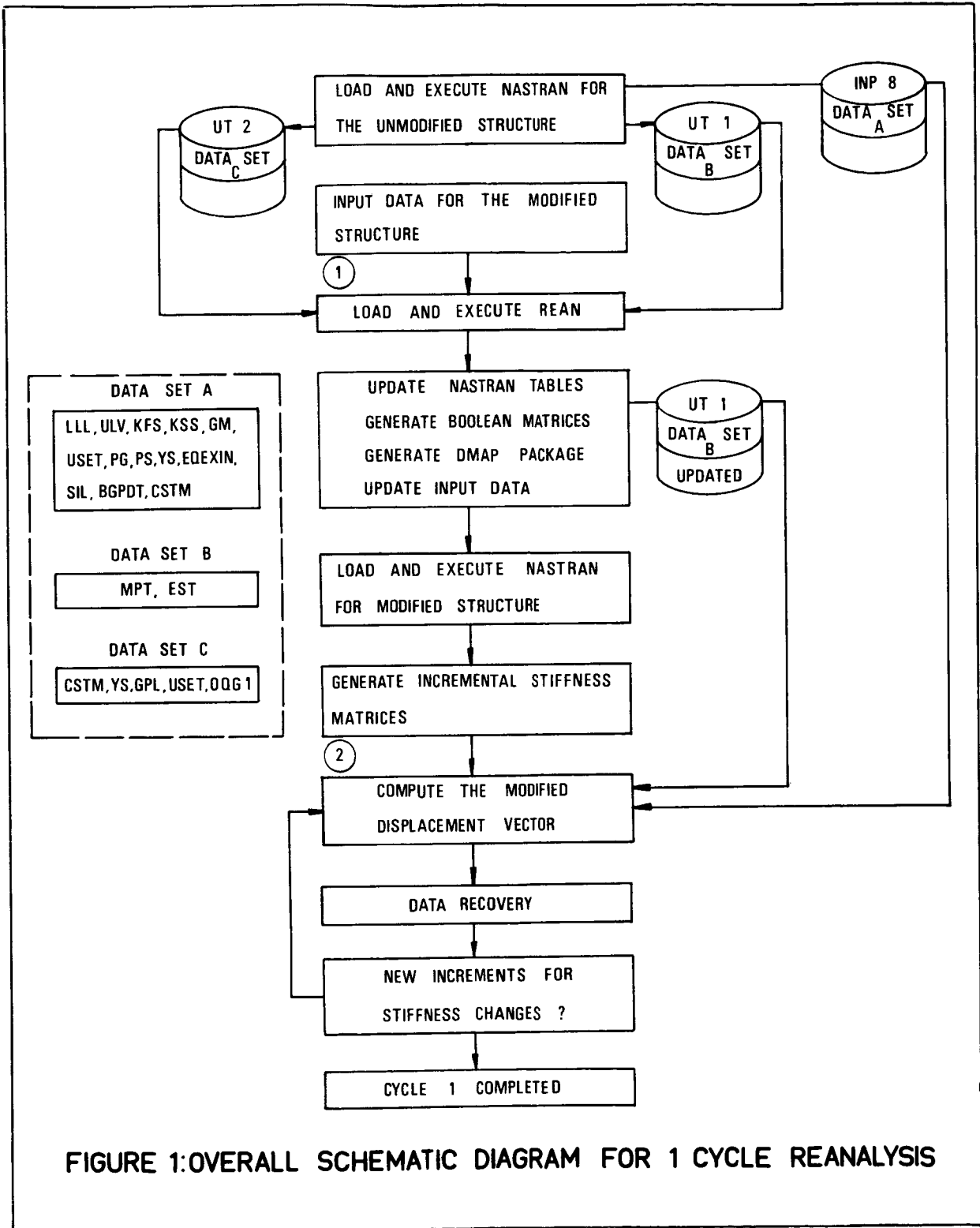
	Regular Analysis (sec.)	Analysis using the Direct Modification Procedure (sec.)					
		Region "A" — 22% Modifications		Region "B" — 5 % Modifications		Region "C" — 14 % Modifications	
		First Stiffness Modification	Each Additional Modification	First Stiffness Modification	Each Additional Modification	First Stiffness Modification	Each Additional Modification
Input Data Processing, Generation and Partitioning of Stiffness Matrix	184	16	0	18	0	35	0
Solution	116	78	5	140	7	430	20
Data Recovery and Output of Results	46	46	46	46	46	46	46
Total	345	140	51	204	53	511	66

