DETERMINATION OF SUBCRITICAL DAMPING BY

MOVING-BLOCK/RANDOMDEC APPLICATIONS

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SUMMARY

Two techniques are described which allow the determination of subcritical dampings and frequencies during aeroelastic testing of flight vehicles. The two techniques are the moving-block technique and the randomdec technique. The moving-block technique is shown to have the advantage of being able to provide damping and frequency information for each mode which might be present in a signal trace, but it has the disadvantage of requiring that the structure be excited transiently. The randomdec technique requires only random turbulence for excitation, but the randomdec signature is difficult to analyze when more than one mode is present. It is shown that by using the moving-block technique to analyze the randomdec signatures the best features of both methods are gained. Examples are presented illustrating the direct application of the moving-block method to model helicopter rotor testing and application of the combined moving-block/randomdec method to flutter studies of two fixed-wing models.

INTRODUCTION

Determination of subcritical damping during flutter tests both in wind tunnels and in flight is a subject which is currently receiving widespread attention. Since flutter is a potentially dangerous aeroelastic instability which can lead to catastrophic structural failure, it is desirable to obtain the flutter boundary without actually experiencing flutter. Traditionally, wind-tunnel flutter model test procedures have been to treat flutter as an event that either occurs or does not occur. The models are actually taken to the flutter condition, and by varying tunnel parameters (Mach number and dynamic pressure), sufficient flutter points are obtained to define the flutter boundary. This practice has in the past led to the total destruction of some very expensive models. Although the need for subcritical damping data has long been recognized, obtaining these data is not an easy task, and subcritical damping techniques have not been routinely used in the past. A large part of the difficulty has been associated with the inability to reduce, analyze, and display model damping data in near real time so that the damping can be continuously monitored during approach to the flutter boundary. The installation of the computer controlled data acquisition system has now made it practical to apply subcritical damping methods to flutter tests in the Langley transonic dynamics tunnel.

This paper describes two techniques which are currently being used in wind-tunnel aeroelastic model tests of both helicopter rotors and fixed wings. The two techniques to be described are the so-called moving-block technique and the randomdec technique. Although neither of the techniques is new, it is felt that the combined application of the two analyses is unique, particularly with respect to fixed-wing flutter testing.

SYMBOLS

A	amplitude of transient response
$F(\omega)$	Fourier transform at frequency ω
i	imaginary number, $\sqrt{-1}$
k	an integer in equation (14)
N	an integer in equation (10); total number of data samples in equation (13)
$\bar{N}$	number of data samples in block used for frequency optimization
T	period of boxcar function
t	time
Δt	time between discrete data samples
$u(t)$	boxcar function, equation (2)
$y(t)$	transient response of single-degree-of-freedom system, equation (1)
ζ	damping ratio
τ	start time of boxcar function
ϕ	phase angle
ω	damped natural frequency
ω_n	undamped natural frequency

MOVING-BLOCK TECHNIQUE

The moving-block technique was originally developed by the Lockheed-California Company, and its use in rotary wing aeroelastic stability testing has been reported in references 1 and 2. A formulation of this technique has

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been developed at the Langley Research Center, and it has been implemented on the recently installed data acquisition system for the Langley transonic dynamics tunnel.

The moving-block technique is a method which allows the determination of modal dampings and frequencies from a response signal of a structure which has been excited transiently. The transient excitation may consist of a sinusoidal input which has been abruptly terminated or it may be an impulsive excitation. In any event, if the damping and frequency of a particular mode are desired, it is necessary that this mode be excited by the type of excitation chosen. This requirement that the structure be excited and then be allowed to decay freely is one of the disadvantages of the method, but for many applications it is not an overly burdensome requirement. This is particularly true in helicopter applications where the existing control system can be used to supply the necessary excitation of the rotor system.

In order to illustrate how the moving-block technique works, consider the transient response of a single-degree-of-freedom system which may be written as

$$y(t) = Ae^{-\zeta\omega_n t} \sin(\omega t + \phi) \tag{1}$$

where

$$\omega^2 = \omega_n^2(1 - \zeta^2)$$

Now compute the finite Fourier transform of this response at the damped frequency ω from time τ to time $\tau + T$. This is the same as multiplying the response by the boxcar function

$$u(t) = \begin{cases} 0 & \text{for } t < \tau \\ 1 & \text{for } \tau \le t \le \tau + T \\ 0 & \text{for } t > \tau + T \end{cases} \tag{2}$$

as shown in figure 1 and computing the infinite transform. The significance of the starting time τ is discussed subsequently. The finite transform is given by

$$F(\omega) = \int_{\tau}^{\tau+T} Ae^{-\zeta\omega_n t} \sin(\omega t + \phi) e^{-i\omega t} dt \tag{3}$$

This integration may be performed in closed form and the result is

$$F(\omega) = \frac{A}{\zeta\omega_n(\zeta\omega_n + i2\omega)} \left(e^{-(i\omega+\zeta\omega_n)\tau} [(i\omega + \zeta\omega_n) \sin(\omega\tau + \phi) + \omega \cos(\omega\tau + \phi)] \right.$$

$$\left. - e^{-(i\omega+\zeta\omega_n)(\tau+T)} \{(i\omega + \zeta\omega_n) \sin[\omega(\tau + T) + \phi] + \omega \cos[\omega(\tau + T) + \phi]\} \right) \tag{4}$$

