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BOPACE

(THE BOEING PLASTIC ANALYSIS CAPABILITY FOR ENGINES)

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1.0 INTRODUCTION

BOPACE is the acronym for the Boeing Plastic Analysis Capability for Engines. It is a nonlinear stress analysis program, based on a very general family of isoparametric (curved boundary) finite elements. Although BOPACE development has been strongly influenced by the requirements for analysis of engines, in particular the space shuttle main engine, it is a general program applicable to many types of nonlinear structures. The current BOPACE Version 5 is based on the earlier BOPACE 2-D and 3-D codes [1,2]†, but it provides major improvements in the areas of increased problem size capability, additional analysis features, and added user conveniences. This document describes the current BOPACE program and includes theoretical, user, programmer, preprocessing, and example problem sections.

BOPACE has been developed by The Boeing Company for the NASA Marshall Space Flight Center, based on the following general requirements.

1) Analysis of very high temperature, large plastic-creep effects, and geometric nonlinearities.

2) Treatment of cyclic thermal and mechanical loads.

3) Improved material constitutive theory which closely follows actual behavior under variable temperature conditions.

4) A stable numerical solution approach which avoids cumulative errors.

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† Brackets denote references given in Appendix A.

5) Capability for efficient handling of up to 4500 degrees of freedom (1500 DOF front), within 64K words of computer core.

The BOPACE research and development efforts have led to an improved hardening theory for cyclic plasticity, a method for representing general cases of load reversal, and techniques for improving the accuracy and controlling convergence of highly nonlinear solutions. New features in the current program version include substructuring, an out-of-core Gauss wavefront equation solver, multi-point constraints, combined material and geometric nonlinearities, automatic calculation of inertia effects, provision for distributed as well as concentrated mechanical loadings, follower forces, singular crack-tip elements, the SAIL automatic data generation capability, and expanded user control over input quantity definition, output selection, and program execution.

BOPACE is written in FORTRAN IV and is currently available for both the IBM 360/370 and the UNIVAC 1108 machines.

The BOPACE Programming effort has been led by D. L. Beste of Boeing Computer Services.

BOPACE

PART I: THEORETICAL MANUAL

2.0 MATERIAL CONSTITUTIVE THEORY

The basic purpose of classical constitutive theory in an elasto-visco-plastic program such as BOPACE is to provide incremental relations between stresses and strains. BOPACE uses these relations with the finite-element stiffness method to provide a convenient and efficient approach for solution of an important class of nonlinear problems.

BOPACE accounts for elastic, plastic, thermal and creep deformations, and the nonlinear dependence of all deformations on temperature. The material constitutive theory includes a combined isotropic/kinematic plastic hardening theory, and a generalized approach to cyclic load reversal. The BOPACE constitutive theory is developed by a tensorial apporach which provides all relations in a form which is invariant under coordinate transformations.

2.1 ELASTICITY EQUATIONS

This section defines the cumulative and incremental forms of the relations for temperature-dependent elasticity, which are used in BOPACE for initially isotropic materials. Anisotropic elasticity is discussed in Section 2.7.

General Concepts and 3-D Relations - The basic cumulative stress-strain relation, for either temperature-dependent or temperature-independent elasticity, is

$$\sigma_{ij} = D^e_{ijk\ell}\ \varepsilon^e_{k\ell} \tag{2.1-1}$$

where σ and ε^e are the 3 x 3 tensors of stress and elastic (recoverable) strain, respectively, and D^e is the tensor of elastic coefficients which may depend on temperature. For convenience we will use the equivalent single-subscript notation

$$\sigma_i = D^e_{ij}\ \varepsilon^e_j \tag{2.1-2}$$

where subscripts i and j range over all nine of the tensor components. For 3-dimensional analysis the relation 2.1-2 is taken as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{yx} \\ \sigma_{zx} \\ \sigma_{zy} \end{Bmatrix} = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon^e_{xx} \\ \varepsilon^e_{yy} \\ \varepsilon^e_{zz} \\ \varepsilon^e_{xy} \\ \varepsilon^e_{xz} \\ \varepsilon^e_{yz} \\ \varepsilon^e_{yx} \\ \varepsilon^e_{zx} \\ \varepsilon^e_{zy} \end{Bmatrix} \tag{2.1-3}$$

where

$$D_{11} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix}$$

and

$$D_{22} = D_{33} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-2\nu & 0 & 0 \\ 0 & 1-2\nu & 0 \\ 0 & 0 & 1-2\nu \end{bmatrix}$$

Here E is Young's modulus and ν is Poisson's ratio.

Note that the elasticity matrix in Equation 2.1-3 is consistent with the tensorial definition of shear strains (e.g. $\varepsilon^e_{xy} = \gamma^e_{xy}/2$, where γ_{xy} is the engineering definition of shear strain). Tensorial definitions are used in the BOPACE program in order to easily formulate constitutive theory which is invariant with respect to coordinate transformations.

The last three of Equations 2.1-3 are somewhat redundant and may be discarded to give an abbreviated 6-component form (e.g., $\sigma_{yx} = \sigma_{xy}$ and $\varepsilon_{yx} = \varepsilon_{xy}$). It should be emphasized, however, that in performing later summations all non-zero values of the nine tensor components must be accounted for.

<u>Relations for Special Cases</u> - Equations 2.1-3 can be used to determine the cumulative stress components, given all of the cumulative elastic strain components. These equations can therefore be applied in the following analysis cases.

1. 3-dimensional (all strain components computed from displacements)
2. generalized plane strain (specified zero or non-zero value of one normal strain)
3. axisymmetric (circumferential strain computed from radial displacement)
4. confined rod (specified zero or non-zero values of two normal strains).

Special cases exist, however, in which one or more stress components are specified while their corresponding strain components are unknown. These include:

1. generalized plane stress or partially confined rod (specified zero or non-zero value of one normal stress)
2. unconfined rod (specified zero or non-zero values of two normal stresses).

Where one normal stress (say, σ_{zz}) is zero, the stress-strain relation is

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-x \end{bmatrix} \begin{Bmatrix} \varepsilon^e_{xx} \\ \varepsilon^e_{yy} \\ \varepsilon^e_{xy} \end{Bmatrix} \qquad (2.1\text{-}4a)$$

and if the normal stress is non-zero, the corresponding strain can be computed as

$$\varepsilon_{zz} = (\sigma_{zz}(1-2\nu)/G - (\varepsilon_{xx} + \varepsilon_{yy})\nu) / (1-\nu) \qquad (2.1\text{-}4b)$$

Where two normal stresses (say, σ_{yy} and σ_{zz}) are zero, the stress-strain relation is simply

$$\sigma_{xx} = E\ \varepsilon_{xx} \qquad (2.1\text{-}5a)$$

and if the normal stresses are non-zero, the corresponding strains can be computed as

$$\left.\begin{aligned} \varepsilon_{yy} &= ((1-\nu)\sigma_{yy} - \nu\sigma_{zz})/G - \nu\varepsilon_{xx} \\ \varepsilon_{zz} &= ((1-\nu)\sigma_{zz} - \nu\sigma_{yy})/G - \nu\varepsilon_{xx} \end{aligned}\right\} \qquad (2.1\text{-}5b)$$

Once all strain components are known or computed, the stresses can be found if desired from the complete 3-dimensional stress-strain relation.

Incremental Relations - For the case of temperature-indepdendent elasticity the incremental stress-strain relations are simply

$$\Delta\sigma_i = D^e_{ij} \ \ \Delta\varepsilon^e_j \qquad (2.1\text{-}6)$$

where Δ denotes an incremental quantity and D^e is the appropriate elasticity matrix.

When temperature dependence is considered, the incremental relation may be written either as

$$\Delta\sigma_i = \sigma_i^1 - \sigma_i^0 = D_{ij}^{e1}\,\varepsilon_j^{e1} - D_{ij}^{e0}\,\varepsilon_j^{e0} \qquad (2.1\text{-}7a)$$

or

$$\Delta\sigma_i = \Delta D_{ij}^{e}\,\varepsilon_j^{e0} + D_{ij}^{e1}\,\Delta\varepsilon_j^{e} \qquad (2.1\text{-}7b)$$

where the superscripts 0 and 1 denote quantities evaluated at the beginning and end of the increment, respectively, and $\Delta D^e = D^{e1} - D^{e0}$ is the change in elasticity matrix from beginning to end of the increment. The first term in Equation 2.1-7b accounts for stress change due only to change in elastic properties, while the second term accounts for additional stress change due to the increment of elastic strain.

2.2 THERMAL STRAIN

<u>Alternate Formulations</u> - The conventional description of thermal strain is given, for isotropic materials, by

$$\Delta \begin{Bmatrix} \varepsilon_{xx}^t \\ \varepsilon_{yy}^t \\ \varepsilon_{zz}^t \end{Bmatrix} = \gamma\,\Delta T \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \qquad (2.2\text{-}1)$$

where ε^t denotes thermal strain, T is the temperature, and γ is the thermal coefficient of expansion which may be a function of temperature.

An alternate integrated description of thermal strain is

$$\begin{Bmatrix} \epsilon^t_{xx} \\ \epsilon^t_{yy} \\ \epsilon^t_{zz} \end{Bmatrix} = \epsilon^t(T) \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \qquad (2.2\text{-}2)$$

where here ϵ^t gives the thermal strain directly as a function of temperature. For analysis purposes, ϵ^t may be taken as zero at any convenient reference temperature.

BOPACE Formulation - BOPACE uses the direct form 2.2-2. This form is preferred over that involving a thermal expansion coefficient because accumulated errors in thermal strain are not introduced. These errors could arise with the form 2.2-1, in case γ varied with temperature and the specified heating and cooling sequences used different temperature increments. BOPACE takes the structural fabrication temperature of each element as the reference temperature for zero thermal strains. (If the material data defines a non-zero thermal strain value at the fabrication temperature, all thermal strains computed for the element are adjusted by subtracting out that value.)

2.3 PLASTICITY

This section defines the incremental elasto-plastic relations used in the BOPACE program. (See also Section 2.6 for the elasto-plastic interative algorithm.) BOPACE employs a new concept of combined isotropic and kinematic hardening, and accounts for temperature-dependent elasto-plastic behavior as well as a generalized form of cyclic load reversal. In order to develop the constitutive theory in a straightforward manner, discussion of the effects of temperature-dependent elasticity on the elasto-plastic relations is deferred until Section 2.5.

Definitions - The following nomenclature is defined.

σ = total stress

α = stress center (of yield surface in kinematic hardening)

s = deviatoric (total - hydrostatic) stress

a = deviatoric stress center

$\hat{s}$ = $s - a$ = relative deviatoric stress

ε^e = elastic (recoverable) strain

ε^p = plastic (time-independent non-stress-inducing) strain

ε^c = creep (time-dependent non-stress-inducing) strain

General Concepts - The basic concepts in most elasto-plastic theories are those of a yield surface, the dependence of yield on only the deviatoric stress components, incompressibility under plastic strains, and normality of the plastic-strain-rate vector to the yield surface. The definition of a particular theory requires assumptions for three basic constituents:

1) a surface relating the stress components at yield
2) a flow rule defining a direction for the incremental plastic-strain vector
3) a hardening rule.

Yield Surface - BOPACE employs the Huber-Mises yield surface [3], defined by the relative deviatoric stresses as

$$F = \hat{s}_i \hat{s}_i - \hat{s}_i^o \hat{s}_i^o = 0 \qquad (2.3\text{-}1)$$

where the summation is again taken over all nine tensor components of s. The $\hat{s}_i^o$ are components of a point on the yield surface at a known condition of temperature and plastic deformation, e.g., from a uniaxial test.

Equation 2.3-1 holds whenever the material is plastic, i.e., whenever the components of s are on the yield surface. Function F may be thought of geometrically as defining a hypersphere in 9-dimensional deviatoric stress space. Alternately, when expressed in the 3-D space of principal stresses, this yield surface can be shown to be an open-ended circular cylinder whose axis passes through the origin and makes equal angles with each of the three principal stress axes. The Huber-Mises yield surface is generally used to describe plasticity in metals because it agrees reasonably well with test results and it gives a smooth surface which is convenient for calculations.

Flow Rule - BOPACE uses the Prandtl-Reuss flow rule, which is the usual rule associated with the Huber-Mises yield surface. The assumptions are that the material is incompressible under plastic flow, and that the plastic strain rate is normal to the yield surface at the stress point.

These assumptions provide the relation

$$\Delta\varepsilon_i^p = \lambda \hat{s}_i \qquad (2.3\text{-}2)$$

where λ is a flow parameter (or plastic proportionality constant).

Basic Hardening Concepts - An elastic-plastic material which work hardens in the plastic range is commonly analyzed using either of two classical hardening theories. Isotropic hardening [4], which assumes a uniform expansion of the yield surface during plastic flow, accounts for change in size of the hysteresis loop during cycling. Kinematic hardening [5], which assumes a rigid translation of the yield surface in the direction of the plastic strain increment, accounts for the pronounced Bauschinger effect which is evident in cyclic behavior of most metals. In general, an actual cyclic behavior can be more accurately described by a combination of isotropic and kinematic hardening. A combined hardening theory has been given by Hodge [6] for materials which satisfy the Tresca yield condition. Because a better representation for most metals is provided by the Huber-Mises yield surface, a corresponding combined hardening theory [7] has been developed for the BOPACE program.

Hardening Parameters - A simple combined hardening theory such as that presented in Reference 7 makes two basic assumptions:

1) Size of the yield surface is a function of a cumulative hardening parameter, κ. This means that the isotropic hardening, i.e. the incremental change in size of the yield surface, depends on the initial value of κ and its change $\Delta\kappa$.

2) Yield surface translation is related (but only in an incremental manner) to a kinematic hardening parameter, κ^k. The kinematic hardening, i.e. the incremental translation of the yield surface, depends on the initial value of κ^k and its change $\Delta\kappa^k$.

For a simple uniaxial load case, the yield surface size at any time is measured by one half the algebraic difference between the current yield stresses in tension (positive) and compression (negative), while the cumulative kinematic hardening is measured by one half the algebraic sum of the yield stresses in tension and compression.

It will be evident in the discussion to follow that isotropic hardening can be related to κ on either a cumulative or incremental basis, while kinematic hardening can be related to κ^k only on an incremental basis. In addition to the parameters κ and κ^k, hardening is also a function of temperature.

Figure 2.3-1 shows hysteresis loops for the first two strain-controlled cycles of a typical material which exhibits combined isotropic and kinematic hardening. Here σ denotes yield stress and α denotes yield stress center. The Bauschinger kinematic hardening effect is apparent in that the initial yielding in tension causes a reduced yield stress in compression, i.e. a shift of the yield center by an amount α. Successive yielding in compression causes a reduced yield stress in tension, and so forth. Isotropic hardening causes the increase in size of the hysteresis loop with continued cycling. The hysteresis loops for many materials become stabilized after a number of cycles, and they may begin to decrease in size as further deformation causes a softening effect.

Figure 2.3-2 shows the stabilized hysteresis loops for a material at various temperature levels. (Different strain ranges are used to separate the loops for purpose of illustration.) The hysteresis loop of a material

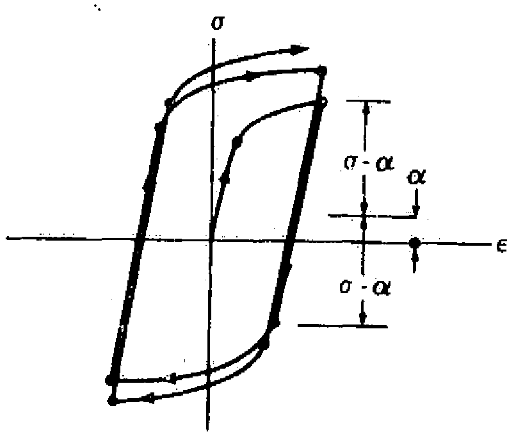


Figure 2.3-1. Combined Hardening Behavior (Non-stabilized)

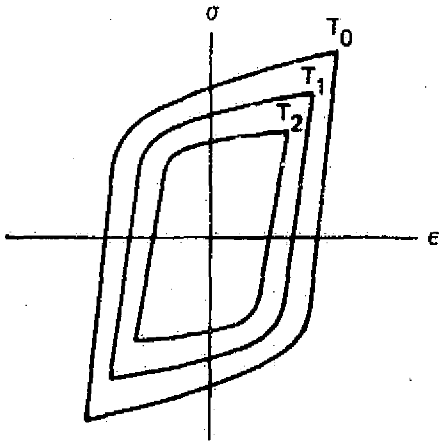


Figure 2.3-2. Variable Temperature Hysteresis Loops (Stabilized)

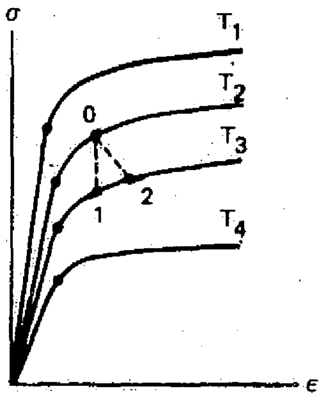


Figure 2.3-3. Variable Temperature Hardening Effects

typically decreases in size with increasing temperature. Note that the size of the yield surface will vary in a similar manner with temperature. Also the rates of isotropic and kinematic hardening with respect to plastic deformation vary with temperature.

The isotropic hardening parameter κ may be appropriately taken as either the cumulative plastic work density, or as the sum of increments of effective plastic strain. The kinematic hardening parameter κ^k must account for the Bauschinger effect in cyclic loading, and it may be taken as an adjusted value of κ. As long as no load reversal occurs and the loading is proportional, κ^k is simply equal to κ. However, κ^k must be set to zero at the start of each increment in which a complete load reversal occurs. (A complete load reversal occurs when the incremental plastic strain vector has a direction exactly reversed from that of the previous plastic increment). For an incomplete load reversal, the BOPACE program computes the starting value for κ^k by multiplying the existing accumulated value of κ^k by the factor (1 + COSINE)/2, where COSINE is the Cosine of the angle between successive incremental plastic strain vectors. At the end of each increment, κ^k becomes $\kappa^k + \Delta\kappa$.

Because the Bauschinger effect varies with cumulative deformation in certain materials (e.g. it may become more pronounced as plastic cycling continues), BOPACE allows an additional option for the kinematic hardening to be defined as a product of two functions. The first is a function of κ^k and defines the shape of the kinematic hardening, while the second is an additional factor which depends on κ and defines the magnitude of the kinematic hardening.

In order to implement the BOPACE hardening theory, it must be determined how the size of the yield surface varies with temperature. In addition, the dependence of isotropic and kinematic hardening on the parameters κ and κ^k must be determined. This is accomplished by performing cyclic tests at several levels of constant temperature. After the cyclic hardening behavior is thus determined at different constant temperatures, an assumption must be made for variable temperature cycling. The hardening effects of variable temperature are illustrated in Figure 2.3-3. As long as temperature remains constant, plastic hardening behavior is defined by following the shape of a stress-strain curve at the given temperature, say to the point 0 on the T_2 curve. If temperature changes to T_3, and then plastic deformation continues, an initial point must be determined on the T_3 curve from which the new yield surface size and initial hardening slopes may be determined. This transfer from curve T_2 to curve T_3 requires a definition of the basis for hardening, i.e. the definition of the parameters κ and κ^k. BOPACE allows the option of either plastic work or the sum of increments of effective plastic strain to be used as the hardening basis. The strain and work options correspond to the respective points 1 and 2 in Figure 2.3-3.

The hardening relationship determined from a series of cyclic tests may depend somewhat on the strain range used in a particular test. If strain range is a significant factor the test conditions should duplicate the approximate expected strain range for which an analysis is to be made. The choice between plastic strain and plastic work as a basis for the hardening parameters κ and κ^k may depend to a large extent on which basis provides the better overall representation of cyclic behavior at various strain ranges.

Multiaxial Hardening Rule - The kinematic hardening rule employed in BOPACE is that due to Prager [5]. It gives the increment of yield surface translation in terms of the incremental plastic strains, as

$$\Delta\alpha_i = C_{ij}\,\Delta\varepsilon_j^p = \frac{2}{3}\,c\,I_{ij}\,\Delta\varepsilon_j^p \qquad (2.3\text{-}3)$$

where c is the kinematic contribution to the slope of the uniaxial stress vs. plastic-strain curve, and I is the identity matrix. An alternate hardening rule due to Ziegler [8] is preferred by some plasticity analysts because the form of Ziegler's rule does not change with reduction in the number of spatial dimensions, and it is therefore supposed to simplify the calculations. Prager's rule is considered more acceptable from a physical point of view, however, and it presents no difficulty when all components of the required tensors are retained as they are in the BOPACE programs. Note that for Prager's kinematic hardening rule, the deviatoric stress center is equal to the stress center, i.e. $a_i = \alpha_i$.

The isotropic hardening, i.e. change in size of the yield surface due to plastic deformation, is defined for a proportional test loading by

$$\Delta\hat{s}_i^o = R_{ij}^o\,\Delta\varepsilon_j^p = \frac{2}{3}\,r\,I_{ij}\,\Delta\varepsilon_j^p \qquad (2.3\text{-}4)$$

where r is the isotropic contribution to the slope of the uniaxial stress vs. plastic-strain curve.

The necessary condition that stresses remain on the yield surface is satisfied by taking the differential of Equation 2.3-1. The condition is $\dot{F} = 0$, which to a first order approximation can be shown to give

$$\hat{s}_i \, \Delta\sigma_i - \hat{s}_i \, \Delta\alpha_i - \hat{s}_i^o \, \Delta\hat{s}_i^o = 0 \qquad (2.3\text{-}5)$$

or

$$\hat{s}_i \, \Delta\sigma_i - A\lambda = 0 \qquad (2.3\text{-}6)$$

where

$$A = \frac{1}{\lambda} \hat{s}_i \, \Delta\alpha_i + \frac{1}{\lambda} \hat{s}_i^o \, \Delta\hat{s}_i^o = C_{ij} \, \hat{s}_i \, \hat{s}_j + R_{ij}^o \, \hat{s}_i^o \, \hat{s}_j^o \qquad (2.3\text{-}7)$$

The key to a successful combined hardening theory is the proper determination of the hardening variable A. BOPACE uses input data hardening tables which give the yield-surface size and the surface translation as functions of the hardening parameters κ and κ^k. These are two-dimensional tables for each material whose ordinates and abscissas are, respectively, temperature and hardening parameter. Given the initial values of κ and κ^k at the beginning of an increment, and estimated values for $\Delta\kappa$ and $\Delta\kappa^k$, the corresponding increments of isotropic and kinematic stress increase are obtained from the hardening tables. (Hardening due to temperature change is included by adding it to the isotropic stress increment.) The hardening slopes c and r are then computed, by dividing the incremental stress increases by the estimated increment of effective plastic strain. This procedure gives average values for the slopes c and r during the increment, and tends to produce an accurate and stable numerical iterative process. Note that, in general, it is only the isotropic and kinematic stress increases, and not the slopes c and r, which the tables relate directly to the hardening parameters. (For a particular loading and stress state, the slopes c and r can at any given time be related indirectly to the hardening parameters.) The choice of a test value for $\hat{s}^o$ in Equation 2.3-7 is arbitrary, as long as it is a point on a yield surface of size corresponding to $\hat{s}$, i.e., a surface with equal values of temperature and parameter κ. It is convenient in BOPACE to take $\hat{s}^o$ equal to s.

Incremental Stress-Strain Relation - The incremental stress-strain relation now follows the development of References 9 and 10. Take

$$\Delta\sigma_i = D^e_{ij}\,\Delta\varepsilon^e_j = D^e_{ij}\,\Delta\varepsilon^{e+p}_j - D^e_{ij}\,\hat{s}_j\lambda \qquad (2.3\text{-}8)$$

where D^e is the appropriate matrix of elastic constants.

Then

$$A\lambda = \hat{s}_i\,\Delta\sigma_i = \hat{s}_i\,D^e_{ij}\,\Delta\varepsilon^{e+p}_j - \hat{s}_i\,D^e_{ij}\,\hat{s}_j\lambda \qquad (2.3\text{-}9)$$

which gives

$$\lambda = \hat{s}_i \, D^e_{ij} \, \Delta\varepsilon^{e+p}_j / (A + \hat{s}_k \, D^e_{k\ell} \, \hat{s}_\ell) \qquad (2.3\text{-}10)$$

Substituting Equation 2.3-10 into Equation 2.3-8 provides the desired relation

$$\Delta\sigma_i = \left(D^e_{ij} - \frac{D^e_{ik} \, \hat{s}_k \, \hat{s}_\ell \, D^e_{\ell j}}{A + \hat{s}_m \, D^e_{mn} \, \hat{s}_n} \right) \Delta\varepsilon^{e+p}_j \qquad (2.3\text{-}11)$$

or

$$\Delta\sigma_i = (D^e_{ij} + D^p_{ij}) \, \Delta\varepsilon^{e+p}_j = D_{ij} \, \Delta\varepsilon^{e+p}_j \qquad (2.3\text{-}12)$$

D is the elasto-plastic Jacobian (tangent-stiffness) matrix relating incremental stresses to incremental elastic+plastic strains. In effect, it separates the elastic and plastic strains and determines the incremental stress corresponding to the incremental elastic strain. D^p is the stiffness reduction due to plastic flow, and becomes zero for the case of infinite hardening, i.e. $A = \infty$, or equivalently the total slope (c+r) of the stress vs. plastic-strain curve is infinite.

To develop the plane strain relation we essentially carry out the 3-D matrix derivation, and then drop out all zz terms because for iterative solution purposes $\Delta\varepsilon_{zz}^{e+p} = 0$. For plane stress, the derivation is carried out using only the plane stress elasticity matrix.

Effective Stress-Strain and Plastic Work - The concepts of "effective stress" and "effective strain" are related to plastic work, and are used in a limited way in the development of constitutive theory for the BOPACE program.

Because they can easily be misapplied, especially in the presence of yield surface translation, the use and limitations of the concepts are briefly discussed here for the Mises plasticity theory.

Due to characteristics of the Prager hardening theory, the following statements of equivalence and proportionality should first be noted.

$$\Delta a_i = \Delta\alpha_i \approx \Delta\varepsilon_i^p \approx \hat{s}_i \not\approx s_i \qquad (2.3\text{-}13)$$

Because of the incremental nature of kinematic hardening, s_i and $\hat{s}_i$ are in general not proportional.

The Mises effective stress $\bar{\sigma}$ is defined by

$$\bar{\sigma}^2 = \frac{3}{2} s_i \; s_i \qquad (2.3\text{-}14)$$

The incremental and cumulative values for plastic work, W^p, are given by

$$\Delta W^p = \sigma_i \; \Delta\varepsilon_i^p \qquad (2.3\text{-}15a)$$

and

$$W^p = \Sigma \; \sigma_i \; \Delta\varepsilon_i^p \qquad (2.3\text{-}15b)$$

where Σ denotes summation over all increments. For the special case of proportional loading (i.e. loading in which all stress components are increased proportionately) followed by a constant stress level (i.e. no plastic hardening), the cumulative plastic work is given by

$$W^p = \sigma_i \, \varepsilon_i^p \qquad (2.3\text{-}15c)$$

As a matter of convenience in computing plastic work, an increment of effective plastic strain, $\Delta\bar{\varepsilon}^p$, has historically been defined by

$$(\Delta\bar{\varepsilon}^p)^2 = \frac{2}{3} \Delta\varepsilon_i^p \, \Delta\varepsilon_i^p \qquad (2.3\text{-}16)$$

At this point, however, care must be exercised in using the historical calculation for plastic work. If kinematic hardening were zero, then $\hat{s}_i = s_i$, and because $\Delta\varepsilon_i^p$ is proportional to $\hat{s}_i$ the use of Equations 2.3-14 and 2.3-16 would give plastic work as

$$\Delta W^p = \bar{\sigma} \, \Delta\bar{\varepsilon}^p \qquad (2.3\text{-}17a)$$

and

$$W^p = \Sigma \, \bar{\sigma} \, \Delta\bar{\varepsilon}^p \qquad (2.3\text{-}17b)$$

If in addition, the condition were one of proportional loading and constant stress, then by defining the cumulative effective plastic strain, $\bar{\varepsilon}^p$, in the same manner as $\Delta\bar{\varepsilon}^p$, we would have

$$W^p = \bar{\sigma}\,\bar{\varepsilon}^p \qquad (2.3\text{-}17c)$$

Of course the Equations 2.3-17 in general <u>are not valid</u>, because of the presence of kinematic hardening and non-proportional loading. Thus plastic work must be computed from Equation 2.3-15a and b, rather than from the product of effective stress and strain quantities.

The quantity $\bar{\varepsilon}^p$ serves little purpose in a general plasticity analysis, although it is a tensorially invariant quantity and does provide a measure of net residual deformation. For a rational measure of deformation history, either the plastic work, W^p, or the sum of increments of effective plastic strain, $\Sigma\,\Delta\bar{\varepsilon}^p$, is appropriate. The difference in concept between the quantities W^p and $\Sigma\,\Delta\bar{\varepsilon}^p$ should, however, be recognized.

2.4 CREEP

<u>Stages</u> - Metals characteristically exhibit the three stages of primary, secondary and tertiary creep. Figure 2.4-1 shows these stages in a typical creep history under conditions of constant temperature and stress. Because creep rate varies considerably during the different stages, the description of actual creep histories is considered to be essential for an accurate analysis. The BOPACE program accounts for

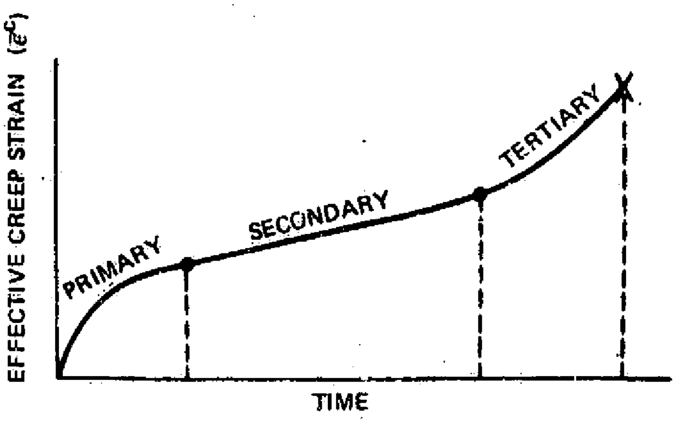


Figure 2.4-1. Typical Creep Stages

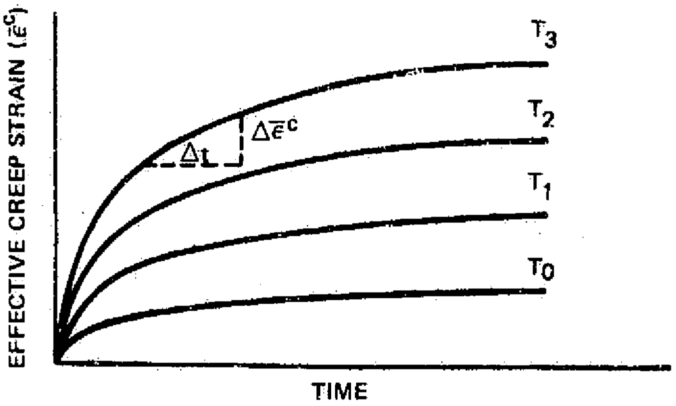


Figure 2.4-2. BOPACE Creep Representation (Example for Variable Temperature and Constant Stress)

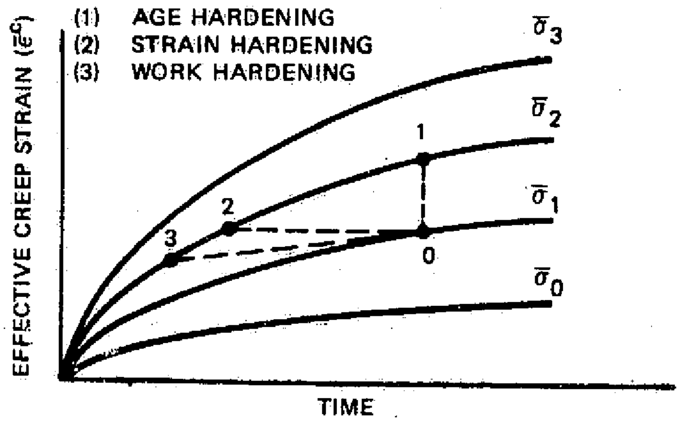


Figure 2.4-3. BOPACE Creep Hardening Options (Example for Constant Temperature and Variable Stress)

the creep time history by allowing the user to define, by a series of input points, the shape of the effective-creep-strain vs. time curve for each material.

Temperature and Stress Effects - Creep rate in most metals is very dependent upon temperature and stress level. The BOPACE approach to creep analysis provides a reasonable description of temperature and stress effects, while avoiding excessive storage and computational requirements. For each material, BOPACE requires a creep curve shape which gives the relative variation of effective-creep-strain vs. time for the various stages considered. This shape is assumed to be valid for all the temperatures and stress levels of the particular material. A table of creep factors for the material is then specified as a function of temperature and effective stress, and a portion of the actual creep curve is determined by multiplying the reference creep curve by the appropriate factor using the average temperature and stress during the increment. Figure 2.4-2 shows portions of typical creep curves for the special case of constant stress level and variable temperature. Note that according to BOPACE assumptions these curves have the same shape.

Hardening - As long as the temperature and stress level remain constant, an increment of creep is determined by following the corresponding creep curve for the given time increment. However, if temperature or stress level changes, an initial point must be identified on the corresponding new creep curve in order to determine the new creep rate. This transfer from one curve to another requires an assumption for creep hardening, which in BOPACE is defined by a single hardening parameter, κ^c. BOPACE allows the option of either age, strain, or work hardening, for which κ^c is defined respectively as the accumulated time, sum of increments of effective creep strain, or creep work. Consider, for example, these options in Figure **2.4-3** for a case of constant temperature. Creep during the preceding increments has progressed to the point 0 on the $\bar{\sigma}_1$ curve. The average effective stress during the present increment is $\bar{\sigma}_2$, which gives the initial points 1, 2 and 3, respectively, for the options of age, strain and work hardening. Incremental creep for the current increment is then determined by continuing along the $\bar{\sigma}_2$ curve from the appropriate initial point, for a distance equal to the specified creep time increment. In the general case both temperature and stress will vary from one increment to the next, but the hardening option still determines in the same manner how the transfer is made between the creep curves.

Load Reversal - The main use of the creep-hardening parameter κ^c comes into play during a load reversal. When a complete reversal occurs, κ^c is set to zero and the initial point on the creep curve is taken as that corresponding to a zero value of κ^c. (A complete load reversal occurs if the incremental creep-strain vector has a direction exactly reversed from that of the preceding creep increment.) For an incomplete load reversal, the BOPACE program computes the starting value for κ^c by multiplying the existing value of κ^c by the factor (1 + COSINE)/2, where COSINE is the Cosine of the angle between successive incremental creep strain vectors. Parameter κ^c then accumulates as before, i.e. at the end of each increment κ^c becomes $\kappa^c + \Delta\kappa^c$.

Multiaxial Flow Rule - The incremental creep-strain vector has historically been taken normal to a Mises type of surface which passes through the stress point. When kinematic plastic hardening is considered, this surface could be taken either as the actual translated yield surface, or as an untranslated surface which passes through the stress point but whose center remains at the origin. The appropriate choice of surface is not clear, and the multiaxial creep flow rule is therefore defined on the basis of programming simplicity. BOPACE defines multiaxial creep under elastic conditions by

$$\Delta\varepsilon_i^c = \left(\frac{3}{2}\,\Delta\bar{\varepsilon}^c/\bar{\sigma}\right) s_i \qquad (2.4\text{-}1)$$

where $\Delta\bar{\varepsilon}^c$ is the increment of effective creep strain defined by

$$(\Delta\bar{\varepsilon}^c)^2 = \frac{2}{3}\,\Delta\varepsilon_i^c \cdot \Delta\varepsilon_i^c \qquad (2.4\text{-}2)$$

while $\bar{\sigma}$ and s are evaluated at the beginning of the increment.

Creep which occurs under plastic conditions is taken in the same direction as that of the plastic strain increment (see Section 2.6).

2.5 COMPLETE STRESS-STRAIN RELATIONS

In Sections 2.1 to 2.4, the basic theory used in BOPACE for elasticity, thermal strains, plasticity and creep has been discussed. The present section describes the complete stress-strain relations, and the manner in which simultaneous elastic, plastic, thermal and creep strains are accounted for. The combined effects of temperature-dependent elasticity and plasticity are included.

General 3-D Relations - For temperature-dependent behavior, an elasto-plastic incremental stress-strain relation follows from Equations 2.1-5b and 2.3-8:

$$\Delta\sigma_i = \Delta D^e_{ij}\ \epsilon^{e0}_j + D^{el}_{ij}\ \Delta\epsilon^{e+p}_j - D^{el}_{ij}\ s_j\lambda \qquad (2.5\text{-}1)$$

Here the first term accounts for stress change due to change in elastic properties, while the second and third terms account for stress change due to change in elastic strain. Following Equation 2.3-9,

$$A\lambda = \hat{s}_i\ \Delta\sigma_i = \hat{s}_i\ \Delta D^e_{ij}\ \epsilon^{e0}_j + \hat{s}_i\ D^{el}_{ij}\ \Delta\epsilon^{e+p}_j - \hat{s}_i\ D^{el}_{ij}\ s_j\lambda \qquad (2.5\text{-}2)$$

where again

$$A = C_{ij}\ \hat{s}_i\ \hat{s}_j + R^o_{ij}\ \hat{s}^o_i\ \hat{s}^o_j \qquad (2.5\text{-}3)$$

For the general case of temperature-dependent plasticity, R^o accounts for isotropic hardening due to both plastic deformation and temperature. Then

$$\lambda = \frac{\hat{s}_i \, \Delta D^e_{ij} \, \varepsilon^{e0}_j + \hat{s}_i \, D^{el}_{ij} \, \Delta\varepsilon^{e+p}_j}{A + \hat{s}_k \, D^{el}_{k\ell} \, \hat{s}_\ell} \qquad (2.5\text{-}4)$$

Substituting Equation 2.5-4 into 2.5-1 gives

$$\Delta\sigma_i = \left(\Delta D^e_{ij} - \frac{D^{el}_{ik} \, \hat{s}_k \, \hat{s}_\ell \, \Delta D^e_{\ell j}}{A + \hat{s}_m \, D^{el}_{mn} \, \hat{s}_n} \right) \varepsilon^{e0}_j + \left(D^{el}_{ij} - \frac{D^{el}_{ik} \, \hat{s}_k \, \hat{s}_\ell \, D^{el}_{\ell j}}{A + \hat{s}_m \, D^{el}_{mn} \, \hat{s}_n} \right) \Delta\varepsilon^{e+p}_j \qquad (2.5\text{-}5)$$

or, using abbreviated notation

$$\Delta\sigma_i = (\Delta D^e_{ij} + \Delta D^p_{ij}) \, \varepsilon^{e0}_j + (D^{el}_{ij} + D^{pl}_{ij}) \, \Delta\varepsilon^{e+p}_j = \Delta D_{ij} \, \varepsilon^{e0}_j + D^1_{ij} \, \Delta\varepsilon^{e+p}_j \qquad (2.5\text{-}6)$$

Thus the increment of stress can be determined as the sum of two products: an incremental matrix times the initial elastic strains, plus an end-of-increment matrix times the incremental elastic+plastic strains.

This formulation was used by the original BOPACE program in the iterative stress-strain algorithm for temperature dependent materials, and is developed here for the sake of clarity. The present BOPACE program, however, employs an improved iterative algorithm, which allows an additional benefit by substituting the simpler Equation 2.1-5a for 2.1-5b. Details of the new algorithm are discussed in Section 2.6. For either approach, the formation of the tangent stiffness matrix is based on Equation 2.3-12, with quantities evaluated at a single appropriate temperature. (In updating the matrix the temperature used is that at the end of the increment).

2.6 IMPROVED ALGORITHM FOR INELASTIC CALCULATIONS

<u>Summary of Basic Concepts</u> - The iterative residual-force procedure is often employed with an incremental solution for inelastic (plasticity and creep) problems, in order to avoid accumulated error. Each iteration in the residual-force procedure involves the following two stages.

1) Equilibrium and Compatibility: Given the current residuals (unbalanced forces or stresses), the equilibrium and compatibility equations are applied in order to predict an improved configuration (of displacements and strains).

2) Separation of Strains: Given the current strains, some algorithm based on the inelastic material theory is applied in order to separate the strains into their elastic, plastic and creep portions, and thus provide the resulting stresses.

When this procedure has converged to the correct result, the following conditions will be met.

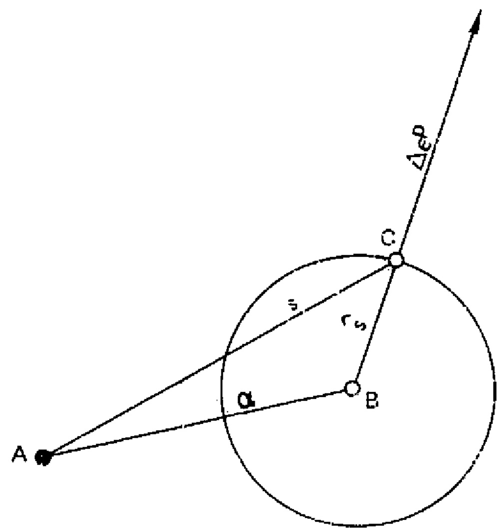
1) Forces in equilibrium
2) Displacements compatible
3) Plastic strain increment satisfies normality rule
4) Size of yield surface consistent with deformation history
5) Translation of yield surface consistent with deformation history

The overall BOPACE solution technique based on the residual-force procedure is summarized in Section 4. The purpose of the present section is to discuss the details of a new algorithm which has been developed and incorporated into BOPACE, for improving the convergence and accuracy of the inelastic stress-strain calculations. This algorithm defines the implementation of stage 2 (separation of strains) in the residual-force iterative procedure.

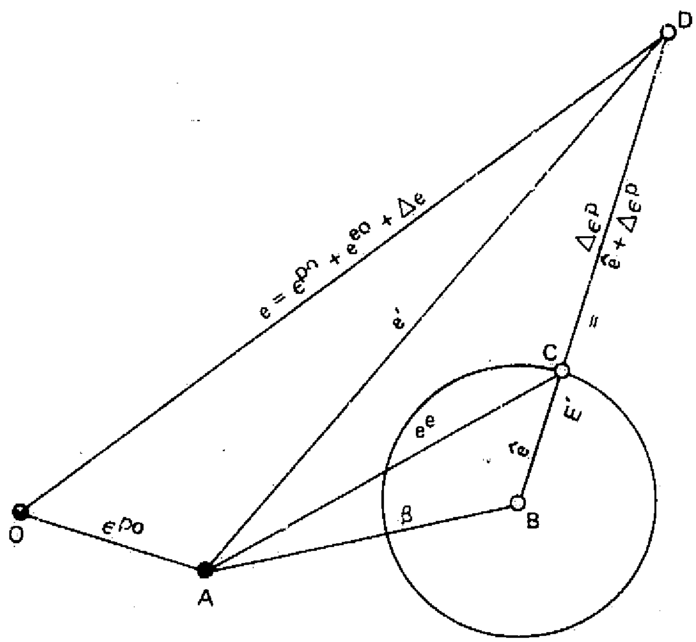
Background - The theory already presented in Sections 2.1 through 2.5 may be employed for both stages of the iterative procedure, and in fact equations of the type 2.5-5 were used for all stress-strain calculations in the initial version of BOPACE. Convergence difficulties resulted from the use of this approach in stage 2, however, when the incremental inelastic strains were large relative to the cumulative elastic strains. These difficulties were substantially eliminated by properly controlling the direction defined for the incremental inelastic strains. (The reason for the difficulties and the method of control were presented in Reference 11). Another quite different approach is based on a "strain-space" concept, and was presented by Barsoum in Reference 12 with the claim of a significant improvement in efficiency. That approach was therefore modified appropriately to make it suitable for a finite element solution procedure, and incorporated into the BOPACE program. Because the method as presented in Reference 12 assumes kinematic hardening only, it was extended to include the combined isotropic and kinematic hardening provided by BOPACE. In addition, some further techniques for accelerating convergence were identified and incorporated into the strain-space method. The resulting BOPACE algorithm appears to be a significant improvement, and it has been made a permanent part of the current program. Although the implementation of the algorithm to include creep,temperature dependent elasticity and plasticity, etc., is somewhat complicated, the basic procedure will be detailed here.

Basic Definitions and Comparison of Algorithms - The new inelastic algorithm involves calculations in the "deviatoric strain space," rather than the more conventional "deviatoric stress space" used in previous BOPACE programs. For the sake of clarity, the previously used stress-space algorithm will again be summarized here, and the elastic-plastic quantitites used in the new strain-space algorithm will be defined and compared with previous quantities.

Note: ● = Point which is fixed during increment
O = Point which moves during increment



a) QUANTITIES IN DEVIATORIC STRESS SPACE



b) QUANTITIES IN DEVIATORIC STRAIN SPACE

Figure 2.6-1. Graphical Representation of Elastic–Plastic Quantities

As described in Section 2.3, the definition of a plasticity theory requires assumptions for three basic constituents: a yield surface, a flow rule, and a hardening assumption. BOPACE development is based on the Mises yield surface, and this surface is represented by a hypercircle in 9-dimensional deviatoric stress space, as shown in Figure 2.6-1a. The surface is defined by the equation

$$F = \hat{s}_i \hat{s}_i - \hat{s}^o_i \hat{s}^o_i = 0 \qquad (2.6\text{-}1)$$

where s is the deviatoric stress, $\hat{s} = s - \alpha$ is the relative deviatoric stress and defines the isotropic hardening, α is the surface translation and defines the kinematic hardening, while $\hat{s}^o$ is a reference value of $\hat{s}$ and must be known as a function of plastic deformation (e.g. from a uniaxial test). Point A in Figure 2.6-1a is the origin of the deviatoric stress space, point B is the current center of the yield surface, and point C represents the current state of deviatoric stress. A stress point on the surface corresponds to a plastic state. According to the Prandtl-Reuss flow rule, the direction of the incremental plastic strain, $\Delta\varepsilon^p$, is normal to the yield surface at the current deviatoric stress state, s. A solid circle (●) in Figure 2.6-1 denotes a point which remains fixed throughout the increment, while an open circle (O) denotes a point which moves during the increment. In order to achieve greater accuracy and allow larger load increments, BOPACE evaluates moving points such as B and C at the midpoint of the plastic increment. Additional details of the BOPACE stress-space algorithm are discussed in Section 2.3.

For the new strain-space algorithm, the three basic constituents of the plasticity theory remain unchanged, and direct use is made of the stress-space theory and nomenclature. However, we now work with a yield surface and associated quantities in strain-space. Thus we compute the deviatoric elastic strain, e^e, in terms of the deviatoric stress, s, by

$$e_i^e = s_i/G \qquad (2.6\text{-}2)$$

where $G = E/(1+\nu)$ is a tensorial shear modulus. Similarly we define a "strain center", β, in terms of the stress center, α, by

$$\beta_i = \alpha_i/G \qquad (2.6\text{-}3)$$

Then the relative deviatoric strain, $\hat{e}$, is defined by

$$\hat{e}_i = e_i^e - \beta_i = (s_i - \alpha_i)/G = \hat{s}_i/G \qquad (2.6\text{-}4)$$

The geometrical interpretation of the new algorithm involving these quantities is provided by a sketch in 9-dimensional deviatoric strain-space, shown in Figure 2.6-1b. There point 0 is the origin, defining the initial undeformed (zero strain) state. Subsequent deformation is caused by a series of load increments, resulting in elastic and plastic strains. A superscript 0 is used to denote the value of a quantity at the beginning of the load increment. Thus, point A defines the cumulative plastic strain, ε^{p0}, which exists at the beginning of the current increment.

(Because of the plastic incompressibility assumption, the plastic strains themselves are deviatoric strains). All other points in Figure 2.6-1b refer to locations at some time during the current increment. In particular, we will be mainly concerned with the location of these points at a defined reference time. This reference time may be taken at the end of the increment, following the approach of Barsoum [12], or greater accuracy may be obtained at the expense of some additional variable storage by taking the reference time at the midpoint of the plastic increment, as is done in the new BOPACE algorithm. Point D defines the total cumulative deviatoric strain, e, at the reference time. The circle is associated with the Mises yield surface, but is a hyper-circle in the deviatoric strain space. A strain point within the surface corresponds to an elastic state, while a strain point <u>outside</u> the surface corresponds to a plastic state. The size of this circle is defined by its radius $\hat{e}_i$ ($\hat{e}_i = \hat{s}_i/G$), whereas the Mises stress-space surface has radius $\hat{s}$. The center of the circle is at point B ($B_i = \varepsilon_i^{p0} + \beta_i = \varepsilon_i^{p0} + \alpha_i/G$), whereas the center of the Mises stress-space surface has components α_i. During plastic deformation, the strain-space surface may undergo both expansion (due to isotropic hardening), and translation (due to kinematic hardening). The cumulative deviatoric elastic strain, e^e, is defined by the vector AC ($e_i^e = s_i/G$). From these comparisons it should be apparent that the basic quantities in Figures 2.6-1a and b, respectively, can be made to coincide, if points A are superimposed and all dimensions in 2.6-1b are divided by the factor G. The incremental plastic strain, $\Delta\varepsilon^p$, is defined by the vector CD. It is normal to the circle because of the Prandtl-Reuss flow rule, and is therefore colinear with the radius $\hat{e}$ to point C. The vector BD = $\hat{e} + \Delta\varepsilon^p$ is denoted by E'. The symbols e', e^e, $\Delta\varepsilon^p$ and E' are consistent with their usage in Reference 12.

Computation Procedure - We now define the new strain-space algorithm for implementing stage 2 of the residual-force iterative procedure. The problem which must be solved can be stated in terms of the various strain vectors. At the beginning of the increment, we have known values for ε^{p0} (which remains constant during the increment), and for β, e^e, and $\hat{e}$. These have been determined such that they are all consistent, i.e., such that the appropriate vectors meet at single points A, B and C. The current estimate for the value of e' at the reference time is also known from stage 1 of the iterative procedure. We must then determine values for β, e^e, $\hat{e}$ and $\Delta\varepsilon^p$ at the reference time, consistent with the convergence requirements. Stated somewhat differently, we are given the locations of points A and D at the reference time, and the locations of points B and C at the beginning of the increment. We must then compute the locations of B and C at the reference time, consistent with the convergence requirements.

The basic steps of the stage 2 algorithm are summarized by the following.

1) Given values at beginning of increment for:

α^0 = stress center

$\hat{s}^0$ = relative deviatoric stress

e^{e0} = elastic strains

2) Given ε^1 = total (elastic + plastic) strain from stage 1.

3) Take values at reference time, based on estimated incremental deformation, for:

$\Delta\alpha$ = kinematic hardening increment

$\Delta|\hat{s}|$ = isotropic hardening increment

4) Compute: $\varepsilon_i^1 \leftarrow \varepsilon_i^1 - \varepsilon_i^{p0}$ = initial elastic strain + total strain increment

e_i' = corresponding deviatoric value

5) Compute: $\beta_i = (\alpha_i^0 + \Delta\alpha_i)/G$

$|\hat{e}| = (|\hat{s}^0| + \Delta|\hat{s}|)/G$

6) Compute: $E_i' = e_i' - \beta_i$

7) Compute: $\lambda = (|E'| - |\hat{e}|)/|E'|$ = plastic proportionality constant

8) Compute: $\Delta\varepsilon_i^p = \lambda E_i'$ = incremental plastic strain at reference time. Adjust $\Delta\varepsilon_i^p \leftarrow \Delta\varepsilon_i^p$ times ratio (ratio = total time increment / reference time increment, to obtain total plastic strain increment. Set α and $|\hat{s}|$ values based on $\Delta\varepsilon^p$.

9) Compute end of increment values for:

$\varepsilon_i^e = \varepsilon_i^1 - \Delta\varepsilon_i^p$ = cumulative elastic strain

$\sigma_i = D_{ij}\,\varepsilon_j^e$ = cumulative stress

10) Use σ to compute residual forces and residual norm, and return to stage 1 if convergence has not been achieved.

The strain-space algorithm presented above corresponds to that given by Barsoum [12] except that here a combined isotropic and kinematic hardening is provided and a reference (midpoint) time calculation of the incremental variables is used to improve accuracy. As noted by Barsoum, greater consistency and better convergence are obtained by utilizing an algorithm in strain space rather than in stress space. This is because the stress-space calculation fixes the $\Delta\varepsilon^p$ vector along the direction of a previous $\hat{s}$ vector, rather than simultaneously fixing the directions of $\hat{s}$ and $\Delta\varepsilon^p$ consistent with the given total strain increment $\Delta\varepsilon$. The stress-space algorithm can cause large tangential oscillations in the location of point C, resulting in divergence if $\Delta\varepsilon^p$ is large relative to the cumulative elastic strain.

Although a strain-space algorithm as described will eliminate most of the inconsistencies and tendencies toward divergence, it should be noted that an inconsistency still exists in the plastic hardening quantities. This is due to updating α and $\hat{s}$ based on the estimated increment of plastic deformation, which will not in general be consistent with the actual deformation. Thus, if another iteration were performed using the same value for the total strain increment $\Delta\varepsilon$, different results would be obtained due to change in β and $\hat{e}$. This inconsistency in the basic strain-space space algorithm often results in poor convergence, with radial oscillations of points B and C from one iteration to the next, espacially if the plastic hardening slopes (c and r) are relatively large.

The present strain-space algorithm eliminates the difficulty by properly modifying the calculation of λ in step 7. In this modification we use the parameters c and r associated with kinematic and isotropic hardening, respectively, in the expressions

$$\Delta\alpha_i = \frac{2}{3} c \, \Delta\varepsilon_i^p$$

$$\Delta|s| = \frac{2}{3} r |\Delta\varepsilon^p| \qquad (2.6\text{-}5)$$

The E' vector can then be written as

$$E_i' = e_i' - \beta_i = e_i' - (\beta_i^0 + \Delta\beta_i)$$

$$= e_i' - \beta_i^0 - \Delta\alpha_i/G = e_i' - \beta_i^0 - \frac{2}{3} c \, \Delta\varepsilon_i^p/G \qquad (2.6\text{-}6)$$

Replacing $\Delta\varepsilon_i^p$ in this equation by $\lambda E_i'$, we may solve for E_i':

$$E_i' = (e_i' - \beta_i^0)/(1 + \frac{2}{3} \lambda c/G) \qquad (2.6\text{-}7a)$$

In a similar manner we may obtain

$$|\hat{e}| = |\hat{e}^0| + \frac{2}{3} \lambda r |E'|/G \qquad (2.6\text{-}7b)$$

The plastic proportionality constant, as already defined, is

$$\lambda = (|E'| - |e|)/|E'| \qquad (2.6\text{-}7c)$$

It is apparent from Equations 2.6-7 that the expression for λ is non-linearly dependent upon λ itself, and this is the reason why consistent λ is not solved for directly. An accurate value for λ, however, can easily be obtained by a "linear intersection method." In this method we take the approximate value of λ from step 7 of the stage 2 algorithm, and substitute into the Equations 2.6-7 to obtain a new computed value

λ_{c0}. We then assume a value of $\lambda + \Delta\lambda$, where $\Delta\lambda$ is a small change (perhaps $.01\lambda$), and again substitute into Equations 2.6-7 to compute another value λ_{c1}. The two pairs of assumed and computed λ values are plotted in Figure 2.6-2. The correct value for λ lies on the 45-degree line (since there the assumed and computed values would be equal), at the intersection of this line with the line connecting the two plotted points. This corrected value of λ is obtained by the following adjustment of λ from step 7.

$$\lambda \leftarrow \lambda + \Delta\lambda(\lambda_{c0} - \lambda)/(\Delta\lambda - \lambda_{c1} + \lambda_{c0}) \qquad (2.6\text{-}8)$$

The incorporation of this adjustment into the strain-space algorithm provides consistent values for all quantities in stage 2 of the iterative process, and results in improved convergence.

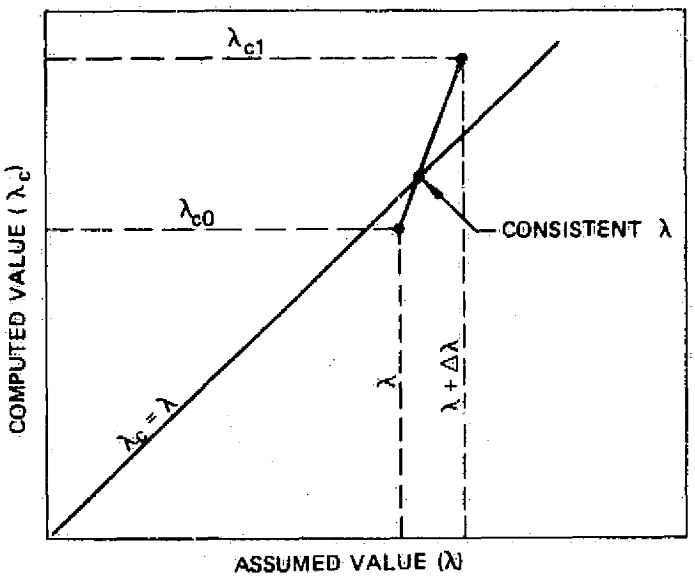


Figure 2.6-2. Linear-Intersection Calculation for λ

Further Extensions and Refinements to the Basic Algorithm - The strain-space algorithm as presented here is employed in BOPACE for plastic analysis. In addition, the BOPACE algorithm treats creep strains in a manner similar to that for the plastic strains. For cases where the material is elastic at the beginning of an increment and then reaches the plastic yield point at some intermediate time during the increment, greater accuracy is obtained by dividing the calculations into two parts. In such cases the initial creep is taken in the direction of the initial deviatoric stress, and creep which occurs after the yield point is taken in the same direction as the plastic strain increment. Other extensions, such as temperature dependent elastic-plastic-creep, have also been incorporated into the BOPACE program.

2.7 ANISOTROPIC ELASTICITY

The anisotropic stress-strain relation for 3-dimensional analysis may be written in the general form

$$\sigma_i = D_{ij}\ \varepsilon_j^e \tag{2.7-1a}$$

where D is a 6 x 6 symmetric matrix of elastic constants. In order to provide a simple form of temperature dependence in its anisotropic elasticity, BOPACE includes a factor, f, which may be specified as a function of temperature. The stress-strain relation then becomes

$$\sigma_i = f(T)\ D_{ij}\ \varepsilon_j^e \tag{2.7-1b}$$

Thermal strains are introduced for the anisotropic material, by specifying each normal thermal strain component as an independent function of temperature. Thus,

$$\left.\begin{aligned} \varepsilon_{xx}^t &= \varepsilon_{xx}^t\ (T) \\ \varepsilon_{yy}^t &= \varepsilon_{yy}^t\ (T) \\ \varepsilon_{zz}^t &= \varepsilon_{zz}^t\ (T) \end{aligned}\right\} \tag{2.7-2}$$

The current BOPACE version does not provide plasticity or creep for anisotropic materials.

2.8 REDEFINITION OF MATERIAL PROPERTIES

It is desirable to provide maximum versitility for definition of material properties, without unnecessarily complicating the form of input and storage. Toward that end, BOPACE allows redefinition of any material properties, at the start of each load increment. This allows the user to modify material properties, for example, to approximate some of the following types of behavior.

1. Differences between tensile and compressive properties (redefine material as function of current stress state).
2. Crack surface and other tension cutoff situations.
3. Treatment of plasticity or creep behavior which does not follow, during the entire deformation, the theoretical behavior of a single material definition.
4. Treatment of temperature dependent anisotropic materials, for which the entries of the elasticity matrix do not all vary proportionately with temperature.

Some caution must of course be used in redefining material properties, in order to prevent significant discontinuities in behavior, which could lead to inconsistent results or difficulties in convergence of the solution.

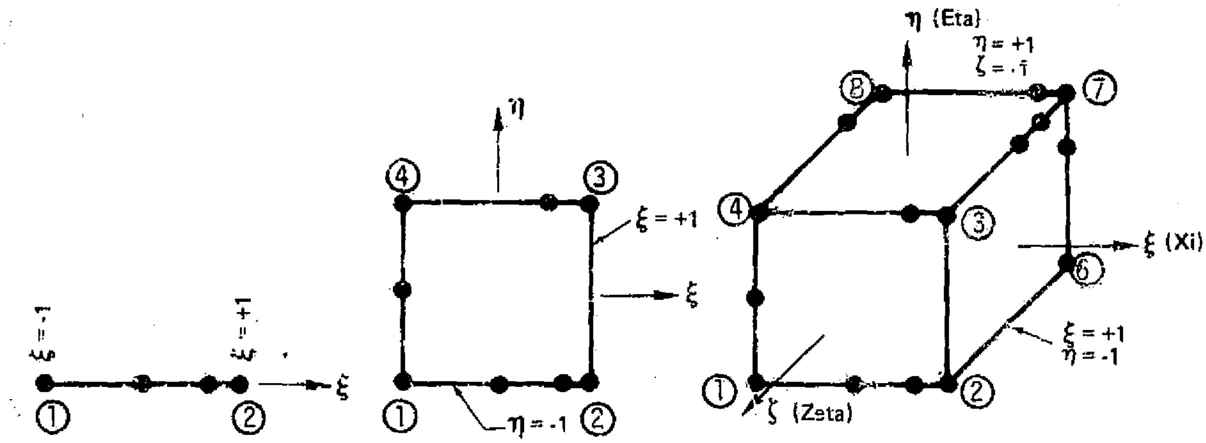
3.0 FINITE ELEMENT FORMULATIONS

The BOPACE program provides a family of isoparametric (curved boundary) finite elements, with a user-selected number of nodes along each element boundary. The simplest elements of this family are the 2-node rod, 4-node quadrilateral and 8-node brick, and various higher order elements of the family are defined by adding additional edge nodes to the basic corner nodes. BOPACE allows each element edge to contain an optionally different number of arbitrarily spaced nodes (from 2 to 5), resulting in a total maximum number of nodes equal to 5, 16 and 44 on the rod, quadrilateral and brick elements, respectively. The arbitrary number and spacing of nodes allowed by this family provides versatility for representing complex geometries, and also makes variable mesh spacing convenient.