Table 5 - Comparison of Regular Analysis vs. The Direct Modification Procedure (\*)

Magnitude of The Modified Part	Wing Model 1			Wing Model 2			Wing Model 3			Fuse lage		
	6%	13.9%	21.9%	4%	8%	14%	0.4%	3%	14%	2.2%	5%	14%
Regular Run (%)	100	100	100	100	100	100	100	100	100	100	100	100
First Increment of Stiffness Modifications (%)	30	61	102	35	70	151	5	18	75	31	52	155
Four Additional Increments of Stiffness Modifications (%)	7	14	33	10	18	46	1	4	22	7	12	36

(\*) The comparison is related to the computing time of the displacement vectors ( i.e. the computing time for the stress recovery is not included )





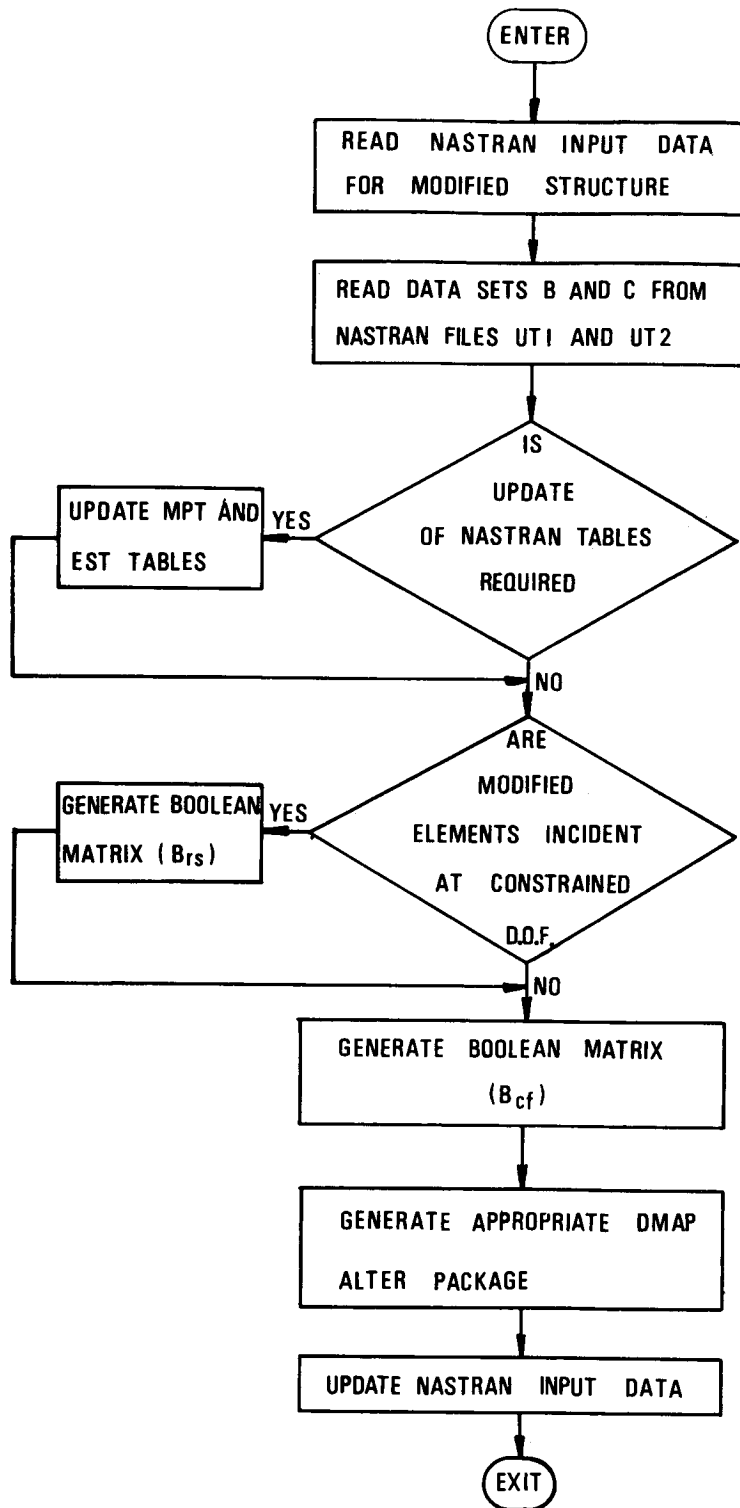


FIGURE 2:FLOW DIAGRAM FOR PROGRAM REAN

(BLOCK 1)