After a considerable amount of algebraic manipulation the amplitude of this transform may be written as

$$|F(\omega)| = \frac{Ae^{-\zeta\omega\tau}}{2\zeta\omega} \{1 - 2e^{-\zeta\omega T} + e^{-2\zeta\omega T} + (1 - e^{-\zeta\omega T})\zeta \sin 2(\omega\tau + \phi)$$

$$- e^{-\zeta\omega T}(1 - e^{-\zeta\omega T})\zeta \sin 2[\omega(\tau + T) + \phi]\}^{1/2} \tag{5}$$

In obtaining this expression, it has been assumed that $\zeta << 1$ and thus $\omega_n \approx \omega$. Also, terms involving ζ^2 have been deleted as being small compared to unity. It is convenient to write this expression in the form

$$|F(\omega)| = \frac{A}{2\omega} e^{-\zeta\omega\tau} \left[\frac{1 + f(\zeta)}{\zeta^2}\right]^{1/2} \tag{6}$$

where

$$f(\zeta) = - 2e^{-\zeta\omega T} + e^{-2\zeta\omega T} + (1 - e^{-\zeta\omega T})\zeta \sin 2(\omega\tau + \phi)$$

$$- e^{-\zeta\omega T}(1 - e^{-\zeta\omega T})\zeta \sin 2[\omega(\tau + T) + \phi] \tag{7}$$

Taking the natural logarithm of equation (6) yields

$$\ln|F(\omega)| = - \zeta\omega\tau + \ln\left(\frac{A}{2\omega}\right) + \frac{1}{2} \ln\left[\frac{1 + f(\zeta)}{\zeta^2}\right] \tag{8}$$

The last term in equation (8) may be expanded in a Maclaurin series to yield

$$\ln|F(\omega)| = - \zeta\omega\tau + \ln\left(\frac{A}{2\omega}\right)$$

$$+ \frac{1}{2} \ln\left((\omega T)^2 + (\omega T)\{\sin 2(\omega\tau + \phi) - \sin 2[\omega(\tau + T) + \phi]\}\right)$$

$$- \frac{\zeta}{4} \omega T \left\{\frac{2\omega T + \sin 2(\omega\tau + \phi) - 3 \sin 2[\omega(\tau + T) + \phi]}{\omega T + \sin 2(\omega\tau + \phi) - \sin 2[\omega(\tau + T) + \phi]}\right\} \tag{9}$$

From this expression it can be seen that if a plot of $\ln|F(\omega)|$ versus τ is made, the resulting curve will be the superposition of a straight line with slope $-\zeta\omega$ and an oscillatory component which oscillates about the straight line with a frequency of 2ω. This fact can be more easily seen if it is assumed that T is an integral multiple of the basic period of oscillation. That is,

$$T = \frac{2\pi N}{\omega} \qquad (N = 1, 2, 3, \ldots) \tag{10}$$

With this assumption, equation (9) becomes

$$\ln|F(\omega)| = -\zeta\omega\tau + \frac{1}{2}\zeta \sin 2(\omega\tau + \phi) + C \tag{11}$$

where C is a constant given by

$$C = \ln\left(\frac{A}{2\omega}\right) + \ln(\omega T) - \frac{\zeta\omega T}{2} \tag{12}$$

Thus if the boxcar shown in figure 1 is started at $\tau = 0$ and successive discrete transforms at frequency ω are performed for increasing values of τ, a plot can be made from which the damping can be determined. It is precisely this process which is used in the moving-block analysis.

This analytical background has, for simplicity, dealt with the response of a single-degree-of-freedom system. The basic strong point of the moving-block method is, however, its ability to provide frequency and damping information for each of the modes in a multimode response signal. If a multimode response is thought of in terms of a Fourier series representation, then the response is simply a summation of several single-degree-of-freedom responses, and the Fourier transform effectively provides the means for isolating the various components of the response.

IMPLEMENTATION OF MOVING-BLOCK TECHNIQUE

The moving-block technique described previously has been implemented on the data acquisition system of the Langley transonic dynamics tunnel. The data system consists of a Xerox Sigma 5 digital computer coupled with a 60-channel analog front end. The system is equipped with a graphics display unit which allows data reduction to be accomplished with as much interaction by the engineer as desired. A more detailed description of the data system is presented in reference 3.

The moving-block technique is set up as a completely interactive program. The sequence of events which are incorporated in the analysis is depicted in figure 2. The first step in the process is to obtain the signal to be analyzed. This signal may be digitized directly from the data stream coming from the model which has been transiently excited, or the signal may be a randomdec signature which is passed from the randomdec analysis to be described subsequently in this paper.

Once the signal to be analyzed is obtained, a fast Fourier transform (FFT) of the signal is computed. This transform is solely for the purpose of providing the analyst with information relative to the frequency content of the signal. The transform also allows the analyst to determine whether or not the mode of interest has been excited. From the FFT results the analyst selects the frequency for the mode to be analyzed. The peak in the FFT results may not correspond to the actual frequency in the signal because of the fact that the frequency resolution available from the FFT is dependent upon both digital sampling rate and number of points in the sample as given by

$$\Delta f = \frac{1}{N \Delta t} \tag{13}$$

Thus a scheme to optimize the selected frequency has been included in the analysis. In accomplishing this optimization a segment, or block, of the input transient response signal is first selected. Generally, the block length is chosen to be one-half the total number of points in the data sample. Let the number of samples in the block be denoted by $\bar{N}$. The algorithm used by FFT analyses for determining the frequencies at which the transform is computed is

$$f = \frac{k}{\bar{N} \Delta t} \qquad (k = 0, 1, 2, \ldots) \tag{14}$$

By using the frequency selected from the original FFT results, the sampling rate, and $\bar{N}$, a value for k can be calculated. Then, if k is held constant and $\bar{N}$ is changed by one data point, a small change in the computed frequency occurs. The optimization then proceeds as follows. Compute the discrete transform at the following three frequencies:

$$\left.
\begin{aligned}
f_{\bar{N}-1} &= \frac{k}{(\bar{N} - 1) \Delta t} \\[2ex]
f_{\bar{N}} &= \frac{k}{\bar{N} \Delta t} \\[2ex]
f_{\bar{N}+1} &= \frac{k}{(\bar{N} + 1) \Delta t}
\end{aligned}
\right\} \tag{15}$$

