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ORBIT DETERMINATION WITH THE TRACKING DATA RELAY SATELLITE SYSTEM

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ABSTRACT

This paper investigates the possibility of employing the tracking data relay satellite system to satisfy the orbit determination demands of future applications missions. To model the effect of relay satellite state error on orbit determination it is necessary to take into account the way in which the relay satellite epoch states were computed. It is shown that when the relay satellites are continuously and independently tracked from ground stations it is possible, using six hour data arcs, to recover user satellite state with an average error of about 25 m radially, 260 m along track, and 20 m cross track. For this arc length range sum data and range sum rate data are equally useful in determining orbits. For shorter arc lengths (20 min) range sum rate data is more useful than range sum data.

When relay satellites are not continuously tracked, user satellite state can be recovered with an average error of about 140 m radially, 515 m along track, and 110 m cross track. These results indicate that the TDRS system can be employed to satisfy the orbit determination demands of applications missions such as the MAGSAT and potential gradiometer missions provided the relay satellites are continuously and independently tracked.

CONTENTS

ILLUSTRATIONS

TABLES

TABLES (Continued)

ORBIT DETERMINATION WITH THE TRACKING DATA RELAY SATELLITE SYSTEM

INTRODUCTION

Future earth applications missions are likely to require satellite orbits with high inclination for global coverage, and low altitude for sensitivity. Orbit determination demands for such missions are expected to be rigid. Unfortunately, high inclination low altitude orbits are difficult to determine accurately. The low altitude feature implies that short arc data processing techniques are necessary to limit the effects of atmospheric drag error and gravity field error. The high inclination feature together with the need for short arc data processing implies a requirement for a large and well distributed set of dedicated ground based tracking stations.

An attractive alternative to the use of ground based stations is provided by the tracking and data relay satellites (TDRS). The TDRS system will consist of two satellites in geosynchronous orbits spaced approximately 130° apart at 41 and 171 degrees west longitude. The satellites will function as relays for range and doppler information from lower altitude satellites to a ground station located within the continental United States. The system provides a means by which low altitude satellites can be tracked on an almost continuous basis. This paper analyzes the possibility of using the TDRS system to accurately recover the orbits of earth applications satellites.

The unfamiliar feature of determining orbits by means of a TDRS system is the presence of the relay satellite states as an error source. In general the user of the system is uninterested in the states of the relay satellites and would rather not burden the numerical procedures with the need for simultaneously estimating relay satellite states along with the user satellite state. Hence in this study, it is assumed that in the reduction of satellite-to-satellite tracking data to estimate user satellite state, the relay satellite states are left unadjusted. Under this assumption, the uncertainties in relay satellite states function as an unmodeled and time varying error source which disturbes the estimate of user satellite state. Some subtleties are encountered in attempting to model the effect of this error source. The time histories of relay satellite state errors are functions of the way in which their epoch states were computed. For instance, suppose each relay satellite is continuously tracked over a given period to estimate an epoch state at the beginning of the period. If that epoch state is propagated to the end of the period using the same dynamic model that was used to process the data, the resultant errors will be constrained by the data fitting criterion implicit in the least squares reduction algorithm. The errors so obtained will be smaller than the errors obtained if either one did not match dynamic models or if one propagated the epoch state beyond the data collection period. The same phenomenon can be understood from a statistical vantage point by observing that when the dynamic models are matched the epoch state errors become correlated with dynamic parameter errors, and that over the data arc these correlations tend to limit the errors in the epoch state propagation. Clearly, in order to simulate the effect of relay satellite state errors on the orbit determination of user satellite state one must include these correlations by imposing a set of assumptions concerning the way in which relay satellite epoch states were computed.

The nominal orbit chosen for the numerical simulations was polar, circular, with a 300 km altitude. The effects of drag error, geopotential error, ground station location error, data noise and bias, and relay satellite state errors were included in the simulations. For the first set of simulations it was assumed that each relay satellite was tracked continuously by two ground stations. The results of these simulations reflect the optimal performance that may be expected from a TDRS system. In the second set of simulations it was assumed that the relay satellites are not continuously tracked and that for long time intervals the epoch state propagations are unconstrained by data. Hence for these simulations the relay satellite epoch state errors are uncorrelated with dynamic parameter errors. These simulations may be a more accurate reflection of the actual performance of the TDRS system in most cases.