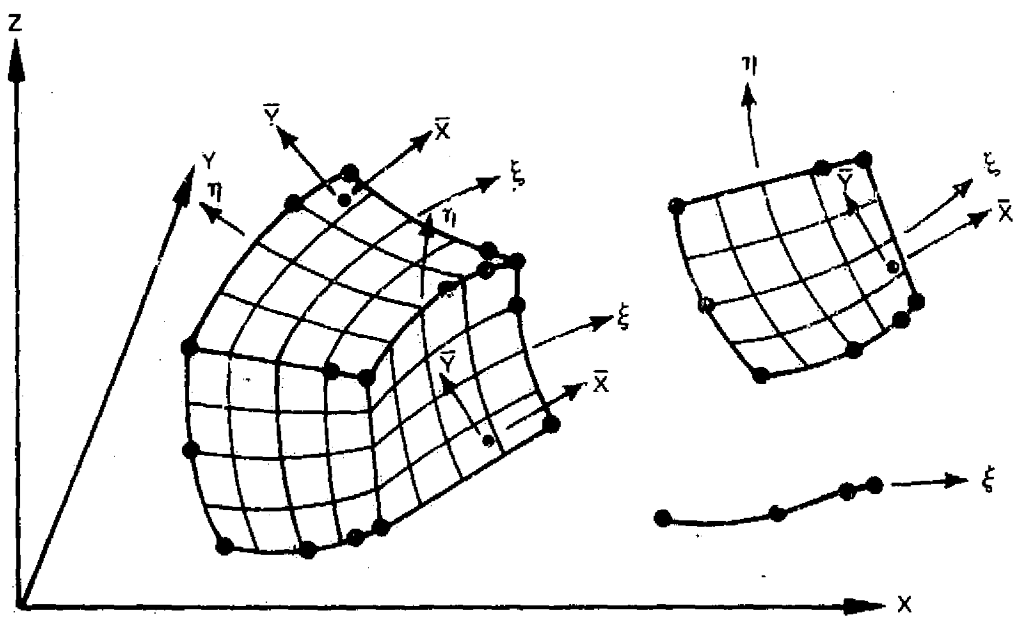
This section discusses the elemental-level formulations, including the isoparametric formulation and shape functions, the calculation of reference-point and nodal quantities, the stiffness matrix generation, and the numerical integration process.

3.1 ISOPARAMETRIC FORMULATION AND SHAPE FUNCTIONS

The isoparametric finite element concept involves the definition of shape functions over a simple "parent" element. These functions then serve a dual purpose: 1) they map the geometry of the parent element into an element of the actual body, and 2) they accomplish the usual task of



(a) PARENT ELEMENTS — 1-D (ROD), 2-D (QUAD), 3-D (BRICK)



(b) ACTUAL ELEMENTS, AS USED FOR 3-DIMENSIONAL PROBLEM

Figure 3.1-1: BOPACE Isoparametric Elements interpolating the field quantities (e.g., the temperature and displacements) at any point within the element in terms of the nodal quantities. A detailed discussion of isoparametric formulations is given in the book by Zienkiewicz [13].

Typical BOPACE isoparametric elements are illustrated in Figure 3.1-1, with both their parent and actual forms. The discussion in this section will be presented mainly in terms of the 3-dimensional BRICK element, but the procedure for other elements follows a similar development via an appropriate reduction in dimensions.

Element Coordinates, Nodes and Reference Points - The parent BRICK element is defined as a 2 x 2 x 2 cube, having an associated Cartesian coordinate system ξ-η-ζ with origin at the center of the cube and axes normal and parallel to the faces (the coordinate normal to a face has a value of ± 1 on that face). The nodes of the actual element define its generally curved boundaries (each edge is a space curve defined by the polynomial through its nodes). Element nodal quantities (forces, displacements, stiffnesses) are referred at this stage to the basic X-Y-Z coordinate system, which is a global Cartesian system for the entire structure. Each element also contains a number of reference points, located in its interior or on its surface. These points include the integration points required for solution, plus an optional additional number of user-selected points. (The additional points are for the purpose of output only, and have no effect on the solution). Element reference-point quantities (strains, stresses, etc.) are referred to the $\bar{x}$-$\bar{y}$-$\bar{z}$ coordinate systems, which are Cartesian systems defined for each point. It should be noted that for ROD or QUAD type elements used in a 3-dimensional problem, the reference-point coordinate system axes must logically be tangent to the centerline or surface of the element. Coordinate systems are discussed in more detail in Section 4.

Element Geometry and Field Quantities - Each element node has an associated shape function given in terms of the ξ-η-ζ coordinates, i.e., at the ith node the shape function is denoted by $N^i(\xi, \eta, \zeta)$. These shape functions define the geometry of the actual element by a pointwise mapping from the parent element. Thus in the actual element the X-coordinate of a point is given by

$$X = X^i N^i(\xi, \eta, \zeta) \qquad (3.1\text{-}1a)$$

Here (ξ, η, ζ) are the coordinates of the corresponding point in the parent element, and X^i is the X-coordinate of the ith node, with summation implied over i. Similar expressions are used for Y and Z coordinates. Field quantities such as temperature, and displacements U-V-W (in the X-Y-Z directions, respectively), of a point in the actual element are defined in the same manner. For example, the U-displacement of a point is defined by

$$U = U^i N^i(\xi, \eta, \zeta) \qquad (3.1\text{-}1b)$$

where U^i is the U-displacement of the ith node.

Displacement Derivative Calculations - At each reference point in the BOPACE element, the spatial displacement derivatives must be expressed in terms of the nodal displacements. To accomplish this, we first define a matrix g at the reference point, consisting of the nodal shape functions differentiated with respect to the ξ-η-ζ coordinates:

$$g = \begin{bmatrix} \frac{\partial N^1}{\partial \xi} & \frac{\partial N^2}{\partial \xi} & \text{---} & \text{---} & \text{---} & \frac{\partial N^n}{\partial \xi} \\ \frac{\partial N^1}{\partial \eta} & \frac{\partial N^2}{\partial \eta} & \text{---} & \text{---} & \text{---} & \frac{\partial N^n}{\partial \eta} \\ \frac{\partial N^1}{\partial \zeta} & \frac{\partial N^2}{\partial \zeta} & \text{---} & \text{---} & \text{---} & \frac{\partial N^n}{\partial \zeta} \end{bmatrix} \tag{3.1-2}$$

A Jacobian matrix, J, at the point is computed as

$$J_{ij} = g_{im} X_{jm} \tag{3.1-3a}$$

where X_{ij} is the X-Y-Z system ith coordinate of the jth node. The Jacobian has the form

$$J = \begin{bmatrix} \frac{\partial X}{\partial \xi} & \frac{\partial Y}{\partial \xi} & \frac{\partial Z}{\partial \xi} \\ \frac{\partial X}{\partial \eta} & \frac{\partial Y}{\partial \eta} & \frac{\partial Z}{\partial \eta} \\ \frac{\partial X}{\partial \zeta} & \frac{\partial Y}{\partial \zeta} & \frac{\partial Z}{\partial \zeta} \end{bmatrix} \tag{3.1-3b}$$

In general the X and $\bar{x}$ coordinate systems are different, in which case a transformation matrix, C, of direction Cosines is defined at the reference point, such that

$$\begin{Bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{Bmatrix} = [C] \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \qquad (3.1\text{-}4a)$$

The Jacobian is then transformed by

$$\bar{J}_{ij} = J_{im} C_{jm} \qquad (3.1\text{-}4b)$$

Finally a transformation is applied to the g matrix, of the form

$$\bar{g}_{ij} = \bar{J}^{-1}_{im} g_{mj} \qquad (3.1\text{-}4)$$

This inverse Jacobian transformation produces the desired form of the partial derivative matrix $\bar{g}$, which is

$$\bar{g} = \begin{bmatrix} \frac{\partial N^1}{\partial \bar{x}} & \frac{\partial N^2}{\partial \bar{x}} & \text{---} & \text{---} & \text{---} & \frac{\partial N^n}{\partial \bar{x}} \\ \frac{\partial N^1}{\partial \bar{y}} & \frac{\partial N^2}{\partial \bar{y}} & \text{---} & \text{---} & \text{---} & \frac{\partial N^n}{\partial \bar{y}} \\ \frac{\partial N^1}{\partial \bar{z}} & \frac{\partial N^2}{\partial \bar{z}} & \text{---} & \text{---} & \text{---} & \frac{\partial N^n}{\partial \bar{z}} \end{bmatrix} \qquad (3.1\text{-}5)$$

A composite matrix is then defined by

$$\bar{G} = \begin{bmatrix} \bar{g} & o & o \\ o & \bar{g} & o \\ o & o & \bar{g} \end{bmatrix} \tag{3.1-6}$$

Now to define displacement derivatives we will find it convenient to use both the vector (single subscript) and matrix (double subscript) forms interchangeably. Thus we define, for example,

$$\theta' = \left(\frac{\partial U}{\partial \bar{x}} \; \frac{\partial U}{\partial \bar{y}} \; \frac{\partial U}{\partial \bar{z}} \; \frac{\partial V}{\partial \bar{x}} \; \frac{\partial V}{\partial \bar{y}} \; \frac{\partial V}{\partial \bar{z}} \; \frac{\partial W}{\partial \bar{x}} \; \frac{\partial W}{\partial \bar{y}} \; \frac{\partial W}{\partial \bar{z}} \right) =$$

$$\begin{bmatrix} \frac{\partial U}{\partial \bar{x}} & \frac{\partial V}{\partial \bar{x}} & \frac{\partial W}{\partial \bar{x}} \\ \frac{\partial U}{\partial \bar{y}} & \frac{\partial V}{\partial \bar{y}} & \frac{\partial W}{\partial \bar{y}} \\ \frac{\partial U}{\partial \bar{z}} & \frac{\partial V}{\partial \bar{z}} & \frac{\partial W}{\partial \bar{z}} \end{bmatrix} \tag{3.1-7a}$$

and

$$\bar{\theta} = \left(\frac{\partial \bar{u}}{\partial \bar{x}} \; \frac{\partial \bar{u}}{\partial \bar{y}} \; \frac{\partial \bar{u}}{\partial \bar{z}} \; \frac{\partial \bar{v}}{\partial \bar{x}} \; \frac{\partial \bar{v}}{\partial \bar{y}} \; \frac{\partial \bar{v}}{\partial \bar{z}} \; \frac{\partial \bar{w}}{\partial \bar{x}} \; \frac{\partial \bar{w}}{\partial \bar{y}} \; \frac{\partial \bar{w}}{\partial \bar{z}} \right) \tag{3.1-7b}$$

If we arrange the vector q of element nodal displacements in the form

$$q = \left(U^1 \; U^2 \; U^3 \; U^4 \; - - - \; U^n, \; V^1 \; - - - \; V^n, \; W^1 \; - - - \; W^n \right) \tag{3.1-8}$$

we may write the important relationship between displacement derivatives and nodal displacements, in the form

$$\theta_j' = \bar{G}_{ij}\, q_j \qquad (3.1\text{-}9)$$

The reference-point system derivatives $\bar{\theta}$ may be obtained from the mixed derivatives θ' by the transformation

$$\bar{\theta}_{ij} = C_{im}\, \theta_{jm}' \qquad (3.1\text{-}10)$$

In constructing the BOPACE program logic it is actually more convenient to rearrange the q-vector so that the U-V-W displacements at a particular node are grouped together. This also requires rearrangement of the columns of the $\bar{G}$ matrix. However the calculation and storage of the $\bar{g}$-matrix for each reference point occurs in the simple form of Equation 3.1-5, and required operations involving the $\bar{G}$ matrix are performed simply by an appropriate indexing procedure, taking full advantage of the evident sparsity in the given form of $\bar{G}$.

<u>General Considerations for the Shape Functions</u> - The shape functions for an element are derived so as to have the following characteristics.

1) Each function is independent, having a unit value at its associated node and zero values at all other nodes.

2) Each function satisfies interelement continuity requirements, having zero values on all edges and faces except those on which its associated node is located.

3) In order to guarantee convergence with mesh refinement, any state of constant displacement derivatives within an element must be obtainable by a linear combination of the shape functions.

The derivation of shape functions which satisfy these requirements has often been accomplished by trial and error or by chance discovery, as in the case of the so-called "serendipity" isoparametric elements of Reference 13. A rational approach to the same functions, however, is provided through the use of Lagrange interpolation, and this approach proves to be more general as well. By means of Lagrange interpolation, two types of elements are provided in BOPACE, depending upon the manner in which the mapping between parent and actual elements is accomplished. The "proportionate" mapping provides elements which usually perform better for general analysis, while the "serendipity" mapping provides elements which are useful for crack-tip analysis. The following paragraph describes the two types of mappings used in BOPACE, and their application to regular and crack-tip types of elements.

Proportionate and Serendipity Mappings (Regular and Crack-Tip Elements) - The edge nodes may be arbitrarily spaced along the edge of the actual element. With the proportionate mapping, the edge node positions on the parent element are determined by using their perpendicular projections onto the straight line connecting the appropriate two corner nodes of the actual element, with a proportionate mapping of this line back onto the edge of the parent element. Thus a variable spacing of the edge nodes generally occurs on the parent element itself. With the serendipity mapping, the edge nodes are uniformly spaced on the parent element, and receive their variable spacing on the actual element through mapping by the shape functions. Then by proper user location of the nodes along appropriate edges of the actual element, a crack-tip analysis capability is created, providing displacements and singular strains which vary in a half-power and inverse half-power relationship, respectively, along these edges. For example, location of midside nodes at the quarter points (one-fourth the distances along the edges from the crack tip to the opposite corner nodes) creates the singular crack-tip element described in Reference 14. It is to be noted that the required strain singularity is exact along the element edges, but only approximate within the element interior. The proportionate mapping is usually more accurate for general analysis, because the displacements and strains vary as polynomials rather than as half powers or other functions. The capability for representing basic lower order strain states is therefore disrupted to a lesser extent with the proportionate mapping. The following discussion of the BOPACE element functions is conveniently presented by first describing the interior edge node functions, and then defining the corner node functions.

Edge Functions - Consider for example, a particular edge of the parent element, which is parallel to the ξ direction and has nodes 1, 2, --- m arbitrarily spaced along its length. A Lagrange interpolation function for the ith node on this edge is formed by taking the product

$$(\xi - \xi_1)(\xi - \xi_2)---(\xi - \xi_{i-1})(\xi - \xi_{i+1})---(\xi - \xi_m)$$

It is to be noted that this product is nonzero at node i and is zero at all other nodes along the edge. It is a polynomial of order m-1. This product is then supplemented by giving it a linear variation in the other two coordinate directions, i.e., by multiplying by the product

$$(\eta - \eta_0)(\zeta - \zeta_0)$$

where η_0 and ζ_0 are coordinates (± 1) of the opposite edges. Finally a unit normalized function is obtained after dividing by the value which the function takes on at its associated node. Thus the final shape function for the ith node of the edge is

$$N^i(\xi, \eta, \zeta) = (\prod_j (\xi - \xi_j))(\eta - \eta_0)(\zeta - \zeta_0)/(\text{normalization factor}) \tag{3.1-11a}$$

where the Π product sum is taken over all nodes on the edge except the ith node.

The derivatives of this function which are required for BOPACE analysis are obtained from the following expressions.

$$\frac{\partial N^i}{\partial \xi} = N^i \sum_{j=1}^{m} \frac{1}{\xi - \xi_j}, \; j \neq i \tag{3.1-11b}$$

$$\frac{\partial N^i}{\partial \eta} = N^i/(\eta - \eta_0) \tag{3.1-11c}$$

$$\frac{\partial N^i}{\partial \zeta} = N^i/(\zeta - \zeta_0) \tag{3.1-11d}$$

The shape functions and their derivatives, for edge nodes of edges parallel to the η and ζ axes, follow directly from Equations 3.1-11 by cyclic permutation of the coordinates ξ-η-ζ.

Corner Functions - The shape function for a corner node is most effectively obtained in two parts -- the "linear" function and the "deviation from linearity." The linear function is the function which would be used for an 8-node brick without edge nodes. For example, this function for the corner node at $\xi = \eta = \zeta = 1$, is

$$N = \frac{1}{8}(1 + \xi)(1 + \eta)(1 + \zeta) \qquad (3.1\text{-}12a)$$

which provides displacement states in which all edges remain straight. The edge nodes during such displacement, of course, undergo nonzero displacements, and these must be eliminated by the addition of the deviation-from-linearity functions. Such functions are simply the edge node functions already discussed. Thus the total shape function for the above corner node is

$$N = \frac{1}{8}(1 + \xi)(1 + \eta)(1 + \zeta) - \Sigma\alpha^i N^i \qquad (3.1\text{-}12b)$$

where the summation is performed over all edge nodes.

Each coefficient α^i is the value of the corner function 3.1-12a evaluated at the ith edge node, and N^i is the shape function of the ith edge node. Shape functions for other corner nodes are obtained in the same manner, after appropriate changes of sign (± 1) in Equation 3.1-10a.

3.2 STRAIN, FORCE AND STIFFNESS QUANTITIES

Strains - The Lagrangian strain, ε, is defined in terms of material displacement derivatives, θ, at a point in the body, by

$$\varepsilon_i = A0_{ij}\theta_j + \frac{1}{2} A1_{ijk}\theta_j\theta_k \qquad (3.2\text{-}1)$$

Here A0 and A1 are constant coefficients which define the strain tensor, with A1 providing the (geometrically) nonlinear portion of the strain. In terms of engineering strain components, this equation may be rewritten with the linear (A0) contribution in expanded form, as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_z \\ v_x \\ v_y \\ v_z \\ w_x \\ w_y \\ w_z \end{Bmatrix} + \frac{1}{2} A1_{ijk}\, \theta_j\theta_k \qquad (3.2\text{-}2a)$$

where x-y-z and u-v-w are the coordinate directions and corresponding displacements, respectively, for the point.

Here the nonzero terms of A1 are

$$
\begin{aligned}
A1_{111} &= A1_{144} = A1_{177} = 1 \\
A1_{222} &= A1_{255} = A1_{288} = 1 \\
A1_{333} &= A1_{366} = A1_{399} = 1 \\
A1_{412} &= A1_{421} = A1_{445} = A1_{454} = A1_{478} = A1_{487} = 1 \\
A1_{513} &= A1_{531} = A1_{546} = A1_{564} = A1_{579} = A1_{597} = 1 \\
A1_{623} &= A1_{632} = A1_{656} = A1_{665} = A1_{689} = A1_{698} = 1
\end{aligned}
\tag{3.2-2b}
$$

and a useful form for the nonlinear contribution is defined by

$$
A1_{ijk}\,\theta_k = \begin{bmatrix}
u_x & 0 & 0 & v_x & 0 & 0 & w_x & 0 & 0 \\
0 & u_y & 0 & 0 & v_y & 0 & 0 & w_y & 0 \\
0 & 0 & u_z & 0 & 0 & v_z & 0 & 0 & w_z \\
u_y & u_x & 0 & v_y & v_x & 0 & w_y & w_x & 0 \\
u_z & 0 & u_x & v_z & 0 & v_x & w_z & 0 & w_x \\
0 & u_z & u_y & 0 & v_z & v_y & 0 & w_z & w_y
\end{bmatrix}
\tag{3.2-2c}
$$

Because of the symmetry of A1 ($A1_{ijk} = A1_{ikj}$), the differentiation of Equation 3.2-2a provides the strain rate, $\dot{\varepsilon}$, in terms of the displacement derivative rate, $\dot{\theta}$, in the simple form

$$
\dot{\varepsilon}_i = (A0_{ij} + A1_{ijk}\,\theta_k)\dot{\theta}_j \tag{3.2-3}
$$

Forces - The principle of virtual work is valid for arbitrary nonlinear materials, and it provides a simple basis for deriving the element force and stiffness relations. The equivalence of external and internal virtual work relates the generalized nodal forces p_i and displacements q_i in the element equilibrium equation

$$\delta q_i p_i = \int_V \delta\varepsilon_i \ \sigma_i \ dV = \int_V \delta\varepsilon_i \ (\sigma_i{}^* + DO^*_{ij} \ \Delta\varepsilon_j) dV \qquad (3.2\text{-}4)$$

which holds along any equilbrium path in the neighborhood of a particular equilibrium (*) configuration. (The integral expression in Equation 3.2-3 is exact, except that only the first order incremental stress-strain relation involving the DO matrix is used. Its use does not lead to inaccurate results in the solution process because of the equilibrium check and midpoint residual-force corrective iterations which are performed.) Here $\delta\varepsilon$ and δq are kinematically consistent variations, and from Equations 3.2-3 and 3.1-9 (ignoring, for simplicity, the overbars which denote coordinate system)

$$\delta\varepsilon_i = (AO_{ij} + A1_{ijk} \ \theta_k) \ \delta\theta_j \equiv A_{ij} \ \delta\theta_j \simeq A_{im} \ G_{mj} \ \delta q_j \equiv B_{ij} \ \delta q_j \qquad (3.2\text{-}5)$$

The theoretical implementation of Equation 3.2-4 requires the use of the second Piola-Kirchoff stress associated with Lagrangian strain, with integration over the undeformed volume (e.g., see Oden and Key, Reference 15). However the small strain assumption is used in the BOPACE formulation, so that the stress may be taken as the usual engineering or true stress. (A general large strain development is presented in the context of perturbation solution methods, in References 16 and 17.)

Substituting for $\delta\varepsilon$ in terms of δq, and realizing that Equation 3.2-4 must be satisfied for arbitrary variations δq, provides the basic equilibrium equation for forces, as

$$p_i = \int_V B_{ai}\sigma_a \, dV = \int_V G_{mi} \, (AO_{am} + A1_{amn} \, \theta_n)(\sigma^*_a + DO^*_{ab} \, \Delta\varepsilon_b)dV \tag{3.2-6a}$$

or at a particular equilibrium (*) configuration, where $\sigma = \sigma^*$ and $\theta = \theta^*$, we have

$$p^*_i = \int_V G_{mi} \, (AO_{am} + A1_{amn} \, \theta^*_n) \, \sigma^*_a dV = \int_V G_{mi} \, A^*_{am} \, \sigma^*_a dV \tag{3.2-6b}$$

Stiffness – Differentiating Equation 3.2-6a and evaluating at the particular equilibrium configuration ($\Delta\varepsilon = 0$, $\theta = \theta^*$, etc.) provides the first order equilibrium rate equation

$$\dot{p}^*_i = KO^*_{ij} \, \dot{q}^*_j \tag{3.2-7}$$

where

$$KO^*_{ij} = \int_V G_{mi} \, (AO_{am} \, DO^*_{ab} \, AO_{bn} + \sigma^*_a \, A1_{amn}) \, G_{nj} \, dV \tag{3.2-8}$$

The "tangent stiffness" matrix KO* as given by Equation 3.2-8 is clearly separable into two parts-- the geometrically linear and the geometrically nonlinear contributions. The first contribution to KO* is due to the incremental stress-strain relation, and its symmetry depends on symmetry of the matrix DO*. The second contribution is due to the initial stresses during changing geometry, and is always symmetric in form. The incremental form of Equation 3.2-7 is of course

$$\Delta p_i = KO^*_{ij}\ \Delta q_j \qquad (3.2\text{-}9)$$

and is the basis for BOPACE solution procedures. In generating the tangent stiffness by Equation 3.2-8, a matrix $H^*_{mn} = AO_{am}\ DO^*_{ab}\ AO_{bn} + \sigma^*_a\ A1_{amn}$ is first computed, after which the product $G_{mi}\ H^*_{mn}\ G_{nj}$ is formed. The geometrically nonlinear part of H is given by

$$\sigma_a\ A1_{amn} = \begin{bmatrix} h & 0 & 0 \\ 0 & h & 0 \\ 0 & 0 & h \end{bmatrix}$$

where

$$h = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$

The stiffness generation procedure described here takes maximum advantage of sparsity in the G matrix, and also allows the inclusion of geometric nonlinearities as a simple optional program step.

3.3 NUMERICAL INTEGRATION

The integrals which define force and stiffness quantities in the BOPACE program are calculated by numerical integration using Gauss product formulas, and must be evaluated over the volume of the actual element. The mechanics of the integration process, however, are best accomplished over the parent element, where there are simple integration limits in terms of the ξ-η-ζ Cartesian coordinate system. Integration is therefore taken in the form

$$\int_V f\, dV = \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} f(\xi,\eta,\zeta)\,|J|\; d\xi d\eta d\zeta \qquad (3.3\text{-}1)$$

where f is the function to be integrated, and $|J|$ is the Jacobian determinant which corrects for the fact that a differential volume (dxdydz) in the actual element is equal to $|J|$ ($d\xi d\eta d\zeta$).

The integral is evaluated numerically by substituting for it a sum over a number of Gauss integration points:

$$\int_V f dV = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} f(\xi_i,\eta_j,\zeta_k)\; W_{ijk} \qquad (3.3\text{-}2)$$

Here m, n, p are the numbers of integration points in the ξ, η, ζ directions, respectively (total number of points = mxnxp), and W_{ijk} is a weighting factor for each point which includes the value of the Jacobian determinant. The Gauss integration scheme is used because it provides higher accuracy for a given number of points than some other methods, through an optimum selection of the point locations. (The use of m Gauss points allows the exact integration of a polynomial of degree 2m-1.) The BOPACE program provides, as a default, the automatic selection of the number of integration points in each coordinate direction, so as to exactly integrate the stiffness matrix. (This stiffness integration is generally exact only if all element edges are straight lines and the H-matrix is constant over the element). However, it has been found that accuracy and convergence are often improved if fewer integration points are used, especially for plasticity analysis. (A 2 x 2 x 2 point rule is often useful for BRICK elements). BOPACE allows the user to select, if he wishes, the number of Gauss points in each direction.

4.0 COORDINATE SYSTEMS

BOPACE coordinate systems are used to locate nodes and element reference points, to define the quantities (e.g., forces, displacements, stresses and strains) associated with these nodes and points, and to help define other program input such as distributed load directions and inertia related vectors.

The definition and use of the element parent coordinate systems have been discussed in Section 3. BOPACE uses several other coordinate systems, each of which is associated with an integer identification number:

- 0 tangent
- 1 basic Cartesian
- 2 basic cylindrical
- 3 basic spherical
- >3 special Cartesian

Each of these systems is described in the remainder of this section.

Tangent Systems - Tangent system coordinates $\bar{x}$-$\bar{y}$-$\bar{z}$ are element associated Cartesian systems, and are used to define reference-point quantities and distributed load directions, for various points within or on the surface of the element. The $\bar{x}$ axis is taken tangent to a parent coordinate ξ axis at the point; the $\bar{y}$ axis is taken normal to $\bar{x}$ and tangent to the parent ξ-η plane, such that $\bar{y}$ has a positive component in the η direction; and $\bar{z}$ is defined such that $\bar{x}$-$\bar{y}$-$\bar{z}$ is a right hand system. The ξ-η coordinates used to define the tangent system correspond to the parent element, or in some cases to a local parent region such as a particular face or edge of the element.

In case the parent system is defined for a 1-dimensional region (e.g., a rod type element, or edge of a membrane or solid type element), the η direction is undefined. The ambiguity for direction $\bar{y}$ is then overcome by taking $\bar{y}$ normal to $\bar{x}$ and in the basic XY plane, such that $\bar{z}$ has a component in the positive Z direction. In the special case where $\bar{x}$ is parallel with Z (i.e., $\bar{x}$ has no component in the XY plane), $\bar{y}$ is simply taken in the Y direction.

Basic Systems - The three basic right hand coordinate systems are shown in Figure 4.0-1. They are general purpose systems used for locating points and defining directions. The Cartesian system X-Y-Z provides the basic reference frame for the entire structure. The local coordinate directions for other systems are defined in terms of X-Y-Z using the usual direction Cosine transformations. Much of the internal program storage and computation is in terms of the basic Cartesian system. The cylindrical system R-Θ-Z has its origin and Z axis coincident with those of the basic Cartesian system, while R is in the X-Y plane, and Θ is measured from X to R. The spherical system R-Θ-Φ is the same as the basic cylindrical system, except that the Z coordinate is replaced by Φ (Φ is in the R-Z plane, and is measured from the X-Y plane to R).

Special Cartesian Systems - These are additional user input coordinate systems, for defining directions of nodal or reference-point quantities. They are not associated with any particular origin, and therefore, can not be used for location of points.

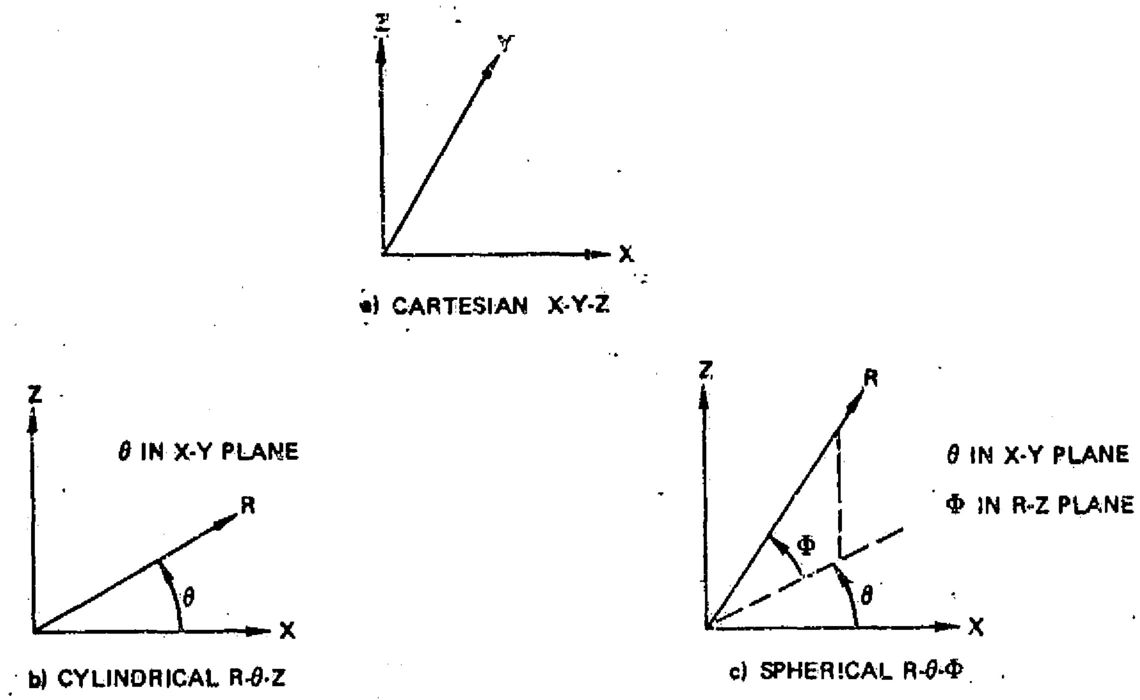


Figure 4.0-I: Basic Coordinate Systems

5.0 LOADS

The BOPACE load options consist of five types:

1) Concentrated mechanical loads
2) Distributed mechanical loads
3) Thermal loads
4) Normal strain or stress loads
5) Inertia loads.

Each of the first four loading types is defined by one or more load sets, which can be combined by means of their respective load factors. The inertia loads are computed from specified concentrated and distributed mass data, along with quantities which define the acceleration behavior of the structure.

The BOPACE forces defined by concentrated mechanical loads are fixed in direction (non follower-force type). The BOPACE distributed mechanical loads and inertia loads, however, may be of the follower-force type. For geometrically nonlinear problems, the direction and line of action of these loads is updated at the beginning of each increment, based on the current displaced configuration. This means that if geometric nonlinearity has been specified, all inertia loads and those distributed mechanical loads which are referenced to tangent coordinate systems, will contain follower-force effects. (Distributed mechanical loads referenced to other coordinate systems, are fixed in direction because the coordinate systems are fixed).

All loads represent cumulative values, so that the change in load for a particular increment of a problem is the difference between the specified cumulative start and end of increment loads.

5.1 CONCENTRATED MECHANICAL LOADS

A concentrated mechanical load set is defined by the zero or non-zero externally applied load value for each independent freedom in the structure. A particular load value will either be a force, or, if the freedom has been constrained via a single-point constraint (SPC) it will be an imposed displacement. If a load value is specified for a dependent freedom constrained via a multi-point constraint (MPC), the value must be a force. The program then automatically distributes this force to the independent MPC freedoms, according to the defined MPC coefficients.

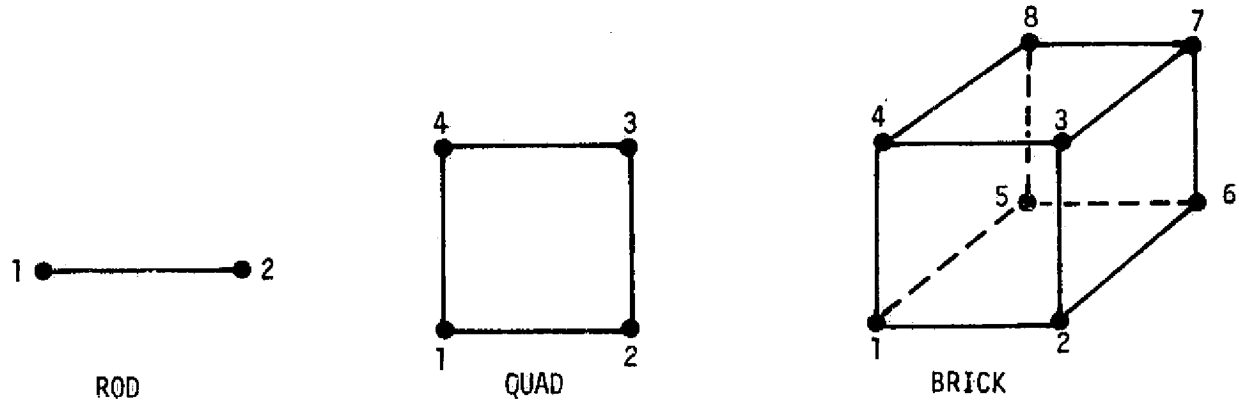
A nodal force is by definition the rate of change of external or internal virtual work, with respect to a virtual displacement of its associated nodal freedom. The equivalent concentrated nodal force corresponding to a general loading condition depends on the loading distribution, as well as the element geometry and shape functions. The relationship may be quite simple as on a rectangular face of an 8-node brick (each corner receives one fourth of the total load), or it may be complicated and physically meaningless, as on the same type of face with midside nodes (the correct corner force values are actually negative). In the more complicated cases it is best to define the distributed loading, and let the program compute the equivalent concentrated values using the distributed or inertia loading routines.

5.2 DISTRIBUTED MECHANICAL LOADS

A distributed mechanical load set is defined by the values of load intensity distributed on the edges and faces of the various elements in the structure. Associated with this set of values is a corresponding set of program computed equivalent concentrated nodal loads. BOPACE provides a very general capability to specify loads of the pressure (normal to surface) type and drag (tangent to surface) type using tangent coordinate systems, as well as load intensity components in directions defined by any other coordinate system.

For loads of the normal and tangent type, local region tangent coordinate systems are required. These are formed using the local region parent (ξ-η-ζ) coordinates as discussed in Section 4. Figure 5.2-1 serves to help define the local parent systems for the edges and faces of the various types of BOPACE elements. There the elements are shown, with their corner nodes numbered according to the scheme used for BOPACE element input data. The edge and face regions for each element are also listed along with their associated corner nodes. The corner node ordering for each region defines the parent coordinate system for that region. For example, edge region 3 of the QUAD element has its local ξ axis in the direction from node 3 to node 4. Face region 2 of the BRICK element has its local ξ axis along the edge nodes 6-5, η axis in the direction 5-8, and ζ axis defined by the right hand rule. Note that for each face region of the BRICK element, the positive normal (ζ direction) is outward.

One or more components of distributed load intensity may be specified at each desired location. To specify a uniform load intensity, only one location (i.e., the edge or face number) is given, along with the component values. For linear variation of load intensity, the corner node values are specified.



EDGE	NODES
1	1-2

EDGE	NODES
1	1-2
2	2-3
3	3-4
4	4-1

FACE	NODES
1	1-2-3-4

EDGE	NODES
1	1-2
2	2-3
3	3-4
4	4-1
5	5-6
6	6-7
7	7-8
8	8-5
9	1-5
10	2-6
11	3-7
12	4-8

FACE	NODES
1	1-2-3-4
2	6-5-8-7
3	1-5-6-2
4	2-6-7-3
5	3-7-8-4
6	4-8-5-1

Figure 5.2-1: Local Region Node Orders For Distributed Loads

(For a quadrilateral area region, the specification of values at four points means that the variation is linear along the edges, but possibly of higher order in the interior.) For a general variation of load intensity, values are specified for the corner nodes and the interior edge nodes of the region. In any case, the user need not be concerned with any particular ordering of the input nodal values, because the program uses the randomly specified node identification numbers to identify the appropriate edge or face and to define if necessary the corresponding local parent coordinate system. For uniform or linear load variation, the program computes any unspecified nodal intensities by proper interpolation from the element shape functions.

The equivalent concentrated nodal loads are computed by an integral involving the region shape functions and the distributed load intensities. An area loading, for example, uses the area integral

$$P_i = \int_A N_i \, d \, dA = \int_A N_i \, d_j \, N_j \, dA \qquad (5.2\text{-}1)$$

where P_i is the equivalent concentrated load in a particular coordinate direction at the ith node of the region, d is the load intensity in this direction, d_j is the load intensity at the jth node, and N are the shape functions for the region. Coordinate transformations are applied in the integral as required. The integration is carried out numerically using Gauss product formulas, with the number of Gauss points in each coordinate direction selected so as to give an accurate result. The number of points used depends on the maximum number of nodes in that direction as well as the order of variation of load intensity, and the resulting integration is exact if all edges of the region are straight lines. For edge regions the number of Gauss points used in a given direction is (n+m)/2, and for area regions it is (n+m)/2 + 1, where n is the maximum number of nodes in that direction and m is the order of distributed load variation (1 = uniform, 2 = linear, etc.).

5.3 THERMAL LOADS

A thermal load set is defined by the value of temperature at each node in the structure. The required temperatures at element reference points are computed from nodal temperatures by

$$T = T_i N_i \tag{5.3-1}$$

where T_i is the ith nodal temperature, and N_i is the shape function for node i evaluated at the reference point. Reference-point temperatures at the beginning of the problem are set by the fabrication temperature of each element.

The BOPACE treatment of element fabrication temperatures and nodal thermal loads can be employed to account for the effects of initial residual stresses, manufacturing tolerance errors, shrink-fit assemblies, and other initial-strain situations. The basic program assumption is that at fabrication time, all elements are at their specified respective fabrication temperatures, and they fit together into a stress-free configuration defined by their given nodal coordinates. Also, for all purposes of computation and output, thermal strains are taken as zero at the fabrication temperature. (If the material data defines a non-zero thermal strain value at the fabrication temperature, all thermal strains computed for the element are adjusted by substracting out that value.)

As an example, take the case of a fastener and drilled plate, manufactured from the same material and intended for a shrink-fit assembly. Assume that thermal strain data have been defined for the material such that the thermal strains are 0 and .002 at respective temperatures of 100 and 400. The BOPACE temperatures for the elements of the fastener and plate have been defined at fabrication as 100 and 400, respectively, and at that time the diameters of both fastener and plate hole are 1.0. The assembly then fits together with no gap, and the fastener/plate interface may be defined by a single set of nodes (or if desired, by pairs of coincident fastener/plate nodes which are fixed together via MPC constraints). A nodal thermal loading is then applied in one or more increments, which brings the entire assembly to a uniform temperature of 100. The result is no thermal strains in the fastener, but thermal strains of -.002 throughout the plate. The BOPACE elastic-plastic-creep analysis provides the accompanying distribution of other strains and stresses within the fastener and plate elements.

5.4 NORMAL STRAIN/STRESS LOADS

A normal strain/stress load set is defined by the zero or non-zero value for each free normal direction of the elements in the structure. Free directions are those for which the strain is not determined by nodal displacements, i.e., the surface normal for membrane type elements and the two centerline normals for rod type elements, as defined by the element parent coordinate systems. In order to provide the most general type of element behavior, BOPACE allows the user to control these otherwise undetermined strains or stresses by means of the normal strain/stress loads. Whether strain or stress values are specified is determined by element property codes.

5.5 INERTIA LOADS

BOPACE provides three sources of acceleration for automatic calculation of inertia loads. Each source applies uniformly to the entire structure, and is defined by a spacial vector, whose magnitude is given by the LFACTOR card and whose direction is defined by the TRANSLATE or ROTATE cards. These vectors are:

1) translational acceleration, a
2) rotational velocity, ω
3) rotational acceleration, α.

Based on these user supplied data, BOPACE first computes the total acceleration components (x-y-z) at each node as

$$\ddot{q} = a + \omega\times(\omega\times R) + \alpha\times R \tag{5.5-1}$$

where R is a vector from the rotational axis to the node.

For a BOPACE isoparametric element, a nodal acceleration in, say, the x direction, causes only x direction accelerations of points within the element. This results in an uncoupling of the x-y-z inertia effects, so that each load component can be computed separately. For the x direction, therefore, a vector $\ddot{Q}$ is formed from the $\ddot{q}$ vectors, such that $\ddot{Q}_i$ is the x direction acceleration at node i. The vector P, of x direction inertia loads is then computed using $\ddot{Q}$ and the concentrated and distributed masses:

$$P_i = -m \ddot{Q}_i - \Sigma \int_V N_i \rho N_j \, dV \ddot{Q}_j \qquad (5.5\text{-}2)$$

Here m is the value of concentrated mass at node i, ρ is the distributed mass density of an element, N_i is the shape function for the ith node of an element, and the summation is taken over the volumes of all elements. The y and z components of inertia loads are computed similarly. The volume integrals are evaluated by the same Gauss product formulas used to compute the element stiffness matrices.

6.0 NONLINEAR SOLUTION METHOD

6.1 BASIC SOLUTION REQUIREMENTS

The exact elasto-plastic-creep analysis of a structure requires the satisfaction, at all points in the structure, of three requirements:

1) Equilibrium of stresses

2) Compatibility of strains

3) Satisfaction of constitutive theory, which is summarized by the appropriate stress-strain rate relation.

The following paragraphs summarize the BOPACE solution approach as it relates to satisfying these three requirements.

<u>Stress-Strain Relation</u> - The stress-strain rate relation is cast into an incremental form, as defined by the material constitutive theory of Section 2. The assumed stress-strain relation is satisfied exactly in the BOPACE solution procedure, provided that the increment is sufficiently small so that incremental quantities can be treated in a differential manner, and that the iteration procedure is sufficient to produce convergence.

<u>Compatibility</u> - Compatibility is satisfied exactly within each isoparametric element as a result of the finite-element derivation. In the global sense, i.e., over the entire structure, compatibility is also satisfied exactly, by merging the element degrees of freedom into global degrees of freedom and thereby establishing the equality of displacements at appropriate adjacent nodes.

Equilibrium - Equilibrium in general is satisfied only approximately within an isoparametric element, because of its variable stress state. Stresses are also not necessarily in equilibrium between adjacent elements, although all stress equilibrium is satisfied in the limit as the finite-element mesh is refined. For any mesh representation of the structure, global equilibrium is satisfied in BOPACE in an average sense, because equilibrium is established between the generalized nodal forces defined according to the usual finite-element procedure.

6.2 COMPARISON OF COMMON SOLUTION METHODS

The common stiffness methods used for solution of elasto-plastic problems can be classified by three general types:

1) The pure "tangent stiffness" method

2) The "constant-stiffness residual-load" method

3) "Combined" methods.

Tangent-Stiffness Method - The pure tangent-stiffness method obtains the solution for each load increment by a single solution of the incremental equilibrium equation:

$$\Delta P_i = KO^*_{ij} \, \Delta Q_j \qquad (6.2\text{-}1)$$

in which ΔP and ΔQ are the global incremental forces and displacements, respectively, and KO^* is the Jacobian (tangent-stiffness) matrix. This is the type of solution used in NASTRAN's "piecewise linear analysis," for example. There is no equilibrium check, and no iteration is performed to improve the incremental solution. The matrix KO* is determined by evaluation or extrapolation at previous solution points. Because in an actual structure the stress-strain slopes, creep rates, direction of the incremental plastic and creep strain vectors, etc., will generally vary within an increment, the pure tangent-stiffness approach can result in a substantial departure from the true force-displacement path unless load increments are kept quite small.

Constant-Stiffness Residual-Load Method - This solution method [10] employs an iterative procedure. In each iteration the residual (unbalanced) forces are computed based on the current estimate for the incremental configuration, and are then applied to the constant elastic stiffness matrix in order to solve for displacement corrections. The approach is computationally efficient because it requires the formation and decomposition of only a single stiffness matrix, but it is not directly applicable to highly nonlinear structures because of convergence difficulties.

Combined Methods - Various combined methods have been employed for solution of elasto-plastic problems, for example that described in Reference 18. These involve the use of an equilibrium check through the calculation of unbalanced forces, as well as various procedures for updating the approximate Jacobian matrix.

BOPACE Approach - BOPACE provides several options for nonlinear solution, based on user specification of the Jacobian updates and the iteration sequence. The most general option uses a combined approach for solution, with the iterative procedure consisting of two stages:

1) Improvement of the solution configuration by using the Jacobian matrix to reduce the residual nodal forces.

2) Calculation of residual forces based on the estimated configuration and "exact" constitutive theory.

Several user controlled options are available in BOPACE for defining and updating the Jacobian matrix.

6.3 CALCULATION OF RESIDUAL FORCES

It is assumed for the present discussion that the exact solution configuration is known at the start of a particular load increment. (Actually the BOPACE program takes any unbalanced forces which might remain from the previous increment and adds them to the present load increment, in order to achieve greater accuracy.) For a given iteration within the present increment, i.e., for a given estimate of the solution, it is necessary to compute the corresponding unbalanced forces. This section summarizes the steps involved in computing these forces, including determination of strains, stresses, and forces. A flowchart for these calculations is given in Figure 6.3-1.

Strains - For the given estimate of end-of-increment global nodal displacements, Q, the corresponding element nodal displacements, q, are obtained by coordinate transformations at the nodes, involving appropriate direction Cosines. For the BOPACE program, all element displacements are referred to the basic X-Y-Z Cartesian coordinate system. Strains, ε are then computed at each reference point by using the relations 3.1-9 and 3.2-1:

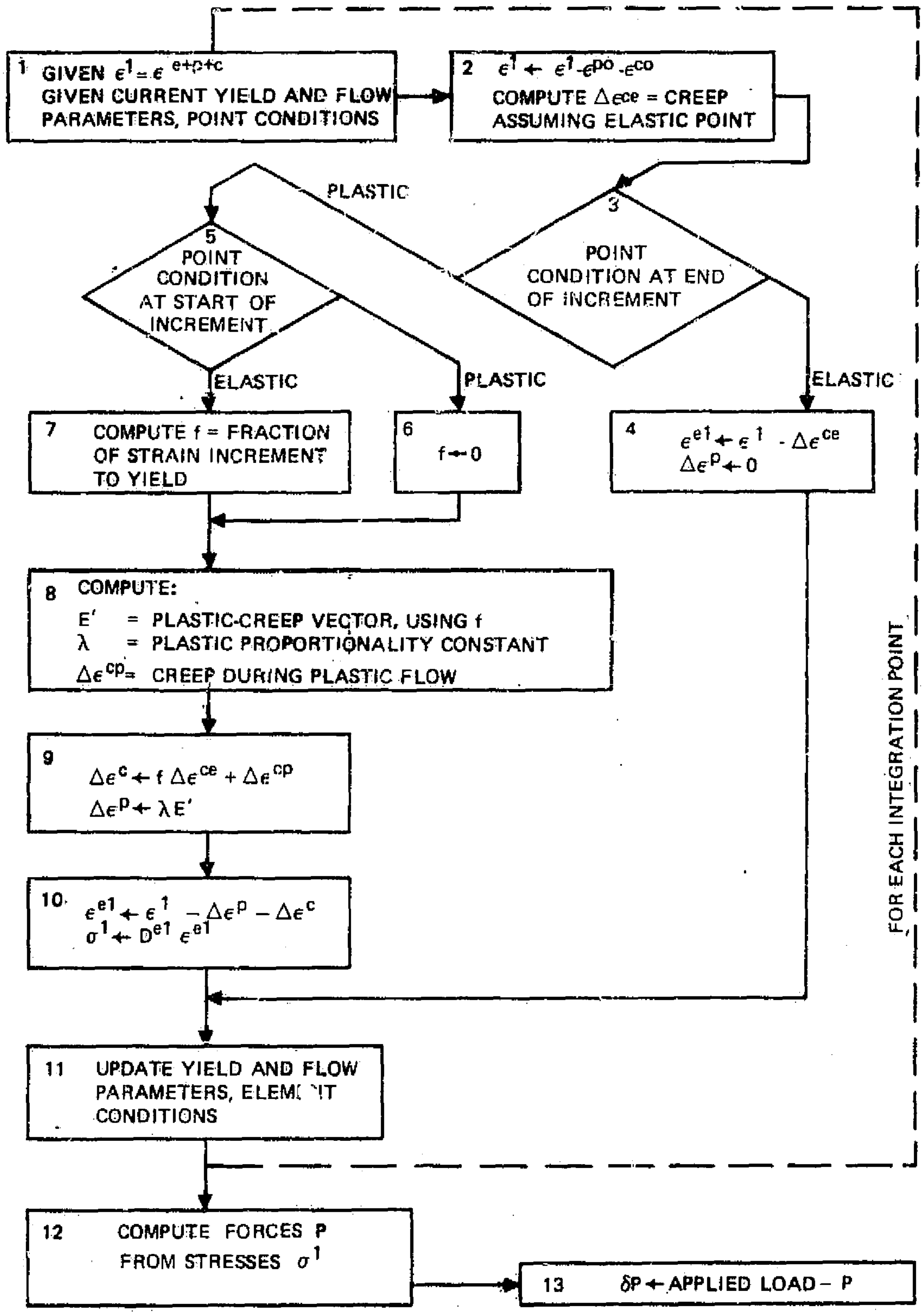


Figure 6.3-1: BOPACE Residual—Force Calculations

$$\theta_i = G_{ij}\ q_j$$

$$\varepsilon_i = A0_{ij}\ \theta_j + \frac{1}{2} A1_{ijk}\theta_j\theta_k \qquad (6.3\text{-}1)$$

The end-of-increment strain ε is the total (physical) strain at the point:

$$\varepsilon_i = \varepsilon_i^e + \varepsilon_i^p + \varepsilon_i^c + \varepsilon_i^t \qquad (6.3\text{-}2)$$

The thermal strains, ε^t, are determined as described in Section 2.2, and are measured relative to the assumed zero strain condition at fabrication time. Subtracting these strains from the total strain, gives:

$$\varepsilon_i^{e+p+c} = \varepsilon_i^e + \varepsilon_i^p + \varepsilon_i^p = \varepsilon_i - \varepsilon_i^t \qquad (6.3\text{-}3)$$

The corresponding incremental strain $\Delta\varepsilon^{e+p+c}$ is determined as the difference from start-of-increment to estimated end-of-increment strain.

<u>Stresses</u> - Elastic strains are then determined using the elasto-plastic-creep algorithm presented in Section 2.6. With the elastic strains known at the end of the increment, the stresses are computed:

$$\sigma_i^1 = D_{ij}^{el}\ \varepsilon_i^{el} \qquad (6.3\text{-}4)$$

where ε^{el} are the known cumulative elastic strains at the end of the increment. The stress-strain calculation may need to be modified, depending on which of three conditions exists at the particular integration point:

Condition I   Point is elastic at end of load increment, i.e., either the point remains elastic or unloading occurs. Compute stress and elastic strains. Plastic strains are zero.

Condition II — Point is plastic throughout load increment. Compute stresses, elastic and plastic strains by algorithm of Section 2.6.

Condition III — Point is initially elastic, but becomes plastic at some point during the load increment. Find intermediate time at which yielding occurs (this requires solving a simple quadratic equation). Compute stresses and elastic strains up to that time. Compute stresses and strains beyond yielding as for Condition II.

The condition at the beginning of the increment is known for each point. The condition at the end of the increment is assumed, for the first iteration, to be Condition I. The end condition is re-evaluated during each iteration, using either the material yield value or the plastic-strain vector. For an elastic point, it is determined whether or not the current material yield has been exceeded. For a plastic point, the plastic strain vector (normal to the yield surface) is observed; an outward vector ($\lambda>0$) implies a plastic condition, while an inward vector ($\lambda\leq 0$) implies elastic unloading.

Element Nodal Forces - The force-stress relation for the BOPACE element is defined by Equations 3.2-5 and 3.2-6:

$$p_i = \int_V B_{ai}\, \sigma_a \, dV \qquad (6.3\text{-}5)$$

where V is the element volume, B is the strain-displacement matrix, and p_i are the element nodal forces.

Global Nodal Unbalanced Forces - Global forces, P, are obtained from the element forces, p, by adding nodal contributions from all elements and applying coordinate transformations. The global unbalanced forces, δP, are then determined by subtracting these computed (internal) forces from the applied (external) loads:

$$\delta P_i = \text{Load}_i - P_i \qquad (6.3\text{-}6)$$

The loads are defined as the sum of applied concentrated and distributed mechanical and inertia loads.

6.4 IMPROVING THE SOLUTION

The basic global relation for incremental forces and displacements corresponds to the element relation 3.2-7:

$$\Delta P_i = KO^*_{ij}\ \Delta Q_j \qquad (6.4\text{-}1)$$

where the incremental global displacements ΔQ are the total physical displacements (including thermal, elastic, plastic and creep effects). KO^* is the elasto-plastic tangent-stiffness (Jacobian) matrix for the increment.

In order to improve a given displacement configuration, the displacement corrections δQ corresponding to unbalanced forces δP, are obtained in BOPACE by solving a set of linear equations of the form

$$\delta P_i = K^J_{ij}\ \delta Q_j \qquad (6.4\text{-}2)$$

The matrix K^J is also a Jacobian (tangent-stiffness) matrix, or some approximation to the Jacobian, but is used for displacement corrections rather than a one-step solution for the displacements of the entire increment. The purpose of this section is to discuss the procedure for relating Equations

6.4-1 and 6.4-2, and describe BOPACE options for updating the Jacobian.

Procedure - In the iterative BOPACE approach, the only global solution employed is the displacement-correction relation 6.4-2. The best approximation for the Jacobian, for iterative purposes, is

$$K^J = KO^1 = K^{el} + K^{pl} \qquad (6.4\text{-}3)$$

where KO^1 is evaluated at the end of the current load increment using Equation 3.2-9, and K^{el} and K^{pl} are its elastic and plastic contributions, respectively. The effects of change in elastic properties as well as the effects of thermal and creep strains, are computed at the integration-point level by the algorithm of Section 2.6, and accounted for by the unbalanced forces. Thus Equation 6.4-1 is satisfied in an iterative fashion.

Updating the Jacobian - In order to account for possible large-scale elastic unloading of the structure under cyclic load conditions, one or more initial iterations are performed for each load increment using only the elastic portion, K^{el}, of the K^J matrix. Succeeding iterations use the total K^J matrix.

Initially the K^J matrix is taken to be the usual elastic stiffness matrix for the structure, with elastic properties evaluated at the fabrication temperature. Whenever convergence is not achieved within a specified number of iterations, the Jacobian matrix is updated. BOPACE allows four options for updating the matrix K^J and/or its component matrix K^{el} (all matrix updates are based on current temperature and geometry):

1) Use only initial elastic matrix K^{el} with no updating. This option corresponds to the constant-stiffness residual-load

method, and is most effective for problems with small plastic strains and elastic properties which do not vary much with temperature.

2) Update only K^{el}. This option is best for problems with small plastic strains and elastic properties which vary considerably with temperature.

3) Update total K^{J} matrix, but not elastic matrix K^{el}. This option may be used for problems with large plastic strains and elastic properties which vary somewhat with temperature.

4) Update both K^{J} and K^{el} matrices. This is the most effective option for problems with large plastic strains and elastic properties which vary considerably with temperature.

6.5 SUMMARY OF BOPACE SOLUTION METHOD

An outline of the BOPACE solution method is given in the flowchart of Figure 6.5-1. In step 1, the Jacobian is initialized to the elastic stiffness matrix, based on elastic properties at the fabrication time.

At the start of each load increment (step 2) the residual forces δP are set equal to the increment of applied loads. Also, if any residual forces ramain from the previous load increment, these are added to δP. The estimate for displacements, Q, is defined by displacements at the end of the previous increment.

The iteration loop involves successive improvement of the solution, by solving for displacement corrections using the unbalanced forces and the Jacobian,

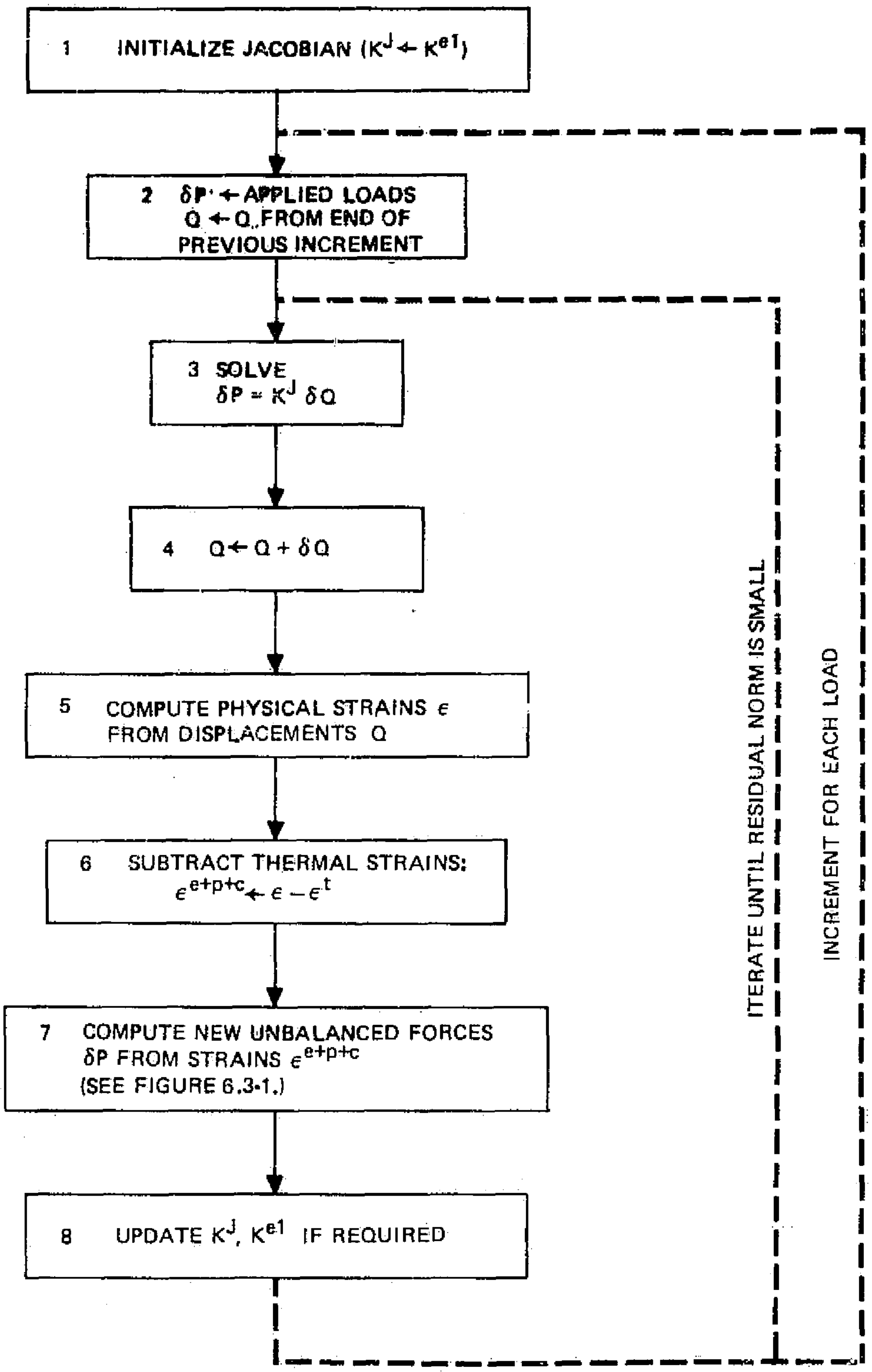


Figure 6.5-1: BOPACE Solution

and then recomputing the unbalanced forces corresponding to the new displacement configuration. The displacement corrections δQ are determined in step 3, and in step 4 the improved configuration Q is updated by addition of δQ. Although convergence of this iterative process is usually quite good, BOPACE has a feature for modifying the process if convergence is not occurring. This involves using only a specified fraction of the computed correction, e. g., $Q \leftarrow Q + 0.5\ \delta Q$. This would increase the numerical stability but could tend to slow down convergence.