Note that the block size is different for each computation. By observing the amplitude of the transform from these three calculations, one can determine how to continue changing the block size to cause the magnitude of the transform to reach a peak. When this peak is reached, the frequency corresponding to that peak is the optimized frequency at which the damping calculations are made.

The damping calculation is made by using the optimized frequency and the block size which resulted in this frequency and by computing successive discrete Fourier transforms as the block is moved down the data record. The block is first positioned at the beginning of the record, the transform is computed, and the logarithm of the transform amplitude is plotted. The block is then moved down the data record one data sample and this process repeated. When the block reaches the end of the data record a plot equivalent to a plot of equation (9) has been made. The damping in the mode being analyzed is obtained from the slope of a linear least-squares fit to this curve.

HELICOPTER TRANSIENT RESPONSE APPLICATION

The moving-block technique was originally implemented at Langley to facilitate subcritical aeroelastic testing of model helicopter rotors in the Langley transonic dynamics tunnel. An in-house model, termed the generalized rotor aeroelastic model (GRAM), is used to test rotors up to 3.4 m (11 ft) in diameter. The model is shown in the left part of figure 3 with the Bell Helicopter Company flex-hinge rotor installed. The right part of figure 3 shows the hydraulic control system of the model which is used both for quasi-static control of the rotor and for transient excitation of the rotor for subcritical damping measurements.

In conducting the rotor tests, the rotor is first trimmed to the desired operating condition, and then the excitation is started either manually or under computer control. The type of excitation, amplitude, frequency, and number of cycles of excitation are options which are manually selectable by the engineer. The computer is programed to begin digitizing data from the channel of interest two or three cycles before the termination of the excitation. The digitized data are then plotted on the graphics display unit (GDU) so that the analyst may select the point on the signal trace where he would like to start the damping analysis. It has been found desirable to have the analyst select the starting point rather than have the computer determine when the excitation terminates and then begin digitizing data because of certain time lags inherent in the system. The analyst also generally feels more confident about the data if he can see some of the forced response in the trace just prior to termination of the excitation.

The Bell Helicopter Company flex-hinge rotor was recently tested on the GRAM. One of the objectives of this test was to examine the amount of in-plane damping available in the rotor system. Figure 4 is a typical GDU display from this particular test. The data trace in the lower left quadrant of this figure was taken from one of the blade chordwise bending gages. Note that this plot begins at the starting point previously selected by the analyst. The plot in the upper left quadrant of the figure is the FFT amplitude plotted out to the Nyquist frequency. Since the frequency of interest may be obscured on this plot, the analyst is provided the capability of interactively changing the frequency range over which the FFT amplitude is plotted. The plot in the lower right quadrant of figure 4 is an expanded scale version of the plot in the upper left quadrant. The frequency of interest is selected by the analyst from either of the FFT plots by use of a light gun. This selected frequency is optimized automatically, and the damping plot is displayed in the upper right quadrant. After the analyst selects the start and stop times for the least-squares fit, the least-squares calculations are made, and the computed frequency and damping are displayed at the bottom of the GDU screen. Options are provided for changing the block size and repeating the analysis at the same frequency and for selecting a new frequency for which the modal damping is desired.

A word of explanation is in order concerning the damping plot in the upper right quadrant of figure 4. The plot is seen to have a portion which approximates a straight line and a later portion which deviates considerably from the

straight line. This deviation from the straight line occurs when the mode being analyzed damps out rapidly. Once the mode of interest damps out, the calculations are influenced by other modes in the signal as well as by noise. This behavior of the damping plot illustrates the desirability of an interactive formulation of this technique.

RANDOMDEC TECHNIQUE

Since a detailed description of the randomdec method is presented in reference 4, only the highlights of the method are described. Simply stated, the randomdec technique provides a means for obtaining damping and frequency information by performing an ensemble average of segments of a random time history of the structural response. The underlying assumption in the method is that the structural response is the linear superposition of the responses to a step force (initial displacement), an impulsive force (initial velocity), and a random force. If the segments used in the ensemble average are chosen so that the initial displacement is the same for all segments and the initial velocities of alternating segments have opposite signs, then the resulting ensemble average, called the randomdec signature, represents the response to a step force, since the averages of the impulse force and random force components approach zero as the number of segments used in the ensemble average increases.

For a single-degree-of-freedom system the damping and frequency can be determined directly from the randomdec signature. The dampings and frequencies of the individual modes of a multi-degree-of-freedom system can be determined either by bandpass filtering the response signal before determining the randomdec signature so that only one mode is present or by further processing of the signature to separate it into its individual frequency components. For example, in the latter case a curve fitting procedure has been presented in reference 5 for determining the individual frequency components of a randomdec signature that contains the responses of several modes.

The randomdec method is very attractive for use in flutter investigations, since no discrete forced excitation is required. The almost always present wind-tunnel turbulence in the case of model tests and atmospheric turbulence in the case of flight tests are sufficient to provide the needed random excitation. Some results from wind-tunnel model studies are presented in reference 6, and some results from a flight flutter clearance study are presented in reference 7.

