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A MATHEMATICAL SIMULATION MODEL OF A 1985-ERA TILT-ROTOR PASSENGER AIRCRAFT

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BOEING VERTOL COMPANY

A DIVISION OF THE BOEING COMPANY

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A-8						E-27					
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ABSTRACT

A mathematical model for use in real-time piloted simulation of a 1985-era tilt rotor passenger aircraft is presented. The model comprises the basic six degrees-of-freedom equations of motion, and a large angle of attack representation of the airframe and rotor aerodynamics, together with equations and functions used to model turbine engine performance, aircraft control system and stability augmentation system.

A complete derivation of the primary equations is given together with a description of the modeling techniques used. Data for the model is included in an Appendix.

FOREWORD

This report was prepared by the Boeing Vertol Company for the National Aeronautics and Space Administration, Ames Research Center, under Contract NAS2-8048. The contract was administered by NASA. Mr. Richard J. Abbott was the Contract Administrator; Messrs George P. Callas, Michael A. Shovlin, T. Galloway were the Technical Monitors. The Boeing Vertol Project Manager was Mr. Harold Alexander and the Project Engineer was Mr. Michael A. McVeigh.

SUMMARY

This report documents the equations, functions, control systems diagrams, and data required in the real-time simulation of a 1985-era tilt rotor passenger aircraft. The simulatic mathematical model was intended for use on the NASA-Ames F Simulator for Advanced Aircraft to study the handling qual of large tilt rotor aircraft. The model could also be use research on advanced terminal area control. rs.

The mathematical model consists of the rigid body equations for motion of the aircraft in roll, pitch, and yaw about a moving center of gravity. The equations differ somewhat from the classical equations because of the necessity of accounting for the motion of the tilting rotors and nacelles. The math model is "full-force", that is, the representation of the aerodynamics of the rotors and airframe is suitable for the large angles of attack encountered in VSTOL flight and can represent pure rearwards and sidewards flight from hover. The aerodynamics of the airframe and the interference between components was estimated from a combination of theory and experimental data. The forces and moments acting on the large 56-foot diameter hingeless rotors were obtained from a regression analysis of test data on a smaller rotor of similar construction and properties. The control system models pilot controls, a thrust management system and a stability augmentation system.

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LIST OF SYMBOLS

Symbol	Definition	Units
A	Rotor disc area (per rotor)	ft^2
AR	Aspect ratio, b^2/S	ND
A_{1c}	Lateral cyclic angle in rotor wind axes	deg
A'_{1c}	Lateral cyclic angle in swashplate axes	deg
A''_{1c}	Lateral cyclic angle in swashplate axes resolved through swashplate phase angle	deg
$\bar{a}$	Speed of sound or acceleration	ft/sec or ft/sec^2
a	Acceleration	ft/sec^2
(a_g/a)	Ratio of lift-curve slope in ground effect to lift-curve slope out of ground effect	ND
$a_o \rightarrow a_{32}$	Coefficients in wing lift and drag equations	--
B_G	Percent brake pedal deflection	ND
B.L.	Aircraft butt line	in.
B_{1c}	Longitudinal cyclic angle in rotor wind axes	deg
B'_{1c}	Longitudinal cyclic angle in swash-plate axes	deg
B''_{1c}	Longitudinal cyclic angle in swash-plate axes resolved through swash-plate phase angle	deg
b	Span of lifting surface (wing, tail, etc)	ft
c	Chord	ft
C_D	Drag coefficient, $\frac{D}{qS}$	ND

Symbol	Definition	Units
C_{D_O}	Drag coefficient at zero lift	ND
ΔC_D	Drag coefficient increment	ND
C_{DS}	Drag coefficient referred to rotor slipstream dynamic pressure, D/q_sS	ND
C_L	Lift coefficient, L/qS	ND
C_{L_O}	Average lift coefficient	ND
ΔC_L	Lift coefficient increment	ND
C_{L_S}	Lift coefficient referred to rotor slipstream dynamic pressure, L/q_sS	ND
C_{L_α}	Lift-curve slope	1/rad
C_{L_δ}	Lift increment due to flap deflection	1/deg
$C_{\mathcal{L}}$	Rolling moment coefficient, $\mathcal{L}/q\,bS$	ND
$C_{\mathcal{L}_S}$	Rolling moment coefficient referred to rotor slipstream dynamic pressure, $\mathcal{L}/q_sbS$	ND
C_M	Pitching moment coefficient, M/qSc	ND
C_{M_O}	Wing pitching moment coefficient as a function of flap deflection; pitching moment coefficient of fuselage or nacelles at zero angle of attack	ND
ΔC_M	Pitching moment coefficient increment	ND
C_{M_S}	Pitching moment coefficient referred to rotor slipstream dynamic pressure, M/q_sSc	
C_{M_δ}	Change in wing/body pitching moment coefficient as a function of flaperon deflection	ND
C_N	Yawing moment coefficient, N/qSb	ND
C_{N_O}	Yawing moment coefficient of fuselage or nacelles at zero angle of attack	ND

Symbol	Definition	Units
C_{N_S}	Yawing moment coefficient referred to rotor slipstream dynamic pressure, $N/q_S Sb$	ND
C_{NF}	Rotor normal force coefficient, $NF/\rho\pi\Omega^2R^4$	ND
C_{NF_O}	Rotor normal force coefficient with zero cyclic pitch	ND
C_P	Rotor power coefficient, $\frac{550RHP}{\rho\pi\Omega^3R^5}$	ND
C_{P_O}	Rotor power coefficient with zero cyclic pitch	ND
C_{PM}	Rotor hub pitching moment coefficient, $PM/\rho\pi\Omega^2R^5$	ND
C_{PM_O}	Rotor hub pitching moment coefficient with zero cyclic pitch	ND
C_{SF}	Rotor side force coefficient, $SF/\rho\pi\Omega^2R^4$	ND
C_{SF_O}	Rotor side force coefficient with zero cyclic pitch	ND
C_T	Rotor thrust coefficient, $T/\rho\pi\Omega^2R^4$	ND
C_{T_O}	Rotor thrust coefficient with zero cyclic pitch	ND
C_{T_S}	Rotor thrust coefficient referred to rotor slipstream dynamic pressure, $T/q_S A$	ND
C_Y	Side force coefficient, Y/qS	ND
C_{YM}	Rotor yawing moment coefficient, $\rho\pi\Omega^2R^5$	ND
C_{YM_O}	Rotor yawing moment coefficient with zero cyclic pitch	ND
C_{Y_α}	Lift-curve slope of vertical tail	1/rad
C_O	Coefficient of equation that defines pitching moment coefficient as a function of flap deflection	ND

Symbol	Definition	Units
C_1	Coefficient of equation that defines pitching moment coefficient as a function of flap deflection	1/rad
C_2	Coefficient of equation that defines pitching moment coefficient as a function of flap deflection	$1/rad^2$
D	Rotor diameter	ft
(D/T)	Aircraft download-to-thrust ratio	ND
$D_{NF_{1\to5}}$	Coefficients in the equation for the change in normal force coefficient with lateral cyclic angle	1/deg
$D_{PM_{1\to6}}$	Coefficients in the equation for the change in hub pitching moment coefficient with lateral cyclic angle	1/deg
$D_{SF_{1\to5}}$	Coefficients in the equation for the change in side force coefficient with lateral cyclic angle	1/deg
D_{ST_n}	Damping coefficients of the landing gear oleo struts	lb/ft/sec
$D_{YM_{1\to6}}$	Coefficients in the equation for the change in hub yawing moment coefficient with lateral cyclic angle	1/deg
dC_{NF}/dA_{1c}	Change in normal force coefficient with lateral cyclic angle	1/deg
dC_{NF}/dB_{1c}	Change in normal force coefficient with longitudinal cyclic angle	1/deg
dC_{PM}/dA_{1c}	Change in hub pitching moment coefficient with lateral cyclic angle	1/deg
dC_{PM}/dB_{1c}	Change in hub pitching moment coefficient with longitudinal cyclic angle	1/deg
dC_{PM}/dQ	Change in hub pitching moment coefficient with pitch rate	1/rad/sec
dC_{SF}/dA_{1c}	Change in side force coefficient with lateral cyclic angle	1/deg

Symbol	Definition	Units
dC_{SF}/dB_{1c}	Change in side force coefficient with longitudinal cyclic angle	1/deg
dC_{YM}/dA_{1c}	Change in hub yawing moment coefficient with lateral cyclic angle	1/deg
dC_{YM}/dB_{1c}	Change in hub yawing moment coefficient with longitudinal cyclic angle	1/deg
dC_{YM}/dR	Change in hub yawing moment coefficient with yaw rate	1/rad/sec
dC_M/dC_L	Change in wing pitching moment with lift coefficient	ND
$d\sigma/d\beta$	Change in fuselage sidewash angle with sideslip angle	ND
EI	Product of modulus of elasticity and moment of inertia	$lb\text{-}in^2$
EI_O	Product of modulus of elasticity and moment of inertia at wing root	$lb\text{-}in^2$
$E_{NF_{1\to5}}$	Coefficients in the equation for the change in normal force coefficient with longitudinal cyclic angle	1/deg
$E_{PM_{1\to6}}$	Coefficients in the equation for the change in hub pitching moment coefficient with longitudinal cyclic angle	1/deg
$E_{SF_{1\to5}}$	Coefficients in the equation for the change in side force coefficient with longitudinal cyclic angle	1/deg
$E_{YM_{1\to6}}$	Coefficients in the equation for the change in hub yawing moment coefficient with longitudinal cyclic angle	1/deg
E_{HT}, E_{VT}	Oswald efficiency of horizontal or vertical tail	ND
F	Generalized force or force on nacelle	lb
FPR	Lateral-directional SAS function	--
FR1	Lateral-directional SAS function	--
$F\phi$	Lateral-directional SAS function	--

Symbol	Definition	Units
$F_{\phi 1}$	Lateral-directional SAS function	--
$F_{\phi 2}$	Lateral-directional SAS function	--
$F_{\psi 1}$	Lateral-directional SAS function	--
$F_{\psi 2}$	Lateral-directional SAS function	--
F_a	Aerodynamic force on nacelle	lb
F_{gzn}	Landing gear oleo strut vertical force	lb
F_{sn}	Landing gear oleo strut lateral force	lb
F_x	Longitudinal generalized force	lb
F_Y	Lateral generalized force	lb
F_z	Vertical generalized force	lb
$F_{\mu n}$	Landing gear oleo strut longitudinal force	lb
f_{NF}	Multiplier on rotor normal force	ND
f_P	Multiplier on rotor power	ND
f_{PM}	Multiplier on rotor hub pitching moment	ND
f_Q	Multiplier on rotor torque	ND
f_{SF}	Multiplier on rotor side force	ND
f_T	Multiplier on rotor thrust	ND
f_{YM}	Multiplier on rotor hub yawing moment	ND
G	Generalized moment	ft-lb
GEF	Ground effect factor	ND
G_{G1}	Governor gain	deg/sec/rad/sec
G_{G2}	Governor gain	deg/sec/rad/sec
G_{G3}	Governor gain	deg/sec/deg

Symbol	Definition	Units
G_p	Lateral directional SAS gain	in/rad/sec
G_{prl}	Lateral directional SAS gain	in/rad/sec
$G_{p\delta_s}$	Lateral directional SAS gain	in/in
G_q	Longitudinal SAS gain	deg/rad/sec
G_r	Lateral directional SAS gain	in/rad/sec
G_{r2}	Lateral directional SAS gain	in/rad/sec
$G_{r_{\delta r}}$	Lateral directional SAS gain	in/rad/sec
$G_{\beta p}$	Lateral directional SAS gain	in/rad
$G_{\beta r}$	Lateral directional SAS gain	in/rad
$G_{\beta\delta r}$	Lateral directional SAS gain	in/in
$G_{\delta B1}$	Longitudinal SAS gain	deg/in
$G_{\delta B2}$	Longitudinal SAS gain	deg/in
$G_{\delta TH}$	Governor throttle gain	deg/in
G_θ	Longitudinal SAS gain	deg/rad/sec
G_ϕ	Lateral directional SAS gain	in/rad/sec
G_ψ	Lateral directional SAS gain	in/in
$G_{\psi\delta r}$	Lateral directional SAS gain	in/in
g	Gravitational constant	ft/sec^2
H	Height	ft
HP	Horsepower	--
$H'_{w'FUEL}$	Horizontal distance between wing mass element center of gravity and fuel center of gravity	ft
$H'_{w'NF}$	Horizontal distance between wing mass element center of gravity and fixed nacelle center of gravity	ft
$H'_{w'w}$	Horizontal distance between wing mass element center of gravity and fixed nacelle center of gravity	ft

Symbol	Definition	Units
h	Height or angular momentum	ft or lb-ft-sec
h_{CG}^{N}	Angular momentum of nacelle about aircraft center of gravity	lb-ft-sec
h_F	Distance from wing pivot plane to fuselage mass element center of gravity	ft
h_P	Height of pivot above wing chord line or angular momentum of nacelle about the pivot	ft
h_T	Landing gear oleo strut deflection during ground contact	ft
h_W	Distance from wing pivot plane to wing mass element center of gravity	ft
h_O	Angular momentum of an element of mass about its own center of gravity	lb-ft-sec
h_1	Wing vertical bending deflection	ft
h/D	Rotor hub height to rotor diameter ratio	ND
h_θ	Distance from aircraft center of gravity to bottom of right main gear following a positive pitch rotation	ft
h_ϕ	Distance from aircraft center of gravity to bottom of right main gear following a positive roll	ft
I	Mass moment of inertia	slug-ft^2
I_{XX}	Vehicle mass roll moment of inertia about center of gravity	slug-ft^2
I_{XX_O}	Mass roll moment of inertia of aircraft components about their own center of gravity	slug-ft^2
$I_{XX}^{(F)}$	Mass roll moment of inertia of fuselage mass element about its center of gravity	slug-ft^2

Symbol	Definition	Units
$I_{xx}^{(W)}$	Mass roll moment of inertia of wing mass element about its center of gravity	slug-ft^2
I'_{xx}	Mass roll moment of inertia of the tilting portion of each nacelle about its center of gravity	slug-ft^2
I_{yy}	Vehicle mass pitch moment of inertia about center of gravity	slug-ft^2
I_{yy_o}	Mass pitch moment of inertia of aircraft components about their centers of gravity	slug-ft^2
$I_{yy}^{(F)}$	Mass pitch moment of inertia of fuselage mass element about its center of gravity	slug-ft^2
$I_{yy}^{(W)}$	Mass pitch moment of inertia of wing mass element about its center of gravity	slug-ft^2
I'_{yy}	Mass pitch moment of inertia of the tilting portion of each nacelle about its center of gravity	slug-ft^2
I_{xz}	Vehicle mass product of inertia about center of gravity	slug-ft^2
I_{xz_o}	Mass product of inertia of aircraft components about their own centers of gravity	slug-ft^2
$I_{xz}^{(F)}$	Mass product of inertia of fuselage mass element about its center of gravity	slug-ft^2
$I_{xz}^{(W)}$	Mass product of inertia of wing mass element about its center of gravity	slug-ft^2
I'_{xz}	Mass product of inertia of the tilting portion of each nacelle about its center of gravity	slug-ft^2
I_{zz}	Vehicle mass yaw moment of inertia about center of gravity	slug-ft^2
I_{zz_o}	Mass yaw moment of inertia of aircraft components about their own centers of gravity	slug-ft^2

Symbol	Definition	Units
$I_{zz}^{(F)}$	Mass yaw moment of inertia of fuselage mass element about its center of gravity	slug-ft^2
$I_{zz}^{(W)}$	Mass yaw moment of inertia of wing mass element about its center of gravity	slug-ft^2
I'_{zz}	Mass yaw moment of inertia of the tilting portion of <u>each</u> nacelle about its center of gravity	slug-ft^2
i	Incidence angle	deg or rad
$\hat{\underline{i}}$	Unit vector in i direction	
J_{xx}	Dummy inertia, I_{zz}-I_{yy}	slug-ft^2
J_{yy}	Dummy inertia, I_{xx}-I_{zz}	slug-ft^2
J_{zz}	Dummy inertia, I_{yy}-I_{xx}	slug-ft^2
$\hat{\underline{j}}$	Unit vector in j direction	--
K'_A	Wing slipstream correction factor	ND
$\frac{K_{D1}}{T} \rightarrow \frac{K_{D4}}{T}$	Coefficients of curve fit equation for wing download as a function of rotor height/diameter ratio	ND
$\frac{K_{M1}}{T} \rightarrow \frac{K_{M4}}{T}$	Coefficients of curve fit equation for wing pitching moment as a function of rotor height/diameter ratio	ND
K_{ℓ}	Multiplier on slipstream rolling moment coefficient	ND
K_n	Miltiplier on slipstream yawing moment coefficient	ND
K_{ST_n}	Landing gear spring constants	lb/ft
$K_{W1} \rightarrow K_{W10}$	Coefficients for wing bending equations	--
K_{δ_B}	Multiplier on longitudinal cyclic pitch available from longitudinal stick	in/in

Symbol	Definition	Units
K_{δ_e}	Ratio between longitudinal stick motion and elevator deflection	deg/in
K_{δ_R}	Multiplier on longitudinal cyclic pitch available from pedal displacement	in/in
$K\delta_{RUD}$	Ratio between pedal and rudder deflection	deg/in
$K_{\delta s}$	Multiplier on longitudinal cyclic pitch and differential collective available from lateral stick	in/in
$K_{\delta ' s}$	Lateral cyclic pitch/degree of differential collective pitch	deg/deg
K_θ	Wing stiffness in torsion	ft-lb/rad
K_o	Coefficient of fuselage drag coefficient equation to account for drag due to sideslip	$1/rad^3$
K_1	Coefficient in fuselage drag coefficient equation	$1/rad^2$
K_2	Coefficient in fuselage drag coefficient equation	1/rad
K_3	Coefficient in fuselage lift coefficient equation	1/rad
K_4	Coefficient in fuselage lift coefficient equation	$1/rad^2$
K_5	Coefficient in fuselage pitching moment coefficient equation	1/rad
K_6	Coefficient in fuselage pitching moment coefficient equation	$1/rad^2$
K_7	Coefficient in fuselage side force coefficient equation	1/rad
K_8	Coefficient in fuselage side force coefficient equation	1/rad
K_9	Coefficient in fuselage yawing moment coefficient equation	1/rad

Symbol	Definition	Units
K_{10}	Coefficient in fuselage yawing moment coefficient equation	$1/\text{rad}^2$
K_{20}	Wing/body interference effects on C_{ℓ_β}	1/rad
K_{21}	Wing planform effects on C_{ℓ_β}	1/rad
K_{22}	Wing planform and lift effects on C_{N_β}	1/rad
K_{30}	Coefficient in nacelle drag coefficient equation	1/rad
K_{31}	Coefficient in nacelle drag coefficient equation	$1/\text{rad}^2$
K_{32}	Coefficient in nacelle lift coefficient equation	1/rad
K_{34}	Coefficient in nacelle pitching moment coefficient equation	1/rad
K_{35}	Coefficient in nacelle pitching moment coefficient equation	$1/\text{rad}^2$
K_{36}	Coefficient in nacelle side force coefficient equation	1/rad
K_{37}	Coefficient in nacelle side force coefficient equation	$1/\text{rad}^2$
K_{38}	Coefficient in nacelle yawing moment coefficient equation	1/rad
K_{39}	Coefficient in nacelle yawing moment coefficient equation	$1/\text{rad}^2$
K_{40}	Coefficient in nacelle yawing moment coefficient equation	1/rad
K_{41}	Coefficient in nacelle yawing moment coefficient equation	$1/\text{rad}^2$
K_{42}	Coefficient in fuselage lift coefficient equation	ND
$\hat{\underline{k}}$	Unit vector in k direction	--
L_s	Nacelle shaft length from pivot to spinner	ft

Symbol	Definition	Units
$\mathcal{L}$	Rolling moment	ft-lb
ℓ	Distance from nacelle pivot to nacelle center of gravity	ft
ℓ'	Horizontal distance from nacelle pivot to aircraft component center of gravity positive - positive forward from pivot	ft
ℓ_{AC}	Horizontal distance from horizontal tail quarter chord to wing aerodynamic center	ft
ℓ_F	Horizontal distance from pivot to center of gravity of fuselage mass element	ft
ℓ_0	Wing root lift/foot	lb/ft
ℓ_{PA}	Horizontal distance from pivot to center of gravity of pilots' station - positive forward from pivot	ft
ℓ_W	Horizontal distance from pivot to wing mass element center of gravity	
M	Pitching moment	ft-lb
m	Pitching moment, or aircraft mass	ft-lb or slugs
M/T	Pitching moment/rotor thrust	ft-lb/lb
m_f	Mass of fuselage structure	slugs
m_N	Mass of one nacelle	slugs
m_W	Mass of wing	slugs
N	Yawing moment	ft-lb
NF	Rotor normal force	lb
N_I	Engine gas generator speed	rev/min
$N_{1\ IND}$	Engine gas generator indicator	--
N_I^*	Engine gas generator speed at sea level standard, static conditions	rev/min

Symbol	Definition	Units
$N_{1\theta\ IND}$	Referred engine gas generator speed indicator	--
N_{II}	Engine power turbine speed	rev/min
N^*_{II}	Engine power turbine speed at sea level standard static conditions	rev/min
P	Body axes roll rate	rad/sec
PC	Horizontal distance from wing leading edge to pivot location	ft
P^N	Nacelle axes roll rate	rad/sec
P^R	Nacelle wind axes roll rate	rad/sec
p	Body axes roll rate	rad/sec
Q	Body axes pitch rate or rotor torque	rad/sec or lb-ft
Q_{IND}	Torque indicator	ND
Q_{MAX}	Maximum engine torque available	lb-ft
Q^N	Nacelle axes pitch rate	rad/sec
Q^R	Nacelle wind axes pitch rate	rad/sec
Q*	Engine torque at sea level standard static condition	lb-ft
q	Body axes pitch rate or freestream dynamic pressure	rad/sec or lb/ft^2
q_s	Dynamic pressure of rotor slipstream	lb/ft^2
R	Body axes yaw rate or rotor resultant force or rotor radius	rad/sec or lb or ft
RHP	Rotor horsepower	--
R^N	Nacelle axes yaw rate	rad/sec
R^R	Nacelle wind axes yaw rate	rad/sec
r	Body axes yaw rate	rad/sec
$\underline{r}$	Radius vector	--

Symbol	Definition	Units
r_n	Landing gear tire radius	ft
S	Surface area	ft^2
SF	Rotor side force	lb
SHP	Shaft horsepower	--
SHP*	Engine shaft horsepower at sea level standard static conditions	--
T	Rotor thrust	lb
TEA	Engine referred turbine inlet temperature	deg
(T_{IGE}/T_{OGE})	Ratio of the rotor thrust in ground effect to the thrust out of ground effect	--
$T_1 \rightarrow T_3$	Coefficients of curve fit equations for rotor/rotor interference	ND
t	Time	sec
U	Body axes longitudinal component of velocity at aircraft center of gravity or rotor hub, wing, horizontal and vertical tail velocities referred to rotor shaft and local surface chord axes, respectively	ft/sec
U'	Body axes longitudinal component of velocity at rotor hub and wing aerodynamic center	ft/sec
U_{PA}	Body axes longitudinal component of velocity at pilot's station	ft/sec
V	Total velocity	ft/sec
V_t	Rotor tip speed	ft/sec
V'	Resultant flow through rotor disc	ft/sec
V_*	Non-dimensional rotor forward velocity	ND
$\underline{V}$	Total velocity vector	ft/sec

Symbol	Definition	Units
V	Body axes lateral component of velocity at aircraft center of gravity or rotor hub wing, horizontal and vertical tail velocities referred to rotor shaft and local surface chord axes, respectively	ft/sec
V'	Body axes lateral component of velocity at rotor hub and wing aerodynamic center	ft/sec
v_i	Rotor induced velocity	ft/sec
V_{PA}	Body axes lateral component of velocity at pilot's station	ft/sec
V_*	Non-dimensional rotor induced velocity	ND
W.L.	Fuselage water line position	in.
W'	Weight of aircraft components	lb
WDTIND	Fuel flow indicator	--
W	Body axes vertical component of velocity at aircraft center of gravity or rotor, hub, wing, horizontal and vertical tail velocities referred to rotor shaft and local surface chord axes, respectively	ft/sec
W'	Body axes vertical component of velocity at rotor hub and wing aerodynamic center	ft/sec
W_{PA}	Body axes vertical component of velocity at pilot's station	ft/sec
$X_{subscript}$	Longitudinal distance, measured positive forward from nacelle pivot along body axes	ft
$\Delta X_{subscript}$	Longitudinal force, measured positive forward along body axes	lb
X_{aero}	Total longitudinal aerodynamic force at center of gravity measured positive forward along body axes	lb

Symbol	Definition	Units
$X^{sprscript}_{subscript}$	Longitudinal force, measured positive forward along body axes	lb
$\dot{X}_{North}$	Longitudinal ground track velocity	ft/sec
$Y_{subscript}$	Lateral distance, measured positive along right wing along body axes	ft
$\Delta Y_{subscript}$	Lateral force, measured positive along right wing in body axes	lb
Y_{aero}	Total lateral aerodynamic force at center of gravity measured positive along right wing in body axes	lb
$Y^{sprscript}_{subscript}$	Lateral force, measured positive along right wing in body axes	lb
$\dot{Y}_{East}$	Lateral ground track velocity	ft/sec
$Z_{subscript}$	Vertical distance, measured positive down nacelle pivot along body axes	ft
$\Delta Z_{subscript}$	Vertical force, measured positive down along body axes	lb
$\bar{Z}_{aero}$	Total vertical aerodynamic force at center of gravity, measured positive down along body axes	lb
$Z^{sprscript}_{subscript}$	Vertical force, measured positive down along body axes	lb
$\dot{Z}_{down}$	Vertical ground track velocity	ft/sec
z	Vertical distance from nacelle pivot to center of gravity of aircraft component, positive down from nacelle pivot along body axes	ft
α	Angle of attack	rad
β	Angle of sideslip	rad
$\Delta'_{w'fuel}$	Vertical distance between wing fuel center of gravity and wing mass element center of gravity	ft

Symbol	Definition	Units
$\Delta'_{w'fuel}$	Vertical distance between fixed nacelle center of gravity and wing mass element center of gravity	ft
$\Delta'_{w'w}$	Vertical distance between wing center of gravity and wing mass element center of gravity	ft
δ	Control element (surface or stick) angular or linear displacement	deg or in.
δ'_{C}	Vertical distance between cargo center of gravity and fuselage mass element center of gravity	ft
δ'_{CR}	Vertical distance between crew center of gravity and fuselage mass element center of gravity	ft
$\delta'_{F'}$	Vertical distance between fuselage center of gravity and fuselage mass element center of gravity	ft
δ'_{HT}	Vertical distance between horizontal tail center of gravity and fuselage mass element center of gravity	ft
δ_{STEER}	Nose wheel steering angle, positive right	deg
δ'_{VT}	Vertical distance between vertical tail center of gravity and fuselage mass element center of gravity	ft
ε	Wing or rotor downwash angle	rad
ε_{O}	Wing downwash angle at zero wing angle of attack	rad
ε_{iLR}	Rotor/rotor interference angle, left rotor on right rotor	rad
ε_{iRL}	Rotor/rotor interference angle, right rotor on left rotor	rad
ε_{W}	Wing on rotor interference	rad
ζ	Rotor sideslip angle or damping ratio	rad or ND
$\zeta_{w1} \rightarrow \zeta_{w4}$	Wing damping ratio	ND

Symbol	Definition	Units
$H'_{w'fuel}$	Horizontal distance between wing fuel center of gravity and wing mass element center of gravity	ft
$H'_{w'NF}$	Horizontal distance between fixed nacelle center of gravity and wing mass element center of gravity	ft
$H'_{w'w}$	Horizontal distance between wing center of gravity and wing mass element center of gravity	ft
η'_{c}	Horizontal distance between cargo center of gravity and fuselage mass element center of gravity	ft
η'_{CR}	Horizontal distance between crew center of gravity and fuselage mass element center of gravity	ft
η'_{F}	Horizontal distance between fuselage center of gravity and fuselage mass element center of gravity	ft
η_{HT}	Horizontal tail efficiency	ND
η'_{HT}	Horizontal distance between horizontal tail center of gravity and fuselage mass element center of gravity	lb
η_{VT}	Vertical tail efficiency factor	ND
η'_{VT}	Horizontal distance between vertical tail center of gravity and fuselage mass element center of gravity	ft
η_{TR}	Transmission efficiency	ND
θ	Aircraft pitch or Euler angle or temperature ratio	rad or ND
θ_{t}	Wing twist angle	rad
$\theta_{.75}$	Rotor collective pitch angle at three-quarter radius	deg
λ	Angle between the rotor shaft and a line drawn through the nacelle center of gravity from the pivot	rad

Symbol	Definition	Units
μ	Rotor advance ratio = $V/\Omega R$	ND
μ_s	Tire sliding coefficient of friction when sliding sidewards (for concrete)	ND
μ_o	Tire rolling coefficient of friction (for concrete)	ND
μ_1	Coefficient of rolling friction for brakes	ND
$\xi_{R1} \rightarrow \xi_{R4}$	Terms in wing immersed area calculation	--
ρ	Ambient air density	slug/ft^3
σ	Fuselage sidewash angle	rad
σ_h	Ambient density ratio	ND
τ	Angle between freestream velocity and rotor resultant force	rad
τ_D	Engine response time constant	sec
τ_E	Engine response time constant	sec
τ_{HT}	Horizontal tail effectiveness	ND
τ_{LAS}	Load alleviation system time constant	sec
τ_{VT}	Vertical tail effectiveness	ND
τ_p	Lateral directional SAS time constant	sec
τ_r	Lateral directional SAS time constant	sec
τ_ϕ	Lateral directional SAS time constant	sec
$\tau_{\phi\delta s}$	Lateral directional SAS time constant	sec
τ_ψ	Lateral directional SAS time constant	sec
$\tau_{\psi\delta r}$	Lateral directional SAS time constant	sec
τ_1	Rotor thrust response time constant	sec
τ_2	Rotor thrust response time constant	sec
ϕ	Aircraft roll angle or Euler angle	rad

Symbol	Definition	Units
ϕ_p	Rotor swashplate phase angle	rad
$\phi_1 \rightarrow \phi_5$	Functions in wing vertical bending equations	--
χ	Rotor wake skew angle	rad
ψ	Aircraft yaw angle or Euler angle	rad
Ω	Rotor or engine rotational speed	rad/sec
$\underline{\Omega}$	Angular velocity vector	rad/sec
ω	Natural frequency	rad/sec
$\omega_{w1} \rightarrow \omega_{w3}$	Wing natural frequencies	rad/sec

Subscripts

A	Available
AC	Aerodynamic center
ACT	Actuator
AERO	Aerodynamic force
a	Aileron
B	Longitudinal stick
c	Cargo
CG	Center of gravity
CR	Crew
C/4	Quarter chord
DUM	Dummy variable
E	Engine
EFF	Effective
e	Elevator or effective
F	Fuselage
FAC	Fuselage aerodynamic center
FUEL	Fuel in wing
$FUEL_{CG}$	Fuel center of gravity
FUS	Fuselage
F'	Fuselage minus landing gear
f	Flap
GLAS	Load alleviation system
GYRO	Gyroscopic
g	Ground or gust
HL	Left rotor hub

Subscripts

HR	Right rotor hub
HT	Horizontal tail
HTCG	Horizontal tail center of gravity
IGE	In ground effect
i	Immersed
L	Left wing or rotor
LAS	Load alleviation system
LE	Left engine
LG	Landing gear
L-L	Rotor lead-lag
LN	Left nacelle
LR	Left rotor
LRH	Left rotor hub
LT	Left wing tip
LW	Left wing
LW_o	Left wing referred to freestream
MAX	Maximum
N	Nacelle or natural frequency
NF	Fixed portion of nacelle
NFCG	Fixed portion of nacelle center of gravity
NL	Left nacelle
NR	Right nacelle
NT	Tilting portion of nacelle
n	Landing gear index, n=1 left gear, n=2 right gear, n=3 nose gear
OGE	Out of ground effect

Subscripts

P	Power, nacelle pivot, or rotor polar moment of inertia
POWER	Power
PA	Pilot station
R	Right wing, rotor or rudder pedal
RE	Right engine
REQ	Required
RIGID	Rigid
RN	Right nacelle
RR	Right rotor
RRH	Right rotor hub
RT	Right wing tip
RUD	Rudder
RW	Right wing
RW_o	Right wing referred to freestream
S	Rotor shaft, side, or lateral stick
SP	Spoiler
STALL	Stall
T	Tail, total or wing tip
TH	Throttle
VT	Vertical tail
VTCG	Vertical tail center of gravity
W	Wing
WAC	Wing aerodynamic center
WCG	Wing center of gravity
x	Along the longitudinal axis, positive forward

Subscripts

y	Along the lateral body axis, positive out right wing
z	Along the vertical body axis, positive down
$\_$	Denotes a vector quantity

Superscripts

(c)	Refers to cargo or payload weight
(CR)	Refers to aircraft crew weight
F	Fuselage
F'	Fuselage less landing gear
HT	Horizontal tail
(HT)	Refers to horizontal tail weight component
IGE	In ground effect
LW	Left wing
N	Nacelle
NL	Left wing tip at pivot
NR	Right wing tip at pivot
RW	Right wing
T	Total of horizontal and vertical tail
VT	Vertical tail
(VT)	Refers to vertical tail weight component
W	Wing
(W'_{FUEL})	Refers to wing fuel weight
(W_f')	Refers to fuselage weight component
(W'_{NF})	Refers to weight of fixed portion of nacelle
(W'_W)	Refers to wing weight component

Superscripts

"	Denotes an interim calculation or coefficient in local wind axes
'''	Denotes an interim calculation
-	Denotes average value
*	Denotes interim calculation or calculation in freestream wind axes
'	Denotes an interim calculation
+	Denotes an interim calculation
∧	Denotes a unit vector

1.0 INTRODUCTION

The rising costs and diminishing availability of fossil fuels, the increasing congestion at major airports, and the growing need to reduce noise and air pollution are strong reasons for evaluating rotary-wing vehicles for the short-haul air travel market.

The low disc loadings of rotor configurations allow vertical or short takeoff and landing for a relatively low installed horsepower. This power economy results in improved fuel efficiency and reduced air pollution. The capability for V/STOL operation greatly reduces runway requirements and provides a means to alleviate air traffic congestion at airports.

The tilt rotor aircraft combines the V/STOL advantages of the helicopter with the speed and altitude advantages of fixed wing aircraft. In Reference 1 a study was made to define a tilt rotor aircraft capable of carrying 100 passengers over a 200 nautical mile stage length at minimum direct operating cost. The configuration emerged as a 4-engined, 33905 kg (74749 lb) aircraft with a wing span of 25 meters (82 ft), a rotor diameter of 17.16 meters (56.3 ft) and a cruise speed of 349 KTAS. This represents a very large increase in size for a tilt-rotor vehicle compared to current (NASA-Army Bell XV-15) and past (XV-3) tilt rotor designs. The question then arises as to the flying qualities of such a rotary wing vehicle and the impact of operating large tilt rotors in projected terminal area navigation systems.

This report presents the development of a mathematical model which could be used in a piloted simulation of a large tilt rotor aircraft. This is a model of the 1985 Tilt Rotor Configuration originally planned for use on the NASA-Ames Flight Simulator for Advanced Aircraft. The model could also be used in research on advanced controllers for terminal area operation.

The aircraft selected for modeling was the design point tilt rotor aircraft described in Reference 1 and detailed in Section 2.0. The math model is full force, with all inertial and aerodynamic terms included. The forces and moments generated by the large, hingeless rotors are calculated from a set of equations derived from a regression analysis of full-scale test data on a rotor of similar design. Direct calculation of the rotor forces and moments in real time is not practicable because of the complexity of the equations required to represent the flap-lag coupling effects of soft-in-plane hingeless rotors.

The aerodynamic interference effects of the rotor on wings and tails, the effect of the wing upwash on the rotor, and the interference of one rotor on the other in edgewise flight, are represented. Turbine engine performance, dynamic response and performance limits, both thermodynamic and mechanical, are included.

The control system elements represented are pilot command, three axis stability augmentation and a thrust management/ governor system. Control system actuator dynamics are included as first and second order lags.

The effect of the tilting rotors and nacelles on the aircraft center of gravity and inertias are calculated. Forces and moments resulting from acceleration of the nacelles during rapid tilting maneuvers are included.

The airframe c.g. and inertia representation permits the location, inertia, and c.g. of major components of the aircraft to be entered. All lengths, overall c.g. and inertias of the aircraft are then calculated.

Wing and nacelle aeroelastic effects are treated on a quasi-static basis, i.e., coupling is through the aerodynamic terms only.

The mathematical model is presented in detail in Appendix E and derivations of important equations are also included in the body of the report and the appendices.

2.0 DESCRIPTION OF AIRCRAFT

The 1985 commercial 100 passenger tilt rotor aircraft is shown in Figures 2.1 and 2.2. Table 2.1 lists the major dimensions and characteristics of the aircraft.

The 1985 tilt rotor has a takeoff gross weight of 33,905 kilograms (74,749 pounds). The rotors are three-bladed and are of hingeless fiberglass composite construction. The rotor diameter is 17.16 meters (56.3 feet) and the solidity ratio is 0.089. In hover and low-speed flight, cyclic pitch control is applied to the rotor to provide control power and trim. These rotors are highly twisted (36 degrees) by comparison with helicopter blades to provide efficient operation at high advance ratio as well as in hover.

