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L. C. Lee

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

SMALL SCALE TURBULENCE IN THE CRAB NEBULA: EVIDENCE OF LOWER HYBRID PARAMETRIC INSTABILITIES DRIVEN BY THE PULSAR WAVE

L. C. Lee *

Theoretical Studies Group

Goddard Space Flight Center

Greenbelt, Maryland

*NAS/NRC Resident Research Associate

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ABSTRACT

Strong small scale turbulence with size $a \sim 50$ km is observed in the Crab Nebula from the temporal pulse broadening data. It is shown that the strong 30 Hz pulsar wave can parametrically excite instabilities near the lower hybrid frequency in the thermal plasma of the Crab Nebula with a characteristic wavelength of the order of the scale size a of the turbulence observed. The growth rate of the excited waves is about 0.2 - 10 sec. These instabilities provide a coupling mechanism between the pulsar wave and the Nebula plasma.

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(I) INTRODUCTION

Recent observations of angular broadening and temporal broadening of the Crab Nebula pulsar's radiation indicate the existence of a scattering region of plasma turbulence within the Crab Nebula (Counselman & Rankin 1972, Rankin & Counselman 1973, Lyne & Thorne 1975, Vandenberg 1976). Using the results of VLBI (very-long-baseline interferometer) data, Vandenberg (1976) was able to show that the angular broadening and the temporal broadening of the pulsar radiation occur in different locations along the line of sight to the Crab pulsar. Vandenberg (1976) then proposed the electron density fluctuation in Filament 159 of the Crab Nebula (Trimble 1970) as the cause of the temporal pulse broadening. However, Vandenberg (1976) made a mistake in estimating the scale size and the amount of plasma irregularity in the filament. Lee (1977) developed a thin screen scintillation theory for a spherical wave and corrected Vandenberg's errors. Lee (1977) pointed out that if the pulse broadening is due to the thermal plasma in the Crab Nebula (of thickness ~0.5 pc), then the rms electron density fluctuation ΔN in the Nebula is

$$\Delta N/\phi_0 \approx 0.01\ \mathrm{cm}^{-3} \tag{1}$$

and the scale size of electron density fluctuation, a, is given by

$$a/\phi_o \approx 15 \text{ km} \tag{2}$$

where ϕ_o is the rms phase fluctuation caused by the Nebula. From the pulse shape observed, one must have $\phi_o \gg 1$. In the next section, we estimate ϕ_o to be $\sim$1-5. Thus $\Delta N \simeq 0.01$-0.05 cm^{-3} and a $\simeq$ 15-75 km. However, if the pulse broadening is due to Filament 159 (with thickness $\sim$0.01 pc) as proposed by Vandenberg, then one has in the filament $\Delta N/\phi_o \simeq 0.08$ cm^{-3} and $a/\phi_o \simeq 15$ km.

It is the purpose of this paper to propose a mechanism to explain the observed small scale turbulence in the Crab Nebula. We will find in the next section that the averaged thermal electron density in the Nebula is $N_{th} \simeq 0.05$ cm^{-3}. Thus, for the small scale turbulence (a $\approx$ 50 km), $\Delta N \simeq N_{th}$ and the turbulence is very strong. We believe that some external driving force is needed to excite such a strong turbulence at a relatively small scale size. The natural candidate for the driving force is the strong 30 Hz pulsar wave. In Section II, we briefly describe the physical environment in the Crab Nebula. In Section III, we show that the 30 Hz pulsar wave can indeed excite several parametric instabilities near the lower hybrid frequency in the Crab thermal plasma with a characteristic wavelength $\approx$ a. Thus these instabilities can explain the observed strong turbulence and, in addition, can provide a coupling mechanism between the pulsar and the nebula. In Section IV, we discuss and sum up the results.