In step 5 the strain-displacement relations are used to compute the total strains ε from displacements Q. In step 6 the thermal strains are subcontracted from total strains to give the elastic+plastic+creep strains required for the calculation of stresses. Step 7 involves the major iteration algorithm, in which the strain is separated into elastic, plastic and creep components. Stresses are determined accorording to the algorithm of Section 2.6, and the corresponding unbalanced forces are computed.

If the maximum allowable iterations have been exceeded, step 8 is used to update the Jacobian matrices according to the specified updating option. The Jacobian update is based on the current estimates of the yield surface and flow parameters for each integration point at the end of the present increment. Iteration is stopped when a residual error norm (determined by a ratio of residual forces to applied forces) is sufficiently small.

7.0 LINEAR EQUATION SOLUTION

The stiffness equations relating force and displacement rates are solved in BOPACE using a modified Gauss wavefront solution procedure [19,20]. For large problems it uses an out-of-core method, which basically requires that core storage be available for only one nodal row of the stiffness matrix at a time. Special features include the use of a sparse-matrix blocked-partition scheme, a fast merging procedure based on a binary tree algorithm, and a method of core space allocation which eliminates searching for partitions during the matrix decomposition process.

The basic stages required for solution of general stiffness equations are:

1) Generation of the stiffness partitions for each element.

2) Merging these partitions together to form the stiffness matrix for the entire system.

3) Decomposition of the stiffness matrix into factored form.

4) A forward/backward substitution process to solve for unknown forces and displacements.

In the true wavefront procedure the generation, merging and decomposition phases are combined, and the system stiffness matrix is never formed as such. To achieve an efficient solution the elements must be numbered in an optimum order while the nodes may be randomly ordered. In the BOPACE modified wavefront procedure the four phases are accomplished individually, and the entire stiffness matrix is formed and made available. To achieve an efficient solution, the nodes must be numbered in an optimum order while the elements may be randomly ordered. It is interesting to note that from an ordering standpoint this modified procedure may be regarded as a true wavefront procedure, if each stiffness partition (i.e. single-node to single-node connection) is considered as being an individual element. The BOPACE procedure provides a simple method for incorporating multi-point constraint effects, by transferring dependent partition contributions during the generation phase.

7.1 GENERATION

The generation stage consists of forming the stiffness partitions for each finite element, accounting for the MPC relations, and writing the partitions onto the generation file. Only the partitions in the upper symmetric half of the element stiffness matrix are formed. Each partition is assigned a packed code defining its row/column position in the system stiffness matrix, and the partition is transposed if necessary (row/column codes correspond to the upper symmetric half of the system matrix). Each code is unique within a particular element matrix, i.e., additional contributions to a partition arising from MPC equations are all added together and grouped with their corresponding code. SPC relations have no effect on the generation. At the end of this stage all partitions have been written with their row/column codes, in more or less random order, onto the generation file. Generation time for an element is approximately proportional to the square of the number of element freedoms, times the number of element integration points. Generation time can be significantly increased by the presence of MPC relations.

7.2 MERGING

The merging stage consists of reading the partitions from the generation

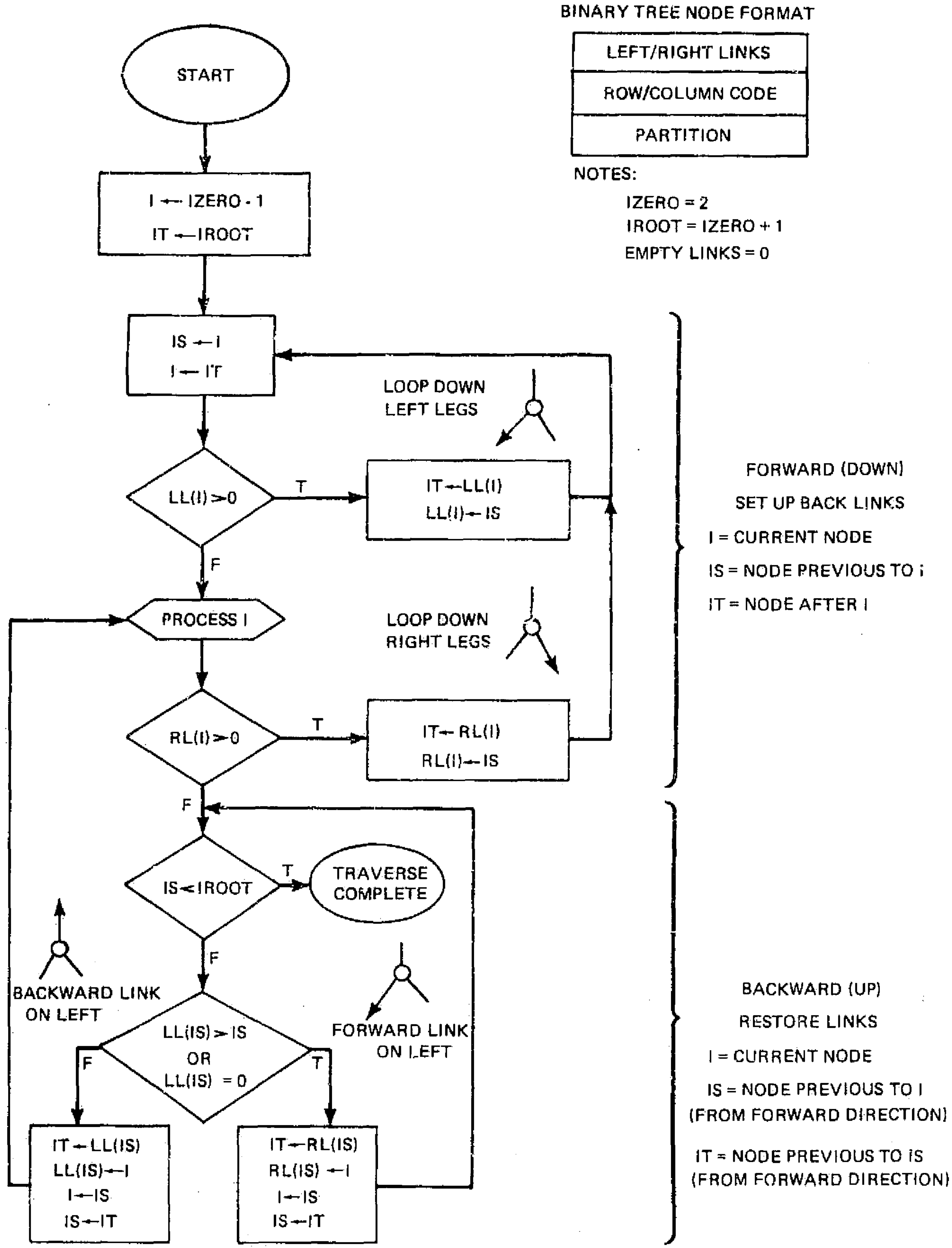


Figure 7.2-1: Merge Traverse of Binary Tree

file, ordering them according to increasing row/column code value (adding together all partitions having the same code), and writing them onto the merge file by rows using a blocked partition form. If no partitions exist for a particular row, a zero diagonal partition is inserted at this time. Core is divided into three areas - a large sorting area, and two smaller input/output areas which serve alternately to store either previously ordered partitions or the partitions currently being ordered. If core is limited then use is made of two scratch files as backup for the two smaller core areas. The merge process is accomplished using the following steps.

1) Read as many partitions as space allows, from the generation file into the next (sequential) available locations of the sorting area. As each partition is read, form its binary-tree pointer (left or right link) to allow its later ordered retrieval by row/column code.

2) Retrieve partitions from sorting area in increasing code order, by traversing binary tree (see Flow Chart in Figure 7.2-1). As these partitions are retrieved, merge them with all previously ordered partitions being read into the input area, and place them into the output area. (When input or output area is filled, perform a read or write to the corresponding scratch area).

3) When sorting area is emptied, prepare to refill it from the generation file and switch input/output areas or corresponding scratch files.

4) Repeat steps 1-3 until all partitions from the generation file have been read and ordered.

5) Transfer ordered partitions from output area or scratch file to final merge file, writing them by rows.

The merge file contains two records for each nodal row of the stiffness matrix. The first record is a single word whose value is the number of equivalent single-precision words in the second record, and the second record contains all partitions for the row in blocked form. Each block contains the number of partitions in the block, the row/column code of the first partition, and then the partitions stored by columns. A new block is started whenever the next partition is not sequential, i.e., its row/column code is greater than one plus the code of the previous partition.

7.3 DECOMPOSITION AND SOLUTION

The decomposition stage accomplishes the factoring of the stiffness matrix, and the solution stage uses a forward and backward substitution to solve for the unknown components of force and displacement. The stiffness equations

$$K_{ij}Q_j = P_i \tag{7.3-1}$$

generally involve a combination of prescribed forces and prescribed displacements. The equations are solved in BOPACE using the modified Gauss wavefront approach. Here the theoretical solution steps are first shown for a system involving only prescribed forces, and then the modifications are shown for a general mixed problem. Finally, the computer implementation of these steps is discussed.

Solution With Prescribed Forces - The decomposed form of Equation 7.3-1 is taken as

$$U^T D^{-1} U Q = P \tag{7.3-2}$$

where U is an upper triangular matrix, and D is a diagonal matrix whose elements are equal to the diagonals of U. The decomposition and solution procedure consists of the following three steps.

1) Decomposition: The elements of K are

$$K_{ij} = \sum_{k=1}^{i-1} U_{ki} D_{kk}^{-1} U_{kj} + U_{ii} D_{ii}^{-1} U_{ij} \tag{7.3-3a}$$

Since $D_{ii} = U_{ii}$, the elements of U are obtained successively by row, as

$$U_{ij} = K_{ij} - \sum_{k-1}^{i-1} D_{kk}^{-1} U_{ki} U_{kj} \tag{7.3-3b}$$

2) Forward substitution: Let $Y = D^{-1}UQ$. Then

$$P_i = \sum_{k-1}^{i-1} U_{ki} Y_k + U_{ii} Y_i \tag{7.3-4a}$$

giving

$$Y_i = D_{ii}^{-1}(P_i - \sum_{k=1}^{i-1} U_{ki} Y_k) \tag{7.3-4b}$$

3) Backward substitution:

$$Y_i = \sum_{k=i+1}^{n} D_{ii}^{-1} U_{ik} Q_k + D_{ii}^{-1} U_{ii} Q_i \tag{7.3-5a}$$

giving

$$Q_i = Y_i - D_{ii}^{-1} \sum_{k=i+1}^{n} U_{ik} Q_k \tag{7.3-5b}$$

Solution of Mixed Problem - For the general case where there is a combination of prescribed forces and displacements, the three steps given above must be modified. The procedure is described here for the case in which a single (rth) displacement is prescribed, but additional prescribed displacements would be treated in the same manner. The modified form of the decomposition (7.3-2) is

$$\begin{bmatrix} U_{11}^T & 0 & 0 \\ U_{1r}^T & 1 & 0 \\ U_{1n}^T & 0 & U_{nn}^T \end{bmatrix} \begin{bmatrix} D_{11}^{-1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & D_{nn}^{-1} \end{bmatrix} \begin{bmatrix} U_{11} & U_{1r} & U_{1n} \\ 0 & U_{rr} & U_{rn} \\ 0 & 0 & U_{n} \end{bmatrix} \begin{Bmatrix} Q_1 \\ \hat{Q}_r \\ Q_n \end{Bmatrix} = \begin{Bmatrix} \hat{P}_1 \\ P_r \\ \hat{P}_n - U_{rn}^T \hat{Q}_r \end{Bmatrix} \tag{7.3-6}$$

where ($\hat{}$) denotes a prescribed quantity. The first and last rows, denoted by 1 and n, respectively, involve prescribed forces. The elements of U are given by Equation 7.3-3b, except that no contribution from U_{rn} is distributed to the elements of U_{nn}. Detailed steps for decomposition, and forward and backward substitution, are given below.

1) Decomposition.

   First rows: Compute each row of U according to Equation 7.3-3b and distribute the contributions to later rows.

   rth row: Compute the rth row of U according to (7.3-3b) but do not distribute to later rows.

   Last rows: Again compute U according to (7.3-3b) and distribute to later rows.

2) Forward substitution.

   First rows: Compute $Y_1 = U_{11}^{-1} P_1$ using Equation 7.3-4b and distribute to later elements of Y. This produces the vector

$$(Y_1, -U_{1r}^T Y_1, -U_{12}^T Y_1)^T \qquad (7.3\text{-}7)$$

rth row: The rth row of (7.3-6) can be expanded and rearranged to give the relation

$$[U_{rr}, U_{rn}] \begin{Bmatrix} \hat{Q}_r \\ Q_n \end{Bmatrix} = \{P_r - U_{1r}^T Y_1\} \qquad (7.3\text{-}8)$$

The quantity $U_{1r}^T Y_1$ is available from the Y vector (7.3-7) and is placed in the force vector P_r. This quantity in the Y vector is then replaced by Q_r, and contributions are distributed to later elements of Y as in Equation 7.3-4b.

Last rows: Continue forward substitution by Equation 7.3-4b to obtain

$$Y_n = U_{nn}^{T-1} (P_n - U_{rn}^T \hat{Q}_r - U_{1n}^T Y_1) \qquad (7.3\text{-}9)$$

3) Backward Substitution

Last rows: From Equation 7.3-5b

$$Q_n = U_{nn}^{-1} D_{nn} Y_n \qquad (7.3\text{-}10)$$

rth row: By Equation 7.3-8 the final contribution $(U_{rr} Q_r + U_{rn} Q_n)$ is added to the existing contribution $U_{1r}^T Y_1$, to give the total

$$P_r = (U_{1r}^T Y_1 + U_{rr} \hat{Q}_r + U_{rn} Q_n) \qquad (7.3\text{-}11)$$

First rows: Continuing backward substitution by Equation 7.3-5b gives

$$Q_1 = U_{11}^{-1} D_{11} (Y_1 - U_{1r}\hat{Q}_r - U_{1n}Q_n)$$

Implementation - The BOPACE decomposition and solution algorithms operate in a blocked-partitition mode, corresponding to the form of the merged stiffness and decomposition matrices. By means of this procedure, the indexing and storage operations can be applied in general to many partitions at a time, thus increasing program efficiency.

For decomposition, the core is divided into three areas. The first area is large enough to store the maximum size nodal row of the stiffness or decomposition matrix. The last two areas are each equal to one-half of the remaining core, and are input/output areas which serve alternately to store either previous decomposition contributions or the updated contributions including effects from the current row. If core is limited, then use is made of two scratch files as backup for the last two core areas.

A dummy decomposition is first performed to determine the maximum wavefront (active decomposition nodes) for the structure. The decomposition process is then accomplished using the following steps (dependent MPC freedoms are treated during decomposition like specified displacements).

1) Read current nodal row of the stiffness matrix from merge file, into end of the row storage area. Add previous decomposition contributions for this row from the input core area, and store resulting completed row of decomposition at start of the row storage area.

2) Decompose the just completed row (i.e., compute its contributions to later rows), merge these contributions with previous contributions from the input area, and store results in the output area. (When input or output area is filled, perform a read or write to the corresponding scratch area.)

3) Switch input/output areas or corresponding scratch files.

4) Output current row onto decomposition file in blocked partition form (same row format as for merge file).

5) Repeat steps 1-4 for each nodal row of matrix.

The above procedure makes it unnecessary to search for any partition or to store a vector for partition addressing, because the next partition needed for calculation is always the next one available in the core storage area.

For solution, enough core is needed to store the maximum size nodal row of the decomposition matrix. The forward substitution involves reading the decomposition matrix one row at a time, while the backward substitution involves a similar reading of the nodal rows in reverse order. Before the solution procedure begins, the MPC coefficients are used to take prescribed forces at the dependent freedoms and distribute them to the independent freedoms. During the solution procedure, dependent MPC freedoms are ignored. After the solution procedure is completed the MPC coefficients are again used to calculate the dependent displacements in terms of the independent displacements.

8.0 SUBSTRUCTURING

Substructuring concepts are used in order to decrease computer run times or core storage requirements, or for the convenience of being able to model and solve several parts of a structure largely independently of each other. Substructuring procedures can be classified generally according to three types of applications:

1) Parameter type studies, where a small part of the structure is modified one or more times. The results of each modification can be determined without a new formation and decomposition of the entire stiffness matrix.

2) Nonlinear problems, where the material or geometric nonlinearity effects are largely concentrated in a particular region of the structure. The incremental iterative solution process can often be accomplished by updating only the portion of the stiffness matrix corresponding to this region.

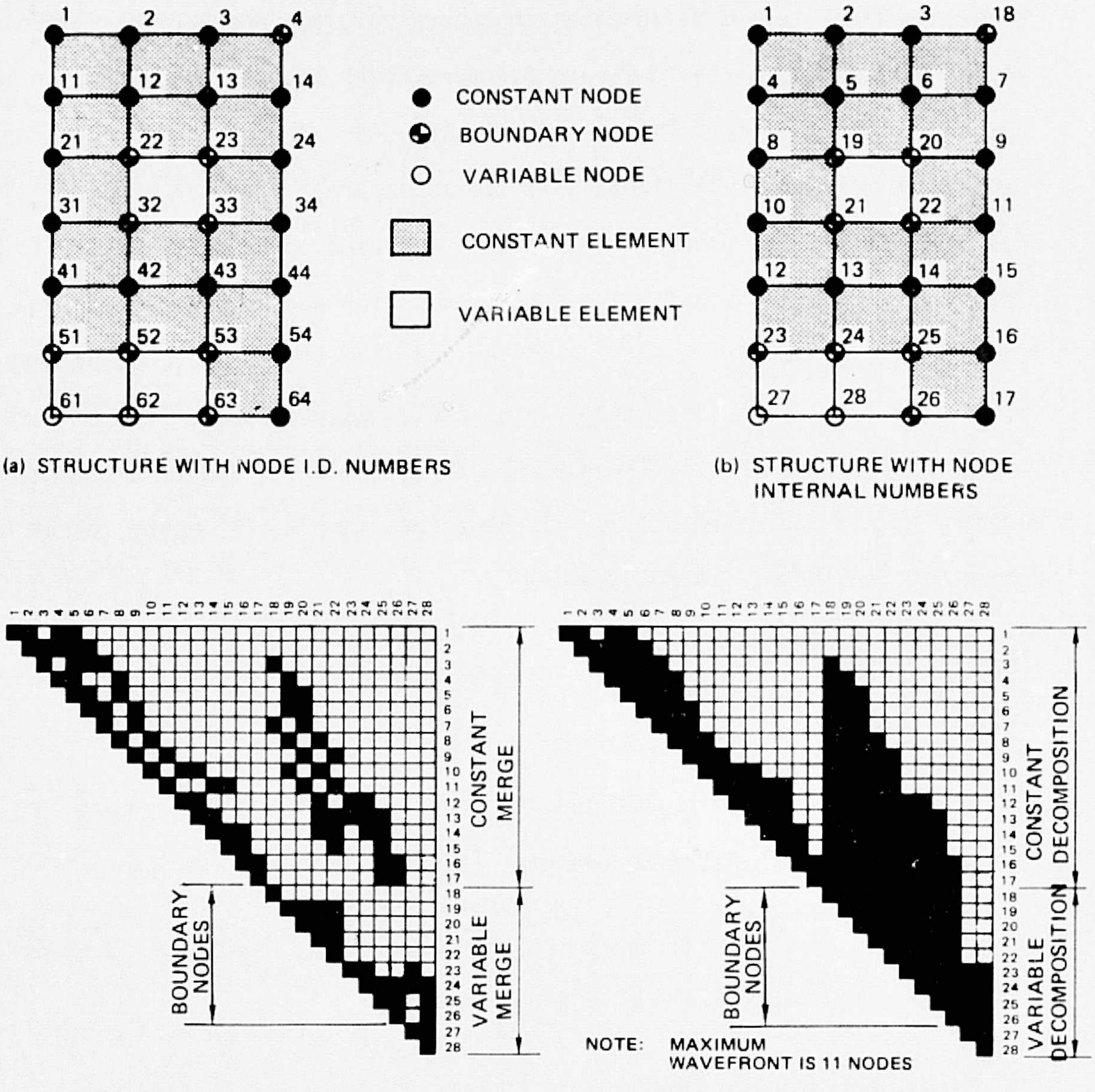
3) Large problems which can be divided into several distinct regions, with the regions connected together at localized interfaces. Each region can be solved largely independently, by reducing out internal freedoms in each region in terms of the connecting boundaries.

The third type of procedure is used largely for convenience, in order to design and analyze individual parts of a structure separately. It does not significantly change the required run time, as compared to that of a similarly efficient non-substructure procedure. It can, however, reduce the maximum wavefront storage (by perhaps as much as a factor of two) for some problems, and core storage is also reduced somewhat by the need to store only part of the force and displacement vector values in core at any particular time.

The BOPACE substructuring approach is directed toward the first and second types of applications. The structure is divided into two parts - a "constant" and a "variable" structure. The variable structure defines those parts of the structure which can be modified, and for which the stiffness matrix can be updated as BOPACE iterates to a solution. For each new variable structure the decomposition process is performed only for the nodal rows corresponding to that variable structure. The forward-backward substitution process, however, is always performed for the entire structure. Although the entire force and displacement vectors must be in core for solution, this approach has the advantage that the "constant" structure is permitted to have some nonlinear material or geometric effects, which are accounted for by iteration in the solution process.

BOPACE defines three types of nodes - constant, boundary and variable, in that order. Boundary nodes are used along interfaces to attach the constant and variable structures together, or in the constant structure area where it is desired to change the SPC definitions from one variable structure case to the next. Constant elements are connected to constant or boundary nodes, and may not be redefined. Variable elements are connected to variable or boundary nodes, and may be redefined (or expanded or decreased in number).

The simple substructure example shown in Figure 8.0-1 will be used to help illustrate the BOPACE procedure. In this example there are 28 nodes, with internal ordering as shown in (b). Nodes 1-17 are constant, nodes 18-26 are boundary, and nodes 27-28 are variable. Node 18 is made a boundary node in



(a) STRUCTURE WITH NODE I.D. NUMBERS

(b) STRUCTURE WITH NODE INTERNAL NUMBERS

(c) MERGED MASTER STIFFNESS MATRIX

(d) DECOMPOSED MASTER STIFFNESS MATRIX

Figure 8.0-1: BOPACE Substructuring Example

CC	CB	
	BB	BV
		VV

NOTE: C = CONSTANT
B = BOUNDARY
V = VARIABLE

Figure 8.0-2: General Stiffness Matrix Substructuring Schematic order to allow its SPC definitions to be changed from one variable structure to the next, while nodes 19-26 are boundary nodes because they are used to connect constant elements with variable elements. The sparse form of the merged and decomposed stiffness matrices is apparent from (c) and (d). (It may be noted that in general a column of the decomposed matrix is full, below the row in which its first non-zero partition occurs in the merged matrix. In some cases, however, a null partition can occur in the decomposed column below this point). A schematic of a general substructure stiffness matrix is shown in Figure 8.0-2. There the letters C, B and V denote constant, boundary and variable, respectively. Thus for example, CB denotes partitions connecting constant to boundary nodes.

The basic steps involved in a BOPACE substructure problem are given as follows:

1) Merge constant portion (CC and CB) of stiffness matrix, and merge constant contributions to boundary (BB).

2) Decompose constant stiffness (CC and CB), and distribute constant decomposition contributions to boundary (BB).

3) Merge variable contributions to boundary (BB and BV), and merge variable stiffness (VV).

4) Add boundary contributions (BB) from Step 2 to stiffnesses from Step 3, to obtain total variable stiffness (BB, BV and VV).

5) Complete decomposition of total variable stiffness from Step 4.

6) Add total variable decomposition (BB, BV and VV) from Step 5, to

6) Continued

constant decomposition (CC and CB) from Step 2, to obtain total structure decomposition.

7) Perform forward/backward substitution using total structure decomposition, to obtain solution for all forces and displacements in structure.

8) Repeat Steps 3-7 for each new variable structure.

Because of the additional substructure overhead costs which arise from the matrix addition and input/output file operations, BOPACE substructuring is not recommended if the variable structure is a large portion of the total structure.

MPC relations require that rows and columns of the stiffness matrix corresponding to the dependent freedoms, be moved to locations corresponding to the independent freedoms. Because of the order in which the above matrix contributions are formed and stored, certain restrictions are placed on substructure MPC relations. The permissible forms of MPC equations for the constant and boundary freedoms can be written symbolically as

$$C = f(C,B) \tag{8.0-1a}$$

$$B = f(B) \tag{8.0-1b}$$

and for the variable freedoms as

$$V = f(V,B) \tag{8.0-1c}$$

Equation 8.0-1a means, for example, that dependent constant structure displacements may be defined as a function of both constant and boundary freedom displacements, via the constant structure MPC equations.

9.0 DEFINITIONS - THEORETICAL MANUAL

This section defines symbols used in the BOPACE Theoretical Manual.

Variables:

a	Deviatoric stress center; Translational acceleration
c	Kinematic hardening slope
d	Distributed load intensity
e	Deviatoric strain
f	Function designation
g	Shape function derivatives matrix
h	Initial stress matrix
m	Concentrated mass
p,q	Local or element nodal forces, displacements
r	Isotropic hardening slope
s	Deviatoric (total - hydrostatic) stress
x,y,z	Cartesian coordinates
u,v,w	Displacements in x,y,z directions
X,Y,Z	Basic Cartesian Coordinates
R,θ,Z	Basic Cylindrical Coordinates
R,θ,Φ	Basic Spherical Coordinates
U,V,W	Displacements in X,Y,Z directions
A	Elasto-plastic hardening parameter; Strain tensor coefficient matrix
B	Strain-displacement matrix
C	Kinematic hardening matrix; Direction Cosines matrix

Variables:

C,B,V	Constant, boundary, variable parts of substructure
D	Elasticity matrix
E	Young's modulus
E'	Basic yield surface normal vector
F	Yield surface function
G	Tensorial shear modulus; Shape function derivatives matrix
H	Intermediate stiffness calculation matrix
I	Identity matrix
J	Jacobian matrix
K	Stiffness matrix
N	Element shape function
P,Q	System nodal forces, displacements
R	Isotropic hardening matrix
T	Temperature
V	Volume
Y	Intermediate equation solution vector
W	Work; Integration point weighting factor
α	Stress center of yield surface; Rotational acceleration
ω	Rotational velocity
β	Strain center of yield surface
γ	Thermal coefficient of expansion; Shear strain
ϵ	Strain
θ	Displacement derivatives
κ	Cumulative plastic hardening parameter

Variables:

κ^c	Creep hardening parameter
κ^k	Kinematic hardening parameter
λ	Plastic proportionality constant
ν	Poisson's ratio
σ	Stress
ρ	Mass density
ξ,η,ζ	Parent element Cartesian coordinate system

Subscripts:

a,b,i,j,k,ℓ,m,n,r

General indices

Superscripts:

0	Start-of-increment quantity
1	End-of-increment quantity
o	Known test value
c	Creep quantity
e	Elastic quantity
p	Plastic quantity
t	Thermal quantity

Special Symbols:

$\delta(\ )$	Residual (corrective) quantity; Virtual quantity
$\partial(\ )$	Partial derivative
$\|\ \|$	Length of vector

Special Symbols:

$\Delta(\ )$	Incremental quantity
$(\ )^*$	Reference equilibrium quantity
$(\ )^T$	Matrix transpose
$(\ )^{-1}$	Matrix inverse
$(\ )'$	Mixed coordinate system quantity
$(\bar{\ })$	Effective quantity; Tangent coordinate system quantity
$(\hat{\ })$	Relative deviatoric quantity; Prescribed equation solution quantity
$(\dot{\ })$	Rate quantity
$(\ddot{\ })$	Second order rate (acceleration) quantity
Σ	Summation
π	Product Sum
$\times$	Vector cross product

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BOPACE

PART II: USER MANUAL

10.0 BOPACE INPUT DATA

10.1 GENERAL ORGANIZATION

The schematic of the BOPACE data deck for a problem is shown in Figure 10.0-1. A problem is defined in general by a constant structure combined with one or more variable structures, and with one or more load increments for each constant-variable structure combination. The BOPACE input data consist of four distinct groups:

1) Overall problem control data (for each cold start or restart),
2) Constant structure data (given for substructure problem),
3) Variable structure data, and
4) Increment data.

The variable structure data and increment data may be redefined an unlimited number of times in any one problem.

The overall problem control data begin with the TITLE card. Following this are parameters to define the basic problem type, and to control the solution method and iteration sequence. Also included are cards to control the effect of diagnostic conditions, to restart or checkpoint the problem, and to allow the user to select the nodal and reference-point results that are to be printed for his problem.

The constant structure data begin with the CTITLE card. Material, coordinate system, node, boundary node, element, multi-point constraint and single-point

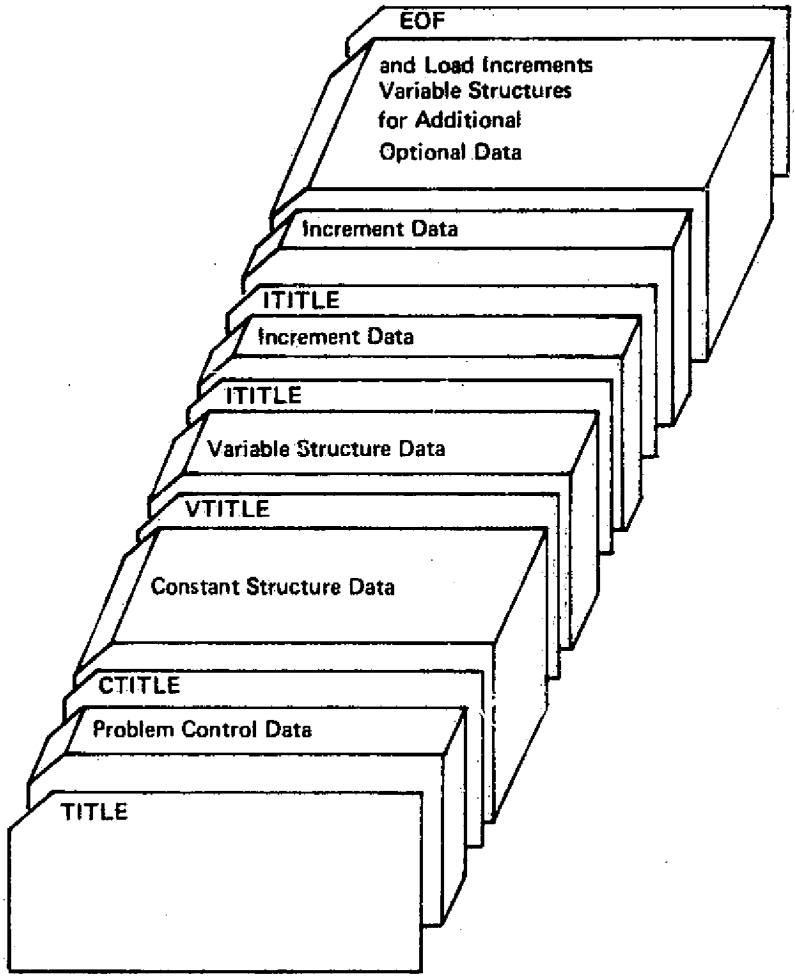

Figure 10.0-1: BOPACE Problem Data Deck

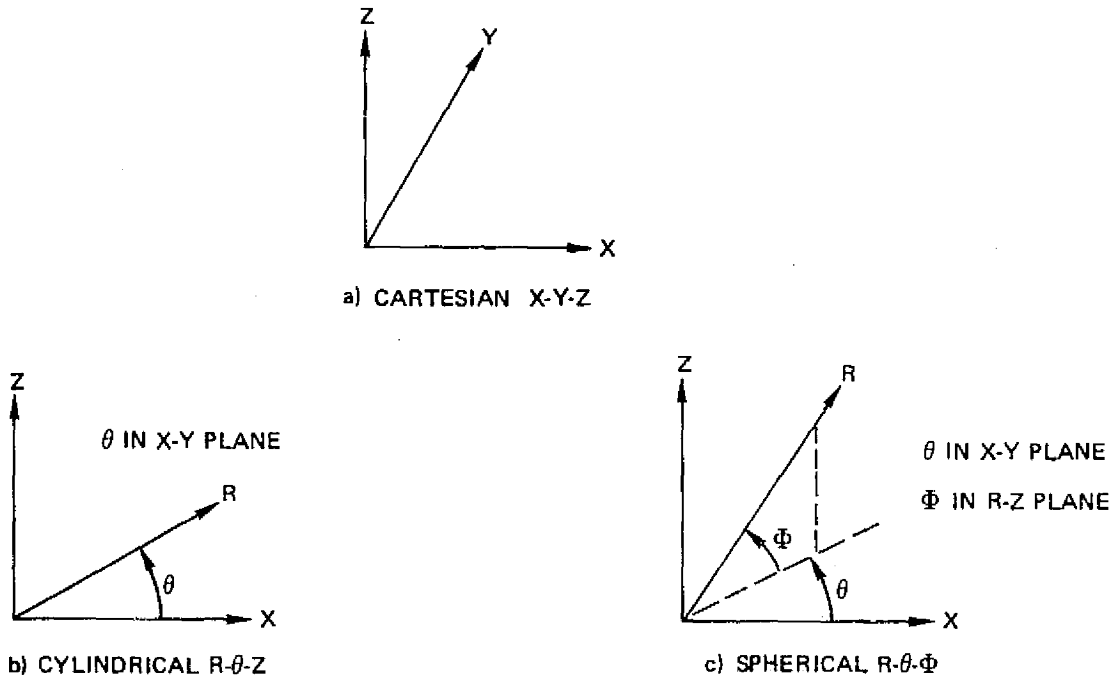

Figure 10.0-2: Basic Coordinate Systems

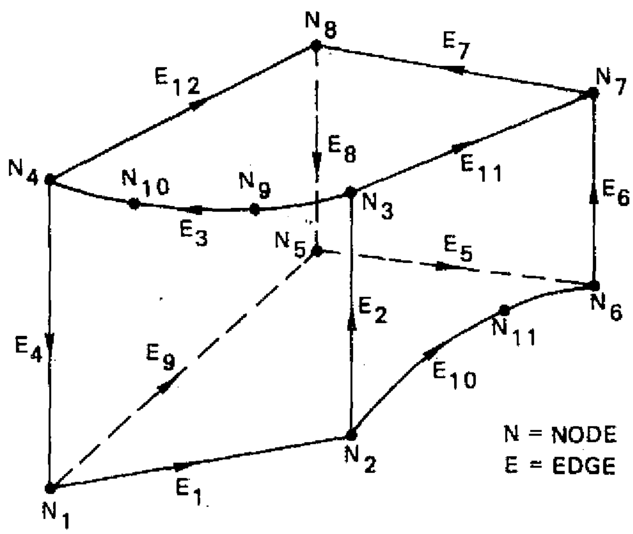

Figure 10.0-3. Isoparametric Element Example

constraint definitions are included. The constant structure data define those parts of the structure for which the stiffness matrix is to be treated as constant during the solution of the problem. BOPACE does not require a problem to have a constant structure, and constant structure data is usually not given unless the problem involves substructuring.

The variable structure data begin with the VTITLE card. Material, coordinate system, node, element, multi-point constraint and single-point constraint definitions are included. The variable structure data define those parts of the structure which can be modified, and for which the stiffness matrix can be updated as BOPACE iterates to a solution. A null variable structure is unusual but is allowed. (In this case, if a VTITLE card is not input, a null variable structure title is generated by the program.) If no constant structure was defined, the variable structure is the entire structure. If both a constant and variable structure exist, they may be connected via the boundary nodes defined under CTITLE. Certain portions of variable structure information involving materials and coordinate systems, may have already been defined by constant structure data, in which case the data need not be repeated.

The increment data begin with the ITITLE card, and consist of cumulative load factors, control data, material and coordinate system data, and load set data. The cumulative load factors are used as multipliers for the load sets, in order to compute the various cumulative mechanical and thermal loads. (A load factor of zero means that a particular loading is not acting on the structure.) The control data given here will override, for the particular increment only, any of the same data defined under TITLE. Material tables may be redefined in the increment data, and will permanently replace any corresponding existing data; the new material data is used immediately during the iteration process, and later when the next stiffness matrix update is performed. Coordinate systems may also be defined or redefined (redefinition produces a warning message). The load set data are used to modify or regenerate the load sets for concentrated, distributed, thermal, normal strain/stress, and inertia loads. (Any load set data not redefined remain unchanged.)

Multiple problems may be run simply by stacking the problem decks consecutively. The last card required after the entire data deck is the EOF card.

10.2 CARD FORMAT

All data cards input to BOPACE are in a free field format. The free field data rules are:

1) Each data card must begin with a name tag, which identifies the data on the rest of the card. The name tag must start in column 1, and consist of alphanumeric characters with no imbedded blanks. Only the first <u>four</u> characters in the name tag must be given correctly.
2) A name tag of <u>CONT</u>INUE indicates the data on the card has been continued from the previous card. There is no limit on the number of continuation cards.
3) A name tag must be immediately followed by one or more blanks. The remainder of the card contains data items associated with the name tag, with each data item followed by a delimiter.

4) A legal delimiter is either a comma, one or more blanks, or a comma with one or more adjacent blanks. The end of the card is equivalent to a comma.

5) A $ causes BOPACE to ignore the columns following the $ on that card. The $ may be used for comments, or to allow the BOPACE input interpreter to stop scanning the card for further data.

6) A null data item causes BOPACE to use a default value. (If a default value does not exist, a zero value is generated.) Null items are input by successive commas which do not enclose a data item, or by completely omitting the last one or more items associated with a name tag.

10.3 BOPACE DATA CARD DEFINITIONS

BOPACE requires most data to be input in a predefined order. Not all data is required to be input. If a particular portion of data is not required for the problem to be solved, then this data may be omitted. Section 10.3.0 gives a summary name-tag list of the BOPACE data card types, shown in the suggested order of input. The remainder of Section 10.3 gives a more detailed explanation of each card type, including the definition of its various individual data items.

10.3.0 Summary Name-Tag List of BOPACE Data Cards

TITLE	
DCONDITION	
PROBLEM	(Cold start only).
SOLUTION	
RESTART	
CHECKPOINT	
SET	Give for each set (cold start only).
PRT1 PRT2	Required to obtain output of problem results.
CTITLE	
MATI IMODULUS IPOISSON ISTRAIN	Isotropic elastic material data group. Repeat for each isotropic material.
PLASTIC PTEMP IHARD KSHAPE KFACTOR	Plastic material data group. Repeat for each isotropic material.
CREEP CSHAPE CTEMP CFACTOR	Creep material data group. Repeat for each isotropic material.
MATA AELASTIC AFACTOR XSTRAIN YSTRAIN ZSTRAIN	Anisotropic material data group. Repeat for each anisotropic material.
CARTESIAN	Repeat for each special coordinate system.
NODE	Repeat for each constant non-boundary node.
BOUNDARY	
NODE	Repeat for each boundary node.
PBRICK PQUAD PQRING	Use as many of these cards as required to define properties for constant elements.
RBRICK RQUAD RQRING	Use as many of these cards as required to define reference points for constant elements.
BRICK QUAD QRING	Give one of these element cards for each constant element.
MPC	Give one card for each multi-point constraint.
SPC	Use as many cards as desired for single-point constraints.

VTITLE
All constant structure card types except BOUNDARY may be given here. Any material data group (MATI, PLASTIC, CREEP, or MATA group) redefined here will permanently replace the corresponding data group defined previously. Redefinition of a previously defined special coordinate system is allowed, but will produce a warning message.

ITITLE
LFACTOR
CTIME

SOLUTION PRT1 PRT2	If given here, these override corresponding problem control cards, for current increment only.
MATI GROUPS PLASTIC GROUPS CREEP GROUPS MATA GROUPS	If redefined here, each of these permanently replaces the material data group defined previously.
CLOAD C1LOAD C2LOAD	Concentrated mechanical load sets group. Use as many cards as desired to redefine loads.
DLOAD D1LOAD D2LOAD	Distributed mechanical load sets group. Use as many cards as desired to redefine loads.
TLOAD T1LOAD T2LOAD	Thermal load set group. Use as many cards as desired to redefine thermal loads.
SLOAD S1LOAD S2LOAD	Normal strain/stress load set group. Use as many cards as desired to redefine normal strain/stress loads.
TAXIS RAXIS CMASS C1MASS C2MASS	Inertia loads data group. Use as many CMASS type cards as desired to redefine concentrated masses.
EOF	Final card after end of data for all problems.

10.3.1 Problem Control - Title Card TITLE

Description: Defines the title for the problem.

Format

TITLE	title

Examples

TITLE	THIS CARD IS REQUIRED

CONTINUE	FOR EVERY BOPACE PROBLEM

Field	Contents
title	Any hollerith characters.

Remarks:

1) This card is required for every BOPACE problem and it must be the first data card.

2) The non-continued part of the title appears on the first line of every page of output.

10.3.2 Problem Control - Diagnostic Condition Card DCONDITION

Description: Diagnostic condition for switching to a diagnostic detection mode only (problem solution is aborted).

Format

DCONDITION	cond

Examples

DCON	1

Field	Content
cond	Condition code.
	0 diagnostic detection only (no problem solution)
	1 a warning message causes a switch to diagnostic detection only
	2 forces a solution when warning messages are present, but an error message causes a switch to diagnostic detection only (default)
	>2 forces a solution when warning or error messages are present

Remarks: 1) If a DCOND card is not input, then cond = 2.

10.3.3 Problem Control - Problem Type Card PROBLEM

Description: Defines basic problem type.

Format

```
PROBLEM  probt geomnl
```

Examples

```
PROB     2       1
```

Field	Contents
probt	Problem type.
	2 two-dimensional space
	3 three-dimensional space (default)
	4 axisymmetric problem
geomnl	Nonlinearity code.
	0 material nonlinearity only (default)
	1 geometric and material nonlinearity

Remarks: 1) If a PROBLEM card is not input, all default values will be assumed.

10.3.4 Problem Control - Solution Parameter Card SOLUTION

Description: Defines solution method and iteration variables for all increments.

Format

SOLUTION	errmax	scode	maxup	$maxit_1$	$maxit_2$	$maxit_3$

CONT	maxie	maxyc	maxcut	cut	afact

Examples

SOLU	.005	1	1	4

Field	Contents
errmax	Maximum allowable residual error norm (default is .001).
scode	Stiffness matrix generation code for updating stiffness matrix during iteration loop. 1 do not update matrix (always use the initial elastic matrix generated for a temperature distribution defined by the element fabrication temperatures) 2 update elastic matrix (based on current temperature and geometry) 3 update total (elastic plus plastic) matrix (default) 4 update both elastic and total matrices
maxup	Maximum number of stiffness matrix updates per increment in order to achieve convergence to within the maximum allowable residual error norm (default is 1).
$maxit_i$	Maximum number of residual-force iterations before update i of the stiffness matrix is computed (default is 10, 10 and 10).

10.3.4 Problem Control - Solution Parameter Card - continued

Field	Contents
maxie	Maximum number of initial iterations for each increment using the elastic matrix (default is 2).
maxyc	Maximum allowable magnitude of the elastic-plastic sum code (default is 2).
maxcut	Maximum number of cuts to be performed (giving a new solution as a fraction of a previously used displacement correction) if residual norm is not decreasing (default is 1).
cut	Cutting fraction to be multiplied times previously used displacement correction (default is .5).
afact	Fraction from end of increment to evaluate stress versus plastic-strain slope in forming total stiffness matrix (default is .1).

Remarks:

1) If SOLUTION is not input, all default values will be assumed.

2) If $maxit_1$ is zero, BOPACE will update the stiffness matrix before the iteration process starts.

3) If $maxit_1$ and maxup are zero, then BOPACE will not perform an incremental solution, but will print the requested nodal and reference-point quantities, computed during the previous increment.

10.3.5 Problem Control - Restart Card RESTART

Description: Directs BOPACE to start a problem from a previous problem that was checkpointed.

Format

```
RESTART  incr vstr  tapno
```

Examples

```
REST     48   3     1
```

Field	Contents
incr	Increment number on the checkpoint tape from the end of which a restart is to be made.
vstr	Variable structure number on the checkpoint tape from the end of which a restart is to be made (default is 1).
tapno	Logical unit containing a checkpoint tape from a previous problem (default is 28).

Remarks:

1) Value of zero for incr causes the restart to occur after variable structure vstr.

2) A value of zero for vstr causes the restart to occur after the constant structure.

3) In case of multiple restarts, the incr and vstr values are cumulative.

10.3.6 Problem Control - Checkpoint Card <u>CHECKPOINT</u>

<u>Description:</u> Directs BOPACE to checkpoint the problem.

<u>Format</u>

```
CHECKPOINT     tapeno
```

<u>Examples</u>

```
CHEC     56
```

<u>Field</u>	<u>Content</u>
tapeno	Logical unit on which BOPACE is to checkpoint the problem (default is 29).

<u>Remarks:</u> 1) This card is required only for problems that are to be checkpointed.

10.3.7 Problem Control - Set Card(s) SET

Description: Defines a set of either nodes or elements for output requests.

Format

SET	sid i_1, i_2, etc, DO i_3 i_4 i_5 MINUS i_6 i_7, etc.	Repeat as required
CONTINUE	DO i_8,i_9,i_{10} PLUS i_{11} i_{12} i_{15}	

Examples

SET	10	1.	3	DO	100	200	MINUS	DO	50	60	PLUS 55

SET	5 PLUS DO,1,75,2

SET	100	90

Field	Contents
sid	Set identification number.
i_1 i_2, etc.	Node or element identification numbers that are to be output.
DO i_3 i_4 i_5	Nodes or elements that are to be output, beginning at i_3, ending at i_4, and those intermediate nodes or elements generated by repeatedly adding i_5 to i_3. Default i_5 = 1.
MINUS	Nodes or elements following the MINUS are removed from the set definition.
PLUS	Nodes or elements following the PLUS are added to the set definition.

Remarks:

1) SET cards are optional.

2) SET cards can be referenced by PRT1 and PRT2 cards.

3) Set generation begins with an implied PLUS operator, which holds until a MINUS is encountered, etc. Redundant PLUS or MINUS operators are optional.

10.3.8 Problem Control - Output Request Card PRT1

Description: Print selected nodal quantities.

Format

PRT1	n	sid

Examples

PRT1	1	-1

PRT1	1	0

PRT1	1	100

Field	Contents
n	Specified type of nodal quantity.
	1 internal forces and displacements
sid	Set identification number.
	-1 print all nodal quantities of specified type
	0 print no nodal quantities of specified type (default)
	>0 print nodal quantities of specified type for only the the nodes included in set sid

Remarks: 1) Caution - if a PRT1 card is not input, then no nodal quantities are printed.

10.3.9 Problem Control - Output Request Card PRT2

Description: Print selected element reference-point quantities.

Format

PRT2	n_1 sid_1 n_2 sid_2 n_3 sid_3 etc.

Examples

PRT2	1,-1 2,0 11,3 6,1

Field	Contents
n_i	Specified type of reference-point quantity. 1 cumulative stresses 2 incremental stresses 3 cumulative elastic strains 4 incremental elastic strains 5 cumulative plastic strains 6 incremental plastic strains 7 cumulative creep strains 8 incremental creep strains 9 cumulative total strains 10 effective plastic and creep strains 11 thermal strains
sid_i	Set identification number. -1 print all element reference-point quantities of specified type 0 print no element reference-point quantities of specified type (default) >0 print element reference-point quantities of specified type for only the elements included in set sid

Remarks: 1) Caution - if a PRT2 card is not input, then no element reference quantities are printed.

10.3.10 Constant Structure - Title Card CTITLE

Description: Defines a title for the constant structure data.

Format

CTITLE	ctitle

Examples

CTITLE	THIS TITLE APPEARS ON THE SECOND LINE

CONT	OF EVERY PAGE OF OUTPUT

Field	Contents
ctitle	Any hollerith characters.

Remarks:

1) This card is required to be the first card of the constant structure data.

2) The non-continued part of the title appears as the second line on every page of output.

3) This card is required only if there is constant structure data.

MATI
IMODULUS
IPOISSON
ISTRAIN

10.3.11 Constant Structure - Isotropic Elastic Properties Cards

Description: Defines Young's modulus, Poisson's ratio and thermal strains for isotropic materials.

Format

MATI	mid	density						Repeat for each isotropic material
IMODULUS	tm_1	mod_1	tm_2	mod_2	tm_3	mod_3	etc.	
IPOISSON	tp_1	poi_1	tp_2	poi_2	tp_3	poi_3	etc.	
ISTRAIN	ts_1	str_1	ts_2	str_2	ts_3	str_3	etc.	

Example

MATI 4	.001
IMOD	0.,1.E6 1.,10.9E6
IPOI	0 .3
ISTR	0.,1.E-6

Field	Contents
mid	Material identification number ($1 \leq mid \leq 5$).
density	Mass density.
tm_i	Temperatures at which Young's modulus is defined.
mod_i	Young's modulus at temperature tm_i.
tp_i	Temperatures at which Poisson's ratio is defined.

10.3.11 Constant Structure - Isotropic Elastic Properties Cards - continued

Field	Contents
poi_i	Poisson's ratio at temperature tp_i.
ts_i	Temperatures at which thermal strain is defined.
str_i	Thermal strain at temperature ts_i.
Remarks:	1) If the ISTRAIN card is not input for a material, then default thermal properties are generated with no thermal strain.

10.3.12 Constant Structure - Isotropic Plastic Properties Cards

PLASTIC
PTEMP
IHARD
KSHAPE
KFACTOR

Description: Define isotropic hardening, kinematic hardening shapes, and kinematic hardening factors for isotropic plastic materials.

Format

PLASTIC	mid ptype ktype					
PTEMP	temp					
IHARD	ip_1	is_1	ip_2	is_2	etc.	
KSHAPE	kp_1	ks_1	kp_2	ks_2	etc.	
KFACTOR	cp_1	f_1	cp_2	f_2		

Repeat for each temperature (PTEMP through KFACTOR)

Repeat for each isotropic plastic material (PLASTIC through KFACTOR)

Examples

PLAS	3	1						
PTEMP	0.							
IHAR	0.	2.	3.	2.	9.	3.5		
KSHA	0.	0.	1.	1.	3.	2.	9.	3.5
PTEMP	1.							
IHAR	0.	2.	3.5	2.2				
KSHA	0.	0.	1.	1.	4.	2.		

10.3.12 Constant Structure - Isotropic Plastic Properties Cards - continued

Field	Contents
mid	Material identification number ($1 \leq mid \leq 5$).
ptype	Plasticity type. 1 strain hardening (hardening parameters = sum of increments of effective plastic strain, default) 2 work hardening (hardening parameters = cumulative plastic work density)
ktype	Kinematic type. 0 kinematic hardening is a function of one parameter (default) 1 kinematic hardening is a function of two parameters
temp	Temperature.
ip_i	Cumulative hardening parameters at which isotropic stress values are defined for temperature temp (must monatonically increase).
is_i	Isotropic stress at cumulative hardening parameter ip_i.
kp_i	Kinematic parameters at which kinematic stress shapes are defined for temperature temp (must monatonically increase).
ks_i	Kinematic stress shape at kinematic parameter kp_i.
cp_i	Cumulative parameters at which kinematic stress factors are defined for temperature temp (must monatonically increase).
f_i	Kinematic stress factor at cumulative parameter cp_i (must monatonically increase).

10.3.12 Constant Structure - Isotropic Plastic Properties Cards - continued

Remarks:

1) If no plasticity data or only the PLASTIC card is input for a material, then default plastic properties are generated with an essentially infinite yield stress.

2) If the kinematic type is zero for a material, then the kinematic stress factors are taken as 1.0 and the KFACTOR card is not input.

3) Temperatures must monatonically increase.

4) For uniaxial tension case, isotropic stress = (t+c)2, and kinematic stress = (t-c)/2, where t = tensile yield stress and c = compressive yield stress.

10.3.13 Constant Structure - Isotropic Creep Properties Cards

CREEP
CSHAPE
CTEMP
CFACTOR

Description: Define a creep curve shape and temperature dependent creep factors.

Format

CREEP	mid ctype	Repeat for each isotropic material
CSHAPE	$time_1$, $strain_1$, $time_2$ $strain_2$ etc.	
CTEMP	temp	Repeat for each temperature
CFACTOR	$stress_1$, $fact_1$, $stress_2$ $fact_2$ etc.	

Examples

CREEP	3 2
CSHAPE	0. 0. 10.1 30. 20. 40.
CTEMP	0.
CFACT	0. 1. 3. 1. 11. 9

Field	Contents
mid	Material identification number ($1 \leq mid \leq 5$).
ctype	Creep type.
	1 age hardening (based on creep time, default)
	2 strain hardening (based on sum of increments of effective creep strain)
	3 work hardening (based on cumulative creep work density)
$time_i$	Times at which creep strains are defined (must monatonically increase).
$strain_i$	Creep strain at time $time_i$.

10.3.13 Constant Structure - Isotropic Creep Properties Cards - continued

Field	Contents
temp	Temperature.
$stress_i$	Stress values at which creep factors are defined (must monatonically increase).
$fact_i$	Creep factor at stress value $stress_i$.

Remarks:

1) If no creep data or only the CREEP card is input for a material, then default creep properties are generated with no creep.

2) Creep is equal to the creep factor (function of temperature and stress) times the creep curve shape. If the CTEMP and CFACTOR cards are not input for a material, then the creep factors are taken as 1.0.

3) Temperatures must monatonically increase.

MATA
AELASTIC
AFACTOR
XSTRAIN
YSTRAIN
ZSTRAIN

10.3.14 Constant Structure - Anisotropic Elastic Properties Cards

Description: Defines anisotropic elasticity matrix and thermal strains.

Format

MATA	mid	density				
AELASTIC	c_{11}	c_{12}	c_{13}	---	c_{66}	
AFACTOR	t_1	f_1	t_2	f_2 ---	etc.	
XSTRAIN	tx_1	sx_1	tx_2	sx_2---	etc.	
YSTRAIN	ty_1	sy_1	ty_2	sy_2---	etc.	
ZSTRAIN	tz_1	sz_1	tz_2	sz_2---	etc.	

Repeat for each anisotropic material

Examples

MATA	6					
AELA1	1	0	0	0	0	0
CONT2	0	2	1	0	0	0
CONT3	0	1	5	0	0	0
CONT4	0	0	0	5	0	0
CONT5	0	0	0	0	5	0
CONT6	0	0	0	0	0	4
AFACT	-1	1	0	1	10	2

10.3.14 Constant Structure - Anisotropic Elastic Properties Cards - continued

Field	Contents
mid	Material identification number ($6 \leq mid \leq 10$).
density	Mass density.
c_{ij}	Entry for the elasticity matrix.
t_i	Temperatures at which elastic matrix factors are defined.
f_i	Elasticity matrix factor for temperature t_i.
tx_i	Temperatures at which thermal strains are defined for direction x (element reference-point displacement coordinate system) in the material.
sx_i	Thermal strain for direction x at temperature tx_i.
ty_i	Temperatures at which thermal strains are defined for direction y in the material.
sy_i	Thermal strain for direction y at temperature tx_i.
tz_i	Temperatures at which thermal strains are defined for direction z in the material.
sz_i	Thermal strain for direction z at temperature ty_i.

Remarks:

1) If the AFACTOR card is not input for a material, then the elasticity matrix factors are taken as 1.0.

2) If the XSTRAIN, YSTRAIN or ZSTRAIN card is not input, then default thermal properties are generated with no thermal strain for the respective x, y or z direction.

3) For an anisotropic elastic material, BOPACE computes cumulative stress from engineering cumulative elastic strain, using the elasticity matrix and its factor as follows.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = f \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

10.3.15 Constant Structure - Cartesian Coordinate System Card(s) CARTESIAN

Description: Defines special Cartesian coordinate systems (used for nodal displacements and forces, and for elemental reference-point quantities).

Format

CARTESIAN	cid	opt	$node_a$	$node_b$	$node_c$								Give one of these for each special Cartesian system
CARTESIAN	cid	opt	rcid	x_a	y_a	z_a	x_b	y_b	z_b	x_c	y_c	z_c	

Examples

CARTESIAN	100	1	4	8	3
CART	53,2	2	1.,30,10.	1.,45,10.	1.,30,12.

Field	Contents
cid	Coordinate system identification number (3<cid).
opt	Option code. 1 three nodes define coordinate system 2 coordinates of three points define coordinate system
$node_a$	Node defining origin of coordinate system.
$node_b$	Node on the x-axis of coordinate system.
$node_c$	Node in the x-y plane of the coordinate system.
rcid	Coordinate system referenced to define the coordinates of three points. 1 basic Cartesian (default) 2 basic cylindrical 3 basic spherical
x_a,y_a,z_a	Coordinates defining the origin.
x_b,y_b,z_b	Coordinates defining a point on the x-axis.
x_c,y_c,z_c	Coordinates defining a point in the x-y plane.

10.3.15 Constant Structure - Cartesian Coordinate System Card(s) - continued

Remarks:

1) Special Cartesian coordinate systems are optional.

2) See Figure 10.0-2 for a schematic of the BOPACE Cartesian, cylindrical and spherical systems.

3) Angle coordinates are input in degrees.

10.3.16 Constant Structure - Node Definition Card(s) NODE

Description: Define the nodes that comprise the constant structure.

Format

NODE	nid	x y z	lid	did	spc

} Repeat for each node

Examples

NODE	51	11.5	90.	0.	2	2	31

Field	Contents
nid	Identification number of node (0<nid).
x, y, z	Coordinates of node.
lid	Coordinate system used to define coordinates of node. 1 basic Cartesian (default) 2 basic cylindrical 3 basic spherical
did	Coordinate system used to compute displacements and nodal forces (0<did, default 1).
spc	Single point constraints (packed number composed of the digits 0, 1, 2 and/or 3). 0 no constraint (default) 1 constrain freedom 1 2 constrain freedom 2 3 constrain freedom 3

Remarks:

1) Each node must have a unique nid number.

2) Boundary nodes may be used to define a constant/variable structure interface. In that case non-boundary nodes are defined first and then boundary nodes. The two groups of nodes are separated by a data card which has only the name tag BOUNDARY on it.

3) Angle coordinates are defined in degrees.

10.3.17 Constant Structure - Element Property Definition Card(s) — PBRICK

Description: Defines properties for isoparametric BRICK elements in the constant structure.

Format

PBRICK	pid	ftemp	mcode	Repeat for each different definition of element properties.

Examples

PBRICK	2	70.

Field	Contents
pid	Identification number of the element property definition, referred to by BRICK card(s).
ftemp	Fabrication temperature of the element.
mcode	Mapping code for element shape functions. 0 proportionate mapping 1 serendipity mapping (crack-tip element)

Remarks: 1) Each element property definition must have a unique pid number within constant or variable structure only. Element property definitions are not recognized across constant/variable structure boundaries.

10.3.18 Constant Structure - Element Property Definition Cards

PQUAD

Description: Defines properties for isoparametric QUAD elements in the constant structure.

Format

PQUAD	pid	thick	nscode	ftemp	mcode	Repeat for each different definition of element properties

Examples

PQUAD	51	1.0	1	70.	1

Field	Contents
pid	Identification number of the element property definition, referred to by QUAD card(s).
thick	Thickness of the element.
nscode	Number of prescribed normal stress directions. 0 prescribed normal strain (generalized plane-strain element) 1 prescribed normal stress (generalized plane-stress element, default)
ftemp	Fabrication temperature of the element.
mcode	Mapping code for element shape functions. 0 proportionate mapping 1 serendipity mapping (crack-tip element).

Remarks: 1) Each element property definition must have a unique pid number within constant or variable structure only. Element property definitions are not recognized across constant/variable structure boundaries.

10.3.19 Constant Structure - Element Property Definition Cards - PQRING

Description: Defines properties for isoparametric QRING elements in the constant structure.

Format

PQRING	pid	ftemp	mcode	Repeat for each different definition of element properties

Examples

PQRING	151	70.	1

Field	Contents
pid	Identification number of the element property definition, referred to by QRING card(s).
ftemp	Fabrication temperature of the element.
mcode	Mapping code for element shape functions. 0 proportionate mapping 1 serendipity mapping (crack-tip element)

Remarks: 1) Each element property definition must have a unique pid number within constant or variable structure only. Element property definitions are not recognized across constant/variable structure boundaries.

10.3.20 Constant Structure - Element Reference-Point Definition Card(s) - RBRICK

Description: Defines reference points for isoparametric BRICK element(s) in the constant structure.

Format

RBRICK	rid	lid	did	rpcode	$icode_0$	$icode_1$	$icode_2$	$icode_3$	Repeat for each different definition of element reference points
CONTINUE	gp_{11}	gp_{21}	gp_{31}	gp_{12}	gp_{22}	gp_{32}	---	gp_{3ngp}	

Examples

RBRICK	2	2,27	25	1,2,2,2	-1,1,.1	-1,1,.2
CONT	-1,1,.5	0,0,.5				

Field	Contents
rid	Identification number of the element reference-point definition, referred to by BRICK card(s).
lid	Coordinate system used to display reference-point locations in BRICK element(s). 1 basic Cartesian (default) 2 basic cylindrical 3 basic spherical
did	Coordinate system used to define the stresses and strains at a reference point for BRICK element(s). 0 tangent system (default) 1 basic Cartesian 2 basic cylindrical 3 basic spherical >3 special Cartesian

10.3.20 Constant Structure - Element Reference-Point Definition Card(s) - continued

Field	Contents
rpcode	Reference-point code defining the point locations at which stresses, strains, etc. are to be computed for printout. This code is a packed number composed of the digits 1, 2, 3, 4 and/or 5. 1 integration points 2 corner points 3 surface center points 4 element center point (default) 5 general user-defined points
$icode_0$	Integration-point type. 0 number of Gauss points in each parent coordinate direction is equal to the maximum number of nodes in that direction 1 number of Gauss points in each direction is to be input as $icode_1$, $icode_2$, and $icode_3$ ($0<icode_i$)
$icode_1$	Number of Gauss points in direction ξ.
$icode_2$	Number of Gauss points in direction η.
$icode_3$	Number of Gauss points in direction ζ.
gp_{ij}	ith parent coordinate of general reference-point j (given only if rpcode includes the digit 5).