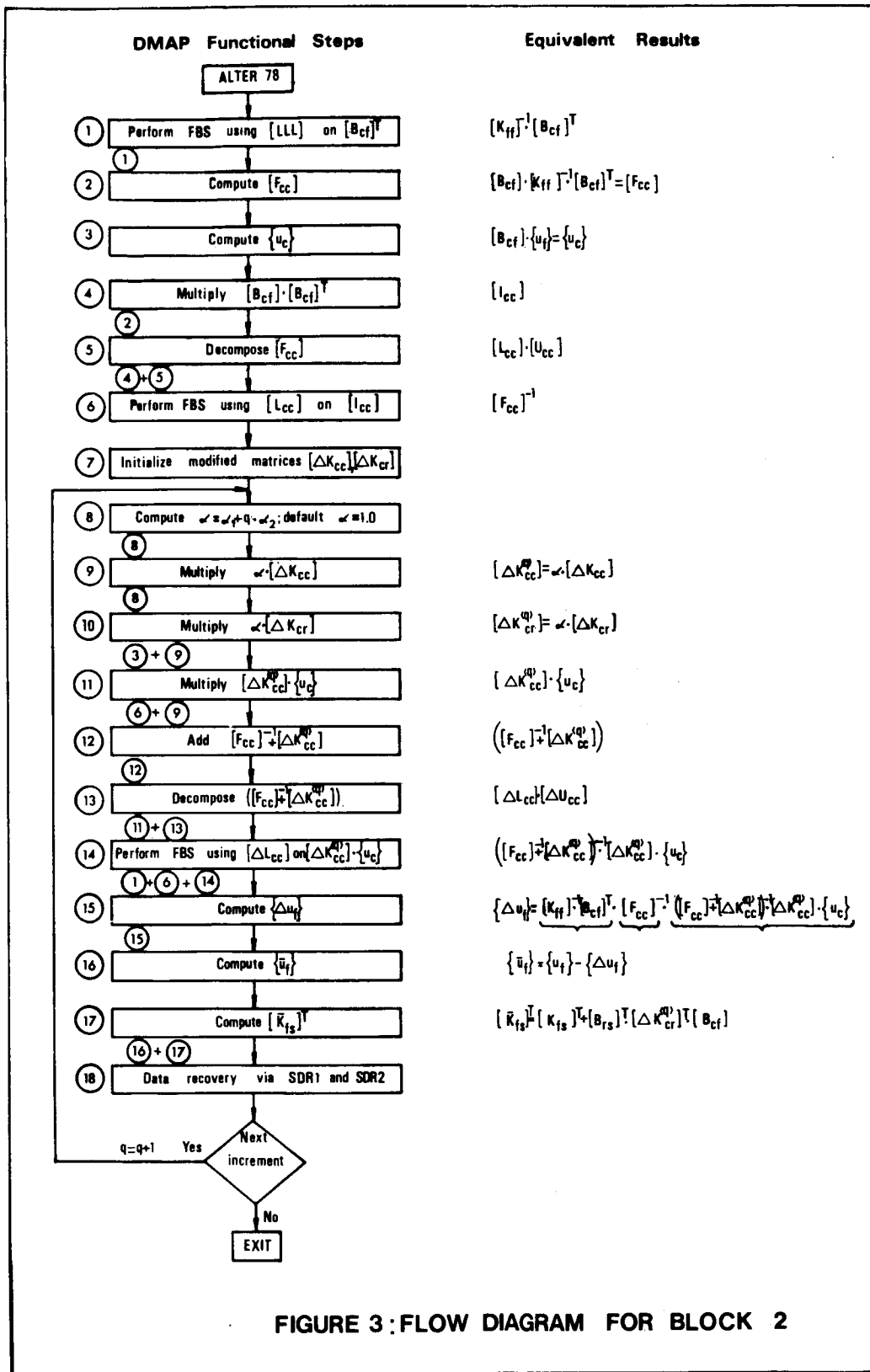


FIGURE 3: FLOW DIAGRAM FOR BLOCK 2

