

A SUMMARY OF NASTRAN FLUID/STRUCTURE
INTERACTION CAPABILITIES

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SUMMARY

A summary of fluid/structure interaction capabilities for the NASTRAN computer program is presented. The paper concentrates on indirect applications of the program towards solving this class of problem; for completeness and comparative purposes, direct usage of NASTRAN will be briefly discussed. The solution technology addresses both steady state and transient dynamic response problems.

INTRODUCTION

A substantial amount of activity is in progress in the general area of applying the NASTRAN computer program to fluid/structure interaction problems. The class of problems under consideration is limited to linear elastic structures in contact with a fluid. The fluid constitutive equation is represented by an acoustical type small deformation relation wherein pressure is proportional to the divergence of the displacement field (per cent change in elemental volume). The time domain character of the problems treated are either transient (usually incident step pressure waves with an exponential decay wave form) or steady state (acoustical induced response resulting from a harmonic train of incident or radiating pressure waves).

The direct use of NASTRAN to solve problems in the category described above is documented in the NASTRAN program manuals, therefore the paper will only briefly mention direct usage for completeness and comparative purposes. Instead, the paper concentrates on nonstandard fluid/structure applications of NASTRAN that range anywhere from employing the program directly (through an analogy argument) to using the program capability indirectly (in conjunction with auxiliary post-processing programs).

Special attention is given to the case where an elastic structure is completely submerged in a limitless fluid domain. Five methods are presented for handling the modeling problem of having to represent an infinite fluid region with only a finite number of elements. Methodology is covered that enables one to either eliminate the need for any fluid elements at all (through the proper handling of the fluid/structure interface) or requires one to only model a tractable finite number of fluid elements. Two of the five methods cover transient problems and the remaining ones are for steady state problems. Exact versus finite element solutions are presented for most of the methods covered in the paper.

STEADY STATE--UNCOUPLED FLUID FIELD

This class of problems treats the case where a totally submerged elastic structure is interacting with an infinite fluid domain. The structure is represented by finite elements whereas fluid field is represented by a continuum. This category of problem is not handled by any of the rigid formats currently in NASTRAN. A special post-processing program called FIST (reference 4) was written that will accept basic mode shape information directly from the NASTRAN rigid format output tape. FIST is designed to process this information into the desired solution for the complete fluid/structure interaction response.

The method centers about the process of obtaining a relation between the fluid/structure interface fluid pressure and the interface normal velocity. Once this relationship is determined, the uncoupling process unfolds. The starting point for formulations of this type (references 6-9) is the Helmholtz integral, reference 5, where for any point, \bar{x} , on the closed submerged surface, S , which interfaces with the fluid, the total pressure $p(\bar{x})$ on the surface is related to the normal velocity, w , on the surface by the integral relation

$$p(\bar{x}) = p^i(\bar{x}) - 2 \int_S p(\bar{y}) \frac{\partial G(\bar{x}, \bar{y})}{\partial n(\bar{y})} dS(\bar{y}) + 2i\omega\rho \int_S w(\bar{y}) G(\bar{x}, \bar{y}) dS(\bar{y}) \quad (4)$$

where \bar{y} is a dummy variable for any position $\bar{x} \in S$, ρ the mass density of the fluid and G is the free space Green's function given by

$$G(\bar{x}, \bar{y}) = \frac{\exp(-i\omega|\bar{x}-\bar{y}|/c)}{4\pi|\bar{x}-\bar{y}|} \quad (5)$$

The development to follow in this subsection on harmonic analysis follows reference 2 (modified for incident pressure by the method given in reference 3) for the first part of this subsection on a direct solution to the problem and follows reference 4 for the modal solution to the problem. The partial derivative of G with respect to $n(\bar{y})$ denotes the rate of change of G in the direction normal to the surface at point \bar{y} , and $|\bar{x}-\bar{y}|$ denotes the distance between the \bar{x} and \bar{y} points.

The next step is to obtain a discrete version of equation (4) which is accomplished by representing the surface pressure and normal velocity in terms of a linear combination of scalar basis functions ψ_i defined as

$$p(\bar{x}) = \sum_{n=1}^N p_n \psi_n(\bar{x}) \quad (6)$$

$$w(\bar{x}) = \sum_{n=1}^N w_n \psi_n(\bar{x})$$

where N denotes the number of surface grid points in contact with the fluid. For example, reference 4 has used a cubic spacial distribution con-

sistent with the finite element displacement fields for the neighboring structural elements. This is in contrast to reference 10 which employs a piecewise constant distribution of pressure over the interface zones of the structure or to reference 11 which employs a quadratic distribution. Employing a higher order basis function has the advantage that the same solution accuracy can be achieved with a coarser interface mesh. This fact ultimately results in a cost-effective computer program that should run more efficiently, while maintaining the same accuracy, when employing the higher order distribution basis functions.

Upon substituting equations (6) into equation (4) and evaluating equation 4 over a discrete set of points (x_j , $j = 1, 2, J$) corresponding to the fluid structure interface node points, one obtains

$$[L]\{P\} = [R]\{W^I\} + \{P^I\} \quad (7)$$

where $[L]$ and $[R]$ are $J \times J$ matrices and $\{P^I\}$ is a known $J \times 1$ column vector, and $\{P\}$ is a column vector of discrete pressure values $p(\bar{x})$; these matrices result from the evaluation of equation (4).

Assuming for the moment that the driving frequency, ω , is not at (or very near) certain characteristic wave numbers of the fluid field enclosing the structure, equation (7) can be solved for $\{P\}$, thus

$$\{P\} = [Z(\omega)]\{W\} + [L]^{-1}\{P^I\} \quad (8)$$

where $[Z(\omega)] \equiv [L]^{-1}[R]$.

Reference 13 has presented a method for arriving at equation (8) even in situations where ω is at or near one of the characteristic cavity resonance frequencies.* Briefly stated, the improved method consists of determining the unique surface pressure, $p(\bar{x})$, that simultaneously satisfied the surface Helmholtz integral equation (4) and the interior Helmholtz integral reference 14. The interior Helmholtz integral is a relation similar to the form of equation (4) except it relates the fact that the fluid pressure for all points in the region of space occupied by the structure is zero. Enforcing this interior Helmholtz integral over a judiciously selected set of M interior points leads to a matrix analogous to equation (7) in the form

$$[L^I]\{P\} = [R^I]\{W\} \quad (9)$$

Thus, equations (7) and (9) result in a set of $(J+M)$ equations for the J unknowns $\{P\}$. Solving the overdetermined set of equations specified by equations (7) and (9) in a least square sense leads to an equation in the same form as equation (8) except that $Z(\omega)$ is determined in a more involved manner.

