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MEASURES OF PILOT PERFORMANCE DURING V/TOL AIRCRAFT LANDINGS ON SHIPS AT SEA

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16. Abstract

Simulation experiments to determine the feasibility of landing V/TOL aircraft on ships at sea, require that the motion and attitude of the aircraft relative to the landing platform be known at the instant of touchdown. The success of these experiments depends on the ability of the experimenter to measure the pilot's performance during the landing maneuver. To facilitate these measurements, the equations describing the motion of the aircraft and its attitude relative to the landing platform are presented in a form which is suitable for simulation purposes. It is assumed that the pilot has achieved a successful landing when the relative motion at each landing wheel, at the instant of touchdown, does not exceed design values for the landing gear. By using this criterion, the equations presented can be used to determine the success or failure of each landing maneuver and hence establish the feasibility of such maneuvers.

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NOMENCLATURE

[A]	a column vector of velocity components relative to aircraft axes
[D]	a direction cosine matrix
d_{ij}	an element in the ith row, and the jth column of the direction cosine matrix
[E]	a column vector of velocity components in a fixed reference system
$\hat{I}$, $\hat{J}$, $\hat{K}$	unit vectors defining a ship axes system
[I]	a unit matrix
$\hat{i}$, $\hat{j}$, $\hat{k}$	unit vectors defining an aircraft axes system
P	the rolling component of the ship's angular velocity vector
p	the rolling component of the aircraft's angular velocity vector
Q	the pitching component of the ship's angular velocity vector
q	the pitching component of the aircraft's angular velocity vector
R	the yawing component of the ship's angular velocity vector
$\bar{R}_p$	the position vector of the landing platform, relative to the origin of the ship coordinate system
r	the yawing component of the aircraft's angular velocity vector
$\bar{r}$	the position vector of an aircraft landing wheel, relative to the center of gravity of the aircraft
[S]	a column vector of velocity components relative to ship axes
[T]	a transformation matrix
U, V, W	ship linear velocity components
u, v, w	aircraft linear velocity components
X, Y, Z	coordinates relative to ship axes
x, y, z	coordinates relative to aircraft axes
Z_{cg}	the perpendicular height of the aircraft center of gravity above the landing platform

$\theta_1\ \theta_2\ \theta_3$	Euler angles corresponding to direction cosines
Φ, Θ, Ψ	ship Euler angles
ϕ, θ, ψ	aircraft Euler angles
$\bar{\Omega}$	ship angular velocity vector
$\bar{\omega}$	aircraft angular velocity vector

Superscript:

T	denotes transposition

Subscripts:

c.g.	center of gravity
EA	a transformation from fixed axes to aircraft body axes
ES	a transformation from fixed axes to ship axes
I	signifies inertial components
ij	the ith row and the jth column of the direction cosine matrix
N	nose wheel
P	landing platform
Pi	the point on the landing platform at which the ith landing wheel makes contact
Pcg	the point on the landing platform beneath the aircraft center of gravity
Ps	the point on the landing platform at which the starboard landing wheel makes contact
p	port landing wheel
R	aircraft motion relative to the landing platform
S	ship motion
SA	a transformation from ship axes to aircraft axes
s	starboard landing wheel

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SUMMARY

Simulation experiments to determine the feasibility of landing V/TOL aircraft on ships at sea, require that the motion and attitude of the aircraft relative to the landing platform be known at the instant of touchdown. The success of these experiments depends on the ability of the experimenter to measure the pilot's performance during the landing maneuver. To facilitate these measurements, the equations describing the motion of the aircraft and its attitude relative to the landing platform are presented in a form which is suitable for simulation purposes. It is assumed that the pilot has achieved a successful landing when the relative motion at each landing wheel, at the instant of touchdown, does not exceed design values for the landing gear. By using this criterion, the equations presented can be used to determine the success or failure of each landing maneuver and hence establish the feasibility of such maneuvers.

INTRODUCTION

Landing V/TOL aircraft on ships at sea is complicated by the motion of the landing platform which moves in response to the motions of the sea. In order to achieve a successful landing, the pilot has to induce in his aircraft a state which is compatible with the state of the ship, at the instant of touchdown. In the present context, the state refers specifically to the components of linear and angular motion and the corresponding spatial orientation. The relative states must be such that the rate at which the aircraft approaches the landing platform does not exceed design limits for the landing gear. Moreover, in the event that the aircraft attitude is not compatible with the attitude of the ship, the total landing energy is initially being absorbed by fewer than the total number of shock absorbers available.

In order to determine if the relative motions and attitudes during the landing maneuver are within prescribed limits, it is necessary to know the motion of the aircraft relative to the landing platform. The equations describing the relative motions during touchdown require the formulation of a matrix that defines the attitude of the aircraft. A similar matrix defining the attitude of the ship is also required. The elements of these matrices are functions of the conventional Euler angles for the aircraft and the ship. The elements of the aircraft attitude matrix are computed in the process of solving the mathematical model of the aircraft. In order to determine the elements of the attitude matrix for the ship, the components of the ship's angular velocity vector are measured and inserted into Euler's geometrical equations, which are then solved to obtain the Euler angles for the ship. Subsequent to formulation, the ship's attitude matrix is transposed and premultiplied by the aircraft attitude matrix to obtain the required matrix defining the transformation from ship axes to aircraft body axes. The result is then used to transform the motion components of the landing platform from ship axes to aircraft body axes.