The rotors and forward rotor transmission tilt; however, the engines, mounted outboard of the tilt package, remain stationary. This arrangement does not require the engines to be requalified for vertical operation and reduces the inertia of the tilt package.

The aircraft has four engines, two on each wing tip. The rotors and engines are connected by means of a cross-shaft which provides the torque transmission across the aircraft in event of engine failure. The location of the engines outboard of the tilt package provides easy access to the engine bays for maintenance or engine removal.

The span of the aircraft is 25 meters (82 feet) measured from outboard of one nacelle to outboard of the other. The wing is straight and uptapered with a NACA 63_4221 section with a wing setting angle of 2° relative to the fuselage. The wing aspect ratio is 7.14.

The wing has full-span 30-percent-chord plain flaperons used as both flaps and ailerons. A leading edge umbrella flap is provided which opens for hover and low-speed helicopter-type flight to alleviate the rotor download on the wing. This device is also used to ensure that wing unstalling at end of transition occurs simultaneously on both wings.

The empennage T-tail configuration was selected to reduce the impact of rotor downwash on the horizontal stabilizer in transition flight. The horizontal tail volume ratio is 1.47, and the vertical tail volume ratio is 0.159.

The tricycle landing gear configuration provides good ground handling characteristics and is retractable. The undercarriage provides an overturning angle of 27°.

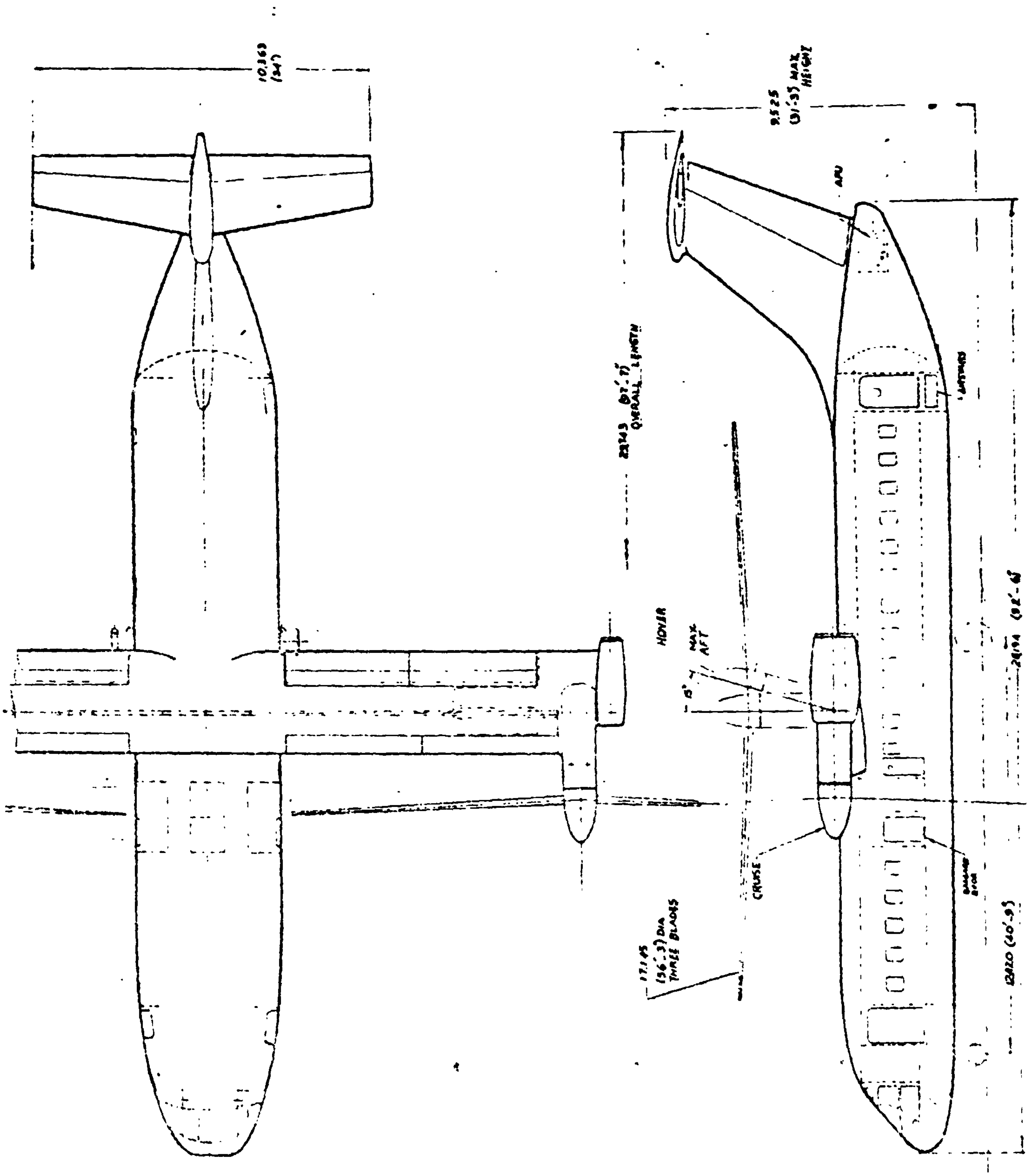


FIGURE 2.1. 1985 COMMERCIAL VTOL, 100 PASSENGER TILT ROTOR - BASELINE DESIGN.

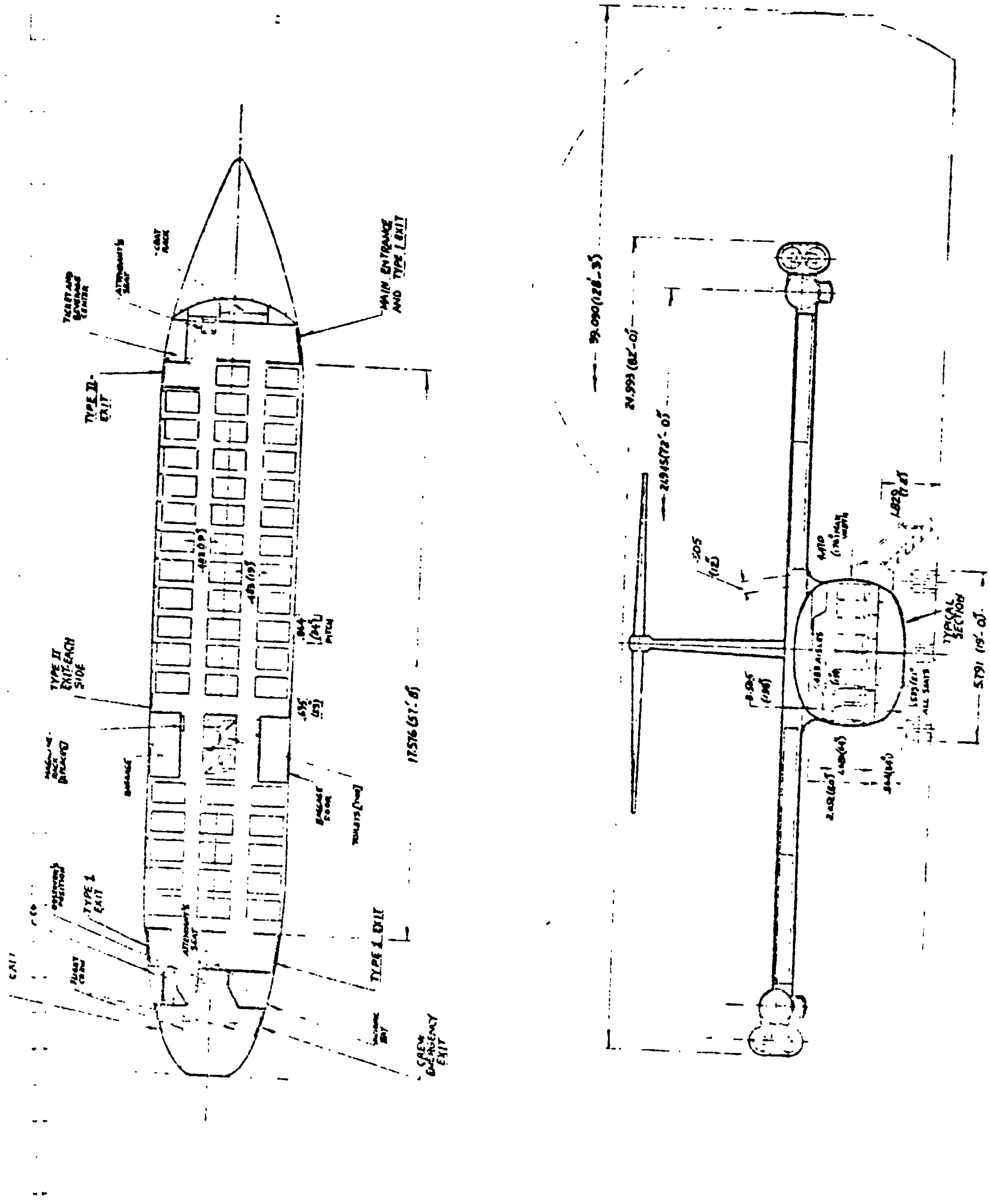


FIGURE 2.2. 1985 COMMERCIAL VTOL, 100 PASSENGER TILT ROTOR - BASELINE DESIGN (CONTINUED).

TABLE 2.1

1985 COMMERCIAL TILT ROTOR - DIMENSIONAL DATA

WING:	
AREA (REFERENCE)	747.5 FT^2
ASPECT RATIO	7.15
SPAN (BETWEEN ROTOR ℄)	73.1 FT
TAPER RATIO	1.00
CHORDS:	
ROOT	10.23 FT
TIP	10.23 FT
MEAN AERODYNAMIC	10.23 FT
SWEEPBACK	0 DEGREES
DIHEDRAL	0 DEGREES
INCIDENCE:	
ROOT	2.0 DEGREES
TIP	2.0 DEGREES
AIRFOIL SECTION:	
ROOT	NACA 63_4221 (MODIFIED)
TIP	NACA 63_4221 (MODIFIED)
FUSELAGE:	
LENGTH	92.5 FT
DEPTH	11.5 FT
WIDTH	14.75 FT
WETTED	3563 FT^2

1985 COMMERCIAL TILT ROTOR - DIMENSIONAL DATA
(Continued)

NACELLES:	
ENGINE:	
LENGTH	8.58 FT
DEPTH	5.5 FT
WIDTH	3.0 FT
WETTED AREA (PER NACELLE)	136. FT^2
TILTING:	
LENGTH	15.5 FT
DEPTH	5.08 FT
WIDTH	3.42 FT
WETTED AREA (PER NACELLE)	122 FT^2
HORIZONTAL TAIL:	
AREA (EXPOSED)	211.5 FT^2
AREA (REFERENCE)	227.5 FT^2
SPAN	35 FT
ASPECT RATIO	5.38
TAPER RATIO	0.625
DISTANCE $(\bar{c}/4)_W$ to $(\bar{c}/4)_{HT}$	50.75 FT
CHORDS:	
ROOT	8.0 FT
TIP	5.0 FT
MEAN AERODYNAMIC	6.62 FT
SWEEPBACK AT 0 PERCENT CHORD	9.75 DEGREES
DIHEDRAL	0 DEGREES

1985 COMMERCIAL TILT ROTOR - DIMENSIONAL DATA
(Continued)

INCIDENCE:	
ROOT	0 DEGREES
TIP	0 DEGREES
AIRFOIL SECTION:	
ROOT	NACA 64A010 (MODIFIED)
TIP	NACA 64A010 (MODIFIED)
VERTICAL TAIL:	
AREA (EXPOSED, EXCLUDES DORSAL)	210 FT^2
AREA (REFERENCE)	278 FT^2
SPAN (REFERENCE)	20.67 FT
ASPECT RATIO	1.536
TAPER RATIO	0.482
DISTANCE $(\bar{c}/4)_W$ to $(\bar{c}/4)_{VT}$	40.5 FT
CHORDS:	
ROOT	18.33 FT
TIP	8.83 FT
MEAN AERODYNAMIC	14.1 FT
SWEEPBACK AT 0 PERCENT CHORD	39 DEGREES
AIRFOIL SECTION	NACA 64A008 (MODIFIED)
CONTROL SURFACES:	
FLAPERON:	
AREA (AFT OF HINGE)	150.4 FT^2
SPAN (LENGTH EACH SIDE)	24.5 FT
CHORD (% OF WING CHORD)	30
SWEEPBACK OF HINGELINE	0 DEGREES

1985 COMMERCIAL TILT ROTOR - DIMENSIONAL DATA
(Continued)

SPOILERS:	
AREA	63.2 FT^2
SPAN (LENGTH EACH SIDE)	24.5 FT
CHORD (% OF WING CHORD)	12.65
POSITION OF LE (% OF WING CHORD)	66.0
SWEEPBACK OF HINGE LINE	0 DEGREES
LEADING EDGE UMBRELLA:	
AREA (PLAN)	102.8 FT^2
CHORD (% OF WING CHORD)	18.6
SWEEPBACK OF HINGE LINE	0 DEGREES
ELEVATORS:	
AREA (AFT OF HINGE LINE)	62.1 FT^2
CHORD (% HORIZONTAL TAIL CHORD)	33
SWEEPBACK OF HINGE LINE	3.0 DEGREES
RUDDER:	
AREA (AFT OF HINGE)	49.3 FT^2
CHORD (% VERTICAL TAIL CHORD)	25
SWEEPBACK OF HINGE LINE	26.7 DEGREES
ROTORS:	
NUMBER OF BLADES (PER ROTOR)	3
DIAMETER	56.3 FT
BLADE AREA (PER BLADE)	70.1 FT^2
GEOMETRIC DISC AREA (TOTAL)	4978 FT^2
SOLIDITY	.089

1985 COMMERCIAL TILT ROTOR - DIMENSIONAL DATA
(Continued)

AIRFOIL SECTION:

ROOT	VR-7
70% RADIUS	VR-8
89% RADIUS	VR-9
TIP	VR-9

3.0 EQUATIONS OF MOTION

This section presents the derivation of the airframe equations of motion and the simplifications that were made in order to obtain the final equations as presented in Appendix E. The treatment accounts for all six rigid-body degrees-of-freedom including the effects of the tilting nacelles and rotors. The principal features of the derivation are:

- o Assumption of X-Z plane of symmetry
- o The basic equations are derived about the instantaneous center of gravity of the aircraft since the center of gravity is strongly dependent on nacelle incidence.
- o Rotor and engine gyroscopic terms are included.
- o The wing elastic degrees of freedom do not couple inertially. The coupling occurs only through the aerodynamic terms.
- o Wing aeroelastic effects are not included in the center of gravity calculations.

3.1 AXES SYSTEM

A set of right-handed orthogonal axes OXYZ is placed at the center of mass of the aircraft and is fixed in the aircraft such that OX lies in the lateral plane of symmetry and is positive forward parallel to the fuselage water line zero. The remaining axes are placed as shown in Figure 3.1.

The orientation of the aircraft is defined with respect to a set of earth-fixed axes C X'Y'Z'. With the axes OXYZ initially parallel to C X'Y'Z', the aircraft is yawed to the right about O through an angle ψ, then pitched up about OZ through the angle θ, and finally rolled right about OX through the angle ϕ.

If $\underline{V}$ and $\underline{\Omega}$ are the aircraft velocity and angular velocity vectors relative to the earth-fixed axes, the projections of these vectors on the moving axes are U, V, and W for the components along OX, OY, and OZ, and P, Q, and R for the angular velocity components.

Thus,

$$\underline{V} = U\underline{i} + V\underline{j} + W\underline{k} \tag{3.1}$$

$$\underline{\Omega} = P\underline{i} + Q\underline{j} + R\underline{k} \tag{3.2}$$

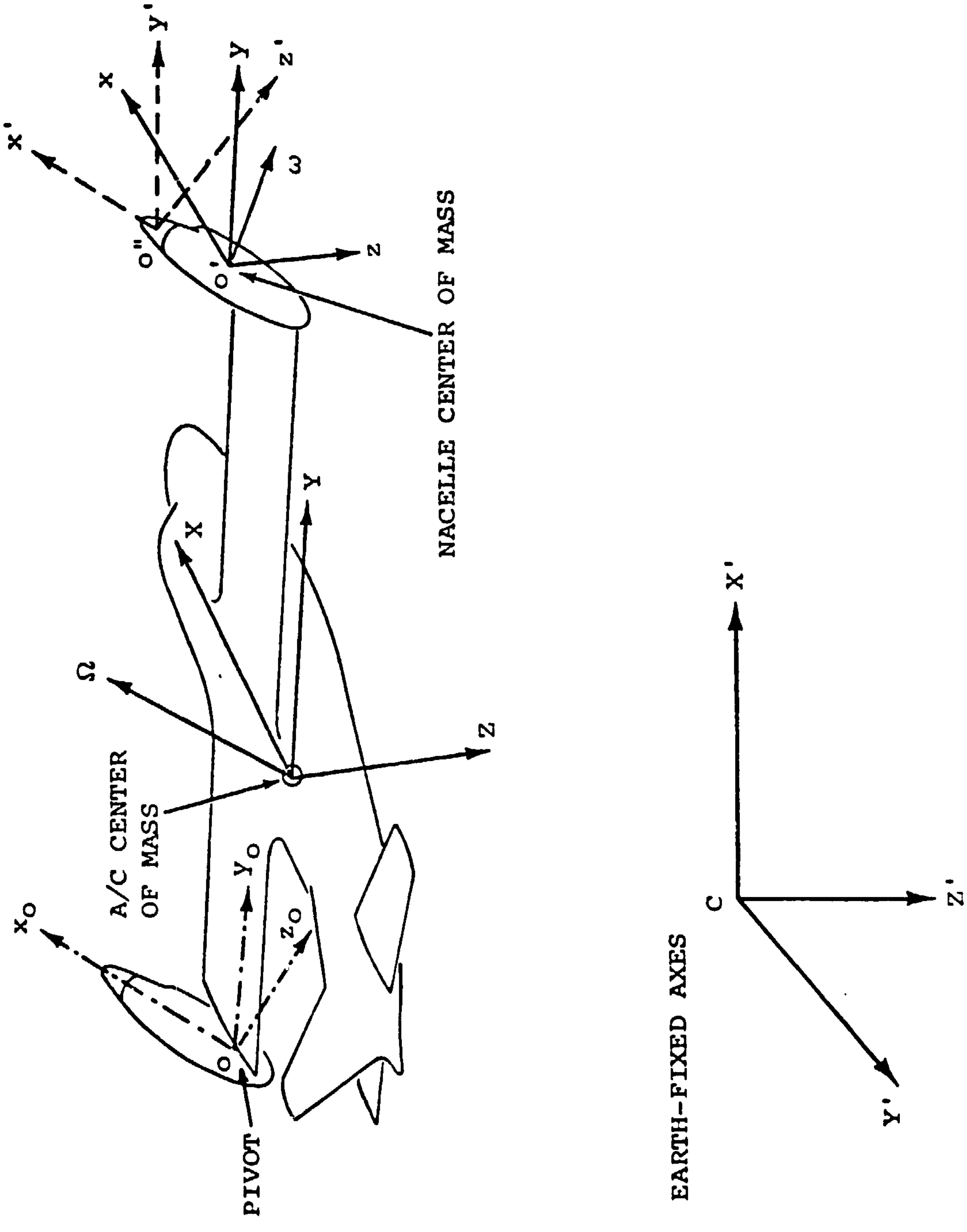


Figure 3.1 *Axes Systems*

where the unit vectors $\underline{i}$, $\underline{j}$, and $\underline{k}$ lie along OX, OY, and OZ.

3.2 AIRCRAFT GROUND TRACK

The components of $\underline{V}$ relative to the earth-fixed axes are obtained in terms of U, V, W and ψ, θ, ϕ as, (See Reference 2),

$$\frac{dX'}{dt} = U\cos\theta\cos\psi + V(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) + W(\cos\phi\sin\theta\cos\psi + \sin\theta\sin\psi)$$

$$\frac{dY'}{dt} = U\cos\theta\sin\psi + V(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) + W(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \qquad (3.3)$$

$$\frac{dZ'}{dt} = -U\sin\theta + V\sin\phi\cos\theta + W\cos\phi\cos\theta$$

Integration of these equations gives the aircraft ground track. A further relationship may be obtained between the rate of change of the Euler angles (ψ, θ, ϕ) and the components of the angular velocity in the moving axes system, viz,

$$\dot{\psi} = (R\cos\phi + Q\sin\phi)\sec\theta$$

$$\dot{\theta} = Q\cos\phi - R\sin\phi \qquad (3.4)$$

$$\dot{\phi} = P + \dot{\psi}\sin\theta$$

3.3 FORCE EQUATION

The total external force, $\underline{F}$, acting at the aircraft center of mass is given by

$$\underline{F} = \frac{d}{dt}(m\underline{V}) = m\left[\frac{\delta\underline{V}}{\delta t} + \underline{\Omega} \times \underline{V}\right] \qquad (3.5)$$

where m is the mass of the aircraft and $\frac{\delta\underline{V}}{\delta t}$ is the rate of change of $\underline{V}$ with respect to the moving reference frame OXYZ, i.e.

$$\frac{\delta\underline{V}}{\delta t} = \dot{U}\,\hat{\underline{i}} + \dot{V}\hat{\underline{j}} + \dot{W}\hat{\underline{k}} \qquad (3.6)$$

If $\underline{F}$ has components F_x, F_y, and F_z along the respective axes then

$$\underline{F} = F_x \hat{\underline{i}} + F_y \hat{\underline{j}} + F_z \hat{\underline{k}} = m\left\{\dot{U}\hat{\underline{i}} + \dot{V}\hat{\underline{j}} + \dot{W}\hat{\underline{k}} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P & Q & R \\ U & V & W \end{vmatrix}\right\}$$

thus

$$\begin{aligned} F_x &= m\ (\dot{U} + QW - RV) \\ F_y &= m\ (\dot{V} + RU - PW) \\ F_z &= m\ (\dot{W} + PV - QU) \end{aligned} \qquad (3.7)$$

The forces F_x, F_y and F_z are given by

$$\begin{aligned} F_x &= X_{AERO} - mg \sin\theta \\ F_y &= Y_{AERO} + mg \sin\phi \cos\theta \\ F_z &= Z_{AERO} + mg \cos\phi \cos\theta \end{aligned} \qquad (3.8)$$

Where X_{AERO}, etc., are the components of the total aerodynamic force acting at the aircraft center of mass.

Substituting equations (3.5) in equations (3.7), the following equations are obtained for the aircraft accelerations,

$$\begin{aligned} \dot{U} &= \frac{X_{AERO}}{m} - g \sin\theta - QW + RV \\ \dot{V} &= \frac{Y_{AERO}}{m} + g \cos\theta \sin\phi - RU + PW \\ \dot{W} &= \frac{Z_{AERO}}{m} + g \cos\theta \cos\phi + QU - PV \end{aligned} \qquad (3.9)$$

3.4 MOMENT EQUATION

The derivation of the equations for the total moment acting about the aircraft center of mass is complicated by the fact that the center of mass changes position due to the tilting nacelles. Thus the centers of gravity of the principal aircraft component masses of the wings (m_W), fuselage (including tails) (m_f), and nacelles (m_N), move with respect to the reference axes OXYZ placed at the instantaneous overall center of gravity of the aircraft. The equation of motion for such a mass element will first be obtained and the total moment found by adding the contributions of all the elements.

3.5 EQUATION OF MOTION FOR A MASS ELEMENT

With reference to Figure (3.1) O'xyz is a right-handed set of axes placed at the center of gravity of the representative mass. The axes are parallel to the set OXYZ. The mass, m, rotates about its own center of gravity with angular velocity $\underline{\omega}$ which, in general, differs from $\underline{\Omega}$ the angular velocity of the aircraft. If $\underline{r}$ is the radius vector from O to O' then the velocity of the center of mass of the element is

$$\underline{V} = \frac{\delta \underline{r}}{\delta t} + \underline{\Omega} \times \underline{r} \tag{3.10}$$

The angular momentum of this mass about O is

$$\underline{h} = m\,(\underline{r} \times \underline{V}) + \underline{ho} \tag{3.11}$$

where $\underline{ho}$ is the angular momentum of m about its own center of mass and is given by

$$\underline{ho} = \bar{I}\,\underline{\omega}$$

where

$$\bar{I} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \tag{3.13}$$

and I_{xx}, etc., are the moments and products of inertia of the mass about O'xyz.

The total moment, $\underline{G}$, about the aircraft center of mass is given by

$$\underline{G} = \frac{d\underline{h}}{dt} = \frac{\delta \underline{h}}{\delta t} + \underline{\Omega} \times \underline{h}$$

Using equations (3.10), (3.11), and (3.12) in (3.14), the moment becomes

$$\underline{G} = m\left[\frac{\delta \underline{r}}{\delta t} \times \left(\frac{\delta \underline{r}}{\delta t} + \underline{\Omega} \times \underline{r}\right) + \underline{r} \times \frac{\delta}{\delta t}\left(\frac{\delta \underline{r}}{\delta t} + \underline{\Omega} \times \underline{r}\right)\right] + \frac{\delta}{\delta t}(\bar{I}\underline{\omega})$$
$$+ m\,\underline{\Omega} \times \left[\underline{r} \times \left(\frac{\delta \underline{r}}{\delta t} + \underline{\Omega} \times \underline{r}\right)\right] + \underline{\Omega} \times (\bar{I}\underline{\omega}) \tag{3.15}$$

which reduces to

$$\underline{G} = 2m\underline{\Omega}\left(\underline{r}\cdot \frac{\delta \underline{r}}{\delta t}\right) + m\underline{r} \times \frac{\delta^2 \underline{r}}{\delta t^2} + m\,\frac{\delta \underline{\Omega}}{\delta t}\,(\underline{r}\cdot\underline{r}) - m\underline{r}\left(\underline{r}\cdot \frac{\delta \underline{\Omega}}{\delta t}\right) \tag{3.16}$$

$$-2m\,\frac{\delta \underline{r}}{\delta t}\,(\underline{\Omega}\cdot\underline{r}) - m(\underline{r}\cdot\underline{\Omega})(\underline{\Omega}\times\underline{r}) + \bar{I}\,\frac{\delta \underline{\omega}}{\delta t} + \underline{\Omega}\times\left(\bar{I}\,\underline{\omega}\right)$$

The only masses that possess angular velocities different from that of the aircraft are the nacelles, which are free to pitch about O' with angular rate $\dot{i} = \frac{di_N}{dt}$. Thus $\underline{\omega}$ may be written generally as

$$\underline{\omega} = P\hat{\underline{i}} + (Q + \dot{i}_N)\,\hat{\underline{j}} + R\,\hat{\underline{k}} \tag{3.17}$$

Now, with $\underline{r} = X\hat{\underline{i}} + Y\hat{\underline{j}} + Z\hat{\underline{k}}$, where X, Y, and Z are the instantaneous coordinates of the individual mass center relative to the aircraft mass center, the various terms of equation (3.16) are, in component form,

$$\underline{r}\cdot\frac{\delta \underline{r}}{\delta t} = XX + YY + ZZ$$

$$\underline{r} \times \frac{\delta^2 \underline{r}}{\delta t^2} = (YZ - ZY)\hat{\underline{i}} - (XZ - ZX)\hat{\underline{j}} + (XY - YX)\hat{\underline{k}}$$

$$\frac{\delta \underline{\Omega}}{\delta t}\,(\underline{r}\cdot\underline{r}) = (X^2 + Y^2 + Z^2)(\dot{P}\hat{\underline{i}} + \dot{Q}\hat{\underline{j}} + \dot{R}\hat{\underline{k}}) \tag{3.18}$$

$$r\cdot\frac{\delta \underline{\Omega}}{\delta t} = X\dot{P} + Y\dot{Q} + Z\dot{R}$$

$$\underline{\Omega}\cdot\underline{r} = XP + YQ + ZR$$

$$(\underline{r}\cdot\underline{\Omega})(\underline{\Omega}\times\underline{r}) = (XP+YQ+XR)\left[(QZ-RY)\hat{\underline{i}} - (PZ-RX)\hat{\underline{j}} + (PY-XQ)\hat{\underline{k}}\right]$$

$$\bar{I}\,\frac{\delta \underline{\omega}}{\delta t} = (I_{xx}\dot{P} - I_{xz}R)\hat{\underline{i}} + I_{yy}(\dot{Q} + \ddot{i}_N)\hat{\underline{j}} + (I_{zz}\dot{R} - I_{xz}\dot{P})\hat{\underline{k}}$$

$$\underline{\Omega}\times(\bar{I}\underline{\omega}) = (QR\,I_{zz} - QPI_{xz} - RQI_{yy} - R\dot{i}_N I_{yy})\hat{\underline{i}}$$

$$-(PR\,I_{zz} - P^2 I_{xz} - PR\,I_{xx} + R^2 I_{xz})\hat{\underline{j}}$$

$$+(QR\,I_{xz} + PQI_{yy} + P\dot{i}_N I_{yy} - PQ\,I_{xx})\hat{\underline{k}}$$

where, in the last two terms, the products of inertia I_{xy} and I_{yz} are zero from symmetry considerations.

Substituting the above relations into equation (3.16) and noting that $\dot{Y}$ and $\ddot{Y}$ are always zero (no lateral motion of th individual masses) the following expressions are obtained for the components of the moment $\underline{G} = \Delta L\underline{i} + \Delta M\underline{j} + \Delta N\underline{k}$:

$$
\begin{aligned}
\Delta L = {} & \dot{P}[I_{xx} + m(Y^2 + Z^2)] - (\dot{R}+PQ)[I_{xz} + m\,XZ] \\
& + RQ[I_{zz}-I_{yy}+m(Y^2-Z^2)] + m\,YZ(R^2-Q^2) - I_{yy}R\,\dot{i}_N \\
& + m\,(Y\ddot{Z}-2\dot{X}YR - 2\dot{X}ZR + 2Z\dot{Z}P - XY\,(\dot{Q} - PR))
\end{aligned}
$$

$$
\begin{aligned}
\Delta M = {} & \dot{Q}\,[I_{yy}+m(X^2+Z^2)] - (R^2-P^2)[I_{xz} + mXZ] \\
& + PR\,[I_{xx}-I_{zz} + m(Z^2-X^2)] + I_{yy}\dot{i}_N \\
& + m\,[\ddot{X}Z-X\ddot{Z} + 2Q(Z\dot{Z}+X\dot{X})-XY\,(\dot{P}+RQ)+YZ(PQ-\dot{R})]
\end{aligned}
\tag{3.19}
$$

$$
\begin{aligned}
\Delta N = {} & \dot{R}\,[I_{zz}+m(X^2+Y^2)] - (\dot{P}-RQ)\,[I_{xz}+m\,XZ] \\
& + PQ\,[I_{yy}-I_{xx} + m(X^2-Y^2)] + I_{yy}P\,\dot{i}_N \\
& + m\,[2X\dot{X}R - Y\ddot{X} - 2XZP - 2Y\dot{Z}Q - YZ\,(\dot{Q}+PR)+XY(Q^2-P^2)]
\end{aligned}
\tag{3.20}
$$

Summing the rolling moment equation:

$$
\begin{aligned}
L = {} & I_{XX}\,\dot{P}-I_{XZ}\,(\dot{R}+PQ) + (I_{ZZ}-I_{YY})RQ \\
& + m_N(R^2-Q^2)(Z_{NR}-Z_{NL})Y_N + m_N\Big\{Y_N(\ddot{Z}_{NR}-\ddot{Z}_{NL}) \\
& -2Q(\dot{X}_{NR}-\dot{X}_{NL})Y_N-2R(\dot{X}_{NR}Z_{NR} + \dot{X}_{NL}Z_{NL}) + 2P(\dot{Z}_{NR}Z_{NR} + {} \\
& \dot{Z}_{NL}Z_{NL})-(\dot{Q}-PR)(X_{NR}-X_{NL})\,Y_N\Big\}+ 2m_f Z_f(P\dot{Z}_f - {} \\
& R\dot{X}_f) + 2m_w Z_w(P\dot{Z}_w - R\dot{X}_w)-R\,I^N_{yy}\,(\dot{i}_{NL} + \dot{i}_{NR})
\end{aligned}
\tag{3.21}
$$

where I_{XX}, I_{XZ}, I_{ZZ}, and I_{YY} are the inertias of the aircraft about its center of gravity, and the subscripts f, w, NL and NR stand for fuselage, wing, left nacelle and right nacelle. The remaining symbols are defined in the List of Symbols. Similar expressions are obtained for the pitching moment and yawing moment. In the interests of brevity the remainder of the discussion will be limited to equation (3.21).

Evaluation of the terms of the rolling moment equation indicate that this equation may be simplified considerably without a significant change in accuracy. For example, terms containing $(\dot{X}_{NR}-\dot{X}_{NL})$ may be dropped because $\dot{X}_{NR}$ is normally identical to

$\dot{X}_{NL}$, i.e. the nacelles are raised or lowered together at the same rate. Equation (3.21) may thus be written

$$L=I_{XX}\dot{P}-I_{XZ}(\dot{R}+PQ) + (I_{ZZ}-I_{YY})RQ + m_N Y_N(\ddot{X}_{NR}-\ddot{Z}_{NL}) \qquad (3.22)$$

where the last term has been retained in consideration of the high differential nacelle accelerations encountered during hover maneuvers.

From the relationships presented in Appendix C the last term of Equation (3.22) may be rewritten as

$$\begin{aligned} -\ell m_N Y_N \,[\ddot{i}_{NR} \cos (i_{NR}-\lambda) + \dot{i}_{NL}^2 \sin (i_{NL}-\lambda) \\ -\dot{i}_{NR}^2 \sin (i_{NR}-\lambda) - \ddot{i}_{NL} \cos (i_{NL}-\lambda)] \end{aligned} \qquad (3.23)$$

which may be approximated to

$$-\ell m_N Y_N \,[\ddot{i}_{NR} \cos (i_{NR}-\lambda) - \ddot{i}_{NL} \cos (i_{NL}-\lambda)] \qquad (3.24)$$

since the nacelle rates appear as squared terms.

Similar treatment of the pitching moment and yawing moment equations results in the following final form of the moment equations.

$$\begin{aligned}
L_{AERO} &= I_{XX}\dot{P}-I_{XZ}(\dot{R}+PQ) + (I_{ZZ}-I_{YY})RQ \\
&\quad -\ell m_N Y_N \,[\ddot{i}_{NR} \cos (i_{NR}-\lambda) - \ddot{i}_{NL} \cos (i_{NL}-\lambda)] \\
M_{AERO} &= I_{YY}\dot{Q} - I_{XZ}(R^2-P^2) + (I_{XX}-I_{ZZ})PR \qquad (3.25) \\
&\quad + \ddot{i}_{NR} \left\{ I^N_{YY_o} + \ell m_N \,[X_R \cos (i_{NR}-\lambda) - Z_R \sin (i_{NR}-\lambda)] \right\} \\
&\quad + \ddot{i}_{NL} \left\{ I^N_{YY_o} + \ell m_N \,[X_L \cos (i_{NL}-\lambda) - Z_L \sin (i_{NL}-\lambda)] \right\} \\
N_{AERO} &= I_{ZZ}\dot{R}-I_{XZ}(\dot{P}-RQ) + (I_{YY}-I_{XX})PQ \\
&\quad + \ell m_N Y_N \,[\ddot{i}_{NR} \sin (i_{NR}-\lambda) - \ddot{i}_{NL} \sin (i_{NL}-\lambda)]
\end{aligned}$$

where the moments L_{AERO}, M_{AERO}, and N_{AERO} represent the sum of the aerodynamic moments and rotor/engine gyroscopic moments about the aircraft center of mass. $I^N_{YY_o}$ is the nacelle pitch inertia referred to the nacelle-fixed axes system described in Appendix C. Equations for the aircraft inertias are also presented in that Appendix.

3.6 EQUATIONS OF MOTION FOR NACELLES

The equation of motion for a nacelle is required in order to obtain the moment exerted by the nacelle on the wing tip at the pivot. This moment is then used in the equations for wing twist.

The angular momentum of a nacelle about its pivot point is given by

$$\underline{h}_p = (\underline{r}-\underline{r}_p) \times m_N\underline{V} + \underline{h}_{O_N} \tag{3.26}$$

$$= m_n\ (\underline{r}\times\underline{V}) + \underline{h}_o - m_n\underline{r}_p \times \underline{V}$$

where $\underline{r}$ is the radius vector from aircraft c.g. to nacelle c.g.

$\underline{V}$ is the velocity of the nacelle c.g.