One of the disadvantages of the randomdec method to date has been the difficulty in determining the damping when more than one mode is present in the randomdec signature. The great advantage of the moving-block technique is, on the other hand, the ability to analyze signals which may have several modes present and to allow the analyst to determine the damping present in each of the modes. It seemed only natural, then, to use the moving-block technique to analyze randomdec signatures. That is, the randomdec signature is used as the transient response input to the moving-block analysis. Some results of applications of the combined moving-block/randomdec method are discussed in the subsequent section.

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The combined moving-block/randomdec method has been used during several wind-tunnel model studies in the Langley transonic dynamics tunnel. Some results from two of these applications are described in the following discussion.

The first application to be described was in the testing of a high-aspect-ratio subsonic-transport wing model. A photograph of this cantilever-mounted model is shown in figure 5. The flutter boundary for this model was determined during testing and also is shown in figure 5. Some subcritical damping data were obtained as the flutter boundary was approached along the path indicated by the dashed line in the figure. The conditions at which damping and frequency were evaluated are indicated by the circle symbols on the figure. At these six conditions the wind-tunnel conditions were held constant, and a randomdec signature was determined and then processed through the moving-block analysis to determine the damping and frequency. One of the randomdec signatures from this test and the results of applying the moving-block analysis to this signature are shown in figure 6. The resulting subcritical damping results are presented in figure 7 in the form of the variation of damping in the critical flutter mode with Mach number and dynamic pressure. It was necessary to plot the damping versus both of these parameters since both were being varied as the flutter boundary was approached. The actual flutter point is indicated by the square symbols on the figure. Note that the flutter point predicted by extrapolating the subcritical damping results is very close to the actual flutter condition.

The second application described was to a low-aspect-ratio arrow-wing model. A photograph of this model is presented in figure 8. Some subcritical damping and frequency data were obtained for this model by using the moving-block/randomdec method as the flutter boundary was approached in a manner similar to that described for the transport-type wing model. Subcritical damping data for the arrow-wing model are presented in figure 9 in the form of the variations of damping ratio with dynamic pressure and Mach number. The measured flutter condition is indicated by the square symbols on the figure. Here again an extrapolation of the subcritical damping results predicts a flutter condition that is very close to that determined experimentally.

As the results presented show, the flutter conditions for both the subsonic-transport wing and arrow-wing models were predicted with sufficient accuracy by extrapolating moving-block/randomdec subcritical damping data. However, it should be pointed out that the method is still in a developmental stage and has not yet replaced the traditional method of actually determining flutter points in defining the flutter boundary during model tests in the Langley transonic dynamics tunnel.

CONCLUDING REMARKS

Two techniques have been discussed for determining damping and frequency information during subcritical aeroelastic testing of fixed-wing aircraft and helicopters. The moving-block technique has the advantage of being able to determine the damping and frequency for each of the modes which might be present in a response signal, but it has the disadvantage of requiring that the structure be excited transiently. This disadvantage has not presented any particular difficulties in the helicopter rotor tests conducted to date, however, since the helicopter control system may be used to provide the necessary excitation. In a fixed-wing test the requirement for transient excitation could be rather troublesome. The randomdec technique has the distinct advantage of providing frequency and damping information with random turbulence being the only excitation required. The disadvantage of the randomdec method is that frequency and damping data for a particular mode are difficult to obtain if the randomdec signature is made up of more than one mode. In order to capitalize on the strong points of each of these powerful methods, the two techniques have been used in series. That is, the moving-block technique has been used to analyze the randomdec signatures. The two examples presented to illustrate the application of this combined procedure indicate that the procedure can, in fact, be used for subcritical flutter testing. The method is, however, still in a developmental stage and it has not yet replaced the traditional method of actually determining flutter points in defining the flutter boundary during model tests in the Langley transonic dynamics tunnel.