(II) CR AB NEBULA

(a) THERMAL PLASMA

The Crab Nebula contains three components: relativistic charged particles, magnetic field and thermal plasma. The magnetic field in the Crab Nebula has been observed to be $B_{neb} \sim 3 \times 10^{-4}$ Gauss (e.g., Fazio et al. 1972, Fazio 1973) and the total energy of relativistic electrons is about 10^{49} erg (Trimble & Rees 1970). The thermal electrons in most filaments have a density, $N_{th} \simeq 10^3$ cm^{-3} and a temperature, $T_e \simeq 10^4$ °K (Woltjer 1958, Trimble 1970). The thermal plasma density outside filaments is much lower than that in filaments. Shklovskii (1966) pointed out that in the Crab Nebula the averaged thermal plasma density, $N_{th} \leqslant 1$ cm^{-3}, because of the absence of the gross Faraday depolarization of radio emission. From the motion of wisps, Scargle (1969) argued that $N_{th} \simeq 5 \times 10^{-4}$ cm^{-3} near Wisp 1 and $N_{th} \simeq 0.05$ cm^{-3} at the northwest edge. From the fact that the expansion speed of the nebula at the outer edge, $U_{neb} \sim 1.5 \times 10^8$ cm/sec, much less than the speed of light, one would obtain $N_{th} \simeq 0.05$ cm^{-3} for $B_{neb} \approx 10^{-4}$ Gauss near the outer edge of the nebula (Kahn 1971). Assume that $N_{th} > \Delta N$ (Lee & Jokipii 1976). From Eq. (1), we also find that $N_{th} \geqslant 0.01$ cm^{-3}. Thus we take $N_{th} \simeq 0.05$ cm^{-3}. If $\Delta N \leqslant N_{th}$ as assumed, Eq. (1) gives $\phi_o \approx 1$-5. Therefore, $\Delta N \simeq 0.01$-0.05 cm^{-3} and $a \simeq 15$-75 km. Assuming energy equipatition between magnetic field and thermal plasma in the Nebula, we have the thermal plasma temperature $T_e \simeq 5 \times 10^7$ °K. Thus in the following calculations, we will take $B_{neb} \simeq 3 \times 10^{-4}$Gauss, $N_{th} \simeq 0.05\ cm^{-3}$ and $T_e \simeq 5 \times 10^7$ °K.

(b) 30 Hz PULSAR WAVE

Crab pulsar is generally believed to be a compact oblique magnetic rotator and therefore emits strong 30 Hz electromagnetic waves (Ostriker & Gunn, 1969). Just outside the speed-of-light cylinder, the wave electric field E is strong and the parameter

$$\alpha \equiv \frac{eE}{m_e c \omega_o} \gg 1,$$

where c is the speed of light, e the electron charge, M_e the electron mass and ω_o = 30 Hz is the frequency of the pulsar wave. This 30 Hz wave can accelerate particles, leaking from the pulsar magnetosphere, to relativistic velocity (Ostriker & Gunn 1969, Ferrari & Trussoni 1975). The lack of circular polarization in the optical continuum radiation from the nebula strongly suggests that the pulsar wave does not propagate freely more than $\simeq$20" (or 6×10^{17} cm) from the pulsar (Landstreet & Angel 1971, Rees 1971). Observations of a 'valley' in the brightness near the pulsar at both radio and optical frequencies suggest there exists a cavity around the pulsar swept clear of thermal gas and static magnetic field by the 30 Hz wave (Scargle 1971).

Thus we have a picture for the Crab Nebula: the 30 Hz pulsar wave propagates from the pulsar and interacts, at a distance of several 10^{17} cm away from the pulsar, with the Crab Nebula, which is composed of thermal plasma and magnetic field. At the edges of the central pulsar cavity, the wave pressure becomes comparable to the thermal and magnetic pressure of the Nebula and the wave field $E \approx B_{neb}$. As the wave propagates into the Nebula, the pulsar wave will be damped and/or reflected such that $E \ll B_{neb}$ inside the nebula.

Several authors have considered the interaction between the pulsar wave and the nebula. Rees & Gunn (1974) suggested that the 30 Hz wave would be absorbed due to synchrotron absorption by the downstream shocked high energy plasma with a mean free path $\sim 3 \times 10^{12}$ cm. Ferrari (1974) and Dobrowolny & Ferrari (1976) discussed the plasma instabilities excited by the interaction between the 30 Hz pulsar wave and the relativistic wind leaking from the pulsar magnetosphere. These instabilities occur only near the equatorial plane of the pulsar spin axis where wisps are observed. Max (1973) discussed parametric instabilities excited by the 30 Hz wave in the nebula thermal plasma, neglecting effects of the nebula magnetic field. The existence of a static magnetic field allows more possible modes to be excited. In the next section, we will consider the interaction between the 30 Hz wave and the thermal plasma in the presence of the nebula magnetic field.

(III) PARAMETRIC EXCITATION BY THE PULSAR WAVE

In the Crab Nebula where $N_{th} \simeq 0.05\ \mathrm{cm}^{-3}$, $B_{neb} \simeq 3 \times 10^{-4}$ Gauss, and $T_e \simeq 5 \times 10^7$ °K, we have the electron cyclotron frequency $\Omega_e \simeq 840$ Hz, ion cyclotron frequency $\Omega_i \simeq 0.5$ Hz, electron plasma frequency $\omega_{pe} \simeq 1800$ Hz, ion plasma frequency $\omega_{pi} \simeq 40$ Hz, Debye length $\lambda_D \simeq 2$ km, and the ion sound speed $V_s \simeq 8 \times 10^7$ cm/sec. The lower hybrid frequency ω_{LH} is given by

$$\omega_{LH} = \omega_{pi}(1 + \omega_{pe}^2/\Omega_e^2)^{-1/2} \simeq (\Omega_i \Omega_e)^{1/2} \simeq 20 \text{ Hz} \tag{3}$$

Thus the 30 Hz pulsar wave, when propagating in the thermal plasma of the Crab Nebula, acts as a pump wave with frequency ω_o in the vicinity of the lower hybrid frequency ($\omega_o \gtrsim \omega_{LH}$). Parametric decay instabilities with pump frequency near ω_{LH} has recently been studied (Kindel et al. 1972, Porkolab 1974, Hesegawa & Chen 1975, Berger & Perkins 1976, Ott 1975, Tripathi et al. 1977) and observed in the rf heating of tokamak plasma (Hooke & Bernabei 1972, Chu et al. 1973, Chang & Porkolab 1974, Porkolab et al. 1977).