RESULTS OF SIMULATION SET I

In performing a numerical simulation of an orbit determination the following procedure is usually adopted. A nominal value of the state of a spacecraft is assumed at an epoch. A model for the geopotential field and models for other forces which may act on the spacecraft are defined. From this information a nominal orbit is obtained. Next, assumptions are made concerning numbers and locations of tracking stations, data types and data acquisition rates. Using purely geometric considerations the correct or noiseless representation of the data is obtained. A random number generator is used to add stationary white noise with the appropriate standard deviation to the data. The procedure then is to introduce the simulated data into an orbit determination program (ODP) and estimate the state of the spacecraft at epoch along with other parameters in the dynamic or measurement model. In order to be realistic, however, the models used in the ODP should differ from the corresponding models used to generate the data. The differences will reflect a realistic evaluation of the dynamic and measurement modeling errors to be expected in the orbit determination process. Finally the estimated state at epoch and the dynamic model in the ODP are used to obtain an estimated orbit. The differences between the nominal orbit and the estimated orbit plotted as a function of time represent a typical realization of the error sequence of an orbit determination process.

The purpose of this set of simulations is to determine the performance of the TDRS system when the relay satellites are continuously and independently tracked. The first step was to define dynamic and measurement models for data generation. A relay satellite was placed at 41°W longitude. Tracking was provided by the Madrid and White Sands tracking stations. A second relay satellite was positioned at 171°W longitude and tracking was assumed from the White Sands and Australia stations. The gravitational constant was $.3986032(10)^{15}$ m^3/sec^2. Higher degree and order geopotential coefficients were obtained from the Goddard Earth Model 1 gravity field.[1] A radiation coefficient of 1.8 and an area to mass ratio of .0073 m^2/kg were assumed for each relay satellite. The low altitude satellite was placed in a polar, circular, 300 km orbit. The product of the area to mass ratio and the drag coefficient was assumed to be 10^{-5} m^2/gm. The Jacchia model[2] was used to describe the atmospheric density as a function of position. Range and doppler tracking of the low altitude satellite was generated using the TDRS system as a relay and White Sands as the ground station. A random number generator added white noise to each data type. Parameters in the dynamic and measurement models that were employed to generate the data were perturbed to simulate the effect of systematic error sources. The perturbed models and a standard least squares estimator were utilized to estimate relay satellite epoch states from the ranging data obtained from the Australia, White Sands, and Madrid tracking stations. The estimated relay satellite epoch states and the perturbed dynamic and measurement models were then used to process the satellite to satellite range and doppler data and to estimate low altitude epoch state. Tracking and error assumptions for the two orbit determination procedures are provided in Tables I and II.

The epoch state of the relay satellite which was located at 41°W longitude and which was tracked from the Madrid and the White Sands stations was recovered with an accuracy of about 15 m. The epoch state of the other relay satellite which was located at 171°W longitude and which was tracked from the Australia and the White Sands stations was recovered with an accuracy of about 60 m. The epoch time for each orbit determination process was the same. Also it is important to notice that the perturbed dynamic and measurement models used to estimate relay satellite epoch states were also used to estimate the user satellite epoch state from the satellite-to-satellite tracking data. This implies that the relay satellite epoch state errors were correlated with dynamic and measurement parameter errors and that these correlations tended to limit the growth of the relay satellite state errors over the satellite to satellite tracking data span. The orbit determinations were performed with range sum data, with range sum rate data, and with the combination of the data types. The epoch state estimates were propagated over the six hour data period and compared at descrete time points to the true state. The root mean squares of the errors were computed in the along track, cross track, and radial directions. The results are displayed in Table III.

It is useful to compare the results of Table III with the orbit determination needs of typical applications missions. Both the MAGSAT mission and proposed satellite borne gradiometer missions require low altitude, high inclination orbits and an orbit determination accurate to 60 m radially and 300 m horizontally.[3,4]

Table III indicates that with continuous tracking of the relay satellites and with six hour arc reductions of satellite to satellite tracking data, orbit determination requirements of the MAGSAT mission and potential gradiometer missions can be satisfied.

An interesting feature of Table IV is that range sum data and range sum rate data appear to be equally valuable in recovering orbits. This is not the case when much shorter arcs are used. Table IV shows the orbit determination results for a 20 min. data arc. Table IV demonstrates that for short arcs range sum rate data is more useful than range sum data and that the best results are obtained when both data types are used.

The significance of a given error source was determined by repeating the user satellite orbit determination process sequentially with the effect of a single error source included in the simulation. Data bias and survey error were found to be negligable error sources. For six hour arcs, geopotential error and drag error have a comparable effect on the orbit determination of the user satellite. Table V displays the isolated effect of relay satellite epoch state error on the recovery of user satellite state. A comparison of Table V with Table III indicates that under the assumptions of these simulations the relay satellite state errors dominate other error sources in the along track direction but not in the radial or cross track directions.