$\underline{h}_{0_N}$ is the angular momentum of the nacelle about its own c.g.

m_N is the nacelle mass

and $\underline{r}_p$ is the radius vector from aircraft c.g. to nacelle pivot

The term $m_n\ (\underline{r}\times\underline{V}) + \underline{h}_{O_N}$ is the angular momentum of the nacelle about the aircraft c.g. $(= \underline{h}^N_{CG})$.

i.e. $\underline{h}_p = \underline{h}^N_{CG} - m_N(\underline{r}_p \times \underline{V})$

The moment about the pivot is

$$\underline{G}_p = \frac{d\underline{h}_p}{dt} = \frac{d\underline{h}_N}{dt} - m_n \frac{d}{dt} (\underline{r}_p \times \underline{V}) = \underline{G}^N_{CG} - \Delta\underline{G} \tag{3.27}$$

Since the quantity $\underline{G}^N_{cg}$ has already been obtained (equations (3.18), (3.19), and (3.20)), only the remaining term needs to be evaluated.

$$\Delta\underline{G} = m_N \frac{d}{dt} (\underline{r}_p\times\underline{V}) = m_N\left\{\frac{\delta\underline{r}_p}{\delta t} \times \underline{V} + \underline{r}_p \times \frac{\delta\underline{V}}{\delta t} + \underline{\Omega}(\underline{r}_p \times \underline{V})\right\}$$

$$= m_N \left\{\frac{\delta\underline{r}_p}{\delta t} \times\left(\frac{\delta\underline{r}}{\delta t} + \underline{\Omega}\times\underline{r}\right) + \underline{r}_p \times \frac{\delta}{\delta t}\left(\frac{\delta\underline{r}}{\delta t} + \underline{\Omega}\times\underline{r}\right)\right. \tag{3.28}$$

$$\left. + \underline{\Omega} \times \left[\underline{r}_p \times\left(\frac{\delta\underline{r}}{\delta t} + \underline{\Omega} \times \underline{r}\right)\right]\right\}$$

Expansion of these terms results in the following expression

$$\Delta \underline{G} = m_N\left\{\frac{\delta \underline{r}_p}{\delta t} \times \frac{\delta \underline{r}}{\delta t} + \underline{\Omega}\left(\underline{r}\cdot\frac{\delta \underline{r}_p}{\delta t}\right) - \underline{r}\left(\frac{\delta \underline{r}_p}{\delta t}\cdot\underline{\Omega}\right) + \underline{r}_p \times \frac{\delta^2 \underline{r}}{\delta t^2} + \frac{\delta \underline{\Omega}}{\delta t}\left(\underline{r}\cdot\underline{r}_p\right)\right.$$
$$- \underline{r}\left(\underline{r}_p\cdot\frac{\delta \underline{\Omega}}{\delta t}\right) + \underline{\Omega}\left(\frac{\delta \underline{r}}{\delta t}\cdot\underline{r}_p\right) - 2\frac{\delta \underline{r}}{\delta t}\left(\underline{r}_p\cdot\underline{\Omega}\right) \tag{3.29}$$
$$\left. + \underline{r}_p\left(\frac{\delta \underline{r}}{\delta t}\cdot\underline{\Omega}\right) - \left(\underline{r}_p\cdot\underline{\Omega}\right)\left(\underline{\Omega}\times\underline{r}\right)\right\}$$

We require only the $\underline{j}$ component of this vector in order to obtain the nacelle pivot pitching moment.

The components of the vectors $\underline{r}_p$, $\underline{r}$ and $\underline{\Omega}$ are

$$\underline{r}_p = X_p\hat{\underline{i}} + Y_N\hat{\underline{j}} + Z_p\hat{\underline{k}} = -X_{CG}\hat{\underline{i}} + Y_N\hat{\underline{j}} - Z_{CG}\hat{\underline{k}}$$

$$\underline{r} = X_N\hat{\underline{i}} + Y_N\hat{\underline{j}} + Z_N\hat{\underline{k}}$$

$$\underline{\Omega} = P\hat{\underline{i}} + Q\hat{\underline{j}} + R\hat{\underline{k}}$$

Noting that the $\underline{j}$ components of $\frac{\delta \underline{r}_p}{\delta t}$, $\frac{\delta \underline{r}}{\delta t}$ are zero (since Y_N is a constant), the above expression yields

$$\Delta M = m_N\left\{\ddot{X}_N Z_{CG} - \ddot{Z}_N X_{CG} + \dot{Z}_{CG}\dot{X}_N + \dot{Z}_N\dot{X}_{CG} + PQ\, Y_N Z_N \right.$$
$$\left. - RQ\, X_N Y_N\right\} \tag{3.30}$$

Combining this equation with Equation (3.19) and using the transformations given in Appendix C, the final equation for the right-hand nacelle pivot actuator pitching moment becomes, after some simplification,

$$M_{NR} = -\dot{i}_{NR}\left[I^N_{YY_O} + \ell^2 m_N\left(1-\frac{m_N}{m}\right)\right] - \ell^2 m_N\left(1-\frac{m_N}{m}\right)\left[\dot{Q} - PR\cos 2(i_{NR}-\lambda)\right.$$
$$\left. + (R^2-P^2)\sin(i_{NR}-\lambda)\operatorname{Cos}(i_{NR}-\lambda)\right] - (R^2-P^2)\, I^N_{zz_O}\sin i_{NR}\cos i_{NR}$$
$$- I_{YY_O}\dot{Q} + \ell\frac{m_N}{m}\left[X_{AERO}\sin(i_{NR}-\lambda) + Z_{AERO}\cos(i_{NR}-\lambda)\right]$$
$$- \ell m_N Y_N\left\{(\dot{R}-PQ)\sin(i_{NR}-\lambda) - (\dot{P}+RQ)\cos(i_{NR}-\lambda)\right\}$$
$$+ M_{NR_{AERO}} \tag{3.31}$$

where $M_{NR_{AERO}}$ includes the moment resulting from nacelle aerodynamic loads and the rotor gyroscopic moments. The terms X_{AERO} and Z_{AERO} are, respectively, the total aircraft aerodynamic X and Z forces.

The corresponding equation for the left nacelle actuator moment is obtained by substituting $-Y_N=Y_N$ and changing the R subscript to L.

3.7 DETERMINATION OF ROTOR GYROSCOPIC MOMENTS

The gyroscopic moments are most readily obtained as follows. A set of axes O"x'y'z' is taken at the rotor hub (rotor c.g.) parallel to the nacelle-fixed set of axes $Ox_oy_oz_o$. Associated with each axis are the corresponding unit vectors $\underline{i}'$ $\underline{j}'$ and $\underline{k}'$. The angular velocity of the rotor with respect to these axes is the vector

$$\underline{\omega} = \Omega_R \underline{i}' \qquad (3.32)$$

where Ω_R is the rotor rotational speed.

The angular momentum of the rotor with respect to its c.g. is

$$\underline{h}_o = \bar{I}_R \underline{\omega}$$

where $\bar{I}_R$ is the inertia matrix $\begin{bmatrix} I_{R_{x'}} & & \\ & I_{R_{y'}} & \\ & & I_{R_{z'}} \end{bmatrix}$

the off-diagonal terms being zero since the axes O"x'y'z' are principal axes of inertia of the rotor and hub.

In component form the angular momentum of the rotor is

$$\underline{h}_o = I_{R_{y'}} \Omega_R \hat{\underline{i}}' = I_R \Omega_R \hat{\underline{i}}' \qquad (3.34)$$

With respect to the inertial axes OYXZ, the components of $\underline{h}_o$ are

$$\underline{h}_o = I_R \Omega_R \cos i_N \hat{\underline{i}} - I_R \Omega_R \sin i_N \hat{\underline{k}} \qquad (3.35)$$

The hub moment is therefore given by

$$\underline{G}_{HUB} = \frac{d\underline{h}_o}{\delta t} = \frac{\delta \underline{h}_o}{\delta t} + \underline{\Omega} \times \underline{h}_o \qquad (3.36)$$

where $\underline{\Omega} = P\hat{\underline{i}} + Q\hat{\underline{j}} + R\hat{\underline{k}}$ (3.37)

Substitution of equations (3.35) and (3.37) into equation (3.36) results in the following equations for the rotor gyroscopic moments.

$$L_{gyro} = I_R \dot{\Omega}_R \cos i_N - I_R \Omega_R (\dot{i}_N + Q) \sin i_N \tag{3.38}$$

$$M_{gyro} = I_R P \Omega_R \sin i_N + I_R R \Omega_R \cos i_N \tag{3.39}$$

$$N_{gyro} = -I_R \dot{\Omega}_R \sin i_N - I_R \Omega_R (\dot{i}_N + Q) \cos i_N \tag{3.40}$$

The above terms appear in the Computer Representation (Appendix E) as additions to the rotor aerodynamic forces and moments.

4.0 AIRFRAME AERODYNAMICS

This section presents the mathematical equations and representations of the aerodynamic data for the aircraft without rotors. The contribution of the rotors is described in Section 5. The overall airframe aerodynamics are obtained from the following components:

(a) Fuselage

(b) Wings

(c) Horizontal Tail

(d) Vertical Tail

(e) Nacelles

The data and equations for each of the aerodynamic components are discussed below, together with the substantiating methods. The aerodynamic data are presented in local wind axes. Resolution to aircraft body axes is accomplished as described in the mathematical model (Appendix E). Where required, the equations have been written so as to be applicable over the entire range of angle of attack ± 180 degrees.

4.1 FUSELAGE

The aerodynamic lift, drag, and pitching moment coefficients of the fuselage were estimated using the methods of Reference 3. The forces and moments are referred to the point on the fuselage corresponding to the wing quarter chord position. This reference point was selected in order to minimize the number of force and moment transfer equations in the mathematical model. Wing-to-body carryover effects have been included in fuselage loads.

The equations for the fuselage forces and moments are:

Lift:

$$C_{L_F} = K_{42} + K_3 \mathrm{Sin}\alpha_F \mathrm{Cos}\alpha_F + K_4 \mathrm{Sin}\alpha_F \mathrm{Cos}\alpha_F \left| \mathrm{Sin}\alpha_F \mathrm{Cos}\alpha_F \right|$$

Drag:

$$C_{D_F} = C_{DO_F}(1 + K_o|\beta_F|^3) + K_2(\mathrm{Sin}\alpha_F \mathrm{Cos}\alpha_F)^2 + K_1 \left|\mathrm{Sin}\alpha_F \mathrm{Cos}\alpha_F\right| + \Delta C_{D_{LG}}$$

Side Force:

$$C_{Y_F} = K_7 \mathrm{Sin}\beta_F \mathrm{Cos}\beta_F + K_8 \mathrm{Sin}\beta_F \mathrm{Cos}\beta_F \left|\mathrm{Sin}\beta_F \mathrm{Cos}\beta_F\right|$$

Pitching Moment: $C_{M_F} = C_{M_{O_F}} + K_5 \operatorname{Sin}\alpha_F \operatorname{Cos}\alpha_F + K_6 \operatorname{Sin}\alpha_F \operatorname{Cos}\alpha_F |\operatorname{Sin}\alpha_F \operatorname{Cos}\alpha_F| + \Delta C_{M_{LG}}$

Yawing Moment: $C_{N_F} = C_{N_{OF}} + K_9 \operatorname{Sin}\beta_F \operatorname{Cos}\beta_F + K_{10} \operatorname{Sin}\beta_F \operatorname{Cos}\beta_F |\operatorname{Sin}\beta_F \operatorname{Cos}\beta_F|$

Rolling Moment: $C_{\ell_F} = 0$

$$\text{where } \alpha_F = \operatorname{Tan}^{-1}\left(\frac{W}{U}\right),\; C_{L_F} = \frac{L_F}{\frac{1}{2}\rho V_{FUS}^2 S_W} \quad \text{etc.}$$

$$\beta_F = \operatorname{Tan}^{-1}\left[\frac{V}{\sqrt{U^2 + W^2}}\right],\; C_{M_F} = \frac{M_F}{\frac{1}{2}\rho V_{FUS}^2 S_W C_W} \quad \text{etc.}$$

and $\Delta C_{D_{LG}}$, $\Delta C_{M_{LG}}$, are the landing gear contributions to fuselage drag and pitching moment coefficients, when the landing gear is extended.

The fuselage forces and moments are then resolved into body axes at the aircraft C.G.

4.2 NACELLES

The forces and moments acting on the nacelles were estimated using the cross-flow methods of Reference 4. For convenience the resulting forces and moments are referred to the rotor hub, so that they may be added directly to the rotor forces and moments. The following equations are for the forces and moments on two nacelles:

$$C_{L_N} = K_{32} \sin \alpha_N \cos \alpha_N$$

$$C_{D_N} = C_{D_{O_N}} + K_{30} |\alpha_N| + K_{31} \alpha_N^2$$

$$C_{M_N} = C_{M_{O_N}} + K_{34} \sin \alpha_N \cos \alpha_N + K_{35} \sin \alpha_N \cos \alpha_N |\sin \alpha_N \cos \alpha_N|$$

$$C_{Y_N} = K_{36} \sin \beta_N \cos \beta_N + K_{37} \sin \beta_N \cos \beta_N |\sin \beta_N \cos \beta_N|$$

$$C_{N_N} = C_{N_{O_N}} + K_{38} \sin\beta_N \cos\beta_N + K_{39} \sin\beta_N \cos \beta_N |\sin\beta_N \cos \beta_N|$$

$$C_{\ell_N} = 0$$

The nacelle forces and moments in nacelle axes are:

$$\Delta X'_N = q_N S_W[-C_{D_N} \cos \alpha_N + C_{L_N} \sin \alpha_N - C_{Y_N} \sin \beta_N \cos \alpha_N]\frac{1}{2}$$

$$\Delta Y'_N = q_N S_W[C_{Y_N} \cos \beta_N - C_{D_N} \sin \beta_N]\frac{1}{2}$$

$$\Delta Z'_N = q_N S_W[-C_{L_N} \cos \alpha_N - C_{D_N} \cos \beta_N \sin \alpha_N - C_{Y_N} \sin \beta_N \sin \alpha_N]\frac{1}{2}$$

$$\Delta \mathcal{L}'_N = q_N S_W b_W[-\left(\frac{C_W}{b_W}\right) C_{M_N} \sin \beta_N \cos \alpha_N - C_{N_N} \sin \alpha_N]\frac{1}{2}$$

$$\Delta M'_N = q_N S_W C_W[C_{M_N} \cos \beta_N]\frac{1}{2}$$

$$\Delta N'_N = q_N S_W b_W[C_{N_N} \cos \alpha_N - \left(\frac{C_W}{b_W}\right) C_{M_N} \sin \beta_N \cos \alpha_N]\frac{1}{2}$$

4.3 HORIZONTAL TAIL

Aerodynamics of the horizontal tail were obtained using the methods of Reference 3 in combination with test data. The horizontal tail includes a plain elevator.

The angle of attack of the horizontal tail, including interference effects, for zero elevator deflection, is

$$\alpha_{HT} = \mathrm{Tan}^{-1}\left[\frac{w_{HT}}{u_{HT}}\right] - \varepsilon + i_{HT}$$

where ε is the total downwash at the tail due to wing, rotor and ground effects and i_{HT} is the tail incidence angle.

The effect of elevator deflection on the effective tail angle of attack is introduced through the elevator effectiveness parameter, τ_{HT}, which is a function of the elevator and horizontal tail areas. Thus the effective horizontal tail angle of attack is

$$\alpha_{e_{HT}} = \alpha_{HT} + \tau_{HT}\delta_e$$

where δ_e is the elevator deflection.

The tail downwash angle, ε, depends on wing angle of attack and on rotor slipstream deflection. At a given rotor angle of attack, the slipstream deflection is a function of rotor thrust coefficient, C_{T_S}, where the coefficient is based on the slipstream dynamic pressure. Figure 4.1 presents data on downwash angles measured during tests on a tilt rotor wind tunnel model (Reference 5). As can be seen, the downwash at low values of thrust coefficient is the same as the value of the power-off

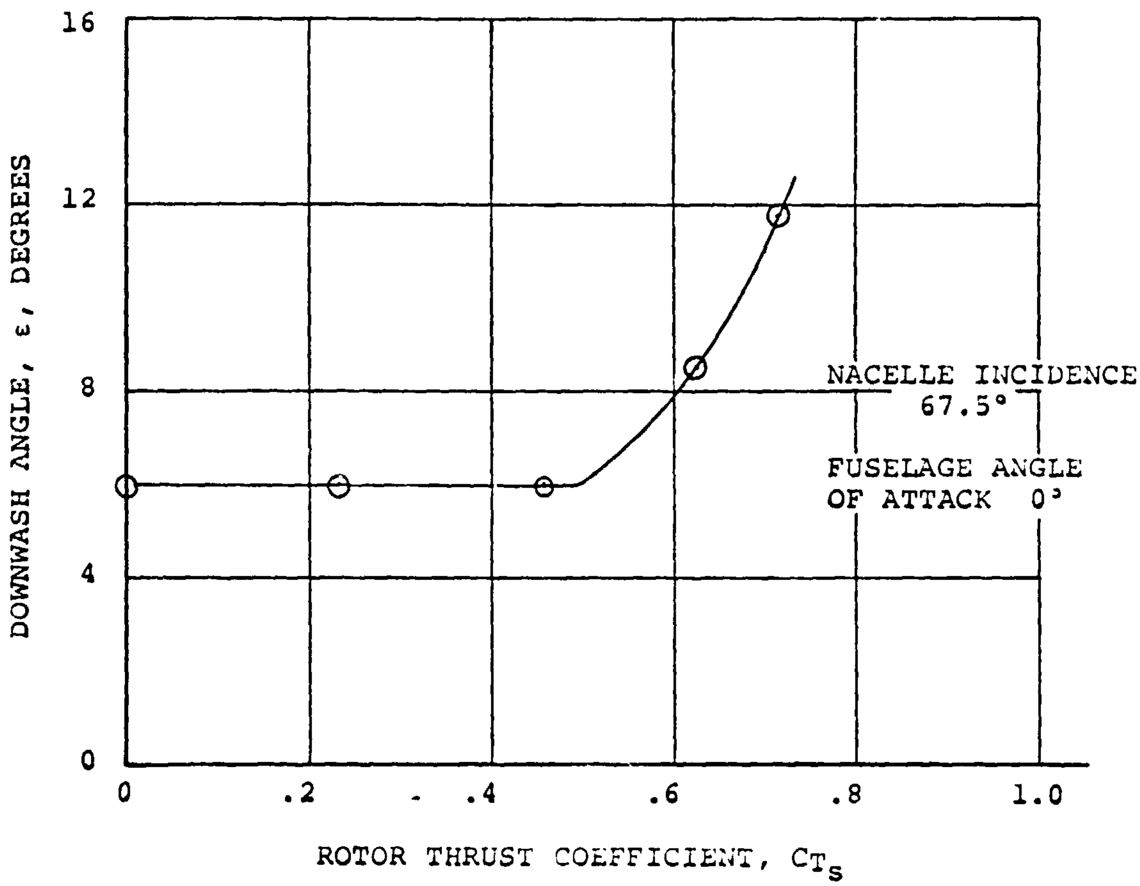


Figure 4.1 Variation of Horizontal Tail Downwash Angle with Thrust Coefficient

wing downwash ($C_{T_S} = 0$). Above values of C_{T_S} in the neighborhood of $C_{T_S}=.5$ the downwash increases with increasing thrust coefficient. The values in the increasing portion of ε vs C_{T_S} were found to correspond approximately to the slipstream deflection angle ε_p. Therefore, the approach adopted in the mathematical model was to test if the rotor slipstream downwash ($\bar{\varepsilon}_p$) exceeded the wing downwash and, if so, to use the computed slipstream downwash value as the tail downwash angle. Otherwise the wing downwash value was used.

Thus if

$$\bar{\varepsilon}_p \geq \varepsilon_o + \frac{d\varepsilon}{d\alpha}\left(\bar{\alpha}_W - \ell_{AC}\frac{\dot{W}}{U^2}\right)$$

then $$\varepsilon = \frac{\bar{\varepsilon}_p(1-GEF)}{\sqrt{1-M^2}}$$

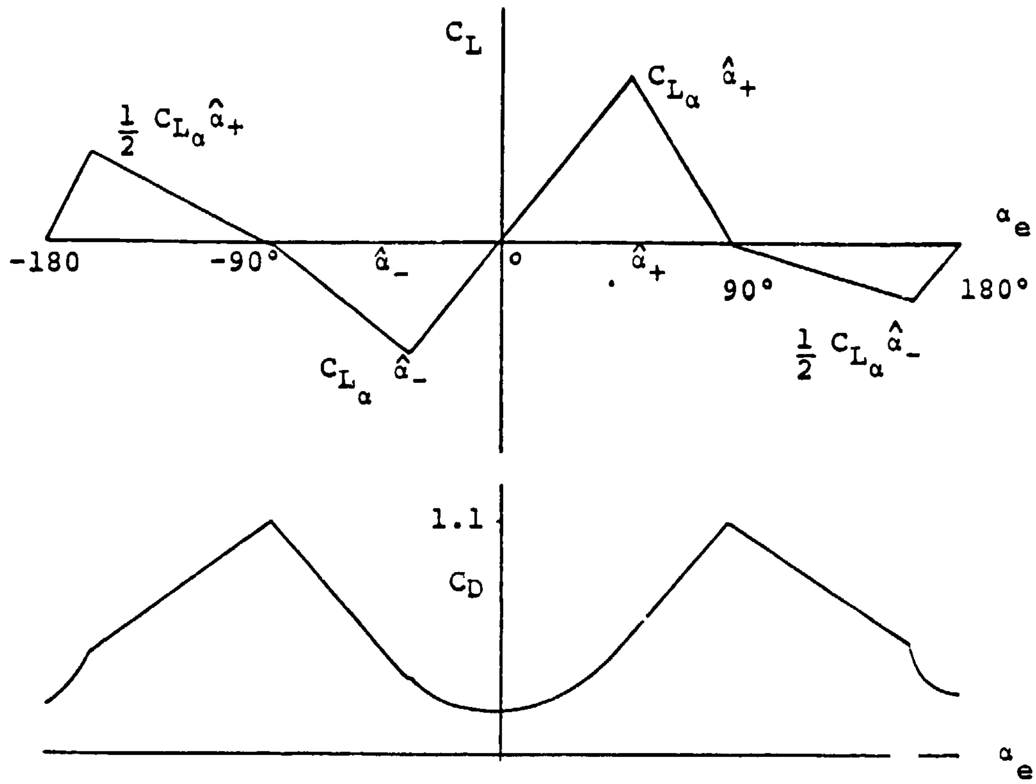
otherwise

$$\epsilon = \left[\epsilon_o + \frac{d\epsilon}{d\alpha}\left(\bar{\alpha}_W - \ell_{AC}\frac{\dot{W}}{U^2}\right)\right]\frac{(1-GEF)}{\sqrt{1-M^2}}$$

In these expressions ϵ_o is the wing downwash angle at zero wing angle of attack, $\frac{d\epsilon}{d\alpha}$ is the downwash derivative, ℓ_{AC} is the distance from the wing to the tail aerodynamic centers, and $\ell_{AC}\frac{\dot{W}}{U^2}$ is the familiar downwash lag term. In general, the quantities ϵ_o and $\frac{d\epsilon}{d\alpha}$ depend on the average of the left and right flaperon deflections. The effect of differential deflection of aileron/spoiler in producing an asymmetrical downwash field at the horizontal tail was not included because of the small contribution this makes to total aircraft rolling moment.

The term (1-GEF) in the above equations is the ground effect factor. This quantity was obtained from Reference 2 and is a function of the wing span and height of the horizontal tail above the ground (Appendix E, Page E-42). This factor, when multiplied by the downwash which would be found out of ground effect, yields the downwash in ground effect. Ground effect is discussed in more detail in Section 8.

The lift and drag forces acting on the horizontal tail are required over the complete range of angle of attack -180° to +180°, since the tilt rotor can fly backwards. The following sketch shows the schematic variation of lift and drag coefficients over this range plotted as a function of the effective horizontail tail angle of attack, α_{eHT}.



The angle of attack for $C_{L_{HT_{MAX}}}$ is denoted by α_{HT_+} and is the value of the effective angle of attack at the stall less 2 degrees i.e.

$$\hat{\alpha}_{HT_+} = (\alpha_{HT_{STALL}} - 2°) + \tau_{HT}\delta_e$$

Similarly the angle of attack for stall at negative angles of attack is

$$\hat{\alpha}_{HT_-} = -(\alpha_{HT_{STALL}} - 2°) + \tau_{HT}\delta_e$$

The slope of the lift curve within this range of positive and negative angles of attack is given by

$$C_{L_\alpha} = \frac{C_{L_{\alpha HT}}\left(\frac{a_g}{a}\right)}{\sqrt{1-M^2}}$$

where a_g/a is the ratio of tail lift-curve slopes in and out of ground effect, and $\sqrt{1-M^2}$ is the Prandtl-Glauert correction factor for the effect of Mach number on lift-curve slope.

Within this region on the lift curve the value of lift coefficient is given by $C_{L_{HT}} = C_{L_\alpha} \alpha_{e_{HT}}$ and the corresponding drag coefficient by

$$C_{D_{HT}} = C_{D_{O_{HT}}} + \frac{C^2_{L_{HT}}}{\pi AR_{HT} \, e_{HT}}$$

After stall angle of attack is passed the lift is assumed to fall linearly to zero at $\alpha_e = \pm 90°$.

In these regions the lift is given by

$$C_{L_\alpha} = C_{L_\alpha} \hat{\alpha}_{\pm} \frac{(\pm 90 - \alpha_{e_{HT}})}{(\pm 90 - \hat{\alpha}_{HT_\pm})}$$

where the appropriate signs are taken depending on the sign of $\alpha_{e_{HT}}$.

The corresponding drag is obtained by assuming a linear variation of drag from the value of $C_{L_{MAX}}$ to a value of $C_D = 1.1$ (flat plate normal to stream) at $\alpha_{e_{HT}} = 90°$. Thus

$$C_{L_{HT_{STALL}}} = C_{L_\alpha} \hat{\alpha}_{HT_\pm}$$

$$C_{D_{HT_{STALL}}} = C_{D_{O_{HT}}} + \frac{C^2_{L_{HT_{STALL}}}}{\pi AR_{HT} \, e_{HT}}$$

and

$$C_{D_{HT}} = C_{D_{HT_{STALL}}} + \frac{(\alpha_{e_{HT}} - \hat{\alpha}_{HT_\pm})(1.1 - C_{D_{HT_{STALL}}})}{(\pm 90 - \hat{\alpha}_{HT_\pm})}$$

If the effective angle of attack of the horizontal tail exceeds $\pm 90°$ the tail will point trailing-edge first into the relative wind. Under this condition early stalling is precipitated because of the sharp "leading edge" and blunt "trailing edge". In order to represent this, it was assumed that the attainable $C_{L_{MAX}}$ of the tail under these conditions is half that occurring in normal flight.

Thus if $\quad 90° < \alpha_{e_{HT}} \leq (180 - \frac{1}{2} \hat{\alpha}_{HT_-})$

or $(-180 + \frac{1}{2} \hat{\alpha}_{HT_+}) \leq \alpha_{e_{HT}} < -90°$

then

$$C_{L_{HT}} = .5C_{L_\alpha}\,\hat{\alpha}_{HT_-}\;\frac{(\alpha_{e_{HT}} - 90°)}{(90° - \frac{1}{2}\hat{\alpha}_{HT_-})}$$

or

$$C_{L_{HT}} = .5C_{L_\alpha}\,\hat{\alpha}_{HT_+}\;\frac{(\alpha_{e_{HT}} + 90°)}{(-90 + \frac{1}{2}\hat{\alpha}_{HT_+})}$$

The corresponding drag coefficients are:

for $90° < \alpha_{e_{HT}} \leq (180 - \frac{1}{2}\hat{\alpha}_{HT_-})$;

$$C_{L_{HT_{STALL}}} = 0.5\,C_{L_\alpha}\,\hat{\alpha}_{HT_-}$$

$$C_{D_{HT_{STALL}}} = \frac{C^2_{L_{HT_{STALL}}}}{\pi AR_{HT}\,e_{HT}} + C_{D_{O_{HT}}}$$

which gives $C_{D_{HT}} = C_{D_{HT_{STALL}}} + \dfrac{(\alpha_{e_{HT}} + 0.5\,\hat{\alpha}_{HT_-} - 180°)(1.1 - C_{D_{HT_{STALL}}})}{(0.5\,\hat{\alpha}_{HT_-} - 90°)}$

and for $(-180 + \frac{1}{2}\hat{\alpha}_{HT_+}) \leq \alpha_{e_{HT}} < -90°$;

$$C_{L_{HT_{STALL}}} = 0.5\,C_{L_\alpha}\,\hat{\alpha}_{HT_+}$$

$$C_{D_{HT_{STALL}}} = \frac{C^2_{L_{HT_{STALL}}}}{\pi AR_{HT}\,e_{HT}} + C_{D_{O_{HT}}}$$

which gives $C_{D_{HT}} = C_{D_{HT_{STALL}}} - \dfrac{(\alpha_{e_{HT}} + 180° - .5\hat{\alpha}_{HT_+})(1.1 - C_{D_{HT_{STALL}}})}{(.5\hat{\alpha}_{HT_+} - 90°)}$

In the range $(180 - .5\hat{\alpha}_{HT_-}) \leq \alpha_{e_{HT}} \leq 180°$ when the tail has unstalled

$$C_{L_{HT}} = C_{L_\alpha}\,(\alpha_{e_{HT}} - 180°)$$

$$C_{D_{HT}} = C_{D_{O_{HT}}} + \frac{C^2_{L_{HT}}}{\pi\,AR_{HT}\,e_{HT}}$$

and similarly for the range $-180° \leq \alpha_{e_{HT}} < (-180 + .5\ \hat{\alpha}_{HT_+})$

$$C_{L_{HT}} = C_{L_\alpha}\ (\alpha_{e_{HT}} + 180°)$$

$$C_{D_{HT}} = C_{D_{O_{HT}}} + \frac{C^2_{L_{HT}}}{\pi AR_{HT} e_{HT}}$$

The above equations define the variation of tail lift and drag over the entire range of angle of attack. The tail pitching moment is not computed since it makes only a small contribution to the total aircraft pitching moment.

4.4 VERTICAL TAIL

The aerodynamic forces and moments acting on the vertical tail were estimated using the methods of Reference 3. The angle of attack of the vertical tail is given by

$$\alpha_{VT} = -\ \mathrm{Tan}^{-1}\left[\frac{v_{VT}}{\sqrt{u^2_{VT} + w^2_{VT}}}\right] + \beta_F \left(\frac{d\sigma}{d\beta}\right)$$

where u_{VT}, v_{VT}, and w_{VT} are the components of the velocity at the vertical tail aerodynamic center as given in Appendix C. The term $\beta_F \left(\frac{d\sigma}{d\beta}\right)$ is the sidewash correction for the presence of the fuselage.

As in the treatment of the horizontal tail, the effect of rudder deflection is obtained using a rudder effectiveness parameter τ_{VT}. Thus the effective angle of attack of the vertical tail when the rudder is deflected is

$$\alpha_{e_{VT}} = \alpha_{VT} + \tau_{VT}\ \delta_{RUD}$$

The treatment of the vertical tail aerodynamics through the complete angle of attack range -180° to +180° then follows the same lines as that for the horizontal tail aerodynamics previously described.

The vertical tail forces and moments in body axes are then obtained from:

$$X^{VT}_{AERO} = \bar{q} S_{VT} \eta_{VT} \Big[-C_{D_{V_T}} \cos(\beta_{VT} - \sigma)\cos(\alpha_{HT} - i_{HT}) - C_{Y_{VT}} \sin(\beta_{VT} - \sigma)\cos(\alpha_{HT} - i_{HT})\Big]$$

$$Y^{VT}_{AERO} = \bar{q}S_{VT}\eta_{VT}\left[C_{Y_{VT}}\cos(\beta_{VT}-\sigma)-C_{D_{VT}}\sin(\beta_{VT}-\sigma)\right]$$

$$Z^{VT}_{AERO} = \bar{q}S_{VT}\eta_{VT}\left[-C_{D_{VT}}\cos(\beta_{VT}-\sigma)\sin(\alpha_{HT}-i_{HT})-C_{Y_{VT}}\sin(\beta_{VT}-\sigma)\sin(\alpha_{HT}-i_{HT})\right]$$

$$M^{VT}_{AERO} = Z^{VT}_{AERO}\ (X_{CG}-X_{VT}) + X^{VT}_{AERO}\ (Z_{VT}-Z_{CG})$$

$$N^{VT}_{AERO} = -Y^{VT}_{AERO}\ (X_{CG}-X_{VT})$$

$$L^{VT}_{AERO} = -\ Y^{VT}_{AERO}\ (Z_{VT}-Z_{CG})$$

4.5 WING AERODYNAMICS

The treatment of the wing aerodynamics is the most complex of all the components. Because wing flexibility must be represented, each wing panel required a separate treatment. The approach adopted for simulation purposes was first to obtain the aerodynamic forces and moments on the complete wing considered as rigid and uninfluenced by slipstream interference effects. With this data as a basis, the effects of elastic deflection were introduced as an increment in the effective angle of attack of each wing panel and the rotor slipstream interference was then calculated. This approach is described in detail below.

4.5.1 BASIC WING AERODYNAMICS

The basic wing lift, drag and pitching moment coefficients for the wing in the presence of the fuselage rotors-off, were calculated using the methods of Reference 3. This data is applicable to low speed flight. Corrections for Mach number effects are introduced through the Prandtl-Glauert factor $\sqrt{1-M^2}$. Beyond stall angle of attack, the lift, drag, and pitching moment curves are extended linearly to $\pm 90°$ angle of attack in order to provide a representation of wing behavior at low transition speeds when wing angles of attack approach 90°. The data was calculated for the complete range of flaperon settings.

The complete wing basic lift, drag, and pitching moment data also applies to each individual wing panel provided the data is obtained at the appropriate panel angle of attack. This approximation is acceptable if the angles of attack of each wing panel are not substantially different. This condition is normally fulfilled.

In addition to the above data, the effects of spoiler deflection on panel lift, drag, and pitching moment are required. These were estimated using the data of Reference 3. As can be seen from the equations presented in Appendix E the spoiler effectiveness is strongly dependent upon flaperon deflection, a result of the spoilers being slot-lip spoilers.

4.5.2 ROTOR SLIPSTREAM INTERFERENCE

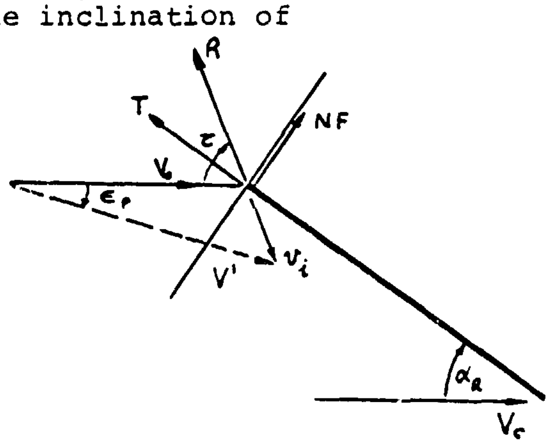
Before the basic wing aerodynamic data can be utilized in the calculation of the wing forces, the effects of the rotor slipstream must be calculated. The calculation procedure presented here has been developed and used at Boeing for some years, and gives acceptable agreement with wind tunnel test data on a wide variety of both tilt rotor and tilt wing configurations.

The method uses momentum theory to obtain the direction and speed of the rotor slipstream in the neighborhood of the wing. From this the effective angle of attack of that part of the wing that is immersed in the slipstream is calculated. The lift, drag, and pitching moment on the wing are then calculated for this angle of attack as if the entire wing were immersed. The area of the wing immersed in the slipstream is now computed and, using the ratio of the immersed to total wing area, the forces acting on the immersed portion are approximated.

At the angle of attack of the wing outside the slipstream, the wing forces and moments are obtained from the basic wing data as if no slipstream effects were present. These forces are then scaled by the ratio of unimmersed to total wing areas to obtain approximately the forces acting on the unimmersed wing. The sum of the approximations to immersed and unimmersed wing forces is now formed. This sum is then multiplied by a correction factor to obtain the final forces.

This correction factor is obtained from a consideration of the mass flows associated with the rotor-wing combination. In the following outline of the method only one rotor is considered.

From the following sketch, which shows the forces acting on the rotor, the inclination of



the resultant force on the rotor to the freestream direction is given by

$$\tau_R = \alpha_R + \mathrm{Tan}^{-1}\left(\frac{NF}{T}\right)$$

The resultant force on the rotor is

$$R = \sqrt{T^2 + NF^2 + SF^2}$$

where T, NF and SF are the thrust, normal force and sideforce, respectively.

The mass flow through the disc is

$$m = \rho\ A\ V'$$

where A is the disc area and V' is obtained from the induced velocity triangle at the disc plane.

$$V' = \sqrt{(V_o + v_i \cos \tau)^2 + (v_i \sin \tau)^2}$$

The resultant force on the rotor is related to the mass flow by

$$R = 2m\ v_i = 2\rho A\ V'\ v_i$$

From these equations the following quartic equation is obtained for the induced velocity at the disc.

$$v_*^4 + 2V_* v_*^3 \cos \tau + v_*^2 V_*^2 = 1$$

where the nondimensional notations

$$v_* = \frac{v_i}{\sqrt{\dfrac{R}{2\rho A}}} \qquad V_* = \frac{V_o}{\sqrt{\dfrac{R}{2\rho A}}}$$

have been introduced.

This equation is then solved for v_* and the direction of the slipstream just behind the rotor disc is calculated from

$$\varepsilon_p = \mathrm{Tan}^{-1}\left[\frac{v_* \sin\tau}{v_* \cos\tau + V_*}\right]$$

The rotor thrust coefficient C_{T_S} is defined as

$$C_{T_S} = \frac{T}{(q + \frac{T}{A})A}$$

NOTE: Because the rotor diameter to wing chord is large the slipstream is considered to be uncontracted in the vicinity of the wing.

with $T = R \cos(\tau - \alpha_R)$

and $q = \frac{1}{2}\rho V^2 = \frac{1}{4} V_*^2 R$

then $C_{T_S} = \dfrac{\cos(\tau-\alpha_R)}{\cos(\tau-\alpha_R) + \frac{V_*^2}{4}}$

The aspect ratio of the slipstream-immersed wing area is given by

$$AR_i = \frac{S_i}{c^2}$$

where S_i is the immersed area calculated by the method described in Appendix D, and c is the wing chord.