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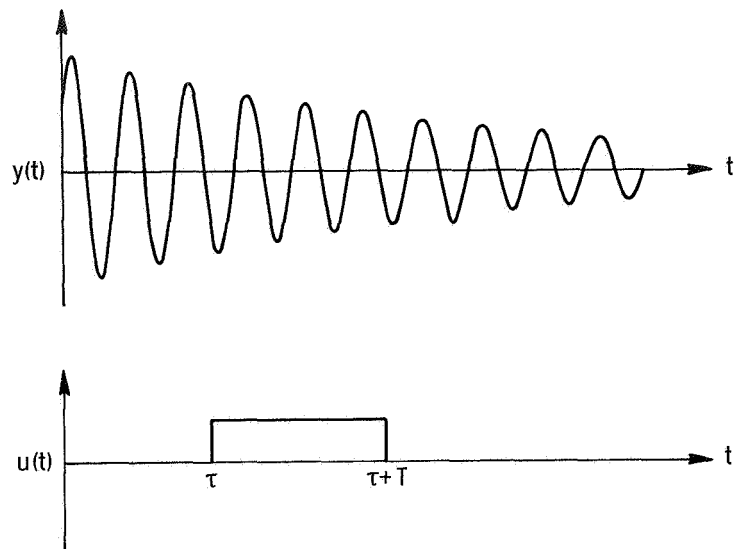
Figure 1.- Single-degree-of-freedom response function and boxcar function used in finite Fourier transform.
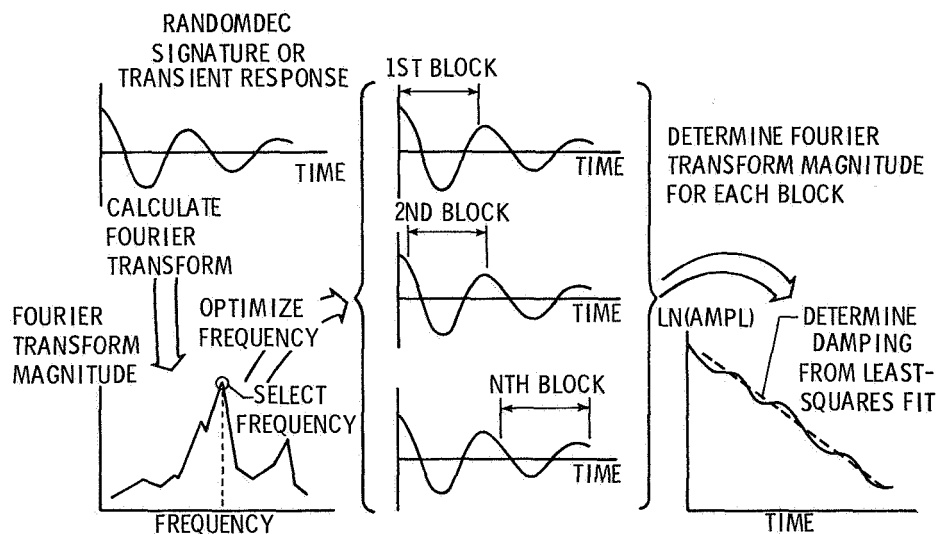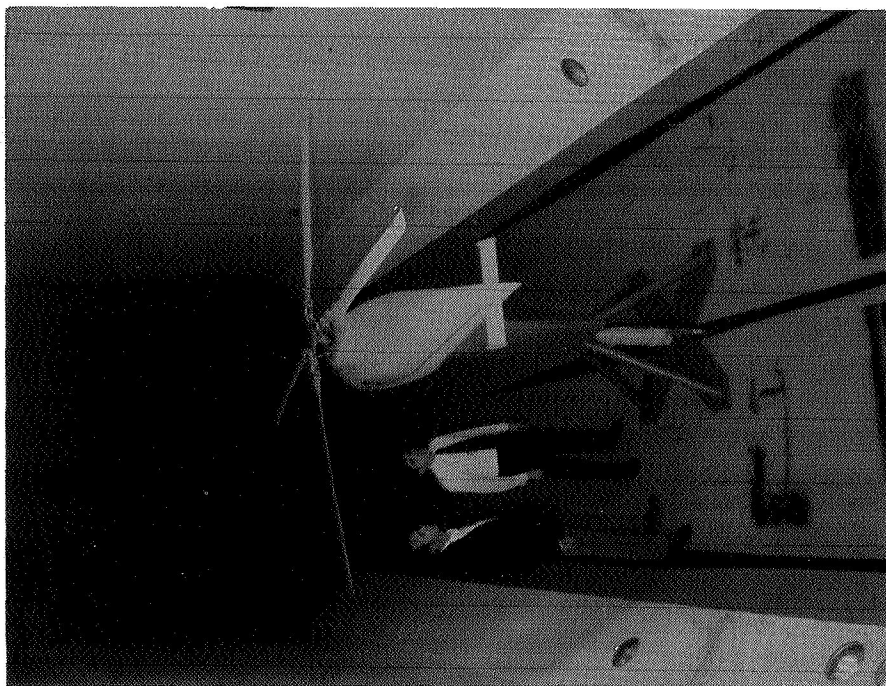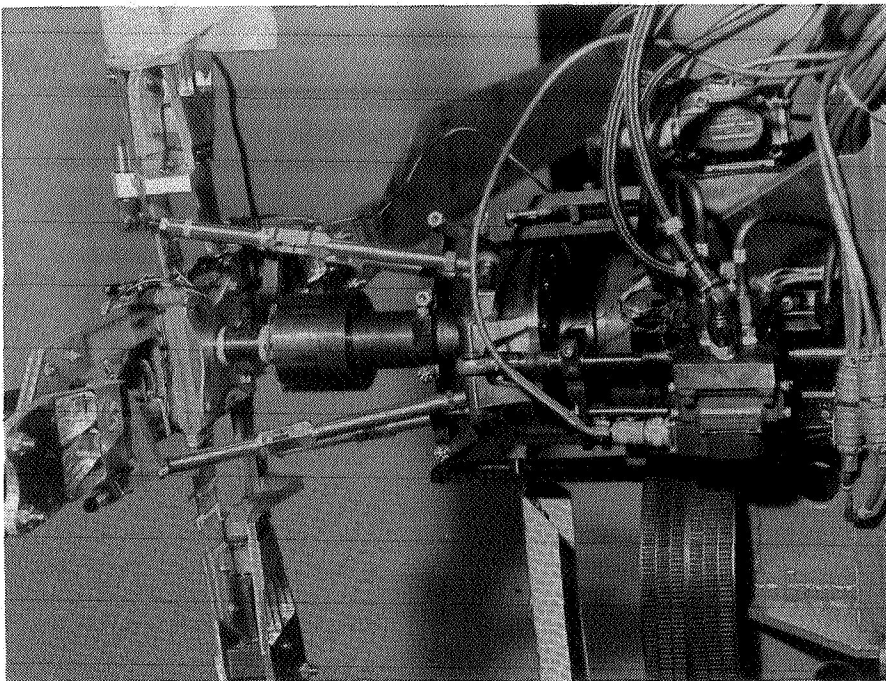

Figure 2.- Moving-block schematic illustration.

