A COMPARISON OF THE TWO NASTRAN DIFFERENTIAL STIFFNESS TECHNIQUES

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SUMMARY

NASTRAN contains two techniques to solve the differential stiffness problems. One is incorporated in a new static analysis rigid format and the other is contained in a new normal modes analysis rigid format. The purpose of this paper is to compare the two techniques relative to computational accuracy and time of execution on Level 16.0.

INTRODUCTION

Through Level 15.5, the Static Analysis with Differential Stiffness (Rigid Format 4) capability was based on a one-step process (Reference 1). This process was a static solution to obtain the linear stiffness matrix and internal element forces followed by an element differential stiffness computation. This procedure was based on the assumption that the internal force is a linear multiple of the applied load and that the applied load remains fixed in magnitude and direction, moving with its point of application. The user provided differential stiffness linear load factors. An iterative technique was introduced (Reference 2) and is now fully described in Reference 3.

The new approach to solve the differential stiffness problem is begun with the iteration of the displacements to compute the differential stiffness matrix K^d from

$$[K + K^d (u_i)] \{u_{i+1}\} = \{P\} \tag{1}$$

where u_i and u_{i+1} are the set of displacements at two successive iterations, K is a stiffness matrix, and P is a load vector. Rearranging terms, $[K^d (u_i)]$ is removed from the left hand side and is replaced with the term $[K^d (u_e)]$ to give

$$[K + K^d (u_e)] \{u_{i+1}\} = \{P\} + [K^d (u_e) - K^d (u_i)] \{u_i\} \tag{2}$$

or

$$(K + K^d (u_e)] \{u_{i+1}\} = \{P\} + [K^d (u_e-u_i)] \{u_i\} \tag{3}$$

where u_e is an estimate initially equal to the linear elastic solution. With this technique the internal loads may change due to differential stiffness effects so that the solution is not linearly related to the applied load. Thus equation (3) treats the change in differential stiffness as a load correction.

Three PARAMeters are provided to control the iterative process. The first, BETAD, limits the number of load corrections before adjusting the differential stiffness. The second, NT, limits the cummulative number of iterations. Thus load correction iterations can be performed up to the limit BETAD, at which time the differential stiffness is adjusted, and then more load correction iterations are performed and an adjustment is made to a new differential stiffness until NT is exhausted. The third, EPSIØ, is a convergence criteria which terminates the process when successive iterations of the differential stiffness are sufficiently small. Convergence occurs when ε_i < EPSIØ where

$$\varepsilon_i = \frac{|\{u_{i+1}\}^T \{P_{i+1} - P_i\}|}{|\{u_{i+1}\}^T \{P_i\}|} \tag{4}$$

The user either relies on the default values of BETAD=4, NT=10, and EPSIØ=1.0×10^{-5} or prescribes values through a PARAM bulk data card.

Figure 1 is a simplified flow diagram of the procedure. The requirements of the rigid format are that two subcases be used to define the static output requests and the differential stiffness requests. Loads and constraints are defined above the subcase level and plot requests are last in the Case Control Deck.

A new normal Normal Modes Analysis with Differential Stiffness (Rigid Format 13) capability was described (Reference 2) which combines static, differential stiffness, and normal modes analyses.

Presently, this technique is based on the original differential stiffness approach (Reference 1), but is limited to one loop (or load factor) through the Rigid Format.

Figure 2 is a simplified flow diagram of this process. The rigid format is utilized via three subcases. The first pertains to the static analysis where the load is defined, the second prescribes one load factor for differential stiffness, and the third contains a method for a real eigenvalue analysis. Individual output requests can be made at the subcase level and plot requests are last in the Case Control deck.

TEST CASE

The test case used is the standard NASTRAN Demonstration Problem for Rigid Format 4. The structure is a hanging cable acted upon by its own weight, which is an equilibrium position, assumes the shape of a catenary. The original shape of the cable is circular. The final shape of the cable is readily predictable from equations developed in Reference 5.