The lift on the wing, if the slipstream were absent, is obtained by calculating the effective angle of attack of the wing

$$\alpha_o = \sin^{-1}\left[\frac{w_W}{\sqrt{u_W^2 + w_W^2}}\right] + \theta_t$$

where w_W, u_W are the velocities at the wing aerodynamic center and θ_t is the elastic twist at the point. The lift coefficient (C_L^*) for this angle of attack is obtained from the aerodynamic data for the appropriate flaperon/spoiler deflection.

Similarly the lift (C_L'') and drag (C_D'') coefficients of the wing in the slipstream (assuming wing is completely immersed) are obtained fromthe aerodynamic data at the angle of attack

$$\alpha_s = \alpha_o - \varepsilon$$

The total lift coefficient of the wing with slipstream is therefore

$$C_{L_S} = K'_A \left[\frac{S_i}{S} (C''_L \cos \varepsilon - C''_D \sin \varepsilon) + C^*_L (1-C_{T_S})(1-\frac{S_i}{S})\right]$$

where

$$C_{L_S} = \frac{L}{q_S S_W}$$

in which q_S is the nominal slipstream dynamic pressure, defined by $q_S = q + \frac{T}{A}$.

The factor K'_A is a correction factor to account for the fact that the lift-sharing between the immersed and unimmersed portions of the wing is not simply proportional to the respective areas.

From considerations of the mass flows associated with the wing-rotor combination the factor K'_A was obtained in the form

$$K'_A = \frac{V_* + \frac{C_{L_{\alpha i}}}{C_{L_\alpha}} v_*}{V_* + v_*}$$

where, from wing theory,

$$\frac{C_{L_{\alpha i}}}{C_{L_\alpha}} = \frac{1}{1 + \frac{C_{L_\alpha}}{\pi}\left[\frac{1}{AR_i} - \frac{1}{AR}\right]}$$

The drag and pitching moments for the wing with slipstream are obtained similarly and are given by:

$$C_{D_S} = K'_A \left\{ \frac{S_i}{S} (C''_L \sin \varepsilon + C''_D \cos \varepsilon) + C^*_D (1-C_{T_S})(1-\frac{S_i}{S})\right\}$$

$$C_{M_S} = K'_A \left\{ \frac{S_i}{S} C''_M + C^*_M (1-C_{T_S})(1-\frac{S_i}{S})\right\}$$

The rolling moment and yawing moment coefficients for the wing are given by:

$$C_{\ell_S} = (K_{20} + K_{21} \bar{C}_L)(1-\bar{C}_{T_S})\beta_F + \bar{Y}_{AC}\left(\frac{1-C_{T_S}}{2b_W}\right)\left(C^*_{L_{LW}} - C^*_{L_{RW}}\right) + \Delta C_{\ell_{S_{POWER}}}$$

$$C_{n_S} = K_{22} \bar{C}_L^2 (1-C_{T_S})\beta_F + \bar{Y}_{AC}\frac{(1-C_{T_S})}{2b_W}(C^*_{D_{RW}} - C^*_{D_{LW}}) + \Delta C_{n_{S_{POWER}}}$$

where the increment in rolling moment due to power is

$$\Delta C_{\ell_{S_{POWER}}} = \frac{1}{4}\Bigg\{\left[C_{L_{S_{LW}}} - (1-\bar{C}_{T_S})C^*_{L_{LW}}\right]\left[1-\frac{1}{2}\left(\frac{S_i}{S}\right)_{LW}\right] - \left[C_{L_{S_{RW}}} - (1-\bar{C}_{T_S})\, C^*_{L_{RW}}\right]\left[1-\frac{1}{2}\left(\frac{S_i}{S}\right)_{RW}\right]\Bigg\}$$

and the increment in yawing moment is

$$\Delta C_{n_{S_{POWER}}} = \frac{1}{4}\Bigg\{\left[C_{D_{S_{RW}}} - (1-\bar{C}_{T_S})C^*_{D_{RW}}\right]\left[1-\frac{1}{2}\left(\frac{S_i}{S}\right)_{RW}\right] - \left[C_{D_{S_{LW}}} - (1-\bar{C}_{T_S})C^*_{D_{LW}}\right]\left[1-\frac{1}{2}\left(\frac{S_i}{S}\right)_{LW}\right]$$

Figure 4.2 shows a correlation between the wing-in-slipstream method described above and experimental results for the Boeing Model 160 tilt rotor aircraft. As may be seen the simple treatment gives acceptable predictions of wing forces and moments.

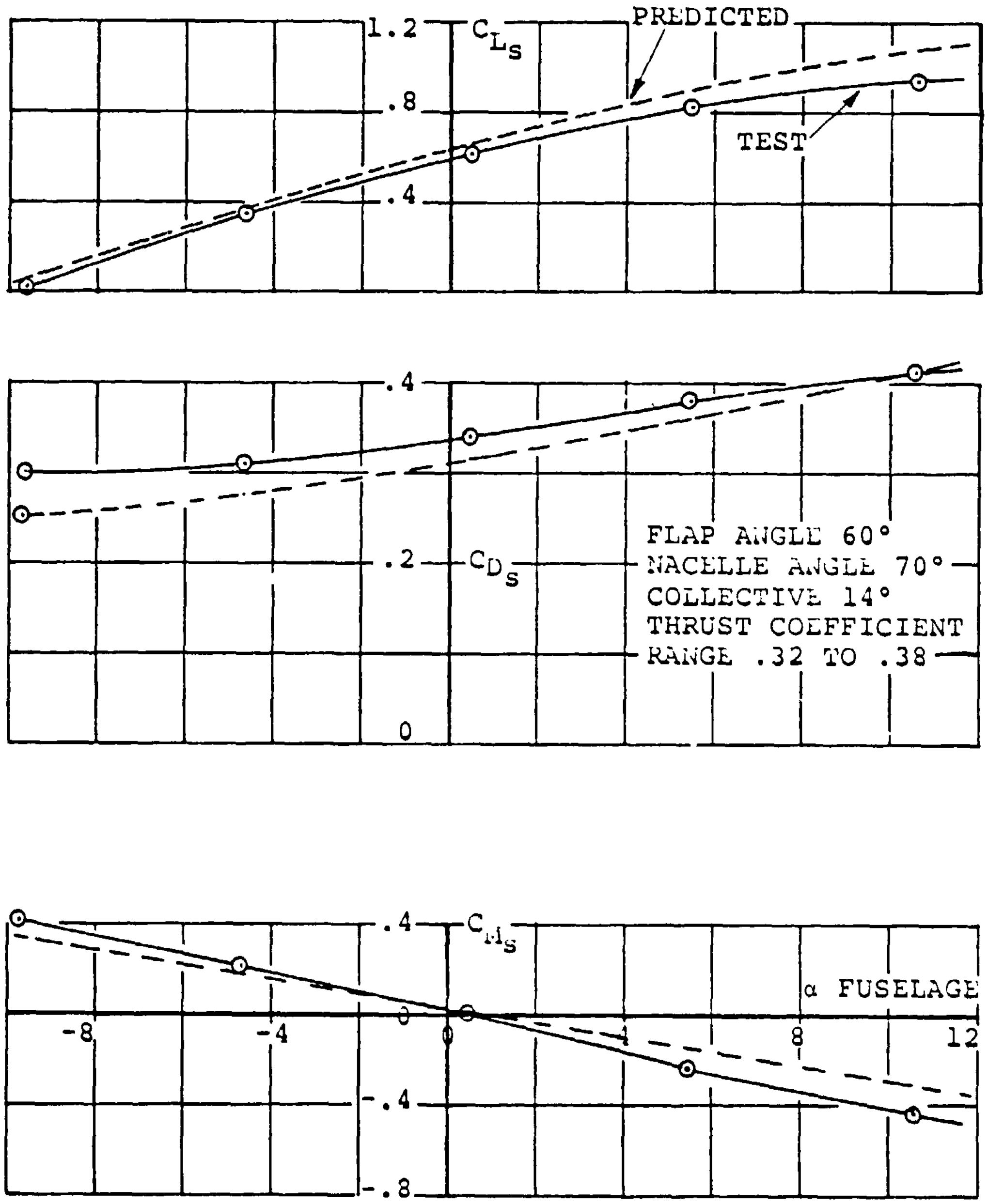


Figure 4.2 Correlation of Theory with Test for Predictions of Slipstream Forces and Moments

5.0 ROTOR REPRESENTATION

The mathematical representation of the rotor for the 1985 transport is based upon full scale test data obtained on the 26 foot rotor for the Boeing Model 222 tilt rotor. The test data was curve-fitted and scaled by solidity to yield equations suitable for representing the 1985 transport rotor. Where test data was not available, the rotor performance was calculated using Boeing rotor performance computer programs. The use of mathematical expressions for the rotor forces and moments results in maximum computational efficiency and minimum cycle times. This method is preferrable to table look-up procedures.

5.1 Sign Convention

The sign convention for rotor forces and moments is defined in Figure 5.1, which shows the rotors under combined pitch ($\alpha_{T.L.} = i_N + \alpha_F$) and sideslip β. The resultant rotor angle of attack is given by

$$\alpha_R = \cos^{-1} (\cos \alpha_{T.L.} \cos \beta)$$

and the rocor disc "sideslip" angle is

$$\zeta_H = \mathrm{Tan}^{-1} \left[\frac{\mathrm{Tan}\ \beta}{\mathrm{Sin}\ \alpha_{T.L.}} \right]$$

The resulting rotor forces and moments are defined with respect to the plane containing the resultant rotor angle of attack e.g. normal force lies in this plane while rotor side force is perpendicular to it.

5.2 Isolated Rotor Aerodynamics

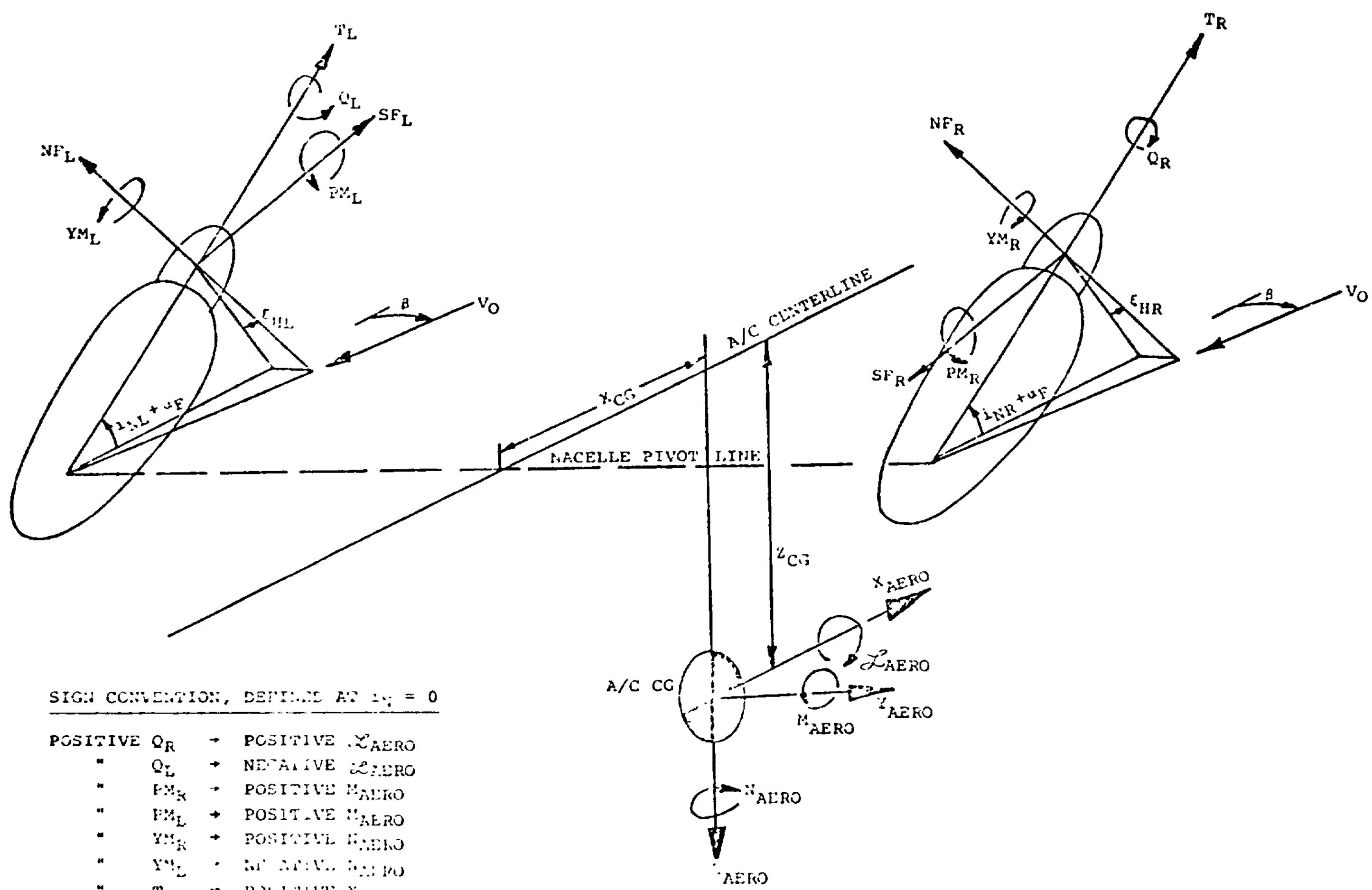
The equations used to represent the isolated rotor aerodynamics are presented below. The equations are used to compute the rotor wind-axes forces which are then resolved through the rotor sideslip angle into nacelle axes and hence transferred to aircraft body axes for use in the equations of motion.

5.2.1 Thrust Vs θ_{75}

The thrust produced by the rotor at any flight condition is obtained from the following equations

$$\phi = \theta_{75} - \tan^{-1}\left[\frac{\mu \cos \alpha}{0.75}\right] - 6.3015\mu + 5.5816\mu^2$$

$$- 8 \mu \sin \alpha + 1.115 \qquad (1)$$



SIGN CONVENTION, DEFINED AT $i_N = 0$

POSITIVE	Q_R	→	POSITIVE $\mathcal{L}_{AERO}$
"	Q_L	→	NEGATIVE $\mathcal{L}_{AERO}$
"	PM_R	→	POSITIVE M_{AERO}
"	PM_L	→	POSITIVE M_{AERO}
"	YM_R	→	POSITIVE N_{AERO}
"	YM_L	→	NEGATIVE N_{AERO}
"	$T_{L,R}$	→	POSITIVE X_{AERO} (CRUISE)
"	NF_L	→	NEGATIVE Z_{AERO}
"	NF_R	→	NEGATIVE Z_{AERO}
"	SF_R	→	NEGATIVE Y_{AERO}
"	SF_L	→	POSITIVE Y_{AERO}

FIGURE 5.1 ROTOR FORCE AND MOMENT SIGN CONVENTIONS

and C_T is given by

$$C_T = 0.000679\ \phi + 0.000015\ \phi^2 + 0.0022\ \mu\phi + 0.000211\mu^2\phi \tag{2}$$

5.2.2 Thrust Vs Power

Once thrust has been established the power coefficient is given by

$$\begin{aligned} C_P = {} & 0.00006 + 0.00057\ \mu + 0.000085\ \mu^2 + 1.12\ C_T^{3/2} \\ & - 0.024075\ C_T + \mu C_T\ (0.53 + 0.456\ \mu - 39.937\ \mu C_T \\ & + 31.79\ C_T) + [0.0115\mu - 0.03\mu^2 - C_T(3.4\mu - 8\mu^2)]\frac{(\alpha \mathrm{RAD})}{\pi} \\ & - 0.22064\mu\ (C_T + 0.001971)\ \sin\alpha \\ & + (0.3082\mu - 2.18\mu^2)\ C_T\ \sin\alpha \end{aligned} \tag{3}$$

5.2.3 Normal Force

Normal force is obtained as the sum of three terms

$$C_{NF} = F(\mu, \alpha, C_T) + \frac{\partial C_{NF}}{\partial A_1} A_1 + \frac{\partial C_{NF}}{\partial B_1} B_1 \tag{4}$$

where the cyclic pitch derivatives are functions of α, μ, and C_T.

In performing the analysis the cyclic derivatives were first defined as:

$$\begin{aligned} \frac{\partial C_{NF}}{\partial A_1} = {} & 0.00002175 + 0.0014483\mu^2 - 0.0000734\mu \\ & - 0.0006\mu\ \sin 2\alpha + 0.00425\ C_T \end{aligned} \tag{5}$$

and

$$\begin{aligned} \frac{\partial C_{NF}}{\partial B_1} = {} & 0.0000425 - 0.0010492\mu - 0.0017028\mu^2 \\ & + 0.0017892\mu\ \sin\alpha - 0.0245\ C_T \end{aligned} \tag{6}$$

The following expressions may be used to calculate normal force with zero cyclic pitch.

For $0 \leq \mu \leq 0.6$

$$C_{NF} = C_{NF_1} = 0.068\mu^3 \sin 2\alpha + [0.133695\mu C_T + 56.111\mu\, C_T^2 (1-\mu)]K \tag{7}$$

where $K = \sin \alpha$ for $\alpha > 20°$

and $K = \sin \alpha(10-0.45\alpha°)$ for $0 \leq \alpha \leq 20$

For $0.6 < \mu$

$$C_{NF} = (C_{NF_1})(1-0.8(\mu-0.6)) \tag{8}$$

5.2.4 Side Force

Side force is defined in a similar manner to normal force

$$C_{SF} = F(\mu, C_T, \alpha) + \frac{\partial C_{SF}}{\partial A_1} A_1 + \frac{\partial C_{SF}}{\partial B_1} B_1 \tag{9}$$

where the cyclic derivatives are given by:

$$\frac{\partial C_{SF}}{\partial A_1} = -0.0000328 + 0.0008119\mu + 0.0013178\mu^2 + 0.0189\, C_T - 0.001342\, \mu \sin \alpha \tag{10}$$

and

$$\frac{\partial C_{SF}}{\partial B_1} = 0.00001683 + 0.0011208\mu^2 - 0.0000568\mu + 0.00328\, C_T - 0.00052438\, \mu \sin 2\alpha \tag{11}$$

The side force at zero cyclic is given by the following equations:

$$C_{SF} = 0.00430\, \mu \sin \alpha - 0.0028827\, \mu\, (\alpha_{RAD})^2 + 0.012\, \mu \sin \alpha\, C_T\, (90-\psi°) + 2.19\mu^3 \sin \alpha\, C_T \tag{12}$$

where $\psi° = \tan^{-1}\left[\frac{\mu-\mu_i \cos \alpha}{\mu_i \sin \alpha}\right]$ (13)

and $\mu_i = \left[\left((\mu^4 + C_T^2)^{1/2} - \mu^2\right) /2\right]^{1/2}$ (14)

5.2.5 Hub Pitching Moment

Pitching moment is computed in the same manner as normal force and side force.

$$C_{PM} = F(\alpha, \mu, RPM, C_T) + \frac{\partial C_{PM}}{\partial A_1} A_1 + \frac{\partial C_{PM}}{\partial B_1} B_1 \qquad (15)$$

where the cyclic pitch derivatives are functions of α, μ, RPM and C_T.

$$\frac{\partial C_{PM}}{\partial A_1} = 0.0001620 + 0.00086652\mu - 0.00056151\mu^2 - 0.00000591\mu\,(RPM-298) + 0.0002826\mu \sin 2\alpha + 0.0015\,C_T \qquad (16)$$

and

$$\frac{\partial C_{PM}}{\partial B_1} = -0.000086093 + 0.0000564\mu + 0.0003385\mu^2 - 0.0019\,C_T - 0.00000551\mu\,(RPM-298) + 0.00048791\mu \sin\alpha \qquad (17)$$

$$C_{PM} = 0.009950\,\mu \sin\alpha - 0.010960\mu^2 \sin\alpha + 0.0028126\,\mu \sin 2\alpha - 0.0057743\,\mu \sin\alpha \left(\frac{RPM}{298}\right) + (1.802\,\mu \sin\alpha - 7.56\,(\mu \sin\alpha)^2)\,C_T \qquad (18)$$

5.2.6 Hub Yawing Moment

The yawing moment derivatives due to cyclic pitch are similar to the pitching moment derivatives and are given by

$$\frac{\partial C_{YM}}{\partial A_1} = -0.000086093 + 0.0000612\,\mu \quad 0.0003385\,\mu^2 - 0.0019\,C_T - 0.00000551\,\mu\,(RPM-298) + 0.0003\,\mu \sin\alpha \qquad (19)$$

and

$$\frac{\partial C_{YM}}{\partial B_1} = -0.0001620 - 0.00086652\,\mu$$
$$+0.00056151\,\mu^2 + 0.00000591\,\mu\,(RPM-298) \qquad (20)$$
$$-0.001\,C_T - 0.0003638\,\mu \sin 2\alpha$$

The yaw moment at zero cyclic pitch is given by the following equations

For $0 \leq \mu \leq 0.37$

$$C_{YM} = (0.018369\,\mu - 0.0007)\,\mu \sin\alpha - 1.2\,\mu^2 C_T \sin\alpha$$
$$+ \left[0.00631 - 0.002604\mu - 0.004877\left(\frac{RPM}{298} - 1\right)\right]\left(\frac{RPM}{298} - 1\right)\sin\alpha \qquad (21)$$

and for $\mu > 0.37$

$$C_{YM} = (0.01916 - 0.15321\,(\mu - 0.5435)^2)\sin\alpha$$
$$- 1.2\,\mu^2 C_T \sin\alpha \qquad (22)$$

5.2.7 Pitching Moment due to Pitch Rate

$$1000\,\frac{dC_{PM}}{dQ} = 1.5 + \mu \qquad 0 \leq \mu \leq .2$$

$$= 0.25 + 7.26\,\mu \qquad .2 < \mu \leq .39$$

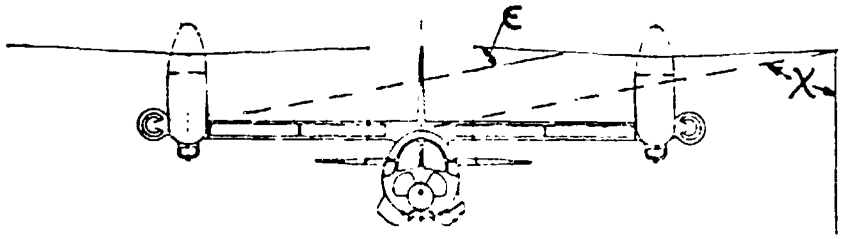
$$= 4.1681 - 2.79\,\mu \qquad \mu > .39$$

5.2.8 Yawing Moment due to Yaw Rate

$$\frac{dC_{YM}}{dR} = -\frac{dC_{PM}}{dQ}$$

5.3 Rotor/Rotor Interference

A procedure for calculating rotor-on-rotor interference effects is included in the mathematical model. Rotor-on-rotor interference arises during sideward flight at low airspeeds with the rotors up and, to a lesser extent, during slipped flight in the transition configurations. The basis for the method is as follows.



The above sketch depicts the tilt rotor aircraft flying sidewards at low speed. The wake of the upwind rotor interferes with the inflow to the downwind rotor producing a change in this rotor's forces and moments.

Reference 6 presents calculated values of the normal component of the induced velocity near a rotor having a triangular disc loading, for different wake skew angles, χ. This data is used to compute an interference angle at the downwind rotor. The interference angle is subtracted from the isolated rotor angle of attack and the resulting angle of attack is used in the calculation of the forces and moments. The rotor/rotor interference effect is washed out with nacelle angle and sideslip angle so that there is no interference at the high end of transition and in cruise. The equations used to calculate interference are presented in Appendix E under the rotor/rotor interference section.

5.4 Effect of Wing Upwash on Rotor Performance

The rotor operates in the upwash field associated with the lifting wing. Thus the rotor behaves as if it were operating at an increased angle of attack. The effective upwash angles were calculated using lifting line theory. In the mathematical model the upwash angles are input in the form of a table of upwash angles as a function of wing lift coefficient, and nacelle incidence angle.

6.0 CONTROL SYSTEM

6.1 Control Arrangement

Control of the 1985 tilt rotor aircraft is accomplished by utilization of longitudinal cyclic, differential longitudinal cyclic, collective and differential collective pitch, in conjunction with the airplane control surfaces. The airplane control surfaces consist of conventional elevator and rudder and a flaperon and spoiler arrangement. The primary controls in each axis for each regime of flight are shown in Table 6.1.

The rotor controls provide a major portion of the control capability from hover through the low transition speed range, but airplane surface controls are operative in all regimes of flight, including hover. The rotor controls are phased out during transition as nacelle incidence decreases, speed increases, and the airplane controls become more effective.

6.2 Longitudinal Control

Longitudinal control in hover is provided by longitudinal cyclic pitch. This is phased out through transition as the elevator becomes more effective. The elevator provides longitudinal control in the cruise mode.

6.3 Lateral/Directional Control

Roll control in hover is accomplished by differential collective (thrust) and yaw control by differential longitudinal cyclic (thrust vector tilt). Differential engine power is provided (via the governor) to ensure maintenance of roll control in the event of cross-shaft failure and also to minimize cross-shaft torque.

In transition, differential collective and differential cyclic per inch of control movement are scheduled as functions of nacelle incidence. Longitudinal and lateral cyclic, elevator angle and flap deflection are also scheduled with nacelle angle to provide a controls-fixed trimmed condition through transition.

In cruise, lateral control is provided by flaperon/spoilers and rudder. The flaps are full-span, single-slotted of 30 percent chord with a fixed hinge point 14.6 percent below the wing chord line. The flaps act as flaperons for roll control and deflect downward to a maximum of 20 degrees from the nominal flap setting. Maximum incremental lift from the flaps is attained at approximately 35 degrees deflection and the maximum rolling moment occurs at the same time, so the flaperon deflection for roll control is limited to a maximum total flap deflection of 35 degrees. If, for example, the flaps are symmetrically deflected 30 degrees, only 5 degrees additional deflection is utilized for roll control. Full-span spoilers

TABLE 6.1 FLIGHT CONTROLS

FLIGHT MODE	PRIMARY CONTROLS
Helicopter (Hover)	
Pitch	Longitudinal Cyclic
Roll	Differential Collective
Yaw	Differential Longitudinal Cyclic
Height Control	Collective/Engine Power
Transition	
Pitch	Longitudinal Cyclic and Elevator
Roll	Differential Collective, Differential Longitudinal Cyclic, Aileron and Spoiler
Yaw	Differential Longitudinal Cyclic, and Rudder
Airplane	
Pitch	Elevator
Roll	Aileron and Spoiler
Yaw	Rudder

of 12.7 percent chord are located forward of the flaps and hinged to the rear spar. The spoilers are "slot-lipped", i.e., they open up the slot forward of the flap with the flaps extended resulting in a large increase in roll control as compared to the control power with flaps closed. Maximum deflection of the spoilers for roll control is 45 degrees from the closed position.

Maximum spoiler rolling moment coefficient is also attained with flaps deflected approximately 35 degrees. Spoiler effectiveness with the flaps retracted is approximately one-third that attainable with the flaps extended.

The spoilers and flaps are also used in conjunction with download alleviation devices, referred to as umbrellas, mounted on the leading edge of the wing for download relief in the hover and low speed range. The umbrellas are 18.6 percent chord on the upper and lower wing surfaces. Maximum deflections of the surfaces for download alleviation are: flaps 70 degrees, spoilers 110 degrees from closed, and umbrellas aft-edge-of-the-upper surface up to 20 degrees from vertical and aft-edge-of-lower-surface down to 10 degrees from vertical. The umbrellas and spoilers retract at 50 knots automatically.

6.4 Thrust/Collective Control

In hover, forward motion of the thrust/collective lever mechanically commands both increased collective pitch and increased power. The governor provides a fine adjustment to the collective pitch to maintain rpm. Over-travel of the pilot's lever, beyond the normal max power position, provides a collective pitch landing flare capability. The over-travel is entered by going through a "gate", which shuts down the rotor governor and leaves the pilot's lever directly connected to collective pitch, as in a helicopter collective pitch lever.

The collective pitch is also scheduled through transition as a function of nacelle incidence, minimizing the adjustment needed from the governor.

In cruise the mechanical interconnection of the thrust/collective lever with collective pitch is phased out completely so that a pure power demand system with governed pitch, like a conventional fixed wing airplane, is provided. The control system block diagrams are shown in Appendix E.

6.5 Control Feel

Control force gradient variation with dynamic pressure prevents excessive sensitivity of control at high speed. The force gradients of the primary controls (longitudinal and lateral stick, and pedals) are varied linearly with dynamic pressure.

The rudder and elevator deflections vary linearly with pilot's rudder pedal and longitudinal stick travel. Aileron deflection is programmed linearly and spoiler deflection non-linearly with lateral stick deflection, to provide near-linear rolling moment effectiveness to near cruise speed. As mentioned earlier, spoiler deflection is limited at high speed by limiting the actuator capacity. The control breakout forces and force gradients are shown in Appendix E.

6.6 Stability Augmentation Systems

Stability augmentation systems are provided to enhance aircraft flying qualities. The system consists of longitudinal, lateral and directional SAS. The longitudinal stability augmentation system incorporates a pitch rate feedback and a longitudinal stick pickoff. In addition, a pitch attitude signal is incorporated to provide some degree of attitude stabilization without the autopilot. (An autopilot is not represented in this simulation.) These signals are shaped and put through an authority limit. The longitudinal SAS commands longitudinal cyclic pitch to provide the required damping in hover and transition. It is not required in the cruise mode and is phased out at 175 knots. The block diagram of the longitudinal SAS in given in Appendix E.

The lateral stability augmentation system is operative in all flight modes. It consists of roll rate feedback for increased damping in roll, a roll attitude feedback to provide roll attitude stability, and a lateral stick pickoff. In addition, a sideslip feedback is incorporated to compensate for dihedral effect. A lateral SAS authority limit is incorporated in the circuit. The output of the lateral stability augmentation system is input to the control system in terms of equivalent lateral stick, since the drive actuator is in series with, and commands the same control as, the pilots lateral stick control linkage. The lateral SAS never opposes the pilots' command. The block diagram of this system is shown in Appendix E.

A directional stability augmentation system is provided and operates in all flight regimes. The yaw channel consists of yaw rate feedback for increased directional damping in hover and low speed flight modes, yaw attitude feedback to provide yaw attitude stability, and a rudder pedal pickoff for quickening. Directional damping provided by the rotors is quite high in the higher transition and cruise speed ranges. No additional yaw rate damping is therefore needed in cruise. A feedback is provided to modify the effective yawing moment due to roll rate which exists in the basic unaugmented aircraft configuration in the cruise speed range. A directional SAS authority limit is incorporated. The SAS command is input to the control system in terms of equivalent inches of rudder pedal. The block diagram for the directional stability augmentation system is shown in Appendix E.

6.7 Thrust Management System

The thrust and power management system for a tilt rotor aircraft must be compatible with both the helicopter and airplane configurations. Thrust control for the hover task, rpm control, gust response (especially in the cruise flight regime), and effect on aircraft flying qualities must all be considered. Classically, helicopters have used collective pitch demand to control thrust and fuel governing to control rpm while fixed-wing aircraft have used fuel flow demand to control thrust and collective pitch governing to control rpm. Each system has its advantages. For a tilt rotor aircraft it is desirable from a practical viewpoint to have one type of governing for both the helicopter and fixed-wing flight regimes. Collective pitch governing was chosen for the 1985 tilt rotor for several reasons:

- It is more readily adapted to the hover flight regime than the fuel governor is to cruise
- It has better gust response characteristics
- It is fast acting and has high accuracy
- Thrust response to pilot control can be easily shaped with feed forward loops
- It has been demonstrated successfully in hover, transition and cruise in the CL-84 aircraft

With collective pitch governing there are two areas in the thrust management system to be considered: (1) design of the collective pitch governor; and (2) the feed forward loops for shaping pilot thrust control. The block diagram for this system is shown in Appendix E.

The governor was designed to meet the following objectives: (1) 0.3 percent steady state error in 2.5 to 3 seconds; (2) 2 percent rpm overshoot; and (3) satisfactory effect on aircraft flying qualities in the all-operational mode (i.e., all aircraft components operational and performing as designed). A single governor reference that uses the rpm signal from each rotor and averages them satisfies the design criteria. To achieve the required accuracy and transient response goals, integral as well as proportional feedback of rpm is necessary in both the hover and cruise regimes. Governor gain is scheduled with nacelle incidence to maintain a near optimum level of governing throughout the flight envelope. Gains are varied linearly as the rotor rpm is changed from hover to cruise. The second requirement of the governor system is shaping the rotor thrust output for a pilot throttle input. Considerations in determining the proper shaping include:

(1) throttle sensitivity

(2) time constant to reach 63% of steady-state thrust

(3) allowable thrust overshoot

Variable pilot's control sensitivity is employed to give the optimum sensitivity in the hover power range yet maintain full power control within a reasonable throttle throw (8 inches). Shaping of the pilot command with collective quickening is used to improve the thrust time constant and thrust response transient shaping so that the pilot may perform the precision hover task with a minimum of difficulty. In the cruise regime, shaping of the thrust output is unnecessary and is phased out during transition.

The thrust/collective pitch control system is designed in such a manner that, during hover, when the pilot moves his control, he commands both a change in engine fuel setting and, mechanically, a change in collective setting. The governor then operates with a time lag to trim the collective to the value required to maintain rpm. The mechanical collective change feature is washed out as a function of nacelle incidence so that when nacelle incidence is decreased to zero, the pilot commands only engine fuel.

7.0 ENGINE MODEL

This section describes the representation of engine performance and dynamics. The basic engine cycle performance data consists of tabulated values of four variables: power, fuel flow, gas generator shaft rpm, and power turbine shaft rpm. These parameters are a function of Mach number and turbine inlet temperature. All data are in referred, normalized format as shown in Table 7.1. Because of the normalized, referred format, all data are valid for any ambient conditions. The effects on engine performance of operating at non-optimum power turbine speed are included in the model. The referred format also facilitates the inclusion of engine thermodynamic and mechanical limits. Limitations on engine cycle operation may be input in any combination of the following: fuel flow, torque, gas generator speed, gas generator referred rpm or output shaft speed. The flow charts which describe this routine mathematically are shown in Appendix E, and the performance data in Appendix F.

A simplified dynamic model of the Lycoming LTC4V-1 engine was formulated for use in the tilt rotor mathematical model. The model basically consists of two first-order lags in series with variable time constants and gains. The output of the model is rate-limited to reflect actual engine performance. This simplified model gives satisfactory results for both large and small power transients. The block diagram for this system is shown as part of the thrust management system block diagram shown in Appendix E.

TABLE 7.1 ENGINE CYCLE DATA FORMAT

VARIABLE	SYMBOL	REFERRED, NORMALIZED FORM
Thrust	F_N	$F_N/\delta F_N^*$
Power	SHP	$SHP/\delta\sqrt{\theta}SHP^*$
Gas Generator rpm	N_I	$N_I/\sqrt{\theta}N_I^*$
Power Turbine rpm	N_{II}	$N_{II}/\sqrt{\theta}N_{II}^*$
Fuel Flow	W_f	$W_f/\delta\sqrt{\theta}F_N^*$
		$W_f/\delta\sqrt{\theta}SHP^*$
Turbine Inlet Temperature	T	T/θ

Where: * = Max. Power Setting, Static, Sea Level, Standard Day
θ = Ambient Temperature (°R) Divided by 518.69°R
δ = Ambient Pressure (psia) Divided by 14.696 psia

8.0 GROUND EFFECTS

The effects of operating near the ground on the rotors and airframe are included in this model. The presence of the ground on the airframe imposes a boundary condition which inhibits the downward flow of air normally associated with the lifting action of the wing and tail. The reduced downwash has three main effects;

- o A reduction in the downwash angle at the tail
- o An increase in the wing lift-curve slope
- o An increase in the tail lift-curve slope

These have been accounted for by the methods given in Reference 2, Appendix B-7. The data given in the reference for the change in wing and tail lift-curve slope has been used directly. The equation specified for the change in downwash angle at the tail due to ground proximity was modified for convenience. The equation as stated is:

$$\frac{(\Delta\varepsilon)_g}{\varepsilon} = \frac{b_1^2 + 4(h-H)^2}{b_1^2 + 4(h+H)^2}$$

where $(\Delta\varepsilon)_g$ = the change in tail downwash angle due to ground proximity

ε = the downwash remote from ground

h = the height of the tail root quarter-chord point above the ground

H = the height of the wing root quarter-chord point above the ground

b_1 = a function of wing lift and wing flap geometry

For this mathematical model, the b_1 in the above equation was taken to be equal to the wing span, b_w. This results in a small error in the change in horizontal tail downwash. It is, however, sufficiently accurate for this simulation.

Ground effects on the rotor are difficult to predict analytically, especially in forward flight. Wind tunnel test data for the Boeing Model 160 powered model, Reference 5, was plotted as a thrust ratio versus effective rotor height/diameter ratio, for two rotor advance ratios. This data, shown in Figure 8.1, was curve fitted and linearly interpolated for advance ratio. The resulting equation is as follows: - (for the right rotor.