RESULTS OF SIMULATION SET II

In these simulations it is assumed that the relay satellites are not continuously tracked and consequently that relay satellite epoch states used to process SST data were obtained by an orbit propagation which was unconstrained by data. Under this assumption the relay satellite epoch state errors are statistically independent from other error sources which enter into the reduction of SST data. When error sources are independent, covariance analysis techniques are convenient for the study of orbit determination errors.

The difference in approach between a simulation study and a covariance analysis can be described as follows: in a simulation, data are generated and a least squares adjustment process is actually performed. The estimated state is then compared to an assumed true state and conclusions are obtained concerning the accuracy of the process. In a covariance analysis mode, the least squares adjustment process is postulated rather than actually performed, and under the assumption that over the range of expected errors, perturbations of orbital estimates are approximately linear functions of perturbations of the error sources, the associated covariance matrix is computed.

A covariance analysis program was used to generate the results of simulation set II. The program assumes as input a normal matrix for a set of parameters and a set of state transition matricies for various points along the arc as generated by an orbit determination program. By manipulating rows and columns of a normal matrix the parameters are effectively divided into two categories, a "solve for" category and a "consider" category. Parameters in the solve for category are assumed to be adjusted in the postulated least squares process. Parameters in the consider category are assumed to influence the functional relationship between the observations and the solve for parameters but to be left unadjusted in the postulated least squares process. Although it is not mathematically necessary, programming considerations require that errors in unadjusted parameters be treated as statistically independent from each other.

Not only is the covariance propagation mode of studying problems less expensive than the numerical simulation mode, but it also provides more information. For any point along the data arc the covariance propagation program can display a tabulation of the contribution to the uncertainty of each satellite state component due to the uncertainty of each consider parameter and due to the noise on the data. The tracking and error assumptions for simulation set II are included in Table VI. The magnitudes of error sources should be interpreted as standard deviations of the misrepresented parameters. The model for geopotential coefficient errors to degree and order 8 were obtained from ref. (5).

In that paper the Goddard Earth Model 5 geopotential field was calibrated against actual observations of 15° by 15° mean gravity anomalies and nominal standard deviation values were scaled to be consistent with the residuals. Numerical simulations were performed to determine the expected errors in relay satellite states when the satellites are not continuously tracked. The results showed that when a relay satellite epoch state is propagated as far as two to three days beyond the data arc used in its computation, errors are well into the kilometer region. To test the validity of the simulation results, a real data reduction was attempted. During July of 1975, the ATS-6 geosynchronous satellite was tracked by remote transponders for two 24 hour periods separated by ten days. The remote transponders were located at Madrid, Ascension Island, and Johannesburg with the master transmitter at Madrid. Each 24 hour pass of data was processed to estimate an ATS-6 epoch vector for an epoch time at the beginning of the pass. The epoch vectors were propagated to the end of the second pass and the resulting orbits were compared during the 24 hour overlap period. The mean difference in the two orbits was between 1 and 2 kilometers. Hence for simulation set II, relay satellite state errors were assumed to have a standard deviation of 600 m per component of position and 6 cm/sec standard deviation per component of velocity. The results can be scaled to reflect any other accuracy level. The nominal orbits for relay and user satellites are the same as in simulation set I. Table VII provides the average along track, cross track and radial standard deviations of the user satellite orbit determination over the six hour data arc. The table also displays the average contribution due to relay satellite state error. Figures 1, 2, and 3 show the time history of orbit determination errors for the user satellite.

Table VII clearly shows that for a six hour orbit determination, relay satellite state error is the dominant error source. The effect of relay satellite state error is proportional to the assumed errors of the relay satellite epoch states. This fact permits a scaling of the results of Table VII to reflect any given accuracy level. For instance, assuming the usual root sum square law for combining errors, if the relay satellite epoch states were uncertain to 1200 m per component rather than 600 m per component the numbers in the second row of Table VII would increase by a factor of 2 and the resulting total errors in the radial, along track, and cross track directions would be 281 m, 1016 m, and 218 m respectively.

The covariance propagation was repeated with the SST data arc increased to 12 hours. The errors were significantly larger than the errors shown on Table VII. It is likely that a covariance analysis of a shorter arc data reduction would indicate orbit determination errors smaller than those shown on Table VII. But the numerical stability of short arc data reductions may be marginal. The six hour data arc reduction was poorly conditioned with several correlations of magnitude greater than .9. Shorter arc data reductions would be less stable.