Next, by employing the principal of virtual work, the total surface pressures can be related to a set of consistent interaction nodal forces, $\{F\}$, thus

$$\{F\} = [C^I]\{P\} \quad (10)$$

* This is sometimes referred to as the cavity resonance problem.

For harmonic steady state problems, the continuous velocity, $w(t)$, and displacement, $u(t)$, amplitudes are related by $w(t) = i\omega u(t)$. Making use of this relation in conjunction with the surface geometry relating normal components of motion into the Cartesian components employed in equation (1) results in the expression

$$\{W\} = i\omega[S]\{U\} \quad (11)$$

Thus, combining equations (10), (8), and (11) leads to

$$\{F\} = [T]\{U\} + [C^I][L]^{-1}\{P^i\} \quad (12)$$

where $[T] \equiv i\omega[C^I][Z][S]$ is typically a fully populated matrix that relates the interaction forces to the boundary displacement field.

For steady state harmonic motion, all response quantities are proportional to $\exp(+i\omega t)$. Thus, $\{U\} = \{U\}^\circ \exp(i\omega t)$, $\{F\} = \{F\}^\circ \exp(i\omega t)$ and $\{F_E\} = \{F_E\}^\circ \exp(i\omega t)$ and the corresponding equation of motion for the structure (equation (1)) becomes

$$(-\omega^2[M] + i\omega[C] + [K])\{U\}^\circ = -\{F\}^\circ + \{F_E\}^\circ \quad (13)$$

where $e^{i\omega t}$ has been canceled out on both sides of the equation. Thus, substituting equation (12) into equation (13) results in the relation

$$[V]\{U\}^\circ = \{F_A\}^\circ \quad (14)$$

where $[V] \equiv -\omega^2[M] + i\omega[C] + [K] + [T]$ and $\{F_A\}^\circ = \{F_E\}^\circ - [C^I][L]^{-1}\{P^i\}$.

It is to be noted that equation (13) contains matrices that are the size of the entire structure whereas the matrix size in equation (12) is only a size corresponding to the nodes in contact with the fluid. Thus, when substituting equation (12) into equation (13), allowances must be made in filling out the $[T]$ and product matrix $[C^I][L]^{-1}$ with zeros in the appropriate place to account for the matrix size mismatch.

Formally, one may now state the solution to the interaction problem as finding the inverse of the highly populated $[V]$ matrix. Thus,

$$\{U\}^\circ = [V]^{-1}\{F_A\}^\circ \quad (15)$$

Once $\{U\}^\circ$ is determined all other response quantities can be routinely computed. Substituting the solution $\{U\}^\circ$ into equation (6) and equation (11) and then equation (11) into equation (8) provides the total pressure, $\{P\}$, at the interface. Then substituting the surface pressure and surface velocity into the exterior form of the Helmholtz integral, reference 3, the pressure in any far field point in the media can easily be computed. Premultiplying the surface motion, $\{U\}^\circ$, by the individual (unassembled) stiffness matrix for each element produces the individual structural nodal forces which in turn can be converted to element stresses.

For large size problems, the nonsymmetry and highly populated form of the complex $[V]$ matrix makes its inversion somewhat of a problem when $[V]$ is large. In some situations, $[V]$ is ill-conditioned for certain frequency ranges due to the presence of large size $[K]$ terms in the $[V]$ expression in comparison to the rest of the terms comprising $[V]$. To get around these problems, an alternate modal analysis approach is sometimes taken, references 4 and 15.

For the modal approach, let $[\phi]$ be the $N \times M$ matrix of M undamped, in vacuo modes of the structural vibrations having N degrees of freedom, thus

$$[\phi] = [\{\psi_1\}, \{\psi_2\}, \dots, \{\psi_M\}] \quad (16)$$

where $\{\psi_m(\bar{x})\}$ is the m^{th} mode column vector which is normalized to the $M \times M$ unit identity matrix $[I]$ such that

$$[\phi]^T [M] [\phi] = [I] \quad (17)$$

The modes $[\phi]$ have the property that

$$[\phi]^T [K] [\phi] = [\lambda] \quad (18)$$

where $[\lambda]$ is a $M \times M$ diagonal eigenvalue matrix whose non-zero elements are the squares of the natural frequencies (rad/sec) of the structure. The displacement field can be expressed in terms of the modes by the relation

$$\{U\}^\circ = [\phi] \{Q\}^\circ \quad (19)$$

Next, upon substituting equation (19) into equation (14) and premultiplying the result by $[\phi]^T$, one obtains

$$[\phi]^T [V] [\phi] \{Q\}^\circ = [\phi]^T \{F_A\}^\circ \quad (20)$$

which can be rewritten in short notation as

$$[\bar{V}] \{Q\}^\circ = \{F_G\}^\circ \quad (21)$$

where $[\bar{V}] \equiv -\omega^2 [I] + [\lambda] + [\phi]^T (i\omega [C] + [T]) [\phi]$ (22)

and $\{F_G\}^\circ \equiv [\phi]^T \{F_A\}^\circ$ (23)

Generally, the $M \times M$ $[\bar{V}]$ matrix is complex, nonsymmetric and only under special situations is the $[\bar{V}]$ matrix fully diagonal (note only the first two contributions to equation (22) are diagonal). When $[\bar{V}]$ is fully diagonal, its inversion is trivial, however, the general case must usually be considered where one is faced with the inversion of the $[\bar{V}]$ matrix in order to solve the system of equations defined by equation (21). Formally, then, the solution to the fluid structure interaction problem can be written as

$$\{Q\}^\circ = [\bar{V}]^{-1} \{F_G\}^\circ \quad (24)$$

where we have traded having to invert a $N \times N$ $[V]$ matrix in the direct approach for having to invert $M \times M$ $[\bar{V}]$ matrix in the modal approach. Strictly speaking, there is one mode shape for each degree of freedom, consequently, if M is set equal to N , one is right back where one started in being faced with the inversion of a $N \times N$ complex matrix. However, one can usually judiciously relate the important modes of vibrations based on certain symmetries of loading or based on the customary omission of the higher modes of vibration. After the selection process, one is usually left with a $[\bar{V}]$ matrix that is substantially smaller in size than the original $[V]$ matrix encountered in the direct approach.