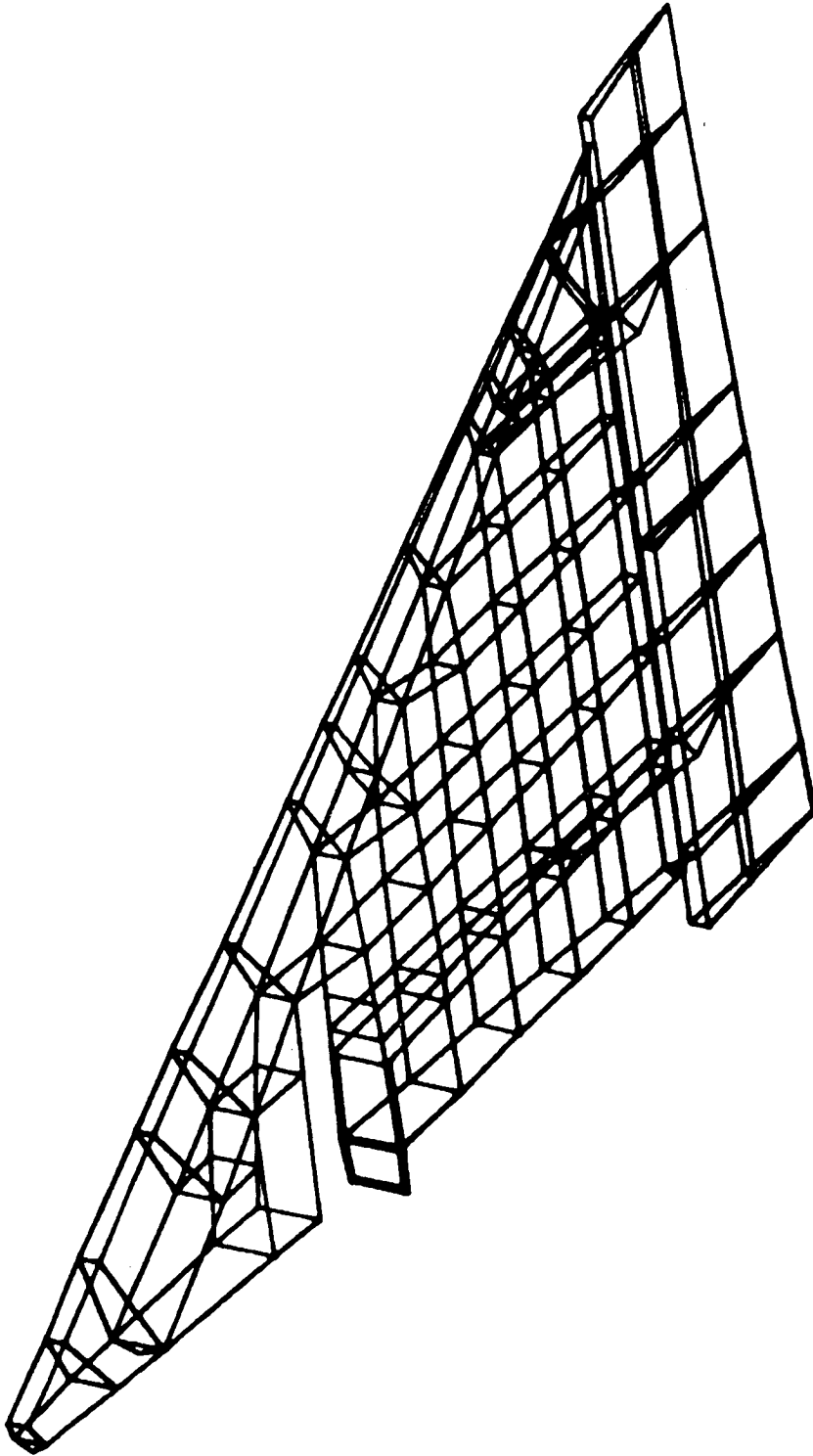


FIGURE 4: IDEALIZATION OF AIRCRAFT WING (MODEL 1)