Remarks: 1) Reference points consist of integration points plus additional user selected points for output purposes.

2) Each element reference-point definition must have a unique rid number within constant or variable structure only. Element reference-point definitions are not recognized across constant/variable structure boundaries.

10.3.21 Constant Structure - Element Reference-Point Definition Cards(s) - RQUAD

Description: Defines reference points for isoparametric QUAD element(s) in the constant structure.

Format

RQUAD	rid	lid	did	rpcode	$icode_0$	$icode_1$	$icode_2$	Repeat for each different definition of element reference points.
CONTINUE	gp_{11}	pg_{21}	pg_{12}	gp_{22}	---	gp_{2ngp}		

Examples

RQUAD	1	1,0	]245	0,,,	.5,.1	.5,.2
CONT	.5,.5	.5,.9	.1,0	.8,.6		

Field	Contents
rid	Identification number of the element reference-point definition, referred to by QUAD card(s).
lid	Coordinate system used to display reference-point locations in QUAD element(s). 1 basic Cartesian (default) 2 basic cylindrical 3 basic spherical
did	Coordinate system used to define the stresses and strains at a reference point for QUAD element(s). 0 tangent system (default) 1 basic Cartesian 2 basic cylindrical >3 special Cartesian

10.3.21 Constant Structure - Element Reference-Point Definition Card(s) - continued

Field	Contents
rpcode	Reference-point code defining the point locations at which stresses, strains, etc. are to be computed for printout. This code is a packed number composed of the digits 1, 2, 4 and/or 5. 1 integration points 2 corner points 4 element center point (default) 5 general user-defined points
$icode_0$	Integration-point type. 0 number of Gauss points in each parent coordinate direction is equal to the maximum number of nodes in that direction 1 number of Gauss points in each direction is to be input as $icode_1$ and $icode_2$ ($0<icode_i$)
$icode_1$	Number of Gauss points in direction ξ.
$icode_2$	Number of Gauss points in direction η.
gp_{ij}	ith parent coordinate of general reference-point j (given only if rpcode includes the digit 5).

Remarks:

1) Reference points consist of integration points plus additional user selected points for output purposes.

2) Each element reference-point definition must have a unique rid number within constant or variable structure only. Element reference-point definitions are not recognized across constant/variable structure boundaries.

3) For QUAD elements used in 3-D problem, did must be 0.

10.3.22 Constant Structure - Element Reference-Point Definition Card(s) - RQRING

Description: Defines reference points for isoparametric axisymmetric quadrilateral element(s) in the constant structure.

Format

RQRING	rid	lid	did	rpcode	$icode_0$	$icode_1$	$icode_2$	Repeat for each different definition of element reference points
CONTINUE	gp_{11}	gp_{21}	gp_{12}	gp_{22}	---	gp_{2ngp}		

Examples

RQRING	1	1,7	1	1,2,2

Field	Contents
rid	Identification number of the element reference-point definition, referred to by QRING card(s).
lid	Coordinate system used to display reference-point locations in QRING element(s). 1 basic Cartesian (default) 2 basic cylindrical

Remarks: 1) See description of RQUAD for remaining fields and remarks.

10.3.23 Constant Structure - Element Definition Card(s) BRICK

Description: Defines isoparametric brick elements in the constant structure. These elements are composed of eight corner nodes, and from zero to three interior nodes per edge (12 edges) in any combinations, for a maximum of 44 total nodes.

Format

BRICK	eid	mid	pid	rid	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_e	Repeat for each brick element
CONTINUE	n_9	---	n_ℓ											

Examples

BRICK	10	3,2,2	11,13,9,4,102,106,101,100

Field	Contents
eid	Element identification number (0<eid).
mid	Material identification number ($1 \leq mid \leq 10$).
pid	Property card identification number.
rid	Reference-point card identification number.
n_1 --- n_8	Corner nodes of brick (see Figure 10.0-3).
n_e	Maximum number of interior nodes per edge.
n_9 -- n_ℓ	Edge nodes where $\ell = 8 + 12 \times n_e$. Edge nodes are defined for each edge in the order shown in Figure 10.0-3. Edges having less than n_e nodes, have zeros inserted so that the number of edge node entries for each edge is the same. If $n_e = 0$, fields n_9 --- n_ℓ are blank.

Remarks:
1) Each element must have a unique eid number.
2) Default element properties are assumed if pid = 0.
3) Default reference-point properties are assumed if rid = 0.

10.3.24 Constant Structure - Element Definition Cards QUAD

Description: Defines an isoparametric quadrilateral membrane element in the constant structure. These elements are composed of four corner nodes, and from zero to three interior nodes per edge (4 edges) in any combinations, for a maximum of 16 total nodes.

Format

QUAD	eid	mid	pid	rid	n_1	n_2	n_3	n_4	n_e	n_5	n_6	$---n_\ell$

} Repeat for each QUAD element

Examples

QUAD 201	4,51,1	101,102,103,104	1	0,106,0,107

Field	Contents
eid	Element identification number (0<eid).
mid	Material identification number ($1 \leq mid \leq 10$).
pid	Property card identification number.
rid	Reference-point card identification number.
$n_1 -- n_4$	Corner nodes of element (see Figure 10.0-3).
n_e	Maximum number of nodes per edge.
$n_5 -- n_\ell$	Edge nodes where $\ell = 4 + 4 \times n_e$. Edge nodes are defined for each edge in the order shown in Figure 10.0-3. Edges having less than n_e nodes, have zeros inserted so that the number of edge node entries for each edge is the same. If $n_e = 0$, fields $n_5 --- n_\ell$ are blank.

Remarks:

1) Each element must have a unique eid number.

2) Default element properties are assumed if pid = 0.

3) Default reference-point properties are assumed if rid = 0.

4) Curved QUAD elements may be used in 3-D problems, e.g. as face skins combined with BRICK core elements, and for membrance analysis of shells. Note that membrane shell analysis will generally require double curvature in each QUAD to prevent singular mechanisms, because the membrane QUAD has no bending stiffness.

10.3.25 Constant Structure - Element Definition Cards QRING

Description: Defines an isoparametric axisymmetric quadrilateral element (nodes as in QUAD element).

Format

QRING	eid	mid	pid	rid	n_1	n_2	n_3	n_4	n_e	n_5	n_6	---n_ℓ	} Repeat for each QRING element

Remarks: 1) See description of QUAD for examples, fields and remarks.

10.3.26 Constant Structure - Multi-Point Constraint Cards <u>MPC</u>

Description: Defines the displacement for one freedom as a function of the displacements at other freedoms of the structure. The form of the equation is:

$$Q_i = a_{ij} Q_j$$

where Q_i and Q_j are the displacements at freedoms i and j, and a_{ij} are coefficients to be multiplied times the displacements of freedoms j (summation on j).

Format

MPC	n_1 c_1 n_2 c_2 a_2 n_3 c_3 a_3	Repeat for each multi-point constraint equation.
CONTINUE	n_4 c_4 a_4 etc.	

Examples

MPC	10, 3 11,3,.5 13,2,2.5

Field	Contents
n_1	Dependent node.
c_1	Component number for the dependent node (1, 2 or 3).
n_2, n_3, etc.	Independent nodes.
c_2, c_3, etc.	Component numbers for independent nodes (1, 2 or 3).
a_2, a_3, etc.	Coefficients for independent nodes.

Remarks:

1) On a constant structure MPC card, only freedoms at the constant and boundary nodes can be referenced.

2) If the dependent freedom is at a boundary node, then the independent freedoms must also be at boundary nodes.

3) MPC's can be used to define sliding boundaries (equal normal displacements at pairs of nodes), to simulate rigid connectors (constant distance between given nodes), to enforce straight lines or plane surfaces (by proper combination of normal displacements), and for many other purposes.

10.3.27 Constant Structure - Single-Point Constraint Cards SPC

Description: Defines displacement freedoms of the structure. The displacements are assumed to be zero unless they are defined via a concentrated load set.

Format

SPC	n_1	c_1	n_2	c_2	n_3	c_3	etc.	} Repeat as required

Example

SPC	100,1	201,3	5,2

Field	Contents
n_i, ci	The freedom corresponding to node n_i and component c_i is to be constrained to zero displacement, unless defined as non zero via the concentrated load sets.

Remarks: 1) Only freedoms at the constant and boundary nodes can be referenced on the constant structure SPC card.

10.3.28 Variable Structure - Title Card VTITLE

Description: Defines a title for the variable structure.

Format

VTITLE	vtitle

Examples

VTITLE	The VTITLE CARD IS THE FIRST CARD

CONTINUE	OF THE VARIABLE STRUCTURE DATA

Field	Contents
vtitle	Any hollerith characters.

Remarks:

1) This card is required as the first card of the variable structure data. If there is no variable structure data, this card need not be input (a null variable structure title will be generated by BOPACE).

2) The non-continued part of the title is printed as either the second or third line on every page of output.

10.3.29 Variable Structure - General Cards

The variable structure data is defined in the same order as the constant structure data. All data card types, except BOUNDARY, defined in the constant structure data section can be used to define the variable structure.

Variable structure elements may be connected to only boundary or variable structure nodes.

Materials and coordinate systems defined for the constant structure can be referenced by the variable structure, without redefinition. Any material group which is redefined will permanently replace the corresponding previously defined group. Coordinate systems may be redefined, but if this is done a warning message will be produced.

In the variable structure, the dependent freedom on an MPC card cannot be a boundary node. Also, all freedoms referenced on the variable structure MPC and SPC cards must be at nodes which are either boundary or variable structure nodes (not constant structure nodes). Boundary node SPC's may be redefined in the variable structure.

10.3.30 Increment - Title Card ITITLE

Description: Define the title for the increment.

Format

```
ITITLE    ititle
```

Examples

```
ITITLE    THIS CARD IS THE FIRST CARD
```

```
CONTINUE OF THE INCREMENT DATA
```

Field	Contents
ititle	Any hollerith characters.

Remarks:

1) This card is required as the first card of the increment data.

2) The non-continued part of the title is printed as either the third or fourth line on every page of increment output data.

3) If there is no increment data, no incremental solutions will be performed.

10.3.31 Increment - Cumulative Load Factor Card LFACTOR

Description: Defines cumulative load factors to be applied to the various load sets, as well as acceleration quantities, for the increment.

Format

LFACTOR	cl_1,	cl_2,	dl_1,	dl_2,	tl	sl	a	ω	α

Example

LFACTOR	0.	1.	5.5	0.	1.0	0.5	0.,	100.,	0.

Field	Contents
cl_i	Coefficient for concentrated nodal load set i.
dl_i	Coefficient for distributed load set i.
tl	Coefficient for thermal nodal load set.
sl	Coefficient for normal stress/strain element load set.
a	Translational acceleration (length/time/time).
ω	Angular velocity (revolutions/time).
α	Angular acceleration (revolutions/time/time).

Remarks:

1) The LFACTOR card causes cumulative loads to be applied for the increment, which are equal to the various factors (coefficients) times their respective load sets.

2) Acceleration quantities defined here are taken as constant over the entire structure. They cause cumulative inertia loads to be applied for the increment, based on defined masses, translational acceleration direction, and axis of rotation.

3) In case geometric nonlinearity was specified on the PROBLEM card, inertia loads and follower-type distributed loads (e.g. pressure or drag loads) are based on the displaced configuration.

4) Caution - if an LFACTOR card is not input, then all its data items are set to zero for that increment.

10.3.32 Increment - Creep Time Card CTIME

Description: Defines incremental creep time for the increment.

Format

CTIME	creept

Example

CTIME	5.0

Field	Contents
Creept	Incremental creep time.

Remarks: 1) If no CTIME card is input, or a zero value is given for creept, then no creep occurs during the increment.

10.3.33 Increment - Repeated Card Types

Increment - Solution Parameter Card SOLUTION

Increment - Output Request Card PRT1

Increment - Output Request Card PRT2

Description: These cards have been described in the Problem Control Data section and may also be used in the Increment Data Section. If they are defined in the Increment Data Section, then they override the corresponding Problem Control card for that particular increment only.

Increment - Isotropic Elastic Properties Cards MATI GROUPS

Increment - Isotropic Plastic Properties Cards PLASTIC GROUPS

Increment - Isotropic Creep Properties Cards CREEP GROUPS

Increment - Anisotropic Elastic Properties Cards MATA GROUPS

Description: These cards have been described in the Constant Structure Data. Material groups may be added or redefined in the Increment Data. Any material group redefined here will permanently replace the corresponding previously defined group. The new material properties are used immediately during the iteration process and later when the next stiffness matrix update is performed.

Increment - Cartesian coordinate system card(s) CARTESIAN

Description: These cards have been defined in the Constant Structure Data. Coordinate systems may be added or redefined in the Increment Data, however redefinition of a system will produce a warning message.

10.3.34 Increment - Concentrated Load Set Card(s) {CLOAD, C1LOAD, C2LOAD}

Description: Define concentrated loads at the nodes. The actual applied loading is equal to the concentrated load factors defined on the LFACTOR card times their respective load sets.

Format

				Repeat as required
CLOAD	clsid	nid_1 c_1 v_1	nid_2 c_2 v_2 etc.	
C1LOAD	clsid	c v	nid_1 nid_2 nid_3 etc.	
C2LOAD	clsid	sid c v		

Examples

CLOAD	1	101,3,.5	100,3,.5	120,1,2.75
C2LOAD	-2	-1,2,0.		

Field	Contents
clsid	± Number of concentrated load set (1 or 2).
nid	Node number.
c	Component number for load direction (1, 2 or 3).
v	Value of the concentrated load.
sid	Node set number. -1 all nodes in structure -2 all nodes in constant structure plus boundary -3 all nodes in variable structure >0 all nodes in set sid

10.3.34 Increment - Concentrated Load Set Card(s) - continued

Remarks:

1) The order of the concentrated load cards is a user option. BOPACE forms each concentrated load in the order defined by the user.

2) Any nodal components for which loads are not defined for an increment, are equal to their values in the previous increment. Before the first increment, all concentrated loads are equal to zero.

3) Positive clsid denotes an add mode, i.e. each specified load is simply added to the set of concentrated loads already existing. Negative clsid denotes a replace mode, i.e., any existing concentrated loads corresponding to a specified load are first deleted from the load set, and then the specified load is added to the load set. Corresponding loads for deletion purposes are those with identical node and component number.

4) A zero c on the C2LOAD card may be used to denote all components of load at nodes in sid.

10.3.35 Increment - Distributed Load Set Card(s) {DLOAD, D1LOAD, D2LOAD}

Description: Define distributed loads over line (edge) or area (surface) regions of elements. The actual applied loading is equal to the distributed load factors defined on the LFACTOR card times their respective load sets.

Format

DLOAD	dlsid	dim	cid	c	eid	id_1	d_1	id_2	d_2	etc.	Repeat as required
D1LOAD	dlsid	dim	cid	c	id	d	eid_1	eid_2	etc.		
D2LOAD	dlsid	dim	cid	c	sid	id	d				

Examples

DLOAD	2	2,0,3	95	10,.2	11,.2	20,.3	21,.3	
D1LOAD	-1	1,1,1,	1	0.	105,106,108,109			
D2LOAD	1	2,1,2	5	6	1.0			

Field	Contents
dlsid	$\pm$ Number of distributed load set (1 or 2).
dim	Dimension of loaded region (1 = line, 2 = area).
cid	Coordinate system used to define load intensity direction. 0 edge or face tangent system 1 basic Cartesian 2 basic cylindrical 3 basic spherical >3 special Cartesian
c	Component number for load intensity direction (1, 2 or 3).

10.3.35 Increment - Distributed Load Set Card(s) - continued

Field	Contents
eid	Element number.
id	Node, edge or face number of element at which load intensity is defined.
d	Distributed load intensity. Units are force per length, or force per area.
sid	Element set number. -1 all elements in structure -2 all elements in constant structure -3 all elements in variable structure >0 all elements in set sid

Remarks:

1) The order of the distributed load cards is a user option. BOPACE forms each distributed load in the order defined by the user.

2) Any regions (edges or faces) for which loads are not defined for an increment, are equal to their values in the previous increment. Before the first increment, all distributed loads are equal to zero.

3) Positive dlsid denotes an add mode, i.e. each specified load is simply added to the set of distributed loads already existing. Negative dlsid denotes a replace mode, i.e. any existing distributed loads corresponding to a specified load are first deleted from the load set, and then the specified load is added to the load set. Corresponding loads for deletion purposes are those with identical dimension, element and region number.

4) On the DLOAD card, a constant (uniform) load intensity is specified by giving only one region (edge or face) number for field id, along with its corresponding intensity d. A linear load intensity variation is specified by giving the corner node identification numbers id_i of the loaded region (two for line or four for area load), along with their corresponding intensities d_i. A nonlinear (general) load variation is specified by giving all node identification numbers of the loaded region, along with their corresponding intensities. Order of the nodes given for a region is arbitrary.

10.3.35 Increment - Distributed Load Set Card(s) - continued

Remarks:

5) On the D1LOAD and D2LOAD cards, only constant load intensities can be specified.

6) A negative dlsid and zero id on the D2LOAD card may be used to delete existing loads on all edges or faces of elements in sid (no loads are added).

10.3.36 Increment - Thermal Load Set Card(s)

$\left\{\begin{array}{l}\text{TLOAD}\\ \text{T1LOAD}\\ \text{T2LOAD}\end{array}\right\}$

Description: Define thermal loads (temperatures) at the nodes. The actual applied loading is equal to the thermal load factor defined on the LFACTOR card times the thermal load set.

Format

TLOAD	tlsid	nid_1	t_1	nid_2	t_2	etc.	
T1LOAD	tlsid	t	nid_1	nid_2	nid_3	etc.	
T2LOAD	tlsid	sid	t				

Repeat as required

Examples

TLOAD	1	159,900.	160,940.	162,950.
T2LOAD	-1	-2		

Field	Contents
tlsid	± Number of thermal load set (1).
nid	Node number.
t	Temperature value.
sid	Node set number.

-1 all nodes in structure
-2 all nodes in constant structure plus boundary
-3 all nodes in variable structure
>0 all nodes in set sid

Remarks: 1) The order of the thermal load cards is a user option. BOPACE forms each thermal load in the order defined by the user.

10.3.36 Increment - Thermal Load Set Card(s) - continued

Remarks:

2) Any nodes for which thermal loads are not defined for an increment, are equal to their values in the previous increment. Before the first increment, the temperature distribution is defined by the element fabrication temperatures. At the end of the first load increment, any nodal temperatures not defined by the user are equal to zero.

3) Positive tlsid denotes an add mode, i.e. each specified load (temperature) is simply added to the set of thermal loads already existing. Negative tlsid denotes a replace mode, i.e. any existing thermal loads corresponding to a specified load are first deleted from the load set, and then the specified load is added to the load set. Corresponding loads for deletion purposes are those with identical node number.

10.3.37 Increment - Normal Strain/Stress Load Set Card(s) {SLOAD, S1LOAD, S2LOAD}

Description: Define normal strain or stress loads for the surfaces of the ROD or QUAD elements. Whether strain or stress is defined, is a function of the nscode value on the PROD or PQUAD card. The actual applied loading is equal to the normal load factor defined on the LFACTOR card times the normal load set.

Format

SLOAD	slsid	eid_1	c_1	s_1	eid_2	c_2	s_2	etc.		Repeat as required
S1LOAD	slsid	c	s	eid_1	eid_2	eid_3	etc.			
S2LOAD	slsid	sid	c	s						

Examples

SLOAD	1	52,1,.01	52,1,.011	150,1,1000.
S1LOAD	-1	2,1.0	106,107,108,110	
S2LOAD	1	5	0	1000.

Field	Contents
slsid	± Number of normal load set (1).
eid	Element number.
c	Component (normal direction) number.
	1 normal to QUAD surface or first normal direction of ROD 2 second normal direction of ROD
s	Value of normal strain or stress.

10.3.37 Increment - Normal Strain/Stress Load Set Card(s) - continued

Field	Contents
sid	Element set number. -1 all elements in structure -2 all elements in constant structure plus boundary -3 all elements in variable structure >0 all elements in set sid

Remarks:

1) The order of the normal load cards is a user option. BOPACE forms each normal load in the order defined by the user.

2) Any element normal components for which loads are not defined for an increment, are equal to their values in the previous increment. Before the first increment, all normal loads are equal to zero.

3) Positive slsid denotes an add mode, i.e. each specified load is simply added to the set of normal loads already existing. Negative slsid denotes a replace mode, i.e. any existing normal loads corresponding to a specified load are first deleted from the load set, and then the specified load is added to the load set. Corresponding loads for deletion purposes are those with identical element and component number.

4) A zero c on the S2LOAD card may be used to denote all components of load on elements in sid.

10.3.38 Increment - Inertia Load Cards

$$\left\{\begin{array}{l}\text{TAXIS}\\\text{RAXIS}\\\text{CMASS}\\\text{C1MASS}\\\text{C2MASS}\end{array}\right\}$$

Description: Define a translational axis and a rotational axis for the structure, and concentrated masses at the nodes. The load at a particular node due to its concentrated mass is defined by the following equation.

load = - m times (a + ω x (ω x R) + α x R)

where a, ω and α are vectors whose directions are defined by the TAXIS and RAXIS cards, and whose magnitudes are defined by the LFACTOR card. R is a vector from the rotational axis to the node, and m is the concentrated mass at the node. Inertia load contributions due to element distributed mass are computed in a similar manner using a volume integral and the element mass density.

Format

TAXIS	x y z c	
RAXIS	x_1 y_1 z_1 x_2 y_2 z_2 c_1 c_2	
CMASS	cmsid nid_1 m_1 nid_2 m_2 etc.	Repeat as required
C1MASS	cmsid m nid_1 nid_2 nid_3 etc.	
C2MASS	cmsid sid m	

10.3.38 Increment - Inertia Load Cards - continued

Examples

TAXIS	10., 0, 2.5			
RAXIS	3., 30.,0	3.,30.,2.	2,2	
CMASS	1	20,10.	22,10.	14,5.
C2MASS	-1	-1		

Field	Contents
x,y,z	Components of the translational axis for the structure.
c	Coordinate system used to define translational axis (default 1).
x_1,y_1,z_1	Coordinates of point 1 on rotational axis.
x_2,y_2,z_2	Coordinates of point 2 on rotational axis.
c_1,c_2	Coordinate systems used to define points 1 and 2 on rotational axis (1, 2 or 3, default 1).
cmsid	$\pm$ Number of concentrated mass load set (1).
nid	Node number.
m	Value of the concentrated mass.
sid	Node set number. -1 all nodes in structure -2 all nodes in constant structure plus boundary -3 all nodes in variable structure >0 all nodes in set sid

10.3.38 Increment - Inertia Load Cards - continued

Remarks:

1) The order of the concentrated mass cards is a user option. BOPACE forms each concentrated mass load in the order defined by the user.

2) Any nodes for which concentrated masses are not defined for an increment, are equal to their values in the previous increment. Before the first increment, all concentrated masses are equal to zero.

3) Positive cmsid denotes an add mode, i.e. each specified mass is simply added to the set of concentrated mass loads already existing. Negative clsid denotes a replace mode, i.e. any existing concentrated mass loads corresponding to a specified mass are first deleted from the load set, and then the specified mass is added to the load set. Corresponding loads for deletion purposes are those with identical node number.

5) Concentrated masses may be defined only at nodes which are connected to elements.

11.0 SIZE LIMITATIONS

General Limitations - The following program variables have been used in BOPACE, to specify several maximum size limitations.

NMAX1 = 5 = maximum number of isotropic materials
= maximum number of anisotropic materials

NMAX2 = 1500 = maximum number of nodes

NMAX3 = 500 = maximum number of elements

NMAX4 = 5000 = maximum node I.D. number

NMAX5 = 2000 = maximum element I.D. number

NMAX6 = 20 = maximum number of points in an elastic modulus, Poisson's ratio or thermal strain curve

NMAX7 = 6 = maximum number of temperatures (hardening curves) per plastic material

NMAX8A = 30 = maximum number of points per isotropic stress hardening curve

NMAX8B = 20 = maximum number of points per kinematic stress hardening shape curve

NMAX8C = 30 = maximum number of points per kinematic stress hardening factor curve

NMAX9 = 10 = maximum number of points in a creep shape curve

NMAX10 = 6 = maximum number of temperatures (hardening curves) per creep material

NMAX11 = 10 = maximum number of points per creep hardening (strain factor vs. stress) curve

NMAX12 = 1000 = maximum number of special coordinate systems

General Limitations

NMAX13 = 2 = number of concentrated mechanical load sets

= number of distributed mechanical load sets

MAXINT = 1000 = maximum total number of reference points (including integration points) per element

MAXIG = 100 = maximum number of general user-defined reference points per element

Data Items - Each BOPACE input data variable is allowed a maximum of 9 digits (including signs, exponents and decimal points).

Wavefront - For the BOPACE linear equations solution, the maximum allowable wavefront (active decomposition nodes) depends somewhat upon other storage requirements. These requirements are defined by the number of nodal freedoms in the problem, the MPC relations, and whether or not a geometrically non-linear solution has been requested. The maximum allowable wavefront is approximately 500 nodes. This number may be increased if the problem contains less than the maximum 4500 freedoms, but the allowable wavefront is decreased by MPC relations or by the specification of geometric nonlinearity. (Geometric nonlinearity requires the storage of an extra vector of nodal displacements, at certain points in the program logic).

BOPACE

PART III: PROGRAMMER MANUAL

12.0 SUBROUTINES

This section lists each subroutine in the BOPACE program, along with a brief description of its purpose and the subroutines which it calls.

MAIN - Main Calling Program.
Subroutines - BIGSC, BIGSCK, BIGSRS, COND, COVER, DECOMP, DUMMY, ELLOOP, ERCOMP, ETIME, EXIT, GFORMS, HEAD, INDAT, LOADS, MERGE, MRTAPE, OUTDAT, OUTEL, OUTPQ, RCARD, SOLN.

BLDATA (BLOCK DATA) - Block data routine to define basic program variables and sizes.

ATRIA - Calculates area of a triangle.
Subroutines - HEADNG

BIGSC - Control program for reading user data after it has been checked for order and gross errors, data generation statements have been executed and data has been transformed to standard form.
Subroutines - MATERL, RCURVE, READO, READ8, SKIP, STRUCT, TITLE.

BIGSCK - Writes checkpoint (restart) file.
Subroutines - ETIME, INDAT, OUTDAT, SRTAPE.

BIGSRS - Reads restart (checkpoint) file.
Subroutines - ETIME, INDAT, OUTDAT, SRTAPE.

COND - Tests problem condition code for bypassing MERGE, DECOMP and SOLN routines.

COSHAP- Generates isoparametric shape functions and their partials, for corner nodes of region.

COVER - Prints BOPACE output cover page.

CSYS - Calculates coordinate transformations at each node.
Subroutines - CSYS2, HEADNG.

CSYSA - Calculates coordinate transformations at each node (same as CSYS).
Subroutines - CSYS2A, HEADNG.

CSYS1 - Calculates basic coordinates of definition points for all special Cartesian systems.
Subroutines - HEADNG.

CSYS2 - Calculates coordinate transformations via vector cross products.

CSYS2A - Calculates coordinate transformations via vector cross products (same as CSYS2).

DECOMP - Matrix Gauss decomposition routine via modified wavefront method, with out-of-core capability.
Subroutines - ETIME, EXIT, HEADNG, INDAT, OUTDAT.

DFORM - Forms stress-strain constitutive matrix for elastic or elastic-plastic material (engineering strain definition).
Subroutines - ZVAL.

DLFORM - Computes equivalent concentrated nodal loads from distributed load sets.

Subroutines - DLOAD, INDAT, OUTDAT, ROTQ.

DLOAD - Computes equivalent concentrated nodal loads from distributed load intensities, for a particular element region.
Subroutines - COSHAP, CSYS2, EDSHAP, GAUS1.

DUMMY - Dummy decomposition routine to calculate wavefront at each node.
Subroutines - EXIT, HEADNG, INDAT.

DYVAL - Linear interpolation routine for incremental ordinate.

EDSHAP - Generates isoparametric shape functions and their partials, for interior nodes on one edge of a region.

ELLOOP - Calling routine to compute strains, stresses and force contributions at an element reference point.
Subroutines - ETIME, FORCE, INDAT, ITER, OUTDAT, STRAIN.

ERCOMP - Computes residual (unbalanced) forces and corresponding residual norm.
Subroutines - HEADNG.

ETIME - Machine-dependent routine called to compute elapsed CPU time since beginning of BOPACE execution, and clock time.
Subroutines - HEADNG.

EXIT - Routine called to indicate end of problem or end of job, and to print problem error summary.

FORCE - Computes cumulative nodal forces for an element from stresses, and adds them to system forces.

Subroutines - ROTQ.

GAUS1 - Sets region integration points for product Gauss formulas.

GBRICK - Computes shape functions, derivatives of shape functions and reference point transformations and locations for the BOPACE elements.

Subroutines - COSHAP, EDSHAP, HEADNG, OUTDAT.

GENER8 - Forms stiffness matrix for an element, in user nodal coordinates.

Subroutines - INDAT, KBRICK, KQRING, KQUAD, ROTK, ROTQ.

GENR8 - Generation/partitioning routine for element stiffness matrices, to create system stiffness partitions including MPC effects.

Subroutines - ETIME, GENER8, HEADNG.

GETDAT - Reads BOPACE standard form data as directed by the various data input routines.

GFORM - Calling program to compute shape functions and partials, and reference points for all elements. Also initializes reference-point data.

Subroutines - CSYS, GBRICK, INDAT, ISET, ZVAL.

GFORMS - Allocates core space for data input or output by the calling routines DLFORM and GFORM.

Subroutines - DLFORM, ETIME, EXIT, GFORM, HEADNG.

HEAD - Writes heading for a load increment.

Subroutines - HEADNG.

HEADNG - Counts lines and pages, and writes headings for various printed data.

INDAT - Routine for unformatted list read.

ISET - Sets element reference points (locations and weights).
Subroutines - EXIT, GAUS1, HEADNG, ISET1.

ISET1 - Sets special user-requested element reference points.

ITER - Major iteration routine to separate elastic-plastic-creep strains, and to compute unknown stress and strain components, at a material reference point (uses tensorial strain components).
Subroutines - DYVAL, YVAL, ZVAL.

ITER1 - Routine called by ITER to compute improved estimate for plastic strain, using linear intersection calculation.

KBRICK - Generates stiffness matrix for brick element.
Subroutines - DFORM, INDAT, YVAL.

KQRING - Generates stiffness matrix for axisymmetric solid ring element (quadrilateral shape).

KQUAD - Generates stiffness matrix for membrane quadrilateral element.
Subroutines - DFORM, INDAT, YVAL.

LOADS - Calculates equivalent concentrated nodal loads due to inertia loads. Also initializes element reference-point data and computes thermal strains.
Subroutines - ETIME, INDAT, OUTDAT, ROTQ, YVAL.

MATERL - Calling routine to read material data.
Subroutines - GETDAT, HEADNG, SKIP, READA, READTC, READTP.

MERGE - Calling routine to generate element stiffness partitions and merge them into system matrix.

Subroutines - ETIME, EXIT, GENR8, HEADNG, MERSOR.

MERSOR - Merges element stiffness partitions into system matrix.

Subroutines - ETIME, EXIT, HEADNG, INDAT, OUTDAT.

MRTAPE - Merges two stiffness files into a single total file.

Subroutines - EXIT, HEADNG, INDAT, OUTDAT.

OUTCCS - Writes cumulative creep strains.

OUTCES - Writes cumulative elastic strains.

OUTCPS - Writes cumulative plastic strains.

OUTCS - Writes cumulative stresses.

OUTCTS - Writes cumulative total strains.

OUTDAT - Routine for unformatted list write.

OUTEL - Calling routine to collect all output data for an element.

Subroutines - COND, ETIME, EXIT, HEADNG, INDAT, OUTCCS, OUTCES, OUTCPS, OUTCS, OUTCTS, OUTEPC, OUTICS, OUTIES, OUTIPS, OUTIS, OUTTHE, SET.

OUTEPC - Writes cumulative effective plastic and creep quantities.

OUTICS - Writes incremental creep strains.

OUTIES - Writes incremental elastic strains.

OUTIPS - Writes incremental plastic strains.

OUTIS - Writes incremental stresses.

OUTPQ - Writes cumulative internal forces and displacements.

Subroutines - COND, ETIME, HEADNG, SET.

OUTTHE - Writes cumulative thermal strains.

RCURVE - Calling routine to read load sets.

Subroutines - EXIT, HEADNG, INDAT, OUTDAT, READ3, READ4, READ5, READ6, READ10.

READA - Reads anisotropic material property data.

Subroutines - GETDAT, HEADNG.

READC - Reads special Cartesian coordinate systems.

Subroutines - GETDAT, HEADNG.

READEC - Reads element connection definitions.

Subroutines - ATRIA, GETDAT, HEADNG, OUTDAT, VQRING, VTET.

READEP - Reads element property and reference-point data.

Subroutines - GETDAT, HEADNG.

READND - Reads node definitions.

Subroutines - CSYS1, GETDAT, HEADNG.

READTC - Reads creep data.

Subroutines - GETDAT, HEADNG, YVAL.

READTM - Reads isotropic material elastic and thermal strain data.

Subroutines - GETDAT, HEADNG.

READTP - Reads isotropic material plastic data.

Subroutines - GETDAT, HEADNG.

READ0 - Reads data to define basic problem type and incremental/iteration control variables, on overall problem level.

Subroutines - GETDAT, HEADNG, SKIP.

READ2 - Reads SPC definitions.

Subroutines - GETDAT, HEADNG.

READ3 - Reads concentrated load data.

Subroutines - GETDAT, HEADNG.

READ4 - Reads distributed load data.

Subroutines - EXIT, GETDAT, HEADNG, INDAT, SKIP.

READ5 - Reads nodal temperature data.

Subroutines - GETDAT, HEADNG, SKIP.

READ6 - Reads inertia data.

Subroutines - CSYSA, GETDAT, HEADNG, ROTQ, SKIP.

READ7 - Reads MPC definitions.

Subroutines - GETDAT, HEADNG.

READ8 - Reads control and parameter data, on incremental level.

Subroutines - GETDAT, HEADNG.

READ10 - Reads element normal loads data.

ROTK - Transforms element stiffness matrix from basic Cartesian to user nodal coordinates.

Subroutines - ROTKK

ROTKK - Transforms element stiffness partition from basic Cartesian to user nodal coordinates.

ROTQ - Transforms nodal forces or displacements for an element, between basic Cartesian and user nodal systems.

SET - Transform a set definition into internal node or element numbers.

Subroutines - HEADNG.

SKIP - Skips to the next logical record of standard BOPACE input data.

Subroutines - GETDAT, HEADNG.

SOLN - Matrix forward-backward substitution routine for Gauss wavefront solution.

Subroutines - ETIME, EXIT, HEADNG, INDAT

SRTAPE - Routine for I/O processing of system stiffness matrix (merge or decomposition matrix).

Subroutines - EXIT, HEADNG, INDAT, OUTDAT.

STRAIN - Compute element reference-point strains from nodal displacements.

Subroutines - ROTQ.

STRUCT - Calling routine to read structural data.

Subroutines - EXIT, HEADNG, INDAT, OUTDAT, READC, READEC, READEP, READND, READ2, READ7.

TITLE - Reads title card for constant structure, variable structure, or increment.

Subroutines - GETDAT, HEADNG.

VQRING - Computes volume of a quadrilateral solid ring element.

VTET - Computes volume of a tetrahedron.

Subroutines - HEADNG.

YVAL - Linear interpolation routine.

ZVAL - Linear table interpolation routine.

13.0 LABELED COMMON BLOCKS

This section lists each labeled common block used in the BOPACE program, along with its description and the subroutines in which it occurs.

BPARAM - Contains variables to define basic problem type.
Subroutines - MAIN, BIGSCK, BIGSRS, FORCE, GENER8, KBRICK, KQRING, KQUAD, LOADS, READO, STRAIN.

CNTRL1 - Basic solution control variables for incrementation and iteration, on overall problem level.
Subroutines - MAIN, BIGSC, BIGSCK, BIGSRS, COND, DFORM, GBRICK, GFORM, HEADNG, OUTEL, OUTPQ, READC, READEC, READND, READO, READ3, READ5, READ6, READ8, READ10, SET, STRUCT, TITLE.

ELDAT0 - Logical units where element and reference-point data are stored.
Subroutines - MAIN, BLDATA, BIGSCK, BIGSRS, ELLOOP, GBRICK, GENER8, GFORM, GFORMS, ITER, KBRICK, KQRING, KQUAD, LOADS, OUTEL, READEC, READ10, STRUCT.

ELDAT1 - Element data for the current element being processed.
Subroutines - BIGSCK, BIGSRS, DFORM, ELLOOP, FORCE, GBRICK, GENER8, GFORM, ISET, ITER, KBRICK, KQRING, KQUAD, LOADS, OUTEL, OUTCES, OUTCCS, OUTCPS, OUTCS, OUTCTS, OUTEPC, OUTICS, OUTIES, OUTIPS, OUTIS, OUTTHE, READEC, READ10, STRAIN.

ELDAT2 - Data for the current element reference point being processed.
Subroutines - BIGSCK, BIGSRS, DFORM, ELLOOP, FORCE, GBRICK, GFORM, ITER, KBRICK, KQRING, KQUAD, LOADS, OUTEL, STRAIN.

ERRORS - Diagnostic warning and error message counters.

Subroutines - MAIN, BIGSCK, BIGSRS, COND, CSYS, CSYSA, CSYS1, DECOMP, EXIT, GBRICK, MATERL, RCARD, READA, READC, READEC, READEP, READND, READTC, READTM, READTP, READO, READ2, READ3, READ5, READ6, READ7, READ8, READ10, SET, SKIP, TITLE, VTET.

FLAGS - Logical variables indicating whether material tables and the various load types are not defined, defined or redefined.

Subroutines - MAIN, BIGSCK, BIGSRS, LOADS, MATERL, RCURVE, READO, READ3, READ5, READ6, READ10, STRUCT.

GDATA1 - Integer variables defining properties of the current and the next data record to be read by GETDAT.

Subroutines - MAIN, BLDATA, BIGSC, GETDAT, MATERL, READA, READC, READEC, READEP, READND, READTC, READTM, READTP, READO, READ2, READ3, READ5, READ6, READ7, READ8, READ10, SKIP, TITLE, VTET.

GDATA2 - Contains the current and the next data record to be read by GETDAT.

Subroutines - BLDATA, GETDAT, READO.

GENC2 - Creep data read by READTC.

Subroutines - BIGSCK, BIGSRS, ITER, MATERL, READO.

GENPO - Plasticity hardening type codes.

Subroutines - BIGSCK, BIGSRS, ITER, MATERL, READO.

GENP7 - Number of points in isotropic hardening curve.

Subroutines - BIGSCK, BIGSRS, GFORM, ITER, MATERL, READO.

GENP8 - Number of points in kinematic hardening shape curves.

Subroutines - BIGSCK, BIGSRS, ITER, MATERL, READO.

GENP9 - Number of points in kinematic hardening factor curves.

Subroutines - BIGSCK, BIGSRS, ITER, MATERL, READO.

GENP10 - Number of temperatures (plastic hardening curves).

Subroutines - BIGSCK, BIGSRS, GFORM, ITER, MATERL, READO.

GENP11 - Isotropic hardening curve abscissas.

Subroutines - BIGSCK, BIGSRS, GFORM, ITER, MATERL, READO.

GENP12 - Kinematic hardening shape curve abscissas.

Subroutines - BIGSCK, BIGSRS, ITER, MATERL, READO.

GENP13 - Kinematic hardening factor curve abscissas.

Subroutines - BIGSCK, BIGSRS, ITER, MATERL, READO.

GENP14 - Plastic hardening curve ordinates (temperatures).

Subroutines - BIGSCK, BIGSRS, GFORM, ITER, MATERL, READO.

GENP15 - Isotropic hardening tables.

Subroutines - BIGSCK, BIGSRS, GFORM, ITER, MATERL, READO.

GENP16 - Kinematic hardening shape tables.

Subroutines - BIGSCK, BIGSRS, ITER, MATERL, READO.

GENP17 - Kinematic hardening factor tables.

Subroutines - BIGSCK, BIGSRS, ITER, MATERL, READO.

GEN1 - Stiffness matrix elastic/plastic code.

Subroutines - MAIN, BIGSCK, BIGSRS.

GEN6 - Material identification numbers and mass densities.

Subroutines - BIGSCK, BIGSRS, LOADS, MATERL, READA, READEC, READTM, READO.

GEN7 - Elastic modulus and Poisson's ratio data.

Subroutines - BIGSCK, BIGSRS, ITER, KBRICK, KQRING, KQUAD, MATERL, READO.

GEN8 - Thermal strain data.

Subroutines - BIGSCK, BIGSRS, LOADS, MATERL, READO.

GEN11 - Anisotropic thermal strain data.

Subroutines - BIGSCK, BIGSRS, LOADS, MATERL, READO.

GEN12 - Anisotropic elasticity data.

Subroutines - BIGSCK, BIGSRS, ITER, KBRICK, KQRING, KQUAD, MATERL, READO.

ILOADS - Axes of translation and rotation data.

Subroutines - BIGSCK, BIGSRS, LOADS, READ6.

IMAGE - Contains next card image to be read by RCARD.

Subroutines - RCARD.

INCRS - Basic solution control variables for incrementation and iteration, on increment level.

Subroutines - MAIN, BIGSCK, BIGSRS, ELLOOP, LOADS, OUTEL, OUTPQ, READ8.

IOUNIT - File unit numbers for input, output, structural definitions and loads.

Subroutines - MAIN, BLDATA, BIGSCK, BIGSRS, COVER, CSYS, CSYSA, CSYS1, DECOMP, DUMMY, ETIME, EXIT, GBRICK, GENR8, GETDAT, GFORM, GFORMS, HEADNG, ISET, MATERL, MERGE, MERSOR, MRTAPE, OUTEL, OUTCES, OUTCCS, OUTCPS, OUTCS,

IPARM - Plasticity parameters for iterative calculations.

Subroutines - ITER, ITER1.

OUTCTS, OUTEPC, OUTICS, OUTIES, OUTIPS, OUTIS, OUTPQ, OUTTHE, RCARD, RCURVE, READEC, READEP, READND, READO, READ8, READ10, SET, SKIP, SOLN, SRTAPE, STRUCT, TITLE.

JLB - Large area of core used for scratch purposes by many BOPACE subroutines. Subroutines - MAIN, BIGSCK, BIGSRS, DECOMP, DUMMY, GFORMS, MATERL, MERGE, MRTAPE, OUTEL, RCURVE, READO, SET, SOLN, SRTAPE, STRUCT.

JLB1 - Files containing the current stiffness matrices.
Subroutines - MAIN, BLDATA, BIGSCK, BIGSRS, READO, STRUCT.

NELNOY - Number of nodes and reference points per element.
Subroutines - BLDATA, BIGSCK, BIGSRS, ELLOOP, GBRICK, GENER8, GFORM, KBRICK, KQRING, KQUAD, LOADS, OUTEL, OUTCES, OUTCCS, OUTCPS, OUTCS, OUTCTS, OUTEPC, OUTICS, OUTIES, OUTIPS, OUTIS, OUTTHE, READEC, READ10.

SIZES - Fixed upper limits for BOPACE, set by BLDATA (BLOCK DATA) routine.
Subroutines - MAIN, BLDATA, BIGSCK, BIGSRS, GBRICK, GFORM, GFORMS, ISET, MATERL, OUTEL, OUTPQ, RCURVE, READEC, READEP, READND, READ10, STRUCT.

SIZESA - Variable sizes, set for particular problem.
Subroutines - MAIN, BIGSCK, BIGSRS, CSYS1, ELLOOP, FORCE, GBRICK, GENER8, GFORM, GFORMS, KBRICK, KQRING, KQUAD, LOADS, OUTEL, RCURVE, READEC, READND, READO, READ2, READ3, READ5, READ6, READ7, READ10, ROTK, ROTQ, STRAIN, STRUCT.

14.0 OVERLAY

The overlay of BOPACE was designed to minimize loading of segments and to maximize the size of common JLB, for a given core size. A schematic of the overlay is shown in Figure 14.0-1.

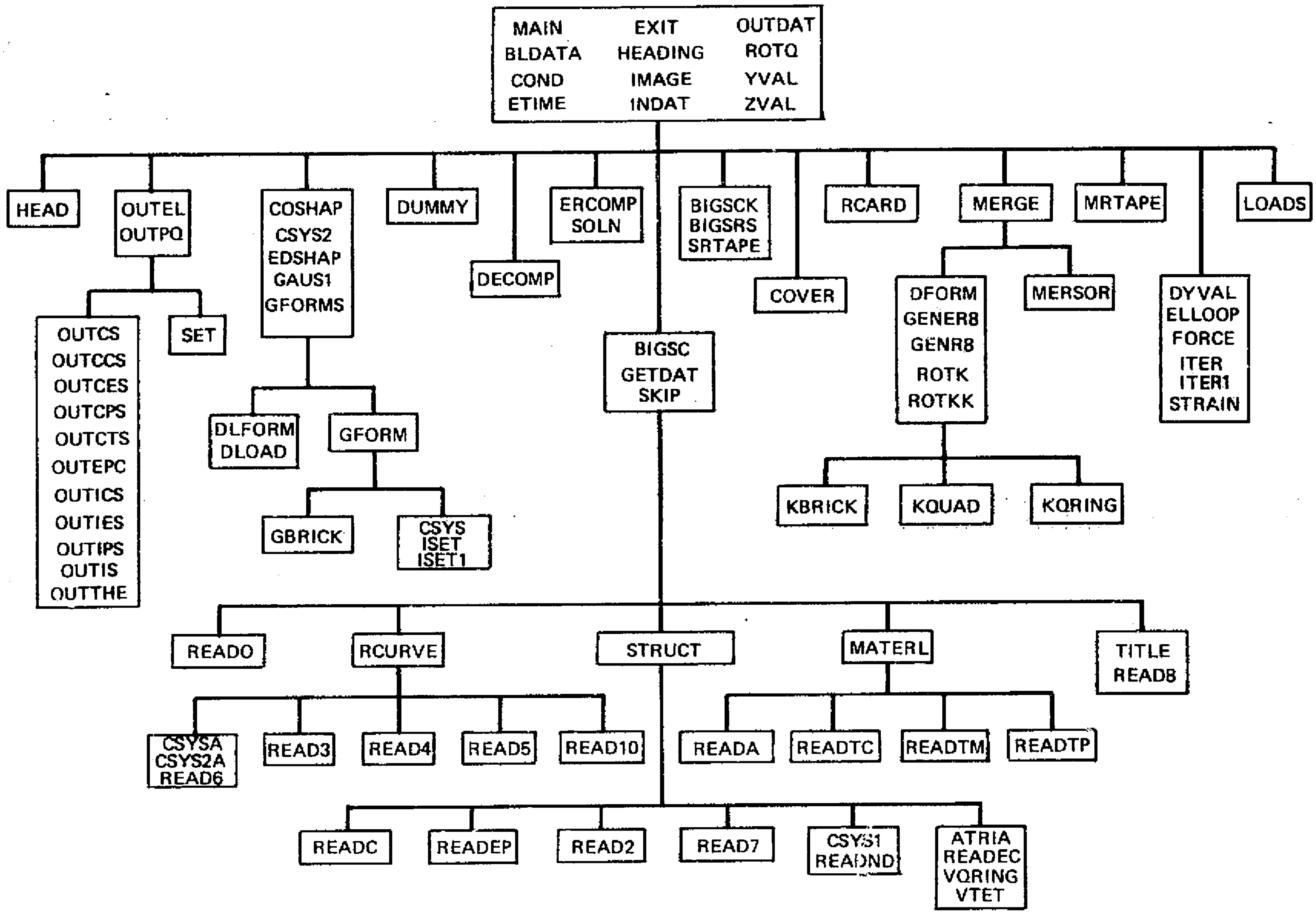

Figure 14.0-1: BOPACE Overlay Schematic

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15.0 FILE USAGE

BOPACE uses Fortran I/O to access a number of files. A current list of files by file name follows:

File	Initial Value	Fixed	Description	Defined by
UIN	5	Yes	input card file.	BLDATA
UOUT	6	yes	output printer file.	BLDATA
UNODAL	18	yes	total nodal displacements and internal nodal forces.	BLDATA
USCR1	19	no	scratch.	BLDATA
USCR2	20	yes	displacement coordinate system number, nodal coordinates and coordinate system definitions.	BLDATA
UCMASS	21	yes	concentrated nodal mass set.	BLDATA
UNTEMP	22	yes	nodal temperature set.	
UPREF	23	yes	concentrated load sets.	BLDATA
UKFMPC	24	yes	constraints.	BLDATA
TRANSF	25	yes	input data in standard form.	BLDATA
TRANSB	26	yes	distributed load sets.	BLDATA
UNOD	27	yes	node numbers and external-internal tables for node and element numbers.	BLDATA
IEDAT	1	no	element data.	BLDATA
IEDIN	2	no	current reference point data.	BLDATA
IEDOUT	3	no	updated reference point data	BLDATA
UDCMPC	12	yes	decomposed stiffness matrix for constant structure	BLDATA

File	Initial Value	Fixed	Description	Defined by
UDCMPB	13	yes	constant structure stiffness matrix after it has been reduced to the boundary nodes.	BLDATA
UDCMPV	14	no	decomposed elastic stiffness matrix for the variable structure.	BLDATA
UMATX3	16	no	scratch	BLDATA
UDCMPP	17	no	decomposed total stiffness matrix for the variable structure.	BLDATA
UMATX1	11	no	scratch.	MAIN
UMATX2	15	no	scratch.	MAIN
UOUTRS	29	no	checkpoint file.	READO or user
UINRS	28	no	restart file.	READO or user

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BOPACE

PART IV. INPUT DATA PREPROCESSING

16.0 SAIL AUTOMATIC DATA GENERATION

The BOPACE data generation capability is patterned after the highly successful BCS SAIL II Language [21] for generating NASTRAN Bulk Data. For BOPACE, the SAIL II Language consists of the standard BOPACE statements, plus SAIL statements and FORTRAN statements. The deck order of the standard BOPACE data cards is the same for SAIL as was previously described for BOPACE. The SAIL statements and the FORTRAN statements can be inserted as needed in the deck.

16.1 STANDARD BOPACE STATEMENTS

SAIL allows an equal sign (=) to follow the name tag on a BOPACE data card. The equal sign tells SAIL that the fields of the card can contain constants, variables or expressions to define the data. If there is no equal sign, all fields of the card are assumed to contain constants. If there is an equal sign, SAIL requires all field delimiters to be commas (with optional adjacent blanks), and SAIL does not allow intermediate null fields (two successive commas with optional blanks).

The equal sign may be placed in all BOPACE data cards except TITLE, CTITLE, VTITLE, ITITLE, SET and CONTINUE.

16.2 SAIL STATEMENTS

Looping - SAIL LOOP statements provide an extended form of the FORTRAN DO statements. They allow BOPACE, SAIL and FORTRAN statements to be executed more than once. The form of the cards is

START LOOP = n, i, j, k, ℓ

END LOOP = n

where

n is the loop identification number. It must be a constant not used as an ID in any other LOOP statement or as a statement label on any FORTRAN statement.

i is a loop parameter.

j is the initial value of i in the looping process.

k is the final value of i in the looping process.

ℓ is the increment to be added to i as the loop progresses.

If ℓ is omitted, 1 is assumed; j, k, and ℓ can be expressions.

START LOOP must be the first statement and END LOOP the last statement in the group of statements to be executed more than once.†

Subdividing Large Decks - For large input data decks to SAIL, the amount of FORTRAN code generated by SAIL can be large enough to cause some compilers difficulty. The user can break up the FORTRAN code into subroutines by using the BREAKPOINT card. An additional subroutine is generated for each BREAKPOINT card. Variables defined before a breakpoint cannot be referenced after a breakpoint unless the variables are redefined. A BREAKPOINT cannot

---

† Certain cards should never be executed more than once. Such cards are the BOPACE standard statements without an equal sign, and BOPACE standard statements which have an equal sign but which contain only constant fields. The user is cautioned to avoid using LOOP, DO, IF, GO TO, etc., operations placed in such a manner that these cards are passed by more than once in the SAIL program logic.

occur within a loop. The form of the card is

BREAKPOINT =

<u>SAIL System Parameters</u> - The SYSTEM statement allows the user to adjust two of the storage arrays in SAIL. The form of the card is

SYSTEM = a, b, c

where a is not used (=0)

b is length of one input/output buffer array (constant)

c is length of the catalog array (constant)

Each BOPACE card generated by SAIL needs 3 words of storage in the catalog array. The buffer array is used to collect BOPACE data cards, and whenever the buffer is full it is transferred to a disk/drum file. The defaults for b and c are 5000 and 21000 respectively. If the SYSTEM card is used, it must be the first card in the data deck.

16.3 FORTRAN STATEMENTS

If FORTRAN statements are in the BOPACE data deck, then they are assumed to obey FORTRAN conventions. That is, labels are in columns 1 - 5, column 6 is the continuation column, columns 73 - 80 are ignored, C in column 1 indicates a comment card, etc. SAIL assumes a statement is FORTRAN if the statement cannot be identified as a BOPACE or SAIL statement.

Any legal FORTRAN statements, including subroutine and function statements, may be used.

16.4 SAIL EXAMPLE PROBLEM

A 2-D rectangular mesh is shown in Figure 16.4-1, with loads and boundary conditions. The BOPACE data for this mesh can be automatically generated using SAIL, as shown below.

```
Col 1     7
     TITLE    RECTANGULAR MESH
     PROB   2
     PRT1   1 -1
     VTITLE   SAIL EXAMPLE
     MATI   1
     IMOD   0 3.E7
     IPOI   0 .3
     START  LOOP = 6, R, 1, 7
     START  LOOP = 4, S, 1, 5
           ID  = 22 + (R-1) * 5 + S
           X   = 11.37 + (S-1) * 3.27
           Y   = 5.19 + (R-1) * 4.23
     NODE = ID, X, Y
     END  LOOP = 4
     END  LOOP = 6
     PQUAD    10, 1.
     RQUAD    10
           IE = -5
     START    LOOP = 20, I, 23, 27
           IE = IE + 5
     START    LOOP = 10, ZZ, 1, 6
```

```
Col 1     7
    QUAD =  I + IE, 1, 10, 10, I + IE, I + IE + 1
    CONT    I + IE + 6, I + IE + 5
    END LOOP = 10
    END LOOP = 20
          DO 30 I = 1, 2
    SPC = 23, I, 27, I, 53, I, 57, I
       30 CONTINUE
    ITITLE   CENTER LINE LOAD
    LFAC  1
          P = -10./SQRT(2.)
    START LOOP = 50, I, 25, 55, 5
    CLOAD = -1, I, 1, P, I, 2, P
    END LOOP = 50
    EOF
```

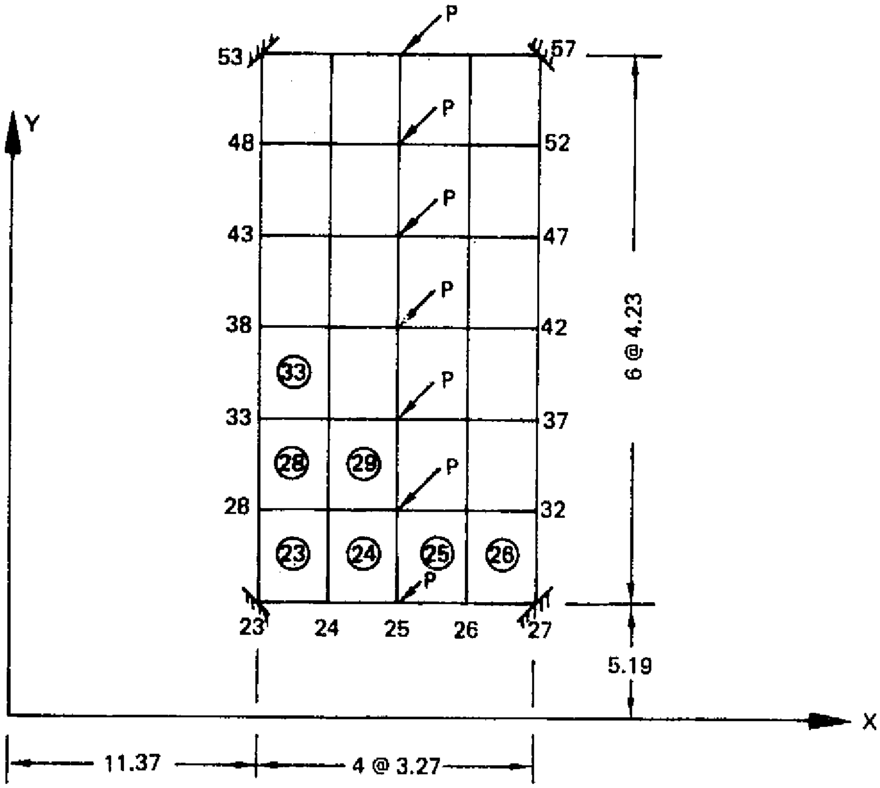

Figure 16.4-1: SAIL Example Problem Mesh

BOPACE

APPENDIX A. REFERENCES

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BOPACE

APPENDIX B. EXAMPLE PROBLEMS

B.1 VARIABLE STRESS (DISTRIBUTED LOAD) PROBLEM

A 1.0 x 1.0 x 1.0 cube is used, to analyze a problem with linear variation of stress and strain. The loading is a uniformly distributed vertical shear, applied on all four vertical sides of the cube. Midside nodes are used to demonstrate equivalent loading values, which are in the ratio of 1:4:1 for the bottom, middle and top nodes, respectively. The input data and results are listed at the end of this section, for an elastic situation.