The previous development is for a general shaped submerged structure. In the special case where the body has an axis of revolution, it is possible to use a Fourier series decomposition in the angular variable of a cylindrical coordinate system centered about the axis of revolution. Thus, one can describe an arbitrary pressure (or velocity) distribution through the relations

$$\begin{aligned}
 p(\bar{x}, \theta) &= \sum_{n=0}^{\infty} p_n(\bar{x}) \text{Cos}n\theta + \sum_{n=1}^{\infty} \tilde{p}(\bar{x}) \text{Sinn}\theta \\
 w(\bar{x}, \theta) &= \sum_{n=0}^{\infty} w_n(\bar{x}) \text{Cos}n\theta + \sum_{n=1}^{\infty} \tilde{w}_n(\bar{x}) \text{Sinn}\theta
 \end{aligned}
 \tag{25}$$

Applying such an expansion to the development just presented for the general three-dimensional case results in problem formulation analogous to equation (14) or to equation (21). The main difference is that the coefficient matrix $[V(n)]$ in equation (15) (or corresponding $[\bar{V}(n)]$ matrix in equation (21)) is now a function of the wave number n corresponding to the expansions in equation (25). Consequently, solving the full three-dimensional problem is equivalent to solving a sequence of $n = 1, 2, \dots, \bar{N}$ smaller sets of linear equations (i.e. equations (14) or (21)). The phrase "smaller sets of linear equations" is used since the elimination of the third spacial dimension (through the introduction of the Fourier expansion) substantially reduces the size of the coefficient matrix $[V]$. From a computational point of view, it is usually more efficient to solve, say, six ($\bar{N}=6$) two-dimensional size problems than one large three-dimensional one. The details of setting up the actual $[V(n)]$ array, for cubic polynomial displacement fields, is presented in more detail in reference 4. Consequently, it will not be repeated here.

The interface of this solution technique with the NASTRAN computer program can be made in one of two ways where the selected approach depends on whether the direct or modal solution technique is used to solve the problem. In the case of the modal formulation, a computer program called FIST (Fluids Interacting with Structures) has been written which directly accepts, as input from NASTRAN, the structural mode shapes for the in vacuo normal modes or mass and stiffness matrices. With the addition of a few simple alter cards in the NASTRAN run stream, the modes are written on tape from NASTRAN using the module OUTPUT 2. For example, in rigid format 3, adding the cards