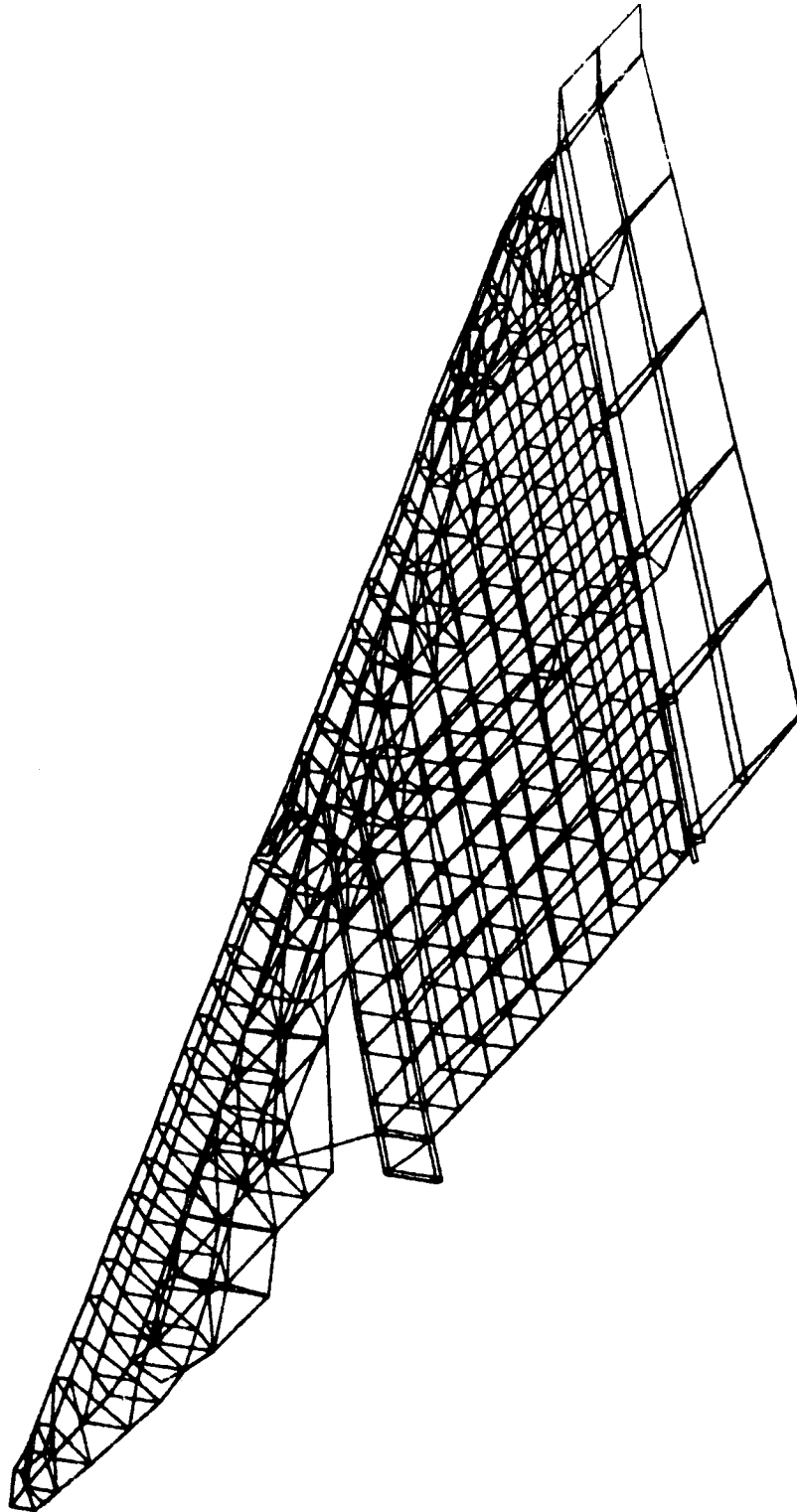


FIGURE 5: IDEALIZATION OF AIRCRAFT WING (MODELS 2 and 3)