(a) GRAM with Bell Helicopter Company flex-hinge rotor installed.

(b) GRAM hydraulic control system.

Figure 3.- The generalized rotor aeroelastic model (GRAM) used for rotor aeroelastic studies in the Langley transonic dynamics tunnel.
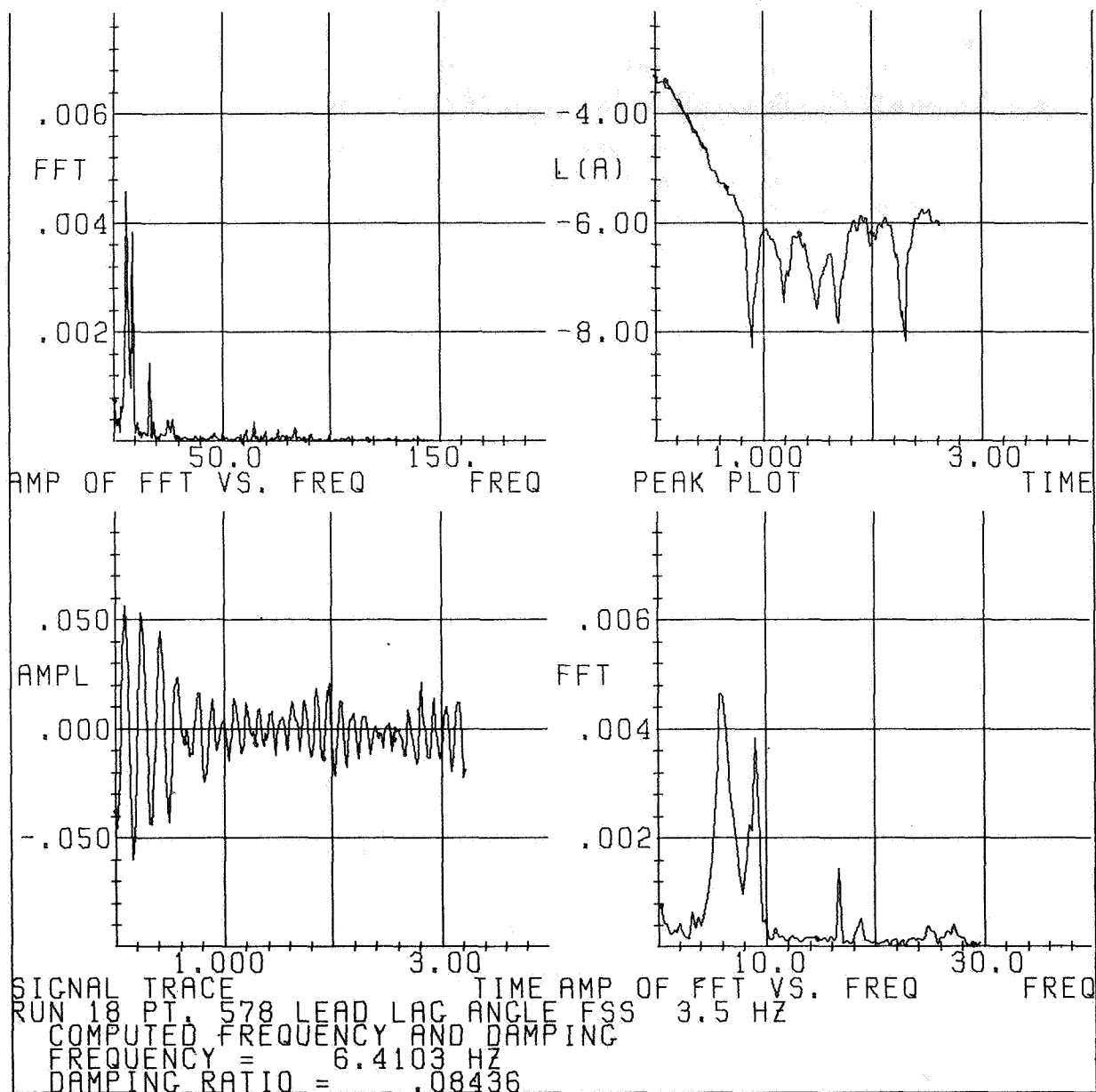
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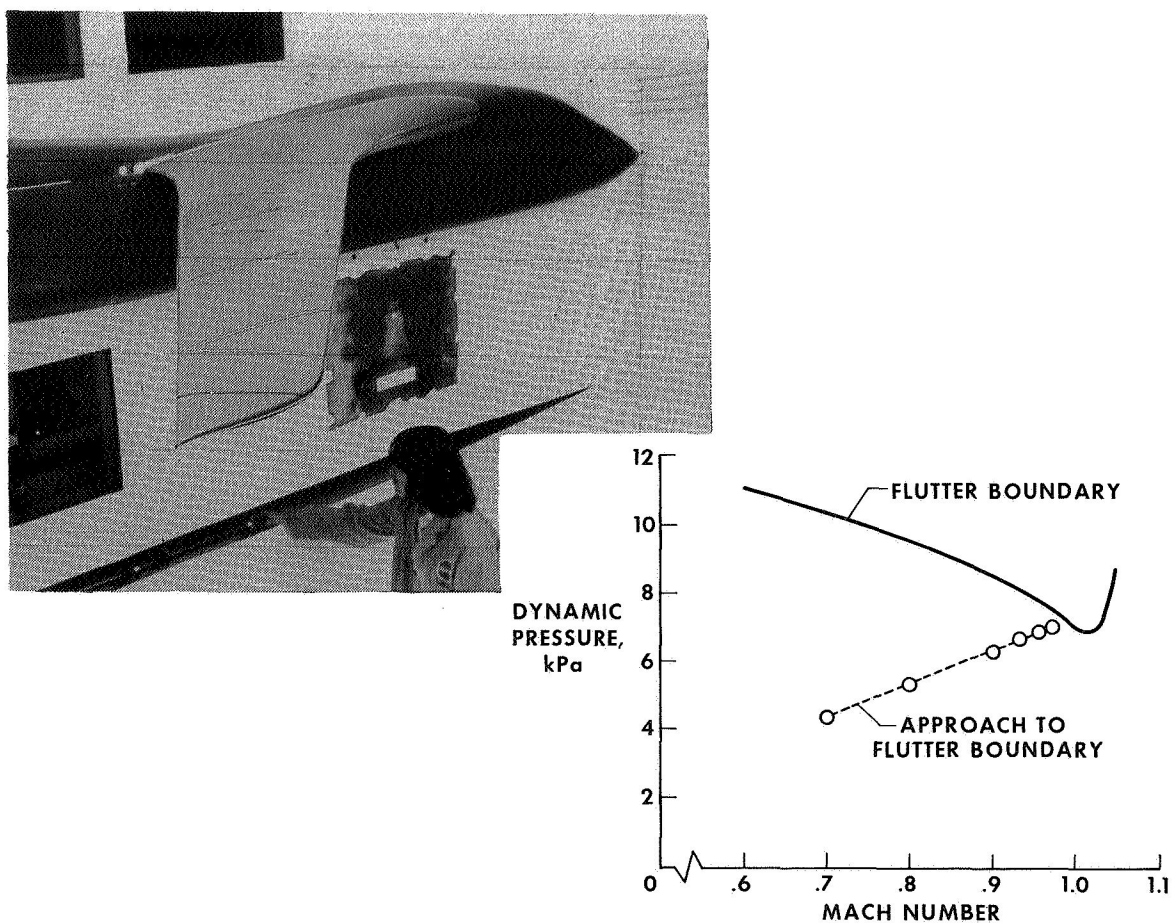Figure 4.- Typical subcritical response data from GRAM
flex-hinge rotor tests.