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ALTER 84
OUTPUT2 KAA,MAA,,,//C,N,1/C,N,0 $
ALTER 94
OUTPUT2 PHIA,MI,,,//C,N,1/C,N,0 $
ALTER 96
OUTPUT2 PHIG,,,,//C,N,1/C,N,0 $
ENDALTER

```

in the EXECUTIVE control deck is all that is needed to have the information needed by FIST to solve the fluid/structure interaction problem. The current version of FIST is currently limited to axisymmetric structures subject to non-axisymmetric loadings.

In the case of the direct formulation, FIST has an option which permits one to solve the system of linear complex simultaneous equations (14). However, the current version is limited to a 40 x 40 nonsymmetric, fully populated matrix. In situations where the [V] matrix is large, the NASTRAN mathematical solution routines can be utilized to solve the problem at hand. The frequency response rigid format number 8 solves the following problem

$$[M]\omega^2 + i\omega[B] + [K]\{X\} = \{P\} \quad (26)$$

for the displacement amplitude {X}, where the multiplying matrix on {X} can be, in general, nonsymmetric and complex. The complex loading vector, {P}, can be introduced into NASTRAN through the RLOAD1 bulk data card and the [M], [B], and [K] arrays need not be computed by NASTRAN but rather can be directly inserted, element by element, through a DMIG card. Since no structure is given to NASTRAN directly, all the [M], [B] and [K] matrices are zero, except for the direct input components (denoted by $[M_{dd}^2]$, $[B_{dd}^2]$, and $[K_{dd}^2]$ in the NASTRAN theoretical manual, pg. 9.3-7). The [M] and [B] arrays are zero by virtue of not defining them in any way. Thus, there remains the $[K_{dd}^2]$ array which is read in (in complex form) via the DMIG card. NASTRAN will proceed in the usual manner for the direct frequency response solution and compute the {X} solution which, of course, corresponds to the desired result {U}^o (i.e. equation (15)).

By employing alter instructions, one who is familiar with DMAP can most likely perform the desired operations in a more direct fashion. The advantage of the dummy stiffness application method is that it can all be done within the current fixed format of the program.

As an illustration of the solution technique, consider the situation where a steel submerged thin wall spherical shell is harmonically driven by a point concentrated force. The shell has a radius of 2.54 cm, wall thickness of 0.127 cm and a nondimensional driving frequency of KA = 0.4 (K ≡ ω/c where ω is the driving frequency in rad/sec and c is dilatational wave speed in the water). The exact solution to this problem is shown in figure 1 by the solid line (reference 12) and is processed using the first 50 axisymmetric modes of a thin sphere. The fine dashed line corresponds to a FIST solution to the problem (employing a cubic distribution pressure variation) which uses the direct solution approach (equation (15)) with the sphere subdivided into eight segments. The coarse dashed line corresponds to a modal solution (equation (24))

that employs three mode shapes and an eighteen segment subdivision. Both FIST solutions employ too coarse of a mesh to predict an accurate response near the point load. Agreement away from the point load is seen to be very good.

STEADY STATE-COUPLED FLUID FIELD

Again, the case of a totally submerged elastic structure is treated, except that in this case the structure as well as the fluid is modeled with finite elements. The first approach, of the two presented in this subsection, is the situation where displacement (or "mock") fluid elements is employed in the problem formulation, references 17, 18. In this situation, we start with the three-dimensional elasticity equations

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} \quad i = 1, 2, 3 \quad (27)$$

$$\text{with } \sigma_{ij} = \lambda \delta_{ij} \frac{\partial u_k}{\partial x_k} + \mu \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] \quad (\delta_{ij} \text{ is the Kronecker delta}) \quad (27a)$$

We start by letting the Lamé constant $\mu \rightarrow 0$; the Lamé constant $\lambda \rightarrow k$; and noting that the reduced equation (27a) now implies that

$$\sigma_{11} = \sigma_{22} = \sigma_{33} \quad (28)$$

$$\text{and } \sigma_{12} = \sigma_{31} = \sigma_{32} = \sigma_{21} = \sigma_{13} = \sigma_{23} = 0.$$

Finally, upon defining $P \equiv -\sigma_{11} = -\sigma_{22} = -\sigma_{33}$, it is seen that the reduced equations (27) and (27a) represent the same field equations as those defined by equations (2, 2a).

Consequently, any solid elements in NASTRAN that are built from working with equations (27, 27a) can be converted into mock fluid elements by appropriately refining the constants in the elasticity stress-strain law

$$\{\sigma\} = [G]\{\epsilon\}$$

For three-dimensional type elements like brick and ring elements, the array of elastic constants for an isotropic material can be written as

$$[G] = \begin{bmatrix} (\lambda+2\mu) & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & (\lambda+2\mu) & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & (\lambda+2\mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \quad (29)$$