CONCLUSIONS

The tracking data relay satellite system offers a means of satisfying the rigid orbit determination demands of applications satellites. The unfamiliar feature of determining orbits by means of a TDRS system is the presence of relay satellite states as an error source. To model the effect of this error source it is necessary to take into account the way in which relay satellite epoch states were computed. For the first set of simulations it was assumed that each relay satellite was continuously tracked by two ground stations. The results show that when 6 hour arcs of SST data are processed user satellite state can be recovered with average error of 25 m radially, 260 m along track, and 20 m cross track. Range sum and range sum rate data are of equal value in computing orbits when arc lengths are six hours. For much shorter arc lengths (20 min) range sum rate data provides more accurate orbits than does range sum data. It is likely, however, that such short arc data processing would prove numerically unstable.

For the second set of simulations the relay satellite epoch error propagation is assumed to be unconstrained by data. This situation applies when the relay satellites are not continuously tracked. The major result of these simulations is that for a six hour data arc reduction, user satellite state can be recovered with an average error of 140 m radially, 515 m along track, and 110 m cross track. These results were obtained by assuming that the relay satellite epoch states were uncertain to 600 m per component. The results can be scaled to reflect a different accuracy level.

The results of this paper indicate that the TDRS system can be employed to satisfy the orbit determination demands of typical applications missions such as the MAGSAT provided the relay satellites are continuously and independently tracked.

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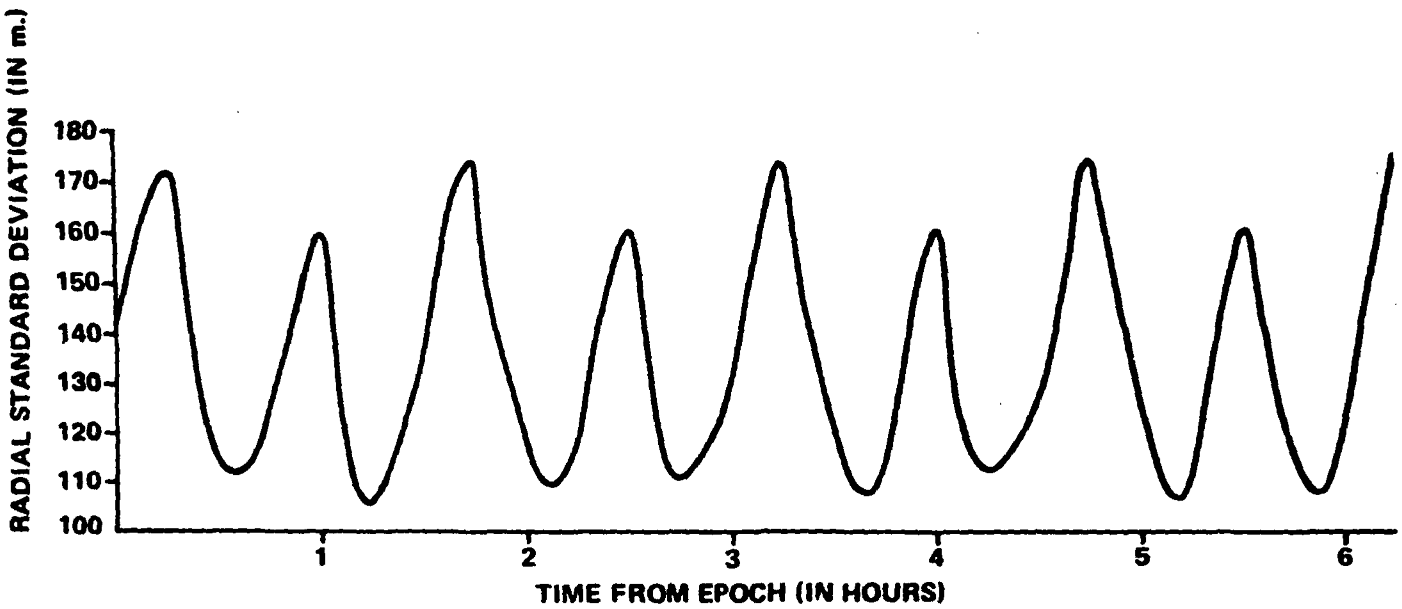