The elements of the matrix defining the transformation from ship axes to aircraft body axes are direction cosines. Although the direction cosines are useful for transformation purposes, they are not convenient measures of attitude. To facilitate the determination of relative attitudes, the equations describing a conversion from direction cosines to the Euler angles representing the attitude of the aircraft relative to the landing platform are formulated.

The feasibility of landing a V/TOL aircraft on a moving ship is determined by using the relative motion and attitude equations to measure pilot performance during the landing maneuver. The performance measured will be the pilot's ability to keep the relative motions and attitudes within prescribed limits. A pilot will be deemed to have achieved a successful landing if the relative motions at all landing wheels do not exceed design values for the landing gear.

VELOCITY OF AIRCRAFT RELATIVE TO THE SHIP

For simulation experiments of this kind, it is necessary to know the velocity of each landing wheel relative to the landing platform at the instant of touchdown. The point at which a given landing wheel makes contact with the landing platform is assumed to have a position vector $\bar{R}_{Pi}$ relative to the origin of the ship's coordinate system. This system originates at a point on the water line, vertically above the center of gravity of the ship. The XZ plane of the ship's coordinate system corresponds to the plane of symmetry of the vessel and the Y axis is normal to it. It is naval practice to have the Y axis emanating from the port side of the ship and the Z axis pointing upward. However, in order to simplify the determination of relative velocities, ship motion will be referred to a coordinate system that originates on the water line but otherwise conforms to the aircraft axes convention.

Relative to this system of coordinates the position vector $\bar{R}_{Pi}$ assumes the form

$$\bar{R}_{Pi} = \left(X_{Pi}\hat{I} + Y_{Pi}\hat{J} + Z_{Pi}\hat{K}\right) \tag{1}$$

where $\hat{I}$, $\hat{J}$, $\hat{K}$ are a triad of mutually orthogonal unit vectors in the directions of the ship's axes, and X_{Pi}, Y_{Pi}, Z_{Pi} are the coordinates of the point at which the ith landing wheel makes contact with the landing platform. The subscript "i" is used to indicate which landing wheel is being considered.

Due to the ship's motion, the velocity of the landing platform at the point at which the ith landing wheel makes contact, is given by the equation (ref. 1)

$$\frac{d\bar{R}_{Pi}}{dt} = \left(\bar{V}_S + \bar{\Omega} \times \bar{R}_{Pi}\right) \tag{2}$$

where

$$\bar{V}_S = (U\hat{I} + V\hat{J} + W\hat{K}) \tag{3}$$

and

$$\bar{\Omega} = (P\hat{I} + Q\hat{J} + R\hat{K}) \tag{4}$$

The components U, V, W are the surging, swaying, and heaving velocities of the ship. These are nautical terms which denote forward, lateral, and vertical components of the ship's motion. P, Q, and R are the rolling, pitching, and yawing components of the ship's angular velocity vector $\bar{\Omega}$.

Equation (2) gives the components of the ship's inertial velocity at the point at which the ith landing wheel makes contact with the landing platform. These are

$$\left.\begin{aligned} U_I &= (U + QZ_{Pi} - RY_{Pi}) \\ V_I &= (V + RX_{Pi} - PZ_{Pi}) \\ W_I &= (W + PY_{Pi} - QX_{Pi}) \end{aligned}\right\} \tag{5}$$

where the subscript I denotes inertial components.

It is next necessary to determine the velocity of each landing wheel in the fully extended position. Let $\bar{r}_s$ be the position vector of the starboard landing wheel. The components of $\bar{r}_s$ relative to a set of aircraft body axes with origin at the center of gravity of the aircraft are

$$\bar{r}_s = (x_s\hat{i} + y_s\hat{j} + z_s\hat{k})$$

where $\hat{i}$, $\hat{j}$, $\hat{k}$ are a triad of mutually orthogonal unit vectors in the directions of the aircraft axes.

The velocity of the starboard landing wheel is

$$\frac{d\bar{r}_s}{ds} = \left(\bar{V}_A + \bar{\omega} \times \bar{r}_s\right) \tag{6}$$

where $\bar{V}_A$ is the linear velocity of the aircraft and $\bar{\omega}$ is its angular velocity.

In component form

$$\left.\begin{aligned} \bar{V}_A &= (u\hat{i} + v\hat{j} + w\hat{k}) \\ \bar{\omega} &= (p\hat{i} + q\hat{j} + r\hat{k}) \end{aligned}\right\} \tag{7}$$

u, v, w are the components of the aircraft's linear velocity vector, and p, q, r are the angular velocity components.

Equation (6) may be used to obtain the aircraft's velocity components at the location of the starboard landing wheel. Relative to the aircraft's body axes, these are

$$\left.\begin{aligned} u_I &= (u + qz_s - ry_s) \\ v_I &= (v + rx_s - pz_s) \\ w_I &= (w + py_s - qx_s) \end{aligned}\right\} \tag{8}$$

where the subscript I again denotes inertial components.

The velocity $\bar{V}_R$ of the starboard landing wheel relative to the landing platform is given by the equation

$$\bar{V}_R = \left[\left(\bar{V}_A + \bar{\omega} \times \bar{r}_s\right) - \left(\bar{V}_s + \bar{\Omega} \times \bar{R}_{Ps}\right)\right] \tag{9}$$

The components of this vector are

$$\left.\begin{aligned} \bar{V}_{R1} &= \left(u + qz_s - ry_s\right)\hat{i} - \left(U + QZ_{Ps} - RY_{Ps}\right)\hat{I} \\ \bar{V}_{R2} &= \left(v + rx_s - pz_s\right)\hat{j} - \left(V + RX_{Ps} - PZ_{Ps}\right)\hat{J} \\ \bar{V}_{R3} &= \left(w + py_s - qx_s\right)\hat{k} - \left(W + PY_{Ps} - QX_{Ps}\right)\hat{K} \end{aligned}\right\} \tag{10}$$

Since the aircraft axes $\hat{i}$, $\hat{j}$, $\hat{k}$ are not, in general, aligned with the ship's reference axes $\hat{I}$, $\hat{J}$, $\hat{K}$ at the instant of touchdown, it is necessary to transform the velocity components of the landing platform from ship axes to aircraft body axes.