Similarly, for two-dimensional solid elements, like membrane elements, the array of elastic constants (corresponding to $\{\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\}$) can be written as

$$[G] = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \quad (30)$$

For the NASTRAN program installation of the mock elements for 2-D membrane solid elements is achieved through a MAT2 card wherein

$$G_{11} = G_{12} = G_{21} = G_{22} = k \equiv \text{fluid bulk modulus}$$

and the remaining $G_{13} = G_{23} = G_{31} = G_{32} = G_{33} = 0.0$.

For employing three-dimensional mock elements, the situation would be straightforward if the [G] matrix were allowed to be input in a general form like the two-dimensional case; or, if the stress-strain matrix were written in terms of Lamé constants rather than in terms of the more common modulus of elasticity, E, and Poisson's ratio, ν . For the latter case, one need only set $\mu = 0$ and $\lambda = k$ and the desired mock element could be formed. In actuality, the NASTRAN [G] array accepts input in the form* of E and ν and internal to the program, the elements of the [G] array are defined as follows

$$\begin{aligned} G_{11} &= G_{22} = G_{33} = E(1-\nu)/[(1-2\nu)(1+\nu)] \\ G_{12} &= G_{21} = G_{13} = G_{31} = G_{23} = G_{32} = E\nu/[(1-2\nu)(1+\nu)] \\ G_{44} &= G_{55} = G_{66} = .5E/(1+\nu) \end{aligned} \quad (31)$$

The problem can be resolved by rewriting a small portion of the Fortran coding that fills out the material constants array in the desired form.

An alternate procedure is to adjust the values of E, ν (or G) on a MAT1 card so that $\lambda = k$ and $\mu = 0$. Setting $G = 0$ (note that $\mu \equiv G$) and solving for the E required to be consistent with the proper fluid bulk modulus, k, will not work because the NASTRAN coding tries to form ν by dividing G (see footnote) and dividing by zero will not be handled properly by the computer.

* The shear modulus, G, can be given in place of E (or ν) in which case the E or ν is computed internally from $E = 2G(1+\nu)$ (or $\nu = \frac{E}{2G} - 1$).

One way to resolve the problem is to relax the strict equalities for the mock element that $\lambda = k$ and $\mu = 0$ but rather enforce them only approximately such that $\lambda \approx k$ and $\mu \approx 0$ (i.e. so long as $\lambda \gg \mu$). By setting

$$\nu = 0.49999 \tag{31a}$$

and
$$E = \frac{k(1-2\nu)(1+\nu)}{\nu} \equiv k/16,664.44$$

on the MAT1 cards, NASTRAN will internally generate a set of G_{ij} constants that will adequately represent (but not be exactly equal to) the desired exact values. As an illustration, consider water that has a bulk modulus of $k = 316,000$ psi (2.18×10^9 N/m²). Thus, for $\mu = 0$, typical proper values of the [G] array would correspond to

$$G_{11} = 316,000, G_{12} = 316,000, \text{ and } G_{44} = 0.0.$$

Applying the suggested approximate approach via equations (31a), NASTRAN invokes equations (31); thus would internally compute the typical [G] array elements as

$$G_{11} = 316,050 \quad G_{12} = 316,037 \quad G_{44} = 6.32$$

which should be sufficiently close to produce fluid response results of the same degree of accuracy had the exact [G] entries been used.

Pressure distribution information is obtained by examining the stress output from NASTRAN. The fluid pressure is obtained by reversing the sign of the normal stress output (since all the normal components are equal for mock elements, the user can select any normal component).

The boundary between the fluid and solid is handled by only forcing the normal component of fluid displacement to be equal to the normal displacement of the interfacing solid. This can be easily done through the introduction of a double node in conjunction with a MPC constraint. The boundary at infinity is handled analogous to the approach used in solids for earthquake problems (reference 19) and later in fluid applications (references 17, 18). This is accomplished by placing the fluid boundary not at infinity, but at a finite distance that is far enough so that interaction waves radiating from the submerged structure will satisfy (or nearly satisfy) the boundary condition

$$p = \rho c \dot{u}_n \tag{32}$$

where ρ is the fluid mass density, c is the fluid sound speed and \dot{u}_n is the velocity normal to the outer boundary. This condition is true for plane waves and asymptotically true for cylindrical and spherical waves. The finite element form of equation (32) is given by

$$\{F_b\} = [C_b]\{\dot{U}\} \tag{33}$$

where $[C_b]$ is a diagonal matrix with zero diagonal values for non-outer boundary points and a value of $\rho c \Delta A_i$ for normal outer boundary degrees-of-freedom where ΔA_i is the pressure-to-force conversion term and corresponds to a segment area A_i at the boundary node. In NASTRAN, the boundary dash pots are applied with CDAMP1 cards.