Figure 1. Radial Standard Deviation for User Satellite Orbit Determination

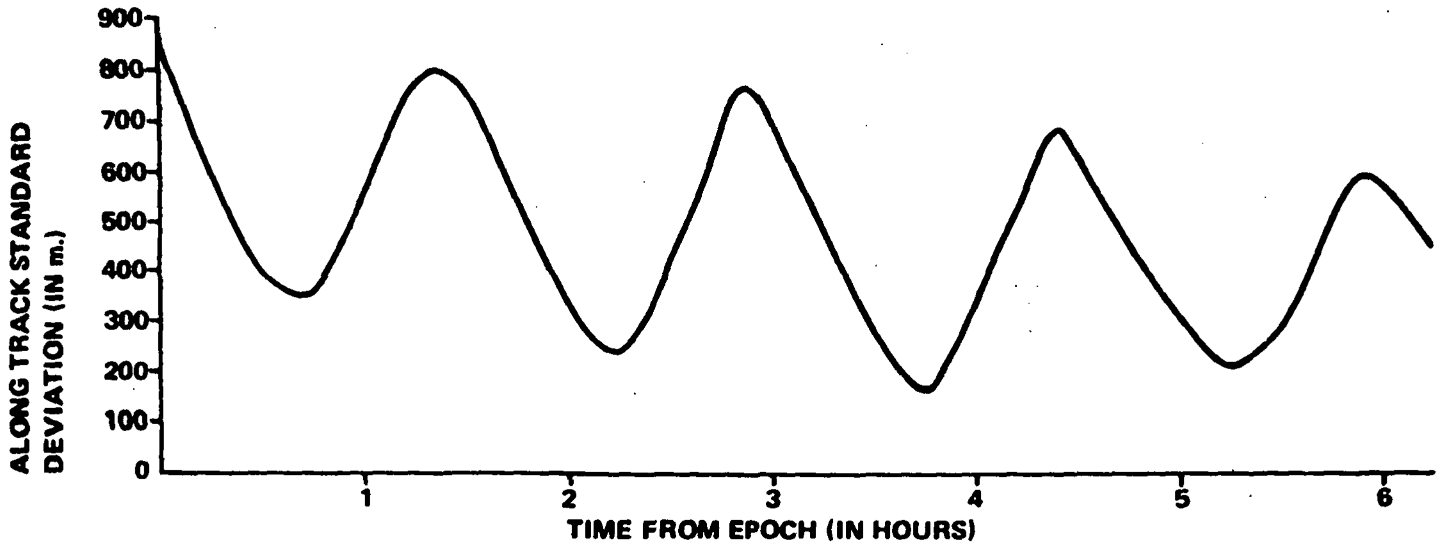

Figure 2. Along Track Standard Deviation for User Satellite Orbit Determination

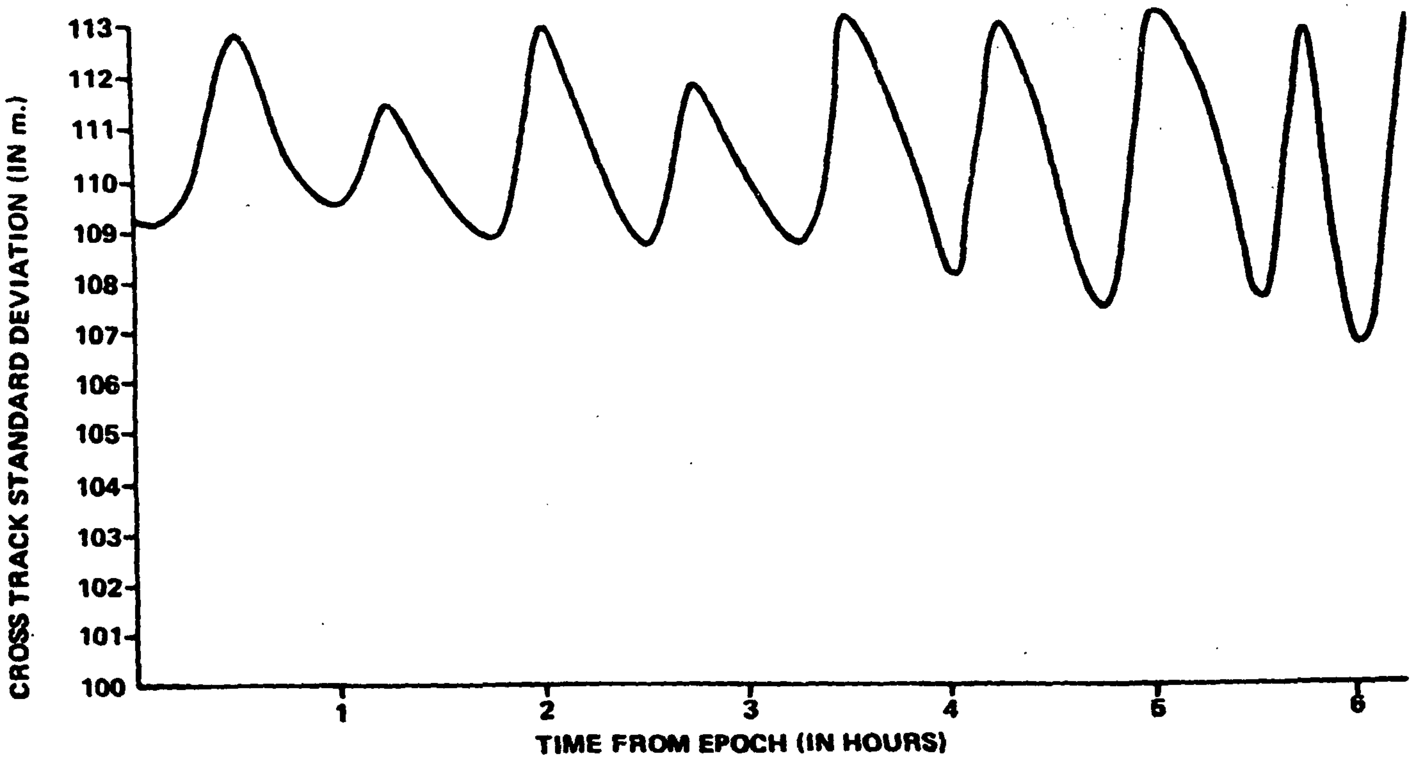

Figure 3. Cross Track Standard Deviation for User Satellite Orbit Determination

Table I

Tracking and Error Assumptions for Relay Satellite Orbit Determination

Tracking Assumptions	
data type	ranging
tracking schedule	1/min for 5 min each hour
arc length	24 hours
data noise	2 m
Systematic Error Assumptions	
error source	magnitude
survey error	10 m per component
data bias	10 m
solar radiation pressure	10% of nominal value
GM	one unit in sixth decimal place
spherical harmonic coefficients to degree and order 8	full difference between GEM 1 field and NWL 4 field

Table II

Tracking and Error Assumptions for TDRS Orbit Determination of A Low Altitude Satellite

Tracking Assumptions	
data type	range sum, range sum rate
tracking schedule	1/min
arc length	6 hours
data noise	2 m, 1 mm/sec
Systematic Error Assumptions	
error source	magnitude
survey error	10 m per component
data bias	10 m, 1 mm/sec
relay satellite state errors	as determined by simulation
GM	one unit in sixth decimal place
spherical harmonic coefficients to degree and order 8	full differences between GEM 1 field and NWL 4 field
atmospheric drag coefficient	10% of nominal value