The coordinates of a point (x,y) on the catenary are defined by

$$x = \frac{H}{w} \sinh^{-1} \left(\frac{ws}{H}\right) \quad , \tag{5}$$

and

$$y = \frac{H}{w} \left[\left(1 + \left(\frac{ws}{H}\right)^2\right)^{1/2} - 1\right] \quad , \tag{6}$$

where H is the tension at the bottom of the catenary, w is the weight per unit length of the cable, and s is the distance along the curve.

The original demonstration problem is in English units and are converted to the Newton-meter system for this discussion. The input data decks are shown in Tables 1 through 5. Notice the alter for Rigid Format 13 necessary to allow multiple load coefficients and plots for the original differential stiffness technique.

RESULTS

Tables 6 and 7 show the deflection results obtained by the two techniques compared to the theoretical expectations. The results computed by Rigid Format 4 were obtained in four iterations when the convergence criteria changed from 4.5×10^{-5} to 4.3×10^{-6}. On the CDC 6600, Functional Module, DSCHK, which performs differential stiffness computations in Rigid Format 4, spent about 2 cpu seconds. Functional Module DSMG1 (used four times) consumed approximately 5 cpu seconds. The results shown for Rigid Format 13 were those obtained after the first load coefficient. (Successive coefficients produced deteriorating answers.) Using Rigid Format 13, Functional Module, DSMG1, used 1.25 cpu seconds and Functional Module, DSMG2, used about one-third of a second.

Figure 3 shows the graphical results of this test.

CONCLUSIONS

For a simple structural element case that can be readily verified, there is no appreciable difference in the results computed. In fact, Rigid Format 13, when using only one differential stiffness coefficient, actually computes the nonlinear solution faster than the Rigid Format 4 counterpart. Thus, the two differential stiffness techniques available in NASTRAN can be utilized equally well depending upon the user's preference.

REFERENCES

1. The NASTRAN Theoretical Manual, NASA SP-221(01), December, 1972, Section 7.1.

2. McDonough, John R., "A Survey of NASTRAN Improvements Since Level 15.5", NASTRAN Users' Experiences, NASA TM X-3278, September, 1975, pp. 11-22.

3. The NASTRAN Theoretical Manual, NASA SP-221(03), March, 1976, Section 7.1.

4. The NASTRAN Demonstration Problem Manual, NASA SP-224(03), March, 1976, Section 4.

5. Spiegel, Murray R., Applied Differential Equations, Prentice-Hall, Inc., 1958, pp. 105-108.

Table 1. Executive Control Deck for Rigid Format 4.

```
ID        DIFFSTIF,RF4
APP       DISPLACEMENT
SOL       4,0
TIME      10
CEND
```

Table 2. Executive Control Deck for Rigid Format 13.

```
ID        DIFFSTIF,RF13
$ REQUIRED ALTER TO CORRECT AN ERROR IN THE RIGID FORMAT
ALTER 29 $
PARAM     //C,N,ADD/V,N,NOMGG/C,N,1/C,N,0 $
$ ALTERS TO CHANGE THE RIGID FORMAT FOR MULTIPLE D. S. FACTORS
ALTER    139 $
JUMP     DSLOOP $
LABEL     DSLOOP $
ALTER     155 $
COND     LBL8D,REPEATD $
REPT     DSLOOP,10 $
PARAM     //C,N,NOT/V,N,TEST/V,N,REPEAT $
LABEL     LBL8D $
ALTER     158, 174 $
ALTER     176, 176 $
PLOT       PLTPAR,GPSETS,ELSETS,CASECC,BGPDT,EQEXIN,SIL,,PUBGV1,GPECT,
           OESB1/PLOTX3/V,N,NSIL/V,N,LUSET/V,N,JUMPPLOT/V,N,PLTFLG/
           V,N,PFILE $
ALTER     187, 188 $
ENDALTER $
APP       DISPLACEMENT
SOL       13,0
TIME      10
CEND
```