Transformation of Motion Vector Components

A set of vector components in a coordinate system that is rotationally fixed is related to the components in the aircraft body axes by a transformation equation of the form

$$[A] = [T]_{EA}[E] \tag{11}$$

where

$[A]$ denotes a column vector of motion components in the aircraft reference system

$[T]_{EA}$ is a matrix which effects a transformation from fixed axes to aircraft body axes

$[E]$ is a column vector of motion components in the fixed reference system

Likewise, the components of a vector in the fixed reference system are related to the components in the moving ship reference system, by a transformation of the same form. That is,

$$[S] = [T]_{ES}[E] \tag{12}$$

where

$[S]$ denotes a column vector of motion vector components relative to ship axes

$[T]_{ES}$ is a matrix that effects a transformation from fixed axes to the moving ship axes

Similarly, a triad of ship axes components can be transformed to aircraft body axes by the transformation equation

$$[A] = [T]_{SA}[S] \tag{13}$$

where

$[T]_{SA}$ is a matrix which effects a tranformation from ship axes to aircraft body axes

Substitution from equation (12) in equation (13) gives a transformation from fixed axes to ship axes, followed by a transformation from ship axes to aircraft axes:

$$[A] = [T]_{SA}[T]_{ES}[E] \tag{14}$$

Finally, substitution from equation (11) in equation (14) yields the following matrix equation:

$$[A] = [T]_{SA}[T]_{ES}[T]_{EA}^{-1}[A]$$

Therefore, $[T]_{SA}[T]_{ES}[T]_{EA}^{-1} = [I]$, where $[I]$ is the unit matrix. Solving this matrix equation for $[T]_{SA}$ yields

$$[T]_{SA} = [T]_{EA}[T]_{ES}^{-1} \tag{15}$$

Since only orthogonal transformations are being considered, the inverse of a transformation matrix equals the transpose of the matrix, and equation (15) simplifies accordingly. That is

$$[T]_{ES}^{-1} = [T]_{ES}^{T} \tag{16}$$

where the superscript T denotes transposition.

Substitution from equation (16) in equation (15) yields the required transformation from ship axes to aircraft body axes. That is

$$[T]_{SA} = [T]_{EA}[T]_{ES}^{T} \tag{17}$$

In terms of the Euler angles ψ, θ, ϕ and using the conventional aeronautical rotation sequence, the required transformation matrices are (ref. 2)

$$[T]_{EA} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[T]_{EA} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \sin\phi\cos\theta \\ \cos\psi\cos\phi\sin\theta + \sin\psi\sin\phi & \sin\psi\cos\phi\sin\theta - \cos\psi\sin\phi & \cos\phi\cos\theta \end{bmatrix} \tag{18}$$

For the transformation from fixed axes to ship axes, the Euler angles will be denoted by the capital Greek letters Ψ, Θ, and Φ. In terms of this notation, the transformation matrix $[T]_{ES}$ assumes a form identical to equation (18).

$$[T]_{ES} = \begin{bmatrix} \cos\Theta\cos\Psi & \cos\Theta\sin\Psi & -\sin\Theta \\ \sin\Phi\sin\Theta\cos\Psi - \sin\Psi\cos\Phi & \sin\Psi\sin\Theta\sin\Phi + \cos\Psi\cos\Phi & \sin\Phi\cos\Theta \\ \cos\Psi\cos\Phi\sin\Theta + \sin\Psi\sin\Phi & \sin\Psi\cos\Phi\sin\Theta - \cos\Psi\sin\Phi & \cos\Phi\cos\Theta \end{bmatrix} \tag{19}$$

The following equation gives the transposed form of this matrix:

$$[T]_{ES}^{T} = \begin{bmatrix} \cos\Theta\cos\Psi & \sin\Phi\sin\Theta\cos\Psi - \sin\Psi\cos\Phi & \cos\Psi\cos\Phi\sin\Theta + \sin\Psi\sin\Phi \\ \cos\Theta\sin\Psi & \sin\Psi\sin\Theta\sin\Phi + \cos\Psi\cos\Phi & \sin\Psi\cos\Phi\sin\Theta - \cos\Psi\sin\Phi \\ -\sin\Theta & \sin\Phi\cos\Theta & \cos\Phi\cos\Theta \end{bmatrix} \tag{20}$$

The computer program, which solves the equations of the mathematical model of the aircraft, evaluates the angles Ψ, Θ, and Φ and hence determines the attitude of the aircraft as a function of time. The Euler angles are then used to compute the elements of the transformation matrix $[T]_{EA}$.