Table III

Root Mean Square Errors for TDRS Orbit Determination (6 Hour Arc)

	Radial	Along Track	Cross Track
range sum data	26 m	250 m	21 m
range sum rate data	25 m	258 m	21 m
combined data	25 m	257 m	21 m

Table IV

Root Mean Square Errors for TDRS Orbit Determination (20 Min. Arc)

	Radial	Along Track	Cross Track
range sum data	96 m	303 m	3 m
range sum rate data	29 m	59 m	4 m
combined data	21 m	48 m	4 m

Table V

Root Mean Square Errors for TDRS Orbit Determination
Due to Relay Satellite State Errors (6 Hour Arc)

	Radial	Along Track	Cross Track
range sum data	16	233	4
range sum rate data	15	241	2
combined data	15	240	1

Table VI

Tracking and Error Assumptions for TDRS Orbit Determination of A Low Altitude Satellite

Tracking Assumptions	
data type	range sum
tracking schedule	1/min
arc length	6 hours
data noise	2 m
Systematic Error Assumptions	
error source	magnitude
survey error	10 m per component
data bias	10 m
relay satellite error	600 m per position component 6 cm/sec per velocity component
GM	one unit in sixth decimal place
spherical harmonic coefficients to degree and order 8	obtained from ref 5
atmospheric drag coefficient	10% of nominal value

Table VII

Average Radial, Along Track, Cross Track Errors for User Satellite Orbit Determination

	Radial	Along Track	Cross Track
total error	142 m	515 m	111 m
effect of relay satellite error	140 m	506 m	108 m

ILLUSTRATIONS