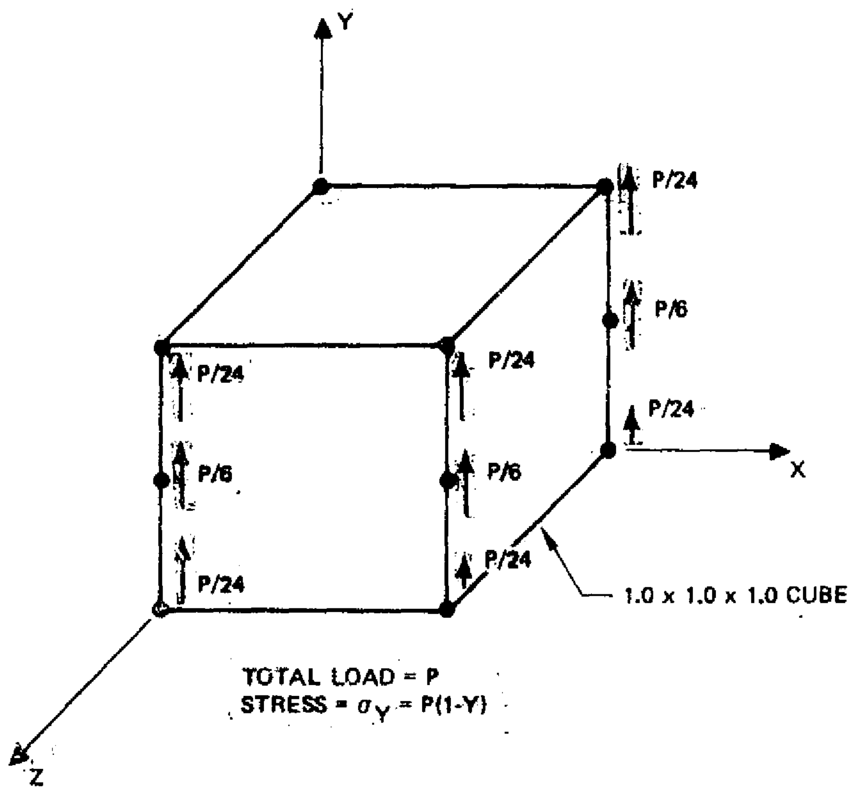

Figure B.1-1: Variable Stress (Distributed Load) Problem

THE BOEING PLASTIC ANALYSIS CAPABILITY FOR ENGINES

DEVELOPED BY

THE BOEING COMPANY

SEATTLE, WASHINGTON

WITH SPONSORSHIP BY

THE NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

GEORGE C. MARSHALL SPACE FLIGHT CENTER

MARSHALL SPACE FLIGHT CENTER, ALABAMA

VERSION 5.2 RELEASE 1/01/77

INPUT DATA

CARD NUMBER	
1	TITLE BOPACE VARIABLE STRESS (DISTRIBUTED SHEAR LOAD) PROBLEM
2	PRT1 1,-1
3	PRT2 1,-1 3,-1
4	VTITLE 3-D ELEMENT WITH MIDSIDE NODES
5	MAT1 1
6	IMODULUS 1.,1.
7	IPOISSON 1.,0.
8	NODE 1000 -.5,-.5,.5
9	NODE 10 .5,-.5,.5
10	NODE 20 -.5,0,.5
11	NODE 30 .5,0,.5
12	NODE 40 -.5,.5,.5
13	NODE 50 .5,.5,.5
14	NODE 2000 -.5,-.5,-.5
15	NODE 1010 .5,-.5,-.5
16	NODE 1020 -.5,0,-.5
17	NODE 1030 .5,0,-.5
18	NODE 1040 -.5,.5,-.5
19	NODE 1050 .5,.5,-.5
20	PBRICK 1
21	RBRICK 1 1,1 23
22	BRICK 1000 1,1,1 1000,10,50,40,2000,1010,1050,1040
23	CONT 1 0,30,0,20,0,1030,0,1020,0,0,0,0
24	SPC 1000,2 10,2 2000,2 1010,2 2000,1 2000,3 1010,3
25	ITITLE UNIFORM SIDE SHEAR LOADING
26	LFACT 0.,1. 0,0 0,0 0.,0.,0.
27	CLOAD 2 20,2,.166667 30,2,.166667 1020,2,.166667 1030,2,.166667
28	CONT 40,2,.041667 50,2,.041667 1040,2,.041667 1050,2,.041667
29	TITLE BOPACE DISTRIBUTED LOADING CHECKOUT (2 INCREMENTS) 10/28/76

```
NUMBER OF DEGREES OF FREEDOM PER NODE = 3

BOPACE WILL ASSUME ONLY MATERIAL NON-LINEARITY TO SOLVE THE PROBLEM

MAXIMUM SPECIFIED ERROR NORM =  1.00000E-03
SOLUTION METHOD CODE =        4
MAXIMUM NO. STIFFNESS UPDATES PER INCREMENT =          1
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE ONE          =  10
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE TWO          =  10
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE THREE AND UP =  10
MAXIMUM ELASTIC ITERATIONS PER INCREMENT =        2
MAXIMUM MAGNITUDE FOR ELASTIC-PLASTIC SUM CODE =          2
MAXIMUM REDUCTIONS =          1
CONVERGENCE REDUCTION FACTOR =  5.00000E-01
FRACTION FROM END OF INCREMENT TO EVALUATE SLOPE = 1.00000E-01
```

MATERIAL NO. 1, MASS DENSITY = 0.0
TEMPERATURE DEPENDENT PROPERTIES

TEMPERATURE	ELASTIC MOD.
1.0000E 00	1.0000E 00

TEMPERATURE	POISSONS RATIO
1.0000E 00	0.0

TITLE BOPACE VARIABLE STRESS (DISTRIBUTED SHEAR LOAD) PROBLEM PAGE 3

VTITLE 3-D ELEMENT WITH MIDSIDE NODES VARIABLE STRUCTURE NUMBER = 1

** NODE ** NO.	I.D.	X1	X2	X3	COORD. LOCATE	COORD. DISPLACE
1	1000	-5.000000D-01	-5.000000D-01	5.000000D-01	1	1
2	10	5.000000D-01	-5.000000D-01	5.000000D-01	1	1
3	20	-5.000000D-01	0.0	5.000000D-01	1	1
4	30	5.000000D-01	0.0	5.000000D-01	1	1
5	40	-5.000000D-01	5.000000D-01	5.000000D-01	1	1
6	50	5.000000D-01	5.000000D-01	5.000000D-01	1	1
7	2000	-5.000000D-01	-5.000000D-01	-5.000000D-01	1	1
8	1010	5.000000D-01	-5.000000D-01	-5.000000D-01	1	1
9	1020	-5.000000D-01	0.0	-5.000000D-01	1	1
10	1030	5.000000D-01	0.0	-5.000000D-01	1	1
11	1040	-5.000000D-01	5.000000D-01	-5.000000D-01	1	1
12	1050	5.000000D-01	5.000000D-01	-5.000000D-01	1	1

```
TITLE     BDPACE VARIABLE STRESS (DISTRIBUTED SHEAR LOAD) PROBLEM                                  PAGE        4
VTITLE .  3-D ELEMENT WITH MIDSIDE NODES                                      VARIABLE STRUCTURE NUMBER =  1

 ELEMENT         ************ CORNER NODES ************     VOLUME    MAP
NO.  I.D. MATL  N1    N2   N3   N4   N5   N6   N7   N8  (ST. LINE) CODE  **************** INTERMEDIATE EDGE NODES ****************
  1 1000   1  1030   10   50   40 2000 1010 1050 1040  1.0000E 00  0      0   30    0   20    0 1030    0 1020    0    0    0    0

SUM OF ELEMENT VOLUMES =     1.0000E 00

BEGIN   GFORMS       CPU = 00:00:00.988      TOD = 22:37:14
```

TITLE BOPACE VARIABLE STRESS (DISTRIBUTED SHEAR LOAD) PROBLEM PAGE 5
VTITLE 3-D ELEMENT WITH MIDSIDE NODES VARIABLE STRUCTURE NUMBER = 1

ELEMENT NO.	I.D.	REFERENCE POINT NO.	TYPE	X1	X2	X3	COORD. LOCATE	COORD. DISPLACE	INTEGRATION SCHEME CODES
1	1000	13	2	-5.000E-01	-5.000E-01	5.000E-01	1	1	0 0 0 0
		14	2	5.000E-01	-5.000E-01	5.000E-01			
		15	2	5.000E-01	5.000E-01	5.000E-01			
		16	2	-5.000E-01	5.000E-01	5.000E-01			
		17	2	-5.000E-01	-5.000E-01	-5.000E-01			
		18	2	5.000E-01	-5.000E-01	-5.000E-01			
		19	2	5.000E-01	5.000E-01	-5.000E-01			
		20	2	-5.000E-01	5.000E-01	-5.000E-01			
		21	3	0.0	0.0	5.000E-01			
		22	3	0.0	0.0	-5.000E-01			
		23	3	0.0	-5.000E-01	0.0			
		24	3	5.000E-01	0.0	0.0			
		25	3	0.0	5.000E-01	0.0			
		26	3	-5.000E-01	0.0	0.0			

END GFORMS CPU = 00:00:01.397 TOD = 22:37:18

BEGIN MERGE CPU = 00:00:01.441 TOD = 22:37:18

BEGIN GENR8 CPU = 00:00:01.447 TOD = 22:37:18

STIFFNESS GENERATION COMPLETED. 78 PARTITIONS WRITTEN.

END GENR8 CPU = 00:00:01.946 TOD = 22:37:21

BEGIN MERSOR CPU = 00:00:01.956 TOD = 22:37:21

END MERSOR CPU = 00:00:02.073 TOD = 22:37:22

END MERGE CPU = 00:00:02.076 TOD = 22:37:22

MAXIMUM WAVEFRONT = 12 NODES AT INTERNAL NODE 1

BEGIN DECOMP CPU = 00:00:02.159 TOD = 22:37:23

END DECOMP CPU = 00:00:02.269 TOD = 22:37:24

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TITLE	BOPACE VARIABLE STRESS (DISTRIBUTED SHEAR LOAD) PROBLEM	PAGE	6
VTITLE	3-D ELEMENT WITH MIDSIDE NODES	VARIABLE STRUCTURE NUMBER =	1
ITITLE	UNIFORM SIDE SHEAR LOADING	INCREMENT NUMBER =	1

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD SET ONE	=	0.0
COEFFICIENT FOR CONCENTRATED LOAD SET TWO	=	1.000000E 00
COEFFICIENT FOR DISTRIBUTED LOAD SET ONE	=	0.0
COEFFICIENT FOR DISTRIBUTED LOAD SET TWO	=	0.0
COEFFICIENT FOR NODAL TEMPERATURE SET	=	0.0
COEFFICIENT FOR NORMAL STRESS/STRAIN SET	=	0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)	=	0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)	=	0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)	=	0.0
CREEP TIME	=	0.0

CONCENTRATED NODAL LOAD SETS

SET NO.	NODE I.D.	COMPONENT	LOAD
2	20	2	1.66667E-01
2	30	2	1.66667E-01
2	40	2	4.16670E-02
2	50	2	4.16670E-02
2	1020	2	1.66667E-01
2	1030	2	1.66667E-01
2	1040	2	4.16670E-02
2	1050	2	4.16670E-02

BEGIN LOADS CPU = 00:00:02.696 TOD = 22:37:29

END LOADS CPU = 00:00:02.858 TOD = 22:37:31

B.1-10

```
BEGIN   SOLN      CPU = 00:00:02.938     TOD = 22:37:32

END     SOLN      CPU = 00:00:03.038     TOD = 22:37:33

BEGIN   ELLOOP    CPU = 00:00:03.041     TOD = 22:37:33

END     ELLOOP    CPU = 00:00:03.268     TOD = 22:37:35

RESIDUAL NORM =  1.13330E-06
```

END OF LOAD INCREMENT 1

```
NO. ELASTIC INTEGRATION POINTS =   12,  NO. PLASTIC INTEGRATION POINTS =    0
    0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,    0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     1, NO. UPDATES PERFORMED =      0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =     1
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-03, ACTUAL ERROR =  1.1333E-06

BEGIN   OUTPUT    CPU = 00:00:03.391     TOD = 22:37:37
```

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **		******** FORCES ********			******** DISPLACEMENTS ********		
NO.	I.D.	U	V	W	U	V	W
1	1000	-1.2752156E-14	-2.0833337E-01	7.6915403E-15	1.3400427E-07	0.0	6.7034426E-07
2	10	-1.4588541E-14	-2.0833361E-01	1.1636039E-14	1.7343325E-07	0.0	1.3408550E-07
3	20	1.9665333E-14	1.6666663E-01	-4.8768879E-10	4.6143857E-08	3.7499982E-01	1.1531602E-06
4	30	2.7544012E-10	1.6666681E-01	-9.8125099E-09	4.4395104E-08	3.7500060E-01	5.9176642E-07
5	40	-7.7488025E-09	4.1666824E-02	1.2419836E-08	-4.7653352E-07	4.9999952E-01	2.2224203E-06
6	50	-1.7587755E-09	4.1666914E-02	2.3749401E-08	-5.1778090E-07	5.0000119E-01	1.7963257E-06
7	2000	2.7636748E-08	-2.0833361E-01	-1.3213321E-07	0.0	0.0	0.0
8	1010	1.0648737E-08	-2.0833397E-01	-6.4988797E-09	-1.2501303E-07	0.0	0.0
9	1020	1.2509815E-08	1.6666657E-01	-2.2563430E-08	-5.0928014E-07	3.7500137E-01	1.1251723E-06
10	1030	1.0344443E-08	1.6666687E-01	-3.1512407E-08	-5.2012763E-07	3.7500233E-01	5.5806936E-07
11	1040	-1.3897925E-08	4.1666910E-02	1.6783065E-07	-1.2455821E-06	5.0000232E-01	3.1742911E-06
12	1050	-2.2180721E-08	4.1666955E-02	5.7503263E-08	-1.2924138E-06	5.0000346E-01	1.5851947E-06

ELEMENT NO.	ELEMENT I.D.	POINT NO.	POINT TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	1000	13	2	1.0000E 00	4.4474E-08	1.0000E 00	6.7034E-07	1.2954E-07	-2.0112E-07	1.8959E-07
		14	2	1.0000E 00	4.4474E-08	1.0000E 00	1.3409E-07	7.9956E-08	-1.1488E-07	8.4242E-08
		15	2	1.4037E-06	-4.1242E-08	1.1921E-06	2.1113E-07	5.8245E-08	1.7427E-07	4.4551E-07
		16	2	8.8830E-07	-4.1242E-08	-7.1526E-07	-9.5187E-07	9.4374E-08	1.7147E-07	-3.8227E-08
		17	2	1.0000E 00	-1.2801E-07	1.0000E 00	6.7034E-07	-3.9577E-07	6.7005E-08	6.6320E-07
		18	2	1.0000E 00	-1.2801E-07	1.0000E 00	1.3409E-07	-2.0203E-07	1.5325E-07	3.2354E-07
		19	2	1.2784E-06	-4.6832E-08	8.3447E-07	2.1113E-07	-3.9613E-07	-4.0723E-07	1.2917E-07
		20	2	3.0028E-06	-4.6832E-08	1.4901E-06	-9.5187E-07	-2.8357E-07	-4.1003E-07	1.1104E-06
		21	3	5.0000E-01	-1.7488E-09	5.0000E-01	3.0842E-08	6.0727E-08	-7.1018E-10	-3.0886E-08
		22	3	5.0000E-01	-1.0847E-08	5.0000E-01	3.0842E-08	-9.5856E-08	-3.5648E-09	3.5541E-07
		23	3	1.0000E 00	-4.1770E-08	1.0000E 00	4.0221E-07	-9.7075E-08	-2.3938E-08	3.1514E-07
		24	3	5.0000E-01	-6.2981E-09	5.0000E-01	3.3697E-08	-1.8131E-08	1.3720E-10	-8.2211E-08

ELEMENT NO.	ELEMENT I.D.	POINT NO.	POINT TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1000	13	2	4.4474E-08	1.0000E 00	6.7034E-07	1.2954E-07	-2.0112E-07	1.8959E-07
		14	2	4.4474E-08	1.0000E 00	1.3409E-07	7.9956E-08	-1.1488E-07	8.4242E-08
		15	2	-4.1242E-08	1.1921E-06	2.1113E-07	5.8245E-08	1.7427E-07	4.4551E-07
		16	2	-4.1242E-08	-7.1526E-07	-9.5187E-07	9.4374E-08	1.7147E-07	-3.8227E-08
		17	2	-1.2801E-07	1.0000E 00	6.7034E-07	-3.9577E-07	6.7005E-08	6.6320E-07
		18	2	-1.2801E-07	1.0000E 00	1.3409E-07	-2.0203E-07	1.5325E-07	3.2354E-07
		19	2	-4.6832E-08	8.3447E-07	2.1113E-07	-3.9613E-07	-4.0723E-07	1.2917E-07
		20	2	-4.6832E-08	1.4901E-06	-9.5187E-07	-2.8357E-07	-4.1003E-07	1.1104E-06
		21	3	-1.7488E-09	5.0000E-01	3.0842E-08	6.0727E-08	-7.1018E-10	-3.0886E-08
		22	3	-1.0847E-08	5.0000E-01	3.0842E-08	-9.5856E-08	-3.5648E-09	3.5541E-07
		23	3	-4.1770E-08	1.0000E 00	4.0221E-07	-9.7075E-08	-2.3938E-08	3.1514E-07
		24	3	-6.2981E-09	5.0000E-01	3.3697E-08	-1.8131E-08	1.3720E-10	-8.2211E-08

ELEMENT NO.	ELEMENT I.D.	POINT NO.	POINT TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	1000	25	3	1.2271E-06	-4.4037E-08	7.0035E-07	-3.7037E-07	-1.3177E-07	-1.1788E-07	4.1171E-07
		26	3	5.0000E-01	-6.2981E-09	5.0000E-01	2.7988E-08	-1.6998E-08	-4.4122E-09	4.0673E-07

ELEMENT NO.	ELEMENT I.D.	POINT NO.	POINT TP.	CUMULATIVE ELASTIC STRAINS X	YY	ZZ	XY	XZ	YZ
1	1000	25	3	037E-08	7.0035E-07	-3.7037E-07	-1.3177E-07	-1.1788E-07	4.1171E-07
		26	3	981E-09	5.0000E-01	2.7988E-08	-1.6998E-08	-4.4122E-09	4.0673E-07

END OUTPUT CPU = 00:00:03.607 TOD = 22:37:39

END OF BOPACE PROBLEM

B. 2 MULTI-ELEMENT CURVED BOUNDARY PROBLEM

A plane-strain problem is analyzed using a 1.0 x 1.0 square, but idealized by four elements including curved interior boundaries. The loading is a uniform distributed vertical load at the top of the structure. Because of the state of constant stress and strain throughout the cube, all reference points in all elements have equal values for stress and strain. The input data listing and results are included at the end of this section.

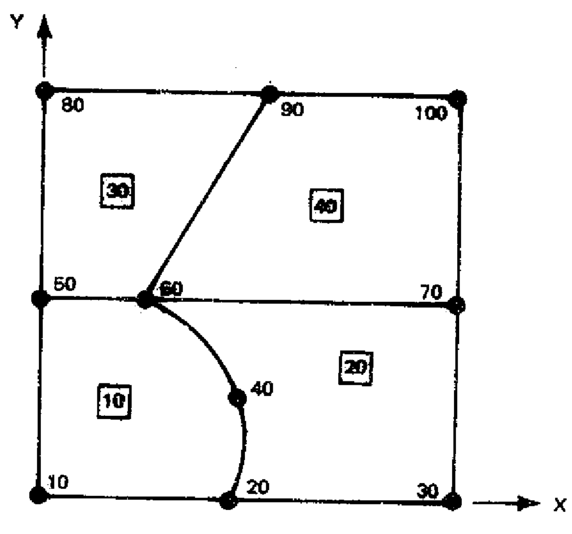

Figure B.2-1: Multi-Element Curved Boundary Problem

INPUT DATA

```
CARD
NUMBER
   1      TITLE  MULTI-ELEMENT CURVED BOUNDARY PROBLEM
   2      CONT  QUAD ELEMENTS
   3      PROB 2
   4      PRT1  1,-1
   5      PRT2  1,-1  3,-1
   6      VTITLE
   7      MATI  1
   8      IMOD  1,1
   9      IPGIS  0,.3
  10      NODE  10  0,0,,,, 12
  11      NODE  20  .5,0,,,, 2
  12      NODE  30  1,0,,,, 2
  13      NODE  40   .5,.25
  14      NODE  50  0,.5
  15      NODE  60  .25,.5
  16      NODE  70  1,.5
  17      NODE  80  0,1
  18      NODE  90  .5,1
  19      NODE  100  1,1
  20      POUAD  1  1.0
  21      ROUAD  1  1,1   2
  22      QUAD  10   1,1,1   10,20,60,50  1  0,40
  23      QUAD  20   1,1,1   20,30,70,60  1  0,0,0,40
  24      QUAD  30   1,1,1   50,60,90,80
  25      QUAD  40   1,1,1   60,70,100,90
  26      ITITLE
  27      LFACT  0,0   1,0
  28      DILOAD  1   1,1,2   3,1.0    30,40
  29      TITLE  BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
```

```
TITLE      MULTI-ELEMENT CURVED BOUNDARY PROBLEM                                   PAGE     1
CONTINUE   QUAD ELEMENTS

NUMBER OF DEGREES OF FREEDOM PER NODE = 2

BOPACE WILL ASSUME ONLY MATERIAL NON-LINEARITY TO SOLVE THE PROBLEM

MAXIMUM SPECIFIED ERROR NORM =  1.00000E-03
SOLUTION METHOD CODE =         4
MAXIMUM NO. STIFFNESS UPDATES PER INCREMENT =          1
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE ONE              =  10
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE TWO              =  10
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE THREE AND UP =  10
MAXIMUM PLASTIC ITERATIONS PER INCREMENT =          2
MAXIMUM MAGNITUDE FOR ELASTIC-PLASTIC SUM CODE =          2
MAXIMUM REDUCTIONS =          1
CONVERGENCE REDUCTION FACTOR =  5.00000E-01
FRACTION FROM END OF INCREMENT TO EVALUATE SLOPE = 1.00000E-01
```

TITLE MULTI-ELEMENT CURVED BOUNDARY PROBLEM
VTITLE

MATERIAL NO. 1, MASS DENSITY = 0.0
TEMPERATURE DEPENDENT PROPERTIES

TEMPERATURE	ELASTIC MOD.
1.0000E 00	1.0000E 00

TEMPERATURE	POISSONS RATIO
0.0	3.0000E-01

TITLE MULTI-ELEMENT CURVED BOUNDARY PROBLEM

VTITLE

VARIABLE STRUCTURE NUMBER = 1

** NODE ** NO.	I.D.	X1	X2	X3	COORD. LOCATE	COORD. DISPLACE
1	10	0.0	0.0	0.0	1	1
2	20	5.000000-01	0.0	0.0	1	1
3	30	1.000000 00	0.0	0.0	1	1
4	40	5.000000-01	2.500000-01	0.0	1	1
5	50	0.0	5.000000-01	0.0	1	1
6	60	2.500000-01	5.000000-01	0.0	1	1
7	70	1.000000 00	5.000000-01	0.0	1	1
8	80	0.0	1.000000 00	0.0	1	1
9	90	5.000000-01	1.000000 00	0.0	1	1
10	100	1.000000 00	1.000000 00	0.0	1	1

TITLE MULTI-ELEMENT CURVED BOUNDARY PROBLEM

VTITLE

PAGE 4

VARIABLE STRUCTURE NUMBER = 1

ELEMENT NO.	I.D.	MATL	N1	N2	N3	N4	N5	N6	N7	N8	VOLUME (ST. LINE)	MAP CODE	INTERMEDIATE EDGE NODES			
1	10	1	10	20	60	50					1.8750E-01	0	0	40	0	0
2	20	1	20	30	70	60					3.1250E-01	0	0	0	0	40
3	30	1	50	60	90	80					1.8750E-01	0				
4	40	1	60	70	100	90					3.1750E-01	0				

SUM OF ELEMENT VOLUMES = 1.0000E 00

BEGIN GFORMS CPU = 00:00:00.768 TOD = 21:39:23

```
TITLE     MULTI-ELEMENT CURVED BOUNDARY PROBLEM                                                PAGE          5
VTITLE                                                                          VARIABLE STRUCTURE NUMBER =  1

  ELEMENT    REFERENCE POINT                                                COORD.  COORD.    INTEGRATION
 NO.  I.D.       NO.   TYPE         X1           X2           X3            LOCATE DISPLACE   SCHEME CODES
  1    10         7     2        0.0          0.0          0.0                1        1          0 0 0
                  8     2        5.000E-01    0.0          0.0
                  9     2        2.500E-01    5.000E-01    0.0
                 10     2        0.0          5.000E-01    0.0
  2    20         7     2        5.000E-01    0.0          0.0                1        1          0 0 0
                  8     2        1.000E 00    0.0          0.0
                  9     2        1.000E 00    5.000E-01    0.0
                 10     2        2.500E-01    5.000E-01    0.0
  3    30         5     2        0.0          5.000E-01    0.0                1        1          0 0 0
                  6     2        2.500E-01    5.000E-01    0.0
                  7     2        5.000E-01    1.000E 00    0.0
                  8     2        0.0          1.000E 00    0.0
  4    40         5     2        2.500E-01    5.000E-01    0.0                1        1          0 0 0
                  6     2        1.000E 00    5.000E-01    0.0
                  7     2        1.000E 00    1.000E 00    0.0
                  8     2        5.000E-01    1.000E 00    0.0

END      GFORMS       CPU = 00:00:01.064      TOD = 21:39:38

BEGIN    MERGE        CPU = 00:00:01.091      TOD = 21:39:38

BEGIN    GENR8        CPU = 00:00:01.094      TOD = 21:39:39

STIFFNESS GENERATION COMPLETED.         50 PARTITIONS WRITTEN.

END      GENR8        CPU = 00:00:01.218      TOD = 21:39:40

BEGIN    MERSOR       CPU = 00:00:01.218      TOD = 21:39:40

END      MERSOR       CPU = 00:00:01.274      TOD = 21:39:42

END      MERGE        CPU = 00:00:01.274      TOD = 21:39:42

MAXIMUM WAVEFRONT =      6 NODES AT INTERNAL NODE      2

BEGIN    DECOMP       CPU = 00:00:01.321      TOD = 21:39:43

END      DECOMP       CPU = 00:00:01.394      TOD = 21:39:43
```

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD SET ONE	=	0.0
COEFFICIENT FOR CONCENTRATED LOAD SET TWO	=	0.0
COEFFICIENT FOR DISTRIBUTED LOAD SET ONE	=	1.000000E 00
COEFFICIENT FOR DISTRIBUTED LOAD SET TWO	=	0.0
COEFFICIENT FOR NODAL TEMPERATURE SET	=	0.0
COEFFICIENT FOR NORMAL STRESS/STRAIN SET	=	0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)	=	0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)	=	0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)	=	0.0
CREEP TIME	=	0.0

DISTRIBUTED LOAD SETS

SET	DIM	ELEMENT	COORD. SYSTEM	EDGE OR FACE	LOAD TYPE	NODE, EDGE OR FACE	LOAD INTENSITY COMPONENTS X1	X2	X3
1	1	30	1	3	UNIF	3	0.0	1.0000E 00	0.0
1	1	40	1	3	UNIF	3	0.0	1.0000E 00	0.0

```
BEGIN   GFORMS     CPU = 00:00:01.647     TOD = 21:39:55
END     GFORMS     CPU = 00:00:01.757     TOD = 21:40:01
BEGIN   LOADS      CPU = 00:00:01.843     TOD = 21:40:03
END     LOADS      CPU = 00:00:01.960     TOD = 21:40:06
```

```
BEGIN   SOLN        CPU = 00:00:02.003     TOD = 21:40:06
END     SOLN        CPU = 00:00:02.055     TOD = 21:40:07
BEGIN   ELLOOP      CPU = 00:00:02.060     TOD = 21:40:07
END     ELLOOP      CPU = 00:00:02.193     TOD = 21:40:09
RESIDUAL NORM =  7.53703E-07

                 E N D   O F   L O A D   I N C R E M E N T    1

NO. ELASTIC INTEGRATION POINTS =   20,  NO. PLASTIC INTEGRATION POINTS =     0
      0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,     0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =      1, NO. UPDATES PERFORMED =      0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      1
SPECIFIED MAX. UNBALANCED FORCE ERROR =  1.0000E-03, ACTUAL ERROR =  7.5370E-07

BEGIN   OUTPUT      CPU = 00:00:02.266     TOD = 21:40:13
```

TITLE MULTI-ELEMENT CURVED BOUNDARY PROBLEM
VTITLE
ITITLE

VARIABLE STRUCTURE NUMBER = 1
INCREMENT NUMBER = 1

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **		**************** FORCES ****************			************* DISPLACEMENTS *************		
NC.	I.D.	U	V	W	U	V	W
1	10	3.7920138E-08	-2.4999982E-01		0.0	0.0	
2	20	4.7250701E-08	-4.9999964E-01		-1.4999998E-01	0.0	
3	30	3.8153800E-09	-2.4999988E-01		-2.9999995E-01	0.0	
4	40	2.1408486E-08	-2.1199315E-07		-1.5000004E-01	2.4999994E-01	
5	50	-3.6902950E-08	-1.2071166E-07		-2.0231579E-07	4.9999988E-01	
6	60	-4.0021582E-08	-3.0910621E-07		-7.5000167E-02	4.9999994E-01	
7	70	4.7180624E-08	-2.3073699E-07		-3.0000007E-01	5.0000000E-01	
8	80	-1.5011032E-08	2.4999994E-01		-2.8212310E-07	1.0000000E 00	
9	90	-5.8213009E-08	4.9999976E-01		-1.5000027E-01	1.0000000E 00	
10	100	-7.4266615E-09	2.4999994E-01		-3.0000025E-01	1.0000000E 00	

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	XX	YY	ZZ	XY	XZ	YZ
					CUMULATIVE STRESSES					
1	10	7	2	1.0000E 00	-1.3100E-07	1.0000E 00	0.0	-1.5563E-07	0.0	0.0
		8	2	1.0000E 00	-6.5500E-08	1.0000E 00	0.0	-7.0123E-08	0.0	0.0
		9	2	1.0000E 00	1.3100E-07	1.0000E 00	0.0	-1.2695E-07	0.0	0.0
		10	2	1.0000E 00	0.0	1.0000E 00	0.0	-6.3928E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	XX	YY	ZZ	XY	XZ	YZ
				CUMULATIVE ELASTIC STRAINS					
1	10	7	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-2.0232E-07	0.0	0.0
		8	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-9.1160E-08	0.0	0.0
		9	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-1.6504E-07	0.0	0.0
		10	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-8.3106E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	XX	YY	ZZ	XY	XZ	YZ
					CUMULATIVE STRESSES					
2	20	7	2	1.0000E 00	-6.5500E-08	1.0000E 00	0.0	-7.0123E-08	0.0	0.0
		8	2	1.0000E 00	0.0	1.0000E 00	0.0	-9.1699E-08	0.0	0.0
		9	2	1.0000E 00	6.5500E-08	1.0000E 00	0.0	-6.1133E-08	0.0	0.0
		10	2	1.0000E 00	1.3100E-07	1.0000E 00	0.0	-2.0087E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	XX	YY	ZZ	XY	XZ	YZ
				CUMULATIVE ELASTIC STRAINS					
2	20	7	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-9.1160E-08	0.0	0.0
		8	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-1.1921E-07	0.0	0.0
		9	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-7.9473E-08	0.0	0.0
		10	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-2.6113E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	XX	YY	ZZ	XY	XZ	YZ
					CUMULATIVE STRESSES					
3	30	5	2	1.0000E 00	1.3100E-07	1.0000E 00	0.0	3.0309E-08	0.0	0.0
		6	2	1.0000E 00	1.3100E-07	1.0000E 00	0.0	-1.8078E-08	0.0	0.0
		7	2	1.0000E 00	0.0	1.0000E 00	0.0	-8.5584E-08	0.0	0.0
		8	2	1.0000E 00	0.0	1.0000E 00	0.0	-6.1590E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	XX	YY	ZZ	XY	XZ	YZ
				CUMULATIVE ELASTIC STRAINS					
3	30	5	2	-3.0000E-01	1.0000E 00	-3.0000E-01	3.9402E-08	0.0	0.0
		6	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-2.3502E-08	0.0	0.0
		7	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-1.1126E-07	0.0	0.0
		8	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-7.9807E-08	0.0	0.0

CUMULATIVE STRESSES

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	XX	YY	ZZ	XY	XZ	YZ
4	40	5	2	1.0000E 00	1.3100E-07	1.0000E 00	0.0	-7.6416E-06	0.0	0.0
		6	2	1.0000E 00	6.5500E-08	1.0000E 00	0.0	-1.0698E-07	0.0	0.0
		7	2	1.0000E 00	0.0	1.0000E 00	0.0	-1.3755E-07	0.0	0.0
		8	2	1.0000E 00	0.0	1.0000E 00	0.0	-9.1699E-08	0.0	0.0

CUMULATIVE ELASTIC STRAINS

ELEMENT NO.	I.D.	POINT NO.	TP.	XX	YY	ZZ	XY	XZ	YZ
4	40	5	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-9.9341E-08	0.0	0.0
		6	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-1.3908E-07	0.0	0.0
		7	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-1.7881E-07	0.0	0.0
		8	2	-3.0000E-01	1.0000E 00	-3.0000E-01	-1.1921E-07	0.0	0.0

END OUTPUT CPU = 00:00:02.419 TOD = 21:40:15

B.3 THERMAL RATCHET

This is a thermal ratchet problem, involving thermal cycling in conjunction with a sustained mechanical load. The finite-element idealization and mechanical loading are shown in Figure 3.3-1. The thermal loading consists of an alternate heating and cooling of the left half of the structure (element 1).

Because the stresses and thermal strains differ in the left and right halves of the structure, the BOPACE MPC capability was used to allow vertical slip at the center. Thus the displacements at nodes 3-11 and 4-12 are constrained to be equal in the X direction, but are allowed to have different values in the Y direction. Poisson's ratio is taken as 0.5 so as to avoid small errors which would otherwise be induced by intermediate yielding within an increment.

Results are summarized in Table B.3-1 for six increments, and the BOPACE input listing and printed output results are included at the end of this section (some of the output pages have been combined to save space). The mechanical loading is applied during the first increment and it then remains on the structure. In the second increment the thermal heating load is applied, and it results in plastic flow within the right side of the structure. Each succeeding heating and cooling cycle (two increments) results in continuing plastic flow and an increase of 0.5 in displacement. Note that this occurs even though a part of the structure is always elastic, because yielding occurs during alternate increments in the left and right sides. This type of behavior must be considered in thermal-mechanical cycling of engines.

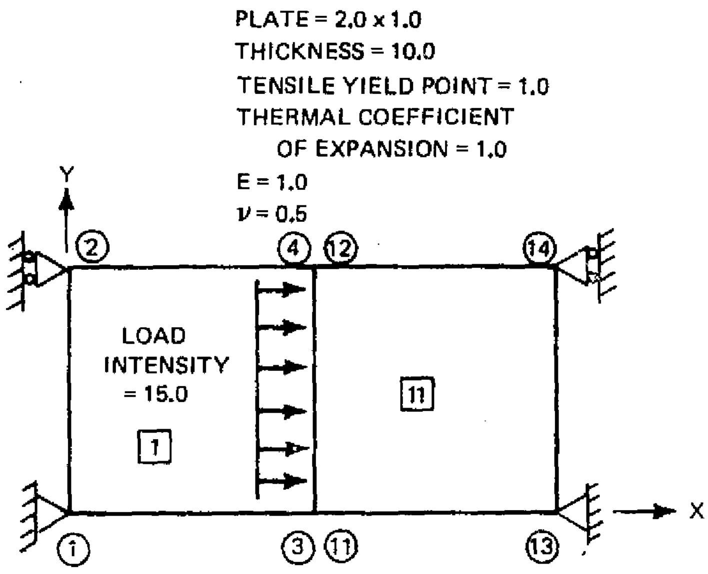

Figure B.3-1: Thermal Ratchet Problem

Table B.3-1: Thermal Ratchet Data

INCREMENT	DISPLACEMENT	TEMP. 1	TEMP. 11
1	0.75	0	0
2	2.0	1.5	0
3	1.5	0	0
4	2.5	1.5	0
5	2.0	0	0
6	3.0	1.5	0

INPUT DATA

```
CARD
NUMBER
   1        TITLE  THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)
   2        PROB 2
   3        $SOLU  ERRMAX  SCODE  MAXUP  MAXIT1 MAXIT2 MAXIT3 MAXIE MAXYC MAXCUT  CUT  AFACT
   4        SOLU   .00001    3     2       ,,     ,      ,      ,     ,     1
   5        PRT1  1,-1
   6        PRT2  1,-1  3,-1  5,-1  6,-1  9,-1  10,-1  11,-1
   7        VTITLE
   8        MAT1  1
   9        IMOD  0,1.
  10        IPOIS  0,.5
  11        ISTRAIN  0,0  10.,10.
  12        PLASTIC  1  2
  13        PTEMP  0.
  14        IHARD  0,1.  100,1.
  15        KSHAPE  0,0  100,0
  16        NODE 1  0,0
  17        NODE 2  0,1
  18        NODE 3  1,0
  19        NODE 4  1,1
  20        NODE 11  1,0
  21        NODE 12  1,1
  22        NODE 13  2,0
  23        NODE 14  2,1
  24        $PQUAD  PID  THICK  NSCODE  FTEMP  MCODE
  25        PQUAD    1    10.     1        0.
  26        $RQUAD  RID  LID  DID  RPCODE  ICODEG  ICODE1 ICODE2  GP---
  27        RQUAD    1    1    1     4        1       2      2
  28        QUAD 1  1,1,1  1,3,4,2
  29        QUAD 11  1,1,1  11,13,14,12
  30        MPC  11,1  3,1,1.
  31        MPC  12,1  4,1,1.
  32        SPC  1,1  1,2  2,1  13,1  13,2  14,1
  33        ITITLE INCREMENT 1 (COLD)
  34        LFACT  0,0   15.0,0   1
  35        $DLOAD  DLSID    DIM  CID  C  EID    ID  D
  36        DLOAD     1        1    1    1    1    2  1.0
  37        ITITLE INCREMENT 2 (HOT)
  38        LFACT  0,0   15.0,0   1
  39        TILOAD  -1   1.5   1,2,3,4
  40        ITITLE INCREMENT 3 (COLD)
  41        LFACT  0,0   15.0,0   1
  42        TILOAD  -1   0.0   1,2,3,4
  43        ITITLE INCREMENT 4 (HOT)
  44        LFACT  0,0   15.0,0   1
  45        TILOAD  -1   1.5   1,2,3,4
  46        ITITLE INCREMENT 5 (COLD)
  47        LFACT  0,0   15.0,0   1
  48        TILOAD  -1   0.0   1,2,3,4
  49        ITITLE INCREMENT 6 (HOT)
  50        LFACT  0,0   15.0,0   1
  51        TILOAD  -1   1.5   1,2,3,4
```

```
TITLE      THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                                    PAGE      1

NUMBER OF DEGREES OF FREEDOM PER NODE = 2

BOPACE WILL ASSUME ONLY MATERIAL NON-LINEARITY TO SOLVE THE PROBLEM

MAXIMUM SPECIFIED ERROR NORM =  1.00000E-05
SOLUTION METHOD CODE =         3
MAXIMUM NO. STIFFNESS UPDATES PER INCREMENT =           2
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE ONE            =  10
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE TWO            =  10
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE THREE AND UP =  10
MAXIMUM ELASTIC ITERATIONS PER INCREMENT =          2
MAXIMUM MAGNITUDE FOR ELASTIC-PLASTIC SUM CODE =          2
MAXIMUM REDUCTIONS =           1
CONVERGENCE REDUCTION FACTOR =  5.00000E-01
FRACTION FROM END OF INCREMENT TO EVALUATE SLOPE = 1.00000E-01
```

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                    PAGE        2
VTITLE                                                              VARIABLE STRUCTURE NUMBER =    1

MATERIAL NO.     1, MASS DENSITY =      0.0
TEMPERATURE DEPENDENT PROPERTIES

 TEMPERATURE    ELASTIC MOD.
  0.0             1.0000E 00

 TEMPERATURE    POISSONS RATIO

*WARNING* POISSONS RATIO IS LESS THAN OR EQUAL TO -.99
       OR GREATER THAN OR EQUAL TO .49 ON CARD     10
  0.0             5.0000E-01

 TEMPERATURE    THERMAL STRAIN
  0.0             0.0
  1.0000E 01      1.0000E 01

MATERIAL NO      1,PLASTICITY TYPE     2,KINEMATIC CODE     0

MATERIAL NO.     1, TEMPERATURE =  0.0

PARAMETER      ISOTROPIC HARDENING
 0.0             1.00000E 00
 1.00000E 02     1.00000E 00

PARAMETER      KINEMATIC HARDENING SHAPE
 0.0             0.0
 1.00000E 02     0.0

TEMPERATURE =   0.0

PARAMETER      ISOTROPIC HARDENING
 0.0             1.00000E 00
 1.00000E 02     1.00000E 00

PARAMETER      KINEMATIC HARDENING SHAPE
 0.0             0.0
 1.00000E 02     0.0
```

TITLE THERMAL RATCHET PROBLEM (2 QUAD ELEMENTS) PAGE 3
VTITLE VARIABLE STRUCTURE NUMBER = 1

** NODE ** NO.	I.D.	X1	X2	X3	COORD. LOCATE	COORD. DISPLACE
1	1	0.0	0.0	0.0	1	1
2	2	0.0	1.000000D 00	0.0	1	1
3	3	1.000000D 00	0.0	0.0	1	1
4	4	1.000000D 00	1.000000D 00	0.0	1	1
5	11	1.000000D 00	0.0	0.0	1	1
6	12	1.000000D 00	1.000000D 00	0.0	1	1
7	13	2.000000D 00	0.0	0.0	1	1
8	14	2.000000D 00	1.000000D 00	0.0	1	1

B.3-6

TITLE THERMAL RATCHET PROBLEM (2 QUAD ELEMENTS) PAGE 4
VTITLE VARIABLE STRUCTURE NUMBER = 1

ELEMENT NO.	I.D.	MATL	N1	N2	N3	N4	N5	N6	N7	N8	VOLUME (ST. LINE)	MAP CODE	***************** INTERMEDIATE EDGE NODES *****************
1	1	1	1	3	4	2					1.0000E 01	0	
2	11	1	11	13	14	12					1.0000E 01	0	

(Column headers N1–N8 are grouped under ************ CORNER NODES ************)

SUM OF ELEMENT VOLUMES = 2.0000E 01

BEGIN GFORMS CPU = 00:00:00.901 TOD = 23:37:15

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                          PAGE       5
VTITLE                                                                   VARIABLE STRUCTURE NUMBER =  1

  ELEMENT   REFERENCE POINT                                       COORD.  COORD.   INTEGRATION
 NO.  I.D.      NO.   TYPE      X1          X2         X3       LOCATE DISPLACE  SCHEME CODES
   1    1        5     4    5.000E-01   5.000E-01   0.0           1       1        1 2 2
   2   11        5     4    1.500E 00   5.000E-01   0.0           1       1        1 2 2

END      GFORMS        CPU = 00:00:01.094      TOD = 23:32:18

BEGIN    MERGE         CPU = 00:00:01.114      TOD = 23:32:18

BEGIN    GENRB         CPU = 00:00:01.114      TOD = 23:32:19

STIFFNESS GENERATION COMPLETED.          31 PARTITIONS WRITTEN.

END      GENRB         CPU = 00:00:01.221      TOD = 23:32:20

BEGIN    MERSOR        CPU = 00:00:01.221      TOD = 23:32:20

END      MERSOR        CPU = 00:00:01.277      TOD = 23:32:21

END      MERGE         CPU = 00:00:01.281      TOD = 23:32:21

MAXIMUM WAVEFRONT =    6 NODES AT INTERNAL NODE    3

BEGIN    DECOMP        CPU = 00:00:01.334      TOD = 23:32:21

END      DECOMP        CPU = 00:00:01.387      TOD = 23:32:22
```

```
TITLE      THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                          PAGE         6
VTITLE                                                                     VARIABLE STRUCTURE NUMBER =  1
ITITLE   INCREMENT 1 (COLD)                                                        INCREMENT NUMBER =   1

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE        =    0.0
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO        =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE         =    1.500000E 01
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO         =    0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET             =    1.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET          =    0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)      =    0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                =    0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)       =    0.0
CREEP TIME                                         =    0.0
```

B.3-8

```
TITLE      THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                          PAGE         7
VTITLE                                                                     VARIABLE STRUCTURE NUMBER =  1
ITITLE   INCREMENT 1 (COLD)                                                        INCREMENT NUMBER =   1
                                 DISTRIBUTED LOAD SETS

                      COORD.   EDGE    LOAD  NODE, EDGE        LOAD INTENSITY COMPONENTS
SET  DIM  ELEMENT    SYSTEM   OR FACE  TYPE   OR FACE            X1            X2            X3
 1    1      1          1        2     UNIF      2          1.0000E 00   0.0           0.0

BEGIN   GFORMS       CPU = 00:00:01.597      TOD = 23:32:36

END     GFORMS       CPU = 00:00:01.720      TOD = 23:32:36

BEGIN   LOADS        CPU = 00:00:01.823      TOD = 23:32:37

END     LOADS        CPU = 00:00:01.920      TOD = 23:32:38
```

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                          PAGE       8
VTITLE                                                                      VARIABLE STRUCTURE NUMBER =   1
ITITLE    INCREMENT 1 (COLD)                                                          INCREMENT NUMBER =   1

BEGIN    SOLN          CPU = 00:00:01.966      TOD = 23:32:39

END      SOLN          CPU = 00:00:02.006      TOD = 23:32:40

BEGIN    ELLOOP        CPU = 00:00:02.020      TOD = 23:32:40

END      ELLOOP        CPU = 00:00:02.093      TOD = 23:32:42

RESIDUAL NORM =  3.13764E-07


                          E N D   O F   L O A D   I N C R E M E N T    1

NO. ELASTIC INTEGRATION POINTS =     8,  NO. PLASTIC INTEGRATION POINTS =     0
     0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,      0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =      2, NO. UPDATES PERFORMED =      0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      1
SPECIFIED MAX. UNBALANCED FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  3.1376E-07

BEGIN    OUTPUT        CPU = 00:00:02.189      TOD = 23:32:43


TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                          PAGE       9
VTITLE                                                                      VARIABLE STRUCTURE NUMBER =   1
ITITLE    INCREMENT 1 (COLD)                                                          INCREMENT NUMBER =   1

                CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS
```

** NODE **		*************** FORCES ***************			************ DISPLACEMENTS ************		
NO.	I.D.	U	V	W	U	V	W
1	1	-3.7499981E 00	-2.5771591E-07		0.0	0.0	
2	2	-3.7499996E 00	1.6916607E-07		0.0	-3.7500000E-01	
3	3	3.7499981E 00	-1.3965013E-07		7.4999994E-01	-1.2561518E-07	
4	4	3.7499990E 00	-2.2819802E-07		7.5000030E-01	-3.7500019E-01	
5	11	3.7500000E 00	2.4365681E-06		7.4999994E-01	6.4267036E-07	
6	12	3.7500000E 00	-5.7241073E-07		7.5000030E-01	3.7500054E-01	
7	13	-3.7499981E 00	1.1376887E-06		0.0	0.0	
8	14	-3.7500010E 00	-3.6038664E-06		0.0	3.7499970E-01	

TITLE THERMAL RATCHET PROBLEM (2 QUAD ELEMENTS) PAGE 10
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE INCREMENT 1 (COLD) INCREMENT NUMBER = 1

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	7.5000E-01	7.5000E-01	7.9473E-08	0.0	8.8548E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	7.5000E-01	-3.7500E-01	-3.7500E-01	1.3282E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	0	-1	7.5000E-01	7.5000E-01	-3.7500E-01	-3.7500E-01	1.3282E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
1	1	5	4	0.0	1.0000E 00	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
1	1	5	4	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	7.5000E-01	-7.5000E-01	-3.9736E-07	0.0	-1.8662E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	-7.5000E-01	3.7500E-01	3.7500E-01	-2.7993E-07	0.0	0.0

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                                          PAGE       11
VTITLE                                                                                     VARIABLE STRUCTURE NUMBER =    1
ITITLE    INCREMENT 1 (COLD)                                                                        INCREMENT NUMBER =    1

    ELEMENT     POINT                         CUMULATIVE    ********************  CUMULATIVE PLASTIC STRAINS  ********************
  NO.  I.D.  NO. TP.                          PLASTIC WORK       XX          YY          ZZ          XY          XZ          YZ
   2    11     5  4                              0.0            0.0         0.0         0.0         0.0         0.0         0.0

    ELEMENT     POINT                         INCREMENTAL   ********************  INCREMENTAL PLASTIC STRAINS  ********************
  NO.  I.D.  NO. TP.                          PLASTIC WORK       XX          YY          ZZ          XY          XZ          YZ
   2    11     5  4                              0.0            0.0         0.0         0.0         0.0         0.0         0.0

    ELEMENT     POINT              E-P   SUM   CUM. EFF.    ********************  CUMULATIVE TOTAL STRAINS  ********************
  NO.  I.D.  NO. TP.              CODE  CODE  TOTAL STRAIN       XX          YY          ZZ          XY          XZ          YZ
   2    11     5  4                 0    -1    7.5000E-01  -7.5000E-01  3.7500E-01  3.7500E-01 -2.7993E-07  0.0         0.0

    ELEMENT     POINT             YIELD         YIELD      **** EFFECTIVE PLASTIC STRAINS ****   ***** EFFECTIVE CREEP STRAINS *****
  NO.  I.D.  NO. TP.          STRESS CTR.  STRESS SIZE   INCREMENTAL  SUM INCR.   CUMULATIVE  INCREMENTAL  SUM INCR.   CUMULATIVE
   2    11     5  4               0.0       1.0000E 00      0.0          0.0         0.0         0.0          0.0         0.0

    ELEMENT     POINT          CUMULATIVE               **** CUMULATIVE THERMAL STRAINS ****
  NO.  I.D.  NO. TP.          TEMPERATURE                    XX          YY          ZZ
   2    11     5  4               0.0                        0.0         0.0         0.0

 END      OUTPUT        CPU = 00:00:02.339      TOD = 23:32:44
```

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                        PAGE      12
VTITLE                                                                 VARIABLE STRUCTURE NUMBER =    1
ITITLE    INCREMENT 2 (HOT)                                                   INCREMENT NUMBER =    2

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE          =    0.0
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO          =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE           =    1.500000E 01
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO           =    0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET               =    1.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET            =    0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)        =    0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                  =    0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)         =    0.0
CREEP TIME                                           =    0.0
```

B-3-12

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                        PAGE      13
VTITLE                                                                 VARIABLE STRUCTURE NUMBER =    1
ITITLE    INCREMENT 2 (HOT)                                                   INCREMENT NUMBER =    2

                    THERMAL NODAL LOAD SET

NODE  I.D.      1            2            3            4          11          12          13          14
TEMP      1.50000E 00 1.50000E 00 1.50000E 00 1.50000E 00 0.0         0.0         0.0         0.0

BEGIN    LOADS          CPU = 00:00:02.665      TOD = 23:32:50

END      LOADS          CPU = 00:00:02.742      TOD = 23:32:54
```

```
BEGIN   SOLN     CPU = 00:00:02.808   TOD = 23:32:55
END     SOLN     CPU = 00:00:02.858   TOD = 23:32:55
BEGIN   ELLOOP   CPU = 00:00:02.865   TOD = 23:32:55
END     ELLOOP   CPU = 00:00:02.981   TOD = 23:32:57
RESIDUAL NORM =  6.25000E-01
BEGIN   SOLN     CPU = 00:00:02.985   TOD = 23:32:57
END     SOLN     CPU = 00:00:03.021   TOD = 23:32:57
BEGIN   ELLOOP   CPU = 00:00:03.028   TOD = 23:32:57
END     ELLOOP   CPU = 00:00:03.134   TOD = 23:32:59
RESIDUAL NORM =  4.96243E-06
BEGIN   SOLN     CPU = 00:00:03.134   TOD = 23:32:59
END     SOLN     CPU = 00:00:03.168   TOD = 23:33:00
BEGIN   ELLOOP   CPU = 00:00:03.171   TOD = 23:33:00
END     ELLOOP   CPU = 00:00:03.261   TOD = 23:33:02
RESIDUAL NORM =  2.50001E-01
BEGIN   ELLOOP   CPU = 00:00:03.288   TOD = 23:33:02
END     ELLOOP   CPU = 00:00:03.404   TOD = 23:33:03
RESIDUAL NORM =  2.50004E-01
BEGIN   ELLOOP   CPU = 00:00:03.417   TOD = 23:33:03
END     ELLOOP   CPU = 00:00:03.524   TOD = 23:33:06
RESIDUAL NORM =  2.50006E-01
BEGIN   SOLN     CPU = 00:00:03.530   TOD = 23:33:06
END     SOLN     CPU = 00:00:03.574   TOD = 23:33:07
```

```
TITLE    THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                    PAGE     15
VTITLE                                                  VARIABLE STRUCTURE NUMBER =     1
ITITLE   INCREMENT 2 (HOT)                                      INCREMENT NUMBER =     2

BEGIN    ELLOOP        CPU = 00:00:03.564      TOD = 23:33:07
END      ELLOOP        CPU = 00:00:03.707      TOD = 23:33:08
RESIDUAL NORM =  1.00001E-01
BEGIN    SOLN          CPU = 00:00:03.707      TOD = 23:33:08
END      SOLN          CPU = 00:00:03.747      TOD = 23:33:09
BEGIN    ELLOOP        CPU = 00:00:03.757      TOD = 23:33:09
END      ELLOOP        CPU = 00:00:03.860      TOD = 23:33:11
RESIDUAL NORM =  4.54551E-02
BEGIN    SOLN          CPU = 00:00:03.863      TOD = 23:33:11
END      SOLN          CPU = 00:00:03.900      TOD = 23:33:11
BEGIN    ELLOOP        CPU = 00:00:03.907      TOD = 23:33:11
END      ELLOOP        CPU = 00:00:03.990      TOD = 23:33:13
RESIDUAL NORM =  2.17399E-02
BEGIN    SOLN          CPU = 00:00:03.996      TOD = 23:33:13
END      SOLN          CPU = 00:00:04.026      TOD = 23:33:13
BEGIN    ELLOOP        CPU = 00:00:04.030      TOD = 23:33:14
END      ELLOOP        CPU = 00:00:04.120      TOD = 23:33:15
RESIDUAL NORM =  1.06395E-02
BEGIN    SOLN          CPU = 00:00:04.123      TOD = 23:33:15
END      SOLN          CPU = 00:00:04.150      TOD = 23:33:15
BEGIN    ELLOOP        CPU = 00:00:04.166      TOD = 23:33:15
END      ELLOOP        CPU = 00:00:04.266      TOD = 23:33:17
RESIDUAL NORM =  5.26435E-03
```

```
BEGIN   MERGE        CPU = 00:00:04.306     TOD = 23:33:17
BEGIN   GENRE        CPU = 00:00:04.316     TOD = 23:33:17
STIFFNESS GENERATION COMPLETED.        31 PARTITIONS WRITTEN.
END     GENR8        CPU = 00:00:04.416     TOD = 23:33:18
BEGIN   MERSOR       CPU = 00:00:04.416     TOD = 23:33:19
END     MERSOR       CPU = 00:00:04.472     TOD = 23:33:20
END     MERGE        CPU = 00:00:04.472     TOD = 23:33:20
MAXIMUM WAVEFRONT =      6 NODES AT INTERNAL NODE     3
BEGIN   DECOMP       CPU = 00:00:04.566     TOD = 23:33:21
END     DECOMP       CPU = 00:00:04.559     TOD = 23:33:21
BEGIN   SOLN         CPU = 00:00:04.599     TOD = 23:33:21
END     SOLN         CPU = 00:00:04.622     TOD = 23:33:22
BEGIN   ELLOOP       CPU = 00:00:04.625     TOD = 23:33:22
END     ELLOOP       CPU = 00:00:04.729     TOD = 23:33:24
RESIDUAL NORM =   8.54052E-07


                     E N D   O F   L O A D   I N C R E M E N T    2

NO. ELASTIC INTEGRATION POINTS =     4,  NO. PLASTIC INTEGRATION POINTS =     4
     4  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,     0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     2, NO. UPDATES PERFORMED =     1
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =     1
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  8.5405E-07

BEGIN   OUTPUT       CPU = 00:00:04.632     TOD = 23:33:25
```

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **		FORCES			DISPLACEMENTS		
NO.	I.D.	U	V	W	U	V	W
1	1	-2.4999866E 00	3.0664842E-06		0.0	0.0	
2	2	-2.4999895E 00	-2.9442353E-06		0.0	1.2500000E 00	
3	3	2.4999876E 00	4.8807979E-06		1.9999962E 00	-7.3349440E-08	
4	4	2.4999895E 00	-5.0050456E-06		1.9999981E 00	1.2499981E 00	
5	11	5.0000076E 00	-1.0712565E-06		1.9999962E 00	1.0201602E-06	
6	12	5.0000153E 00	7.5930553E-07		1.9999981E 00	9.9999934E-01	
7	13	-5.0000076E 00	-1.1142456E-06		0.0	0.0	
8	14	-5.0000153E 00	1.4261932E-06		0.0	9.9999827E-01	

TITLE THERMAL RATCHET PROBLEM (2 QUAD ELEMENTS) PAGE 18
VTITLE , VARIABLE STRUCTURE NUMBER = 1
ITITLE INCREMENT 2 (HOT) INCREMENT NUMBER = 2

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	5.0000E-01	5.0000E-01	-3.1709E-06	0.0	-1.2225E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	5.0000E-01	-2.5000E-01	-2.5000E-01	-1.8337E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	0	-2	5.0000E-01	2.0000E 00	1.2500E 00	1.2500E 00	-1.8337E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
1	1	5	4	0.0	1.0000E 00	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
1	1	5	4	1.5000E 00	1.5000E 00	1.5000E 00	1.5000E 00

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	1.0000E 00	-1.0000E 00	7.9473E-08	0.0	3.1195E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	-1.0000E 00	5.0000E-01	5.0000E-01	4.6792E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	9.9999E-01	-9.9999E-01	5.0000E-01	5.0000E-01	-9.3216E-06	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	9.9999E-01	-9.9999E-01	5.0000E-01	5.0000E-01	-9.3216E-06	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	1	2	2.0000E 00	-2.0000E 00	1.0000E 00	1.0000E 00	-4.6424E-06	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
2	11	5	4	0.0	1.0000E 00	9.9999E-01	9.9999E-01	9.9999E-01	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
2	11	5	4	0.0	0.0	0.0	0.0

END OUTPUT CPU = 00:00:04.978 TOD = 23:33:26

```
TITLE    . THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                   PAGE      20
VTITLE                                                                    VARIABLE STRUCTURE NUMBER =  1
ITITLE   INCREMENT 3 (COLD)                                                        INCREMENT NUMBER =  3

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE        =   0.0
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO        =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE         =   1.500000E 01
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO         =   0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET             =   1.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET          =   0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)      =   0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                =   0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)       =   0.0
CREEP TIME                                         =   0.0
```

B.3-19

```
TITLE      THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                   PAGE      21
VTITLE                                                                    VARIABLE STRUCTURE NUMBER =  1
ITITLE   INCREMENT 3 (COLD)                                                        INCREMENT NUMBER =  3

                     THERMAL NODAL LOAD SET

NODE  I.D.     1        2        3        4        11       12       13       14
TEMP       0.0      0.0      0.0      0.0      0.0      0.0      0.0      0.0

BEGIN   LOADS       CPU = 00:00:05.284    TOD = 23:33:36

END     LOADS       CPU = 00:00:05.377    TOD = 23:33:39
```

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                              PAGE        22
VTITLE                                                                              VARIABLE STRUCTURE NUMBER =  1
ITITLE    INCREMENT 3 (COLD)                                                                 INCREMENT NUMBER =  3

BEGIN    SOLN          CPU = 00:00:05.417      TOD = 23:33:39
END      SOLN          CPU = 00:00:05.457      TOD = 23:33:40
BEGIN    ELLOOP        CPU = 00:00:05.464      TOD = 23:33:40
END      ELLOOP        CPU = 00:00:05.561      TOD = 23:33:43
RESIDUAL NORM =   5.00000E-01
BEGIN    SOLN          CPU = 00:00:05.564      TOD = 23:33:43
END      SOLN          CPU = 00:00:05.604      TOD = 23:33:43
BEGIN    ELLOOP        CPU = 00:00:05.610      TOD = 23:33:43
END      ELLOOP        CPU = 00:00:05.714      TOD = 23:33:46
RESIDUAL NORM =   9.99986E-02
BEGIN    SOLN          CPU = 00:00:05.720      TOD = 23:33:46
END      SOLN          CPU = 00:00:05.757      TOD = 23:33:46
BEGIN    ELLOOP        CPU = 00:00:05.764      TOD = 23:33:47
END      ELLOOP        CPU = 00:00:05.860      TOD = 23:33:49
RESIDUAL NORM =   4.99393E-07

                         E N D  O F  L O A D  I N C R E M E N T   3

NO. ELASTIC INTEGRATION POINTS =    4,  NO. PLASTIC INTEGRATION POINTS =    4
    4  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,    4  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     2, NO. UPDATES PERFORMED =     6
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =     3
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.00000E-05, ACTUAL ERROR =  4.9939E-07

BEGIN    OUTPUT        CPU = 00:00:05.973      TOD = 23:33:50
```

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **		************** FORCES **************			************ DISPLACEMENTS ************		
NO.	I.D.	U	V	W	U	V	W
1	1	5.0000057E 00	3.4671365E-07		0.0	0.0	
2	2	-5.0000057E 00	-9.3926133E-07		0.0	-7.4999541E-01	
3	3	5.0000048E 00	8.4537674E-07		1.4999924E 00	2.5999840E-07	
4	4	-5.0000057E 00	-2.5283123E-07		1.4999924E 00	-7.4999517E-01	
5	11	2.4999886E 00	-1.4207426E-06		1.4999924E 00	-9.3194103E-06	
6	12	2.4999914E 00	5.4463806E-07		1.4999924E 00	7.4999577E-01	
7	13	-2.4999695E 00	5.6697792E-07		0.0	0.0	
8	14	-2.4999695E 00	1.4421830E-06		0.0	7.4999563E-01	

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	1.0000E 00	1.0000E 00	6.3578E-07	0.0	5.9255E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	1.0000E 00	-5.0000E-01	-5.0000E-01	8.8882E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE LASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	4.9999E-01	4.9999E-01	-2.4999E-01	-2.4999E-01	3.5472E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	4.9999E-01	4.9999E-01	-2.4999E-01	-2.4999E-01	3.5472E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	1	2	1.5000E 00	1.5000E 00	-7.5000E-01	-7.5000E-01	1.2435E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
1	1	5	4	0.0	1.0000E 00	4.9999E-01	4.9999E-01	4.9999E-01	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
1	1	5	4	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	5.0000E-01	-5.0000E-01	-3.1789E-07	0.0	8.7611E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	-5.0000E-01	2.5000E-01	2.5000E-01	1.3142E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	9.9999E-01	-9.9999E-01	5.0000E-01	5.0000E-01	-9.3216E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	-1	-2	1.5000E 00	-1.5000E 00	7.5000E-01	7.5000E-01	3.9200E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
2	11	5	4	0.0	1.0000E 00	0.0	9.9999E-01	9.9999E-01	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
2	11	5	4	0.0	0.0	0.0	0.0

END OUTPUT CPU = 00:00:06.103 TOD = 23:33:51

```
TITLE      THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                                   PAGE     26
VTITLE                                                                             VARIABLE STRUCTURE NUMBER =   1
ITITLE    INCREMENT 4 (HOT)                                                                 INCREMENT NUMBER =   4

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE         =   0.0
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO         =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE          =   1.500000E 01
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO          =   0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET              =   1.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET           =   0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)       =   0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                 =   0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)        =   0.0
CREEP TIME                                          =   0.0
```

B.3-24

```
TITLE      THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                                   PAGE     27
VTITLE                                                                             VARIABLE STRUCTURE NUMBER =   1
ITITLE    INCREMENT 4 (HOT)                                                                 INCREMENT NUMBER =   4