In order to determine the attitude of the ship, the components of the ship's angular velocity vector are measured and used to formulate the equations (ref. 3)

$$\left.\begin{aligned} P &= \dot{\Phi} - \dot{\Psi}\sin\Theta \\ Q &= \dot{\Theta}\cos\Phi + \Psi\cos\Theta\sin\Phi \\ R &= \dot{\Psi}\cos\Theta\cos\Phi - \dot{\Theta}\sin\Phi \end{aligned}\right\} \tag{21}$$

Solving these equations for $\dot{\Phi}$, $\dot{\Theta}$, and $\dot{\Psi}$, we obtain

$$
\left.\begin{aligned}
\dot{\Phi} &= P + (Q \sin \Phi + R \cos \Phi)\tan \Theta \\
\dot{\Theta} &= Q \cos \Phi - R \sin \Phi \\
\dot{\Psi} &= (Q \sin \Phi + R \cos \Phi)\sec \Theta
\end{aligned}\right\} \tag{22}
$$

The solution of these equations yields the required Euler angles Φ, Θ, and Ψ, which are then used to determine the elements of the transformation matrix $[T]_{ES}$. Subsequent to the formulation and transposition of this matrix, the product of $[T]_{EA}$ and $[T]_{ES}^{T}$ is formed.

This product matrix yields the required ship-to-aircraft transformation in accordance with equation (17). The result may be used to obtain the velocity of the starboard landing wheel relative to the landing platform. The relative velocities are

$$
\begin{pmatrix} V_{Rx} \\ V_{Ry} \\ V_{Rz} \end{pmatrix}_s = \begin{pmatrix} u + qz_s - ry_s \\ v + rx_s - pz_s \\ w + py_s - qx_s \end{pmatrix} - [T]_{SA} \begin{pmatrix} U + QZ_{Ps} - RY_{Ps} \\ V + RX_{Ps} - PZ_{Ps} \\ W + PY_{Ps} - QX_{Ps} \end{pmatrix} \tag{23}
$$

For experiments of this type, the most important components of ship motion are: heaving, pitching, and rolling. Hence, by assuming that

$$
U = V = R = 0
$$

the amount of computation is reduced, and equation (23) assumes the simpler form

$$
\begin{pmatrix} V_{Rx} \\ V_{Ry} \\ V_{Rz} \end{pmatrix}_s = \begin{pmatrix} u + qz_s - ry_s \\ v + rx_s - pz_s \\ w + py_s - qx_s \end{pmatrix} - [T]_{SA} \begin{pmatrix} QZ_{Ps} \\ -PZ_{Ps} \\ W + PY_{Ps} - QX_{Ps} \end{pmatrix} \tag{24}
$$

where

$$
\begin{pmatrix} V_{Rx} \\ V_{Ry} \\ V_{Rz} \end{pmatrix}_s
$$

are the velocity components of the starboard landing wheel relative to the landing platform, and $[T]_{SA}$ is defined in equation (17).

Similarly, the relative velocity components of the port landing wheel are

$$\begin{pmatrix} V_{Rx} \\ V_{Ry} \\ V_{Rz} \end{pmatrix}_p = \begin{pmatrix} u + qz_p - ry_p \\ v + rx_p - pz_p \\ w + py_p - qx_p \end{pmatrix} - [T]_{SA} \begin{pmatrix} QZ_{Pp} \\ -PZ_{Pp} \\ W + PY_{Pp} - QX_{Pp} \end{pmatrix} \tag{25}$$

In this case, the subscript p denotes the port landing wheel. This is not to be confused with the subscript, P, which refers to the landing platform.

Finally, the relative velocity components of the nosewheel are

$$\begin{pmatrix} V_{Rx} \\ V_{Ry} \\ V_{Rz} \end{pmatrix}_N = \begin{pmatrix} u + qz_N - ry_N \\ v + rx_N - pz_N \\ w + py_N - qx_N \end{pmatrix} - [T]_{SA} \begin{pmatrix} QZ_{PN} \\ -PZ_{PN} \\ W + PY_{PN} - QX_{PN} \end{pmatrix} \tag{26}$$

where the subscript N denotes nosewheel.

Attitude of Aircraft Relative to the Ship

The transformation of vector components from ship axes to aircraft body axes is given by equation (17)

$$[T]_{SA} = [T]_{EA}[T]_{ES}^T$$

In terms of the direction cosines d_{ij}, relating aircraft body axes to ship axes, this matrix equation assumes the form

$$[T]_{SA} = \begin{bmatrix} d_{11} d_{12} d_{13} \\ d_{21} d_{22} d_{23} \\ d_{31} d_{32} d_{33} \end{bmatrix} = [D]$$

The direction cosines d_{ij} have the following values:

$$d_{11} = (\cos\theta \cos\psi \cos\Theta \cos\Psi + \cos\theta \sin\psi \cos\Theta \sin\Psi + \sin\theta \sin\Theta)$$

$$\begin{aligned} d_{12} = [&\cos\theta \cos\psi(\sin\Phi \sin\Theta \cos\Psi - \sin\Psi \cos\Phi) \\ &+ \cos\theta \sin\psi(\sin\Psi \sin\Theta \sin\Phi + \cos\Psi \cos\Phi) - \sin\theta \sin\Phi \cos\Theta] \end{aligned}$$

$$\begin{aligned} d_{13} = [&\cos\theta \cos\psi(\cos\Psi \cos\Phi \sin\Theta + \sin\Psi \sin\Theta) \\ &+ \cos\theta \sin\psi(\sin\Psi \cos\Phi \sin\Theta - \cos\Psi \sin\Phi) - \sin\theta \cos\Phi \cos\Theta] \end{aligned}$$

$$\begin{aligned} d_{21} = [&(\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi)\cos\Theta \cos\Psi \\ &+ (\sin\psi \sin\theta \sin\phi + \cos\psi \cos\phi)\cos\Theta \sin\Psi - \sin\phi \cos\theta \sin\Theta] \end{aligned}$$