A rough guideline is needed for determining how far the fluid boundary should be placed in order for the plane wave approximation, equation (32), to be valid. A steady state solution can be constructed from some distribution of point sources around the structure fluid boundary. For a 3-D problem, a single source will approach (within 98.6%) a plane wave pressure velocity relationship (like equation (32)) after moving one wave length away from the source. The percentage quoted refers to the fact that complex impedance ($z = p/u$) is $.986\rho c$. Moving away $1\ 1/2$ wave lengths, this percentage becomes 99.4%. Thus, we are suggesting that the single source decay information can be used to judge the distance to place the absorbing boundary. For 2-D problems, a line source emitting cylindrical waves will approach a plane wave boundary condition to within 99.2% for one wave length away and to within 99.7% for $1\ 1/2$ wave lengths away. Thus, it is suggested to place the boundary a distance \tilde{D} away from the structure where

$$\tilde{D} = \alpha \tilde{\lambda} \quad (34)$$

and α is a proportionality constant (e.g. 1.5 for a 99.4% correct plane wave assumption) related to the degree of the plane wave boundary condition assumption, and $\tilde{\lambda}$ is the wave length of the steady state driving frequency in water (i.e. $\tilde{\lambda} = 2\pi c/\omega$).

Next, one must consider the size elements to use so that the elements of the mesh do not artificially "ring" at their natural frequencies. To avoid ringing, there exists a minimum element length, ΔL , that is related to some fraction, β , of the wave length of the driving frequency in the fluid, thus

$$\Delta L = \beta \tilde{\lambda} \quad (35)$$

The value of β will depend on the type of elements being used. For example, if one employs CQDMEM elements of the NASTRAN program, $\beta \approx 1/6$ to avoid mesh ringing. Modeling the region from the fluid structure interface out to the mathematical cut in the fluid boundary would result in \tilde{n} elements of length ΔL , thus

$$\tilde{n} = \frac{\tilde{D}}{\Delta L} \quad (36)$$

Substituting equations (34) and (35) into equation (36) results in the expression

$$\tilde{n} = \frac{\alpha}{\beta}$$

which is independent of the driving frequency $\tilde{\omega}$. Thus, employing typical values of $\alpha = 1.5$ and $\beta = 1/6$ into equation (36) shows, for example, that regardless of the driving frequency magnitude, it is possible to model the fluid field with

as few as 9 elements in the direction normal to the surface. In cases where the surface structure elements are coarse, more elements would be needed to blend in the fine surface elements into the coarser field elements.

For radiation type problems (i.e. a structure vibrating and transmitting outgoing waves), one can handle the infinite boundary problem by placing the dash pots around the outer boundary. For scattering type problems where a submerged structure is subject to an incident wave input, the handling of the infinite boundary problem is more complicated in that both the incident wave driving force and the boundary dash pots both appear on the boundary. The manner in which this class problem is handled is discussed in reference 17 and will not be repeated here.

As a demonstration problem consider the two-dimensional problem of an infinitely long cylindrical inclusion imbedded in an acoustical fluid medium. The inclusion is subjected to an incident harmonic plane wave (this is classified as a scattering problem). A sketch of the full model and corresponding finite element sketch is illustrated in figure 2. Constant strain CQDMEM elements are used to construct the model for both the solid and mock fluid elements. Two solutions (for two different inclusion types) are presented in the form of a comparison between an exact and corresponding finite element response. In either case, the exact solution (references 20, 21) is represented by the solid curve and the dots are the corresponding finite element solution. The solution response is given, in non-dimensional form, as the ratio between the total pressure to the incident free field amplitude. The parameters in the upper right corner of the figure denote a set of non-dimensional parameters that characterize the physical parameters of the problem and have the corresponding definitions

$$KA = \text{non-dimensional driving frequency} = \frac{\omega \cdot \text{inclusion radius}}{\text{fluid wave speed}}$$

$$\bar{R} = \frac{\text{radial coordinate}}{\text{inclusion radius}}$$

$$\bar{D} = \frac{\text{fluid dilatational wave speed}}{\text{inclusion dilatational wave speed}}$$

$$\bar{C} = \frac{\text{inclusion dilatational wave speed}}{\text{inclusion shear wave speed}}$$

$$\bar{\rho} = \frac{\text{fluid mass density}}{\text{inclusion mass density}}$$

The response shown in figure 3 corresponds to a vacuous inclusion and the response in figure 4 to an elastic aluminum inclusion. Except for the 0° and 180° (back and front) data points on the aluminum cylinder, the response results agreement was good. Response comparison for other radii (both closer and further away from the results presented) gave equally good results. The mesh size used was pushing the limit regarding the size needed to avoid ringing. It is felt that a finer mesh would have improved the results in the 0° or 180° data points for the aluminum solutions.