                         THERMAL NODAL LOAD SET

NODE I.D.      1            2            3            4           11           12           13           14
TEMP       1.50000E 00 1.50000E 00 1.50000E 00 1.50000E 00 0.0          0.0          0.0          0.0

BEGIN   LOADS        CPU = 00:00:06.376     TOD = 23:33:56

END     LOADS        CPU = 00:00:06.469     TOD = 23:33:58
```

```
BEGIN   SOLN         CPU = 00:00:06.529     TOD = 23:33:58

END     SOLN         CPU = 00:00:06.566     TOD = 23:33:59

BEGIN   ELLOOP       CPU = 00:00:06.576     TOD = 23:33:59

END     ELLOOP       CPU = 00:00:06.675     TOD = 23:34:00

RESIDUAL NORM =  6.81619E-01

BEGIN   SOLN         CPU = 00:00:06.682     TOD = 23:34:00

END     SOLN         CPU = 00:00:06.715     TOD = 23:34:00

BEGIN   ELLOOP       CPU = 00:00:06.729     TOD = 23:34:00

END     ELLOOP       CPU = 00:00:06.838     TOD = 23:34:02

RESIDUAL NORM =  3.89692E-06

BEGIN   SOLN         CPU = 00:00:06.848     TOD = 23:34:02

END     SOLN         CPU = 00:00:06.875     TOD = 23:34:02

BEGIN   ELLOOP       CPU = 00:00:06.882     TOD = 23:34:03

END     ELLOOP       CPU = 00:00:06.965     TOD = 23:34:04

RESIDUAL NORM =  9.99981E-02

BEGIN   ELLOOP       CPU = 00:00:06.975     TOD = 23:34:04

END     ELLOOP       CPU = 00:00:07.065     TOD = 23:34:06

RESIDUAL NORM =  1.00001E-01

BEGIN   ELLOOP       CPU = 00:00:07.075     TOD = 23:34:06

END     ELLOOP       CPU = 00:00:07.168     TOD = 23:34:08

RESIDUAL NORM =  1.00005E-01

BEGIN   SOLN         CPU = 00:00:07.175     TOD = 23:34:09

END     SOLN         CPU = 00:00:07.206     TOD = 23:34:09
```

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                                    PAGE      29
VTITLE                                                                                    VARIABLE STRUCTURE NUMBER =   1
ITITLE    INCREMENT 4 (HOT)                                                                        INCREMENT NUMBER =   4

BEGIN   ELLOOP        CPU = 00:00:07.215     TOD = 23:34:09

END     ELLOOP        CPU = 00:00:07.324     TOD = 23:34:12

RESIDUAL NORM =  7.49026E-07


                        E N D   O F   L O A D   I N C R E M E N T    4

NO. ELASTIC INTEGRATION POINTS =     4,  NO. PLASTIC INTEGRATION POINTS =     4
    4  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,     4  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     2, NO. UPDATES PERFORMED =     0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      6
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  7.4903E-07

BEGIN   OUTPUT        CPU = 00:00:07.421     TOD = 23:34:12



TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                                    PAGE      30
VTITLE                                                                                    VARIABLE STRUCTURE NUMBER =   1
ITITLE    INCREMENT 4 (HOT)                                                                        INCREMENT NUMBER =   4

                 CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **    ****************  FORCES  *****************      *************  DISPLACEMENTS  *************
 NO.  I.D.     U                V                W               U                V                W
   1    1    -2.4999933E 00  7.2161655E-07                     0.0              0.0
   2    2    -2.4999914E 00 -4.3228968E-07                     0.0              1.0000057E 00
   3    3     2.4999933E 00 -1.4134097E-06                     2.4999857E 00 -3.1628677E-08
   4    4     2.4999905E 00  1.6243821E-06                     2.4999866E 00  1.0000049E 00
   5   11     5.0000114E 00  3.1619547E-06                     2.4999857E 00  2.0348871E-08
   6   12     5.0000134E 00 -3.4856166E-06                     2.4999866E 00  1.2499924E 00
   7   13    -5.0000105E 00 -3.5432353E-06                     0.0              0.0
   8   14    -5.0000114E 00 -3.2695725E-06                     0.0              1.2499914E 00
```

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	5.0000E-01	5.0000E-01	-3.0994E-06	0.0	-2.8953E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	5.0000E-01	-2.5000E-01	-2.5000E-01	-4.3429E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	4.9999E-01	4.9999E-01	-2.4999E-01	-2.4999E-01	3.5472E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	-1	-2	9.9999E-01	2.5000E 00	1.0000E 00	1.0000E 00	-7.9572E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
1	1	5	4	0.0	1.0000E 00	0.0	4.9999E-01	4.9999E-01	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
1	1	5	4	1.5000E 00	1.5000E 00	1.5000E 00	1.5000E 00

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	1.0000E 00	-1.0000E 00	-5.5631E-07	0.0	3.2366E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	-1.0000E 00	5.0000E-01	5.0000E-01	4.8549E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	1.5000E 00	-1.5000E 00	7.4999E-01	7.4999E-01	-5.3637E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	4.9999E-01	-4.9999E-01	2.4999E-01	2.4999E-01	3.9579E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	1	2	2.5000E 00	-2.5000E 00	1.2500E 00	1.2500E 00	-5.0873E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
2	11	5	4	0.0	1.0000E 00	4.9999E-01	1.5000E 00	1.5000E 00	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
2	11	5	4	0.0	0.0	0.0	0.0

END OUTPUT CPU = 00:00:07.547 TOD = 23:34:14

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                                       PAGE    33
VTITLE                                                                              VARIABLE STRUCTURE NUMBER =    1
ITITLE    INCREMENT 5 (COLD)                                                                 INCREMENT NUMBER =    5

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE          =    0.0
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO          =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE           =    1.500000E 01
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO           =    0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET               =    1.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET            =    0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)        =    0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                  =    0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)         =    0.0
CREEP TIME                                           =    0.0
```

B.3-29

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                                       PAGE    34
VTITLE                                                                              VARIAB.  STRUCTURE NUMBER =    1
ITITLE    INCREMENT 5 (COLD)                                                                 INCREMENT NUMBER =    5

                         THERMAL NODAL LOAD SET

NODE  I.D.     1          2          3          4          11         12         13         14
TEMP        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0

BEGIN   LOADS       CPU = 00:00:07.967     TOD = 23:34:17

END     LOADS       CPU = 00:00:07.970     TOD = 23:34:20
```

```
BEGIN   SOLN     CPU = 00:00:08.030   TOD = 23:34:20
END     SOLN     CPU = 00:00:08.090   TOD = 23:34:20
BEGIN   ELLOOP   CPU = 00:00:08.083   TOD = 23:34:20
END     ELLOOP   CPU = 00:00:08.183   TOD = 23:34:22
RESIDUAL NORM =  5.00000E-01
BEGIN   SOLN     CPU = 00:00:08.190   TOD = 23:34:22
END     SOLN     CPU = 00:00:08.226   TOD = 23:34:22
BEGIN   ELLOOP   CPU = 00:00:08.233   TOD = 23:34:22
END     ELLOOP   CPU = 00:00:08.333   TOD = 23:34:23
RESIDUAL NORM =  9.99964E-02
BEGIN   SOLN     CPU = 00:00:08.336   TOD = 23:34:23
END     SOLN     CPU = 00:00:08.359   TOD = 23:34:25
BEGIN   ELLOOP   CPU = 00:00:08.363   TOD = 23:34:25
END     ELLOOP   CPU = 00:00:08.469   TOD = 23:34:26
RESIDUAL NORM =  1.04778E-06
```

E N D O F L O A D I N C R E M E N T 5

```
NO. ELASTIC INTEGRATION POINTS =    4, NO. PLASTIC INTEGRATION POINTS =    4
    4  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,    4  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     2, NO. UPDATES PERFORMED =     0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =     3
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  1.04778E-06

BEGIN   OUTPUT   CPU = 00:00:08.552   TOD = 23:34:27
```

TITLE THERMAL RATCHET PROBLEM (2 QUAD ELEMENTS) PAGE 36
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE INCREMENT 5 (COLD) INCREMENT NUMBER = 5

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **		*************** FORCES ***************			************ DISPLACEMENTS ************		
NO.	I.D.	U	V	W	U	V	W
1	1	-5.0000076E 00	7.0119964E-07		0.0	0.0	
2	2	-5.0000165E 00	-2.6608836E-07		0.0	-9.9998626E-01	
3	3	5.0000096E 00	1.0869398E-06		1.9999790E 00	4.2146780E-06	
4	4	-5.0000095E 00	-1.5220563E-06		1.9999790E 00	-9.9998856E-01	
5	11	2.4999796E 00	-2.4116707E-06		1.9999790E 00	2.7695432E-07	
6	12	2.4999819E 00	2.7143651E-06		1.9999790E 00	9.9998957E-01	
7	13	-2.4999796E 00	-2.7549618E-06		0.0	0.0	
8	14	-2.4999819E 00	4513674E-06		0.0	9.9998945E-01	

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	1.0000E 00	1.0000E 00	1.C331E-06	0.0	-4.3511E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	1.0000E 00	-5.0000E-01	-5.0000E-01	-6.5266E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	9.9998E-01	9.9998E-01	-4.9999E-01	-4.9999E-01	1.2973E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	4.9995E-01	4.9999E-01	-2.4999E-01	-2.4999E-01	-3.4175E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	1	2	2.0000E 00	2.0000E 00	-9.9999E-01	-9.9999E-01	-6.3969E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
1	1	5	4	0.0	1.0000E 00	4.9999E-01	9.9998E-01	9.9998E-01	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
1	1	5	4	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	5.0000E-01	-5.0000E-01	-1.5895E-07	0.0	-3.0269E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	-5.0000E-01	2.5000E-01	2.5000E-01	-4.5404E-08	0.0	0.0

TITLE THERMAL RATCHET PROBLEM (2 QUAD ELEMENTS)

VTITLE

ITITLE INCREMENT 5 (COLD)

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VARIABLE STRUCTURE NUMBER = 1

INCREMENT NUMBER = 5

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	1.5000E 00	-1.5000E 00	7.4999E-01	7.4999E-01	-5.3637E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	-1	-2	2.0000E 00	-2.0000E 00	9.9999E-01	9.9999E-01	-9.9041E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
2	11	5	4	0.0	1.0000E 00	0.0	1.5000E 00	1.5000E 00	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
2	11	5	4	0.0	0.0	0.0	0.0

END OUTPUT CPU = 00:00:08.705 TOD = 23:34:28

```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                                    PAGE       39
VTITLE                                                                                   VARIABLE STRUCTURE NUMBER =  1
ITITLE   INCREMENT 6 (HOT)                                                                        INCREMENT NUMBER =  6

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE          =    0.0
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO          =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE           =    1.500000E 01
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO           =    0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET               =    1.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET            =    0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)        =    0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                  =    0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)         =    0.0
CREEP TIME                                           =    0.0
```



```
TITLE     THERMAL RATCHET PROBLEM  (2 QUAD ELEMENTS)                                                    PAGE       40
VTITLE                                                                                   VARIABLE STRUCTURE NUMBER =  1
ITITLE   INCREMENT 6 (HOT)                                                                        INCREMENT NUMBER =  6

                         THERMAL NODAL LOAD SET

NODE  I.D.       1            2            3            4           11          12          13          14
TEMP        1.50000E 00 1.50000E 00 1.50000E 00 1.50000E 00 0.0          0.0         0.0         0.0

BEGIN    LOADS        CPU = 00:00:09.985      TOD = 23:34:31

END      LOADS        CPU = 00:00:09.075      TOD = 23:34:33
```

```
BEGIN   SOLN      CPU = 00:00:09.125    TOD = 23:34:33
END     SOLN      CPU = 00:00:09.161    TOD = 23:34:33
BEGIN   ELLOOP    CPU = 00:00:09.165    TOD = 23:34:33
END     ELLOOP    CPU = 00:00:09.271    TOD = 23:34:35
RESIDUAL NORM =  6.81619E-01
BEGIN   SOLN      CPU = 00:00:09.275    TOD = 23:34:35
END     SOLN      CPU = 00:00:09.308    TOD = 23:34:35
BEGIN   ELLOOP    CPU = 00:00:09.318    TOD = 23:34:35
END     ELLOOP    CPU = 00:00:09.421    TOD = 23:34:37
RESIDUAL NORM =  3.25824E-06
BEGIN   SOLN      CPU = 00:00:09.424    TOD = 23:34:37
END     SOLN      CPU = 00:00:09.461    TOD = 23:34:37
BEGIN   ELLOOP    CPU = 00:00:09.468    TOD = 23:34:37
END     ELLOOP    CPU = 00:00:09.567    TOD = 23:34:39
RESIDUAL NORM =  9.99970E-02
BEGIN   ELLOOP    CPU = 00:00:09.581    TOD = 23:34:39
END     ELLOOP    CPU = 00:00:09.667    TOD = 23:34:40
RESIDUAL NORM =  9.99985E-02
BEGIN   ELLOOP    CPU = 00:00:09.691    TOD = 23:34:40
END     ELLOOP    CPU = 00:00:09.767    TOD = 23:34:42
RESIDUAL NORM =  1.00001E-01
BEGIN   SOLN      CPU = 00:00:09.770    TOD = 23:34:42
END     SOLN      CPU = 00:00:09.804    TOD = 23:34:43
```

TITLE THERMAL RATCHET PROBLEM (2 QUAD ELEMENTS) PAGE 42
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE INCREMENT 6 (HOT) INCREMENT NUMBER = 6

BEGIN ELLOOP CPU = 00:00:09.814 TOD = 23:34:43

END ELLOOP CPU = 00:00:09.933 TOD = 23:34:45

RESIDUAL NORM = 8.71186E-07

E N D O F L O A D I N C R E M E N T 6

NO. ELASTIC INTEGRATION POINTS = 4, NO. PLASTIC INTEGRATION POINTS = 4
4 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 4 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES = 2, NO. UPDATES PERFORMED = 0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10 10 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 6
SPECIFIED MAX. UNBALANCED-FORCE ERROR = 1.0000E-05, ACTUAL ERROR = 8.7119E-07

BEGIN OUTPUT CPU = 00:00:10.030 TOD = 23:34:46

TITLE THERMAL RATCHET PROBLEM (2 QUAD ELEMENTS) PAGE 43
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE INCREMENT 6 (HOT) INCREMENT NUMBER = 6

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE ** NO.	I.D.	FORCES U	FORCES V	FORCES W	DISPLACEMENTS U	DISPLACEMENTS V	DISPLACEMENTS W
1	1	-2.4999847E 00	2.9145358E-06		0.0	0.0	
2	2	-2.4999826E 00	-2.5296823E-06		0.0	7.5001299E-01	
3	3	2.4999847E 00	2.8472441E-06		2.9999704E 00	4.3004894E-07	
4	4	2.4999826E 00	-3.2318976E-06		2.9999704E 00	7.5001234E-01	
5	11	5.0000114E 00	3.3803972E-06		2.9999704E 00	2.6162525E-07	
6	12	5.0000124E 00	-2.2431113E-06		2.9999704E 00	1.4999847E 00	
7	13	-5.0000095E 00	2.1827600E-06		0.0	0.0	
8	14	-5.0000134E 00	-3.3195867E-06		0.0	1.4999836E 00	

TITLE THERMAL RATCHET PROBLEM (2 QUAD ELEMENTS)
VTITLE
ITITLE INCREMENT 6 (HCT)

VARIABLE STRUCTURE NUMBER = 1
INCREMENT NUMBER = 6

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	4.9999E-01	4.9999E-01	-3.0994E-06	0.0	-3.8465E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	4.9999E-01	-2.5000E-01	-2.4999E-01	-5.7696E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	9.9998E-01	9.9998E-01	-4.9999E-01	-4.9999E-01	1.2973E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

B.3-37

ELEMENT NO.	I.D.	POINT NO.	TP.	L-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	1	5	4	-1	-2	1.5000E 00	3.0000E 00	7.5001E-01	7.5002E-01	-5.6401E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
1	1	5	4	0.0	1.0000E 00	0.0	9.9999E-01	9.9998E-01	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
1	1	5	4	1.5000E 00	1.5000E 00	1.5000E 00	1.5000E 00

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	1.0000E 00	-1.0000E 00	-7.9473E-07	0.0	-1.1373E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	-1.0000E 00	5.0000E-01	5.0000E-01	-1.7059E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	2.0000E 00	-2.0000E 00	9.9998E-01	9.9996E-01	-1.3323E-07	0.0	0.0

(CUMULATIVE PLASTIC STRAINS)

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	4.9999E-01	-4.9999E-01	2.4999E-01	2.4999E-01	-7.9595E-08	0.0	0.0

(INCREMENTAL PLASTIC STRAINS)

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	XX	YY	ZZ	XY	XZ	YZ
2	11	5	4	1	2	3.0000E 00	-3.0000E 00	1.5000E 00	1.5000E 00	-3.0382E-07	0.0	0.0

(CUMULATIVE TOTAL STRAINS)

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
2	11	5	4	0.0	1.0000E 00	4.9999E-01	2.0000E 00	2.0000E 00	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	XX	YY	ZZ
2	11	5	4	0.0	0.0	0.0	0.0

(CUMULATIVE THERMAL STRAINS)

END OUTPUT CPU = 00:00:10.176 TOD = 23:34:47

END OF BOPACE PROBLEM

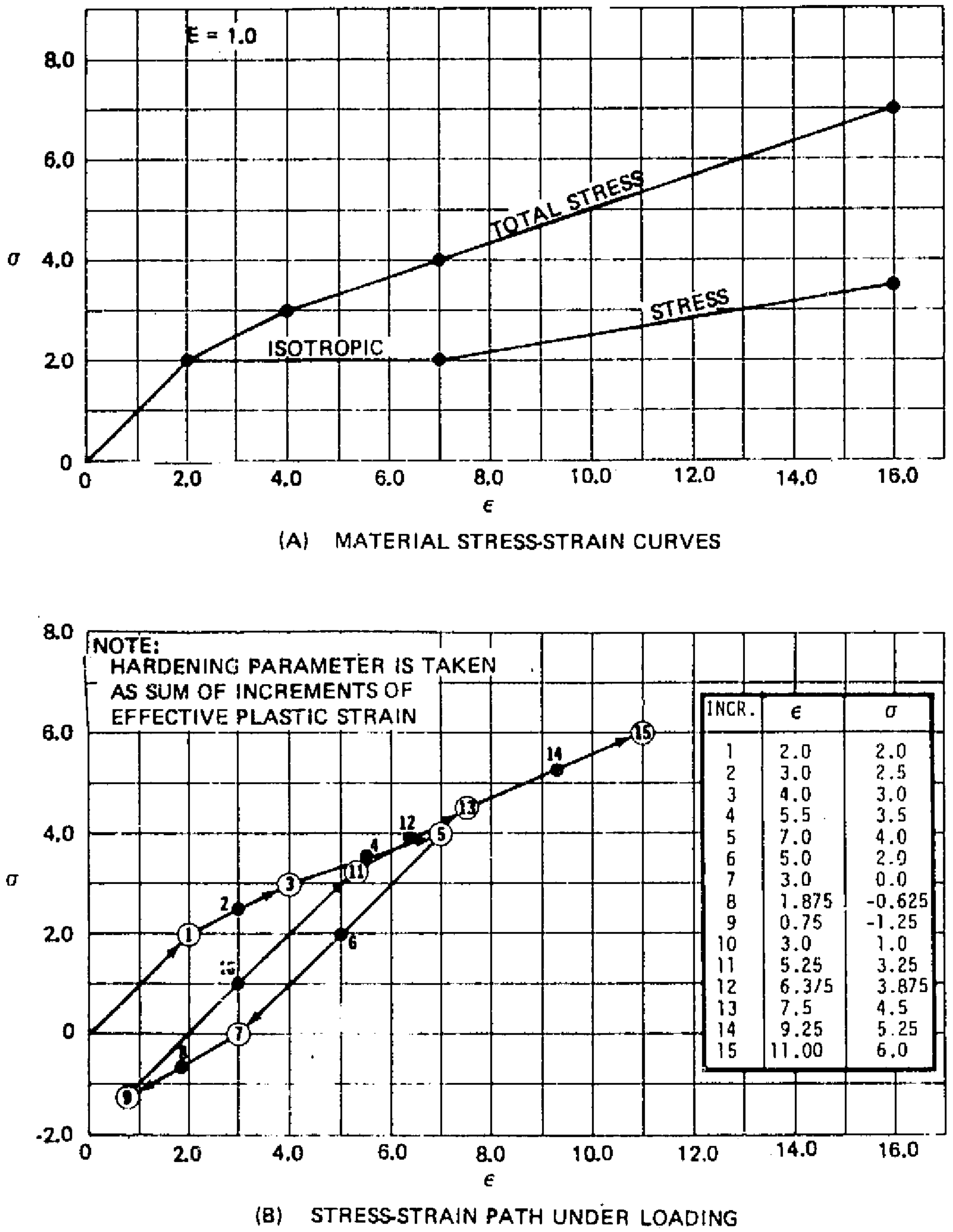
THE NUMBER OF WARNING MESSAGES IS 1

B.4 CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM

Uniaxial Behavior - The basic characteristics of BOPACE combined hardening are shown in Figure B.4.1 for a uniaxial (special case of a plane-stress or 3-D) problem. Figure B.4-1A gives the assumed monotonic stress-strain hardening curves for a hypothetical material with unit elastic modulus. The size of the yield surface is defined by the isotropic stress (= average of tensile and compressive yield stresses), while the Bauschinger kinematic hardening is defined by the difference between the total stress and the isotropic stress. Thus the hardening is completely kinematic out to a strain value of 7.0 (elastic strain = σ/E = 4.0, plastic strain = 3.0), after which there are equal amounts of isotropic and kinematic hardening. For an actual material, these curves would have been determined from cyclic test data.

A resulting cyclic stress-strain curve is given in Figure B.4-1B. The 15 load increments were chosen so as to result in the exact σ-ε points given in the figure insert table. Note that the hardening parameters (κ and κ^k, described in Section 2) in this example were based on effective plastic strain rather than on plastic work, because it makes the relationship between the monotonic and cyclic curves more readily apparent.

This problem may be used as a test problem for various BOPACE elements, by applying appropriate boundary conditions normal to the direction of load, so as to maintain a uniaxial situation.



INCR.	ε	σ
1	2.0	2.0
2	3.0	2.5
3	4.0	3.0
4	5.5	3.5
5	7.0	4.0
6	5.0	2.0
7	3.0	0.0
8	1.875	-0.625
9	0.75	-1.25
10	3.0	1.0
11	5.25	3.25
12	6.375	3.875
13	7.5	4.5
14	9.25	5.25
15	11.00	6.0

Figure B.4-1: Cyclic Plastic—Creep Problem Behavior

Plane-Strain with Additional Options - The uniaxial problem just described was used with a 2-D plane-strain element to illustrate the use of several additional BOPACE options, including temperature-dependent elasticity, creep and use of element normal strain loads. The analysis was performed using the QUAD element and loading given in Figure B.4-2. A listing of the input data and the printed output results are included at the end of this section (some of the output pages have been combined to save space).

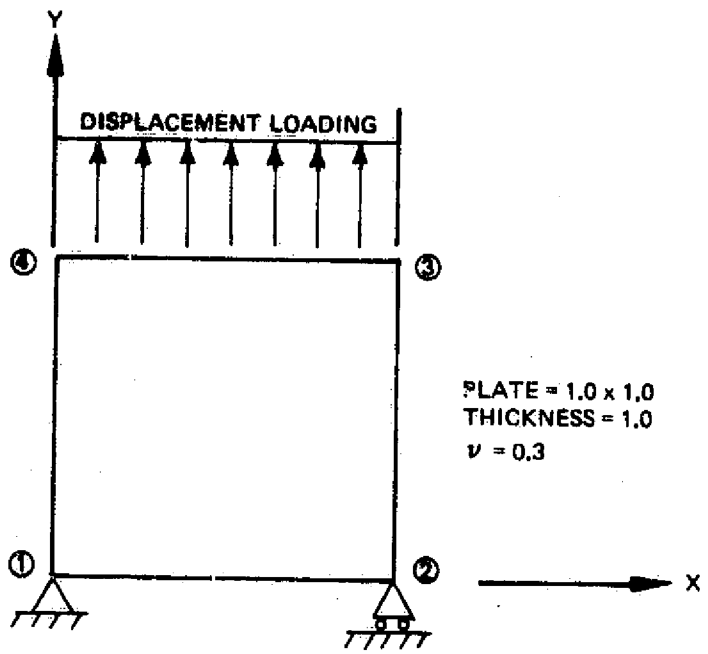


Figure B.4-2: Cyclic Plastic—Creep Checkout Problem Mesh

A summary of the plane-strain problem is provided by Table B.4-1. The 15 increments correspond to those of the previous uniaxial problem. The values of incremental plastic strain, stress, effective stress center, and yield-surface size given in columns 2-5 of Table B.4-1 were kept the same as those of the uniaxial problem, by prescribing proper combinations of creep, thermal strain, and normal strain loads. The stress is equal to the product of the temperature-dependent elastic modulus (column 6) and the elastic strain (column 7).

The creep strain listed in column 9 results from the material creep definition of Figure B.4-3. There the creep curve shape for a strain-hardening material is assumed as shown in (A), while (B) defines the creep factor as a function of average stress level during the increment. The creep strain may be determined using the average stress level (column 10 of Table B.4-1), the creep factor (column 11) and the specified creep time increment (column 12).

In addition, the normal (Z-load) strains given in column 13 and thermal strains in column 14 were imposed. In order to keep the results simple and exact (all numbers in Table B.4-1 are given exactly), the Z-load and thermal-strain values were selected so as to give zero normal stress in each increment. For example, in increment 11:

$$\varepsilon^e_{ZZ} = -0.3$$
$$\varepsilon^p_{ZZ} = -1.0$$
$$\varepsilon^c_{zz} = -0.5$$
$$\varepsilon^t_{ZZ} = 1.5$$

$$\Sigma\varepsilon_{ZZ} \quad -0.3$$

Because the imposed Z-load strain also equals -0.3, a zero value results for the normal stress σ_{ZZ}. Thus it may be noted that this example can be used for detailed checkout of either the plane-stress or the plane-strain capability.

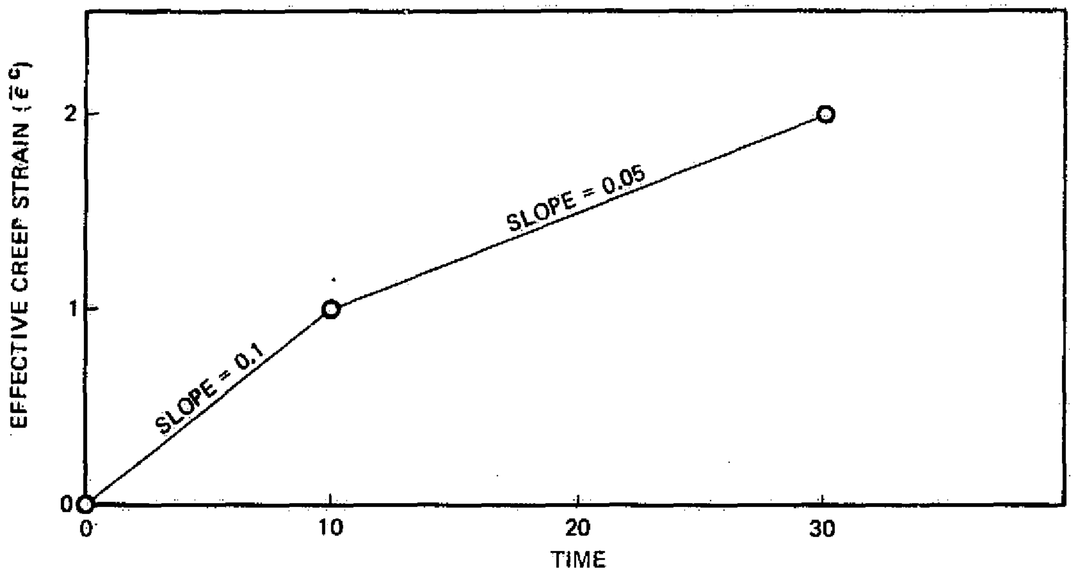
The prescribed vertical displacements shown in column 15 were determined from the various components of the total strain. For example, again in increment 11:

$$\varepsilon^e_{YY} = 1.0$$
$$\varepsilon^p_{YY} = 2.0$$
$$\varepsilon^c_{YY} = 1.0$$
$$\varepsilon^t_{YY} = 1.5$$

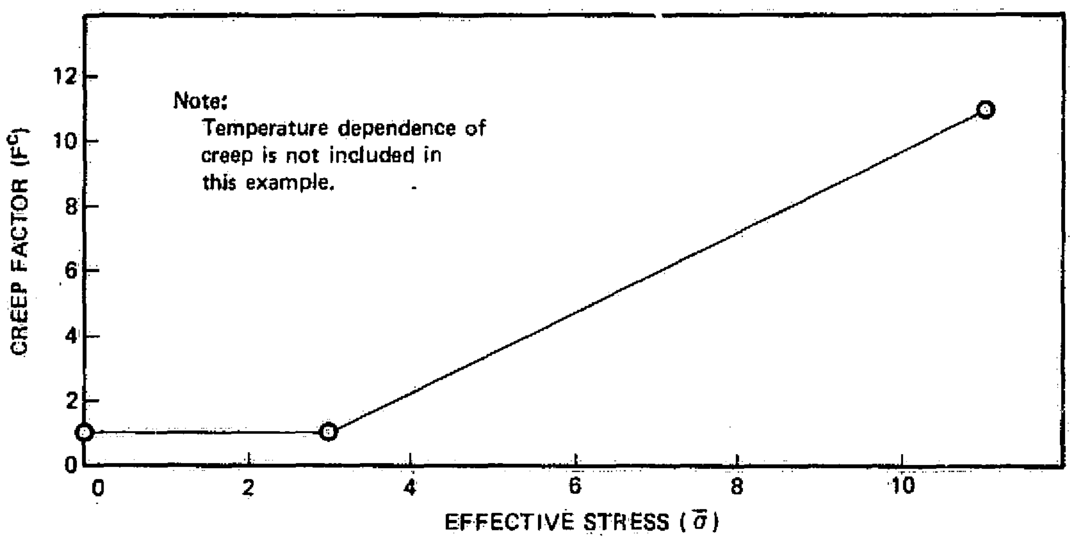
$$Q_{YY} = \Sigma\varepsilon_{YY} = 5.5$$

Table B.4-1: Results for Plane Strain with Normal Z-Loads and Creep

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Incr.	$\Delta\epsilon^p_{YY}$	σ_{YY}	$\frac{3}{2}\alpha_{YY}$	$[\sigma - \alpha]$	E^1	ϵ^e_{YY}	ϵ^p_{YY}	ϵ^c_{YY}	$\bar{\sigma}_{ave}$	F^c	Δt^c	Z-load strain	ϵ^t	Q_{YY}	Temp.
0	–	0	0	2.0	1.0	0	0	0	–	–	–	0	0	0	1.0
1	0	2.0	0	2.0	2.0	1.0	0	0	1.0	1.0	0	0	0.3	1.3	2.0
2	0.5	2.5	0.5	2.0	2.5	1.0	0.5	0.5	2.25	1.0	5.0	0	0.8	2.8	3.0
3	0.5	3.0	1.0	2.0	3.0	1.0	1.0	0.5	2.75	1.0	0	0	1.05	3.55	4.0
4	1.0	3.5	1.5	2.0	3.5	1.0	2.0	1.0	3.25	1.25	4.0	0	1.8	5.8	5.0
5	1.0	4.0	2.0	2.0	4.0	1.0	3.0	1.0	3.75	1.75	0	0	2.3	7.3	6.0
6	0	2.0	2.0	2.0	2.0	1.0	3.0	1.0	3.0	1.0	0	0	2.3	7.3	7.0
7	0	0	2.0	2.0	1.0	0	3.0	1.0	1.0	1.0	0	0	2.0	6.0	5.4
8	-0.5	-0.625	1.5	2.125	1.25	-0.5	2.5	1.0	0.3125	1.0	0	-0.6	1.0	4.0	8.0
9	-0.5	-1.25	1.0	2.25	1.25	-1.0	2.0	0	0.9375	1.0	10.0	-0.2	0.5	1.5	9.0
10	0	1.0	1.0	2.25	2.0	0.5	2.0	0	0.125	1.0	0	0	1.15	3.65	10.0
11	0	3.25	1.0	2.25	3.25	1.0	2.0	1.0	2.125	1.0	10.0	-0.3	1.5	5.5	4.5
12	0.5	3.875	1.5	2.375	3.875	1.0	2.5	1.0	3.5625	1.5625	0	0	2.05	6.55	12.0
13	0.5	4.5	2.0	2.5	4.5	1.0	3.0	1.0	4.1875	2.1875	0	0.2	2.5	7.5	13.0
14	1.0	5.25	2.5	2.75	5.25	1.0	4.0	1.0	4.875	2.875	0	0	2.8	8.8	14.0
15	1.0	6.0	3.0	3.0	3.0	2.0	5.0	4.125	5.625	3.625	10.0	0	5.1625	16.2875	15.0



(A) ASSUMED SHAPE FOR REFERENCE CREEP CURVE



(B) ASSUMED DEPENDENCE OF CREEP FACTOR ON STRESS

Figure B.4-3: Creep Definition for Plastic-Creep Problem

INPUT DATA

CARD NUMBER	
1	TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
2	CONT QUAD ELEMENT, 15 LOAD INCREMENTS
~~3~~	~~$PROB PROBT GEOMNL~~
4	PROB 2
5	SOLU .00001
~~6~~	~~CHECKPOINT 25~~
7	PRT1 1 -1
8	PRT2 1 -1 2 -1 3 -1 4 -1 5 -1 6 -1 7 -1 8 -1 9 -1 10 -1 11 -1
~~9~~	~~VTITLE~~
10	MAT1 1
11	IMODULUS 1,1 2,2 3,2.5 4,2 5,3.5 5.4,1 6,4 7,2 8,1.25
~~12~~	~~CONT 9,1.25 10,2 12,3.875 13,4.5 14,5.25 15,3~~
13	IPOISSON 0,.3
14	ISTRAIN 1,0 2,.3 3,.8 4,1.05 4.5,1.5 5,1.8 6,2.3 7,2.3
~~15~~	~~CONT 8,1 9,.5 10,1.15 12,2.09 13,2.5 14,2.8 15,5.1625~~
16	PLASTIC 1,1
17	PTEMP 0
~~18~~	~~IMAKO 0,2 3,2 9,2.5~~
19	KSHAPE 0,0 1,1 3,2 9,3.5
20	CREEP 1,2
~~21~~	~~CSHAPE 0,0 10,1 30,2~~
22	CTEMP 0
23	CFACTOR 0,1 3,1 11,9
~~24~~	~~NODE 1 0,0, ,,1~~
25	NODE 2 1,0
26	NODE 3 1,1
~~27~~	~~NODE 4 0,1~~
28	RQUAD 10 1.0 0 1.0
29	RQUAD 100 1,0 4 1,2,2
~~30~~	~~QUAD 3 1,10,100 1,2,3,4~~
31	SPC 1,2 2,2 3,2 4,2
32	ITITLE LOAD INCREMENT 1
~~33~~	~~LFACT 1.3 ,0 0,0 2. 0~~
34	CLOAD 1 3,2,1. 4,2,1.
35	TILOAD 1 1.0 1,2,3,4
~~36~~	~~SLOAD 1 3,1,1.0~~
37	ITITLE LOAD INCREMENT 2
38	CTIME 5.
~~39~~	~~LFACT 2.0 ,0 0,0 3. 0~~
40	ITITLE LOAD INCREMENT 3
41	LFACT 3.55 ,0 0,0 4. 0
~~42~~	~~ITITLE LOAD INCREMENT 4~~
43	LFACT 5.8 ,0 0,0 5. 0
44	CTIME 4.
~~45~~	~~ITITLE LOAD INCREMENT 5~~
46	LFACT 7.3 ,0 0,0 6. 0
47	ITITLE LOAD INCREMENT 6
~~48~~	~~LFACT 7.3 ,0 0,0 7. 0~~
49	ITITLE LOAD INCREMENT 7
50	LFACT 6.0 ,0 0,0 5.4 0

```
                  INPUT DATA
  CARD
 NUMBER
    51            SOLU  .COC01  4 1,C,2   $CAUSES IMMEDIATE STIFFNESS UPDATE FOR INCR 7
    52            ITITLE LOAD INCREMENT 8
    53            LFACT      4.0   ,0    0,0  8.    -0.6
    54            ITITLE LOAD INCREMENT 9
    55            LFACT      1.5   ,0    C,0  9.    -0.2
    56            CTIME  10.
    57            ITITLE LOAD INCREMENT 10
    58            LFACT      3.65  ,0    0,0  10.   0
    59            ITITLE LOAD INCREMENT 11
    60            LFACT      5.5   ,0    C,0  4.5   -0.3
    61            CTIME  10.
    62            ITITLE LOAD INCREMENT 12
    63            LFACT      6.55  ,C    C,0  12.   0
    64            ITITLE LOAD INCREMENT 13
    65            LFACT      7.5   ,0    0,0  13.   0.2
    66            ITITLE LOAD INCREMENT 14
    67            LFACT      5.8   ,0    0,0  14.   0
    68            ITITLE LOAD INCREMENT 15
    69            LFACT     10.2875,0    0,0  15.   0
    70            CTIME  10.
    71            END
```

B.4-9

```
TITLE      EDPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                  PAGE      1
CONTINUE   QUAD ELEMENT, 15 LOAD INCREMENTS

NUMBER OF DEGREES OF FREEDOM PER NODE = 2

EDPACE WILL ASSUME ONLY MATERIAL NON-LINEARITY TO SOLVE THE PROBLEM

MAXIMUM SPECIFIED ERROR NORM =  1.00000E-05
SOLUTION METHOD CODE =          4
MAXIMUM NO. STIFFNESS UPDATES PER INCREMENT =             1
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE ONE           =  10
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE TWO           =  10
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE THREE AND UP =  10
MAXIMUM ELASTIC ITERATIONS PER INCREMENT =           2
MAXIMUM MAGNITUDE FOR ELASTIC-PLASTIC SUM CODE =          2
MAXIMUM REDUCTIONS =          1
CONVERGENCE REDUCTION FACTOR =  5.00000E-01
FRACTION FROM END OF INCREMENT TO EVALUATE SLOPE = 1.00000E-01
```

TITLE . SOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
VTITLE

PAGE 2
VARIABLE STRUCTURE NUMBER = 1

MATERIAL NO. 1, MASS DENSITY = 0.0
TEMPERATURE DEPENDENT PROPERTIES

TEMPERATURE	ELASTIC MOD.
1.0000E 00	1.0000E 00
2.0000E 00	2.00[illegible]E 00
3.0000E 00	2.5000E 00
4.0000E 00	3.0000E 00
5.0000E 00	3.5000E 00
5.4000E 00	1.0000E 00
6.0000E 00	4.0000E 00
7.0000E 00	2.0000E 00
8.0000E 00	1.2500E 00
9.0000E 00	1.2500E 00
1.0000E 01	2.[illegible]E 00
1.2000E 01	3.4750E 00
1.3000E 01	4.5000E 00
1.4000E 01	5.2500E 00
1.5000E 01	5.0000E 00

TEMPERATURE	POISSONS RATIO
0.0	3.0000E-01

TEMPERATURE	THERMAL STRAIN
1.0000E 00	0.0
2.0000E 00	3.0000E-01
3.4000E 00	8.[illegible]E-01
4.0000E 00	1.05[illegible]E 00
4.5000E 00	1.5000E 00
5.0000E 00	1.[illegible]E 00
6.0000E 00	2.3[illegible]E 00
7.0000E 00	2.3000E 00
8.0000E 00	1.0000E 00
9.0000E 00	5.0000E-01
1.0000E 01	1.1500E 00
1.2000E 01	2.[illegible]E 00
1.3000E 01	2.5000E 00
1.4000E 01	2.8[illegible]E 00
1.5000E 01	5.1625E 00

MATERIAL NO 1,PLASTICITY TYPE 1,KINEMATIC CODE 0

MATERIAL NO. 1, TEMPERATURE = 0.0

PARAMETER	ISOTROPIC HARDENING
0.0	2.[illegible]E 00
3.0000E 00	2.[illegible]E 00
9.0000E 00	3.5000E 00

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 3
VTITLE VARIABLE STRUCTURE NUMBER = 1

PARAMETER	KINEMATIC HARDENING SHAPE
0.0	0.0
1.00000E 00	1.00000E 00
3.00000E 00	2.00000E 00
9.00000E 00	3.50000E 00

TEMPERATURE = 0.0

PARAMETER	ISOTROPIC HARDENING
0.0	2.00000E 00
3.00000E 00	2.00000E 00
9.00000E 00	3.50000E 00

PARAMETER	KINEMATIC HARDENING SHAPE
0.0	0.0
1.00000E 00	1.00000E 00
3.00000E 00	2.00000E 00
9.00000E 00	3.50000E 00

B.4-11

MATERIAL NO. 1,CREEP TYPE 2

TIME	CREEP STRAIN
0.0	0.0
1.0000E 01	1.0000E 00
3.0000E 01	2.0000E 00

MATERIAL NO. 1, TEMPERATURE = 0.0

STRESS	CREEP FACTOR
0.0	1.0000E 00
3.0000E 00	1.0000E 00
1.0000E 01	4.0000E 00

MATERIAL NO. 1, TEMPERATURE = 0.0

STRESS	CREEP FACTOR
0.0	1.0000E 00
3.0000E 00	1.0000E 00
1.0000E 01	4.0000E 00

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 4
VTITLE VARIABLE STRUCTURE NUMBER = 1

** NODES **

NO.	I.D.	X1	X2	X3	COORD. LOCATE	COORD. DISPLACE
1	1	0.0	0.0	0.0	1	1
2	2	1.0000E 00	0.0	0.0	1	1
3	3	1.00000E 00	1.00000E 00	0.0	1	1
4	4	0.0	1.00000E 00	0.0	1	1

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
VTITLE

ELEMENT NO.	I.D.	MATL	N1	N2	N3	N4	N5	N6	N7	N8	VOLUME (ST. LINE)	MAP CODE	INTERMEDIATE EDGE NODES
1	3	1	1	2	3	4					1.0000E 00	0	

SUM OF ELEMENT VOLUMES = 1.0000E 00

BEGIN GFORMS CPU = 00:00:11.062 TOD = 23:34:53

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
VTITLE

ELEMENT NO.	I.D.	REFERENCE POINT NO.	TYPE	X1	X2	X3	COORD. LOCATE	COORD. DISPLACE	INTEGRATION SCHEME CODES
1	3	5	4	5.000E-01	5.000E-01	0.0	1	0	1 2 2

8.4-12

END GFORMS CPU = 00:00:11.261 TOD = 23:35:00

BEGIN MERGE CPU = 00:00:11.288 TOD = 23:35:01

BEGIN GENRB CPU = 00:00:11.295 TOD = 23:35:01

STIFFNESS GENERATION COMPLETED. 10 PARTITIONS WRITTEN.

END GENRB CPU = 00:00:11.378 TOD = 23:35:02

BEGIN MERSCR CPU = 00:00:11.378 TOD = 23:35:02

END MERSCR CPU = 00:00:11.434 TOD = 23:35:03

END MERGE CPU = 00:00:11.434 TOD = 23:35:03

MAXIMUM WAVEFRONT = 4 NODES AT INTERNAL NODE 1

BEGIN DECOMP CPU = 00:00:11.491 TOD = 23:35:03

END DECOMP CPU = 00:00:11.551 TOD = 23:35:03

BEGIN EIGSCK CPU = 00:00:11.561 TOD = 23:35:04

END EIGSCK CPU = 00:00:11.840 TOD = 23:35:16

BEGIN EIGSCK CPU = 00:00:11.840 TOD = 23:35:16

END EIGSCK CPU = 00:00:11.F24 TOD = 23:35:17

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                    PAGE        7
VTITLE                                                                    VARIABLE STRUCTURE NUMBER =  1
ITITLE   LOAD INCREMENT 1                                                          INCREMENT NUMBER =  1

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE          =    1.299999E 00
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO          =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE           =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO           =    0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET               =    2.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET            =    0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)        =    0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                  =    0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)         =    0.0
CREEP TIME                                           =    0.0
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                    PAGE        8
VTITLE                                                                    VARIABLE STRUCTURE NUMBER =  1
ITITLE   LOAD INCREMENT 1                                                          INCREMENT NUMBER =  1

   CONCENTRATED NODAL LOAD SETS
 SET NO.   NODE I.D.   COMPONENT      LOAD
   1          3            2       1.000000E 00     DISPLACEMENT
   1          4            2       1.000000E 00     DISPLACEMENT
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                    PAGE        9
VTITLE                                                                    VARIABLE STRUCTURE NUMBER =  1
ITITLE   LOAD INCREMENT 1                                                          INCREMENT NUMBER =  1

                    THERMAL NODAL LOAD SET

NODE  I.D.       1            2            3            4
TEMP        1.000000E 00 1.000000E 00 1.000000E 00 1.000000E 00
```

NORMAL STRESS/STRAIN ELEMENT LOAD SET

ELEMENT NUMBER	NSCODE	COMPONENT ONE	COMPONENT TWO
3	0	1.000000E 00	0.0

```
BEGIN   LOADS      CPU = 00:00:12.276     TOD = 23:35:27

END     LOADS      CPU = 00:00:12.350     TOD = 23:35:28
```



```
BEGIN   SOLN       CPU = 00:00:12.396     TOD = 23:35:29

END     SOLN       CPU = 00:00:12.439     TOD = 23:35:29

BEGIN   ELLOOP     CPU = 00:00:12.449     TOD = 23:35:29

END     ELLOOP     CPU = 00:00:12.536     TOD = 23:35:30

RESIDUAL NORM =  3.93750E-01

BEGIN   SOLN       CPU = 00:00:12.543     TOD = 23:35:30

END     SOLN       CPU = 00:00:12.576     TOD = 23:35:30

BEGIN   ELLOOP     CPU = 00:00:12.586     TOD = 23:35:30

END     ELLOOP     CPU = 00:00:12.679     TOD = 23:35:32

RESIDUAL NORM =  2.71552E-01

BEGIN   SOLN       CPU = 00:00:12.682     TOD = 23:35:32

END     SOLN       CPU = 00:00:12.712     TOD = 23:35:32

BEGIN   ELLOOP     CPU = 00:00:12.722     TOD = 23:35:32

END     ELLOOP     CPU = 00:00:12.819     TOD = 23:35:33

RESIDUAL NORM =  3.93750E-01

BEGIN   ELLOOP     CPU = 00:00:12.822     TOD = 23:35:33

END     ELLOOP     CPU = 00:00:12.892     TOD = 23:35:35

RESIDUAL NORM =  1.72561E-07
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                              PAGE      12
VTITLE                                                                          VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 1                                                               INCREMENT NUMBER =   1


                              E N D   O F   L O A D   I N C R E M E N T     1

NO. ELASTIC INTEGRATION POINTS =      4,  NO. PLASTIC INTEGRATION POINTS =      0
     0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,      0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =      1, NO. UPDATES PERFORMED =      0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      4
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  1.7256E-07

BEGIN    BIGSCK        CPU = 00:00:12.942      TOD = 23:35:36

END      BIGSCK        CPU = 00:00:13.075      TOD = 23:35:40

BEGIN    OUTPUT        CPU = 00:00:13.145      TOD = 23:35:41


TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                              PAGE      13
VTITLE                                                                          VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 1                                                               INCREMENT NUMBER =   1

                    CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **     ****************  FORCES  *****************          *************  DISPLACEMENTS  *************
 NO.  I.D.     U                 V                W                 U                V                W
   1    1     -6.4024364E-07   -9.9999970E-01                       0.0              0.0
   2    2      1.6678297E-07   -9.9999911E-01                       7.1525574E-07    0.0
   3    3      5.2095271E-07    9.9999964E-01                       1.2516975E-06    1.2999992E 00
   4    4      2.4960256E-09    9.9999917E-01                       8.2453362E-07   -1.2999992E 00
```

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.0000E 00	2.2925E-07	2.0000E 00	-2.2925E-07	5.2345E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.0000E 00	2.2925E-07	2.0000E 00	-2.2925E-07	5.2345E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-3.0000E-01	1.0000E 00	-3.0000E-01	3.4024E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-3.0000E-01	1.0000E 00	-3.0000E-01	3.4024E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0	-2	8.6667E-01	5.9605E-07	1.3000E 00	0.0	3.4024E-07	0.0	0.0

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 15
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 1 INCREMENT NUMBER = 1

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
1	3	5	4	0.0	2.0000E 00	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
1	3	5	4	2.0000E 00	3.0000E-01	3.0000E-01	3.0000E-01

END OUTPUT CPU = 00:00:13.271 TOD = 23:35:42

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TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 16
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 2 INCREMENT NUMBER = 2

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD SET ONE	=	2.799999E 00
COEFFICIENT FOR CONCENTRATED LOAD SET TWO	=	0.0
COEFFICIENT FOR DISTRIBUTED LOAD SET ONE	=	0.0
COEFFICIENT FOR DISTRIBUTED LOAD SET TWO	=	0.0
COEFFICIENT FOR NODAL TEMPERATURE SET	=	3.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN SET	=	0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)	=	0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)	=	0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)	=	0.0
CREEP TIME	=	5.000000E 00

BEGIN LOADS CPU = 00:00:13.554 TOD = 23:35:47

END LOADS CPU = 00:00:13.641 TOD = 23:35:49

```
TITLE     BCPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                          PAGE        17
VTITLE                                                                   VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 2                                                        INCREMENT NUMBER =   2

BEGIN     SOLN          CPU = 00:00:13.674     TOD = 23:35:49
END       SOLN          CPU = 00:00:13.707     TOD = 23:35:49
BEGIN     ELLCOP        CPU = 00:00:13.714     TOD = 23:35:49
END       ELLCOP        CPU = 00:00:13.801     TOD = 23:35:50
RESIDUAL NORM =  3.81957E-01
BEGIN     SOLN          CPU = 00:00:13.804     TOD = 23:35:50
END       SOLN          CPU = 00:00:13.834     TOD = 23:35:50
BEGIN     ELLCOP        CPU = 00:00:13.840     TOD = 23:35:50
END       ELLCOP        CPU = 00:00:13.930     TOD = 23:35:51
RESIDUAL NORM =  3.33511E-01
BEGIN     SOLN          CPU = 00:00:13.937     TOD = 23:35:51
END       SOLN          CPU = 00:00:13.964     TOD = 23:35:51
BEGIN     ELLCOP        CPU = 00:00:13.964     TOD = 23:35:51
END       ELLCOP        CPU = 00:00:14.060     TOD = 23:35:52
RESIDUAL NORM =  6.71947E-01
BEGIN     ELLCOP        CPU = 00:00:14.070     TOD = 23:35:52
END       ELLCOP        CPU = 00:00:14.160     TOD = 23:35:53
RESIDUAL NORM =  7.73630E-02
BEGIN     SOLN          CPU = 00:00:14.170     TOD = 23:35:53
END       SOLN          CPU = 00:00:14.197     TOD = 23:35:54
BEGIN     ELLCOP        CPU = 00:00:14.203     TOD = 23:35:54
END       ELLCOP        CPU = 00:00:14.290     TOD = 23:35:55
RESIDUAL NORM =  7.12661E-02
```

```
TITLE     SOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                       PAGE     16
VTITLE                                                                       VARIABLE STRUCTURE NUMBER =    1
ITITLE    LOAD INCREMENT 2                                                          INCREMENT NUMBER =    2

BEGIN    SOLN        CPU = 00:00:14.293     TOD = 23:35:55
END      SOLN        CPU = 00:00:14.330     TOD = 23:35:56
BEGIN    ELLOOP      CPU = 00:00:14.330     TOD = 23:35:56
END      ELLOOP      CPU = 00:00:14.423     TOD = 23:35:57
RESIDUAL NORM =  8.69149E-02
BEGIN    ELLOOP      CPU = 00:00:14.426     TOD = 23:35:57
END      ELLOOP      CPU = 00:00:14.503     TOD = 23:35:59
RESIDUAL NORM =  2.62605E-02
BEGIN    SOLN        CPU = 00:00:14.506     TOD = 23:35:59
END      SOLN        CPU = 00:00:14.536     TOD = 23:35:59
BEGIN    ELLOOP      CPU = 00:00:14.536     TOD = 23:35:59
END      ELLOOP      CPU = 00:00:14.653     TOD = 23:36:00
RESIDUAL NORM =  2.96694E-03
BEGIN    ELLOOP      CPU = 00:00:14.662     TOD = 23:36:00
END      ELLOOP      CPU = 00:00:14.746     TOD = 23:36:01
RESIDUAL NORM =  6.66418E-05
BEGIN    SOLN        CPU = 00:00:14.752     TOD = 23:36:02
END      SOLN        CPU = 00:00:14.789     TOD = 23:36:02
BEGIN    ELLOOP      CPU = 00:00:14.792     TOD = 23:36:02
END      ELLOOP      CPU = 00:00:14.879     TOD = 23:36:03
RESIDUAL NORM =  9.17609E-05
BEGIN    MERGE       CPU = 00:00:14.912     TOD = 23:36:03
BEGIN    GENRP       CPU = 00:00:14.915     TOD = 23:36:04
```

```
STIFFNESS GENERATION COMPLETED.        10 PARTITIONS WRITTEN.
END     GENKE      CPU = 00:00:15.009    TOD = 23:36:05
BEGIN   MERSOR     CPU = 00:00:15.009    TOD = 23:36:06
END     MERSOR     CPU = 00:00:15.059    TOD = 23:36:07
END     MERGE      CPU = 00:00:15.059    TOD = 23:36:07
MAXIMUM WAVEFRONT =    4 NODES AT INTERNAL NODE    1
BEGIN   DECOMP     CPU = 00:00:15.105    TOD = 23:36:07
END     DECOMP     CPU = 00:00:15.155    TOD = 23:36:08
BEGIN   BIGSCK     CPU = 00:00:15.162    TOD = 23:36:09
END     BIGSCK     CPU = 00:00:15.212    TOD = 23:36:09
BEGIN   MERGE      CPU = 00:00:15.212    TOD = 23:36:09
BEGIN   GENKB      CPU = 00:00:15.218    TOD = 23:36:09
STIFFNESS GENERATION COMPLETED.        10 PARTITIONS WRITTEN.
END     GENKE      CPU = 00:00:15.308    TOD = 23:36:11
BEGIN   MERSOR     CPU = 00:00:15.311    TOD = 23:36:12
END     MERSOR     CPU = 00:00:15.351    TOD = 23:36:13
END     MERGE      CPU = 00:00:15.351    TOD = 23:36:13
MAXIMUM WAVEFRONT =    4 NODES AT INTERNAL NODE    1
BEGIN   DECOMP     CPU = 00:00:15.405    TOD = 23:36:13
END     DECOMP     CPU = 00:00:15.471    TOD = 23:36:14
BEGIN   BIGSCK     CPU = 00:00:15.475    TOD = 23:36:14
END     BIGSCK     CPU = 00:00:15.508    TOD = 23:36:15
BEGIN   SOLN       CPU = 00:00:15.544    TOD = 23:36:15
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                   PAGE      20
VTITLE                                                                     VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 2                                                         INCREMENT NUMBER =   2

END      SOLN          CPU = 00:00:15.579      TOD = 23:36:16

BEGIN    ELLCOP        CPU = 00:00:15.588      TOD = 23:36:16

END      ELLCOP        CPU = 00:00:15.674      TOD = 23:36:18

RESIDUAL NORM =  1.05262E-05

BEGIN    SOLN          CPU = 00:00:15.681      TOD = 23:36:18

END      SOLN          CPU = 00:00:15.714      TOD = 23:36:19

BEGIN    ELLCOP        CPU = 00:00:15.721      TOD = 23:36:19

END      ELLCOP        CPU = 00:00:15.821      TOD = 23:36:21

RESIDUAL NORM =  9.07626E-07


                        E N D   O F   L O A D   I N C R E M E N T   2

NO. ELASTIC INTEGRATION POINTS =    0,  NO. PLASTIC INTEGRATION POINTS =    4
    4  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,     0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     1, NO. UPDATES PERFORMED =     1
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      2
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  9.0762E-07

BEGIN    BIGSCK        CPU = 00:00:15.867      TOD = 23:36:21

END      BIGSCK        CPU = 00:00:15.954      TOD = 23:36:25

BEGIN    OUTPUT        CPU = 00:00:16.014      TOD = 23:36:26


TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                   PAGE      21
VTITLE                                                                     VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 2                                                         INCREMENT NUMBER =   2

                   CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **    ****************  FORCES  ****************          *************  DISPLACEMENTS  *************
 NO.  I.D.     U                 V                W                U               V               W
   1    1    1.9595609E-06  -1.2499981E 00                        0.0             0.0
   2    2   -1.9047679E-06  -1.2500000E 00                       -1.7403427E-06   0.0
   3    3   -1.3140379E-06   1.2500000E 00                       -3.3221113E-07   2.7999992E 00
   4    4    1.3142995E-06   1.2499990E 00                       -6.2216108E-07   2.7999992E 00
```

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	XX	YY	ZZ	XY	XZ	YZ
					CUMULATIVE STRESSES					
1	3	5	4	2.5000E 00	-4.8715E-06	2.5000E 00	-4.5850E-06	2.6039E-10	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	XX	YY	ZZ	XY	XZ	YZ
					INCREMENTAL STRESSES					
1	3	5	4	5.0000E-01	-5.1008E-06	5.0000E-01	-4.3557E-06	-5.2319E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	XX	YY	ZZ	XY	XZ	YZ
				CUMULATIVE ELASTIC STRAINS					
1	3	5	4	-3.0000E-01	1.0000E 00	-3.0000E-01	1.3540E-10	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	XX	YY	ZZ	XY	XZ	YZ
				INCREMENTAL ELASTIC STRAINS					
1	3	5	4	-3.4305E-06	3.5763E-07	-7.7486E-07	-3.4011E-07	0.0	0.0

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ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	XX	YY	ZZ	XY	XZ	YZ
					CUMULATIVE PLASTIC STRAINS					
1	3	5	4	1.1250E 00	-2.5000E-01	5.0000E-01	-2.5000E-01	9.8186E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	XX	YY	ZZ	XY	XZ	YZ
					INCREMENTAL PLASTIC STRAINS					
1	3	5	4	1.1250E 00	-2.5000E-01	5.0000E-01	-2.5000E-01	9.8186E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	XX	YY	ZZ	XY	XZ	YZ
					CUMULATIVE CREEP STRAINS					
1	3	5	4	1.1250E 00	-2.5000E-01	5.0000E-01	-2.5000E-01	9.8186E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	XX	YY	ZZ	XY	XZ	YZ
					INCREMENTAL CREEP STRAINS					
1	3	5	4	1.1250E 00	-2.5000E-01	5.0000E-01	-2.5000E-01	9.8186E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	XX	YY	ZZ	XY	XZ	YZ
							CUMULATIVE TOTAL STRAINS					
1	3	5	4	1	2	1.8667E 00	-7.1526E-07	2.5000E 00	0.0	1.9651E-07	0.0	0.0

```
TITLE     EOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                          PAGE      23
VTITLE                                                                  VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 2                                                       INCREMENT NUMBER =   2

  ELEMENT      POINT              YIELD        YIELD       **** EFFECTIVE PLASTIC STRAINS ****   ***** EFFECTIVE CREEP STRAINS *****
 NO.  I.D.   NO. TP.          STRESS CTR. STRESS SIZE    INCREMENTAL  SUM INCR.   CUMULATIVE INCREMENTAL  SUM INCR.   CUMULATIVE
  1     3      5  4             5.0000E-01  2.0000E 00    5.0000E-01  5.0000E-01  5.0000E-01  5.0000E-01  5.0000E-01  5.0000E-01

  ELEMENT      POINT            CUMULATIVE                 **** CUMULATIVE THERMAL STRAINS ****
 NO.  I.D.   NO. TP.          TEMPERATURE                     XX          YY          ZZ
  1     3      5  4             3.0000E 00                9.0000E-01  8.0000E-01  8.0000E-01