Consider now the parametric decay of the pump wave (with $\omega_o = 30$ Hz and wavenumber $\underset{\sim}{k}_o$) into two daughter waves (ω_1, $\underset{\sim}{k}_1$) and (ω_2, $\underset{\sim}{k}_2$). Since the wavelength of the incoming pump wave is much greater than that of the daughter waves ($\sim 10^6$ cm), the pump will be treated in the dipole approximation ($\underset{\sim}{k}_o \approx 0$),

$$\underset{\sim}{E} = \underset{\sim}{E}_o \cos(\omega_o t). \tag{4}$$

We then have $\underset{\sim}{k}_1 + \underset{\sim}{k}_2 = \underset{\sim}{k}_o \approx 0$. Under the dipole approximation, Porkolab (1974) has investigated possible decay channels for a pump wave near lower hybrid frequency. According to Kindel et al. (1972) and Porkolab (1974), the pump wave will excite (a) purely growing and (b) decay parametric instabilities. In both cases, one of the two daughter waves is a lower hybrid wave. Let (ω_2, $\underset{\sim}{k}_2$) be the excited lower hybrid wave satisfying the following dispersion relation

$$\omega_2 = \omega_{Lk} \equiv \omega_{LH} \left(1 + \frac{k_{2\parallel}^2}{k_2^2} \frac{M_i}{M_e}\right)^{1/2} , \tag{5}$$

where ω_{LH} the lower hybrid frequency is given by Eq. (3), M_i is the proton mass and $k_{2\parallel}$ is the component of the wavevector along the magnetic field.

(a) PURELY GROWING PARAMETRIC INSTABILITIES

For the purely growing instabilities which occurs for $\omega_0 < \omega_2$, the real part of ω_1 is zero and the minimum threshold is given by

$$\frac{U}{V_s} = 2 \frac{\omega_o}{\omega_{pe}} \left[\frac{2\gamma_2}{\omega_2}\left(H \frac{\omega_{pe}^2}{\Omega_e^2}\right)\right]^{1/2} \tag{6}$$

where $U = CE_o/B_{neb}$ is the $\underset{\sim}{E} \times \underset{\sim}{B}$ drift velocity, $V_s = (kT\ /m_i)^{1/2}$ is the ion sound speed and γ_2 is the linear damping rate of the lower hybrid wave. The damping rate γ_2 for the lower hybrid wave is

$$\gamma_2 = \sqrt{\pi}\, \frac{\omega^2 - \omega_{LH}^2}{\omega}\, y^3 \exp(-y^2) \tag{7}$$

where $y \equiv \omega/k_\parallel V_{te}$ and V_{te} is the electron thermal velocity (Tripathi et al. 1977). Since $y \gg 1$, γ_2 is very small and therefore the condition in Eq. (6) is easily satisfied for the pulsar wave propagating in the Nebula. The growth rate for the purely growing instabilities well above threshold is given by

$$\gamma \simeq (\sqrt{-\beta}\, X\, \omega_2 \omega_{LH}/2)^{1/2} \tag{8}$$

where $\beta = (\omega_o - \omega_{Lk})/\omega_{Lk}$ and $x = kU/\omega_2$. Taking $x = 1$, $(-\beta) = 10^{-4}$ for example, we have $\gamma \simeq 0.07\ \omega_{LH} \simeq 1.4\ \text{sec}^{-1}$. In general the time scale for the purely growing instabilities in the nebula is of order of 1 sec.

(b) DECAY PARAMETRIC INSTABILITIES

For decay instabilities, the pump wave beating with a lower hybrid wave form low-frequency ion quasimodes with frequencies near or above the ion cyclotron frequency (Porkolab 1974). These quasimodes may be of two types: (i) kinetic modes such that their parallel phase velocities are near the electron thermal velocity ($\omega/k_{\parallel} \simeq V_{te}$), in which case the driving mechanism is essentially nonlinear electron Landan damping; (ii) nonresonant fluid-like modes such that $\omega/k_{\parallel} \gg V_{te}$.