Reference 22 presents another completely different approach to solving steady state problems with NASTRAN. In this approach the submerged structure is surrounded with a sphere shaped region of finite elements. The proper boundary condition for handling the infinite fluid domain beyond the bounding sphere surface is treated with an eigenvalue expansion approach. The details are presented in reference 22 (paper in this colloquium), therefore will not be repeated here.

TRANSIENT-UNCOUPLED FLUID FIELD

This class of problem treats a totally submerged structure subject to dynamic loading usually in the form of an incident pressure wave. Reference 23 presents an application of the NASTRAN program towards solving transient fluid/structure interaction problems with the DAA (doubly asymptotic approximation) method. This method involves imposing an analytical decoupling relation (in differential equation form) describing the relationship between the pressure at the interface and the corresponding interface motion. The decoupling approximation eliminates the need for modeling the fluid field with finite elements. A brief outline of the method is presented in reference 23. A more detailed discussion of the implementation into NASTRAN is presented as a paper in this colloquium (reference 24), therefore the reader is referred to that paper for more details.

TRANSIENT-COUPLED FLUID FIELD

The class of problem considered here is the same as the previous transient category except in this case the fluid is modeled as part of the finite element network. The first NASTRAN application in this category considers the case where pressure type fluid elements are used to model the fluid field. These types of elements are different from the displacement type elements discussed earlier in that there is only one degree of freedom per node (namely pressure); this is in contrast* to one, two or three degrees of freedom per node for mock fluid elements which have displacements as the basic unknowns. The implementation of this method into NASTRAN requires one to dummy the construction of the stiffness and mass matrices of a conventional displacement type finite element so that only one displacement component is active (the remaining ones are zero); and further, the remaining nonzero component plays the role of pressure. The proper units are handled through redefining the elements of the [G] stress-strain matrix. This approach to solving fluid/structure interaction problems with NASTRAN was first introduced in the 4th NASTRAN User's Colloquium (reference 23). The complete details of the implementation of the method is presented in the current 5th NASTRAN User's Colloquium (reference 24).

The second application of NASTRAN (employing the coupled fluid field approach) is that of using the mock fluid elements. These displacement type elements were already discussed in the previous section on harmonic analysis. The method of implementing them via the stress-strain matrix [G] is done in

* The actual number depends on whether one is solving a one, two, or three-dimensional problem.

exactly the same manner as for the transient type problems as well. Transient problems using mock elements are solved using rigid format number 9. The infinite fluid boundary problem can be handled by temporal truncation. This is the most straightforward approach and is readily adaptable to both the pressure or mock element type transient solutions. In construction of the finite element mesh, one models the fluid surrounding the solid cutout to, say, 2 structure lengths. For scattering or radiation type problems, one can take advantage of the fact that the continuous equations are hyperbolic in nature. Thus for, say, a radiation problem, the solution will be such that the response in front of a radiating wave is zero, thus the problem does not know a boundary to the mesh even exists until the radiating wave actually gets there (due to the discretization of the problem, the governing equations do not exactly behave like hyperbolic equations, but the idea of traveling waves are still roughly approximated by the governing discretized differential equations). Thus, the solution to the problem can be obtained in the same manner as a finite boundary case, except that the solution response must be truncated at the time when the radiated wave reaches the mesh boundary.

Scattering problems can be treated in a similar manner. The free-field incident wave solution starts the problem, i.e. the initial conditions for the problem solution are obtained by setting the response field, behind the incident wave, equal to the free-field solution. The equations of motion are integrated in time in the usual manner but must be truncated when scattered waves off the structure reach the finite fluid mesh boundary.