$$\begin{aligned} d_{22} = [&(\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi)(\sin\Phi \sin\Theta \cos\Psi - \sin\Psi \cos\Phi) \\ &+ (\sin\psi \sin\theta \sin\phi + \cos\psi \cos\phi)(\sin\Psi \sin\Theta \sin\Phi + \cos\Psi \cos\Phi) \\ &+ \sin\phi \cos\theta \sin\Phi \cos\Theta] \end{aligned}$$

$$\begin{aligned} d_{23} = [&(\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi)(\cos\Psi \cos\Phi \sin\Theta + \sin\Psi \sin\Phi) \\ &+ (\sin\psi \sin\theta \sin\phi + \cos\psi \cos\phi)(\sin\Psi \cos\Phi \sin\Theta - \cos\Psi \sin\Phi) \\ &+ \sin\phi \cos\theta \cos\Phi \cos\Theta] \end{aligned}$$

$$\begin{aligned} d_{31} = [&(\cos\psi \cos\phi \sin\theta + \sin\psi \sin\phi)(\cos\Theta \cos\Psi) \\ &+ (\sin\psi \cos\phi \sin\theta - \cos\psi \sin\phi)\cos\Theta \sin\Psi - \cos\phi \cos\theta \sin\Theta] \end{aligned}$$

$$\begin{aligned} d_{32} = [&(\cos\psi \cos\phi \sin\theta + \sin\psi \sin\phi)(\sin\Phi \sin\Theta \cos\Psi - \sin\Psi \cos\Phi) \\ &+ (\sin\psi \cos\phi \sin\theta - \cos\psi \sin\phi)(\sin\Psi \sin\Theta \sin\Phi + \cos\Psi \cos\Phi) \\ &+ \cos\phi \cos\theta \sin\Phi \cos\Theta] \end{aligned}$$

$$\begin{aligned} d_{33} = [&(\cos\psi \cos\phi \sin\theta + \sin\psi \sin\phi)(\cos\Psi \cos\Phi \sin\Theta + \sin\Psi \sin\Phi) \\ &+ (\sin\psi \cos\phi \sin\theta - \cos\psi \sin\phi)(\sin\Psi \cos\Phi \sin\Theta - \cos\Psi \sin\Phi) \\ &+ \cos\phi \cos\theta \cos\Phi \cos\Theta] \end{aligned}$$

Although the direction cosines are useful for transformation purposes, they are not convenient measures of aircraft attitude. A conversion from direction cosines to a set of Euler angles that represents the attitude of the aircraft relative to the ship can be effected by the method described in reference 4.

For the conventional aeronautical rotation sequence, a direction cosine matrix can be generated as the product of three rotation matrices as follows:

$$[D] = [T_1(\theta_1)][T_2(\theta_2)][T_3(\theta_3)] \tag{27}$$

where

$$[T_1(\theta_1)] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{pmatrix}$$

$$[T_2(\theta_2)] = \begin{pmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix}$$

$$[T_3(\theta_3)] = \begin{pmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The product matrix, equation (27) yields the following direction cosines:

$$d_{11} = \cos\theta_2 \cos\theta_3$$

$$d_{12} = \cos\theta_2 \sin\theta_3$$

$$d_{13} = -\sin\theta_2$$

$$d_{21} = \sin\theta_1 \sin\theta_2 \cos\theta_3 - \sin\theta_3 \cos\theta_1$$

$$d_{22} = \sin\theta_3 \sin\theta_2 \sin\theta_1 + \cos\theta_3 \cos\theta_1$$

$$d_{23} = \sin\theta_1 \cos\theta_2$$

$$d_{31} = \cos\theta_3 \cos\theta_1 \sin\theta_2 + \sin\theta_3 \sin\theta_1$$

$$d_{32} = \sin\theta_3 \cos\theta_1 \sin\theta_2 - \cos\theta_3 \sin\theta_1$$

$$d_{33} = \cos\theta_1 \cos\theta_2$$

The following combinations of these equations are required to convert the direction cosines d_{ij} to the Euler angles θ_1, θ_2, θ_3.

$$d_{11} \sin \theta_3 - d_{12} \cos \theta_3 = 0 \tag{28}$$

$$d_{31} \sin \theta_3 - d_{32} \cos \theta_3 = \sin \theta_1 \tag{29}$$

$$d_{21} \sin \theta_3 - d_{22} \cos \theta_3 = -\cos \theta_1 \tag{30}$$

$$d_{13} = -\sin \theta_2 \tag{31}$$

$$d_{23} \sin \theta_1 + d_{33} \cos \theta_1 = \cos \theta_2 \tag{32}$$

From equation (28):

$$\tan \theta_3 = \frac{d_{12}}{d_{11}} \tag{33}$$

Equations (29) and (30) give:

$$\tan \theta_1 = \left[\frac{d_{31} \sin \theta_3 - d_{32} \cos \theta_3}{d_{22} \cos \theta_3 - d_{21} \sin \theta_3}\right] \tag{34}$$

or

$$\tan \theta_1 = \left[\frac{d_{31} \tan \theta_3 - d_{32}}{d_{22} - d_{21} \tan \theta_3}\right] \tag{35}$$

From equations (31) and (32):

$$\tan \theta_2 = -\left[\frac{d_{13}}{d_{23} \sin \theta_1 + d_{33} \cos \theta_1}\right] \tag{36}$$

Given the nine direction cosines and the rotational sequence, the three Euler angles θ_1, θ_2, and θ_3 can be computed. For the present application, θ_3 corresponds to the aircraft yaw angle relative to the landing platform. The angles θ_2 and θ_1 are pitch and roll angles, respectively, relative to the landing platform. It should be noted that the computed values of θ_i are not unique, since tan θ is a many valued function. That is,

$$\tan \theta = \tan(n\pi + \theta)$$

where n is a positive or negative integer. However, for the case being considered, and the angles anticipated, only those solutions corresponding to $n = 0$ will be required.