The left rotor is identical except for subscripts)

$$\left(\frac{T_{IGE}}{T_{OGE}}\right)_{RR} = \left[\left(\frac{h}{D}\right)^2_{\substack{EFF\\RR}} (.1741 - .6216\mu_{RR}) + \left(\frac{h}{D}\right)_{\substack{EFF\\RR}} (1.4779\ \mu_{RR} - .4143) + 1.2479 - .8806\ \mu_{RR}\right]$$

where $$\left(\frac{h}{D}\right)_{\substack{EFF\\RR}} = \frac{h_{RR}}{2R[|\sin(\theta + i_{N_R})\cos\phi| + .0174]}$$

$$h_{RR} = -Z_{DOWN} + (L_S \cos i_{N_R} - X_{CG}) \sin\theta + [(L_S \sin i_{N_R} + Z_{CG}) \cos\phi - Y_N \sin\phi] \cos\theta$$

= Rotor hub height above the ground

L_S = Distance from the nacelle pivot to the rotor hub

X_{CG} = Longitudinal distance from the pivot to the CG

Z_{CG} = Vertical distance from the pivot to the CG

θ = Aircraft pitch attitude

ϕ = Aircraft roll attitude

i_{N_R} = Right rotor nacelle angle

Y_N = Wing semispan

The equation for the effective rotor height to diameter ratio $(h/D)_{EFF}$ was derived by dividing the rotor hub height by $[\sin(\theta+i_N) \cos\phi]$. This yields the rotor height along the shaft. For the cruise condition the hub height is infinite, $(h/D)_{EFF}$ is infinite and the augmentation ratio due to ground effect is unity. Some special conditions which must be observed when using these equations are noted in Figure 8.1.

$$\left(\frac{T_{IGE}}{T_{OGE}}\right) = \left[\left(\frac{h}{D}\right)^2_{EFF}(.1741-.6216\mu) + \left(\frac{h}{D}\right)_{EFF}(1.4779\mu-.4143) + 1.2479 - .8806\mu\right]$$

<u>NOTE:</u> <u>If</u> $\mu > 0.283$; $\left(\frac{T_{IGE}}{T_{OGE}}\right) = 1.0$

<u>or</u>

<u>If</u> $\left(\frac{h}{D}\right)_{EFF} > 1.3$; $\left(\frac{T_{IGE}}{T_{OGE}}\right) = 1.0$

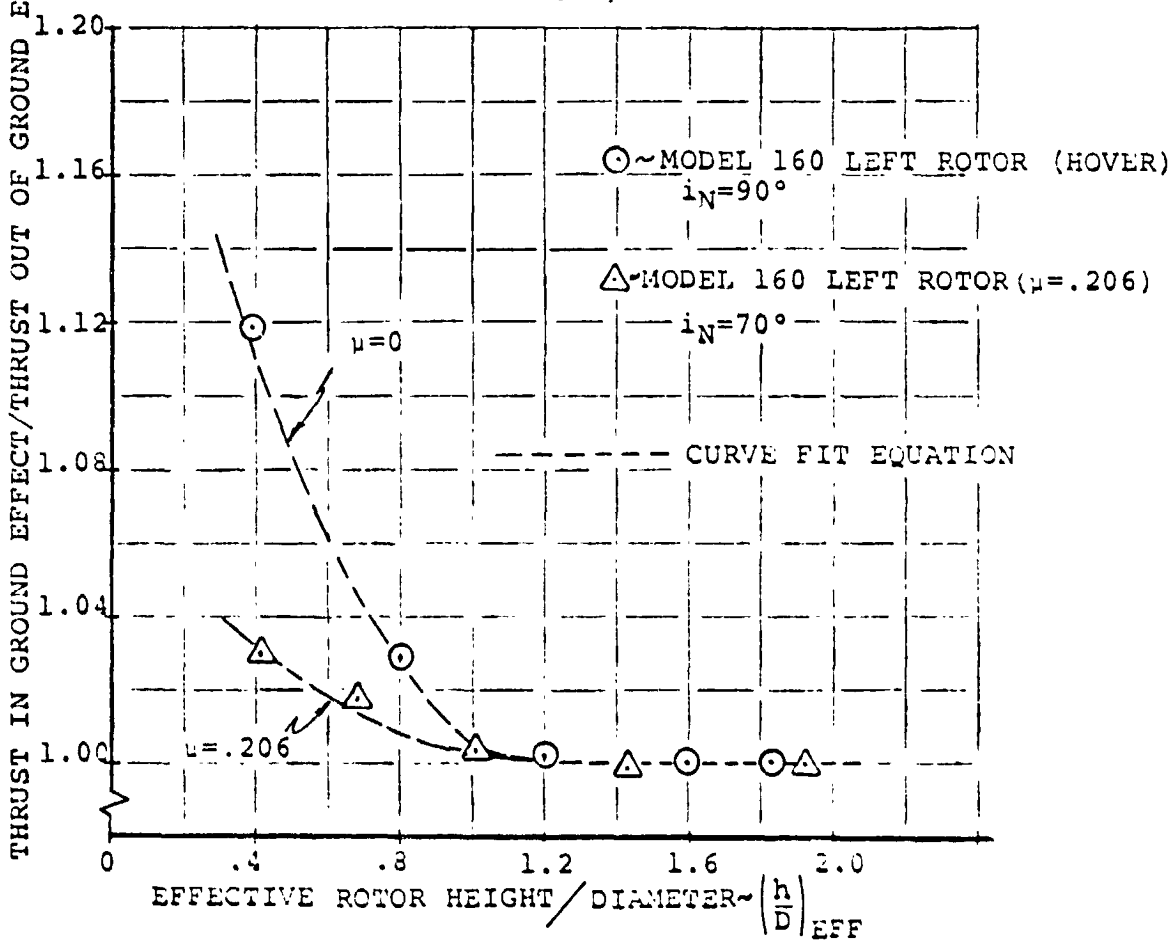


Figure 8.1 *Effect of Rotor Height on Thrust Augmentation Ratio*

9.0 AIRFRAME REPRESENTATION

An airframe representation/preprocessor calculation is included in the mathematical model that enables the user to input the location of major structural elements of the aircraft in terms of water line, butt line and station line location. All lengths, center of gravity distances and inertias used in the equations are then calculated. This feature enables the user to quickly change the location of major structural elements to assess their impact on vehicle response.

In the derivation of the basic equations of motion, the aircraft was divided into three principal mass elements. The fuselage mass element (m_f), the wing mass element (m_W), and the tilting nacelle mass element (m_N). The components of the three mass elements are shown below.

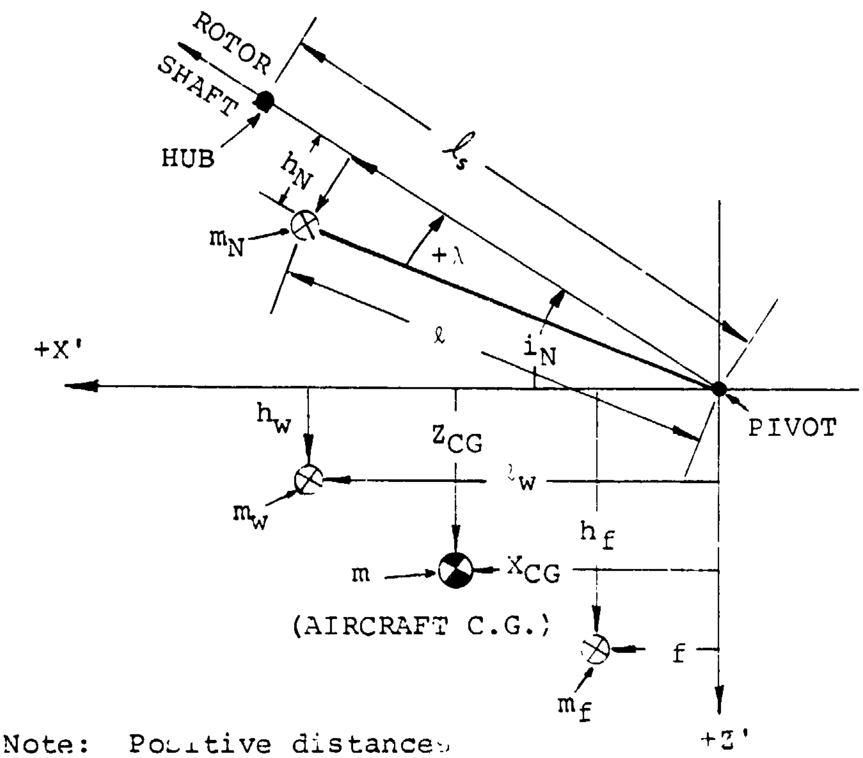
- fuselage mass element (m_f): Fuselage and contents; Horizontal tail and contents; Vertical tail and contents; Crew and trapped liquids; Cargo
- wing mass element (m_W): Wing and contents; Fuel carried in wing; Fixed nacelles and/or engines
- tilting nacelle mass element (m_N): Tilting nacelle (including rotors)

These three mass elements along with their respective distances from the nacelle pivot to the center of each mass element are used to compute the aircraft center of gravity distances with respect to the nacelle pivot. The equations for these center of gravity distances, including the effects of nacelle tilt are:

$$X_{CG} = \frac{m_f \ell_f + m_w \ell_w}{m} + \ell \left(\frac{m_N}{m}\right) \left[\cos\,(i_{NL}-\lambda) + \cos\,(i_{NR}-\lambda)\right]$$

$$Z_{CG} = \frac{m_f h_f + m_w h_w}{m} - \ell \left(\frac{m_N}{m}\right) \left[\sin\,(i_{NL}-\lambda) + \sin\,(i_{NR}-\lambda)\right]$$

The masses and distances used in these equations are defined in the sketch below.



Note: Positive distances are indicated by the direction of the arrows

The quantities required to compute m_f, ℓ_f, m_w, ℓ_w, m, ℓ, m_N, λ, h_f, h_w are available from an aircraft three-view drawing and a standard mass properties buildup. The quantities ℓ and λ (defined in the sketch) are easily obtainable from a drawing. The mass quantities (m, m_N, m_f, m_w) are computed from a mass properties buildup by adding up the components of each mass element as described in the previous paragraph. The lengths ℓ_f, ℓ_w, h_f and h_w are computed by summing the weight moments of the components of each mass element about the nacelle pivot. The equations for these operations have been derived and are presented in Appendix E under the preprocessor equations. The input data to these equations include the weight of each component, and its location in terms of water line, fuselage station line, and butt line.

When the center of gravity distance of each mass element has been determined, the component and total aircraft mass moments of inertia can be computed. The equations for the total aircraft mass moments of inertia are presented in Appendix C. The moments of inertia of each mass element are computed by application of the parallel axis theorem. The moments of inertia of each component about its own center of gravity must be known. The parallel axis theorem states:

$$I_{xx} = \sum_{i=1}^{N} [I_{xxo_i} + m_i \ (y_i^2 + z_i^2)]$$

$$I_{yy} = \sum_{i=1}^{N} [I_{yyo_i} + m_i \ (z_i^2 + x_i^2)]$$

$$I_{zz} = \sum_{i=1}^{N} [I_{zzo_i} + m_i \ (x_i^2 + y_i^2)]$$

$$I_{xz} = \sum_{i=1}^{N} [I_{xzo_i} + m_i (x_i z_i)]$$

where N represents the number of component masses.

These equations have been expanded to compute the moments of inertia of each mass element and are shown in Appendix E under the preprocessor section.

Other lengths required for the mathematical model are computed in this section. The input data for these computations are in terms of the water line, butt line, and fuselage station line locations of the elements in question.

10.0 AEROELASTIC REPRESENTATION

Two aeroelastic degrees of freedom are included in the tilt rotor mathematical model. These are first mode wing vertical bending and first mode wing torsion. The stability and control characteristics of flexible airplanes may be significantly influenced by distortions of the structure under transient loading conditions. When the separation in frequency between the elastic degrees of freedom and the rigid body motions is not large, then significant aerodynamic and inertial coupling can occur between the two. Many of the important effects of elastic distortion, however, can be accounted for simply by modifying the aerodynamic equations. The assumption is made that the changes in aerodynamic loading take place so slowly that the structure is at all times in static equilibrium. This is equivalent to assuming that the natural frequencies of vibration of the structure are much higher than the frequencies of the rigid body motions. Thus a change in load produces a proportional change in the shape of the airplane, which in turn influences the load. This is known as the method of "quasi-static" deflections where all the coupling occurs in the aerodynamic equations.

Since for the 1985 transport, the rigid-body short-period modes are separated from the elastic modes by a substantial margin, the method of "quasistatic" deflection is used to represent the wing bending and torsion modes, with the only coupling in the aerodynamic terms (through angle of attack). The wing twists and bends instantaneously when subjected to an applied load. The assumptions made in deriving the wing bending and torsion relationships are as follows:

- No coupling between bending and torsion modes
- Wings are cantilevered from the fuselage
- Elliptical loading assumed for the rigid untwisted wing
- Aerodynamic loads act at the wing quarter chord
- Wing elastic axis coincident with cross shaft
- Wing center of mass assumed to lie on the elastic axis
- First wing torsional mode assumed linear from tip to root

In the mathematical model, wing twist at the tip is calculated using the following equation:

$$K_{\theta_t}\,\theta_t = M_{ACT} - I_E\Omega_E R + q\,\frac{c_w^2\,b_w}{2}\,C_{m_o}$$

$$+\,q c_w^2\left(\frac{dC_{m_{c/4}}}{dC_\ell} + \frac{x_{WAC}}{c_w}\right)\left(\frac{C_{L_\alpha}\,b_w}{6\pi}\right)\left(4\theta_t + 3\pi\alpha_{RIGID}\right)$$

where:

K_{θ_t} = Wing torsional spring constant

θ_t = Wing twist angle in degrees

M_{ACT} = Nacelle actuator pitching moment

I_E = Engine inertia

Ω_E = Engine speed

R = Body yaw rate

q = Dynamic pressure

c_w = Wing reference chord

b_w = Wing reference span

C_{m_o} = Wing zero lift pitching moment coefficient

$\frac{dC_{m_{c/4}}}{dC_\ell}$ = Wing section pitching moment slope with section lift coefficient

C_{L_α} = Wing lift curve slope

α_{RIGID} = Wing angle of attack without twist

Assuming a linear mode shape from the wing tip to the root and a cantilevered wing (zero twist at root), the wing twist at the aerodynamic center location of the wing is obtained by linear interpolation. The wing twist represents the change in angle of attack of the wing tip and aerodynamic center and are used in the aerodynamic equations.

Wing vertical bending deflection is also treated on a quasi-static basis. The form of the equation used in the mathematical model for the wing tip deflection is as follows:

$$h_1 = K_{W_1} Z^N_{AERO} + K_{W_2} Z^W_{AERO} - K_{W_3} L^N_{AERO} - K_{W_4} \bar{a}_T - K_{W_5} \bar{a}_{WAC}$$

where: h_1 = Wing tip deflection

Z^W_{AERO} = Wing lift

Z^N_{AERO} = Total wing lift

L^N_{AERO} = Nacelle rolling moment

$\bar{a}_T$ = Vertical acceleration of the nacelle

$\bar{a}_{WAC}$ = Vertical acceleration of the wing aerodynamic center

$K_{W_1} \rightarrow K_{W_5}$ = Constants for 1985 transport wing

The form of the equation for the wing deflection at the aerodynamic center is written similarly:

$$h1_{WAC} = K_{W_6} Z^N_{AERO} + K_{W_7} Z^W_{AERO} - K_{W_8} L^N_{AERO} - K_{W_9} \bar{a}_T - K_{W_{10}} \bar{a}_{WAC}$$

The symbols represent the same quantities as the tip deflections except the quantities K_{W_6} to $K_{W_{10}}$ are different from K_1 to K_5.

These equations are derived in Appendix A. Since the wings are assumed cantilevered, these equations may be written for the left and right sides. The equations as used in the mathematical model are written in Appendix E.

The wing tip and aerodynamic center vertical bending velocities are computed by dividing the change in vertical bending deflection by the simulation time frame. The vertical bending deflections and velocities are then added to the velocity components at the wing tip and aerodynamic center. These velocity components are then used in the calculation of the aerodynamic angle of attack.

In addition to the aerodynamic coupling via angle of attack, as discussed above, the wing tip vertical forces and moments act as the driving functions to a set of second order equations that are forced at the wing vertical bending frequency. This results in giving a pilot a "seat of the pants" feel for the vibratory aspects of the wing vertical bending mode.

11.0 CONCLUSIONS

A full-force, 11 degree-of-freedom mathematical model of a 1985-era Tilt Rotor 100 passenger aircraft has been formulated. The mathematical model is complete and includes full-force nonlinear aerodynamics of all components, turbine engine performance and response, thrust management system and governor, pilot's controls and automatic stabilization systems. The model is suitable for implementation on the Ames Flight Simulator for Advanced Aircraft.

12.0 REFERENCES

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APPENDIX A - TREATMENT OF WING FLEXIBILITY

As described in Section 10 the large separation which exists between the natural frequencies of vibration of the wing structure and the aircraft rigid body motions, enables the elastic deformations of the wing structure to be calculated on a quasi-static basis.

In the simple treatment presented below, the bending and torsion modes are considered to be uncoupled. The wing is treated as a cantilever with a built-in root end. The wing is free to twist about the elastic axis which is assumed to coincide with the nacelle pivot line. The center of mass of each chordwise strip is also taken to lie on the pivot line. The unloaded wing has neither geometric nor aerodynamic twist.

WING TWIST

Spanwise twisting of the wing takes place under the action of the nacelle aerodynamic and inertial moments, the wing lift distribution, and the spanwise distribution of aerodynamic pitching moment. The nacelle aerodynamic moments consist of rotor hub loads, transferred to the pivot, together with the aerodynamic loads on the nacelle itself. Nacelle inertial moments include the gyroscopic effects of the rotor drive system.

With reference to Figure A.1, M_N is the moment supplied or absorbed by the nacelle tilt actuator. If K_θ is the wing stiffness as seen by the wing tip, then

$$M_N = K_\theta \ \theta_T \tag{A-1}$$

The total moment about the elastic axis due to wing aerodynamics, nacelle loads and engine gyroscopic torque is

$$T = \int_0^{b/2} m \ dy + M_N + M_{gyro} \tag{A-2}$$

The aerodynamic moment about the elastic axis at any station y is given by

$$M = M_{c/4} + \ell x \tag{A-3}$$

where ℓ is the section lift and x is the distance from the quarter chord to the elastic axis. In terms of the section aerodynamic coefficients,

$$m(y) = \frac{1}{2}\rho V^2 c^2 \ C_{m_{c/4}} + \frac{1}{2}\rho V^2 c^2 C_\ell \frac{x}{c} \tag{A-4}$$

The section lift coefficient, C_ℓ, is given by

$$c_\ell = k \frac{dC_\ell}{d\alpha} (\alpha - \alpha_o) \sqrt{1-\left(\frac{2y}{b}\right)^2}$$
$$= k\, a_o\, (\alpha_R - \varepsilon_p - \alpha_o + \theta_t(y)) \sqrt{1-\left(\frac{2y}{b}\right)^2} \qquad \text{(A-5)}$$

where α_R is the wing root section angle of attack

ε_p is the rotor induced downwash, assumed constant spanwise

α_o is the section zero-lift angle

θ_t is the structural twist at station y

The factor $k \sqrt{1-\left(\frac{2y}{b}\right)^2}$ is introduced so that, for the untwisted wing, the lift distribution is elliptical. The value of k is obtained from the rigid wing elliptical loading as

$$k = \frac{4}{\pi} \frac{C_{L_\alpha}}{a_o} \qquad \text{(A-6)}$$

Thus the equation for C_ℓ becomes, with $\alpha_{RIGID} = \alpha_R - \varepsilon_p - \alpha_o$,

$$C_\ell = \frac{4}{\pi} C_{L_\alpha} \left[\alpha_{RIGID} \sqrt{1-\left(\frac{2y}{b}\right)^2} + \theta_t \sqrt{1-\left(\frac{2y}{b}\right)^2}\right] \qquad \text{(A-7)}$$

In equation (A-4) we can write, for low angles of attack,

$$C_{m_{c/4}} = C_{m_o} + \frac{dC_{m_{c/4}}}{dC_\ell} C_\ell \qquad \text{(A-8)}$$

and therefore

$$m(y) = \frac{1}{2} \rho V^2 c^2 \left\{ C_{m_o} + \left(\frac{dC_{m_{c/4}}}{dC_\ell} + \frac{x}{c}\right) C_\ell \right\} \qquad \text{(A-9)}$$

The equation for the total wing twisting moment, equation (A-2), can now be written as,

$$T = M_{actuator} + M_{GYRO} + \frac{1}{4} \rho V^2 c^2 C_{m_o} b + \frac{1}{2} \rho V^2 c^2$$
$$\left(\frac{dC_{m_{c/4}}}{dC_\ell} + \frac{x}{c}\right) \int_0^{b/2} C_\ell\, dy \qquad \text{(A-10)}$$

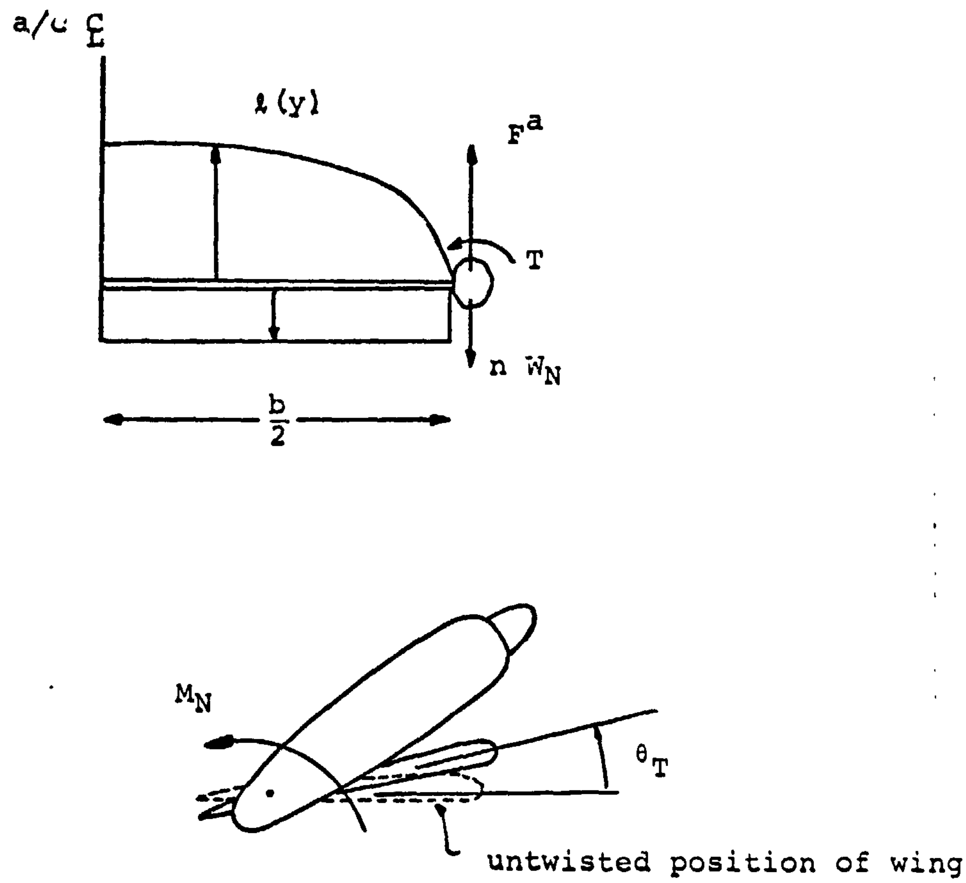


Figure A.1. Wing Geometry for Derivation of Flexibility

Using equation (A-7), assuming a linear structural twist from root to tip and performing the indicated integrations, the equation for total wing twisting moment becomes

$$T = K_\theta \theta_T = M_{actuator} + M_{gyro} + \frac{1}{4}\rho V^2 bc^2 C_{m_o} + \frac{1}{2}\rho V^2 c^2 \left(\frac{dC_{m_{c/4}}}{dC_\ell} + \frac{x}{c}\right)$$

$$\times \frac{C_{L_\alpha} b}{6\pi}\left(3\pi\alpha_{RIGID} + 4\theta_T\right) \qquad \text{(A-11)}$$

The equation for the actuator moment is given in the equations of motion, Section 5.0.

Rearranging, and writing $q = q_s\ (1-C_{T_s}) = \frac{1}{2}\ \rho V^2$

$$\theta_T = \frac{M_N + M_{gyro} + \frac{1}{2}q_s(1-C_{T_s})c_w^2\left[6\pi\alpha_{rigid}\left(\frac{dC_m}{dC_L} + \frac{x}{c}\right) + b_w C_{m_o}\right]}{K_\theta - \frac{2}{3\pi}\ q_s b_w c_w^2\ C_{L_\alpha}\ (1-C_{T_s})\ \left(\frac{dC_m}{dC_L} + \frac{x}{c}\right)} \qquad \text{(A-12)}$$

where C_{M_O} , the zero-lift wing section pitching moment coefficient, is a function of flap deflection:

$$C_{m_O} = C1 + C2\delta f + C3\delta f^2 \qquad \text{(A-13)}$$

Knowing the tip value of twist, the twist at any other spanwise station is obtained by assuming a linear variation of twist from zero at the root to the tip value.

WING VERTICAL BENDING

The spanwise bending moment at any spanwise station y, on the wing is the sum of the bending moments due to wing aerodynamic lift, wing weight, nacelle lift, nacelle weight and net torque on the nacelle. The expressions for each contribution to the bending moments are derived below.

- Bending moment due to wing loading.

  Assuming an elliptical distribution of lift the bending moment is given by

$$M^a\ (y_1) = \int_{y_1}^{b/2} \ell(y)\ (y-y_1)dy \qquad \text{(A-14)}$$

$$= \frac{\ell_o b^2}{4} \int_{y_1}^{b/2} \sqrt{1-\left(\frac{2y}{b}\right)^2}\ \left(\frac{2y}{b} - \frac{2y_1}{b}\right) d\ \left(\frac{2y}{b}\right)$$

where ℓ_o is the lift per unit length at the wing root. Introducing the spanwise variable $\theta=\cos^{-1}\left|\frac{2y}{b}\right|$ making the required substitutions and integrating, the bending moment at any point y is:

$$M(y) = \frac{\ell_o b^2}{4} \left[\frac{1}{2}(\sin\theta-\theta\cos\theta) - \frac{1}{6}\sin^3\theta\right] \quad \text{(A-15)}$$

o <u>Bending due to nacelle net vertical load</u>.

The net vertical force on nacelle is

$F=F^a - nW_N$

where F^a is the aerodynamic force and nW_N is the inertial load on the nacelle. The bending moment due to nacelle force is

$$M^N(y) = \frac{Fb}{2}(1-\cos\theta) \quad \text{(A-16)}$$

o <u>Bending due to wing weight</u>.

Assuming a uniform distribution of wing weight

$$M^W(y_1) = -n\int_{y_1}^{b/2} w(y)\,(y-y_1)\,dy$$

and $w(y) = 2W/b$ where W is the weight of one wing panel

$$\therefore\ M^W(y_1) = \frac{2nW}{b}\int_{y_1}^{b/2}(y-y_1)\,dy \quad \text{(A-17)}$$

$$\text{i.e.}\quad M^W(y) = -\frac{nWb}{2}\left(1-\cos\theta - \frac{1}{2}\sin^2\theta\right)$$

o <u>Bending due to nacelle torque (rolling moment)</u>

$$T(y) = \text{constant} = T \quad \text{(A-18)}$$

Total bending moment at station y is therefore

$$M(y) = M^a(y) + M^N(y) + M^W(y) + T \quad \text{(A-19)}$$

Assuming a linear variation of EI from root to tip given by

$$EI(y) = EI_o\left[1-a\left|\frac{2y}{b}\right|\right] = EI_o\,(1-a\cos\theta), \quad \text{(A-20)}$$

the curvature of the wing due to bending is

$$\frac{M(y)}{EI(y)} = \frac{d^2z}{dy^2} = \frac{\ell_o b^2}{8EI_o}\left[\frac{(\sin\theta-\theta\cos\theta)-1/3\sin^3\theta)}{1-a\cos\theta}\right] + \frac{F^a b}{2EI_o}\left[\frac{1-\cos\theta}{1-a\cos\theta}\right]$$

$$-\frac{nW_N b}{2EI_o}\left[\frac{1-\cos\theta}{1-a\cos\theta}\right] - \frac{nW_w b}{2EI_o}\left[\frac{1-\cos\theta-\frac{1}{2}\sin^2\theta}{1-a\cos\theta}\right]$$

$$+\frac{T}{EI_o}\left[\frac{1}{(1-a\cos\theta)}\right] \qquad \text{(A-21)}$$

Double integration of this equation yields the following expression for the bending deflection of the wing at any point y on the span:-

$$z(y) = \frac{Lb^3}{8\pi EI_o}\phi_1 + \frac{b^3F^a}{8EI_o}\phi_2 - \frac{nW_N b^3}{8EI_o}\phi_3$$

$$-\frac{nW_w b^3}{8EI_o}\phi_4 + \frac{Tb^3}{4EI_o}\phi_5 \qquad \text{(A-22)}$$

where $\phi_1 = \frac{b^2}{4}\int_o^y\left\{\int_o^y \frac{(\sin\theta-\theta\cos\theta)-\frac{1}{3}\sin^3\theta}{1-a\cos\theta}dy\right\}dy$

$$\phi_2 = \phi_3 = \frac{b^2}{4}\int_o^y\left\{\int_o^y \frac{1-\cos\theta}{1-a\cos\theta}dy\right\}dy$$

$$\phi_4 = \frac{b^2}{4}\int_o^y\left\{\int_o^y \frac{1-\cos\theta-\frac{1}{2}\sin^2\theta}{1-a\cos\theta}dy\right\}dy$$

$$\phi_5 = \frac{b^2}{4}\int_o^y\left\{\int_o^y \frac{dy}{1-a\cos\theta}\right\}dy$$

and where the wing lift (2 wing panels) $L = \frac{\pi}{4}\ell_o b$. The function ϕ_1 through ϕ_5 were obtained numerically and are presented in Figure A.2.

Since $L = -2\, Z^W_{AERO}$

$F^a = -\, Z^N_{AERO}$

$T = -\, L^N_{AERO}$

$$nW_w = \frac{1}{2} m_w \frac{Z_{AERO}}{m} = \frac{1}{2} m_w \bar{a}_{WAC}$$

$$nW_N = m_N \bar{a}_T$$

where m_w is the mass of two wing panels

m is the total aircraft mass

$\bar{a}_{WAC}$ is the acceleration of the wing aerodynamic center

$\bar{a}_T$ is the acceleration of the wing tip

and since the values of ϕ_1 through ϕ_5 are constant for any given station y on the wing we can write the final equation for wing bending in the form

$$h_1 = K_{W_1} Z^N_{AERO} + K_{W_2} Z^W_{AERO} - K_{W_3} L^N_{AERO} - K_{W_4} \bar{a}_T - K_{W_5} \bar{a}_{WAC}$$

where $h_1 = -z$

$$K_{W_1} = \frac{b^3\phi_2}{8EI_o}$$

$$K_{W_2} = \frac{b^3\phi_1}{4\pi EI_o}$$

$$K_{W_3} = \frac{b^3\phi_5}{4EI_o}$$

$$K_{W_4} = \frac{m_N b^3\phi_2}{8EI_o}$$

$$K_{W_5} = \frac{m_W b^3\phi_4}{8EI_o}$$

This is the form given in the computer representation. The bending deflection at the aerodynamic center and at the wing tip are obtained using the values of $\phi_1 \rightarrow \phi_5$ appropriate to these stations.

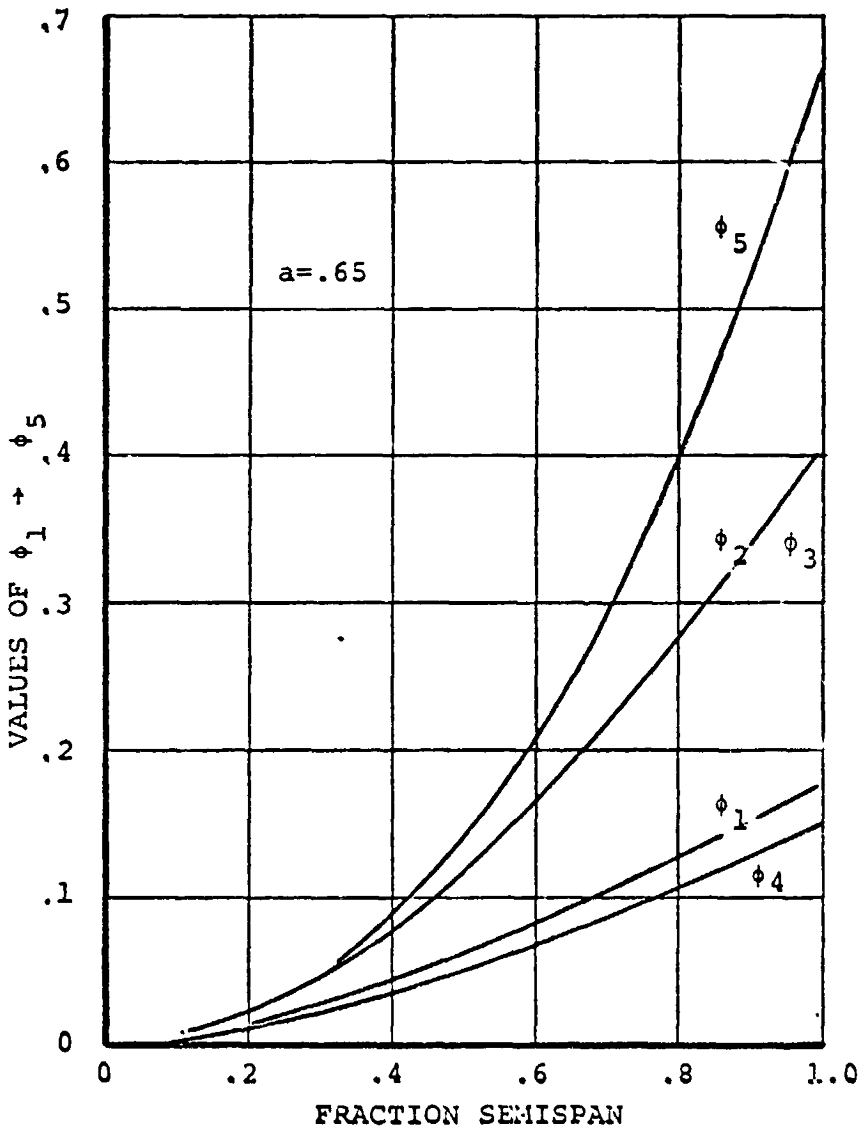


Figure A.2. Wing Bending Functions

APPENDIX B - DERIVATION OF LANDING GEAR EQUATIONS

Presented below are the equations for landing gear forces and moments arising from ground contact. The derivation accounts for brake and friction forces together with a simplified representation of the oleo dynamics. Nose wheel steering is not included.

With reference to Figure B-1 the distance from the center of gravity to the bottom of the right main wheel following a positive pitch rotation is

$$h_\theta = X \sin\theta - Z \cos\theta - r \qquad \text{(B-1)}$$

where X and Z are the coordinates of the hub of the wheel relative to the C.G. and r is the tire radius. If the aircraft is now rolled right, through the angle ϕ, the bottom of the right gear moves through a distance.

$$h_\phi = \left[Y \sin\phi + (Z+r)\ (\cos\phi - 1) \right] \cos\theta \qquad \text{(B-2)}$$

The height of the bottom of the wheel above the ground is therefore

$$h = H_{CG} + h_\theta - h_\phi \qquad \text{(B-3)}$$

and the oleo deflection during ground contact is given by

$$h_T = \frac{H_{CG} + h_\theta - h_\phi}{\cos\phi\ \cos\theta} \qquad \text{(B-4)}$$

By differentiation of equation B-4 and making small angle assumptions regarding the aircraft pitch and roll angles during touchdown, the rate of change of oleo strut deflection is obtained as

$$\dot{h}_T = \frac{\dot{H}_{CG}}{\cos\phi\ \cos\theta} + XQ - YP \qquad \text{(B-5)}$$

Assuming that the oleo response is that of a second order system, the equation of motion for the landing gear is

$$F_G = K_{ST}\ h_T + D_{ST}\ \dot{h}_T \qquad \text{(B-6)}$$

where K_{ST} and D_{ST} are the equivalent spring rates and damping for the oleo, and F_G is the force on the landing gear strut.