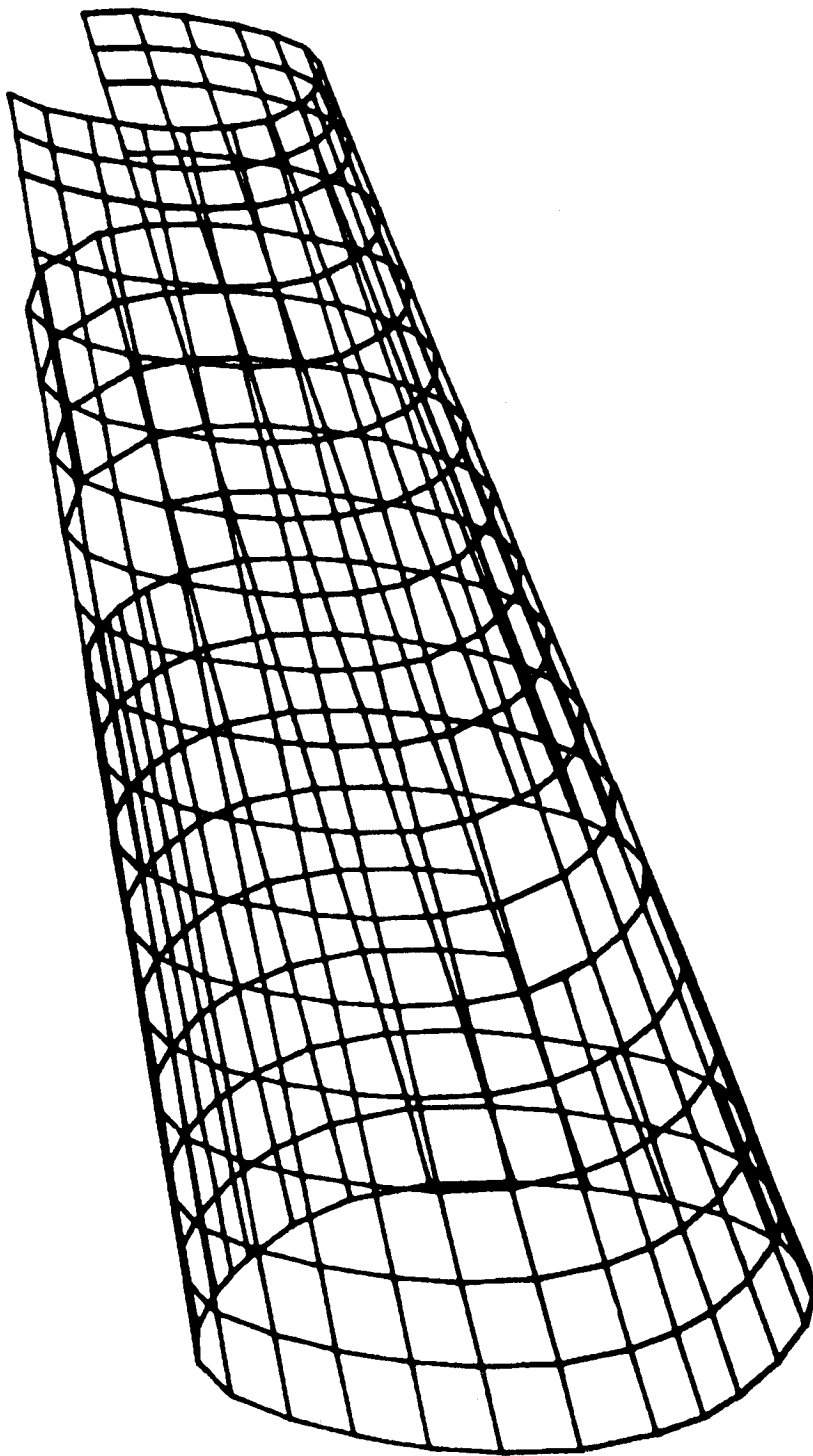


FIGURE 6: IDEALIZATION OF AIRCRAFT FUSELAGE