For kinetic modes, the threshold is the same as that for the purely growing instabilities in Eq. (6). Let $\omega_1 = \omega_R + i\omega_I$. The growth rate ω_I well above threshold is

$$\omega_I/\omega_2 \simeq \frac{U^2}{8V_s^2}\left(\frac{\omega_{pi}}{\omega_o}\right)^2 \frac{\sqrt{\pi}y\exp(-y^2)}{(1+\omega_{pe}^2/\Omega_e^2)}, \tag{9}$$

where $y \equiv \omega_R/k_{\parallel}V_{te} \simeq 1$. However, from the numerical calculation of Porkolab (1974), ω_I is peaked around $U/V_s \approx 1$. The maximum growth rate can then be obtained from Eq. (7) by setting $U/V_s = 1$, $\omega_I(\max) \simeq 0.02\ \omega_2 \simeq 0.6\ \sec^{-1}$. In general the time scale for the growth of the kinetic modes is about 1-10 sec.

The fluid-like quasimodes ($\omega_R/k_{\parallel} \gg V_{te}$) can easily be excited even in weak pump fields (Porkolab 1974). The growth rate ω_I is given by

$$\omega_I = \left(\frac{3}{4}\right)^{1/2}\left(\frac{\omega_o M_i}{2M_e}\right)^{1/3}\left(\frac{Uk_{\parallel}}{1+M_i k_{\parallel}^2/M_e k^2}\right)^{2/3}. \tag{10}$$

Again the growth rate ω_I is peaked at $U/V_s \simeq 1$. From Eq. (10), the maximum growth rate is $\omega_I(\max) \simeq 0.2\ \omega_o \simeq 6\ \sec^{-1}$. Thus the time scale for fluid-like modes in the nebula is about 0.2-5 sec.

The wavelength of the excited waves for both purely growing and decay parametric instabilities is given by $k\lambda_D \simeq 0.1\text{-}0.5$ (Porkolab 1974, Tripathi et al. 1977). In the Crab Nebula, we have $\lambda_D \simeq 2$ km, and therefore the wavelength $\lambda = 2\pi/k \simeq 20\text{-}120$ km, which is approximately the scale size a (~15-75 km) of the turbulence observed.

(IV) DISCUSSIONS

In the above calculation, we assume that the 30 Hz pulsar wave reaches the lower hybrid resonant region ($\omega_o \simeq \omega_{LH}$) directly. If the thermal plasma density increases gradually near the inner edge of the Crab Nebula, then before reaching the resonant region, the 30 Hz pulsar wave will pass through a layer where $\omega_o \simeq \omega_{pe}$ and may be reflected. However, since the pulsar wave is strong ($\alpha \gg 1$), it can penetrate through this layer (Max and Perkins 1971) and reach regions where $\omega_o \simeq \omega_{LH}$. We note that, whereas in the tokamak plasma the lower hybrid resonant region is only a thin layer, the lower hybrid resonant region in the Nebula is very broad since in the Nebula ω_{LH} is determined mostly by the magnetic field ($\omega_{LH} \simeq (\Omega_e \Omega_i)^{1/2}$), not by the plasma density (or ω_{pi}).

Next we consider if the temporal pulse broadening is due to the thermal plasma fluctuations in Filament 159 as proposed by Vandenberg (1976). In

Filament 159 where $T_e \simeq 10^4$ °K and $N_{th} \simeq 100\ \text{cm}^{-3}$, the Debye length is $\lambda_D \simeq 70$ cm. If waves are parametrically excited in the filament by the pulsar wave, then the wavelength λ would be only ~(1–4) × 10^4 cm which is too small to account for the observed turbulence.

Thus the 30 Hz pulsar wave can excite parametric instabilities near the lower hybrid frequency in the thermal plasma of the Crab Nebula with a characteristic wavelength of the order of the scale size a (~15-75 km) of the turbulence observed. The time scale for the growth of the excited waves is about 0.2-10 sec. The excited waves propagate nearly perpendicular to the Nebula field ($k_{\parallel} \ll k$) and therefore will propagate radially outward with a phase velocity ~ 10^8 cm/sec superposed on the convective motion of the Nebula plasma. Since the damping rate of the lower hybrid waves as given by Eq. (7) is very low, these waves will propagate to the outer part of the Nebula. We note that as in the rf tokamak plasma heating, the electrons and ions in the nebula will be heated by this mechanism. If we identify the scale size a of the observed turbulence with the wavelength of the excited waves ($a \simeq \lambda/2\pi \sim (0.1\text{-}0.5) \times 2\pi\lambda_D$), then λ_D gives the plasma temperature in the Nebula, $T_e \sim 10^7$-10^8 °K, which is consistent with that determined by energy equipartition in Section II. Thus the lower hybrid instabilities also provide a coupling mechanism between the 30 Hz wave and the surrounding Nebula, in which the energy and momentum of the pulsar wave are transferred to the plasma in the Nebula.

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