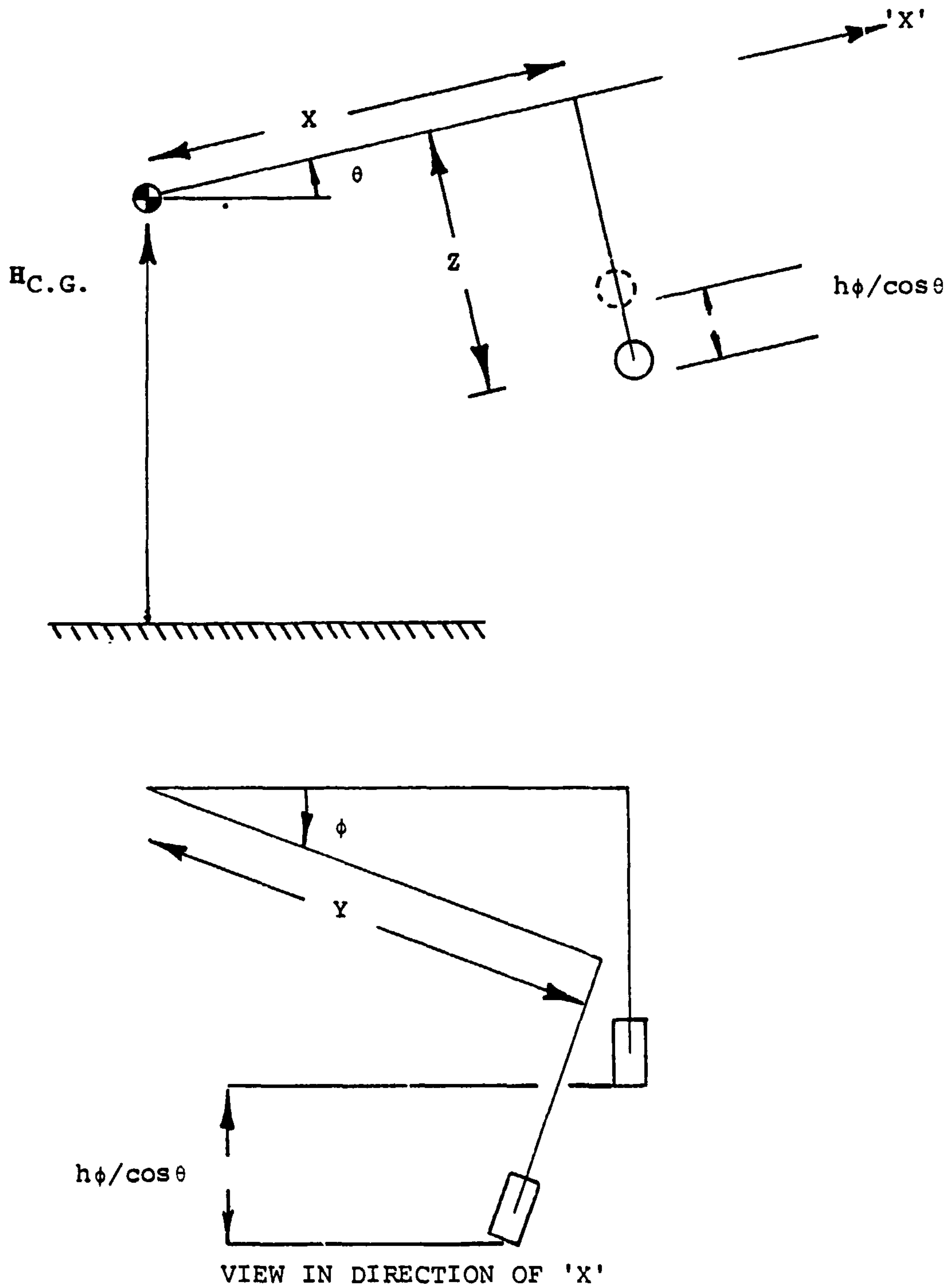


FIGURE B.1 GEOMETRY OF LANDING GEAR

Tire Friction and Side Force

The friction force acting on each tire during ground contact is resolved into a force F_μ along the line of intersection of the plane of the wheel and the ground plane, positive forward, and a side force F_μ at right angles to F_μ lying in the ground plane and positive to starboard. The friction force F_s is assumed to be proportional to oleo force and the amount of braking exerted by the pilot. The side force is proportional to the oleo force.

The components of tire friction are:

$$F_\mu = (\mu_0 + \mu_1 B_G) F_{GZ} \frac{u}{|u|} \tag{B-7}$$

$$F_s = \mu_s F_{GZ} \frac{v}{|v|} \tag{B-8}$$

where μ_0, μ_1 and μ_s are the coefficients for rolling friction, brake friction and sliding friction. B_G is expressed as a percentage of full brake pedal deflection. The signs of the forward and sidewards velocity are introduced to properly orient the tire forces.

The force and moment contributions of each landing gear to the aircraft total forces and moments are, assuming small angles;

$$\Delta X_n = F_{\mu_n} - F_{GZ_n} \theta \tag{B-9}$$

$$\Delta Y_n = F_{s_n} + F_{GZ_n} \phi \tag{B-10}$$

$$\Delta Z_n = F_{\mu_n} \theta - F_{s_n} \phi + F_{GZ_n} \tag{B-11}$$

$$\Delta M_n = -\Delta Z_n X_n + \Delta X_n (Z_n + r_n + h_{T_n}) \tag{B-12}$$

$$\Delta L_n = \Delta Z_n Y_n - \Delta Y_n (Z_n + r_n + h_T) \tag{B-13}$$

$$\Delta N_n = -\Delta X_n Y_n + X_n \Delta Y_n \tag{B-14}$$

where n=1, 2 and 3 denote the left main gear, right main gear and nose gear, respectively.

The total contribution of the landing gear forces to the forces and moments at the center of gravity of the aircraft are:

$$\Delta X_{LG} = \sum_{n=1}^{3} \Delta X_n$$

$$\Delta Y_{LG} = \sum_{n=1}^{3} \Delta Y_n$$

$$\Delta Z_{LG} = \sum_{n=1}^{3} \Delta Z_n$$

$$\Delta L_{LG} = \sum_{n=1}^{3} \Delta L_n$$

$$\Delta M_{LG} \quad \sum_{n=1}^{3} \Delta M_n$$

$$\Delta N_{LG} \quad \sum_{n=1}^{3} \Delta N_n$$

APPENDIX C - VELOCITY AND ACCELERATION TRANSFORMATIONS AND CENTER OF GRAVITY/INERTIA EQUATIONS

C.1 Velocity Transformations

The calculation of aerodynamic forces on wings, fuselage, nacelles, and tail surfaces requires that the angle of attack and relative wind velocity at these surfaces be known. These velocities are obtained most conveniently in terms of the velocity of the pivot reference point.

With reference to Figure C.1, the velocity of a general point in the aircraft relative to the airplane center of gravity is

$$\underline{V} = \frac{\delta \underline{r}}{\delta t} + \underline{\Omega} \times \underline{r} \tag{C-1}$$

where $\underline{r}$ is the radius vector from the c.g. to the point and $\underline{\Omega}$ is the angular velocity of the aircraft. Thus, expanding equation C-1, the velocity of the pivot relative to the c.g. is

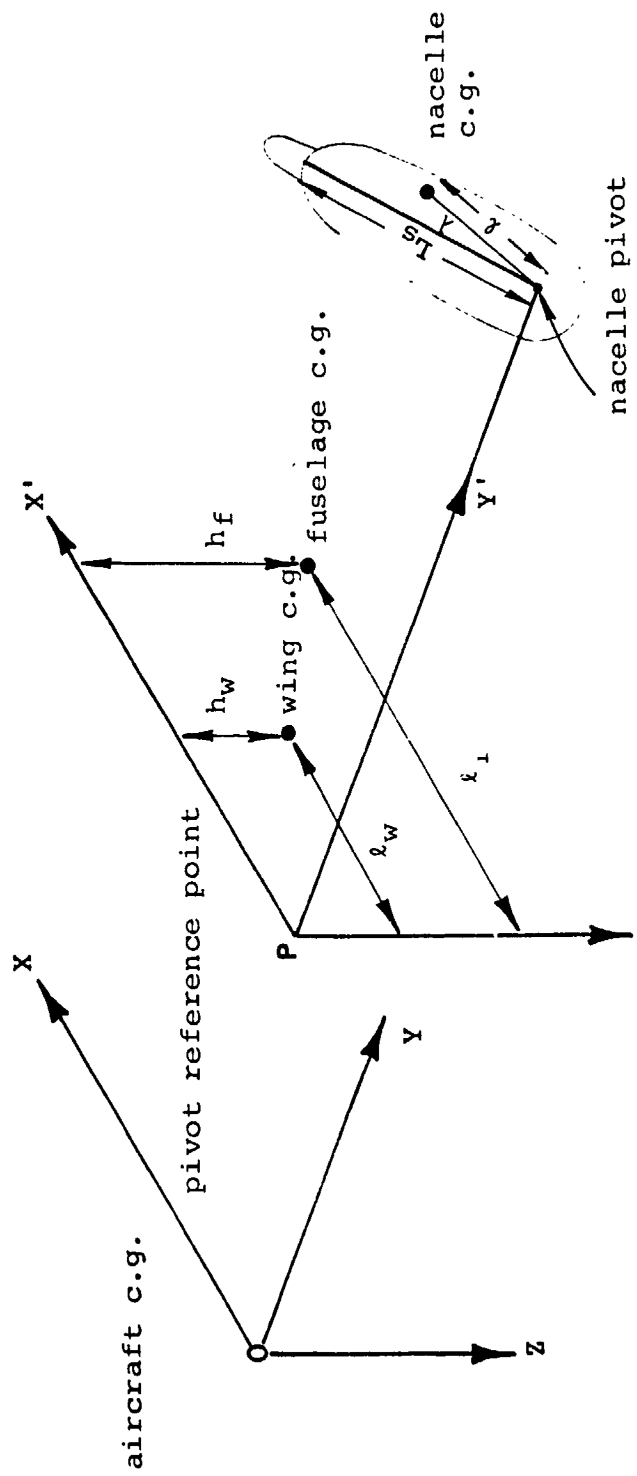
$$\begin{aligned} u_p' &= \dot{X}_p + QZ_p - Y_pR \\ v_p' &= \dot{Y}_p - PZ_p - X_pR \\ w_p' &= \dot{Z}_p + PY_p - QX_p \end{aligned} \tag{C-2}$$

where X_p, Y_p and Z_p are the distances of the pivot from the c.g., measured positively forward, to the right and downwards, respectively. If we measure all distances from the pivot location then $X_p = -X_{CG}$, $Y_p = -Y_{CG} = 0$, $Z_p = -Z_{CG}$ and the velocity of the pivot relative to inertial space can be written,

$$\begin{aligned} u_p &= U + u_p' = U - \dot{X}_{CG} - QZ_{CG} \\ v_p &= V + v_p' = V + PZ_{CG} - X_{CG}R \\ w_p &= W + w_p' = W + QX_{CG} - \dot{Z}_{CG} \end{aligned} \tag{C-3}$$

where U, V, and W are the components of the velocity of the airplane center of gravity.

The velocity of a point in the aircraft relative to the pivot is



O and P are coplanar

FIGURE C.1 REFERENCE AXES SYSTEMS

$$u = \dot{X} + QZ - YR$$
$$v = \dot{Y} + RX - PZ \qquad \text{(C-4)}$$
$$w = \dot{Z} + PY - QX$$

where X, Y, and Z are measured from the pivot to the point. By adding equations (C-3) and (C-4) the velocities of the following components are obtained relative to inertial space. The indicated distances are measured relative to the pivot.

Velocity of Horizontal Tail Aerodynamic Center

$$u_{HT} = u_P + Z_{HT}Q$$
$$v_{HT} = v_P + X_{HT}R - Z_{HT}P \qquad \text{(C-5)}$$
$$w_{HT} = w_P - X_{HT}Q$$

Velocity of Vertical Tail Aerodynamic Center

$$u_{VT} = u_P + Z_{VT}Q$$
$$v_{VT} = u_P + X_{VT}R - Z_{VT}P \qquad \text{(C-6)}$$
$$w_{VT} = w_P + X_{VT}Q$$

Velocity of Left Wing Aerodynamic Center - Body Axes

$$u'_{LW} = u_P + Q\,(Z_{WAC} + h_{1_{LWAC}}) + Y_{WAC}R$$
$$v'_{LW} = u_P + X_{WAC}R - P(Z_{WAC} + h_{1_{LWAC}}) \qquad \text{(C-7)}$$
$$w'_{LW} = w_P - Y_{WAC}P - X_{WAC}Q + \dot{h}_{1_{LWAC}}$$

where $h_{1_{LWAC}}$ is the elastic deflection of the left wing aerodynamic center. The equations for the right wing are obtained by substituting

$$Y_{RWAC} = -Y_{LWAC}$$

and $h_{1_{RWAC}} = h_{1_{LWAC}}$

Velocity of Left Wing Aerodynamic Center-Chord Axes

In order to compute wing angle-of-attack the velocity components are required relative to the wing chord line. If the wing chord makes an angle i_w with the body centerline then

$$
\begin{aligned}
u_{LW} &= u'_{LW} \cos i_w - w'_{LW} \sin i_w \\
v_{LW} &= v'_{LW} \qquad\qquad\qquad\qquad \text{(C-8)} \\
w_{LW} &= w'_{LW} \cos i_w + w'_{LW} \sin i_w
\end{aligned}
$$

The equations for the right wing are obtained by changing the subscript.

Velocity of Left Rotor Hub - Body Axes

$$
\begin{aligned}
u'_{RL} &= u_P + RY_N - L_s (\dot{i}_{NL} + Q) \sin i_{NL} + Qh_{1_L} \\
v'_{RL} &= v_P + L_s (R \cos i_{NL} + P \sin i_{NL}) - Ph_{1_L} \qquad \text{(C-9)} \\
w'_{RL} &= w_P - PY_N - L_s (\dot{i}_{NL} + Q) \cos i_{NL} + \dot{h}_{1_L}
\end{aligned}
$$

where L_s is the distance from the rotor pivot point to the rotor hub and h_{1_L} is the deflection of the wing tip. The equations for the right hub are obtained by changing subscripts and substituting $Y_N = -Y_N$.

Velocity of Left Rotor Hub - Shaft Axes

Since the rotor aerodynamic forces and moments are functions of the shaft angle of attack and sideslip, the velocity components are required relative to shaft axes.

$$
\begin{aligned}
u_{RL} &= u'_{RL} \cos i_{NL} - w'_{RL} \sin i_{NL} \\
v_{RL} &= v'_{RL} \qquad\qquad\qquad\qquad \text{(C-10)} \\
w_{RL} &= w'_{RL} \sin i_{NL} + w'_{RL} \cos i_{NL}
\end{aligned}
$$

The corresponding equations for the right hub are obtained by changing the subscript.

C.2 Center of Gravity and Inertia Equations

Equations are required that express the overall aircraft center of gravity position and inertias in terms of the centers of gravity and inertias of the individual mass components. In order to do this a fixed reference point is chosen in the aircraft defined by the intersection of the line joining the nacelle pivots and the vertical plane of symmetry of the aircraft, see Figure C.1. A set of axes $Px'y'z'$ is taken at this pivot reference point, parallel to the axes OXYZ at the aircraft center of gravity. If the location of the aircraft center of gravity with respect to the pivot reference axes is $(X'_{CG}, Y'_{CG}, Z'_{CG})$ and if (ℓ_f, h_f) and (ℓ_w, h_w) are the x and z coordinates of the fuselage and wing masses measured from the pivot, then the following relationships are obtained between the centers of mass of the components and the aircraft center of gravity.

Fuselage CG Relative to Aircraft CG

$$X_f = \ell_f - X'_{CG}$$

$$X_f = h_f - Z_{CG} \qquad \text{(C-11)}$$

Wing CG Relative to Aircraft CG

$$X_w = \ell_w - X'_{CG}$$

$$Z_w = h_w - Z'_{CG} \qquad \text{(C-12)}$$

Nacelle CG Relative to Aircraft CG

$$X_{NR} = \ell \cos (i_{NR} - \lambda) - X'_{CG}$$

$$X_{NL} = \ell \cos (i_{NL} - \lambda) - X'_{CG} \qquad \text{(C-13)}$$

$$Z_{NR} = \ell \sin (i_{NR} - \lambda) - Z'_{CG}$$

$$Z_{NL} = \ell \sin (i_{NL} - \lambda) - Z'_{CG}$$

where ℓ is the distance from the nacelle pivot point to the nacelle c.g., and λ is the angular depression of the nacelle center of mass below the nacelle pivot, when the nacelle is in the down position, see Figure C.1.

Aircraft Center of Gravity Position

By taking moments about the pivot, the aircraft center of gravity is given by

$$x'_{CG} = \frac{m_f \ell_f + m_w \ell_w}{m} + \ell \left(\frac{m_N}{m}\right) \left[\cos(i_{NL}-\lambda) + \cos(i_{NR}-\lambda)\right]$$

$$z'_{CG} = \frac{m_f h_f + m_w h_w}{m} - \ell \left(\frac{m_N}{m}\right) \left[\sin(i_{NL}-\lambda) + \sin(i_{NR}-\lambda)\right] \qquad \text{(C-14)}$$

The equations of motion (Section 3) require the first and second time derivatives of the center of gravity position. They are as follows:

Center of Gravity Velocity Relative to Pivot Point

$$\dot{x}'_{CG} = -\ell \left(\frac{m_N}{m}\right)\left[\dot{i}_{NR}\sin(i_{NR}-\lambda) + \dot{i}_{NL}\sin(i_{NL}-\lambda)\right]$$

$$\dot{z}'_{CG} = -\ell \left(\frac{m_N}{m}\right)\left[\dot{i}_{NR}\cos(i_{NR}-\lambda) + \dot{i}_{NL}\cos(i_{NL}-\lambda)\right] \qquad \text{(C-15)}$$

Center of Gravity Acceleration Relative to Pivot Point

$$\ddot{x}'_{CG} = -\ell\left(\frac{m_N}{m}\right) \left[\ddot{i}_{NR}\sin(i_{NR}-\lambda) + \ddot{i}_{NL}\sin(i_{NL}-\lambda) + \dot{i}_{NL}^2 \cos (i_{NL}-\lambda) + \dot{i}_{NR}^2 \cos (i_{NR}-\lambda)\right]$$

$$\ddot{z}'_{CG} = -\ell \left(\frac{m_N}{m}\right) \left[\ddot{i}_{NR}\cos(i_{NR}-\lambda) + \ddot{i}_{NL}\cos(i_{NL}-\lambda) - \dot{i}_{NL}^2\sin (i_{NL}-\lambda) - \dot{i}_{NR}^2\sin (i_{NR}-\lambda)\right] \qquad \text{(C-16)}$$

Pilot Station Velocities - Body Axes

The velocities at the pilot's station are required in order to drive the visual display. From Equations (C-3) and (C-4) the components of velocity of the pilot's station in body axes are:

$$u_{PA} = u_P + QZ_{PA} - RY_{PA}$$

$$v_{PA} = v_P + R\ell_{PA} - PZ_{PA}$$

$$w_{PA} = w_P + PY_{PA} - Q\ell_{PA}$$

C-3 Pilot Station Acceleration - Body Axes

The pilot station acceleration is also required to drive the visual display. These accelerations are derived here.

The velocity at the pilot's station is

$$\underline{V}_{PA} = \underline{V}_{CG} + \underline{\Omega} \times \underline{r}_{PA} + \frac{\delta \underline{r}_{PA}}{\delta t}$$

where $\underline{r}_{PA}$ is the vector from the aircraft CG to the pilot's station and $\frac{\delta \underline{r}_{PA}}{\delta t}$ is the rate of change of the pilot's station with respect to the aircraft CG.

The pilot's station acceleration is

$$\underline{a}_{PA} = \frac{d\underline{V}_{PA}}{dt} = \frac{d\underline{V}_{CG}}{dt} + \frac{d}{dt}(\underline{\Omega} \times \underline{r}_{PA}) + \frac{d}{dt}\left(\frac{\delta \underline{r}_{PA}}{\delta t}\right)$$

$$= \underline{a}_{CG} + \frac{\delta}{\delta t}(\underline{\Omega} \times \underline{r}_{PA}) + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{PA}) + \frac{\delta^2 \underline{r}_{PA}}{\delta t^2} + \underline{\Omega} \times \frac{\delta \underline{r}_{PA}}{\delta t}$$

$$= \underline{a}_{CG} + \frac{\delta \underline{\Omega}}{\delta t} \times \underline{r}_{PA} + 2\underline{\Omega} \times \frac{\delta \underline{r}_{PA}}{\delta t} + \underline{\Omega}\,(\underline{r}_{PA} \cdot \underline{\Omega}) - \Omega^2 \underline{r}_{PA} + \frac{\delta^2 \underline{r}_{PA}}{\delta t^2}$$

with $\underline{\Omega} = P\hat{\underline{i}} + Q\hat{\underline{j}} + R\hat{\underline{k}}$

$$\frac{\delta \Omega}{\delta t} = \dot{P}\hat{\underline{i}} + \dot{Q}\hat{\underline{j}} + \dot{R}\hat{\underline{k}}$$

$$\underline{r}_{PA} = (X_{PA} - X_{CG})\,\hat{\underline{i}} + (Y_{PA} - Y_{CG})\,\hat{\underline{j}} + (Z_{PA} - Z_{CG})\,\hat{\underline{k}}$$

$$\frac{\delta \underline{r}_{PA}}{\delta t} = (\dot{X}_{PA} - \dot{X}_{CG})\,\hat{\underline{i}} + (\dot{Y}_{PA} - \dot{Y}_{CG})\,\hat{\underline{j}} + (\dot{Z}_{PA} - \dot{Z}_{CG})\,\hat{\underline{k}}$$

and noting that Y_{CG} and the time derivatives of X_{PA}, Y_{PA}, Z_{PA} are always zero, the above equation yields the pilot's station accelerations as:

$$a_{x_{PA}} = \frac{X_{AERO}}{m} + (\dot{Q} + PR)(Z_{PA} - Z_{CG}) + (Q^2 + R^2)(X_{CG} - \ell_{PA})$$

$$+ Y_{PA}(PQ - \dot{R}) - 2Q\dot{Z}_{CG} - \ddot{X}_{CG}$$

$$a_{y_{PA}} = \frac{Y_{AERO}}{m} + (\dot{P} - QR)(Z_{CG} - Z_{PA}) + (\dot{R} + PQ)(\ell_{PA} - X_{CG})$$

$$- Y_{PA}(R^2 + P^2) + 2(P\dot{Z}_{CG} - R\dot{X}_{CG})$$

$$a_{z_{PA}} = \frac{Z_{AERO}}{m} + (\dot{Q} - PR)(X_{CG} - \ell_{PA}) + (P^2 + Q^2)(Z_{CG} - Z_{PA})$$

$$+ Y_{PA}(\dot{P} + QR) + 2Q\dot{X}_{CG} - \ddot{Z}_{CG}$$

where $a_{x_{CG}} = \frac{Z_{AERO}}{m}$ etc.

and $X_{PA} = \ell_{PA}$, the distance from the pivot to the pilot's station

C.4 Aircraft Inertias

The aircraft roll inertia about the aircraft center of gravity is, from the parallel axis theorem,

$$I_{xx} = I^f_{xx} + I^W_{xx} + I^{NL}_{xx} + I^{NR}_{xx} + m_f Z_f^2 + m_w Z_w^2 + 2m_N Y_N^2 + m_N Z_{NL}^2 + m_N Z_{NR}^2 \quad \text{(C-1)}$$

where I^f_{xx}, etc., are the inertias of the various components about their individual centers of gravity.

In the case of the nacelles the inertias I_{xx}^{NL}, I_{xx}^{NR} are dependent on the nacelle tilt angle, i_N. These inertias are related to the inertics of the nacelle with respect to a set of nacelle-fixed axes O"xyz placed as shown in Figure 3.1. The relationships are

$$
\begin{aligned}
I_{xx}^N &= I_{xx_o}^N + (I_{zz_o}^N - I_{xx_o}^N)\sin^2 i_N - I_{xz_o}\sin 2i_N \\
I_{yy}^N &= I_{yy_o}^N \\
I_{zz}^N &= I_{zz_o}^N + (I_{xx_o}^N - I_{zz_o}^N)\sin^2 i_N + I_{xz_o}\sin 2i_N \\
I_{xz}^N &= I_{xz_o}^N \cos 2i_N + \frac{1}{2}(I_{xx_o} - I_{zz_o})\sin 2i_N
\end{aligned}
\tag{C-18}
$$

Using equations (C-18) together with (C-13), (C-11), and (C-12), in equation (C-17), the roll inertia becomes

$$
\begin{aligned}
I_{xx} = {} & I_{xx}^f + I_{xx}^w + 2I_{xx_o}^N + (I_{zz_o}^N - I_{xx_o}^N)(\sin^2 i_{NL} + \sin^2 i_{NR}) \\
& - I_{xz_o}^N (\sin 2i_{NL} + \sin 2i_{NR}) + 2\, m_N Y_N^2 + m_f h_f Z_f \\
& + m_w h_w z_w - m_f Z_f Z'_{CG} - m_w Z_w Z'_{CG} \\
& - m_N Z_{NL} Z'_{CG} - m_N Z_{NR} Z'_{CG} \\
& - \ell m_N \left[Z_{NR}\sin(i_{NR} - \lambda) + Z_{NL}\sin(i_{NL} - \lambda)\right] \\
= {} & I_{xx}^f + I_{xx}^w + 2I_{xx_o}^N + (I_{zz_o}^N - I_{xx_o}^N)(\sin^2 i_{NL} + \sin^2 i_{NR}) \\
& - I_{xz_o}^N (\sin 2i_{NL} + \sin 2i_{NR}) + 2\, m_N Y_N^2 + m_f h_f Z_f \\
& + m_w h_w Z_w - \ell m_N \left[Z_{NR}\sin(i_{NR} - \lambda) + Z_{NL}\sin(i_{NL} - \lambda)\right]
\end{aligned}
$$

since the terms containing Z'_{CG} sum to zero.

Similarly

$$I_{xz} = I^{f}_{xz} + I^{w}_{xz} + I^{N}_{xz} \; (\cos 2i_{NL} + \cos 2i_{NR})$$

$$+ \frac{1}{2} \, (I^{N}_{xx_o} - I^{N}_{zz_o}) \; (\sin 2i_{NL} + \sin 2i_{NR}) + m_f \ell_f Z_f$$

$$+ m_w Z_w \ell_w + \ell m_N \left[Z_{NR} \cos \, (i_{NR} - \lambda) + Z_{NL} \; \cos \, (i_{NL} - \lambda) \right]$$

$$(I_{zz} - I_{yy}) = I^{f}_{zz} - I^{f}_{yy} + I^{w}_{zz} - I^{w}_{yy} + 2(I^{N}_{zz_o} - I^{N}_{yy_o})$$

$$+ \; (I^{N}_{xx_o} - I^{N}_{zz_o})(\sin^2 i_{NL} + \sin^2 i_{NR}) + I^{N}_{xz_o}(\sin 2i_{NL}$$

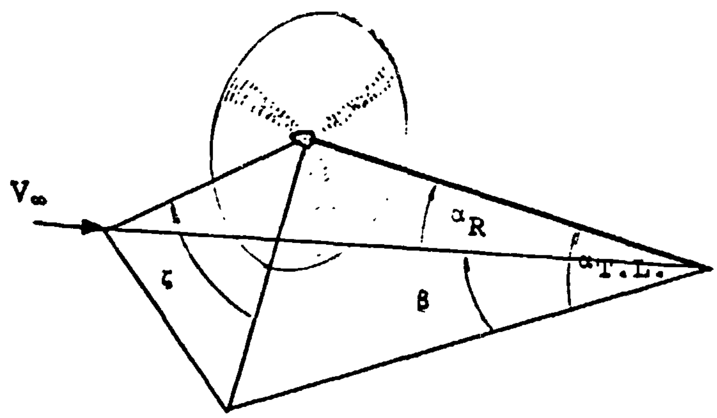
$$+ \sin 2i_{NR}) \;\; -(m_f h_f Z_f + m_w h_w Z_w) + m_N \ell \; \left[Z_{NL} \sin \, (i_{NL} - \lambda) \right.$$

$$\left. + \; Z_{NR} \sin \, (i_{NR} - \lambda) \right] \;\; + 2 m_N Y_N^2$$

Similar expressions are obtained for I_{yy} and I_{zz} and these are presented in Appendix E.

APPENDIX D - CALCULATION OF SLIPSTREAM-IMMERSED WING AREAS

The wing areas washed by the rotor slipstreams are required in the calculation of wing lift and drag. These immersed areas depend on rotor shaft inclination, wing angle of attack and side-slip, and rotor thrust. The equations presented in Appendix E for the immersed areas Si_L and Si_R were obtained as follows.



The above sketch shows a rotor under conditions of combined angle of attack ($\alpha_{T.L.}$) and sideslip (β). The resultant angle of attack of the shaft is given by

$$\alpha_R = \cos^{-1}(\cos \alpha_{T.L.} \cos \beta) \tag{D-1}$$

If the rotor shaft is inclined to the fuselage centerline at angle i_N and the fuselage is at angle of attack α_f then

$$\alpha_{T.L.} = \alpha_f + i_N \tag{D-2}$$

The rotor "sideslip" angle, ζ, is defined by

$$\zeta = \mathrm{Tan}^{-1} \frac{\mathrm{Tan}\ \beta}{\mathrm{Sin}\ \alpha_{T.L.}} \tag{D-3}$$

and is the angle shown in the sketch.

Figure D.1 presents four views of the geometry of rotor slip-stream/wing planform interaction.

Figure D.1[a] is a view of the plane taken through the rotor shaft parallel to the aircraft vertical plane of symmetry. The line PT is the wing chord, the distances PC and h_p are the horizontal and vertical coordinates of the pivot measured from the wing leading edge, and ℓ is the spinner-to-pivot shaft length.

Figure D.1[b] is a view taken normal to the rotor disc plane. In this view, the traces of the slipstream on planes taken through the wing leading and trailing edges parallel to the disc plane appear as circles. This assumes that the slipstream is a sheared circular cylinder.

Figure D.1[c] is a section taken in the plane containing the rotor shaft and the freestream velocity vector V_∞. The angle ε is the deflection of the slipstream relative to the freestream direction. Planes are taken through the wing leading and trailing edges parallel to the rotor disc. These intersect the rotor shaftline at the points O and T, and intersect the slipstream centerline at the points O' and O". These points enable the slipstream traces shown in (b) to be constructed.

Figure D.1[d] is a view taken perpendicular to the wing surface showing the areas washed by the slipstream. For convenience, this view combines the immersed areas of both left and right wings. In general, the imprint of the slipstream on the wing will be bounded in the chordwise direction by curves lines; however, the approximation is made that these lines are straight.

The immersed area of the right wing panel is (assuming that the tip is immersed),

$$S_{i_R} = \frac{1}{2}(PM + TN)c$$

$$= \frac{1}{2}(PR + RM + TS + SN)c \qquad (D\text{-}4)$$

From Figure D.1[b] $\quad PR = OO' \sin \zeta \qquad$ (D-5)

From Figure D.1[c] $\quad OO' = (\ell - OD) \operatorname{Tan} (\alpha_R - \varepsilon) \qquad$ (D-6)

From Figure D.1[a] $\quad OD = PC \cos (i_N - i_W) - h_p \sin(i_N - i_W) \qquad$ (D-7)

From Figure D.1[b] $\quad RM = R'M' = \sqrt{\frac{D_S^2}{4} - O'R'^2} \qquad$ (D-8)

From Figure D.1[b] $\quad O'R' = OO' \cos \zeta + OP \qquad$ (D-9)

From Figure D.1[a] $\quad OP = PC \sin (i_N - i_W) + h_p \cos (i_N - i_W) \qquad$ (D-10)

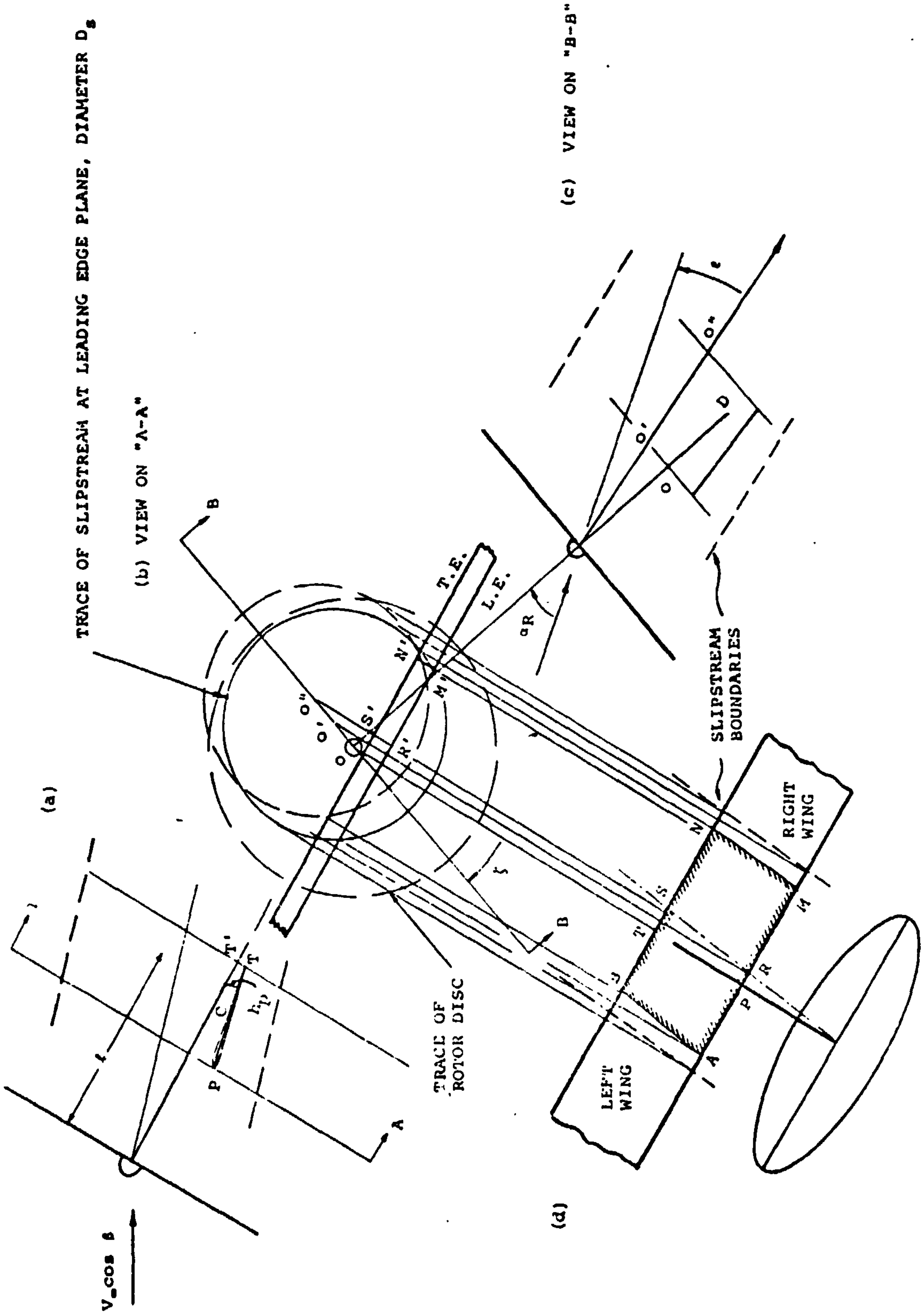


Figure D.1. Geometry of Rotor Slipstream/Wing Planform Interaction

These equations define the leading edge intersection PM. If RM is zero or negative, the slipstream does not intersect the leading edge and the wing is considered to be unaffected by the slipstream.

For the trailing edge intersection, TN:

$$TS = OO'' \sin \zeta \qquad \text{(D-11)}$$

$$OO'' = (\ell + c \cos (i_N - i_W) - OD) \operatorname{Tan} (\alpha_R - \varepsilon) \qquad \text{(D-12)}$$

$$SN = S'N' = \sqrt{\frac{D_s^2}{4} - O''S'^2} \qquad \text{(D-13)}$$

$$O''S' = OO'' \cos \zeta + TT' \qquad \text{(D-14)}$$

$$TT' = OP - c \sin (i_N - i_W) \qquad \text{(D-15)}$$

If we write

$$\xi_1 = PR, \; \xi_2 = RM, \; \xi_3 = TS, \text{ and } \xi_4 = SN$$

then, using the above equations,

$$\xi_1 = [\ell - PC \cos (i_N - i_W) + h_p \sin(i_N - i_W)] \operatorname{Tan}(\alpha_R - \varepsilon) \sin \xi \qquad \text{(D-16)}$$

and

$$\xi_2 = \sqrt{\frac{D_s^2}{4} - \{[\ell - PC \cos(i_N - i_W) + h_p \sin(i_N - i_W)] \operatorname{Tan}(\alpha_R - \varepsilon) \cos \zeta + PC \sin(i_N - i_W) + h_p \cos(i_N - i_W)\}^2} \qquad \text{(D-17)}$$

The corresponding equations for ξ_3 and ξ_4 are obtained by replacing PC in (D-16) and (D-17) and (PC-c)

Thus the immersed area of the right wing panel is given by

$$S_{i_R} = \frac{1}{2} c \, (\xi_1 + \xi_2 + \xi_3 + \xi_4) \qquad \text{(D-18)}$$

From the symmetry of Figure D.1[d], SN=BS and RM=AR. The total immersed area of both wing panels is

$$S_{i_T} = \frac{1}{2} c \, (AM + BN) = \frac{1}{2} c \, (2\xi_2 + 2\xi_4) = c(\xi_2 + \xi_4) \qquad \text{(D-19)}$$

and therefore the immersed area of the left wing is obtained from

$$S_{i_L} = S_{i_T} - S_{i_R} \qquad \text{(D-20)}$$

The above equations correspond to those presented in Appendix E for calculating immersed wing area.

APPENDIX E

The equations, data, and control diagrams required for FSAA simulation of the Boeing Vertol 1985 Tilt Rotor Transport are presented in the following pages. The simulation block diagram is shown on page E-6. Each element of this diagram is numbered. The reference table on page E-2 lists the block diagram element number, the function of the element, and the starting number of the pages containing the equations for the element.

Data for the simulation is provided in Appendix F.