END       OUTPUT        CPU = 00:00:16.120      TOD = 23:36:28
```

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```
TITLE     EOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                          PAGE      24
VTITLE                                                                  VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 3                                                       INCREMENT NUMBER =   3

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE        =    3.549999E 00
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO        =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE         =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO         =    0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET             =    4.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET          =    0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)      =    0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                =    0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)       =    0.0
CREEP TIME                                         =    0.0

BEGIN    LOADS          CPU = 00:00:16.410      TOD = 23:36:32

END      LOADS          CPU = 00:00:16.503      TOD = 23:36:33
```

```
BEGIN   SOLN      CPU = 00:00:16.559    TOD = 23:36:34
END     SOLN      CPU = 00:00:16.596    TOD = 23:36:35
BEGIN   ELLOOP    CPU = 00:00:16.596    TOD = 23:36:35
END     ELLOOP    CPU = 00:00:16.679    TOD = 23:36:36
RESIDUAL NORM =  2.56493E-01
BEGIN   SOLN      CPU = 00:00:16.679    TOD = 23:36:36
END     SOLN      CPU = 00:00:16.713    TOD = 23:36:36
BEGIN   ELLOOP    CPU = 00:00:16.716    TOD = 23:36:37
END     ELLOOP    CPU = 00:00:16.802    TOD = 23:36:38
RESIDUAL NORM =  1.69400E-01
BEGIN   SOLN      CPU = 00:00:16.802    TOD = 23:36:38
END     SOLN      CPU = 00:00:16.832    TOD = 23:36:38
BEGIN   ELLOOP    CPU = 00:00:16.836    TOD = 23:36:39
END     ELLOOP    CPU = 00:00:16.935    TOD = 23:36:40
RESIDUAL NORM =  5.46494E-02
BEGIN   SOLN      CPU = 00:00:16.939    TOD = 23:36:40
END     SOLN      CPU = 00:00:16.974    TOD = 23:36:40
BEGIN   ELLOOP    CPU = 00:00:16.974    TOD = 23:36:40
END     ELLOOP    CPU = 00:00:17.069    TOD = 23:36:42
RESIDUAL NORM =  3.44137E-03
BEGIN   SOLN      CPU = 00:00:17.075    TOD = 23:36:42
END     SOLN      CPU = 00:00:17.105    TOD = 23:36:42
BEGIN   ELLOOP    CPU = 00:00:17.112    TOD = 23:36:42
```

```
END     ELLOOP      CPU = 00:00:17.208     TOD = 23:36:44
RESIDUAL NORM =  1.1C757E-03
BEGIN   SOLN        CPU = 00:00:17.218     TOD = 23:36:44
END     SOLN        CPU = 00:00:17.242     TOD = 23:36:44
BEGIN   ELLOOP      CPU = 00:00:17.246     TOD = 23:36:44
END     ELLOOP      CPU = 00:00:17.322     TOD = 23:36:45
RESIDUAL NORM =  7.263626E-05
BEGIN   SOLN        CPU = 00:00:17.332     TOD = 23:36:45
END     SOLN        CPU = 00:00:17.355     TOD = 23:36:46
BEGIN   ELLOOP      CPU = 00:00:17.361     TOD = 23:36:46
END     ELLOOP      CPU = 00:00:17.445     TOD = 23:36:47
RESIDUAL NORM =  2.373191E-05
BEGIN   SOLN        CPU = 00:00:17.445     TOD = 23:36:47
END     SOLN        CPU = 00:00:17.475     TOD = 23:36:48
BEGIN   ELLOOP      CPU = 00:00:17.476     TOD = 23:36:48
END     ELLOOP      CPU = 00:00:17.558     TOD = 23:36:49
RESIDUAL NORM =  1.997961E-06


                 E N D   O F   L O A D   I N C R E M E N T     3

NO. ELASTIC INTEGRATION POINTS =     2,  NO. PLASTIC INTEGRATION POINTS =     4
    2 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,     0 INTEGRATION POINTS PLASTIC-TO-ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     1, NO. UPDATES PERFORMED =      0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =     8
SPECIFIED MAX. UNBALANCED FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  1.9980E-06
```

```
TITLE    BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                PAGE      27
VTITLE                                                              VARIABLE STRUCTURL NUMBER =   1
ITITLE   LOAD INCREMENT 3                                                   INCREMENT NUMBER =   3

BEGIN   BIGSCK        CPU = 00:00:17.604      TOD = 23:36:49

END     BIGSCK        CPU = 00:00:17.695      TOD = 23:36:50

BEGIN   OUTPUT        CPU = 00:00:17.744      TOD = 23:36:51
```



```
TITLE    BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                PAGE      28
VTITLE                                                              VARIABLE STRUCTURE NUMBER =   1
ITITLE   LOAD INCREMENT 3                                                   INCREMENT NUMBER =   3

                CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS
```

** NODE **		**************** FORCES	*****************		*************	DISPLACEMENTS	*************
NO.	I.D.	U	V	W	U	V	W
1	1	4.34616951E-06	-1.50000000E 00		0.0	0.0	
2	2	4.34620694E-06	-1.50000194E 00		-3.79125418E-06	0.0	
3	3	3.62077C5E-06	1.50000194E 00		-3.19055115E-06	3.54999992E 00	
4	4	-3.62074300E-06	1.50000000E 00		-9.67542006E-06	3.54999992E 00	

TITLE BOFACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
VTITLE
ITITLE LOAD INCREMENT 3

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VARIABLE STRUCTURE NUMBER = 1
INCREMENT NUMBER = 3

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	3.0000E 00	-4.8142E-06	3.0000E 00	0.0	-2.3716E-11	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.0000E-01	5.7311E-08	5.0000E-01	4.5850E-06	-2.8410E-10	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-3.0000E-01	1.0000E 00	-3.0000E-01	-1.0274E-11	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	-8.9407E-07	1.6689E-06	-1.4566E-10	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.5000E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	7.6186E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.3750E 00	-2.5000E-01	5.0000E-01	-2.5000E-01	-2.1998E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.1250E 00	-2.5000E-01	5.0000E-01	-2.5000E-01	9.8186E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0	2	2.3667E 00	-2.8610E-06	3.5500E 00	1.9073E-06	1.7436E-07	0.0	0.0

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                              PAGE    30
VTITLE                                                                                        VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 3                                                                             INCREMENT NUMBER =   3

  ELEMENT       POINT                   YIELD         YIELD        **** EFFECTIVE PLASTIC STRAINS ****   ***** EFFECTIVE CREEP STRAINS *****
  NO.  I.D.   NO. TP.              STRESS CTR. STRESS SIZE     INCREMENTAL   SUM INCR.   CUMULATIVE INCREMENTAL   SUM INCR.   CUMULATIVE
   1     3      5  4                1.0000E 00  2.0000E 00      5.0000E-01  1.0000E 00  1.0000E 00  0.0          5.0000E-01  5.0000E-01

  ELEMENT       POINT                CUMULATIVE            **** CUMULATIVE THERMAL STRAINS ****
  NO.  I.D.   NO. TP.               TEMPERATURE                XX          YY          ZZ
   1     3      5  4                4.0000E 00              1.0500E 00  1.0500E 00  1.0500E 00

END     OUTPUT        CPU = 00:00:17.667    TOD = 23:36:52
```

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```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                              PAGE    31
VTITLE                                                                                        VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 4                                                                             INCREMENT NUMBER =   4

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE          =   5.799999E 00
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO          =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE           =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO           =   0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET               =   5.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET            =   0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)        =   0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                  =   0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)         =   0.0
CREEP TIME                                           =   4.000000E 00

BEGIN   LOADS         CPU = 00:00:18.154    TOD = 23:36:59

END     LOADS         CPU = 00:00:18.240    TOD = 23:37:00
```

```
BEGIN   SOLN      CPU = 00:00:18.293     TOD = 23:37:01
END     SOLN      CPU = 00:00:18.333     TOD = 23:37:01
BEGIN   ELLOOP    CPU = 00:00:18.353     TOD = 23:37:01
END     ELLOOP    CPU = 00:00:18.416     TOD = 23:37:03
RESIDUAL NORM =  4.287181E-01
BEGIN   SOLN      CPU = 00:00:18.423     TOD = 23:37:03
END     SOLN      CPU = 00:00:18.443     TOD = 23:37:03
BEGIN   ELLOOP    CPU = 00:00:18.450     TOD = 23:37:03
END     ELLOOP    CPU = 00:00:18.526     TOD = 23:37:04
RESIDUAL NORM =  3.553161E-01
BEGIN   SOLN      CPU = 00:00:18.530     TOD = 23:37:04
END     SOLN      CPU = 00:00:18.563     TOD = 23:37:05
BEGIN   ELLOOP    CPU = 00:00:18.566     TOD = 23:37:06
END     ELLOOP    CPU = 00:00:18.650     TOD = 23:37:07
RESIDUAL NORM =  3.11299E-01
BEGIN   SOLN      CPU = 00:00:18.656     TOD = 23:37:07
END     SOLN      CPU = 00:00:18.689     TOD = 23:37:08
BEGIN   ELLOOP    CPU = 00:00:18.699     TOD = 23:37:08
END     ELLOOP    CPU = 00:00:18.789     TOD = 23:37:10
RESIDUAL NORM =  3.13393E-01
BEGIN   ELLOOP    CPU = 00:00:18.796     TOD = 23:37:10
END     ELLOOP    CPU = 00:00:18.886     TOD = 23:37:12
RESIDUAL NORM =  1.02620E-01
```

```
BEGIN   SOLN      CPU = 00:00:18.869   TOD = 23:37:12
END     SOLN      CPU = 00:00:18.929   TOD = 23:37:13
BEGIN   ELLCOP    CPU = 00:00:18.936   TOD = 23:37:13
END     ELLCOP    CPU = 00:00:19.025   TOD = 23:37:14
RESIDUAL NORM =  5.16993E-02
BEGIN   SOLN      CPU = 00:00:19.029   TOD = 23:37:15
END     SOLN      CPU = 00:00:19.055   TOD = 23:37:15
BEGIN   ELLCOP    CPU = 00:00:19.069   TOD = 23:37:15
END     ELLCOP    CPU = 00:00:19.169   TOD = 23:37:18
RESIDUAL NORM =  1.06628E-02
BEGIN   SOLN      CPU = 00:00:19.172   TOD = 23:37:18
END     SOLN      CPU = 00:00:19.195   TOD = 23:37:19
BEGIN   ELLCOP    CPU = 00:00:19.202   TOD = 23:37:19
END     ELLCOP    CPU = 00:00:19.288   TOD = 23:37:21
RESIDUAL NORM =  1.91236E-03
BEGIN   SOLN      CPU = 00:00:19.292   TOD = 23:37:21
END     SOLN      CPU = 00:00:19.308   TOD = 23:37:22
BEGIN   ELLCOP    CPU = 00:00:19.317   TOD = 23:37:22
END     ELLCOP    CPU = 00:00:19.402   TOD = 23:37:25
RESIDUAL NORM =  3.15490E-04
BEGIN   SOLN      CPU = 00:00:19.408   TOD = 23:37:25
END     SOLN      CPU = 00:00:19.435   TOD = 23:37:26
BEGIN   ELLCOP    CPU = 00:00:19.439   TOD = 23:37:26
```

```
END      ELLOCP      CPU = 00:00:19.538     TOD = 23:37:29
RESIDUAL NORM =  5.05974E-05
BEGIN    MERGE       CPU = 00:00:19.585     TOD = 23:37:30
BEGIN    GENRB       CPU = 00:00:19.585     TOD = 23:37:30
STIFFNESS GENERATION COMPLETED.        10 PARTITIONS WRITTEN.
END      GENRB       CPU = 00:00:19.668     TOD = 23:37:32
BEGIN    MERSOR      CPU = 00:00:19.674     TOD = 23:37:32
END      MERSOR      CPU = 00:00:19.736     TOD = 23:37:33
END      MERGE       CPU = 00:00:19.736     TOD = 23:37:33
MAXIMUM WAVEFRONT =    4 NODES AT INTERNAL NODE     1
BEGIN    DECOMP      CPU = 00:00:19.788     TOD = 23:37:34
END      DECOMP      CPU = 00:00:19.841     TOD = 23:37:35
BEGIN    BIGSCK      CPU = 00:00:19.847     TOD = 23:37:35
END      BIGSCK      CPU = 00:00:19.961     TOD = 23:37:36
BEGIN    MERGE       CPU = 00:00:19.961     TOD = 23:37:37
BEGIN    GENRB       CPU = 00:00:19.967     TOD = 23:37:37
STIFFNESS GENERATION COMPLETED.        10 PARTITIONS WRITTEN.
END      GENRB       CPU = 00:00:19.977     TOD = 23:37:38
BEGIN    MERSOR      CPU = 00:00:19.981     TOD = 23:37:38
END      MERSOR      CPU = 00:00:20.040     TOD = 23:37:38
END      MERGE       CPU = 00:00:20.040     TOD = 23:37:38
MAXIMUM WAVEFRONT =    4 NODES AT INTERNAL NODE     1
BEGIN    DECOMP      CPU = 00:00:20.06-     TOD = 23:37:39
```

```
END     DECOMP    CPU = 00:00:20.137    TOD = 23:37:42
BEGIN   BIGSCK    CPU = 00:00:20.160    TOD = 23:37:42
END     BIGSCK    CPU = 00:00:20.2?4    TOD = 23:37:43
BEGIN   SOLN      CPU = 00:00:2?.240    TOD = 23:37:44
END     SOLN      CPU = 00:00:20.267    TOD = 23:37:45
BEGIN   ELLOOP    CPU = 00:00:20.273    TOD = 23:37:45
END     ELLOOP    CPU = 00:00:20.370    TOD = 23:37:50
RESIDUAL NORM =  7.31443E-06

                    E N D  O F  L O A D  I N C R E M E N T   4

NO. ELASTIC INTEGRATION POINTS =     0, NO. PLASTIC INTEGRATION POINTS =     4
    0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,    0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     1, NO. UPDATES PERFORMED =     1
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =     1
SPECIFIED MAX. UNBALANCED FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  7.3144E-06

BEGIN   BIGSCK    CPU = 00:00:20.420    TOD = 23:37:51
END     BIGSCK    CPU = 00:00:20.496    TOD = 23:37:52
BEGIN   OUTPUT    CPU = 00:00:20.546    TOD = 23:37:52
```

B.4-32

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE ** NO.	I.D.	FORCES U	FORCES V	FORCES W	DISPLACEMENTS U	DISPLACEMENTS V	DISPLACEMENTS W
1	1	1.66032??E-05	-1.7499965E 00		0.0	0.0	
2	2	-1.6601738E-05	-1.7499965E 00		-1.5709433E-05	0.0	
3	3	-1.6600447E-05	1.7499965E 00		-1.6277417E-05	5.7999992E 00	
4	4	1.6599???E-05	1.7499965E 00		1.0506410E-06	5.7999992E 00	

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	3.5000E 00	-4.9747E-05	3.5000E 00	-3.2095E-05	-1.4544E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.0000E-01	-4.4933E-05	4.9996E-01	-3.2095E-05	-1.4307E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-3.0001E-01	9.9999E-01	-3.0000E-01	-5.4021E-10	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-6.4373E-06	-3.9935E-06	-2.3246E-06	-5.2994E-10	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.7500E 00	-1.0000E 00	2.0000E 00	-1.0000E 00	4.1575E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	3.2500E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	-3.4613E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.7500E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	8.0879E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.6250E 00	-2.5000E-01	5.0000E-01	-2.5000E-01	-1.7307E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0	2	3.8667E 00	-1.4305E-05	5.0000E 00	2.8010E-06	1.2191E-07	0.0	0.0

```
TITLE     SCPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                   PAGE      38
VTITLE                                                                                   VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 4                                                                        INCREMENT NUMBER =   4

   ELEMENT     POINT              YIELD       YIELD       **** EFFECTIVE PLASTIC STRAINS ****   ***** EFFECTIVE CREEP STRAINS *****
  NO.  I.D.  NO. TP.           STRESS CTR. STRESS SIZE   INCREMENTAL  SUM INCR.   CUMULATIVE INCREMENTAL  SUM INCR.   CUMULATIVE
   1    3     5  4              1.5000E 00  2.0000E 00    1.0000E 00  2.0000E 00  2.0000E 00  5.0000E-01  1.0000E 00  1.0000E 00

   ELEMENT     POINT           CUMULATIVE             **** CUMULATIVE THERMAL STRAINS ****
  NO.  I.D.  NO. TP.           TEMPERATURE                XX          YY          ZZ
   1    3     5  4              5.0000E 00            1.9000E 00  1.8000E 00  1.8000E 00

END      OUTPUT      CPU = 00:00:20.673      TOD = 23:37:54
```

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```
TITLE     SCPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                   PAGE      39
VTITLE                                                                                   VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 5                                                                        INCREMENT NUMBER =   5

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE        =   7.299999E 00
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO        =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE         =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO         =   0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET             =   6.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET          =   0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)      =   0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                =   0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)       =   0.0
CREEP TIME                                         =   0.0

BEGIN    LOADS       CPU = 00:00:20.926      TOD = 23:37:59

END      LOADS       CPU = 00:00:21.019      TOD = 23:38:01
```

```
BEGIN   SOLN      CPU = 00:00:21.082     TOD = 23:38:01
END     SOLN      CPU = 00:00:21.115     TOD = 23:38:02
BEGIN   ELLOOP    CPU = 00:00:21.122     TOD = 23:38:02
END     ELLOOP    CPU = 00:00:21.219     TOD = 23:38:03
RESIDUAL NORM =  3.53994E-01
BEGIN   SOLN      CPU = 00:00:21.222     TOD = 23:38:03
END     SOLN      CPU = 00:00:21.252     TOD = 23:38:03
BEGIN   ELLOOP    CPU = 00:00:21.259     TOD = 23:38:04
END     ELLOOP    CPU = 00:00:21.365     TOD = 23:38:05
RESIDUAL NORM =  2.04209E-01
BEGIN   SOLN      CPU = 00:00:21.368     TOD = 23:38:05
END     SOLN      CPU = 00:00:21.395     TOD = 23:38:06
BEGIN   ELLOOP    CPU = 00:00:21.398     TOD = 23:38:06
END     ELLOOP    CPU = 00:00:21.485     TOD = 23:38:08
RESIDUAL NORM =  1.46058E-02
BEGIN   SOLN      CPU = 00:00:21.488     TOD = 23:38:08
END     SOLN      CPU = 00:00:21.515     TOD = 23:38:09
BEGIN   ELLOOP    CPU = 00:00:21.518     TOD = 23:38:10
END     ELLOOP    CPU = 00:00:21.601     TOD = 23:38:11
RESIDUAL NORM =  5.65962E-04
BEGIN   SOLN      CPU = 00:00:21.608     TOD = 23:38:11
END     SOLN      CPU = 00:00:21.635     TOD = 23:38:12
BEGIN   ELLOOP    CPU = 00:00:21.641     TOD = 23:38:12
```

TITLE · BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
VTITLE
ITITLE LOAD INCREMENT 5

PAGE 41
VARIABLE STRUCTURE NUMBER = 1
INCREMENT NUMBER = 5

```
END      ELLOOP        CPU = 00:00:21.731     TOD = 23:38:14
RESIDUAL NORM =  1.62866E-05
BEGIN    SOLN          CPU = 00:00:21.741     TOD = 23:38:14
END      SOLN          CPU = 00:00:21.768     TOD = 23:38:15
BEGIN    ELLOOP        CPU = 00:00:21.769     TOD = 23:38:15
END      ELLOOP        CPU = 00:00:21.854     TOD = 23:38:16
RESIDUAL NORM =  7.68373E-07
```

END OF LOAD INCREMENT 5

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```
NO. ELASTIC INTEGRATION POINTS =     0, NO. PLASTIC INTEGRATION POINTS =     4
    0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,    0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     1, NO. UPDATES PERFORMED =     0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =     6
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  7.6837E-07

BEGIN    BIGSOK        CPU = 00:00:21.907     TOD = 23:38:17
END      BIGSOK        CPU = 00:00:21.977     TOD = 23:38:18
BEGIN    OUTPUT        CPU = 00:00:22.031     TOD = 23:38:19
```

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
VTITLE
ITITLE LOAD INCREMENT 5

PAGE 42
VARIABLE STRUCTURE NUMBER = 1
INCREMENT NUMBER = 5

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE ** NO.	I.D.	FORCES U	FORCES V	FORCES W	DISPLACEMENTS U	DISPLACEMENTS V	DISPLACEMENTS W
1	1	1.4274651E-06	-1.9999933E 00		0.0	0.0	
2	2	-1.4524250E-06	-1.9999933E 00		-4.1739430E-06	0.0	
3	3	-2.4447918E-06	1.9999933E 00		-8.9237583E-06	7.2999992E 00	
4	4	2.4597517E-06	1.9999933E 00		-4.2554916E-08	7.2999992E 00	

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 43
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 5 INCREMENT NUMBER = 5

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	4.0000E 00	-2.1091E-05	4.0000E 00	-3.2553E-05	-3.5641E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.0001E-01	2.8656E-05	5.0003E-01	-4.5850E-07	-3.3586E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-3.0000E-01	1.0000E 00	-3.0001E-01	-1.1388E-06	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.7817E-06	5.2452E-06	-3.9339E-06	-1.0648E-06	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	9.5000E 00	-1.5000E 00	3.0000E 00	-1.5000E 00	3.7063E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	3.7500E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	-3.7869E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.7500E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	8.0879E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0	2	4.6667E 00	-7.6294E-06	7.3000E 00	9.5367E-07	7.3197E-08	0.0	0.0

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                              PAGE      44
VTITLE                                                                                                   VARIABLE STRUCTURE NUMBER =  1
ITITLE    LOAD INCREMENT 5                                                                                       INCREMENT NUMBER =  5

 ELEMENT     POINT                   YIELD        YIELD       **** EFFECTIVE PLASTIC STRAINS ****  ***** EFFECTIVE CREEP STRAINS *****
 NO.  I.D.  NO. TP.              STRESS CTR. STRESS SIZE    INCREMENTAL  SUM INCR.   CUMULATIVE INCREMENTAL  SUM INCR.   CUMULATIVE
  1    3     5  4                 2.0000E 00  2.0000E 00    1.0000E 00  3.0000E 00  3.0000E 00  0.0         1.0000E 00  1.0000E 00

 ELEMENT     POINT               CUMULATIVE                 **** CUMULATIVE THERMAL STRAINS ****
 NO.  I.D.  NO. TP.              TEMPERATURE                   XX          YY          ZZ
  1    3     5  4                 6.0000E 00                2.3000E 00  2.3000E 00  2.3000E 00

END      OUTPUT      CPU = 00:00:22.160      TOD = 23:38:20
```

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```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                              PAGE      45
VTITLE                                                                                                   VARIABLE STRUCTURE NUMBER =  1
ITITLE    LOAD INCREMENT 6                                                                                       INCREMENT NUMBER =  6

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE          =    7.299999E 00
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO          =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE           =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO           =    0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET               =    7.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET            =    0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)        =    0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                  =    0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)         =    0.0
CREEP TIME                                           =    0.0

BEGIN    LOADS       CPU = 00:00:22.417      TOD = 23:38:25

END      LOADS       CPU = 00:00:22.500      TOD = 23:38:27
```

```
TITLE     EDPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                  PAGE      46
VTITLE                                                              VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 6                                                   INCREMENT NUMBER =   6

BEGIN    SOLN        CPU = 00:00:22.560     TOD = 23:38:27

END      SOLN        CPU = 00:00:22.596     TOD = 23:38:28

BEGIN    ELLOOP      CPU = 00:00:22.600     TOD = 23:38:28

END      ELLOOP      CPU = 00:00:22.686     TOD = 23:38:30

RESIDUAL NORM =  1.7817E-06



                        E N D   O F   L O A D   I N C R E M E N T   6

NO. ELASTIC INTEGRATION POINTS =     4,  NO. PLASTIC INTEGRATION POINTS =     0
     0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,    4  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =      1, NO. UPDATES PERFORMED =     0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      1
SPECIFIED MAX. UNBALANCED FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  1.7817E-06

BEGIN    SIGSCK      CPU = 00:00:22.736     TOD = 23:38:31

END      SIGSCK      CPU = 00:00:22.816     TOD = 23:38:32

BEGIN    OUTPUT      CPU = 00:00:22.876     TOD = 23:38:33




TITLE     EDPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                  PAGE      47
VTITLE                                                              VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 6                                                   INCREMENT NUMBER =   6

                   CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **   ****************  FORCES  *****************        *************  DISPLACEMENTS  *************
 NO.  I.D.        U               V               W                  U               V               W
   1    1    2.1570495E-06  -1.0000000E 00                       0.0             0.0
   2    2   -2.1603951E-06  -9.9999905E-01                      -5.9209345E-06   0.0
   3    3   -2.4819601E-06   9.9999901E-01                      -8.0735659E-06   7.2999992E 00
   4    4    2.4844903E-06   1.0000000E 00                      -4.0552260E-07   7.2999992E 00
```

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
VTITLE
ITITLE LOAD INCREMENT 6

VARIABLE STRUCTURE NUMBER = 1
INCREMENT NUMBER = 6

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.0000E 00	-7.7445E-06	2.0000E 00	-1.6506E-05	2.5012E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.0000E 00	1.3296E-05	-2.0000E 00	1.6047E-05	3.7542E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-3.0000E-01	1.0000E 00	-3.0001E-01	1.6258E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	8.9407E-07	9.5367E-07	-5.9605E-07	1.3614E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	9.5000E 00	-1.5000E 00	3.0000E 00	-1.5000E 00	3.7063E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.7500E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	8.0879E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-1	-1	4.6667E 00	-6.6757E-06	7.3000E 00	9.5367E-07	8.6211E-06	0.0	0.0

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 49
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 6 INCREMENT NUMBER = 6

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	**** EFFECTIVE PLASTIC STRAINS **** INCREMENTAL	SUM INCR.	CUMULATIVE	***** EFFECTIVE CREEP STRAINS ***** INCREMENTAL	SUM INCR.	CUMULATIVE
1	3	5	4	2.0000E 00	2.0000E 00	0.0	3.0000E 00	3.0000E 00	0.0	1.0000E 00	1.0000E 00

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	**** CUMULATIVE THERMAL STRAINS **** XX	YY	ZZ
1	3	5	4	7.0000E 00	2.3000E 00	2.3000E 00	2.3000E 00

END OUTPUT CPU = 00:00:23.019 TOD = 23:38:35

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 50
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 7 INCREMENT NUMBER = 7

PARAMETERS FOR THIS INCREMENT

MAXIMUM SPECIFIED ERROR NORM = 1.000000E-05
SOLUTION METHOD CODE = 4
MAXIMUM NO. STIFFNESS UPDATES PER INCREMENT = 1
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE ONE = 0
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE TWO = 2
MAXIMUM NUMBER OF ITERATIONS BEFORE UPDATE THREE AND UP = 10
MAXIMUM ELASTIC ITERATIONS PER INCREMENT = 2
MAXIMUM MAGNITUDE FOR ELASTIC-PLASTIC SUM CODE = 2
MAXIMUM REDUCTIONS = 1
CONVERGENCE REDUCTION FACTOR = 5.000000E-01
FRACTION FROM END OF INCREMENT TO EVALUATE SLOPE = 1.000000E-01

COEFFICIENT FOR CONCENTRATED LOAD SET ONE = 6.000000E 00
COEFFICIENT FOR CONCENTRATED LOAD SET TWO = 0.0
COEFFICIENT FOR DISTRIBUTED LOAD SET ONE = 0.0
COEFFICIENT FOR DISTRIBUTED LOAD SET TWO = 0.0
COEFFICIENT FOR NODAL TEMPERATURE SET = 5.400000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN SET = 0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME) = 0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME) = 0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME) = 0.0
CREEP TIME = 0.0

BEGIN LOADS CPU = 00:00:23.205 TOD = 23:38:47

END LOADS CPU = 00:00:23.300 TOD = 23:38:50

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                    PAGE      51
VTITLE                                                                  VARIABLE STRUCTURE NUMBER =  1
ITITLE    LOAD INCREMENT 7                                                       INCREMENT NUMBER =  7

BEGIN   MERGE        CPU = 00:00:23.462     TOD = 23:38:50
BEGIN   GENR8        CPU = 00:00:23.475     TOD = 23:38:50
STIFFNESS GENERATION COMPLETED.       10 PARTITIONS WRITTEN.
END     GENR8        CPU = 00:00:23.555     TOD = 23:38:53
BEGIN   MERSOR       CPU = 00:00:23.558     TOD = 23:38:53
END     MERSOR       CPU = 00:00:23.615     TOD = 23:38:54
END     MERGE        CPU = 00:00:23.615     TOD = 23:38:54
MAXIMUM WAVEFRONT =      4 NODES AT INTERNAL NODE     1
BEGIN   DECCMP       CPU = 00:00:23.658     TOD = 23:38:54
END     DECCMP       CPU = 00:00:23.725     TOD = 23:38:55
BEGIN   EIGSCK       CPU = 00:00:23.730     TOD = 23:38:55
END     EIGSCK       CPU = 00:00:23.768     TOD = 23:38:56
BEGIN   MERGE        CPU = 00:00:23.791     TOD = 23:38:56
BEGIN   GENR8        CPU = 00:00:23.791     TOD = 23:38:56
STIFFNESS GENERATION COMPLETED.       10 PARTITIONS WRITTEN.
END     GENR8        CPU = 00:00:23.864     TOD = 23:38:58
BEGIN   MERSOR       CPU = 00:00:23.871     TOD = 23:38:58
END     MERSOR       CPU = 00:00:23.934     TOD = 23:38:59
END     MERGE        CPU = 00:00:23.934     TOD = 23:38:59
MAXIMUM WAVEFRONT =      4 NODES AT INTERNAL NODE     1
BEGIN   DECCMP       CPU = 00:00:23.981     TOD = 23:38:59
END     DECCMP       CPU = 00:00:24.044     TOD = 23:38:59
BEGIN   EIGSCK       CPU = 00:00:24.057     TOD = 23:39:00
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                        PAGE     52
VTITLE                                                                         VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 7                                                              INCREMENT NUMBER =   7

END      BIGSCK         CPU = 00:00:24.104      TOD = 23:39:00

BEGIN    SCLN           CPU = 00:00:24.144      TOD = 23:39:00

END      SCLN           CPU = 00:00:24.170      TOD = 23:39:01

BEGIN    ELLOOP         CPU = 00:00:24.186      TOD = 23:39:01

END      ELLOOP         CPU = 00:00:24.290      TOD = 23:39:04

RESIDUAL NORM =   5.25000F-01

BEGIN    SCLN           CPU = 00:00:24.294      TOD = 23:39:04

END      SCLN           CPU = 00:00:24.320      TOD = 23:39:04

BEGIN    ELLOOP         CPU = 00:00:24.327      TOD = 23:39:04

END      ELLOOP         CPU = 00:00:24.407      TOD = 23:39:06

RESIDUAL NORM =   3.00347E-01

                         E N D   O F   L O A D   I N C R E M E N T    7

NO. ELASTIC INTEGRATION POINTS =     4,  NO. PLASTIC INTEGRATION POINTS =     0
    0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,     0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =      1, NO. UPDATES PERFORMED =      1
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =     0   2   10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      2
SPECIFIED MAX. UNBALANCED FORCE ERROR =  1.0000F-05, ACTUAL ERROR =  3.0035E-01

BEGIN    BIGSCK         CPU = 00:00:24.433      TOD = 23:39:07

END      BIGSCK         CPU = 00:00:24.510      TOD = 23:39:09

BEGIN    OUTPUT         CPU = 00:00:24.563      TOD = 23:39:09


TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                        PAGE     53
VTITLE                                                                         VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 7                                                              INCREMENT NUMBER =   7

                   CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **     ***************  FORCES  ****************           *************  DISPLACEMENTS  *************
 NO.  I.D.        U               V               W                U              V                W
   1    1    -3.4211227E-07  -8.8452851E-07                   0.0            0.0
   2    2     2.4007511E-07  -1.2956651E-07                  -8.3446503E-06  0.0
   3    3     4.7635586E-07   2.6262349E-07                  -7.6678131E-06  0.0000000E 00
   4    4    -2.2374088E-07   7.3181598E-07                   6.9729006E-07  0.0000000E 00
```

TITLE   BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
VTITLE
ITITLE   LOAD INCREMENT 7

VARIABLE STRUCTURE NUMBER = 1
INCREMENT NUMBER = 7

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.2272E-06	2.0862E-06	3.8285E-06	-2.0403E-06	1.5304E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.0000E 00	9.6806E-06	-2.0000E 00	1.4466E-05	1.5054E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.5497E-06	3.8147E-06	-3.8147E-06	1.9695E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	3.0000E-01	-1.0000E 00	3.0000E-01	1.9732E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	9.5000E 00	-1.5000E 00	3.0000E 00	-1.5000E 00	3.7063E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.7500E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	8.0879E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUP CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0	-2	4.0000E 00	-6.6757E-06	6.0000E 00	9.5367E-07	2.8353E-07	0.0	0.0

TITLE BCPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 55
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 7 INCREMENT NUMBER = 7

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	**** EFFECTIVE PLASTIC STRAINS **** INCREMENTAL	SUM INCR.	CUMULATIVE	***** EFFECTIVE CREEP STRAINS ***** INCREMENTAL	SUM INCR.	CUMULATIVE
1	3	5	4	2.0000E 00	2.0000E 00	0.0	3.0000E 00	3.0000E 00	0.0	1.0000E 00	1.0000E 00

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	**** CUMULATIVE THERMAL STRAINS **** XX	YY	ZZ
1	3	5	4	5.4000E 00	2.0000E 00	2.0000E 00	2.0000E 00

END OUTPUT CPU = 00:00:24.663 TOD = 23:39:12

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TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 56
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 8 INCREMENT NUMBER = 8

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD SET ONE	=	4.000000E 00
COEFFICIENT FOR CONCENTRATED LOAD SET TWO	=	0.0
COEFFICIENT FOR DISTRIBUTED LOAD SET ONE	=	0.0
COEFFICIENT FOR DISTRIBUTED LOAD SET TWO	=	0.0
COEFFICIENT FOR NODAL TEMPERATURE SET	=	4.000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN SET	=	-6.000000E-01
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)	=	0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)	=	0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)	=	0.0
CREEP TIME	=	0.0

BEGIN LOADS CPU = 00:00:24.946 TOD = 23:39:19

END LOADS CPU = 00:00:25.032 TOD = 23:39:23

```
BEGIN   SOLN     CPU = 00:00:25.086   TOD = 23:39:24
END     SOLN     CPU = 00:00:25.129   TOD = 23:39:24
BEGIN   ELLCOP   CPU = 00:00:25.130   TOD = 23:39:24
END     ELLCOP   CPU = 00:00:25.212   TOD = 23:39:28
RESIDUAL NORM =  7.34997E-01
BEGIN   SOLN     CPU = 00:00:25.219   TOD = 23:39:28
END     SOLN     CPU = 00:00:25.249   TOD = 23:39:28
BEGIN   ELLCOP   CPU = 00:00:25.255   TOD = 23:39:28
END     ELLCOP   CPU = 00:00:25.335   TOD = 23:39:32
RESIDUAL NORM =  4.22959E-01
BEGIN   SOLN     CPU = 00:00:25.342   TOD = 23:39:32
END     SOLN     CPU = 00:00:25.365   TOD = 23:39:32
BEGIN   ELLCOP   CPU = 00:00:25.369   TOD = 23:39:32
END     ELLCOP   CPU = 00:00:25.458   TOD = 23:39:35
RESIDUAL NORM =  2.94467E-01
BEGIN   SOLN     CPU = 00:00:25.465   TOD = 23:39:35
END     SOLN     CPU = 00:00:25.505   TOD = 23:39:35
BEGIN   ELLCOP   CPU = 00:00:25.512   TOD = 23:39:35
END     ELLCOP   CPU = 00:00:25.592   TOD = 23:39:39
RESIDUAL NORM =  4.43793E-02
BEGIN   SOLN     CPU = 00:00:25.595   TOD = 23:39:39
END     SOLN     CPU = 00:00:25.625   TOD = 23:39:39
BEGIN   ELLCOP   CPU = 00:00:25.631   TOD = 23:39:39
```

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```
END     ELLOOP      CPU = 00:00:25.708     TOD = 23:39:43
RESIDUAL NORM =  6.95347E-03
BEGIN   SOLN        CPU = 00:00:25.721     TOD = 23:39:43
END     SOLN        CPU = 00:00:25.745     TOD = 23:39:43
BEGIN   ELLOOP      CPU = 00:00:25.751     TOD = 23:39:43
END     ELLOOP      CPU = 00:00:25.828     TOD = 23:39:46
RESIDUAL NORM =  9.88962E-04
BEGIN   SOLN        CPU = 00:00:25.831     TOD = 23:39:46
END     SOLN        CPU = 00:00:25.871     TOD = 23:39:46
BEGIN   ELLOOP      CPU = 00:00:25.874     TOD = 23:39:47
END     ELLOOP      CPU = 00:00:25.956     TOD = 23:39:51
RESIDUAL NORM =  1.41337E-04
BEGIN   SOLN        CPU = 00:00:25.964     TOD = 23:39:51
END     SOLN        CPU = 00:00:25.984     TOD = 23:39:52
BEGIN   ELLOOP      CPU = 00:00:25.991     TOD = 23:39:52
END     ELLOOP      CPU = 00:00:26.077     TOD = 23:39:55
RESIDUAL NORM =  2.06943E-05
BEGIN   SOLN        CPU = 00:00:26.081     TOD = 23:39:55
END     SOLN        CPU = 00:00:26.111     TOD = 23:39:55
BEGIN   ELLOOP      CPU = 00:00:26.114     TOD = 23:39:55
END     ELLOOP      CPU = 00:00:26.201     TOD = 23:39:58
RESIDUAL NORM =  2.50723E-06
```

```
TITLE    BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                    PAGE      59
VTITLE                                                                                 VARIABLE STRUCTURE NUMBER =   1
ITITLE   LOAD INCREMENT 6                                                                       INCREMENT NUMBER =   6

                                  E N D   O F   L O A D   I N C R E M E N T    6

NO. ELASTIC INTEGRATION POINTS =     0, NO. PLASTIC INTEGRATION POINTS =     4
     4  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,     0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     1, NO. UPDATES PERFORMED =     0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      9
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  2.5072E-06

BEGIN    BIGSCK         CPU = 00:00:26.240     TOD = 23:39:59

END      BIGSCK         CPU = 00:00:26.307     TOD = 23:40:00

BEGIN    OUTPUT         CPU = 00:00:26.360     TOD = 23:40:00
```



```
TITLE    BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                    PAGE      60
VTITLE                                                                                 VARIABLE STRUCTURE NUMBER =   1
ITITLE   LOAD INCREMENT 6                                                                       INCREMENT NUMBER =   6

                    CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **     ****************  FORCES  *****************          *************  DISPLACEMENTS  *************
 NO.  I.D.     U                V                W                   U                V                W
   1    1      -1.1645547E-06   3.1250185E-01                        0.0              0.0
   2    2      1.1720944E-06    3.1250256E-01                        -5.9999973E-01   0.0
   3    3      9.7710927E-07    -3.1250256E-01                       -5.9999943E-01   4.0000000E 00
   4    4      -9.8400453E-07   -3.1250191E-01                       8.0420250E-08    4.0000000E 00
```

TITLE BUPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 61
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 1 INCREMENT NUMBER = 8

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	6.2500E-01	5.3745E-06	-6.2500E-01	-3.5820E-06	-7.7552E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	6.2500E-01	3.7863E-06	-6.2500E-01	-1.5417E-06	-1.6079E-07	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.5001E-01	-5.0001E-01	1.5000E-01	-8.0654E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.5000E-01	-5.0000E-01	1.5000E-01	-2.0701E-07	0.0	0.0

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ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	9.6562E 00	-1.2500E 00	2.5000E 00	-1.2500E 00	2.1797E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.5625E-01	2.5000E-01	-5.0000E-01	2.5000E-01	1.8691E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.7500E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	8.0879E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1	2	3.0667E 00	-6.0000E-01	4.0000E 00	-6.0000E-01	9.4611E-08	0.0	0.0

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                    PAGE      62
VTITLE                                                                                   VARIABLE STRUCTURE NUMBER =  1
ITITLE    LOAD INCREMENT 8                                                                      INCREMENT NUMBER =   8

  ELEMENT      POINT              YIELD        YIELD       **** EFFECTIVE PLASTIC STRAINS ****   ***** EFFECTIVE CREEP STRAINS *****
 NO.  I.D.  NO. TP.            STRESS CTR. STRESS SIZE    INCREMENTAL  SUM INCR.   CUMULATIVE INCREMENTAL  SUM INCR.   CUMULATIVE
  1     3     5  4              1.5000E 00  2.1250E 00     5.0000E-01  3.5000E 00  2.5000E 00  0.0         1.0000E 00  1.0000E 00

  ELEMENT      POINT             CUMULATIVE              **** CUMULATIVE THERMAL STRAINS ****
 NO.  I.D.  NO. TP.             TEMPERATURE                   XX          YY          ZZ
  1     3     5  4               8.0000E 00              1.0000E 00  1.0000E 00  1.0000E 00

END      OUTPUT        CPU = 00:00:26.487      TOD = 23:40:03
```

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```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                    PAGE      63
VTITLE                                                                                   VARIABLE STRUCTURE NUMBER =  1
ITITLE    LOAD INCREMENT 9                                                                      INCREMENT NUMBER =   9

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE        =   1.500000E 00
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO        =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE         =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO         =   0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET             =   9.000000E 00
COEFFICIENT FOR NODAL STRESS/STRAIN  SET           =  -2.000000E-01
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)      =   0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                =   0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)       =   0.0
CREEP TIME                                         =   1.000000E 01

BEGIN   LOADS          CPU = 00:00:26.746      TOD = 23:40:10

END     LOADS          CPU = 00:00:26.836      TOD = 23:40:14
```

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
VTITLE
ITITLE LOAD INCREMENT 9

VARIABLE STRUCTURE NUMBER = 1
INCREMENT NUMBER = 9

```
BEGIN   SOLN     CPU = 00:00:26.899    TOD = 23:40:14
END     SOLN     CPU = 00:00:26.938    TOD = 23:40:14
BEGIN   ELLOOP   CPU = 00:00:26.946    TOD = 23:40:14
END     ELLOOP   CPU = 00:00:27.033    TOD = 23:40:19
RESIDUAL NORM = 3.61??1E-01
BEGIN   SOLN     CPU = 00:00:27.036    TOD = 23:40:19
END     SOLN     CPU = 00:00:27.072    TOD = 23:40:19
BEGIN   ELLOOP   CPU = 00:00:27.079    TOD = 23:40:19
END     ELLOOP   CPU = 00:00:27.166    TOD = 23:40:23
RESIDUAL NORM = 2.99930E-01
BEGIN   SOLN     CPU = 00:00:27.169    TOD = 23:40:23
END     SOLN     CPU = 00:00:27.186    TOD = 23:40:24
BEGIN   ELLOOP   CPU = 00:00:27.192    TOD = 23:40:24
END     ELLOOP   CPU = 00:00:27.279    TOD = 23:40:27
RESIDUAL NORM = 1.20916E-01
BEGIN   SOLN     CPU = 00:00:27.285    TOD = 23:40:27
END     SOLN     CPU = 00:00:27.322    TOD = 23:40:28
BEGIN   ELLOOP   CPU = 00:00:27.322    TOD = 23:40:28
END     ELLOOP   CPU = 00:00:27.409    TOD = 23:40:31
RESIDUAL NORM = 1.67929E-03
BEGIN   SOLN     CPU = 00:00:27.415    TOD = 23:40:31
END     SOLN     CPU = 00:00:27.452    TOD = 23:40:31
BEGIN   ELLOOP   CPU = 00:00:27.462    TOD = 23:40:31
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                          PAGE        65
VTITLE                                                        VARIABLE STRUCTURE NUMBER =    1
ITITLE    LOAD INCREMENT 9                                              INCREMENT NUMBER =    9

END       ELLOOP        CPU = 00:00:27.572      TOD = 23:40:37

RESIDUAL NORM =  2.62151E-03

BEGIN     ELLOOP        CPU = 00:00:27.562      TOD = 23:40:57

END       ELLOOP        CPU = 00:00:27.678      TOD = 23:40:42

RESIDUAL NORM =  2.22818E-03

BEGIN     ELLOOP        CPU = 00:00:27.685      TOD = 23:40:42

END       ELLOOP        CPU = 00:00:27.785      TOD = 23:40:46

RESIDUAL NORM =  1.42423E-03

BEGIN     SOLN          CPU = 00:00:27.785      TOD = 23:40:46

END       SOLN          CPU = 00:00:27.818      TOD = 23:40:46

BEGIN     ELLOOP        CPU = 00:00:27.825      TOD = 23:40:46

END       ELLOOP        CPU = 00:00:27.928      TOD = 23:40:49

RESIDUAL NORM =  0.16139E-05

BEGIN     SOLN          CPU = 00:00:27.931      TOD = 23:40:49

END       SOLN          CPU = 00:00:27.978      TOD = 23:40:49

BEGIN     ELLOOP        CPU = 00:00:27.984      TOD = 23:40:50

END       ELLOOP        CPU = 00:00:28.097      TOD = 23:40:53

RESIDUAL NORM =  2.60219E-05

BEGIN     SOLN          CPU = 00:00:28.097      TOD = 23:40:53

END       SOLN          CPU = 00:00:28.121      TOD = 23:40:54

BEGIN     ELLOOP        CPU = 00:00:28.127      TOD = 23:40:54

END       ELLOOP        CPU = 00:00:28.197      TOD = 23:40:57

RESIDUAL NORM =  7.46329E-07
```

```
TITLE     SOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                    PAGE    66
VTITLE                                                                              VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 9                                                                    INCREMENT NUMBER =   9


                              E N D  O F  L O A D  I N C R E M E N T   9

NO. ELASTIC INTEGRATION POINTS =    0,  NO. PLASTIC INTEGRATION POINTS =    4
    0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,    0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     1, NO. UPDATES PERFORMED =     6
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =     10
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  7.4633E-07

BEGIN    BIGSCK        CPU = 00:00:28.264      TOD = 23:40:58

END      BIGSCK        CPU = 00:00:28.340      TOD = 23:41:01

BEGIN    OUTPUT        CPU = 00:00:28.360      TOD = 23:41:02
```

8.4-53

```
TITLE     SOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                    PAGE    67
VTITLE                                                                              VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 9                                                                    INCREMENT NUMBER =   9

                 CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **    ****************  FORCES  *****************        *************  DISPLACEMENTS  *************
NO.  I.D.     U                V                W                U                V                W
   1    1     -5.6994767E-07   6.2499945E-01                     0.0              0.0
   2    2     5.7194427E-07    6.2499797E-01                     -1.9999510E-01   0.0
   3    3     6.4595510E-07    -6.2499797E-01                    -1.9999522E-01   1.5000000E 00
   4    4     -6.4793494E-07   -6.2499645E-01                    3.6074050E-07    1.5000000E 00
```

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.2500E 00	6.8775E-06	-1.2500E 00	1.0030E-06	-1.9969E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	6.2500E-01	1.0030E-06	-6.2499E-01	4.5850E-06	5.7583E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	3.0000E-01	-1.0000E 00	3.0000E-01	-2.0768E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.5000E-01	-5.0000E-01	1.5000E-01	5.9686E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.0125E 01	-1.0000E 00	2.0000E 00	-9.9999E-01	6.3912E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	4.6875E-01	2.5000E-01	-5.0000E-01	2.5000E-01	-1.3406E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	3.6875E 00	-1.4901E-06	4.8280E-06	-3.3379E-06	5.4066E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	9.3749E-01	5.0000E-01	-1.0000E 00	5.0000E-01	-2.6611E-06	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0	2	1.1333E 00	-2.0000E-01	1.5000E 00	-2.0000E-01	6.0383E-06	0.0	0.0

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 69
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 9 INCREMENT NUMBER = 9

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
1	3	5	4	1.0000E 00	2.2500E 00	5.0000E-01	4.0000E 00	2.0000E 00	1.0000E 00	2.0000E 00	4.9448E-06

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
1	3	5	4	9.0000E 00	5.0000E-01	5.0000E-01	5.0000E-01

END OUTPUT CPU = 00:00:29.497 TOD = 23:41:08

8-4-55

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 70
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 10 INCREMENT NUMBER = 10

PARAMETERS FOR THIS INCREMENT

```
COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE       =   3.650000E 00
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO       =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE        =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO        =   0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET            =   1.000000E 01
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET         =   0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)     =   0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)               =   0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)      =   0.0
CREEP TIME                                        =   0.0
```

BEGIN LOADS CPU = 00:00:28.796 TOD = 23:41:15

END LOADS CPU = 00:00:28.886 TOD = 23:41:17

```
BEGIN   SCLN      CPU = 00:00:28.946   TOD = 23:41:18
END     SCLN      CPU = 00:00:28.986   TOD = 23:41:19
BEGIN   ELLOOP    CPU = 00:00:28.989   TOD = 23:41:19
END     ELLOOP    CPU = 00:00:29.076   TOD = 23:41:23
RESIDUAL NORM =  6.034b3E-01
BEGIN   SCLN      CPU = 00:00:29.079   TOD = 23:41:23
END     SCLN      CPU = 00:00:29.112   TOD = 23:41:24
BEGIN   ELLOOP    CPU = 00:00:29.116   TOD = 23:41:24
END     ELLOOP    CPU = 00:00:29.219   TOD = 23:41:27
RESIDUAL NORM =  4.26186E-01
BEGIN   SCLN      CPU = 00:00:29.222   TOD = 23:41:27
END     SCLN      CPU = 00:00:29.259   TOD = 23:41:27
BEGIN   ELLOOP    CPU = 00:00:29.266   TOD = 23:41:27
END     ELLOOP    CPU = 00:00:29.369   TOD = 23:41:29
RESIDUAL NORM =  6.83473E-01
BEGIN   ELLOOP    CPU = 00:00:29.375   TOD = 23:41:29
END     ELLOOP    CPU = 00:00:29.465   TOD = 23:41:32
RESIDUAL NORM =  2.65205E-05
BEGIN   SCLN      CPU = 00:00:29.469   TOD = 23:41:32
END     SCLN      CPU = 00:00:29.502   TOD = 23:41:32
BEGIN   ELLOOP    CPU = 00:00:29.505   TOD = 23:41:32
END     ELLOOP    CPU = 00:00:29.588   TOD = 23:41:34
RESIDUAL NORM =  2.65982E-05
```

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 72
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 10 INCREMENT NUMBER = 10

BEGIN SOLN CPU = 00:00:29.592 TOD = 23:41:34

END SOLN CPU = 00:00:29.642 TOD = 23:41:35

BEGIN ELLOOP CPU = 00:00:29.652 TOD = 23:41:35

END ELLOOP CPU = 00:00:29.748 TOD = 23:41:37

RESIDUAL NORM = 2.64855E-05

BEGIN ELLOOP CPU = 00:00:29.761 TOD = 23:41:38

END ELLOOP CPU = 00:00:29.851 TOD = 23:41:40

RESIDUAL NORM = 1.18255E-06

E N D O F L O A D I N C R E M E N T 10

NO. ELASTIC INTEGRATION POINTS = 4, NO. PLASTIC INTEGRATION POINTS = 0
0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 4 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES = 1, NO. UPDATES PERFORMED = 0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10 10 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 7
SPECIFIED MAX. UNBALANCED FORCE ERROR = 1.0000E-05, ACTUAL ERROR = 1.1825E-06

BEGIN BIGSCK CPU = 00:00:29.908 TOD = 23:41:40

END BIGSCK CPU = 00:00:29.964 TOD = 23:41:43

BEGIN OUTPUT CPU = 00:00:30.020 TOD = 23:41:43

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 73
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 10 INCREMENT NUMBER = 10

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **		****** FORCES ******			****** DISPLACEMENTS ******		
NO.	I.D.	U	V	W	U	V	W
1	1	-7.2472153E-07	-5.0000644E-01		0.0	0.0	
2	2	7.2475121E-07	-5.0000674E-01		-2.4780093E-06	0.0	
3	3	8.1767538E-07	5.0000674E-01		-2.5952177E-06	3.6499996E 00	
4	4	-8.1770503E-07	5.0000644E-01		3.6696429E-07	3.6499996E 00	

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.0000E 00	-5.7312E-06	1.0000E 00	-4.5850E-06	-2.9701E-11	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.2500E 00	-1.2609E-05	2.2500E 00	-5.5679E-06	1.9672E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-1.5000E-01	5.0000E-01	-1.5000E-01	-1.9305E-11	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-4.5001E-01	1.5000E 00	-4.5000E-01	2.3574E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.0125E 01	-1.0000E 00	2.0000E 00	-9.9999E-01	8.3912E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	3.6875E 00	-1.4901E-06	4.6280E-06	-3.3379E-06	5.4068E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-1	-2	2.4333E 00	-9.5367E-07	3.6500E 00	9.5367E-07	6.2440E-08	0.0	0.0

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 75
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 10 INCREMENT NUMBER = 10

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	EFFECTIVE PLASTIC STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE	EFFECTIVE CREEP STRAINS INCREMENTAL	SUM INCR.	CUMULATIVE
1	3	5	4	1.0000E 00	2.2500E 00	0.0	4.0000E 00	2.0000E 00	0.0	2.0000E 00	4.9448E-06

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	CUMULATIVE THERMAL STRAINS XX	YY	ZZ
1	3	5	4	1.0000E 01	1.1500E 00	1.1500E 00	1.1500E 00

END OUTPUT CPU = 00:00:30.151 TOD = 23:41:45

B.4-59

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 76
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 11 INCREMENT NUMBER = 11

PARAMETERS FOR THIS INCREMENT

```
COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE        =   5.5000000E 00
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO        =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE         =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO         =   0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET             =   4.5000000E 00
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET          =  -3.0000000E-01
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)      =   0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                =   0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)       =   0.0
CREEP TIME                                         =   1.0000000E 01
```

BEGIN LOADS CPU = 00:00:30.430 TOD = 23:41:52

END LOADS CPU = 00:00:30.519 TOD = 23:41:54

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                PAGE      77
VTITLE                                                              VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 11                                                  INCREMENT NUMBER =  11

BEGIN     SOLN           CPU = 00:00:30.570     TOD = 23:41:55
END       SOLN           CPU = 00:00:30.603     TOD = 23:41:56
BEGIN     ELLOOP         CPU = 00:00:30.607     TOD = 23:41:56
END       ELLOOP         CPU = 00:00:30.707     TOD = 23:41:57
RESIDUAL NORM =   3.28124E-01
BEGIN     SOLN           CPU = 00:00:30.707     TOD = 23:41:57
END       SOLN           CPU = 00:00:30.723     TOD = 23:41:57
BEGIN     ELLOOP         CPU = 00:00:30.730     TOD = 23:41:58
END       ELLOOP         CPU = 00:00:30.823     TOD = 23:41:59
RESIDUAL NORM =   4.42395E-01
BEGIN     SOLN           CPU = 00:00:30.826     TOD = 23:41:59
END       SOLN           CPU = 00:00:30.856     TOD = 23:41:59
BEGIN     ELLOOP         CPU = 00:00:30.860     TOD = 23:41:59
END       ELLOOP         CPU = 00:00:30.946     TOD = 23:42:01
RESIDUAL NORM =   6.05576E-01
BEGIN     ELLOOP         CPU = 00:00:30.956     TOD = 23:42:01
END       ELLOOP         CPU = 00:00:31.066     TOD = 23:42:07
RESIDUAL NORM =   6.17440E-02
BEGIN     SOLN           CPU = 00:00:31.076     TOD = 23:42:07
END       SOLN           CPU = 00:00:31.106     TOD = 23:42:08
BEGIN     ELLOOP         CPU = 00:00:31.109     TOD = 23:42:08
END       ELLOOP         CPU = 00:00:31.202     TOD = 23:42:10
RESIDUAL NORM =   2.44050E-02
```

B.4-60

```
BEGIN    SOLN          CPU = 00:00:31.206      TOD = 23:42:10
END      SOLN          CPU = 00:00:31.249      TOD = 23:42:10
BEGIN    ELLOOP        CPU = 00:00:31.252      TOD = 23:42:10
END      ELLOOP        CPU = 00:00:31.346      TOD = 23:42:12
RESIDUAL NORM =  3.315521E-01
BEGIN    ELLOOP        CPU = 00:00:31.346      TOD = 23:42:12
END      ELLOOP        CPU = 00:00:31.445      TOD = 23:42:14
RESIDUAL NORM =  3.521535E-01
BEGIN    ELLOOP        CPU = 00:00:31.452      TOD = 23:42:14
END      ELLOOP        CPU = 00:00:31.555      TOD = 23:42:17
RESIDUAL NORM =  3.653895E-01
BEGIN    SOLN          CPU = 00:00:31.559      TOD = 23:42:17
END      SOLN          CPU = 00:00:31.575      TOD = 23:42:17
BEGIN    ELLOOP        CPU = 00:00:31.582      TOD = 23:42:17
END      ELLOOP        CPU = 00:00:31.692      TOD = 23:42:19
RESIDUAL NORM =  3.593845E-01
BEGIN    SOLN          CPU = 00:00:31.695      TOD = 23:42:19
END      SOLN          CPU = 00:00:31.732      TOD = 23:42:19
BEGIN    ELLOOP        CPU = 00:00:31.738      TOD = 23:42:19
END      ELLOOP        CPU = 00:00:31.838      TOD = 23:42:21
RESIDUAL NORM =  5.21676E-01
BEGIN    MERGE         CPU = 00:00:31.875      TOD = 23:42:21
BEGIN    GENRS         CPU = 00:00:31.861      TOD = 23:42:21
```

```
STIFFNESS GENERATION COMPLETED.        10 PARTITIONS WRITTEN.
END      GENRB        CPU = 00:00:31.971     TOD = 23:42:23
BEGIN    MERSOR       CPU = 00:00:31.981     TOD = 23:42:23
END      MERSOR       CPU = 00:00:32.028     TOD = 23:42:23
END      MERGE        CPU = 00:00:32.029     TOD = 23:42:23
MAXIMUM WAVEFRONT =     4 NODES AT INTERNAL NODE     1
BEGIN    DECOMP       CPU = 00:00:32.084     TOD = 23:42:24
END      DECOMP       CPU = 00:00:32.134     TOD = 23:42:24
BEGIN    BIGSCK       CPU = 00:00:32.141     TOD = 23:42:25
END      BIGSCK       CPU = 00:00:32.174     TOD = 23:42:26
BEGIN    MERGE        CPU = 00:00:32.176     TOD = 23:42:26
BEGIN    GENRB        CPU = 00:00:32.176     TOD = 23:42:26
STIFFNESS GENERATION COMPLETED.        10 PARTITIONS WRITTEN.
END      GENRB        CPU = 00:00:32.257     TOD = 23:42:28
BEGIN    MERSOR       CPU = 00:00:32.261     TOD = 23:42:28
END      MERSOR       CPU = 00:00:32.321     TOD = 23:42:29
END      MERGE        CPU = 00:00:32.321     TOD = 23:42:29
MAXIMUM WAVEFRONT =     4 NODES AT INTERNAL NODE     1
BEGIN    DECOMP       CPU = 00:00:32.364     TOD = 23:42:30
END      DECOMP       CPU = 00:00:32.417     TOD = 23:42:31
BEGIN    BIGSCK       CPU = 00:00:32.427     TOD = 23:42:31
END      BIGSCK       CPU = 00:00:32.467     TOD = 23:42:33
BEGIN    SOLN         CPU = 00:00:32.497     TOD = 23:42:34
```

```
END       SOLN        CPU = 00:00:32.527     TOD = 23:42:34

BEGIN     ELLOOP      CPU = 00:00:32.540     TOD = 23:42:34

END       ELLOOP      CPU = 00:00:32.643     TOD = 23:42:36

RESIDUAL NORM =  2.44127E-01

BEGIN     SOLN        CPU = 00:00:32.650     TOD = 23:42:36

END       SOLN        CPU = 00:00:32.667     TOD = 23:42:37

BEGIN     ELLOOP      CPU = 00:00:32.690     TOD = 23:42:37

END       ELLOOP      CPU = 00:00:32.777     TOD = 23:42:39

RESIDUAL NORM =  7.26602E-02

BEGIN     SOLN        CPU = 00:00:32.780     TOD = 23:42:40

END       SOLN        CPU = 00:00:32.803     TOD = 23:42:40

BEGIN     ELLOOP      CPU = 00:00:32.807     TOD = 23:42:40

END       ELLOOP      CPU = 00:00:32.876     TOD = 23:42:41

RESIDUAL NORM =  2.13806E-02

BEGIN     SOLN        CPU = 00:00:32.883     TOD = 23:42:41

END       SOLN        CPU = 00:00:32.906     TOD = 23:42:42

BEGIN     ELLOOP      CPU = 00:00:32.906     TOD = 23:42:42

END       ELLOOP      CPU = 00:00:32.983     TOD = 23:42:44

RESIDUAL NORM =  5.54278E-03

BEGIN     SOLN        CPU = 00:00:32.986     TOD = 23:42:44

END       SOLN        CPU = 00:00:33.013     TOD = 23:42:44

BEGIN     ELLOOP      CPU = 00:00:33.025     TOD = 23:42:44

END       ELLOOP      CPU = 00:00:33.099     TOD = 23:42:45
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                              PAGE      81
VTITLE                                                                          VARIABLE STRUCTURE NUMBER =    1
ITITLE    LEAP INCREMENT 11                                                            INCREMENT NUMBER =   11

RESIDUAL NORM =  1.44271E-03

BEGIN     SOLN          CPU = 00:00:33.109      TOD = 23:42:46

END       SOLN          CPU = 00:00:33.139      TOD = 23:42:46

BEGIN     ELLOOP        CPU = 00:00:33.139      TOD = 23:42:46

END       ELLOOP        CPU = 00:00:33.219      TOD = 23:42:48

RESIDUAL NORM =  3.71974E-04

BEGIN     SOLN          CPU = 00:00:33.226      TOD = 23:42:48

END       SOLN          CPU = 00:00:33.256      TOD = 23:42:48

BEGIN     ELLOOP        CPU = 00:00:33.259      TOD = 23:42:48

END       ELLOOP        CPU = 00:00:33.329      TOD = 23:42:51

RESIDUAL NORM =  9.59117E-05

BEGIN     SOLN          CPU = 00:00:33.332      TOD = 23:42:51

END       SOLN          CPU = 00:00:33.352      TOD = 23:42:51

BEGIN     ELLOOP        CPU = 00:00:33.362      TOD = 23:42:51

END       ELLOOP        CPU = 00:00:33.449      TOD = 23:42:52

RESIDUAL NORM =  2.52176E-05

BEGIN     SOLN          CPU = 00:00:33.452      TOD = 23:42:53

END       SOLN          CPU = 00:00:33.479      TOD = 23:42:53

BEGIN     ELLOOP        CPU = 00:00:33.482      TOD = 23:42:53

END       ELLOOP        CPU = 00:00:33.575      TOD = 23:42:55

RESIDUAL NORM =  6.76477E-06
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                        PAGE      82
VTITLE                                                                     VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 11                                                        INCREMENT NUMBER =  11