Performance Measures at Touchdown

In terms of a known c.g., height above the landing platform, and the direction cosines relating aircraft axes to ship axes, the relative velocities required for performance measures are those prevailing when

$$Z_{cg} = (d_{13}x_{pw} + d_{23}y_{pw} + d_{33}z_{pw}) \tag{37}$$

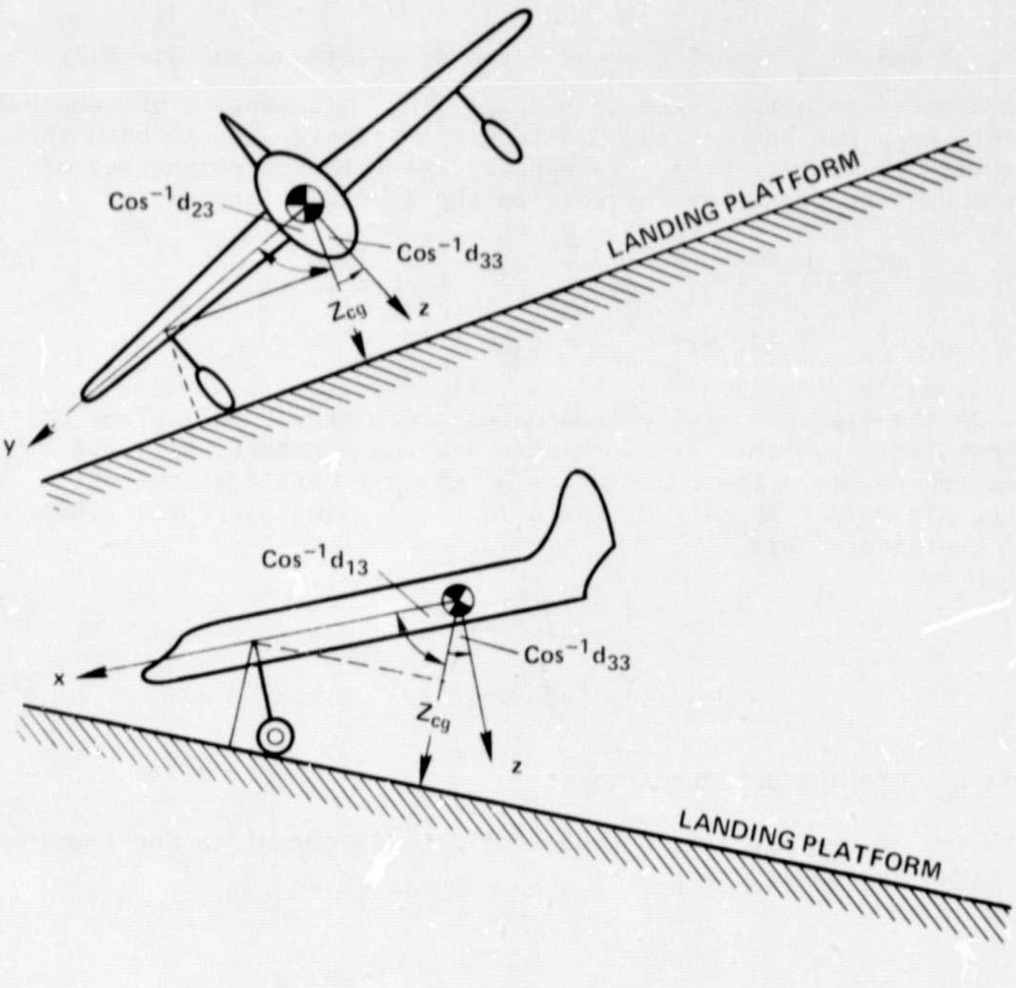
at the port landing wheel;

$$Z_{cg} = (d_{13}x_{sw} + d_{23}y_{sw} + d_{33}z_{sw}) \tag{38}$$

at the starboard landing wheel; and

$$Z_{cg} = (d_{13}x_{nw} + d_{23}y_{nw} + d_{33}z_{nw} \tag{39}$$

at the nosewheel. (See diagram.)



Equations (24) through (26) give the components of aircraft velocity relative to the landing platform. These are referred to aircraft body axes. In order to determine the corresponding components in ship axes, the aircraft velocity components must be transformed to ship axes. Subsequent to transformation, equation (24) assumes the form

$$[T]_{SA}^{T}\begin{pmatrix} V_{Rx} \\ V_{Ry} \\ V_{Rz} \end{pmatrix} = [T]_{SA}^{T}\begin{pmatrix} u \\ v \\ w \end{pmatrix} - \begin{pmatrix} QZ_{Pcg} \\ -PZ_{Pcg} \\ W + PY_{Pcg} - QX_{Pcg} \end{pmatrix}$$

where

$$[T]_{SA}^{T} = \begin{pmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \end{pmatrix}$$

and X_{Pcg}, Y_{Pcg}, and Z_{Pcg} are the coordinates of points on the aircraft trajectory projected onto the plane $(Z + Z_{Pcg}) = 0$. Moreover, a perpendicular from the point Z_{Pcg} on the landing platform will always pass through the center of gravity of the aircraft. Therefore, the velocity components of the aircraft's center of gravity relative to the landing platform are