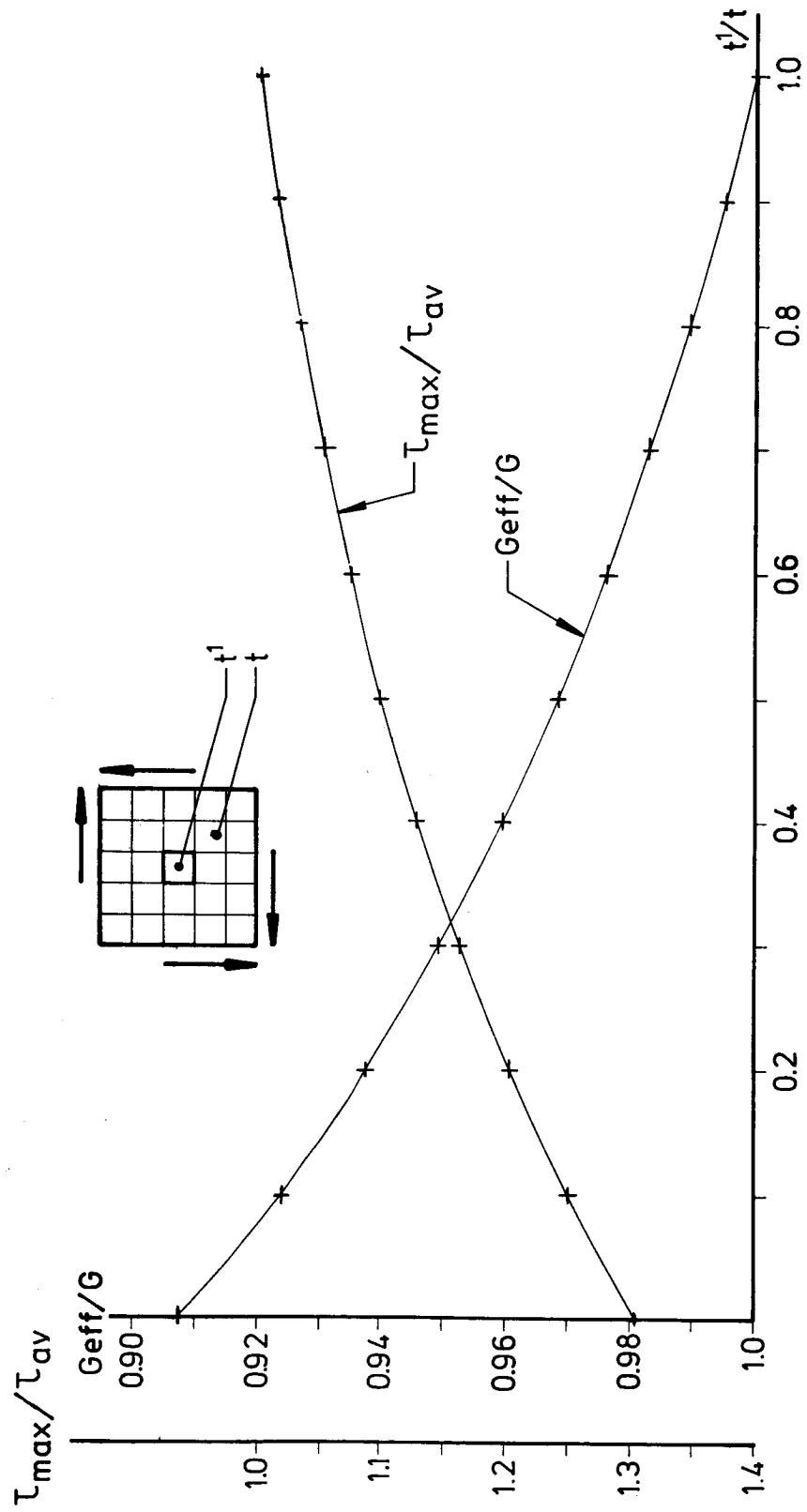


FIGURE 7: PANEL UNDER SHEAR - THE EFFECT OF LOCAL REDUCTION OF THE THICKNESS