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(Cont'd)

TABLE OF CONTENTS
(Cont'd)

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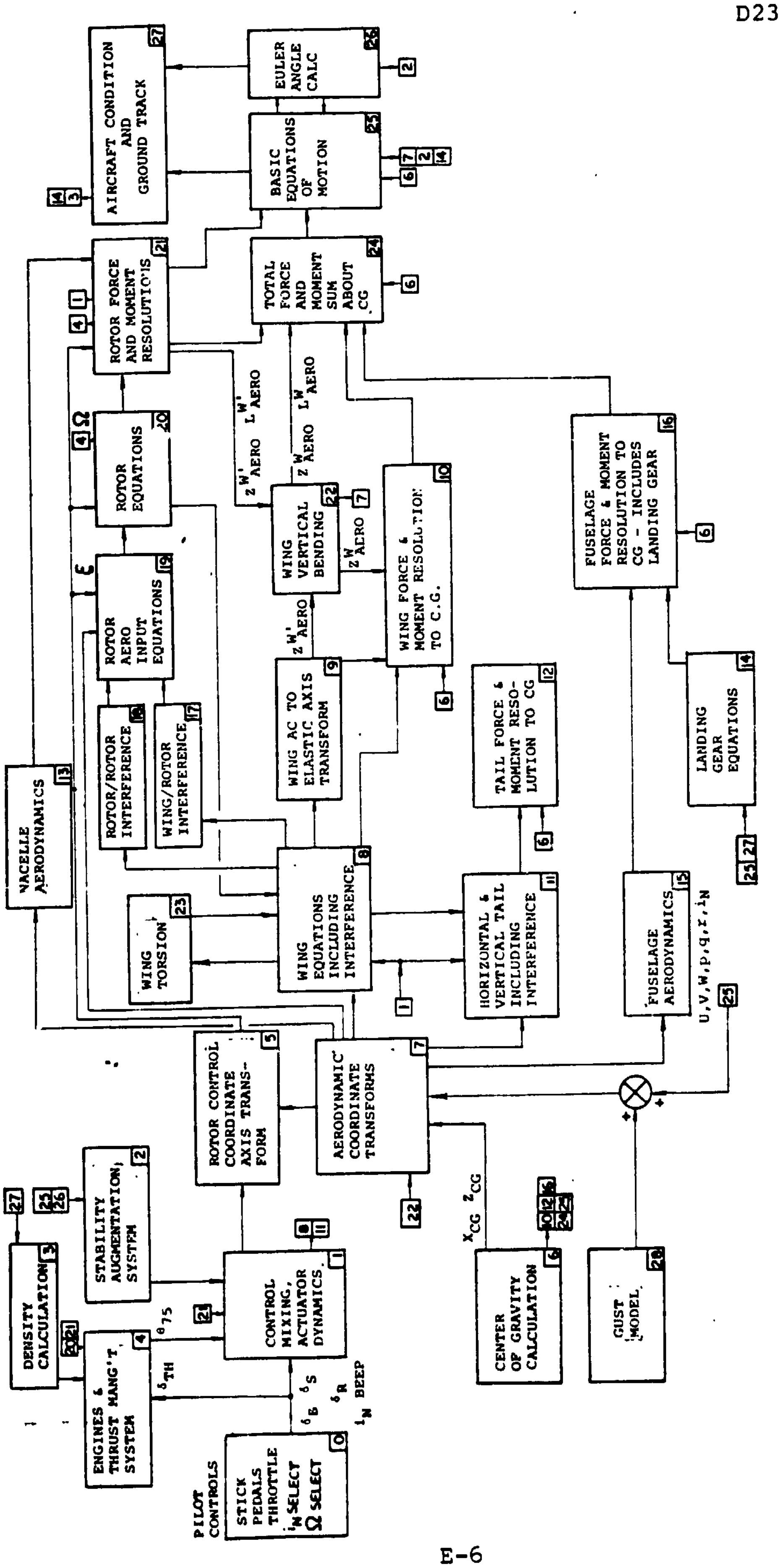
PILOT CONTROLS
STICK
PEDALS
THROTTLE
i_N SELECT
Ω SELECT
0
ENGINES & THRUST MANG'T. SYSTEM
4
DENSITY CALCULATION
3
STABILITY AUGMENTATION SYSTEM
2
CONTROL MIXING, ACTUATOR DYNAMICS
1
ROTOR CONTROL COORDINATE AXIS TRANSFORM
5
AERODYNAMIC COORDINATE TRANSFORMS
7
CENTER OF GRAVITY CALCULATION
6
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28
NACELLE AERODYNAMICS
13
ROTOR/ROTOR INTERFERENCE
18
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17
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ROTOR AERO INPUT EQUATIONS
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ROTOR FORCE AND MOMENT RESOLUTIONS
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WING FORCE & MOMENT RESOLUTION TO C.G.
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12
LANDING GEAR EQUATIONS
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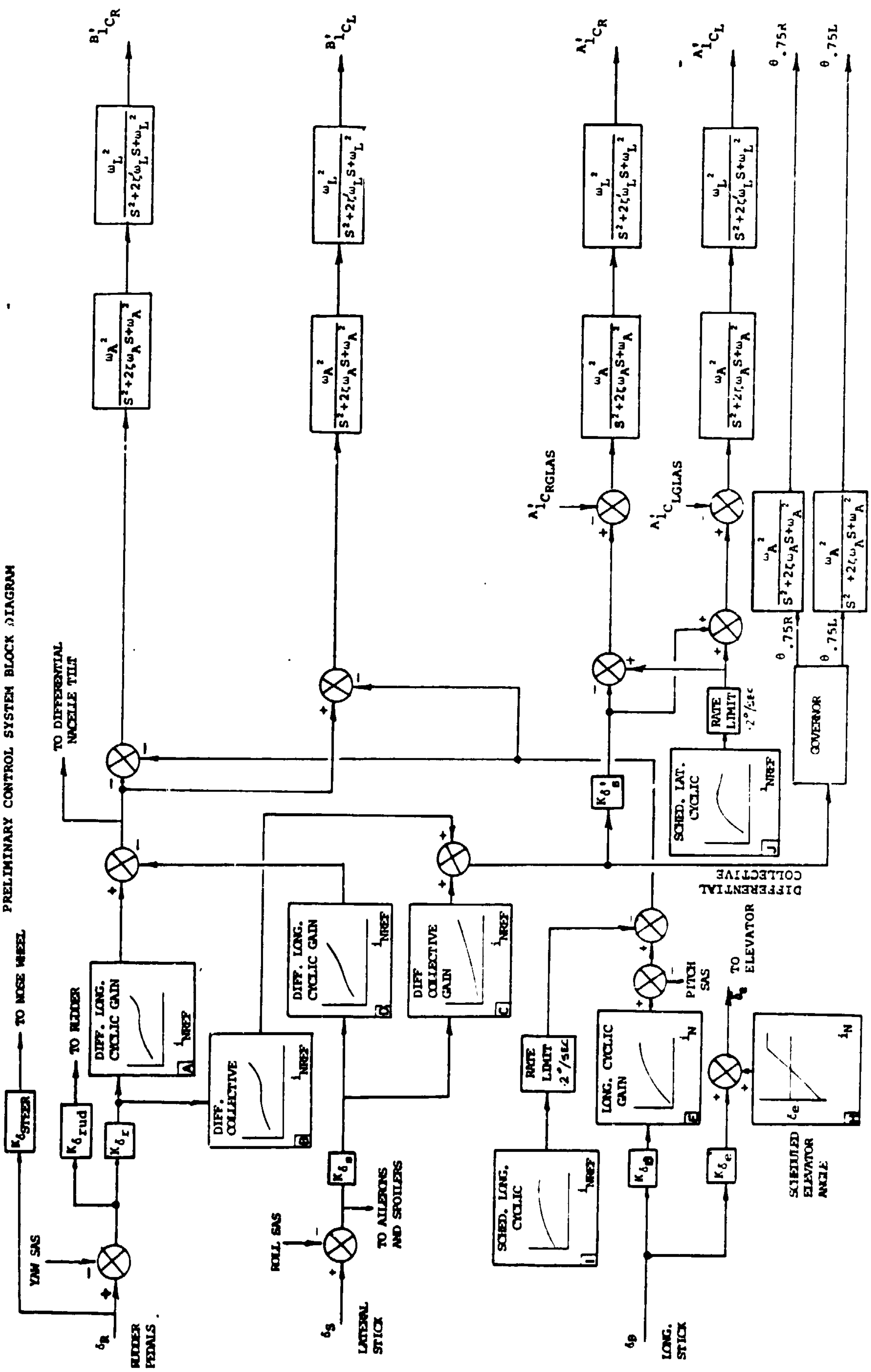
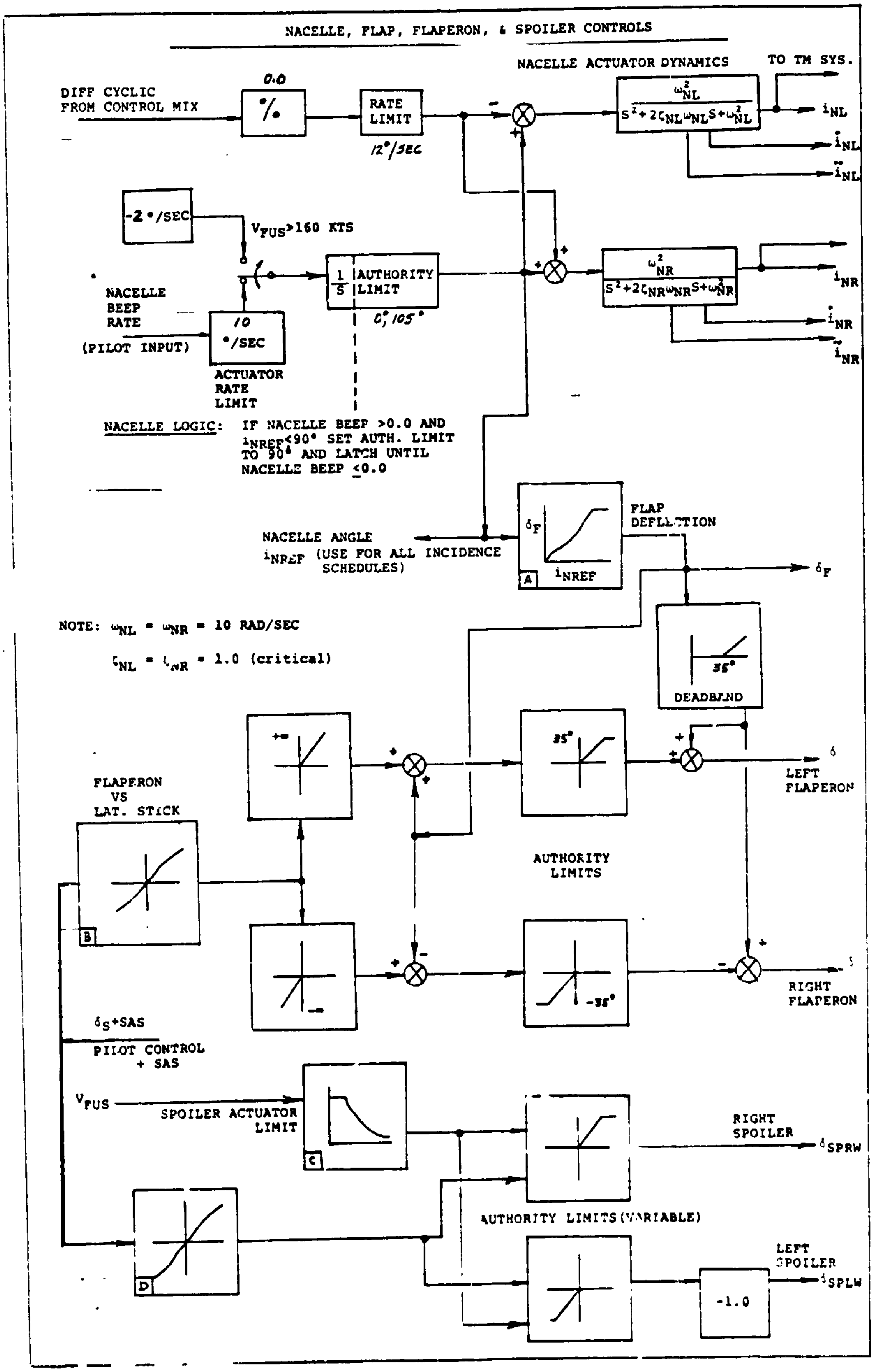
FUSELAGE FORCE & MOMENT RESOLUTION TO CG - INCLUDES LANDING GEAR
16
TOTAL FORCE AND MOMENT SUM ABOUT CG
24
BASIC EQUATIONS OF MOTION
25
EULER ANGLE CALC
26
AIRCRAFT CONDITION AND GROUND TRACK
27
δ_{TH}
θ_{75}
δ_B
δ_S
δ_R
i_N BEEP
X_{CG}
Z_{CG}
U,V,W,p,q,r,i_N
$Z^{W'}_{AERO}$
$L^{W'}_{AERO}$
Z^{W}_{AERO}
L^{W}_{AERO}
Ω
ε

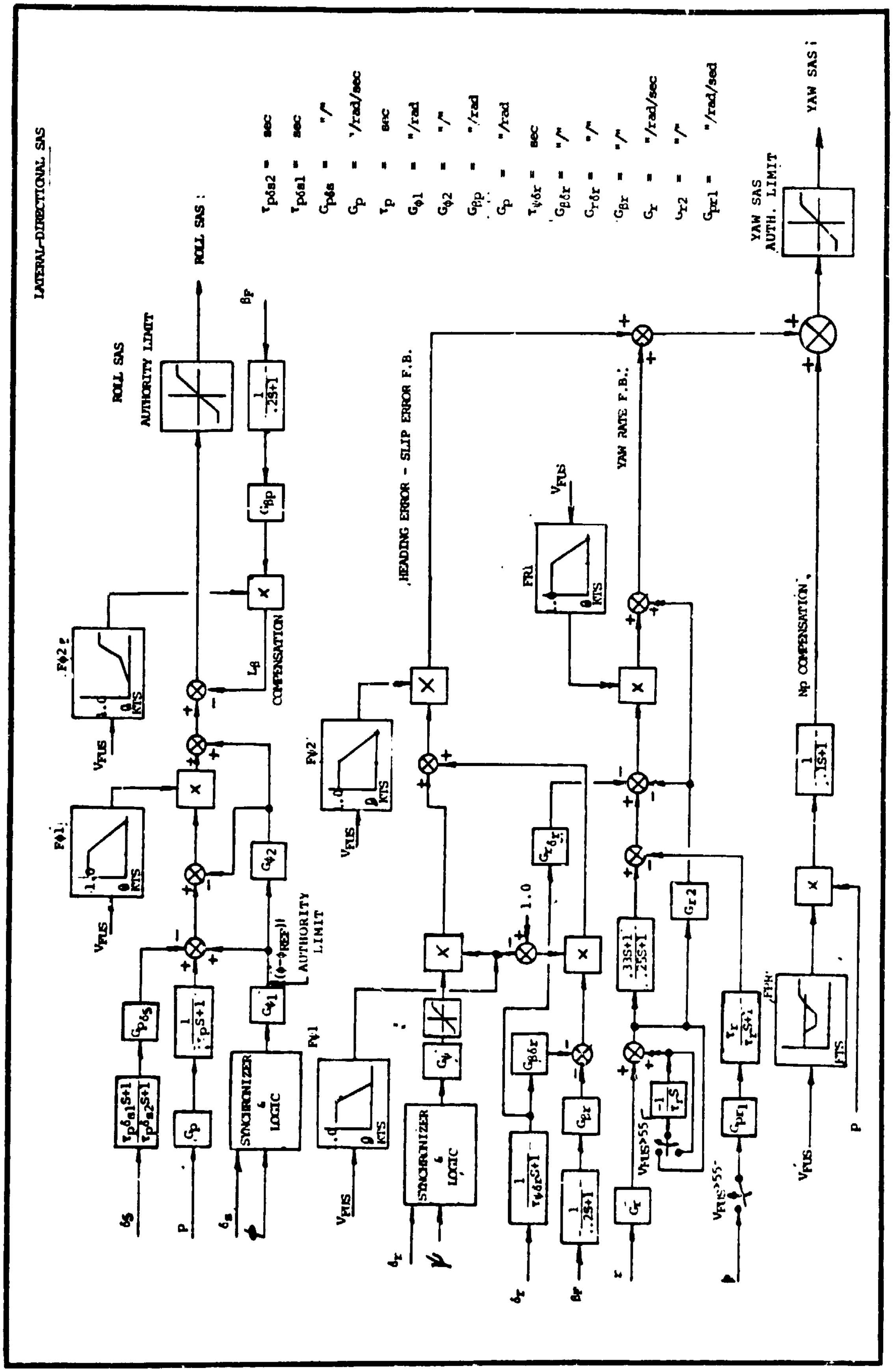


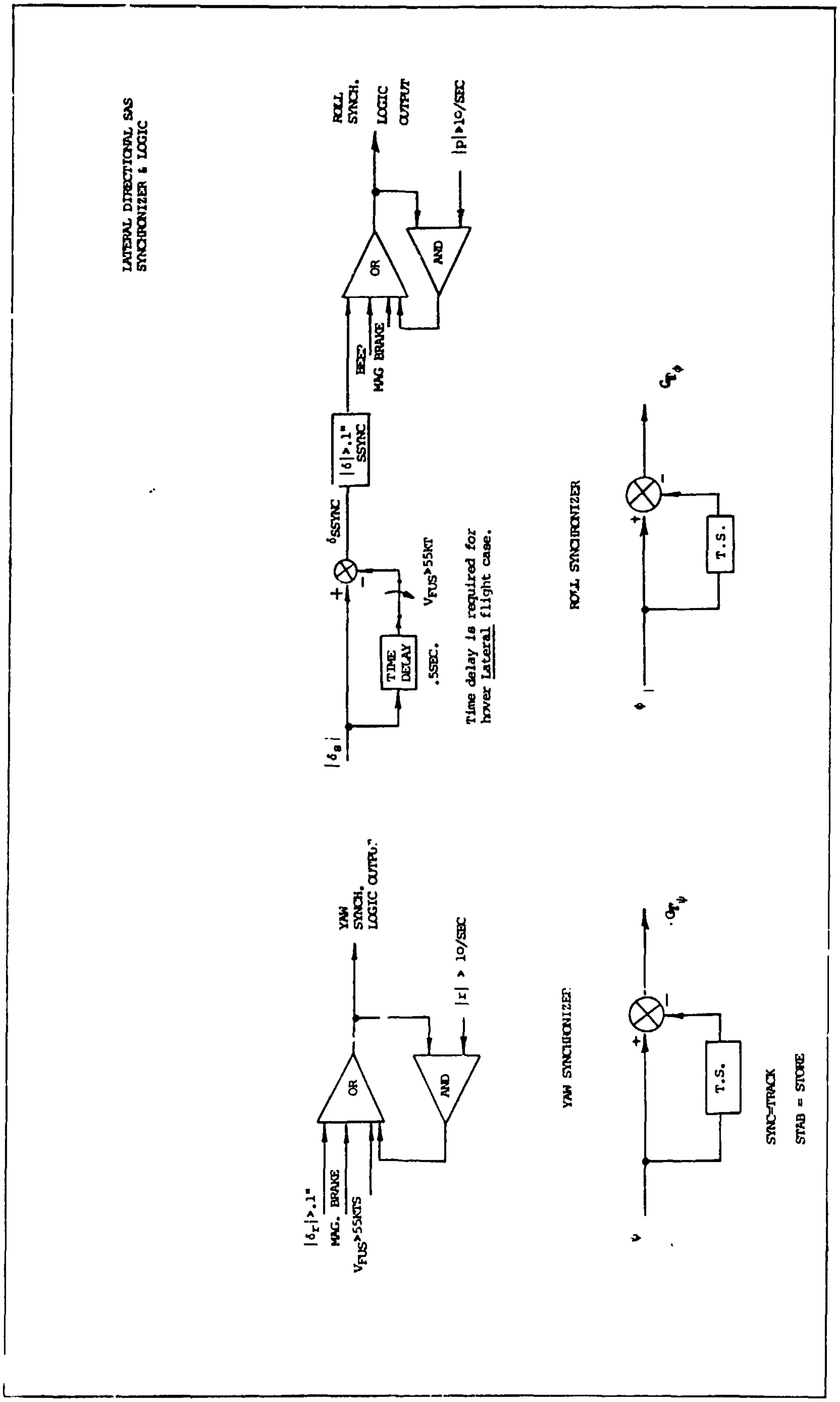
PRELIMINARY CONTROL SYSTEM BLOCK DIAGRAM



$$\frac{\omega_{NL}^2}{S^2+2\zeta_{NL}\omega_{NL}S+\omega_{NL}^2} \qquad \frac{\omega_{NR}^2}{S^2+2\zeta_{NR}\omega_{NR}S+\omega_{NR}^2}$$

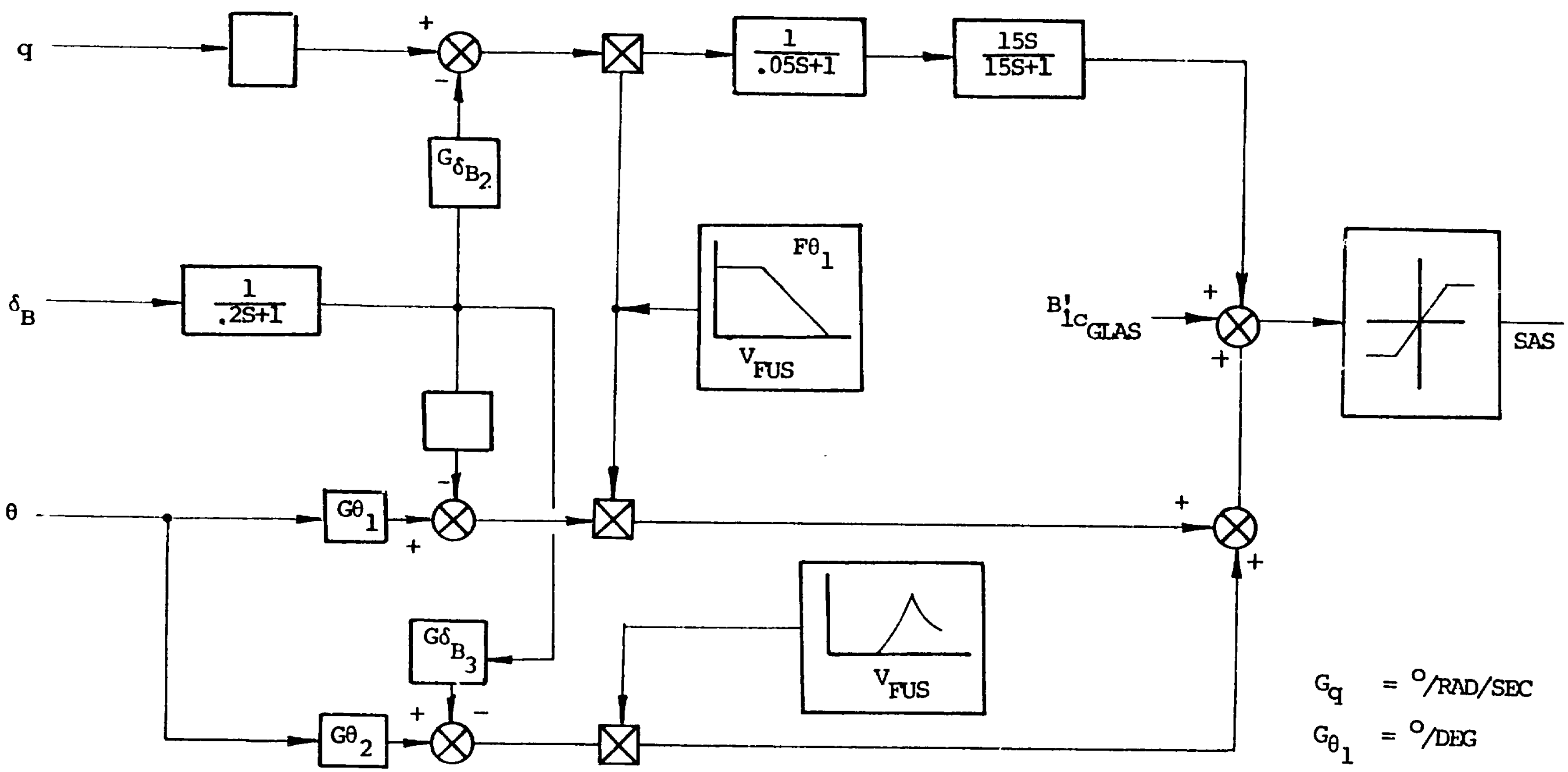
Outputs: i_{NL}, $\dot{i}_{NL}$, $\ddot{i}_{NL}$, i_{NR}, $\dot{i}_{NR}$, $\ddot{i}_{NR}$, δ_F


LATERAL-DIRECTIONAL SAS
ROLL SAS
ROLL SAS AUTHORITY LIMIT
YAW SAS
YAW SAS AUTH. LIMIT
HEADING ERROR - SLIP ERROR F.B.
YAW RATE F.B.
Np COMPENSATION
COMPENSATION
AUTHORITY LIMIT
SYNCHRONIZER & LOGIC


LATERAL DIRECTIONAL SAS
SYNCHRONIZER & LOGIC
ROLL
SYNCH.
LOGIC
OUTPUT
|p|>1°/SEC
OR
AND
BEEP
MAG BRAKE
|δ|>.1"
SSYNC
δSSYNC
VFUS>55KT
TIME
DELAY
.5SEC.
Time delay is required for
hover Lateral flight case.
|δa|
ROLL SYNCHRONIZER
T.S.
YAW
SYNCH.
LOGIC OUTPUT
|r| > 1°/SEC
|δr|>.1"
MAG. BRAKE
VFUS>55KTS
YAW SYNCHRONIZER
SYNC=TRACK
STAB = STORE

SHEET

LONGITUDINAL SAS



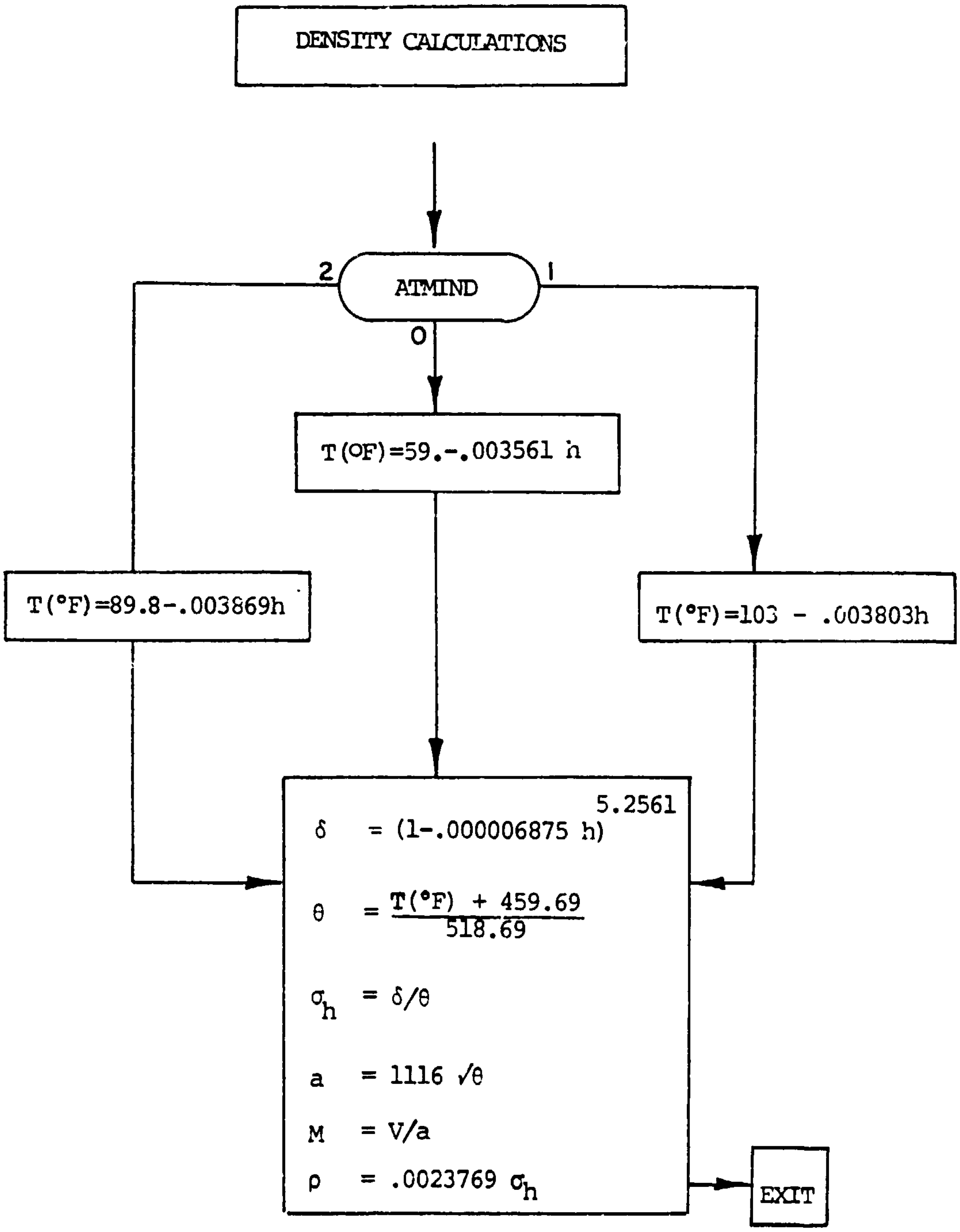
G_q = °/RAD/SEC

G_{θ_1} = °/DEG

$G_{\delta_{B_1}}$ = °/INCH

$G_{\delta_{B_2}}$ = °/INCH

$G_{\delta_{B_3}}$ = °/INCH



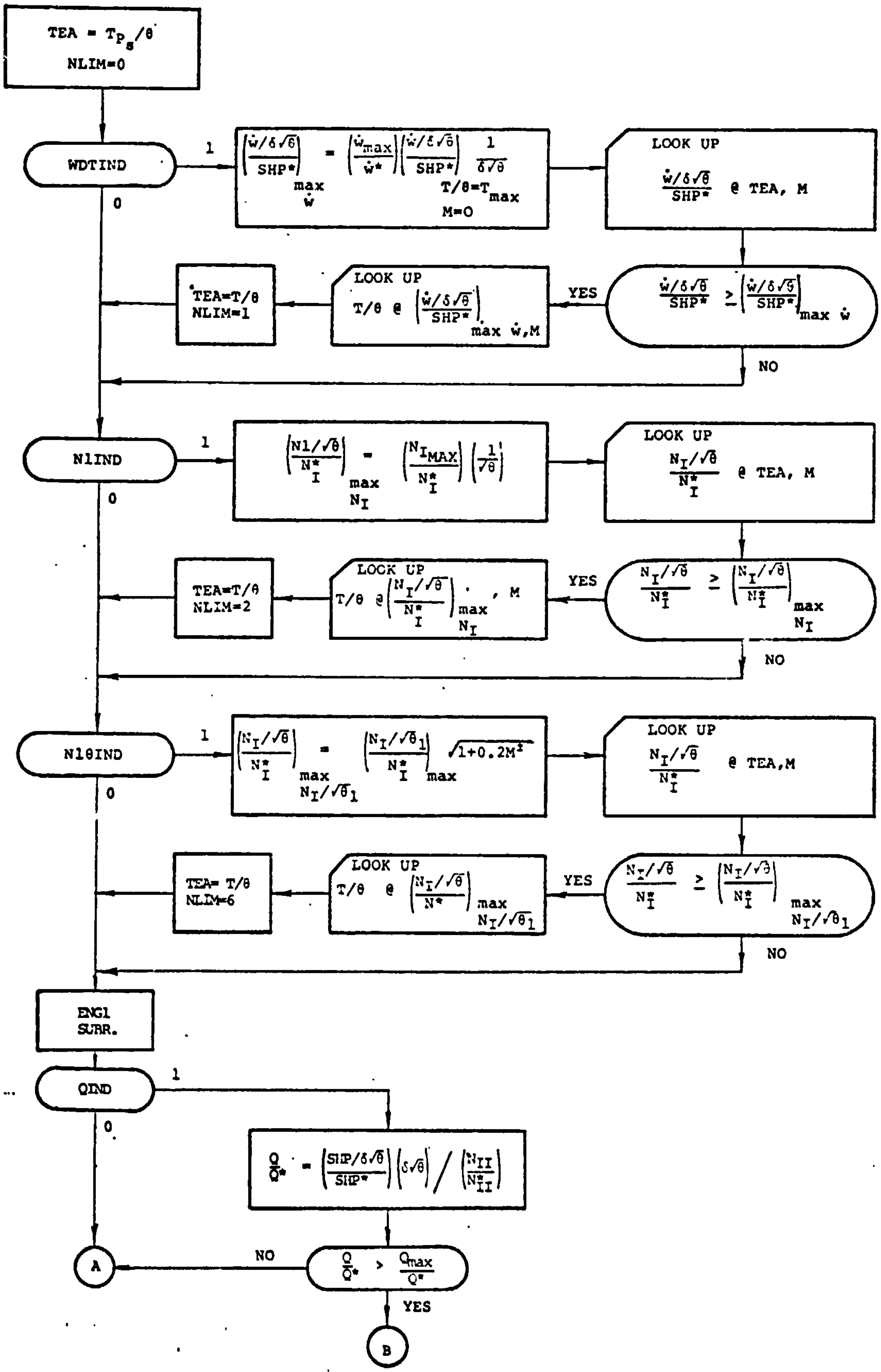
INPUT: h

ATMIND 0 STD ATMOS

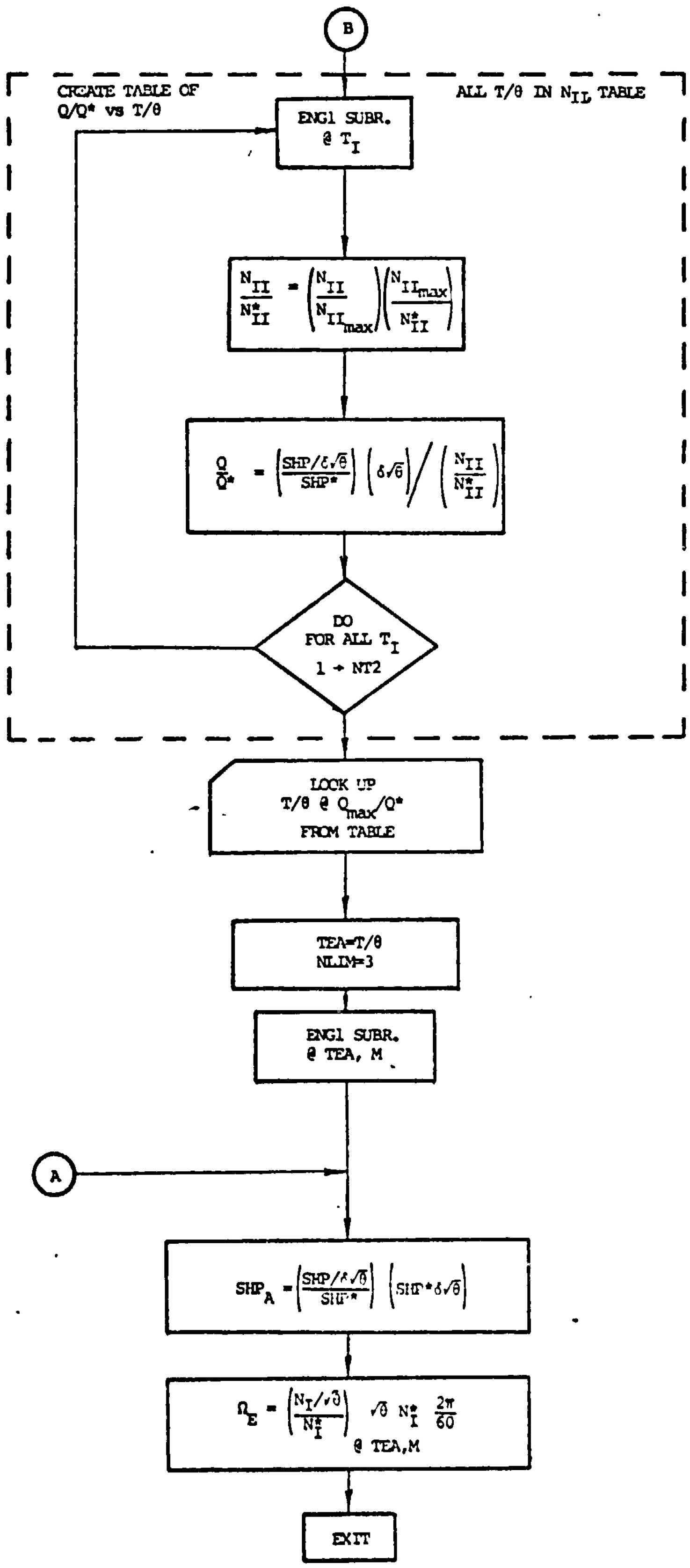
1 HOT ATMOS

2 TROPICAL ATMOS

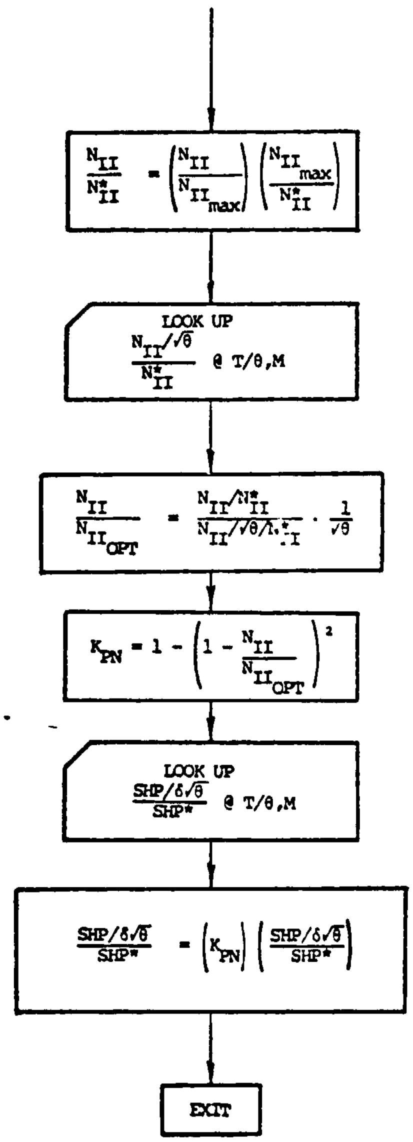
OUTPUT: δ, θ, σ_h, a, M, ρ



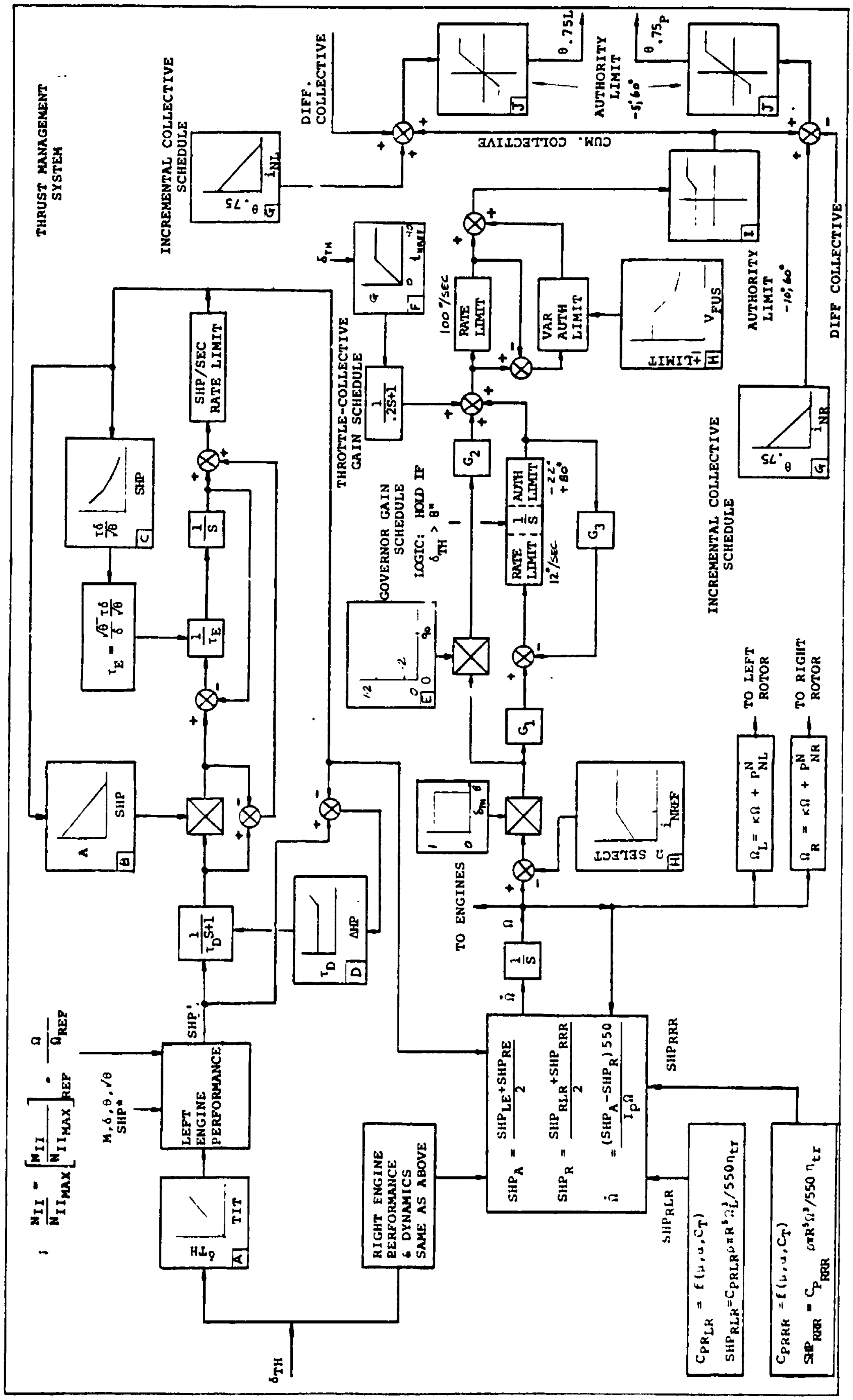
ENGINE ROUTINE POWER AVAILABLE



ENGINE ROUTINE POWER AVAILABLE



FLOW CHART FOR SUBROUTINE ENG 1 OF ENGINE ROUTINE


THRUST MANAGEMENT SYSTEM
INCREMENTAL COLLECTIVE SCHEDULE
DIFF. COLLECTIVE
CUM. COLLECTIVE
AUTHORITY LIMIT
DIFF COLLECTIVE
SHEET
THROTTLE-COLLECTIVE GAIN SCHEDULE
SHP/SEC RATE LIMIT
RATE LIMIT
VAR AUTH LIMIT
GOVERNOR GAIN SCHEDULE
LOGIC: HOLD IF
AUTH LIMIT
TO LEFT ROTOR
TO RIGHT ROTOR
TO ENGINES
Ω SELECT
LEFT ENGINE PERFORMANCE
RIGHT ENGINE PERFORMANCE & DYNAMICS SAME AS ABOVE
SHPRRR
SHPRLR