$$\dot{X}_{Pcg} = (d_{11}u + d_{21}v + d_{31}w - QZ_{Pcg}) \tag{40}$$

$$\dot{Y}_{Pcg} = (d_{12}u + d_{22}v + d_{32}w + PZ_{Pcg}) \tag{41}$$

where Z_{Pcg} is the distance of the landing platform from the XY plane of the ship's coordinate system. Since this is a known constant, $\dot{X}_{Pcg}$ and $\dot{Y}_{Pcg}$ can be integrated to yield the coordinates of the aircraft's center of gravity in the plane $(Z + Z_{Pcg}) = 0$, which is the landing platform. Therefore, the required coordinates are

$$X_{Pcg} = X_o + \int \dot{X}_{Pcg}\, dt \tag{42}$$

$$Y_{Pcg} = Y_o + \int \dot{Y}_{Pcg}\, dt \tag{43}$$

where X_o and Y_o are initial conditions.

The velocity of the aircraft's center of gravity normal to the landing platform is given by the equation

$$\dot{Z}_{cg} = [(d_{13}u + d_{23}v + d_{33}w) - (W + PY_{Pcg} - QX_{Pcg})] \tag{44}$$

Because X_{Pcg} and Y_{Pcg} are known from equations (42) and (43), equation (44) can be integrated to yield

$$Z_{cg} = Z_o - Z_{Pcg} + \int \dot{Z}_{cg}\, dt \tag{45}$$

where Z_o is the initial distance from the XY plane of the ship's coordinate system.

The initial values X_o, Y_o, and Z_o are ship axes components. When the initial conditions are given in earth-fixed axes, a transformation from these axes to ship axes is required.

Given that $\bar{X}_o$, $\bar{Y}_o$, and $\bar{Z}_o$ are initial components in earth-fixed axes, the corresponding ship axes components are

$$\begin{pmatrix} X_o \\ Y_o \\ Z_o \end{pmatrix} = [T]_{ES} \begin{pmatrix} \bar{X}_o \\ \bar{Y}_o \\ \bar{Z}_o \end{pmatrix}$$

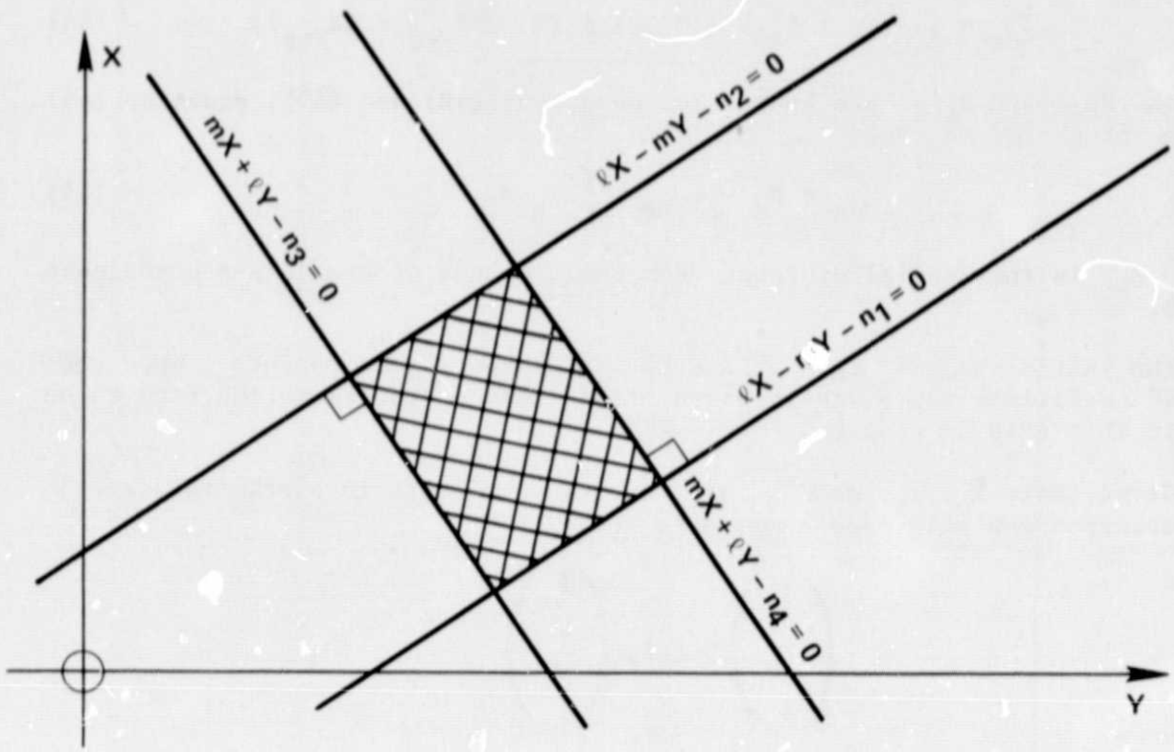
When equations (42) through (45) are modified in accordance with this transformation, we obtain

$$\begin{pmatrix} X_{Pcg} \\ Y_{Pcg} \\ Z_{Pcg} + Z_{cg} \end{pmatrix} = [T]_{ES} \begin{pmatrix} \bar{X}_o \\ \bar{Y}_o \\ \bar{Z}_o \end{pmatrix} + \int \begin{pmatrix} \dot{X}_{Pcg} \\ \dot{Y}_{Pcg} \\ \dot{Z}_{cg} \end{pmatrix} dt$$

The landing platform is assumed to be rectangular and bounded by the lines

$$\left.\begin{aligned} (\ell X - mY - n_1) &= 0 \\ (\ell X - mY - n_2) &= 0 \\ (mX + \ell Y - n_3) &= 0 \\ (mX + \ell Y - n_4) &= 0 \end{aligned}\right\} \tag{46}$$

where ℓ, m, and n_i are constants: i = 1, 2, 3, 4. (See diagram.)