One-dimensional wave propagation examples using mock elements in NASTRAN are presented in reference 17*. A solution employing mock elements for a three-dimensional problem is shown in figure 5. The problem corresponds to a plane step wave traveling through a solid elastic homogeneous medium and interacting with a spherical cavity filled with fluid. At the time these runs were made, a level of NASTRAN capable of solving axis of revolution solid structures subject to nonsymmetric loading was not yet available, consequently, a program (reference 25) other than NASTRAN was used to generate the result shown in figure 6. The program uses a harmonic decomposition in the angular coordinate of a cylindrical coordinate system to reduce the full three-dimensional problem to that of solving a set of smaller two-dimensional ones (with the harmonic wave number as a parameter in each two-dimensional subproblem). The full three-dimensional response is obtained by superposition over the angular harmonics (usually 5 terms are adequate). The response shown in figure 6 is the pressure at the center of the fluid sphere for both the exact solution (reference 26) and the corresponding finite element solution. For reference, the pressure in the free field (negative of the average normal stress in the solid) is also shown. Transient finite element solutions of this type tend to ring about the true solution. The frequency of the ringing is associated with the highest natural frequency of the mesh. Reference 27 discusses this point in detail and provides a digital filtering technique for eliminating some of the ringing problem.

* In reference 17, there are several sign errors that should be pointed out to avoid confusion with the development presented here; namely, on page 74 of reference 17, replace k with $-k$ in equation (3), replace $-k$ with $+k$ on the third line from bottom and , finally, replace $-k$ with $+k$ at the bottom of page 75.

For completeness, a brief discussion of the direct application of the NASTRAN computer program for solving fluid/structure interaction problems is presented next. The NASTRAN program has built-in pressure type elements (called CFLUID1 elements) and are described through RINGFL, PRESPT and FREEPT type fluid nodes. The elements are designed to operate in contained tanks that may have either rigid or elastic walls. These special elements have the following restrictions:

1. The user may not apply loads, constraints, sequencing or omitted coordinate directly on the fluid nodes involved. Instead, the user supplies information related to the boundaries and NASTRAN internally generates the correct constraints, sequencing, and matrix terms.
2. The input data to NASTRAN may include all of the existing operations except the axisymmetric structural element data (e.g., axisymmetric shell elements cannot be used).
3. The fluid must lie within the walls of an open or closed tank.
4. The first 6 rigid formats of NASTRAN may not be used in conjunction with these elements. NASTRAN assumes the walls of the container are rigid for these first 6 rigid formats but allows elasticity for the remaining 6 (fortunately direct frequency and direct transient response are included in the remaining 6).
5. No means are provided for the direct input of applied loads on the fluid. Loading must come through the motion of the walls.

The list of constraints that are placed on the usage of these elements rather severely limits the range of application, particularly in the case where unbounded fluid regions are of interest. Even within these constraints (reference 28), however, some rather interesting applications to acoustic noise problems associated with automobiles have been found.

Another unique feature that the NASTRAN program has is the ability to treat the free surface problem and include gravity terms into the fluid equations of motion.

CONCLUSIONS

The latest version of the NASTRAN computer program, as of this writing, does not handle a very large class of fluid/structure interaction problems via the direct rigid format application of the program. This paper presents a variety of nonstandard usage of the program to broaden the scope of problem application in the area of fluid/structure interaction. The implementation of the techniques presented here varies from one extreme of requiring the user to only make slight modifications to the standard problem input of NASTRAN-to-another extreme of requiring a substantially sized auxiliary support computer program to handle pre- and/or post-processing of the input and output data. The implementation of these methods depends, to some degree, upon the ingenuity of the user. Hopefully, future versions of NASTRAN will have more automatic procedures for solving problems of the type addressed in this paper.

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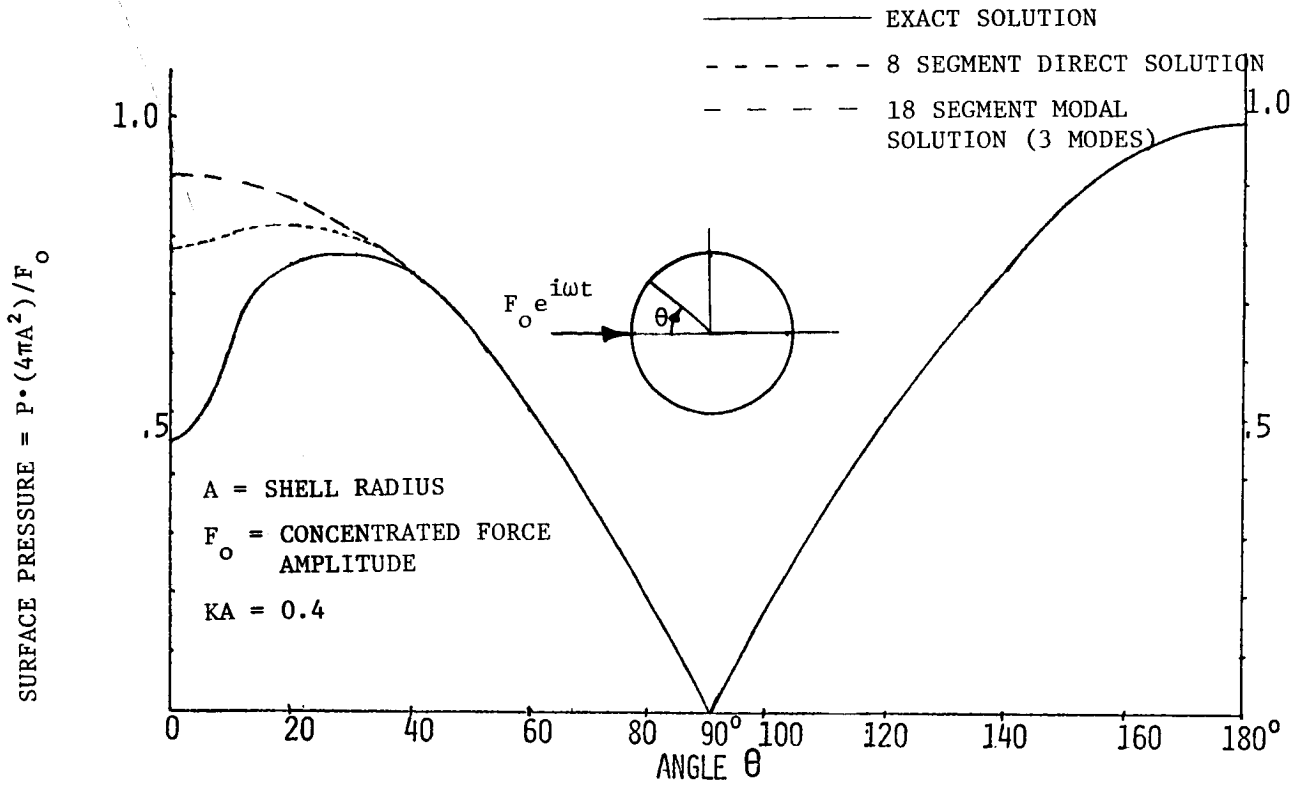


Figure 1. - Surface Pressure on a Point Loaded Thin Wall Sphere

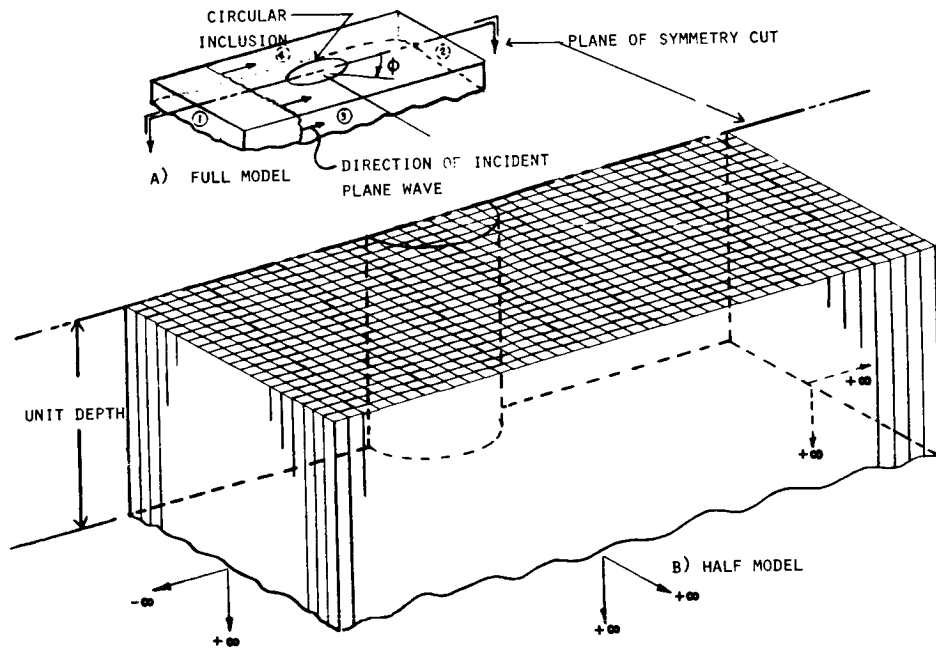


Figure 2. - Finite Element Mesh for Cylindrical Inclusions

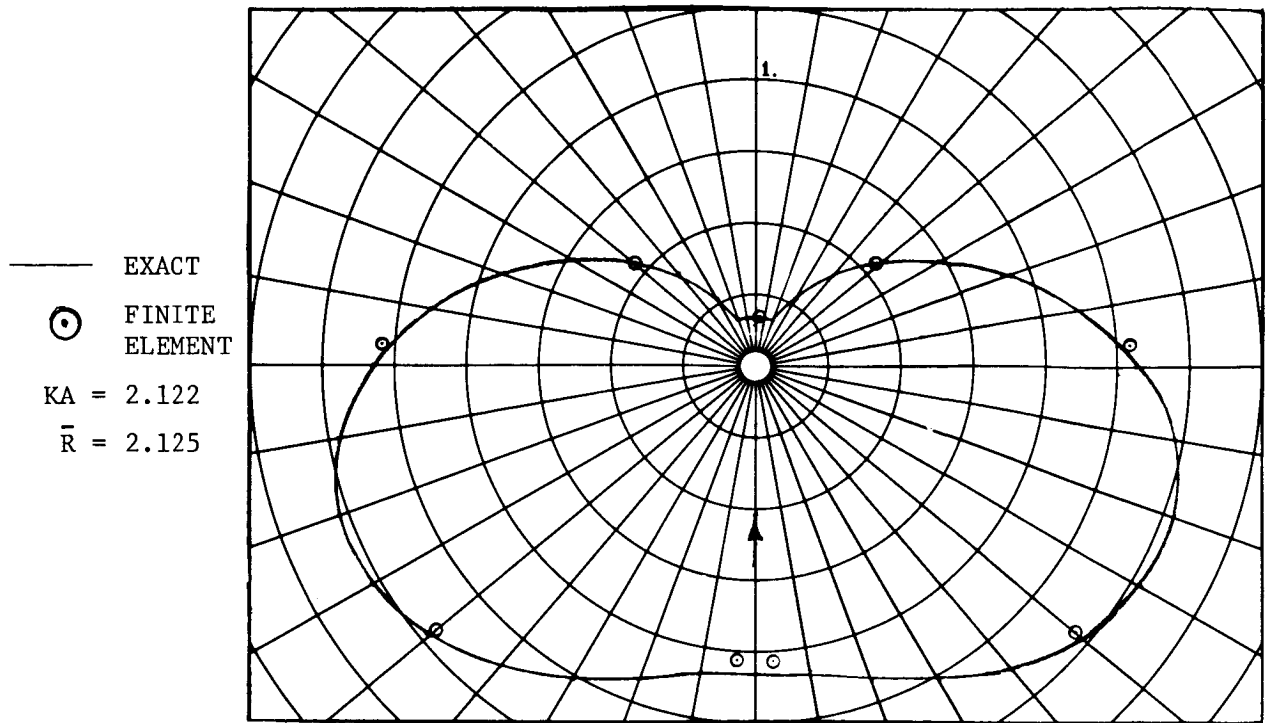


Figure 3. - Total (Incident & Scattered) Steady State Pressure Response of a Cylindrical Void in Water

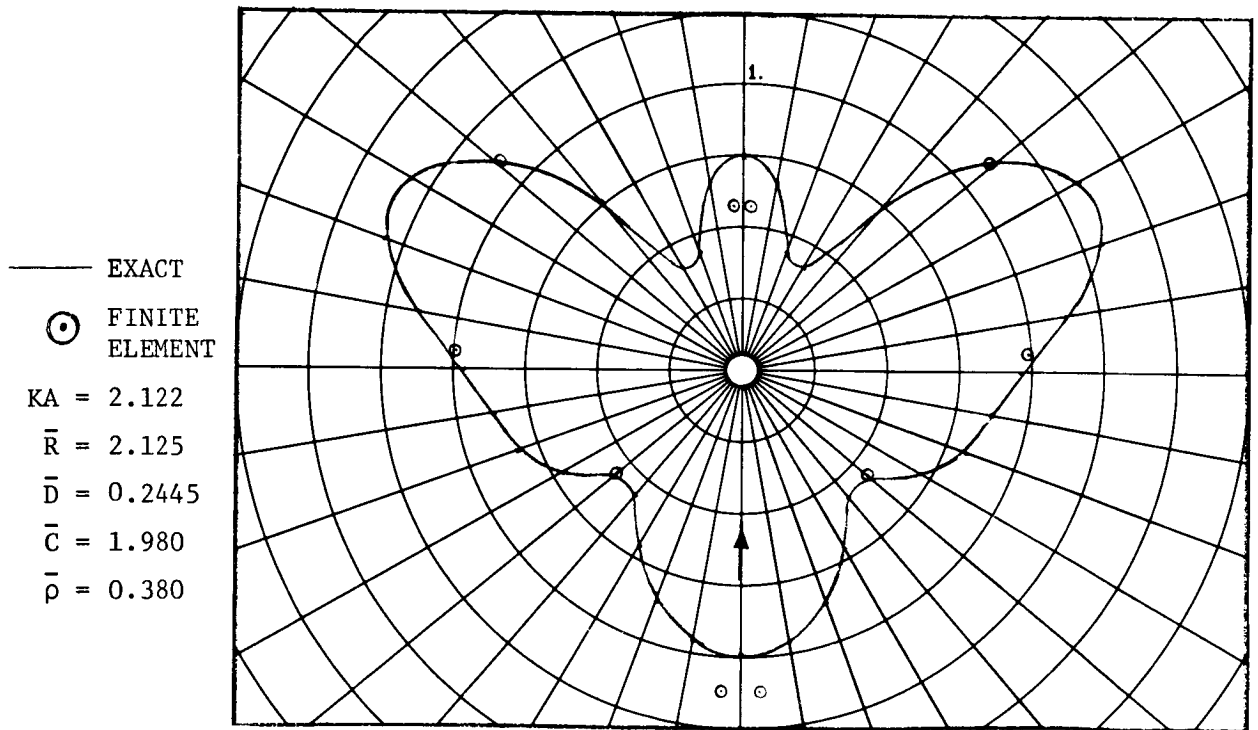


Figure 4. - Total (Incident & Scattered) Steady State Pressure Response of a Solid Elastic Aluminum Cylindrical Inclusion In Water

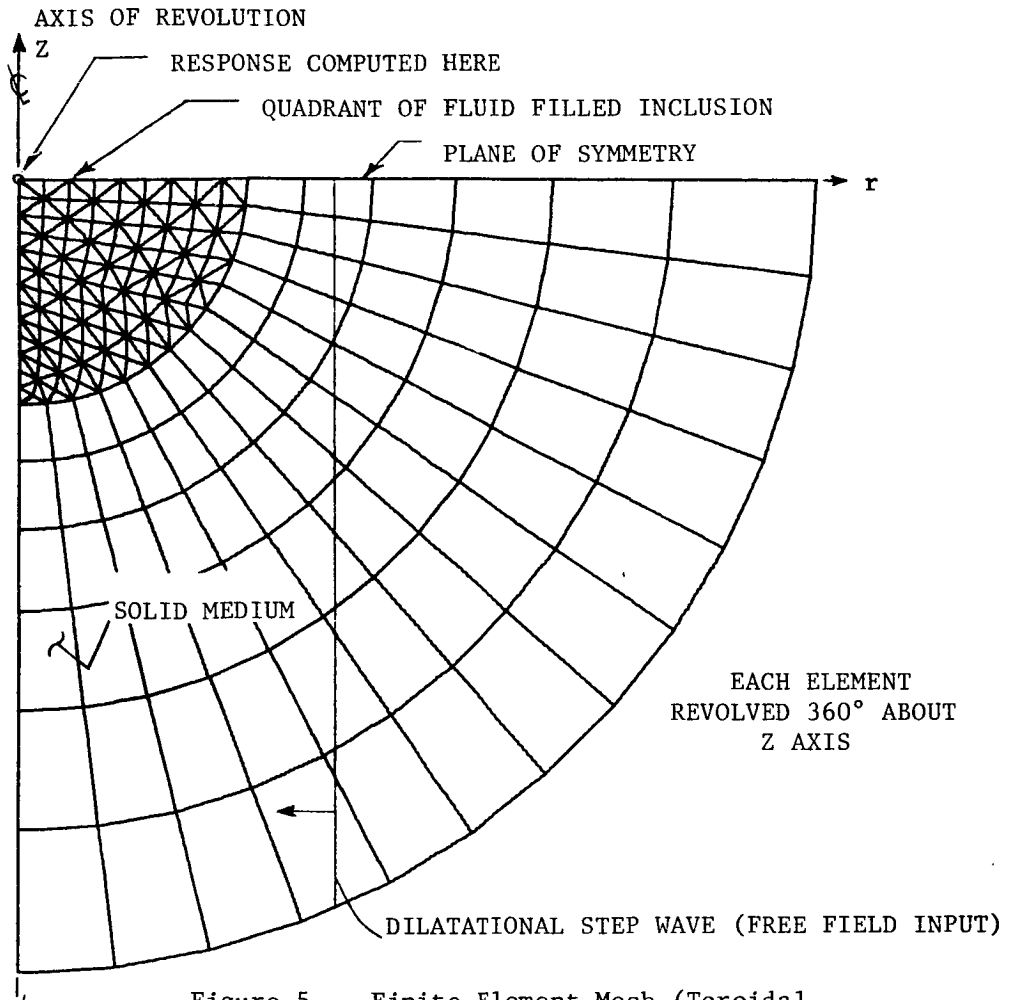


Figure 5. - Finite Element Mesh (Toroidal Element Construction)

CENTER PRESSURE ÷ (STATIC PRESSURE RESPONSE)

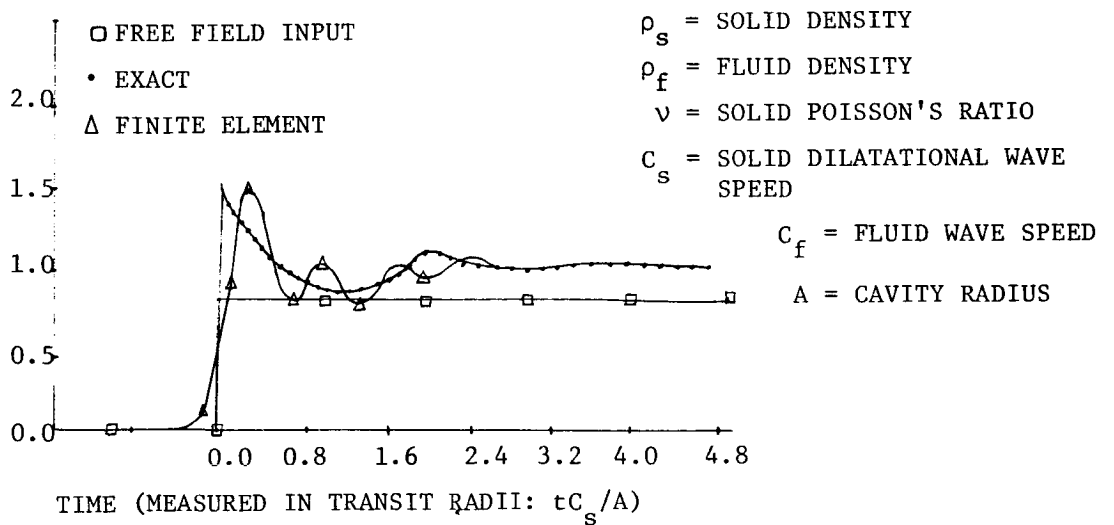


Figure 6. - Pressure Transient Response at Fluid Cavity Center
 $(\rho_s/\rho_f = 1; C_s/C_f = 1; \nu = 1/4)$