Figure 5.– Flutter boundary and subcritical approach
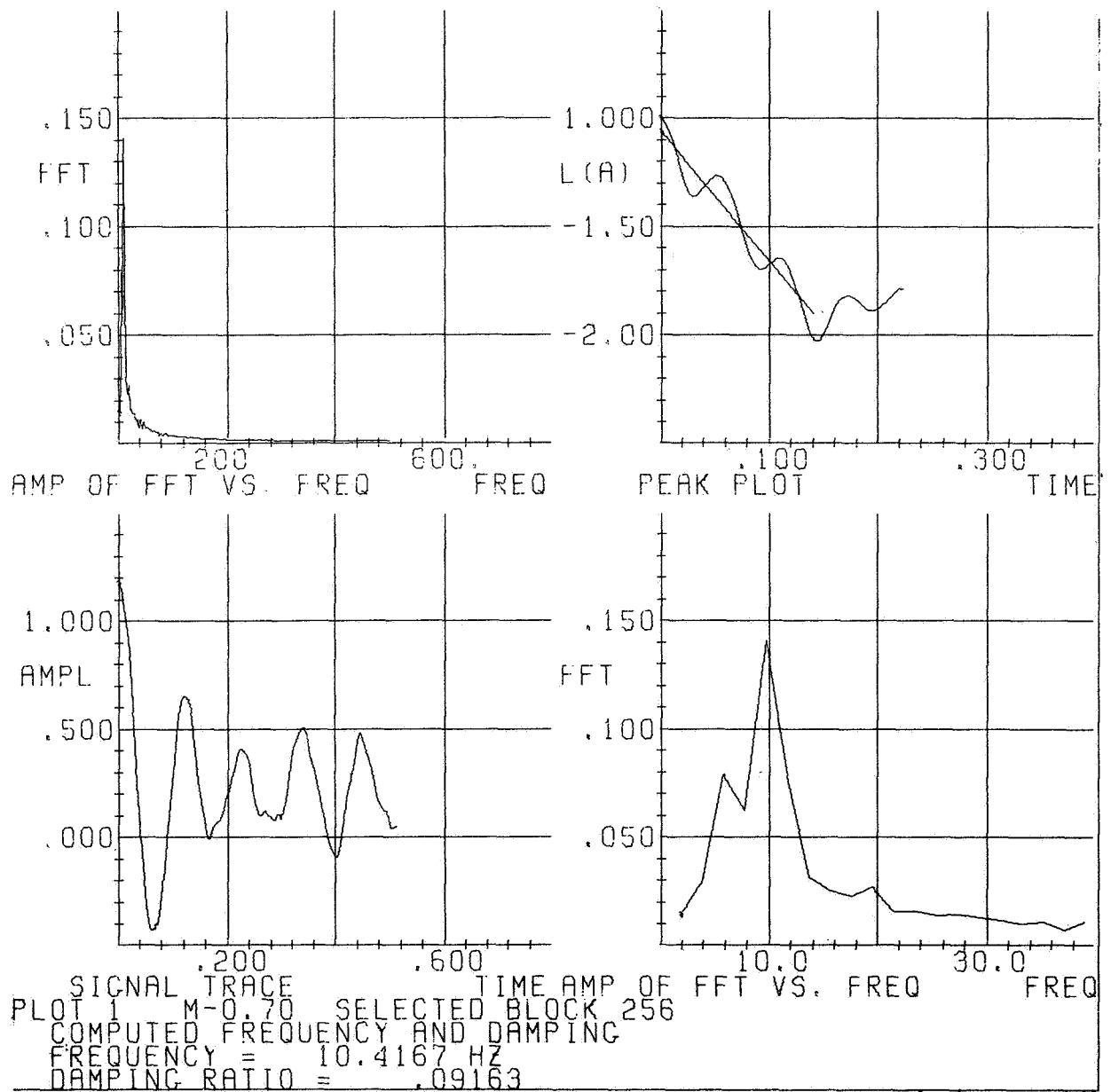to flutter boundary for the subsonic-transport
wing model.