For this and all other geometrical shapes, Z_{cg} should be computed continuously from the initiation of each run, but the information generated is only required when appropriate geometrical conditions are met. For the configuration assumed, these are

$$
\begin{aligned}
(\ell X_{Pcg} - mY_{Pcg}) &\geq n_1 \\
(\ell X_{Pcg} - mY_{Pcg}) &\leq n_2 \\
(mX_{Pcg} + \ell Y_{Pcg}) &\geq n_3 \\
(mX_{Pcg} + \ell Y_{Pcg}) &\leq n_4
\end{aligned}
\qquad (47)
$$

When these conditions are satisfied, the aircraft is over the landing platform.

For a rectangular platform, with sides parallel to the ship's XY coordinate axes, the coefficient m vanishes and equations (46) assume the simpler form:

$$\left.\begin{aligned}(\ell X - n_1) &= 0\\(\ell X - n_2) &= 0\\(\ell Y - n_3) &= 0\\(\ell Y - n_4) &= 0\end{aligned}\right\}\tag{48}$$

Given that the platform has length L and breadth B, these equations can be interpreted as follows:

$$\left.\begin{aligned}(X - X_P) &= 0\\(Y - Y_P) &= 0\\(X - X_P - L) &= 0\\(Y - Y_P - B) &= 0\end{aligned}\right\}\tag{49}$$

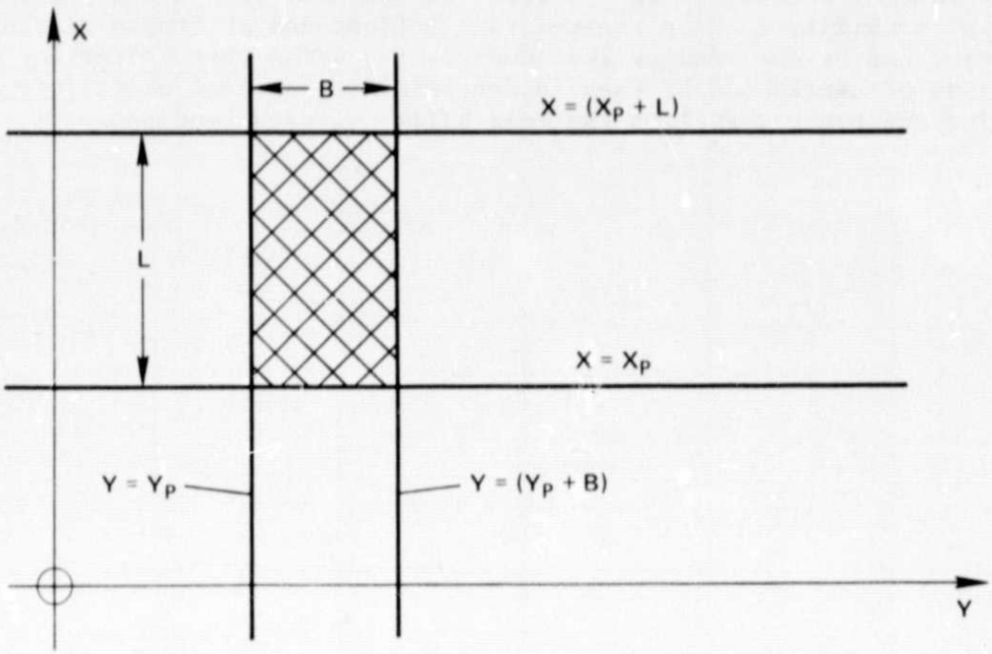
where X_P and Y_P are platform boundary lines, and

$$X_P = n_1/\ell$$

$$Y_P = n_3/\ell$$

$$X_P + L = n_2/\ell$$

$$Y_P + B = n_4/\ell$$



In this case, the aircraft will be over the landing platform when the following geometrical conditions are met:

$$\left.\begin{aligned} X_p \le X_{Pcg} \le (X_p + L) \\ Y_p \le Y_{Pcg} \le (Y_p + B) \end{aligned}\right\} \qquad (50)$$

When either these conditions or the conditions in (47), as appropriate, are satisfied, the computed value of Z_{cg} is used to determine when equations (37) through (39) are satisfied. At this juncture, X_{Pi} and Y_{Pi} are evaluated from the equations

$$X_{Pi} = X_{Pcg} + (d_{11}x_i + d_{21}y_i + d_{31}z_i)$$

$$Y_{Pi} = Y_{Pcg} + (d_{12}x_i + d_{22}y_i + d_{32}z_i)$$

where x_i, y_i, and z_i are the coordinates of the ith landing wheel referred to aircraft body axes. The relative velocities required for performance measures are then obtained from equations (24) through (26), as appropriate.

CONCLUSIONS

Simulation experiments to determine the feasibility of landing V/TOL aircraft on ships at sea, require that the motion and the attitude of the aircraft relative to the landing platform be known. The equations describing the relative motions and attitudes have been derived in a form that is suitable for simulation purposes and the measurement of pilot performance during the landing maneuver. The performance measures can be used to determine a pilot's ability to keep the relative motions and attitudes within limits prescribed by the landing gear design. By using this criterion, the equations presented can be used to determine the success or failure of each landing and hence establish the feasibility of such landings.

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