                              E N D   O F   L O A D   I N C R E M E N T   11

NO. ELASTIC INTEGRATION POINTS =     4,  NO. PLASTIC INTEGRATION POINTS =     0
    0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,    0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     1, NO. UPDATES PERFORMED =     1
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =    10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      9
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  6.7647E-06

BEGIN   BIGSCK        CPU = 00:00:33.632      TOD = 23:42:55

END     BIGSCK        CPU = 00:00:33.692      TOD = 23:42:58

BEGIN   OUTPUT        CPU = 00:00:33.752      TOD = 23:42:59
```

8.4-65

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                        PAGE      83
VTITLE                                                                     VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 11                                                        INCREMENT NUMBER =  11

               CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **    ****************  FORCES  *****************        *************  DISPLACEMENTS  *************
 NO.  I.D.    U                V                 W               U                V                W
   1    1     1.4818579E-05  -1.6250010E 00                      0.0              0.0
   2    2    -1.4678111E-05  -1.6250029E 00                     -3.5001146E-01    0.0
   3    3    -1.4541964E-05   1.6250029E 00                     -3.5001803E-01    5.5000000E 00
   4    4     1.4541206E-05   1.6250019E 00                     -1.1220011E-06    5.5000000E 00
```

TITLE EOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM
VTITLE
ITITLE LOAD INCREMENT 11

ELEMENT NO.	POINT I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	3.2500E 00	-4.5821E-05	3.2500E 00	-2.5332E-05	-7.1648E-10	0.0	0.0

ELEMENT NO.	POINT I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.2500E 00	-4.0090E-05	2.2500E 00	-2.0747E-05	-6.8678E-10	0.0	0.0

ELEMENT NO.	POINT I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-3.0001E-01	1.0000E 00	-3.0000E-01	-2.8739E-10	0.0	0.0

ELEMENT NO.	POINT I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-1.5001E-01	5.0000E-01	-1.5000E-01	-2.6809E-10	0.0	0.0

ELEMENT NO.	POINT I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.0125E 01	-1.0000E 00	2.0000E 00	-9.9999E-01	8.3912E-09	0.0	0.0

ELEMENT NO.	POINT I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	POINT I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.8125E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	5.4024E-08	0.0	0.0

ELEMENT NO.	POINT I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.1250E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	-4.4551E-11	0.0	0.0

ELEMENT NO.	POINT I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0	-2	3.8667E 00	-3.0001E-01	5.5000E 00	-3.0000E-01	6.2128E-06	0.0	0.0

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                  PAGE      85
VTITLE                                                                                    VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 11                                                                         INCREMENT NUMBER =  11

 ELEMENT     POINT              YIELD        YIELD       **** EFFECTIVE PLASTIC STRAINS ****  ***** EFFECTIVE CREEP STRAINS *****
 NO.  I.D.   NO. TP.        STRESS CTR. STRESS SIZE    INCREMENTAL  SUM INCR.   CUMULATIVE INCREMENTAL  SUM INCR.   CUMULATIVE
  1     3     5  4           1.0000E 00  2.2500E 00     0.0          4.0000E 00  2.0000E 00  1.0000E 00  3.0000E 00  1.6000E 00

 ELEMENT     POINT              CUMULATIVE              **** CUMULATIVE THERMAL STRAINS ****
 NO.  I.D.   NO. TP.            TEMPERATURE                XX           YY           ZZ
  1     3     5  4               4.5000E 00              1.5000E 00  1.5000E 00  1.5000E 00

END       OUTPUT        CPU = 00:00:33.671      TOD = 23:43:01
```

8.4-67

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                  PAGE      86
VTITLE                                                                                    VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 12                                                                         INCREMENT NUMBER =  12

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE        =     6.549999E 00
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO        =     0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE         =     0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO         =     0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET             =     1.200000E 01
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET          =     0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)      =     0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                =     0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)       =     0.0
CREEP TIME                                         =     0.3

BEGIN  LOADS        CPU = 00:00:34.161      TOD = 23:43:08

END    LOADS        CPU = 00:00:34.224      TOD = 23:43:10
```

```
BEGIN    SOLN          CPU = 00:00:34.274      TOD = 23:43:11
END      SOLN          CPU = 00:00:34.314      TOD = 23:43:13
BEGIN    ELLOOP        CPU = 00:00:34.317      TOD = 23:43:13
END      ELLOOP        CPU = 00:00:34.404      TOD = 23:43:19
RESIDUAL NORM =   4.115251E-01
BEGIN    SOLN          CPU = 00:00:34.411      TOD = 23:43:19
END      SOLN          CPU = 00:00:34.437      TOD = 23:43:19
BEGIN    ELLOOP        CPU = 00:00:34.444      TOD = 23:43:19
END      ELLOOP        CPU = 00:00:34.557      TOD = 23:43:21
RESIDUAL NORM =   1.934275E-01
BEGIN    SOLN          CPU = 00:00:34.557      TOD = 23:43:21
END      SOLN          CPU = 00:00:34.587      TOD = 23:43:21
BEGIN    ELLOOP        CPU = 00:00:34.590      TOD = 23:43:21
END      ELLOOP        CPU = 00:00:34.697      TOD = 23:43:22
RESIDUAL NORM =   1.05631E-01
BEGIN    SOLN          CPU = 00:00:34.700      TOD = 23:43:22
END      SOLN          CPU = 00:00:34.720      TOD = 23:43:22
BEGIN    ELLOOP        CPU = 00:00:34.727      TOD = 23:43:22
END      ELLOOP        CPU = 00:00:34.813      TOD = 23:43:24
RESIDUAL NORM =   2.089087E-02
BEGIN    SOLN          CPU = 00:00:34.917      TOD = 23:43:24
END      SOLN          CPU = 00:00:34.843      TOD = 23:43:24
BEGIN    ELLOOP        CPU = 00:00:34.850      TOD = 23:43:24
```

```
END      ELLOOP        CPU = 00:00:34.953     TOD = 23:43:25
RESIDUAL NORM =  7.66636E-03
BEGIN    SOLN          CPU = 00:00:34.966     TOD = 23:43:25
END      SOLN          CPU = 00:00:34.993     TOD = 23:43:25
BEGIN    ELLOOP        CPU = 00:00:35.000     TOD = 23:43:25
END      ELLOOP        CPU = 00:00:35.083     TOD = 23:43:26
RESIDUAL NORM =  2.01087E-03
BEGIN    SOLN          CPU = 00:00:35.083     TOD = 23:43:26
END      SOLN          CPU = 00:00:35.119     TOD = 23:43:27
BEGIN    ELLOOP        CPU = 00:00:35.123     TOD = 23:43:27
END      ELLOOP        CPU = 00:00:35.216     TOD = 23:43:29
RESIDUAL NORM =  5.35356E-04
BEGIN    SOLN          CPU = 00:00:35.216     TOD = 23:43:29
END      SOLN          CPU = 00:00:35.266     TOD = 23:43:29
BEGIN    ELLOOP        CPU = 00:00:35.273     TOD = 23:43:29
END      ELLOOP        CPU = 00:00:35.379     TOD = 23:43:31
RESIDUAL NORM =  1.41275E-04
BEGIN    SOLN          CPU = 00:00:35.386     TOD = 23:43:31
END      SOLN          CPU = 00:00:35.412     TOD = 23:43:31
BEGIN    ELLOOP        CPU = 00:00:35.419     TOD = 23:43:31
END      ELLOOP        CPU = 00:00:35.529     TOD = 23:43:33
RESIDUAL NORM =  3.73194E-05
BEGIN    SOLN          CPU = 00:00:35.529     TOD = 23:43:33
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                   PAGE          89
VTITLE                                                                      VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 12                                                      INCREMENT NUMBER =  12

END       SOLN          CPU = 00:00:35.559       TOD = 23:43:33

BEGIN     ELLOOP        CPU = 00:00:35.569       TOD = 23:43:33

END       ELLOOP        CPU = 00:00:35.669       TOD = 23:43:37

RESIDUAL NORM =   1.01074E-05

BEGIN     MERGE         CPU = 00:00:35.709       TOD = 23:43:38

BEGIN     GENR8         CPU = 00:00:35.709       TOD = 23:43:38

STIFFNESS GENERATION COMPLETED.          10 PARTITIONS WRITTEN.

END       GENR8         CPU = 00:00:35.772       TOD = 23:43:39

BEGIN     MERSOR        CPU = 00:00:35.772       TOD = 23:43:39

END       MERSOR        CPU = 00:00:35.848       TOD = 23:43:39

END       MERGE         CPU = 00:00:35.848       TOD = 23:43:39

MAXIMUM WAVEFRONT =     4 NODES AT INTERNAL NODE     1

BEGIN     DECOMP        CPU = 00:00:35.892       TOD = 23:43:40

END       DECOMP        CPU = 00:00:35.955       TOD = 23:43:40

BEGIN     BIGSCK        CPU = 00:00:35.968       TOD = 23:43:40

END       BIGSCK        CPU = 00:00:36.018       TOD = 23:43:42

BEGIN     MERGE         CPU = 00:00:36.018       TOD = 23:43:42

BEGIN     GENR8         CPU = 00:00:36.031       TOD = 23:43:43

STIFFNESS GENERATION COMPLETED.          10 PARTITIONS WRITTEN.

END       GENR8         CPU = 00:00:36.115       TOD = 23:43:44

BEGIN     MERSOR        CPU = 00:00:36.115       TOD = 23:43:44

END       MERSOR        CPU = 00:00:36.176       TOD = 23:43:45

END       MERGE         CPU = 00:00:36.181       TOD = 23:43:45
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                    PAGE      90
VTITLE                                                                                VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 12                                                                  INCREMENT NUMBER =  12

MAXIMUM WAVEFRONT =     4 NODES AT INTERNAL NODE     1

BEGIN     DECOMP        CPU = 00:00:36.234      TOD = 23:43:45

END       DECOMP        CPU = 00:00:36.281      TOD = 23:43:46

BEGIN     BIGSCK        CPU = 00:00:36.291      TOD = 23:43:46

END       BIGSCK        CPU = 00:00:36.321      TOD = 23:43:48

BEGIN     SOLN          CPU = 00:00:36.354      TOD = 23:43:48

END       SOLN          CPU = 00:00:36.387      TOD = 23:43:49

BEGIN     ELLOOP        CPU = 00:00:36.391      TOD = 23:43:49

END       ELLOOP        CPU = 00:00:36.497      TOD = 23:43:53

RESIDUAL NORM =  1.07523E-06



                              E N D   O F   L O A D   I N C R E M E N T   12

NO. ELASTIC INTEGRATION POINTS =     0, NO. PLASTIC INTEGRATION POINTS =     4
    4  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,     0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =      1, NO. UPDATES PERFORMED =      1
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      1
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  1.0752E-06

BEGIN     BIGSCK        CPU = 00:00:36.531      TOD = 23:43:54

END       BIGSCK        CPU = 00:00:36.664      TOD = 23:43:59

BEGIN     OUTPUT        CPU = 00:00:36.666      TOD = 23:44:01

TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                    PAGE      91
VTITLE                                                                                VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 12                                                                  INCREMENT NUMBER =  12

                 CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **    ****************  FORCES  *****************           *************  DISPLACEMENTS  *************
 NO.  I.D.     U                V                W                   U                V                W
   1    1     2.9932969E-06 -1.9374914E 00                          0.0              0.0
   2    2     2.9931996E-06 -1.9374905E 00                         -9.4791300E-06    0.0
   3    3     2.6699527E-06  1.9374905E 00                         -6.0630707E-06    6.5499992E 00
   4    4     2.6698535E-06  1.9374914E 00                         -2.6165621E-07    6.5499992E 00
```

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM — PAGE 92
VTITLE — VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 12 — INCREMENT NUMBER = 12

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	3.8750E 00	-8.8834E-06	3.8750E 00	-6.6625E-06	9.8750E-11	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	6.2495E-01	3.6938E-05	6.2498E-01	1.8669E-05	8.1723E-10	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-3.0000E-01	9.9999E-01	-3.0000E-01	3.3129E-11	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.1086E-05	-9.8348E-06	3.9339E-06	3.2652E-10	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.1906E 01	-1.2500E 00	2.5000E 00	-1.2500E 00	-5.1410E-10	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.7813E 00	-2.5001E-01	5.0001E-01	-2.5006E-01	-8.9053E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.6125E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	5.4524E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1	2	4.3667E 00	-4.7694E-06	6.5500E 00	9.5367E-07	5.3543E-08	0.0	0.0

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                  PAGE      93
VTITLE                                                                              VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 12                                                                  INCREMENT NUMBER =  12

  ELEMENT      POINT              YIELD       YIELD      **** EFFECTIVE PLASTIC STRAINS ****  ***** EFFECTIVE CREEP STRAINS *****
   NO.  I.D.  NO. TP.        STRESS CTR. STRESS SIZE   INCREMENTAL  SUM INCR.  CUMULATIVE INCREMENTAL  SUM INCR.  CUMULATIVE
    1     3      5  4          1.5000E 00  2.3750E 00    5.0001E-01  4.5000E 00  2.5000E 00  0.0          3.0000E 00  1.0000E 00

  ELEMENT      POINT              CUMULATIVE              **** CUMULATIVE THERMAL STRAINS ****
   NO.  I.D.  NO. TP.           TEMPERATURE                 XX          YY          ZZ
    1     3      5  4            1.2000E 01              2.0500E 00  2.0500E 00  2.0500E 00

END      OUTPUT          CPU = 00:00:36.777       TOD = 23:44:03
```

B.4-73

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                  PAGE      94
VTITLE                                                                              VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 13                                                                  INCREMENT NUMBER =  13

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE        =    7.500000E 00
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO        =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE         =    0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO         =    0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET             =    1.300000E 01
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET          =    2.000000E-01
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)      =    0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                =    0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)       =    0.0
CREEP TIME                                         =    0.0

BEGIN    LOADS          CPU = 00:00:37.066       TOD = 23:44:09

END      LOADS          CPU = 00:00:37.156       TOD = 23:44:10
```

```
TITLE     BOFACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                   PAGE       95
VTITLE                                                              VARIABLE STRUCTURE NUMBER =    1
ITITLE    LOAD INCREMENT 13                                                  INCREMENT NUMBER =   13

BEGIN    SOLN          CPU = 00:00:37.216     TOD = 23:44:11
END      SOLN          CPU = 00:00:37.253     TOD = 23:44:11
BEGIN    ELLOOP        CPU = 00:00:37.256     TOD = 23:44:11
END      ELLOOP        CPU = 00:00:37.349     TOD = 23:44:12
RESIDUAL NORM =  3.7646SE-01
BEGIN    SOLN          CPU = 00:00:37.353     TOD = 23:44:12
END      SOLN          CPU = 00:00:37.386     TOD = 23:44:12
BEGIN    ELLOOP        CPU = 00:00:37.389     TOD = 23:44:12
END      ELLOOP        CPU = 00:00:37.476     TOD = 23:44:13
RESIDUAL NORM =  1.71736E-01
BEGIN    SOLN          CPU = 00:00:37.479     TOD = 23:44:13
END      SOLN          CPU = 00:00:37.506     TOD = 23:44:14
BEGIN    ELLOOP        CPU = 00:00:37.516     TOD = 23:44:14
END      ELLOOP        CPU = 00:00:37.599     TOD = 23:44:15
RESIDUAL NORM =  3.83068E-02
BEGIN    SOLN          CPU = 00:00:37.602     TOD = 23:44:15
END      SOLN          CPU = 00:00:37.629     TOD = 23:44:15
BEGIN    ELLOOP        CPU = 00:00:37.642     TOD = 23:44:15
END      ELLOOP        CPU = 00:00:37.735     TOD = 23:44:17
RESIDUAL NORM =  1.29982E-03
BEGIN    SOLN          CPU = 00:00:37.739     TOD = 23:44:17
END      SOLN          CPU = 00:00:37.755     TOD = 23:44:17
BEGIN    ELLOOP        CPU = 00:00:37.759     TOD = 23:44:17
```

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                              PAGE      96
VTITLE                                                                          VARIABLE STRUCTURE NUMBER =    1
ITITLE    LOAD INCREMENT 13                                                             INCREMENT NUMBER =   13

END       ELLOOP          CPU = 00:00:37.855      TOD = 23:44:19

RESIDUAL NORM =   3.785711E-04

BEGIN     SOLN            CPU = 00:00:37.866      TOD = 23:44:19

END       SOLN            CPU = 00:00:37.909      TOD = 23:44:19

BEGIN     ELLOOP          CPU = 00:00:37.918      TOD = 23:44:19

END       ELLOOP          CPU = 00:00:38.005      TOD = 23:44:21

RESIDUAL NORM =   2.02509E-06


                         E N D   O F   L O A D   I N C R E M E N T   13

NO. ELASTIC INTEGRATION POINTS =     0,  NO. PLASTIC INTEGRATION POINTS =     4
    0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,    0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =      1, NO. UPDATES PERFORMED =      0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      6
SPECIFIED MAX. UNBALANCED-FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  2.0251E-06

BEGIN     BIGSCK          CPU = 00:00:38.045      TOD = 23:44:22

END       BIGSCK          CPU = 00:00:38.111      TOD = 23:44:25

BEGIN     OUTPUT          CPU = 00:00:38.175      TOD = 23:44:26


TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                              PAGE      97
VTITLE                                                                          VARIABLE STRUCTURE NUMBER =    1
ITITLE    LOAD INCREMENT 13                                                             INCREMENT NUMBER =   13

                    CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **     ****************  FORCES  *****************          *************  DISPLACEMENTS  *************
 NO.  I.D.       U                V                W                 U               V                W
   1    1     -5.2435971E-06  -2.2499857E 00                      0.0             0.0
   2    2      5.2599053E-06  -2.2499857E 00                      1.9999373E-01   0.0
   3    3      6.4747463E-06   2.2499857E 00                      1.9999427E-01   7.5000000E 00
   4    4     -6.4910575E-06   2.2499857E 00                     -3.7650631E-07   7.5000000E 00
```

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	4.5000E 00	-3.6107E-06	4.5000E 00	-1.5474E-06	-1.6308E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	6.2499E-01	5.2727E-06	6.2500E-01	5.1151E-06	-1.6407E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-3.0000E-01	9.9999E-01	-3.0000E-01	-4.7113E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.5497E-06	-1.7881E-06	1.1921E-06	-4.7444E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.4000E 01	-1.5000E 00	3.0000E 00	-1.5000E 00	-9.3286E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	2.0937E 00	-2.5000E-01	5.0000E-01	-2.5000E-01	-8.8145E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.8125E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	5.4024E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0	2	4.8667E 00	2.0[illegible]-01	7.5000E 00	2.0000E-01	3.9984E-08	0.0	0[illegible]

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 99
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 13 INCREMENT NUMBER = 13

ELEMENT NO.	I.D.	POINT NO.	TP.	YIELD STRESS CTR.	YIELD STRESS SIZE	**** EFFECTIVE PLASTIC STRAINS **** INCREMENTAL	SUM INCR.	CUMULATIVE	***** EFFECTIVE CREEP STRAINS ***** INCREMENTAL	SUM INCR.	CUMULATIVE
1	3	5	4	2.0000E 00	2.5000E 00	5.0000E-01	5.0000E 00	3.0000E 00	0.0	3.0000E 00	1.0000E 00

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE TEMPERATURE	**** CUMULATIVE THERMAL STRAINS **** XX	YY	ZZ
1	3	5	4	1.3000E 01	2.5000E 00	2.5000E 00	2.5000E 00

END OUTPUT CPU = 00:00:38.311 TOD = 23:44:28

B.4-77

TITLE BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM PAGE 100
VTITLE VARIABLE STRUCTURE NUMBER = 1
ITITLE LOAD INCREMENT 14 INCREMENT NUMBER = 14

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD SET ONE	=	8.799999E 00
COEFFICIENT FOR CONCENTRATED LOAD SET TWO	=	0.0
COEFFICIENT FOR DISTRIBUTED LOAD SET ONE	=	0.0
COEFFICIENT FOR DISTRIBUTED LOAD SET TWO	=	0.0
COEFFICIENT FOR NODAL TEMPERATURE SET	=	1.400000E 01
COEFFICIENT FOR NORMAL STRESS/STRAIN SET	=	0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)	=	0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)	=	0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)	=	0.0
CREEP TIME	=	0.0

BEGIN LOADS CPU = 00:00:38.597 TOD = 23:44:34

END LOADS CPU = 00:00:38.680 TOD = 23:44:36

```
BEGIN   SOLN      CPU = 00:00:38.740     TOD = 23:44:37
END     SOLN      CPU = 00:00:38.780     TOD = 23:44:37
BEGIN   ELLOOP    CPU = 00:00:38.767     TOD = 23:44:37
END     ELLOOP    CPU = 00:00:38.850     TOD = 23:44:39
RESIDUAL NORM =  2.67252E-01
BEGIN   SOLN      CPU = 00:00:38.853     TOD = 23:44:39
END     SOLN      CPU = 00:00:38.883     TOD = 23:44:39
BEGIN   ELLOOP    CPU = 00:00:38.897     TOD = 23:44:39
END     ELLOOP    CPU = 00:00:38.997     TOD = 23:44:42
RESIDUAL NORM =  2.26963E-01
BEGIN   SOLN      CPU = 00:00:39.003     TOD = 23:44:42
END     SOLN      CPU = 00:00:39.023     TOD = 23:44:42
BEGIN   ELLOOP    CPU = 00:00:39.030     TOD = 23:44:42
END     ELLOOP    CPU = 00:00:39.136     TOD = 23:44:44
RESIDUAL NORM =  8.43294E-02
BEGIN   SOLN      CPU = 00:00:39.140     TOD = 23:44:44
END     SOLN      CPU = 00:00:39.166     TOD = 23:44:45
BEGIN   ELLOOP    CPU = 00:00:39.170     TOD = 23:44:45
END     ELLOOP    CPU = 00:00:39.253     TOD = 23:44:46
RESIDUAL NORM =  1.04967E-02
BEGIN   SOLN      CPU = 00:00:39.259     TOD = 23:44:46
END     SOLN      CPU = 00:00:39.283     TOD = 23:44:46
BEGIN   ELLOOP    CPU = 00:00:39.289     TOD = 23:44:47
```

```
END      ELLOCP        CPU = 00:00:39.373     TOD = 23:44:48

RESIDUAL NORM =  1.49375E-02

BEGIN    SOLN          CPU = 00:00:39.386     TOD = 23:44:48

END      SOLN          CPU = 00:00:39.413     TOD = 23:44:48

BEGIN    ELLOCP        CPU = 00:00:39.423     TOD = 23:44:48

END      ELLOCP        CPU = 00:00:39.509     TOD = 23:44:49

RESIDUAL NORM =  1.97546E-04

BEGIN    SOLN          CPU = 00:00:39.512     TOD = 23:44:49

END      SOLN          CPU = 00:00:39.539     TOD = 23:44:49

BEGIN    ELLOCP        CPU = 00:00:39.542     TOD = 23:44:50

END      ELLOCP        CPU = 00:00:39.642     TOD = 23:44:51

RESIDUAL NORM =  2.57856E-05

BEGIN    SOLN          CPU = 00:00:39.649     TOD = 23:44:51

END      SOLN          CPU = 00:00:39.672     TOD = 23:44:51

BEGIN    ELLOCP        CPU = 00:00:39.662     TOD = 23:44:51

END      ELLOCP        CPU = 00:00:39.762     TOD = 23:44:53

RESIDUAL NORM =  2.60094E-06



                         E N D   O F   L O A D   I N C R E M E N T   14

NO. ELASTIC INTEGRATION POINTS =     0,  NO. PLASTIC INTEGRATION POINTS =     4
     0  INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,    0  INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =      1, NO. UPDATES PERFORMED =      0
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE =      8
SPECIFIED MAX. UNBALANCED FORCE ERROR =  1.0000E-05, ACTUAL ERROR =  2.6009E-06
```

```
TITLE    EOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                        PAGE    103
VTITLE                                                      VARIABLE STRUCTURE NUMBER =    1
ITITLE   LOAD INCREMENT 14                                           INCREMENT NUMBER =   14

BEGIN    BIGSCK       CPU = 00:00:39.825      TOD = 23:44:53
END      BIGSCK       CPU = 00:00:39.845      TOD = 23:44:56
BEGIN    OUTPUT       CPU = 00:00:39.956      TOD = 23:44:57
```

B4-80

```
TITLE    EOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                        PAGE    104
VTITLE                                                      VARIABLE STRUCTURE NUMBER =    1
ITITLE   LOAD INCREMENT 14                                           INCREMENT NUMBER =   14

                CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

** NODE **   ***************  FORCES  ****************      *************  DISPLACEMENTS  *************
NO.  I.D.    U                V                 W           U                V                W
  1    1    -8.8620929E-06  -2.6250319E 00                   0.0              0.0
  2    2     8.8012556E-06  -2.6250334E 00                   2.7951610E-06    0.0
  3    3     1.0304576E-05   2.6250334E 00                   3.1354430E-06    8.7499992E 00
  4    4    -1.0303243E-05   2.6250319E 00                  -2.1541524E-07    8.7499992E 00
```

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE CUM. STRESS	CUMULATIVE STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.2501E 00	1.8053E-05	5.2501E 00	-4.9142E-06	1.2327E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	EFFECTIVE INCR. STRESS	INCREMENTAL STRESSES XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	7.5009E-01	2.1664E-05	7.5010E-01	-3.2065E-06	1.7541E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-3.0000E-01	1.0000E 00	-3.0000E-01	3.0524E-10	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL ELASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	-1.8477E-06	1.8299E-05	-7.2122E-06	5.0165E-09	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE PLASTIC WORK	CUMULATIVE PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	1.6875E 01	-2.0000E 00	4.0000E 00	-2.0000E 00	-2.3112E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL PLASTIC WORK	INCREMENTAL PLASTIC STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	4.8749E 00	-4.9999E-01	9.9998E-01	-4.9999E-01	-1.3784E-08	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	CUMULATIVE CREEP WORK	CUMULATIVE CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	5.8125E 00	-5.0000E-01	1.0000E 00	-5.0000E-01	5.4024E-06	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	INCREMENTAL CREEP WORK	INCREMENTAL CREEP STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ELEMENT NO.	I.D.	POINT NO.	TP.	E-P CODE	SUM CODE	CUM. EFF. TOTAL STRAIN	CUMULATIVE TOTAL STRAINS XX	YY	ZZ	XY	XZ	YZ
1	3	5	4	0	2	5.8667E 00	4.7694E-06	8.5000E 00	9.5367E-07	3.1217E-06	0.0	0.0

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                PAGE    106
VTITLE                                                                           VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 14                                                               INCREMENT NUMBER =  14

  ELEMENT     POINT                    YIELD        YIELD      **** EFFECTIVE PLASTIC STRAINS ****  ***** EFFECTIVE CREEP STRAINS *****
 NO.  I.D.  NO. TP.               STRESS CTR. STRESS SIZE    INCREMENTAL  SUM INCR.   CUMULATIVE INCREMENTAL  SUM INCR.   CUMULATIVE
  1     3     5  4                 2.5000E 00  2.7500E 00     9.9996E-01  6.0000E 00  4.0000E 00  0.0          3.0000E 00  1.0000E 00

  ELEMENT     POINT              CUMULATIVE               **** CUMULATIVE THERMAL STRAINS ****
 NO.  I.D.  NO. TP.              TEMPERATURE                   XX          YY          ZZ
  1     3     5  4                1.4000E 01              2.8000E 00  2.8000E 00  2.8000E 00

END       OUTPUT         CPU = 00:00:40.095       TOD = 23:44:58
```

8.4.82

```
TITLE     BOPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                PAGE    107
VTITLE                                                                           VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 15                                                               INCREMENT NUMBER =  15

PARAMETERS FOR THIS INCREMENT

COEFFICIENT FOR CONCENTRATED LOAD  SET  ONE        =   1.628749E 01
COEFFICIENT FOR CONCENTRATED LOAD  SET  TWO        =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  ONE         =   0.0
COEFFICIENT FOR DISTRIBUTED LOAD  SET  TWO         =   0.0
COEFFICIENT FOR NODAL TEMPERATURE  SET             =   1.500000E 01
COEFFICIENT FOR NORMAL STRESS/STRAIN  SET          =   0.0
TRANSLATIONAL ACCELERATION (LENGTH/TIME/TIME)      =   0.0
ANGULAR VELOCITY (REVOLUTIONS/TIME)                =   0.0
ANGULAR ACCELERATION (REVOLUTIONS/TIME/TIME)       =   0.0
CREEP TIME                                         =   1.000000E 01

BEGIN     LOADS          CPU = 00:00:40.354       TOD = 23:45:05

END       LOADS          CPU = 00:00:40.448       TOD = 23:45:09
```

```
TITLE     BOPACL CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                      PAGE    108
VTITLE                                                                                  VARIABLE STRUCTURE NUMBER =    1
ITITLE    LOAD INCREMENT 15                                                                      INCREMENT NUMBER =   15

BEGIN   SOLN          CPU = 00:00:40.501      TOD = 23:45:11
END     SOLN          CPU = 00:00:40.541      TOD = 23:45:12
BEGIN   ELLOOP        CPU = 00:00:40.544      TOD = 23:45:12
END     ELLOOP        CPU = 00:00:40.641      TOD = 23:45:14
RESIDUAL NORM =  6.60574E-01
BEGIN   SOLN          CPU = 00:00:40.641      TOD = 23:45:14
END     SOLN          CPU = 00:00:40.677      TOD = 23:45:15
BEGIN   ELLOOP        CPU = 00:00:40.677      TOD = 23:45:15
END     ELLOOP        CPU = 00:00:40.760      TOD = 23:45:17
RESIDUAL NORM =  3.21451E-01
BEGIN   SOLN          CPU = 00:00:40.767      TOD = 23:45:17
END     SOLN          CPU = 00:00:40.790      TOD = 23:45:17
BEGIN   ELLOOP        CPU = 00:00:40.796      TOD = 23:45:17
END     ELLOOP        CPU = 00:00:40.883      TOD = 23:45:19
RESIDUAL NORM =  3.47219E-01
BEGIN   ELLOOP        CPU = 00:00:40.897      TOD = 23:45:19
END     ELLOOP        CPU = 00:00:40.980      TOD = 23:45:20
RESIDUAL NORM =  2.90025E-01
BEGIN   SOLN          CPU = 00:00:40.980      TOD = 23:45:20
END     SOLN          CPU = 00:00:41.013      TOD = 23:45:20
BEGIN   ELLOOP        CPU = 00:00:41.013      TOD = 23:45:20
END     ELLOOP        CPU = 00:00:41.123      TOD = 23:45:21
RESIDUAL NORM =  5.37855E-02
```

```
BEGIN   SOLN      CPU = 00:00:41.130    TOD = 23:45:21
END     SOLN      CPU = 00:00:41.160    TOD = 23:45:22
BEGIN   ELLOOP    CPU = 00:00:41.190    TOD = 23:45:22
END     ELLOOP    CPU = 00:00:41.296    TOD = 23:45:23
RESIDUAL NORM =  5.72994E-02
BEGIN   ELLOOP    CPU = 00:00:41.299    TOD = 23:45:23
END     ELLOOP    CPU = 00:00:41.405    TOD = 23:45:24
RESIDUAL NORM =  6.38256E-02
BEGIN   ELLOOP    CPU = 00:00:41.409    TOD = 23:45:24
END     ELLOOP    CPU = 00:00:41.493    TOD = 23:45:26
RESIDUAL NORM =  8.11595E-02
BEGIN   SOLN      CPU = 00:00:41.486    TOD = 23:45:26
END     SOLN      CPU = 00:00:41.519    TOD = 23:45:26
BEGIN   ELLOOP    CPU = 00:00:41.522    TOD = 23:45:26
END     ELLOOP    CPU = 00:00:41.612    TOD = 23:45:28
RESIDUAL NORM =  3.12196E-02
BEGIN   SOLN      CPU = 00:00:41.619    TOD = 23:45:28
END     SOLN      CPU = 00:00:41.646    TOD = 23:45:28
BEGIN   ELLOOP    CPU = 00:00:41.649    TOD = 23:45:29
END     ELLOOP    CPU = 00:00:41.729    TOD = 23:45:30
RESIDUAL NORM =  1.44403E-02
BEGIN   MERGE     CPU = 00:00:41.759    TOD = 23:45:30
BEGIN   GENKP     CPU = 00:00:41.772    TOD = 23:45:30
```

```
STIFFNESS GENERATION COMPLETED.        10 PARTITIONS WRITTEN.
END      GENRB       CPU = 00:00:41.865    TOD = 23:45:32
BEGIN    MERSCR      CPU = 00:00:41.872    TOD = 23:45:32
END      MERSCR      CPU = 00:00:41.912    TOD = 23:45:32
END      MERGE       CPU = 00:00:41.912    TOD = 23:45:32
MAXIMUM WAVEFRONT =    4 NODES AT INTERNAL NODE     1
BEGIN    DECOMP      CPU = 00:00:41.952    TOD = 23:45:33
END      DECOMP      CPU = 00:00:42.011    TOD = 23:45:33
BEGIN    BIGSCK      CPU = 00:00:42.032    TOD = 23:45:34
END      BIGSCK      CPU = 00:00:42.075    TOD = 23:45:36
BEGIN    MERGE       CPU = 00:00:42.078    TOD = 23:45:36
BEGIN    GENRB       CPU = 00:00:42.082    TOD = 23:45:36
STIFFNESS GENERATION COMPLETED.        10 PARTITIONS WRITTEN.
END      GENRB       CPU = 00:00:42.165    TOD = 23:45:38
BEGIN    MERSCR      CPU = 00:00:42.166    TOD = 23:45:38
END      MERSCR      CPU = 00:00:42.225    TOD = 23:45:39
END      MERGE       CPU = 00:00:42.225    TOD = 23:45:39
MAXIMUM WAVEFRONT =    4 NODES AT INTERNAL NODE     1
BEGIN    DECOMP      CPU = 00:00:42.275    TOD = 23:45:40
END      DECOMP      CPU = 00:00:42.329    TOD = 23:45:40
BEGIN    BIGSCK      CPU = 00:00:42.348    TOD = 23:45:41
END      BIGSCK      CPU = 00:00:42.391    TOD = 23:45:43
BEGIN    SEEN        CPU = 00:00:42.414    TOD = 23:45:43
```

```
TITLE     BCPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                   PAGE    111
VTITLE                                                        VARIABLE STRUCTURE NUMBER =      1
ITITLE    LOAD INCREMENT 15                                   INCREMENT NUMBER =             15

END       SOLN        CPU = 00:00:42.446      TOD = 23:45:43
BEGIN     ELLCOP      CPU = 00:00:42.464      TOD = 23:45:43
END       ELLCOP      CPU = 00:00:42.547      TOD = 23:45:45
RESIDUAL NORM =  2.73724E-03
BEGIN     SOLN        CPU = 00:00:42.551      TOD = 23:45:45
END       SOLN        CPU = 00:00:42.584      TOD = 23:45:45
BEGIN     ELLCOP      CPU = 00:00:42.591      TOD = 23:45:45
END       ELLCOP      CPU = 00:00:42.677      TOD = 23:45:47
RESIDUAL NORM =  1.03623E-03
BEGIN     SOLN        CPU = 00:00:42.677      TOD = 23:45:47
END       SOLN        CPU = 00:00:42.711      TOD = 23:45:47
BEGIN     ELLCOP      CPU = 00:00:42.717      TOD = 23:45:47
END       ELLCOP      CPU = 00:00:42.797      TOD = 23:45:49
RESIDUAL NORM =  9.44475E-05
BEGIN     SOLN        CPU = 00:00:42.797      TOD = 23:45:49
END       SOLN        CPU = 00:00:42.836      TOD = 23:45:49
BEGIN     ELLCOP      CPU = 00:00:42.837      TOD = 23:45:49
END       ELLCOP      CPU = 00:00:42.937      TOD = 23:45:50
RESIDUAL NORM =  9.54186E-05
BEGIN     ELLCOP      CPU = 00:00:42.940      TOD = 23:45:51
END       ELLCOP      CPU = 00:00:43.033      TOD = 23:45:52
RESIDUAL NORM =  1.06239E-04
BEGIN     ELLCOP      CPU = 00:00:43.043      TOD = 23:45:52
```

8-4-86

```
TITLE     BLFACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                         PAGE    112
VTITLE                                                                        VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 15                                                       INCREMENT NUMBER =   15

END       ELLOOP        CPU = 00:00:43.153      TOD = 23:45:53
RESIDUAL NORM =  1.569201E-04
BEGIN     SOLN          CPU = 00:00:43.167      TOD = 23:45:53
END       SOLN          CPU = 00:00:43.183      TOD = 23:45:53
BEGIN     ELLOOP        CPU = 00:00:43.193      TOD = 23:45:53
END       ELLOOP        CPU = 00:00:43.280      TOD = 23:45:54
RESIDUAL NORM =  2.894556E-05
BEGIN     SOLN          CPU = 00:00:43.286      TOD = 23:45:54
END       SOLN          CPU = 00:00:43.306      TOD = 23:45:54
BEGIN     ELLOOP        CPU = 00:00:43.313      TOD = 23:45:54
END       ELLOOP        CPU = 00:00:43.409      TOD = 23:45:55
RESIDUAL NORM =  1.14418E-05
BEGIN     SOLN          CPU = 00:00:43.413      TOD = 23:45:55
END       SOLN          CPU = 00:00:43.433      TOD = 23:45:56
BEGIN     ELLOOP        CPU = 00:00:43.436      TOD = 23:45:56
END       ELLOOP        CPU = 00:00:43.519      TOD = 23:45:57
RESIDUAL NORM =  7.41206E-07

                        E N D   O F   L O A D   I N C R E M E N T   15

NO. ELASTIC INTEGRATION POINTS =     0,  NO. PLASTIC INTEGRATION POINTS =     4
    0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC,     0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT
SPECIFIED MAX. NO. STIFFNESS UPDATES =     1,  NO. UPDATES PERFORMED =     1
SPECIFIED MAX. NO. ITERATIONS PER UPDATE =   10  10  10,  NO. ITERATIONS PERFORMED SINCE LAST UPDATE =     9
SPECIFIED MAX. UNBALANCED FORCE ERROR =  1.0000E-05,  ACTUAL ERROR =  7.41215-07
```

```
TITLE     EDPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                          PAGE    113
VTITLE                                                       VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 15                                      INCREMENT NUMBER =  15

BEGIN   BIGSCK        CPU = 00:00:43.579     TOD = 23:45:57

END     BIGSCK        CPU = 00:00:43.659     TOD = 23:46:00

BEGIN   OUTPUT        CPU = 00:00:43.764     TOD = 23:46:01
```

```
TITLE     EDPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                          PAGE    114
VTITLE                                                       VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 15                                      INCREMENT NUMBER =  15

               CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS
```

** NODE **		******************	FORCES ******************		*************	DISPLACEMENTS *************	
NO.	I.D.	U	V	W	U	V	W
1	1	4.[illegible]E-06	-2.9999723E 00		0.0	0.0	
2	2	-3.566077[illegible]E-06	-2.9999714E 00		-8.6763321E-06	0.0	
3	3	-2.4615[illegible]E-06	2.9999714E 00		-8.2691840E-06	1.6267491E 01	
4	4	2.4[illegible]E-06	2.9999723E 00		-4.8016079E-07	1.6287491E 01	

```
TITLE     SCPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                          PAGE    115
VTITLE                                                                  VARIABLE STRUCTURE NUMBER =    1
ITITLE    LOAD INCREMENT 15                                                     INCREMENT NUMBER =    15

ELEMENT      POINT              EFFECTIVE    ********************** CUMULATIVE STRESSES **********************
NO.  I.D.  NO. TP.             CUM. STRESS      XX          YY          ZZ          XY        XZ          YZ
 1     3     5  4              6.0000E 00  -2.6134E-05  5.9999E 00 -1.2723E-05 -1.4778E-06  0.0         0.0

ELEMENT      POINT              EFFECTIVE    ********************** INCREMENTAL STRESSES **********************
NO.  I.D.  NO. TP.             INCR. STRESS     XX          YY          ZZ          XY        XZ          YZ
 1     3     5  4              7.4996E-01  -4.4188E-05  7.4967E-01 -7.9091E-06 -1.6011E-08  0.0         0.0

ELEMENT      POINT                           ********************** CUMULATIVE ELASTIC STRAINS **********************
NO.  I.D.  NO. TP.                              XX          YY          ZZ          XY        XZ          YZ
 1     3     5  4                           -6.0000E-01  2.0000E 00 -5.9999E-01 -6.4038E-09  0.0         0.0

ELEMENT      POINT                           ********************** INCREMENTAL ELASTIC STRAINS **********************
NO.  I.D.  NO. TP.                              XX          YY          ZZ          XY        XZ          YZ
 1     3     5  4                           -3.0000E-01  9.9997E-01 -2.9999E-01 -6.7090E-09  0.0         0.0

ELEMENT      POINT              CUMULATIVE   ********************** CUMULATIVE PLASTIC STRAINS **********************
NO.  I.D.  NO. TP.             PLASTIC WORK     XX          YY          ZZ          XY        XZ          YZ
 1     3     5  4              2.4500E 01  -2.5000E 00  5.0000E 00 -2.5000E 00 -3.3479E-08  0.0         0.0

ELEMENT      POINT              INCREMENTAL  ********************** INCREMENTAL PLASTIC STRAINS **********************
NO.  I.D.  NO. TP.             PLASTIC WORK     XX          YY          ZZ          XY        XZ          YZ
 1     3     5  4              5.6250E 00  -5.0000E-01  1.0000E 00 -5.0000E-01 -1.0366E-08  0.0         0.0

ELEMENT      POINT              CUMULATIVE   ********************** CUMULATIVE CREEP STRAINS **********************
NO.  I.D.  NO. TP.             CREEP WORK       XX          YY          ZZ          XY        XZ          YZ
 1     3     5  4              2.3391E 01  -2.0625E 00  4.1250E 00 -2.0625E 00  2.1629E-08  0.0         0.0

ELEMENT      POINT              INCREMENTAL  ********************** INCREMENTAL CREEP STRAINS **********************
NO.  I.D.  NO. TP.             CREEP WORK       XX          YY          ZZ          XY        XZ          YZ
 1     3     5  4              1.7578E 01  -1.5625E 00  3.1250E 00 -1.5625E 00 -3.2395E-08  0.0         0.0

ELEMENT      POINT        E-P  SUM  CUM. EFF.  ********************** CUMULATIVE TOTAL STRAINS **********************
NO.  I.D.  NO. TP.        CODE COM  TOTAL STRAIN    XX          YY          ZZ          XY        XZ          YZ
 1     3     5  4          5    2   1.0859E 01  -6.6757E-06  1.6287E 01  0.0        -1.8253E-06  0.0         0.0
```

```
TITLE     BCPACE CYCLIC PLASTIC-CREEP CHECKOUT PROBLEM                                                              PAGE    116
VTITLE                                                                                               VARIABLE STRUCTURE NUMBER =   1
ITITLE    LOAD INCREMENT 15                                                                                  INCREMENT NUMBER =  15

  ELEMENT      POINT              YIELD        YIELD        **** EFFECTIVE PLASTIC STRAINS ****    ***** EFFECTIVE CREEP STRAINS *****
  NO.  I.D.   NO. TP.           STRESS CTR. STRESS SIZE    INCREMENTAL   SUM INCR.   CUMULATIVE INCREMENTAL   SUM INCR.   CUMULATIVE
   1     3      5  4             3.0000E 00  3.0000E 00     1.0000E 00  7.0000E 00  5.0000E 00  3.1250E 00  6.1250E 00  4.1250E 00

  ELEMENT      POINT            CUMULATIVE                 **** CUMULATIVE THERMAL STRAINS ****
  NO.  I.D.   NO. TP.           TEMPERATURE                    XX          YY          ZZ
   1     3      5  4             1.5000E 01                5.1625E 00  5.1625E 00  5.1625E 00

END      OUTPUT        CPU = 00:00:43.835       TOD = 23:46:02

END OF BCPACE JOB
```

B.5 STAINLESS STEEL CYCLIC TEST/ANALYSIS CORRELATION

This example is intended to aid the user in reducing cyclic test data into a form suitable for BOPACE input. The stainless steel specimen used in the example exhibits a pronounced variation in isotropic and kinematic hardening as the cycling progresses, and thus provides a severe test of the BOPACE capability for combined hardening.

The cyclic test data are shown in Figure B.5-1 by the solid lines.† The plotted points and dashed lines represent the analytical results obtained from a BOPACE run. The hysteresis loops are denoted by dash numbers, with the first number denoting the cycle number and the second denoting the first and second half of the cycle.

Because of the obvious variation in magnitude of kinematic hardening over the test cycles, the BOPACE option for variable kinematic hardening was employed in addition to the usual combined isotropic and kinematic hardening. It is to be noted that this type of material input data is somewhat complicated, and it is used here to demonstrate the accuracy which can be achieved through a careful representation of the material cyclic properties. For most analyses a simpler hardening representation will be sufficient.

The assumed hardening curves are given in Figure B.5-2, and the BOPACE input data are listed at the end of this section. A summary of the analytical results is shown in Table B.5-1, including the effective kinematic stress

† The test hysteresis loops were furnished by Dr. R. H. Mallett of the Advanced Reactor Division, Westinghouse Electric Corp.

hardening ($\frac{3}{2}$ α_{xx}), isotropic stress hardening $|\bar{\sigma}-\bar{\alpha}|$, and actual stress σ_{xx}.

The test/analysis correlation is quite good, and could be further improved if one were willing to accept some amount of non-smoothness in the input hardening curves of Figure B.5-2. An exact match would require hardening curves with discontinuous slopes at points corresponding to the strain range used in the cyclic test. This is not justifiable in general, because a proper test/analysis match would not result for other strain ranges. Although the test/analysis correlation is considered to be very satisfactory, cyclic results for other strain ranges would have to be compared before any definite conslusions can be drawn.

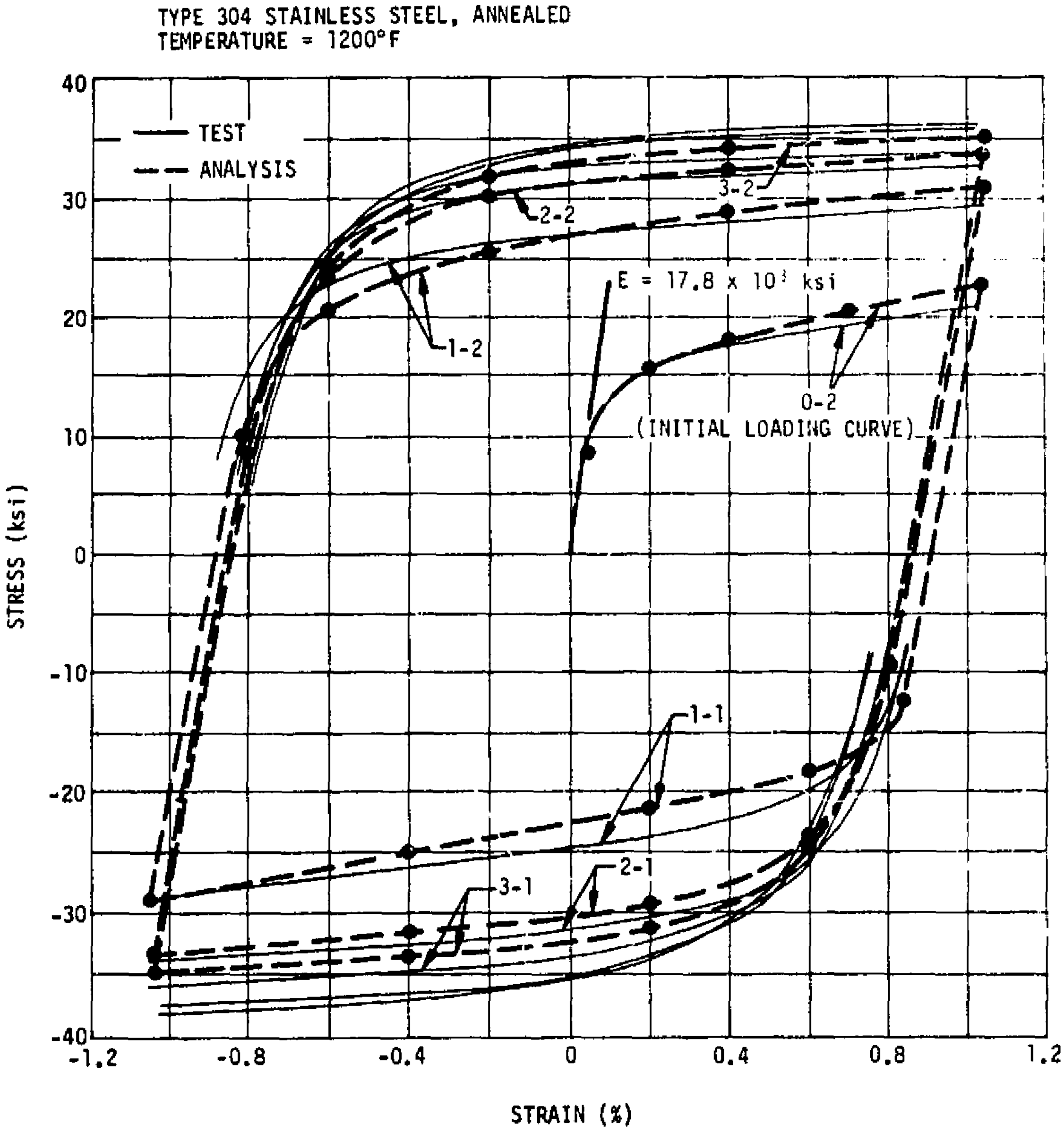

Figure B.5-1: Stainless Steel Cyclic Test/Analysis Correlation

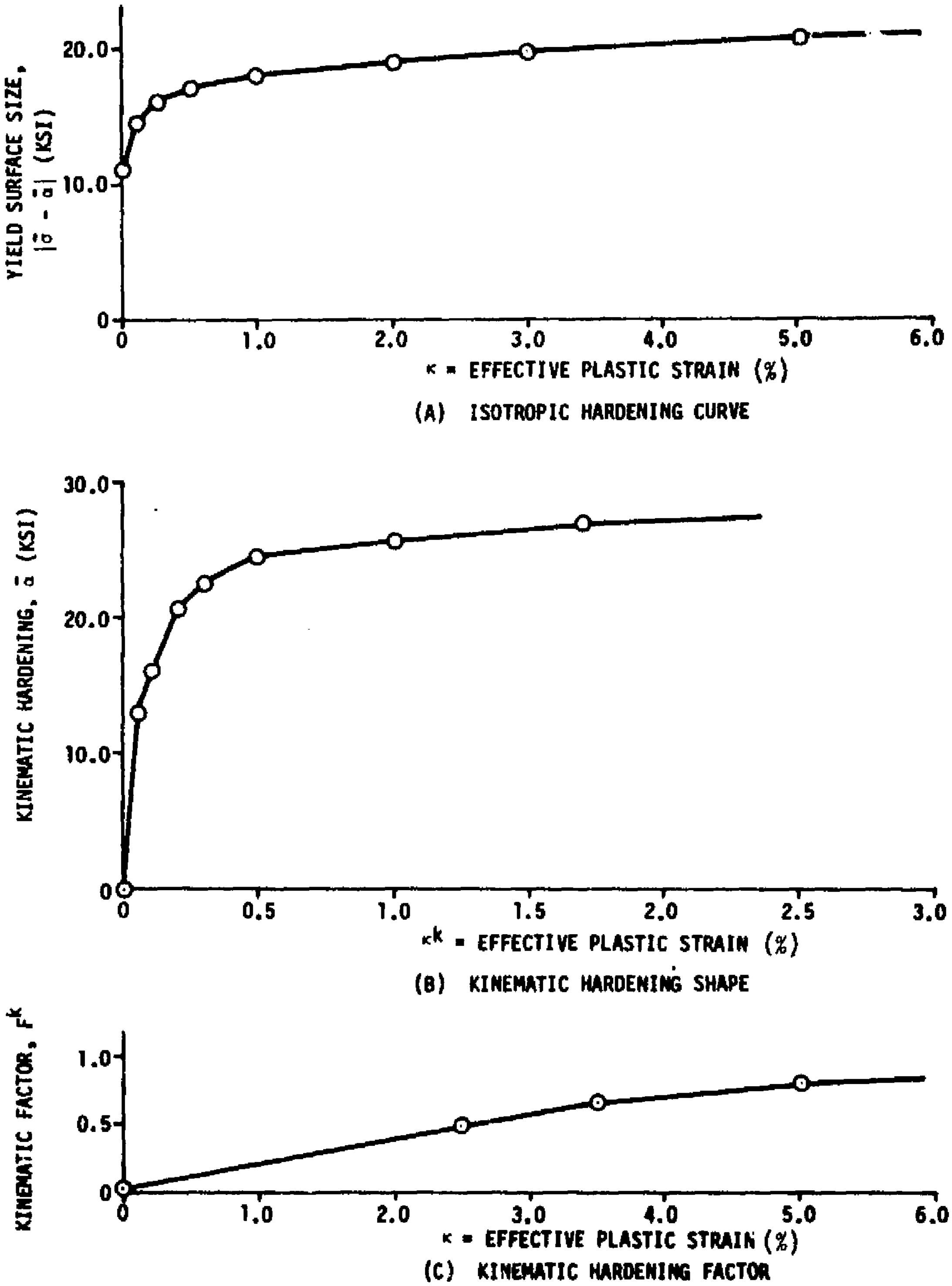

Figure B.5-2: Stainless Steel Hardening Assumptions

Table B.5-1: Results for Stainless Steel Hardening

INCR.	CYCLE	DISPLACEMENT	$\frac{3}{2}\alpha_{XX}$	$\lvert\bar{\sigma}-\bar{\alpha}\rvert$	σ_{XX}	ITERATIONS
1	0-2	.05	0.0	11.0	8.9	1
2	0-2	.2	0.7	14.6	15.3	4
3	0-2	.4	1.8	16.2	18.0	5
4	0-2	.7	3.3	17.1	20.4	5
5	0-2	1.04	5.0	17.9	22.9	5
6	1-1	.9	5.0	17.9	-2.1	1
7	1-1	.8	3.4	17.9	-14.5	4
8	1-1	.6	0.1	18.1	-18.0	5
9	1-1	.2	2.7	18.5	-21.2	5
10	1-1	-.4	-6.0	19.0	-25.0	5
11	1-1	-1.04	-9.5	19.5	-29.0	5
12	1-2	-.9	-9.5	19.5	-4.1	1
13	1-2	-.8	-7.9	19.5	11.6	5
14	1-2	-.6	1.0	19.6	20.6	6
15	1-2	-.2	5.9	19.8	25.7	5
16	1-2	.4	8.6	20.1	28.7	5
17	1-2	1.04	10.8	20.5	31.3	5
18	2-1	.9	10.8	20.5	6.4	1
19	2-1	.8	9.9	20.5	-10.6	4
20	2-1	.6	-2.5	20.5	23.0	6
21	2-1	.2	-8.7	20.7	-29.4	5
22	2-1	-.4	-10.5	20.9	-31.4	5
23	2-1	-1.04	-12.4	21.1	-33.5	5
24	2-2	-.9	-12.4	21.1	-8.6	1
25	2-2	-.8	-12.1	21.1	9.0	4
26	2-2	-.6	2.3	21.1	23.4	7
27	2-2	-.2	9.1	21.3	30.4	5
28	2-2	.4	11.0	21.5	32.5	5
29	2-2	1.04	12.4	21.6	34.0	5
30	3-1	.9	12.4	21.6	9.1	1
31	3-1	.8	12.4	21.6	-8.7	1
32	3-1	.6	-2.8	21.6	-24.4	7
33	3-1	.2	-10.0	21.8	-31.8	5
34	3-1	-.4	-11.7	21.9	-33.6	5
35	3-1	-1.04	-13.1	22.1	-35.2	5
36	3-2	-.9	-13.1	22.1	-10.3	1
37	3-2	-.8	-13.1	22.1	7.5	1
38	3-2	-.6	2.2	22.1	24.3	7
39	3-2	-.2	9.8	22.3	32.1	5
40	3-2	.4	11.5	22.3	33.8	5
41	3-2	1.04	12.7	22.5	35.2	5

INPUT DATA

```
CARD
NUMBER
   1     TITLE STAINLESS STEEL CYCLIC TEST/ANALYSIS CORRELATION
   2     CONT GENERAL COMBINED HARDENING WITH 2-PARAMETER KINEMATIC REPRESENTATION
   3     PROB 2
   4     SOLU -.001   3
   5     CHECKPOINT   29
   6     PRT1  1,-1
   7     PRT2  1,-1  3,-1  5,-1  9,-1
   8     VTITLE
   9     MAT1  1
  10     $ MODULUS 17,800 KSI DIVIDED BY 100 TO GIVE STRAIN IN PER CENT
  11     IMOD  1200.,178.
  12     IPOIS  1200.,.3
  13     PLASTIC  1  1  1
  14     FTEMP  1200.
  15     $ ALL STRAINS GIVEN IN PER CENT
  16     IHARD  0,11.  .10,14.5  .25,16.0  .5,17.0  1.0,18.0  2.0,19.0  3.0,19.7
  17     CONT  5.0,20.7  7.0,21.4  10.0,22.2  15.0,23.0  20.0,23.5  100.0,23.5
  18     KSHAPE  0,0  .05,13.0  .1,16.0  .2,20.5  .3,22.5  .5,24.5  1.0,25.7  1.7,27.0
  19     CONT  5.0,28.6
  20     KFACT  0,.02  2.5,.5  3.5,.67  5.0,.8  7.0,.9  10.0,.95  20.0,1.0  100.0,1.0
  21     NODE  1  0,0
  22     NODE  2  1,0
  23     NODE  3  1,1
  24     NODE  4  0,1
  25     $PQUAD  PID  THICK  NSCODE  FTEMP
  26     PQUAD    1    1.0      1    1200.
  27     $KQUAD  KID  LID  DID  RPCODE
  28     KQUAD    1    1    1     4    1,2,2
  29     QUAD  1  1,1,1  1,2,3,4
  30     SPC  1,1  1,2  2,2  3,2  4,2
  31     ITITLE  CYCLE 0-2
  32     LFACT  .05
  33     $CLOAD  CLSID    NID  C  V    NID  C  V
  34     CLOAD     1       3  2  1.     4  2  1.
  35     ITITLE  CYCLE 0-2
  36     LFACT  .2
  37     ITITLE  CYCLE 0-2
  38     LFACT  .4
  39     ITITLE  CYCLE 0-2
  40     LFACT  .7
  41     ITITLE  CYCLE 0-2 TIP
  42     LFACT  1.04
  43     ITITLE  CYCLE 1-1
  44     LFACT  .9
  45     ITITLE  CYCLE 1-1
  46     LFACT  .8
  47     ITITLE  CYCLE 1-1
  48     LFACT  .6
  49     ITITLE  CYCLE 1-1
  50     LFACT  .2
```

INPUT DATA

CARD NUMBER	
51	ITITLE CYCLE 1-1
52	LFACT -.4
53	ITITLE CYCLE 1-1 TIP
54	LFACT -1.0
55	ITITLE CYCLE 1-2
56	LFACT -.9
57	ITITLE CYCLE 1-2
58	LFACT -.8
59	ITITLE CYCLE 1-2
60	LFACT -.6
61	ITITLE CYCLE 1-2
62	LFACT -.2
63	ITITLE CYCLE 1-2
64	LFACT .4
65	ITITLE CYCLE 1-2 TIP
66	LFACT 1.0
67	ITITLE CYCLE 2-1
68	LFACT .9
69	ITITLE CYCLE 2-1
70	LFACT .8
71	ITITLE CYCLE 2-1
72	LFACT .6
73	ITITLE CYCLE 2-1
74	LFACT .2
75	ITITLE CYCLE 2-1
76	LFACT -.4
77	ITITLE CYCLE 2-1 TIP
78	LFACT -1.04
79	ITITLE CYCLE 2-2
80	LFACT -.9
81	ITITLE CYCLE 2-2
82	LFACT -.8
83	ITITLE CYCLE 2-2
84	LFACT -.6
85	ITITLE CYCLE 2-2
86	LFACT -.2
87	ITITLE CYCLE 2-2
88	LFACT .4
89	ITITLE CYCLE 2-2 TIP
90	LFACT 1.04
91	ITITLE CYCLE 3-1
92	LFACT .9
93	ITITLE CYCLE 3-1
94	LFACT .8
95	ITITLE CYCLE 3-1
96	LFACT .6
97	ITITLE CYCLE 3-1
98	LFACT .2
99	ITITLE CYCLE 3-1
100	LFACT -.4

INPUT DATA

```
CARD
NUMBER
  101        ITITLE  CYCLE 3-1 TIP
  102        LFACT  -1.04
  103        ITITLE  CYCLE 3-2
  104        LFACT  -.9
  105        ITITLE  CYCLE 3-2
  106        LFACT  -.8
  107        ITITLE  CYCLE 3-2
  108        LFACT  -.6
  109        ITITLE  CYCLE 3-2
  110        LFACT  -.2
  111        ITITLE  CYCLE 3-2
  112        LFACT  .4
  113        ITITLE  CYCLE 3-2 TIP
  114        LFACT  1.04
  115        EOF
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