Figure 6.- Sample moving-block/randomdec analysis results
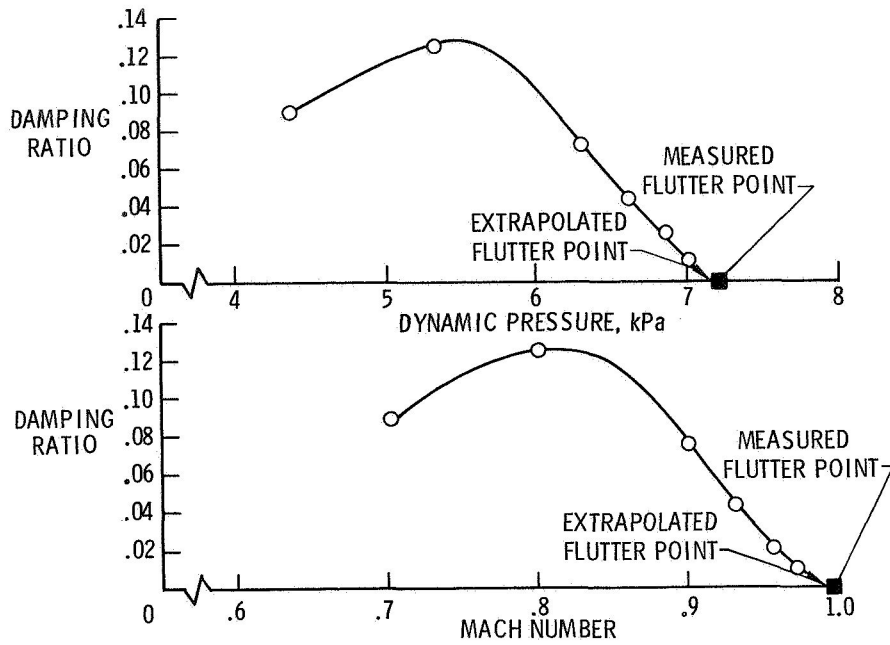for subsonic-transport wing model.

Figure 7.- Moving-block/randomdec subcritical damping
results for subsonic-transport wing model.

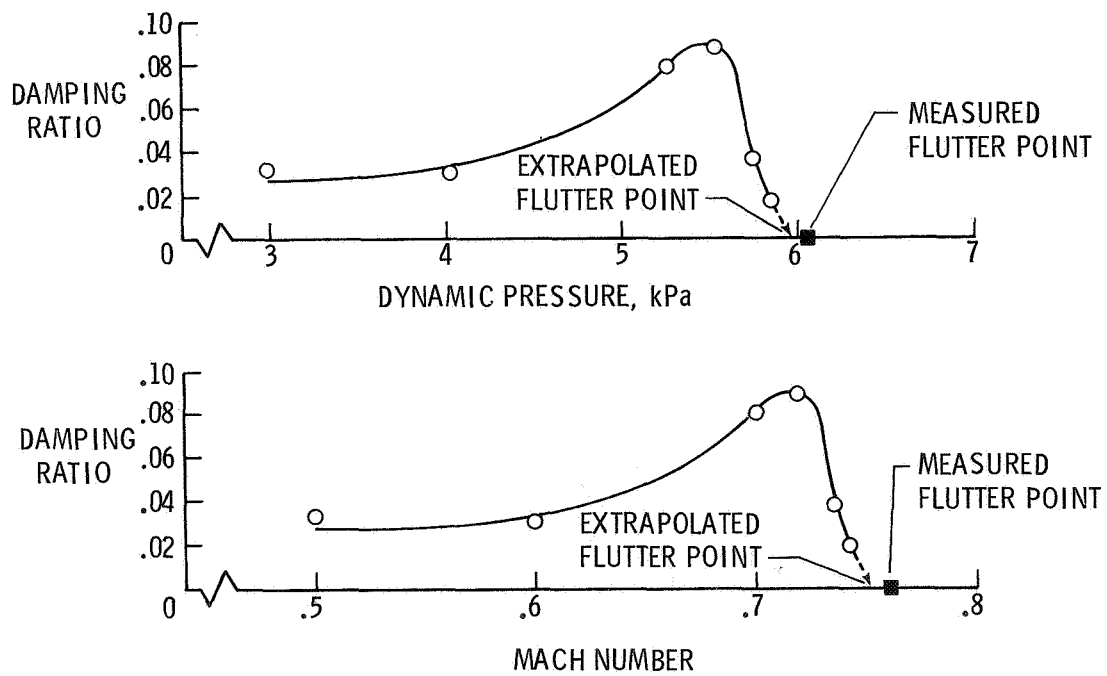Figure 8.- Photograph of arrow-wing model mounted in wind tunnel.

Figure 9.- Moving-block/randomdec subcritical damping
results for arrow-wing configuration.