Table 3. Case Control Deck for Rigid Format 4.

```
TITLE = DIFFERENTIAL STIFFNESS ANALYSIS FOR A HANGING CABLE
SUBTITLE = RIGID FORMAT 4 SOLUTION
LABEL = INITIAL SHAPE IS A CIRCLE, FINAL SHAPE IS A CATENARY
LOAD = 32
SPC = 2
DISPLACEMENT = ALL
SPCFORCE = ALL
STRESS = ALL
FORCE = ALL
OLOAD = ALL
SUBCASE 1
LABEL = LINEAR SOLUTION
SUBCASE 2
LABEL = NONLINEAR SOLUTION
BEGIN BULK
```

Table 4. Case Control Deck for Rigid Format 13.

```
TITLE = DIFFERENTIAL STIFFNESS ANALYSIS FOR A HANGING CABLE
SUBTITLE = RIGID FORMAT 13 SOLUTION
LABEL = INITIAL SHAPE IS A CIRCLE, FINAL SHAPE IS A CATENARY
SPC = 2
DISPLACEMENT = ALL
SPCFORCE = ALL
STRESS = ALL
FORCE = ALL
SUBCASE 1
LABEL = LINEAR SOLUTION
LOAD = 32
OLOAD = ALL
SUBCASE 2
LABEL = NONLINEAR SOLUTION
DSCOEFFICIENT = 50
BEGIN BULK
```

Table 5. Bulk Data Deck

```
$ GEOMETRY IN CYLINDRICAL SYSTEM MEASURED IN METERS
$
CORD2C  10      0       0.0     0.0     0.0     0.0     0.0     1.0     +COORD
+COORD  1.0     0.0     0.0
GRID    10              3.048   0.0                                     TOP
GRID    11              3.048   10.0
GRID    12              3.048   20.0
GRID    13              3.048   30.0
GRID    14              3.048   40.0
GRID    15              3.048   50.0
GRID    16              3.048   60.0
GRID    17              3.048   70.0
GRID    18              3.048   80.0
GRID    19              3.048   90.0                                    BOTTOM
$
$ CONNECTIONS VIA BARS
$
BAROR                                   -1.2    1.0     0.0     1
CBAR    10      10      10      11                                      TOP
CBAR    11      10      11      12
CBAR    12      10      12      13
CBAR    13      10      13      14
CBAR    14      10      14      15
CBAR    15      10      15      16
CBAR    16      10      16      17
CBAR    17      10      17      18
CBAR    18      10      18      19                                      BOTTOM
$
$ GRAVITY LOAD FOR NEWTON-METER SYSTEM
$
GRAV    32      0       9.8     0.0     1.0     0.0
$
$ CONSTRAINTS
GRDSET                                          0       345
SPC     2       10      12      0.0     19      1       0.0
$
$ MATERIAL AND PROPERTY DEFINITIONS
$
MAT1    1       2.63+7          .3      1.78-2
PBAR    10      1       9.29-3  6.87-6  6.87-6
$
$ DIFFERENTIAL STIFFNESS PARAMETERS FOR RF4
$
PARAM   BETAD   8                                                       RF4
PARAM   EPSIO   1.0-5                                                   RF4
PARAM   NT      18                                                      RF4
$
$ DIFFERENTIAL STIFFNESS COEFFICIENTS FOR RF13
$                                                                       RF13
DSFACT  50      1.0     2.0     3.0     4.0     5.0
$
ENDDATA
```

Table 6. Horizontal Deflections

Grid Point	s	Ux - Horizontal		
		Theory	NASTRAN RF4	NASTRAN RF13
11 (10°)	4.25	-0.1480	-0.1445	-0.1441
13 (30°)	3.19	-0.2452	-0.2337	-0.2325
15 (50°)	2.12	-0.1577	-0.1406	-0.1394
17 (70°)	1.06	-0.0338	-0.0267	-0.0264
19 (90°)	0.0	0.0	0.0	0.0

Table 7. Vertical Deflections

Grid Point	s	Uy - Vertical		
		Theory	NASTRAN RF4	NASTRAN RF13
11 (10°)	4.25	-0.00341	-0.01245	-0.01241
13 (30°)	3.19	-0.00696	-0.03865	-0.03432
15 (50°)	2.12	0.000914	0.04478	0.09006
17 (70°)	1.06	0.1737	0.2430	0.2418
19 (90°)	0.0	0.2846	0.3707	0.3679

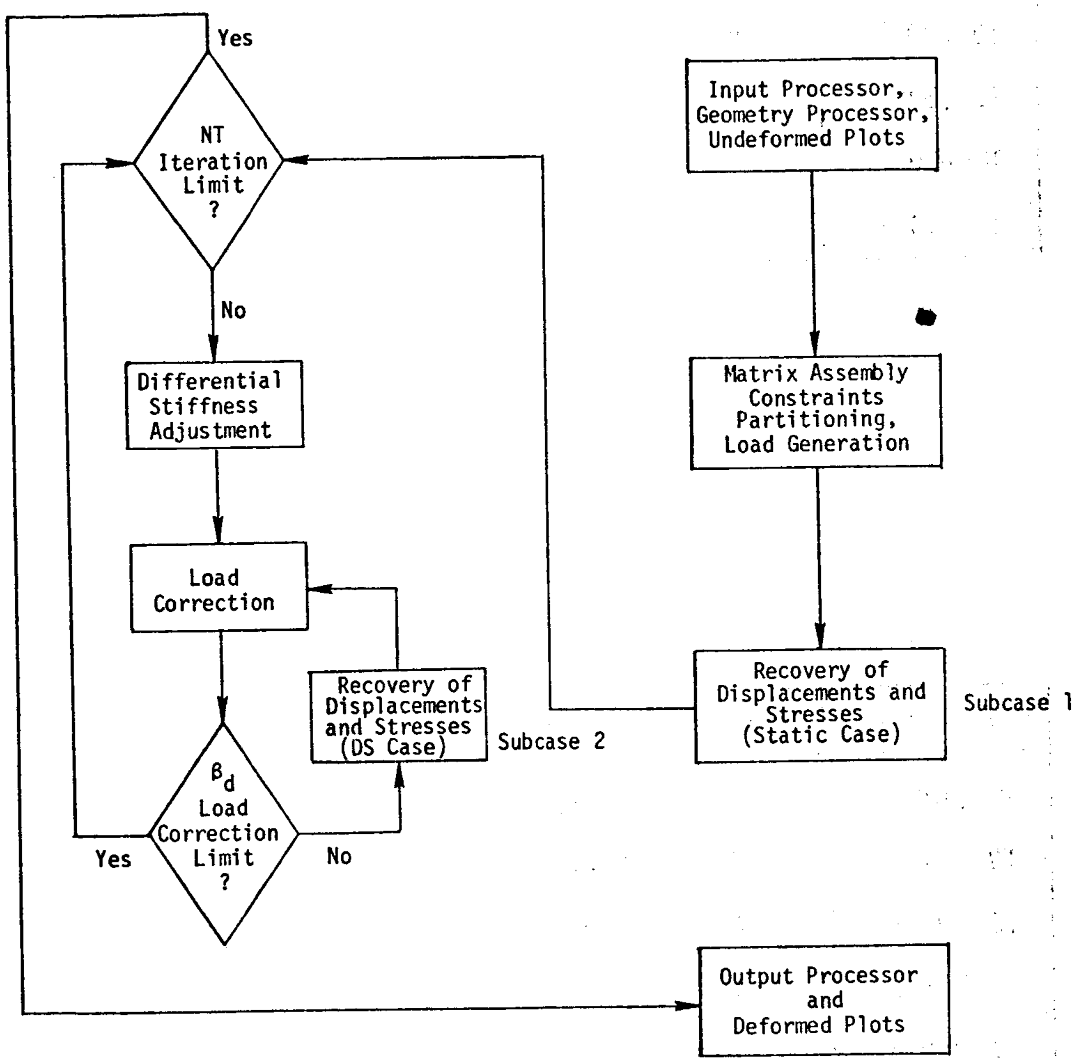

Figure 1. Simplified flow diagram of Rigid Format 4 procedure.

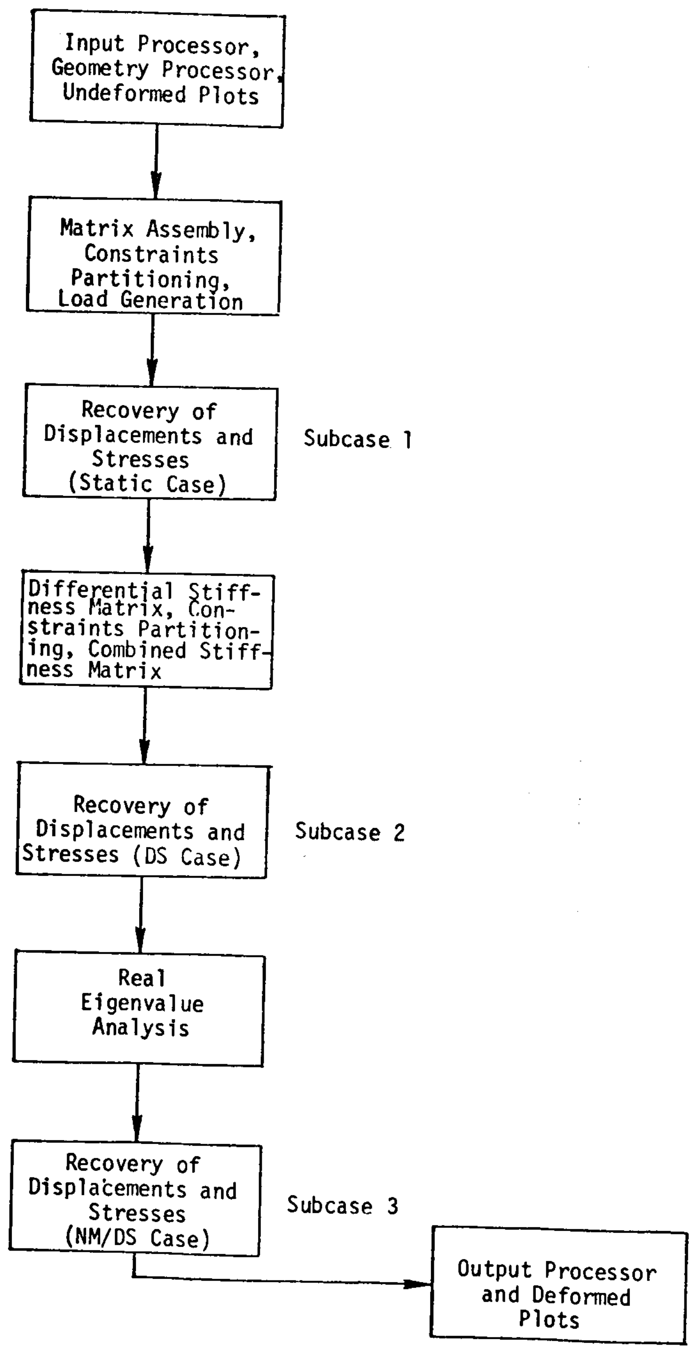

Figure 2. Simplified flow diagram of Rigid Format 13 procedure.

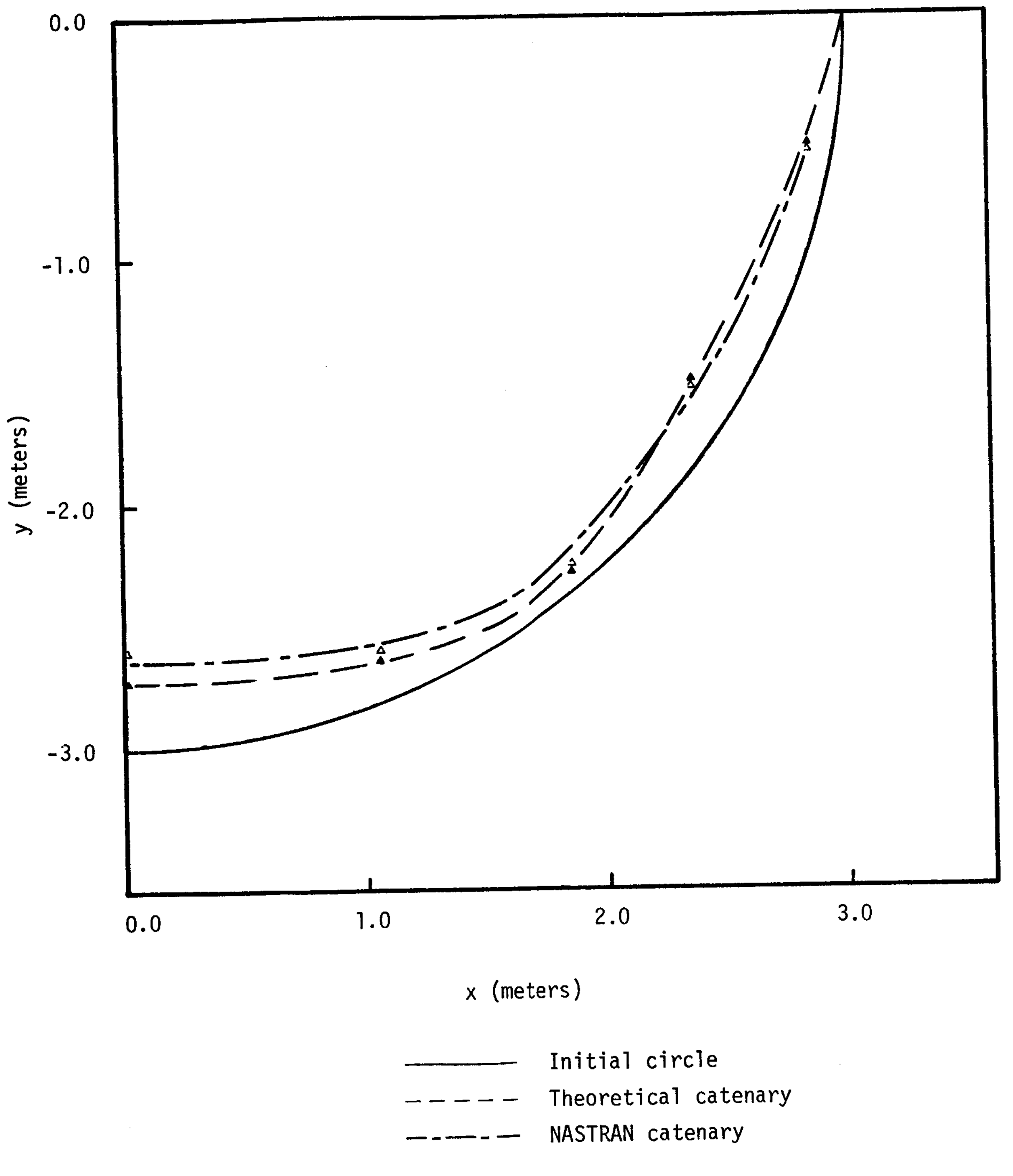

Figure 3. Hanging Cable Test.