ROTOR CONTROL COORDINATE AXIS TRANSFORM

LEFT

$$A_{1CL}'' = A_{1CL}' \cos \phi_P + B_{1CL}' \sin \phi_P$$

$$B_{1CL}'' = -A_{1CL}' \sin \phi_P + B_{1CL}' \cos \phi_P$$

$$A_{1CL} = A_{1CL}'' \cos \xi_{HL} - B_{1CL}'' \sin \xi_{HL}$$

$$B_{1CL} = A_{1CL}'' \kappa \sin \xi_{HL} + B_{1CL}'' \cos \xi_{HL}$$

NOTE: ϕ_P is the control phase angle. ϕ_P is positive for the control axis moved opposite to rotor rotation.

RIGHT

$$A_{1CR}'' = A_{1CR}' \cos \phi_P + B_{1CR}' \sin \phi_P$$

$$B_{1CR}'' = -A_{1CR}' \sin \phi_P + B_{1CR}' \cos \phi_P$$

$$A_{1CR} = A_{1CR}'' \cos \xi_{HR} + B_{1CR}'' \kappa \sin \xi_{HR}$$

$$B_{1CR} = -A_{1CR}'' \kappa \sin \xi_{HR} + B_{1CR}'' \cos \xi_{HR}$$

CENTER OF GRAVITY CALCULATION

C.G. LOCATION RELATIVE TO PIVOT

$$X_{CG} = \frac{m_f \ell_f + m_w \ell_w}{m} + \ell \left(\frac{m_N}{m}\right) \left[\cos (i_{NL} - \lambda) + \cos (i_{NR} - \lambda)\right]$$

$$Z_{CG} = \frac{m_f h_f + m_w h_w}{m} - \ell \left(\frac{m_N}{m}\right) \left[\sin (i_{NL} - \lambda) + \sin (i_{NR} - \lambda)\right]$$

C.G. VELOCITY RELATIVE TO PIVOT

$$\dot{X}_{CG} = - \ell \left(\frac{m_N}{m}\right) \left[\dot{i}_{NL} \sin (i_{NL} - \lambda) + \dot{i}_{NR} \sin (i_{NR} - \lambda)\right]$$

$$\dot{Z}_{CG} = - \ell \left(\frac{m_N}{m}\right) \left[\dot{i}_{NL} \cos (i_{NL} - \lambda) + \dot{i}_{NR} \cos (i_{NR} - \lambda)\right]$$

C.G. ACCELERATION RELATIVE TO PIVOT

$$\ddot{X}_{CG} = - \ell \left(\frac{m_N}{m}\right) \Big[\ddot{i}_{NL} \sin (i_{NL} - \lambda) + \dot{i}^2_{NL} \cos (i_{NL} - \lambda)$$

$$+ \ddot{i}_{NR} \sin (i_{NR} - \lambda) + \dot{i}^2_{NR} \cos (i_{NR} - \lambda)\Big]$$

$$\ddot{Z}_{CG} = - \ell \left(\frac{m_N}{m}\right) \Big[\ddot{i}_{NL} \cos (i_{NL} - \lambda) - \dot{i}^2_{NL} \sin (i_{NL} - \lambda)$$

$$+ \ddot{i}_{NR} \cos (i_{NR} - \lambda) - \dot{i}^2_{NR} \sin (i_{NR} - \lambda)\Big]$$

FUSELAGE PIVOT VELOCITY

$$U_P = U - Z_{CG}q - \dot{X}_{CG}$$

$$V_P = V + Z_{CG}p - X_{CG}r$$

$$W_P = W + X_{CG}q - \dot{Z}_{CG}$$

VELOCITIES OF AIRCRAFT COMPONENTS

LEFT WING A.C. VELOCITY - BODY AXES

$$U'_{LW} = U_P + Z_{WAC}q + Y_{WAC}r + q\ h_{1_{LWAC}}$$

$$V'_{LW} = V_P + X_{WAC}r - Z_{WAC}p - p\ h_{1_{LWAC}}$$

$$W'_{LW} = W_P - Y_{WAC}p - X_{WAC}q + \dot{h}_{1_{LWAC}}$$

ROTOR WING A.C. VELOCITY - BODY AXES

$$U'_{RW} = U_P + Z_{WAC}q - Y_{WAC}r + q\ h_{1_{RWAC}}$$

$$V'_{RW} = V_P + X_{WAC}r - Z_{WAC}p - p\ h_{1_{RWAC}}$$

$$W'_{RW} = W_P + Y_{WAC}p - X_{WAC}q + \dot{h}_{1_{RWAC}}$$

LEFT ROTOR HUB VELOCITY - BODY AXES

$$U'_{RL} = U_P + r\ Y_N - L_S \sin i_{NL}\ (\dot{i}_{NL} + q) + q\ h_{1_L}$$

$$V'_{RL} = V_P + L_S\ (r \cos i_{NL} + p \sin i_{NL}) - p\ h_{1_L}$$

$$W'_{RL} = W_P - p\ Y_N - L_S(\dot{i}_{NL} + q) \cos i_{NL} + \dot{h}_{1_L}$$

RIGHT ROTOR HUB VELOCITY - BODY AXES

$$U'_{RR} = U_P - r\ Y_N - L_S \sin i_{NR}\ (\dot{i}_{NR} + q) + q\ h_{1_R}$$

$$V'_{RR} = V_P + L_S\ (r \cos i_{NR} + p \sin i_{NR}) - p\ h_{1_R}$$

$$W'_{RR} = W_P + p\ Y_N - L_S\ (\dot{i}_{NR} + q) \cos i_{NR} + \dot{h}_{1_R}$$

LEFT ROTOR HUB VELOCITY - SHAFT AXES

$$U_{RL} = U'_{RL} \cos i_{NL} - W'_{RL} \sin i_{NL}$$

$$V_{RL} = V'_{RL}$$

$$W_{RL} = U'_{RL} \sin i_{NL} + W'_{RL} \cos i_{NL}$$

RIGHT ROTOR HUB VELOCITY - SHAFT AXES

$$U_{RR} = U'_{RR} \cos i_{NR} - W'_{RR} \sin i_{NR}$$

$$V_{RR} = V'_{RR}$$

$$W_{RR} = U'_{RR} \sin i_{NR} + W'_{RR} \cos i_{NR}$$

LEFT WING A.C. VELOCITY - CHORD AXES

$$U_{LW} = U'_{LW} \cos i_W - W'_{LW} \sin i_W$$

$$V_{LW} = V'_{LW}$$

$$W_{LW} = U'_{LW} \sin i_W + W'_{LW} \cos i_W$$

RIGHT WING A.C. VELOCITY - CHORD AXES

$$U_{RW} = U'_{RW} \cos i_W - W'_{RW} \sin i_W$$

$$V_{RW} = V'_{RW}$$

$$W_{RW} = U'_{RW} \sin i_W + W'_{RW} \cos i_W$$

HORIZONTAL STABILIZER A.C. VELOCITY - BODY AXES

$$U_{HT} = U_P + \text{[illegible]}\,q$$

$$V_{HT} = V_P + X_{HT}r - Z_{HT}p$$

$$W_{HT} = W_P - X_{HT}q$$

VERTICAL FIN A.C. VELOCITY: RIGHT FIN (BODY AXES)

$$U_{VT_R} = U_P + Z_{VT}q - Y_{VT}r$$

$$V_{VT_R} = V_P + X_{VT}r - Z_{VT}p$$

$$W_{VT_R} = W_P - X_{VT}q + Y_{VT}p$$

LEFT FIN

$$U_{VT_L} = U_P + Z_{VT}q + Y_{VT}r$$

$$V_{VT_L} = V_P + Y_{VT}r - Z_{VT}p$$

$$W_{VT_L} = W_P - X_{VT}q - Y_{VT}p$$

WING AERODYNAMICS

CALCULATE ROTOR INTERFERENCE TERMS:

$$\tau_{RR} = \alpha_{RR} + \mathrm{Tan}^{-1}\left(\frac{NF_R}{T_R}\right)$$

$$R_{RR} = \sqrt{T_R^2 + NF_R^2 + SF_R^2}$$

$$V_{*_R} = \frac{V_{RR}}{\sqrt{\frac{|R_{RR}|+10}{2\rho A}}}$$

$$v_{*_R}^4 + 2\, V_{*_R}\, v_{*_R}^3 \cos \tau_{RR} + V_{*_R}^2\, v_{*_R}^2 = 1 \text{ (Solve for } v_{*_R}\text{)}$$

$$\varepsilon_{P_{RR}} = \mathrm{Tan}^{-1}\left[\frac{v_{*_R} \sin \tau_{RR}}{V_{*_R} + v_{*_R} \cos \tau_{RR}}\right]$$

$$C_{TS_{RR}} = \frac{\cos(\tau_{RR} - \alpha_{RR})}{\cos(\tau_{RR} - \alpha_{RR}) + \frac{V_{*_R}^2}{4}}$$

$$\tau_{LR} = \alpha_{LR} + \mathrm{Tan}^{-1}\left[\frac{NF_L}{T_L}\right]$$

$$R_{LR} = \sqrt{T_L^2 + NF_L^2 + SF_L^2}$$

$$V_{*_L} = \frac{V_{LR}}{\sqrt{\frac{|R_{LR}|+10}{2\rho A}}}$$

$$v_{*_L}^4 + 2\, V_{*_L}\, v_{*_L}^3 \cos \tau_{LR} + V_{*_L}^2\, v_{*_L}^2 = 1 \text{ (Solve for } v_{*_L}\text{)}$$

$$\varepsilon_{P_{LR}} = \mathrm{Tan}^{-1}\left[\frac{v_{*_L} \sin \tau_{LR}}{V_{*_L} + v_{*_L} \cos \tau_{LR}}\right]$$

$$C_{TS_{LR}} = \frac{\cos(\tau_{LR} - \alpha_{LR})}{\cos(\tau_{LR} - \alpha_{LR}) + \frac{V_{*_L}^2}{4}}$$

$$\xi_{HR} = \mathrm{Tan}^{-1}\left[V_{RR}/(W_{RR} + \varepsilon_{WRR}\ U_{RR})\right]$$

$$\xi_{HL} = \mathrm{Tan}^{-1}\left[V_{RL}/(W_{RL} + \varepsilon_{WRL}\ U_{RL})\right]$$

USED IN ROTOR CONTROL TRANSFORMATIONS

$$\bar{\xi} = (\xi_{HR} + \zeta_{HR})\ .5$$

$$\bar{\alpha}_R = (\alpha_{RR} + \alpha_{LR})\ .5$$

$$\left.\begin{aligned} \bar{\varepsilon}_P &= (\varepsilon_{PRR} + \varepsilon_{PLR}).5 \\ \bar{C}_{TS} &= (C_{TSRR} + C_{TSLR})\ .5 \end{aligned}\right\}$$

USED IN TAIL AERODYNAMICS.

$$\left.\begin{aligned} \bar{i}_N &= (i_{NL} + i_{NR})\ .5 \\ C_{LW} &= \frac{(C_{LSRW} + C_{LSLW})}{(1-\bar{C}_{TS})}\ .5 \end{aligned}\right\}$$

USED IN WING/ROTOR INTERFERENCE.

$$\xi_{R1} = [L_S - PC\cos(\bar{i}_N - i_w) + h_p \sin(\bar{i}_N - i_w)]\tan(\bar{\alpha}_R - \bar{\varepsilon}_P)\sin\bar{\xi}$$

$$\xi_{R2} = \sqrt{\frac{D^2}{4} - \left\{[L_S - PC\cos(\bar{i}_N - i_w) + h_p\sin(\bar{i}_N - i_w)]\tan(\bar{\alpha}_R - \bar{\varepsilon}_P)\cos\bar{\xi} + PC\sin(\bar{i}_N - i_w) + h_p\cos(\bar{i}_N - i_w)\right\}^2}$$

IF: $\xi_{R2} = 0$ or Imaginary; $S_{iRW} = 0$ and $S_{iLW} = 0$,

also $(C_{L\alpha i}/C_{L\alpha})_{RW} = 0$ and $(C_{L\alpha i}/C_{L\alpha})_{LW} = 0.0$

Form ξ_{R3} by replacing PC in ξ_{R1} equation with $(PC - c_w)$

Form ξ_{R4} by replacing PC in ξ_{R2} equation with $(PC - c_w)$

IF: $\xi_{R4} = 0$ or Imaginary; $S_{iRW} = 0$ and $S_{iLW} = 0$,

also $(C_{L\alpha i}/C_{L\alpha})_{RW} = 0$ and $(C_{L\alpha i}/C_{L\alpha})_{LW} = 0.0$

IF: UNBRELLAS OPEN; SET $C_{LW} = 0.0$

UMBRELLA LOGIC:

IF $i_{NREF} < F_{iN}$ or $q_F > 8.479$ lbs/ft^2 set umbrellas closed (hysteresis $F_{iN} \pm 1°$; $q_F \pm .1$ lb/ft^2).

$$S_{i_{RW}} = c_w/2 \; [\xi_{R1} + \xi_{R2} + \xi_{R3} + \xi_{R4}] = S_{i_R}$$

$$(S_i/S)_{RW} = 2 \; (S_{i_R}/S_w)$$

$$S_{i_T} = c_w \; [\xi_{R2} + \xi_{R4}]$$

$$S_{i_{LW}} = S_{j_T} - S_{i_R} = S_{i_L}$$

$$(S_i/S)_{LW} = 2 \; (S_{i_L}/S_w)$$

$$(AR_i)_{LW} = (S_{i_L}/c_w^2)$$

$$(AR_i)_{RW} = (S_{i_R}/c_w^2)$$

$$AR_w = S_w/c_w^2 \qquad \text{(FROM PREPROCESSOR)}$$

$$(C_{L\alpha i}/C_{L\alpha})_{LW} = \frac{\pi}{\pi + C_{L\alpha w} \; [1/(AR_i)_{LW} - 1/AR_w]}$$

$$(C_{L\alpha i}/C_{L\alpha})_{RW} = \frac{\pi}{\pi + C_{L\alpha w} \; [1/(AR_i)_{RW} - 1/AR_w]}$$

$$K'_{AL} = \frac{V_L^* + (C_{L\alpha i}/C_{L\alpha})_{LW} \; v_L^*}{V_L^* + v_L^*}$$

$$K'_{AR} = \frac{V_R^* + (C_{L\alpha i}/C_{L\alpha})_{RW} \; v_R^*}{V_R^* + v_R^*}$$

$$\bar{q}_S = [1/2\ \rho\ (u^2 + v^2 + w^2) + (T_L + T_R)/2A]$$

$$q_{S_{RW}} = [1/2\ \rho\ (u_{RW}^2 + v_{RW}^2 + w_{RW}^2) + T_R/A]$$

$$q_{S_{LW}} = [1/2\ \rho\ (u_{LW}^2 + v_{LW}^2 + w_{LW}^2) + T_L/A]$$

WING ANGLE OF ATTACK AND SIDESLIP

$$\alpha_{LWO} = \sin^{-1}\left[\frac{W_{LW}}{\sqrt{U_{LW}^2 + W_{LW}^2}}\right] + \theta_{tLWAC}$$

$$\alpha_{RWO} = \sin^{-1}\left[\frac{W_{RW}}{\sqrt{U_{RW}^2 + W_{RW}^2}}\right] + \theta_{tRWAC}$$

$$\beta_{LWO} = \sin^{-1}\left[\frac{V_{LW}}{\sqrt{U_{LW}^2 + V_{LW}^2 + W_{LW}^2}}\right]$$

$$\beta_{RWO} = \sin^{-1}\left[\frac{V_{RW}}{\sqrt{U_{RW}^2 + V_{RW}^2 + W_{RW}^2}}\right]$$

$$\alpha_{LWSSO} = \alpha_{LWO} - \epsilon_{PLR}$$

$$\alpha_{RWSSO} = \alpha_{RWO} - \epsilon_{PRR}$$

$$\bar{\alpha}_W = (\alpha_{LWO} + \alpha_{RWO})/2$$

$$\alpha_{LW\ RIGID} = \sin^{-1}\left[\frac{W_{LW}}{\sqrt{U_{LW}^2 + W_{LW}^2}}\right] - \epsilon_{PLR}$$

$$\alpha_{RW\ RIGID} = \sin^{-1}\left[\frac{W_{RW}}{\sqrt{U_{RW}^2 + W_{RW}^2}}\right] - \epsilon_{PRR}$$

$$\alpha'_{LWO} = \alpha_{LWO} - i_W - \theta_{tLWAC}$$

$$\alpha'_{RWO} = \alpha_{RWO} - i_W - \theta_{tRWAC}$$

CALCULATION OF INCREMENTAL LIFT, DRAG AND MOMENT COEFFICIENTS

CALCULATE:

$$C_{LLW_O} = C_L \text{ @ } \alpha = \alpha_{LW_{SS_O}}, \ \delta = \delta_{a_{LW}} + \delta_f, \ \delta_{SP} = \delta_{SP_L}$$

$$C_{DLW_O} = C_D \text{ @ } \alpha = \alpha_{LW_{SS_O}}, \ \delta = \delta_{a_{LW}} + \delta_f, \ \delta_{SP} = \delta_{SP_L}$$

$$C_{LRW_O} = C_L \text{ @ } \alpha = \alpha_{RW_{SS_O}}, \ \delta = \delta_{a_{RW}} + \delta_f, \ \delta_{SP} = \delta_{SP_R}$$

$$C_{DRW_O} = C_D \text{ @ } \alpha = \alpha_{RW_{SS_O}}, \ \delta = \delta_{a_{RW}} + \delta_f, \ \delta_{SP} = \delta_{SP_R}$$

$$C^*_{LLW_O} = C_L \text{ @ } \alpha = \alpha_{LW_O}, \ \delta = \delta_{a_{LW}} + \delta_f, \ \delta_{SP} = \delta_{SP_L}$$

$$C^*_{DLW_O} = C_D \text{ @ } \alpha = \alpha_{LW_O}, \ \delta = \delta_{a_{LW}} + \delta_f, \ \delta_{SP} = \delta_{SP_L}$$

$$C^*_{LRW_O} = C_L \text{ @ } \alpha = \alpha_{RW_O}, \ \delta = \delta_{a_{RW}} + \delta_f, \ \delta_{SP} = \delta_{SP_R}$$

$$C^*_{DRW_O} = C_D \text{ @ } \alpha = \alpha_{RW_O}, \ \delta = \delta_{a_{RW}} + \delta_f, \ \delta_{SP} = \delta_{SP_R}$$

$$C_{L_O} = C_L \text{ @ } \alpha = \alpha_F + i, \ \delta = \delta_f, \ \delta_{SP} = 0$$

USING THE FOLLOWING EQUATIONS:

$$\Delta C_{L_\delta} = a_7 \delta \qquad (0^\circ \leq \delta \leq \delta_2)$$

$$= a_8 + a_9 \delta + a_{10} \delta^2 \qquad (\delta_2 < \delta \leq \delta_3)$$

$$= a_{11} + a_{12} \delta + a_{13} \delta^2 \qquad (\delta > \delta_3)$$

$$\Delta C_{DO_\delta} = a_{29} \delta + a_{30} \delta^2 \qquad (0 \leq \delta \leq \delta_5)$$

$$= a_{31} + a_{32} \delta \qquad (\delta > \delta_5)$$

$$\Delta C_{L_{SP}} = a_{14}\delta_{SP} \qquad (0^\circ \le \delta_{SP} \le \delta_{SP_1})$$

$$= a_{15} + a_{16}\delta_{SP} + a_{17}\delta_{SP}^2 \qquad (\delta_{SP} > \delta_{SP_1})$$

$$\Delta C_{D_{SP}} = b_0\delta_{SP} + b_1\delta_{SP}^2$$

$$F = F_1 + F_2\delta + F_3\delta^2 \qquad (0^\circ < \delta < \delta_4)$$

$$= F_4 + F_5\delta + F_6\delta^2 \qquad (\delta > \delta_4)$$

$$\alpha_{NL}^{+} = a_0 + a_1\delta \qquad (0^\circ \le \delta \le \delta_1)$$

$$= a_2 \qquad (\delta > \delta_1)$$

$$\alpha_{NL}^{-} = a_3 + a_4\delta \qquad (0^\circ \le \delta \le \delta_1)$$

$$= a_5 \qquad (\delta > \delta_1)$$

If $\alpha_{NL}^{-} \le \alpha \le \alpha_{NL}^{+}$ calculate:

$$C_L = a_6 + C'_{L\alpha_W}\,\alpha + \Delta C_{L_\delta} + F\,\Delta C_{L_{SP}}$$

$$C_{LW_1} = C_L - F\,\Delta C_{L_{SP}}$$

$$C_{LW_2} = C_{LW_1} - \Delta C_{L_\delta} \qquad (\alpha \ge a_3)$$

$$C_{LW_2} = a_6 + C'_{L\alpha_w}\,a_3 + a_{23} + a_{24}\,\alpha + a_{25}\alpha^2 \qquad (\alpha < a_3)$$

$$C_D = C_{DO_W} + a_{26}\,C_{LW_2}^2 + a_{27}\,C_{LW_1}^2 + a_{28}\,(C_{LW_1} - C_{LW_2})^2$$

$$+ \Delta C_{DO_\delta} + \Delta C_{D_{SP}}$$

If $\alpha^+_{NL} < \alpha \le \alpha^+_{NL} + a_{18}$ calculate:

$$C'_{L_{NL}} = a_6 + C'_{L\alpha_W}\alpha^+_{NL} + \Delta C_{L\delta} + F\ \Delta C_{L_{SP}}$$

$$\alpha_{DUM} = \alpha - \alpha^+_{NL} + a_0$$

$$\Delta C_{L_{NL}} = a_{20} + a_{21}\ \alpha_{DUM} + a_{22}\ \alpha^2_{DUM}$$

$$C_L = C'_{L_{NL}} + \Delta C_{L_{NL}}$$

$$C_{LW_1} = C_L - F\ \Delta C_{L_{SP}}$$

$$C_{LW_2} = a_6 + C_{L\alpha_W}\ \alpha \qquad (\alpha \le a_0)$$

$$= a_6 + C'_{L\alpha_W}\ a_0 + a_{20} + a_{21}\ \alpha + a_{22}\ \alpha^2 \qquad (\alpha > a_0)$$

$$C_D = C_{DO_W} + a_{26}\ C^2_{LW_2} + a_{27}\ C^2_{LW_1} + a_{28}\ (C_{LW_1} - C_{LW_2})^2$$

$$+ \Delta C_{DO_\delta} + \Delta C_{D_{SP}}$$

and print stall warning.

If $\alpha^+_{NL} + a_{18} < \alpha \le 90^\circ$ calculate:

$$C_L = (a_6 + C'_{L\alpha_W}\ \alpha^+_{NL} + \Delta C_{L\alpha_\delta} + F\ \Delta C_{L_{SP}})(90^\circ - \alpha)/(90^\circ - \alpha^+_{NL} - a_{18})$$

$$\alpha_2 = a_0 + a_{18}$$

$$C_{LW_1} = C_L - F\ \Delta C_{L_{SP}}\ (90^\circ - \alpha)/(90^\circ - \alpha^+_{NL} - a_{18}) \qquad (\alpha \le \alpha_2)$$

$$= C_L - F\ \Delta C_{L_{SP}}\ (90^\circ - \alpha_2)/(90^\circ - \alpha^+_{NL} - a_{18}) \qquad (\alpha > \alpha_2)$$

$$C_{LW_2} = a_6 + C'_{L\alpha_W}\ a_0 + a_{20} + a_{21}\ \alpha + a_{22}\ \alpha^2 \qquad (\alpha \le \alpha_2)$$

$$= a_6 + C_{L\alpha_W}\ a_0 \qquad (\alpha > \alpha_2)$$

$$C_{D_1} = C_{DO_W} + a_{26}\ C^2_{LW_2} + a_{27}\ C^2_{LW_1} + a_{28}\ (C_{LW_1} - C_{LW_2})^2$$

$$+ \Delta C_{DO_\delta} + \Delta C_{D_{SP}}$$

$$C_D = C_{D_1} \qquad (\alpha \le \alpha_2)$$

$$C_D = C_{D_1} + (1 - C_{D_1})\ (\alpha - \alpha_2)/(90^\circ - \alpha_2) \qquad (\alpha > \alpha_2)$$

and print stall warning.

If $a_3 - a_{19} \le \alpha < \alpha_{\overline{N}L}$ calculate:

$$C'_{L_{NL}} = a_6 + C'_{L_{\alpha_w}}\ \alpha_{\overline{N}L} + \Delta C_{L_\delta} + F\ \Delta C_{L_{SP}}$$

$$\alpha_{DUM} = \alpha - \alpha_{\overline{N}L} + a_3$$

$$\Delta C_{L_{NL}} = a_{23} + a_{24}\ \alpha_{DUM} + a_{25}\ \alpha^2_{DUM}$$

$$C_L = C'_{L_{NL}} + \Delta C_{L_{NL}}$$

$$C_{L_{W_1}} = C_L - F\ \Delta C_{L_{SP}}$$

$$C_{L_{W_2}} = a_6 + C'_{L_{\alpha_w}}\ \alpha_{\overline{N}L} + a_{23} + a_{24}\ \alpha + a_{25}\ \alpha^2$$

$$C_D = C_{D_{W_o}} + a_{26}\ C^2_{L_{W_2}} + a_{27}\ C^2_{L_{W_1}} + a_{28}\ (C_{L_{W_1}} - C_{L_{W_2}})^2$$

$$+ \Delta C_{D_{o_\delta}} + \Delta C_{D_{SP}}$$

and print stall warning.

If $-90^\circ \le \alpha \le a_3 - a_{19}$ calculate:

$$\alpha_1 = \alpha_{\overline{N}L} - a_{19}$$

$$C'_{L_{NL}} = a_6 + C'_{L_{\alpha_w}}\ \alpha_{\overline{N}L} + \Delta C_{L_\delta} + F\ \Delta C_{L_{SP}}$$

$$\alpha_{DUM} = \alpha - \alpha_{\overline{N}L} + a_3$$

$$\Delta C_{L_{NL}} = a_{23} + a_{24}\,\alpha_{DUM} + a_{25}\,\alpha^2_{DUM}$$

$$C_L = C'_{L_{NL}} + \Delta C_{L_{NL}} \qquad (\alpha \geq \alpha_1)$$

$$= C'_{L_{NL}}\,(90° + \alpha)/(90° + \alpha_1) \qquad (\alpha < \alpha_1)$$

$$\alpha_{DUM} = 2a_3 - a_{19} - \alpha_{\overline{N}L}$$

$$\Delta C_{L_{NL}} = a_{23} + a_{24}\,\alpha_{DUM} + a_{25}\,\alpha^2_{DUM}$$

$$C_{L_{W_1}} = C'_{L_{NL}} + \Delta C_{L_{NL}} - F\,\Delta C_{L_{SP}}$$

$$C_{L_{W_2}} = a_6 + C_{L_{\alpha_w}}\,a_3$$

$$C_{D_1} = C_{D_{W_o}} + a_{26} C^2_{L_{W_2}} + a_{27}\,C^2_{L_{W_1}} + a_{28}\,(C_{L_{W_1}} - C_{L_{W_2}})^2 + \Delta C_{D_{o_\delta}} + \Delta C_{D_{SP}}$$

$$C_D = C_{D_1} - (1 - C_{D_1})\,(\alpha - a_3 + a_{19})/(90° + a_3 - a_{19})$$

and print stall warning.

CALCULATE:

$$C_{M_{LW}} = C_M \text{ @ } \alpha = \alpha_{LW_{SS_O}},\ \delta = \delta_f + \delta_{a_{LW}}$$

$$C_{M_{RW}} = C_M \text{ @ } \alpha = \alpha_{RW_{SS_O}},\ \delta = \delta_f + \delta_{a_{RW}}$$

$$C^{*'}_{M_{LW_O}} = C_M \text{ @ } \alpha = \alpha_{LW_O},\ \delta = \delta_f + \delta_{a_{LW}}$$

$$C^{*'}_{M_{RW_O}} = C_M \text{ @ } \alpha = \alpha_{RW_O},\ \delta = \delta_f + \delta_{a_{RW}}$$

AS FOLLOWS:

If $\alpha_1 \leq \alpha \leq \alpha_2$

Calculate $C'_M = b_2 + b_3\ \alpha$

$$\Delta C_{M_\delta} = b_4 + b_5\ \delta + b_6\ \delta^2$$

$$C_M = C'_M + \Delta C_{M_\delta}$$

If $\alpha > \alpha_2$

$$C'_M = b_2 + b_3\ \alpha_2 + \Delta C_{M_\delta}$$

$$C_M = C'_M\ (90 - \alpha)/(90 - \alpha_2)$$

If $\alpha < \alpha_1$

$$C'_M = b_2 + b_3\ \alpha_1 + \Delta C_{M_\delta}$$

$$C_M = C'_M\ (90 + \alpha)/(90 + \alpha_1)$$

CALCULATE:

$$C'''_{L_{LW}} = C_{L_{LWO}};\quad C'''_{D_{LW}} = C_{D_{LW}};\quad C''_{M_{LW}} = C_{M_{LW}}$$

$$C'''_{L_{RW}} = C_{L_{RWO}};\quad C'''_{D_{RW}} = C_{D_{RW}};\quad C''_{M_{RW}} = C_{M_{RW}}$$

$$C''_{L_{LWMAX}} = C_{L_{MAX}} + \Delta C_{L_{\delta}} + \Delta C_{L_{SP}}$$

$$C''_{L_{RWMAX}} = C_{L_{MAX}} + \Delta C_{L_{\delta}} + \Delta C_{L_{SP}}$$

$$C^{*\prime}_{L_{RW}} = C^{*}_{L_{RWO}};\quad C^{*\prime}_{D_{RW}} = C^{*\prime}_{D_{RWO}};\quad C^{*}_{M_{RW}} = C^{*\prime}_{M_{RWO}}$$

$$C^{*\prime}_{L_{LW}} = C^{*}_{L_{LWO}};\quad C^{*\prime}_{D_{LW}} = C^{*\prime}_{D_{LWO}};\quad C^{*}_{M_{LW}} = C^{*\prime}_{M_{LWO}}$$

$$C^{*}_{L_{LWMAX}} = C''_{L_{LWMAX}}$$

$$C^{*}_{L_{RWMAX}} = C''_{L_{RWMAX}}$$

$$\overline{C}_L = C_{L_O}\,(ag/a)w \,/\, \sqrt{1 - M^2}$$

$$C''^{IGE}_{L_{LW}} = C'''_{L_{LW}}\,(ag/a)w/\sqrt{1-M^2}\;;\quad C''^{IGE}_{L_{RW}} = C'''_{L_{RW}}\,(ag/a)w/\sqrt{1-M^2}$$

$$C^{*IGE}_{L_{LW}} = C^{*\prime}_{L_{LW}}\,(ag/a)w/\sqrt{1-M^2}\;;\quad C^{*IGE}_{L_{RW}} = C^{*\prime}_{L_{RW}}\,(ag/a)w/\sqrt{1-M^2}$$

$\Delta C''^{IGE}_{DLW} = K_{99} \, (C''^{IGE}_{LLW} - C'''_{LLW})^2/\pi AR_w$; $\Delta C''^{IGE}_{DRW} = K_{99} (C''^{IGE}_{LRW} - C'''_{LRW})^2/\pi AR_w$;

$\Delta C^{*IGE}_{DLW} = K_{99} \, (C^{*IGE}_{LLW} - C^{*\prime}_{LLW})^2/\pi AR_w$; $\Delta C^{*IGE}_{DRW} = K_{99} (C^{*IGE}_{LRW} - C^{*\prime}_{LRW})^2/\pi AR_w$

IF: $C''^{IGE}_{LLW} \geq C''_{LLW\,MAX}$; SET $\Delta C''^{IGE}_{DLW} = 0.0$ & $C''^{IGE}_{LLW} = C''_{LLW\,MAX}$

IF: $C''^{IGE}_{LRW} \geq C''_{LRW\,MAX}$; SET $\Delta C''^{IGE}_{DRW} = 0.0$ & $C''^{IGE}_{LRW} = C''_{LRW\,MAX}$

IF: $C^{*IGE}_{LLW} \geq C^{*}_{LLW\,MAX}$; SET $\Delta C^{*IGE}_{DLW} = 0.0$ & $C^{*IGE}_{LLW} = C^{*}_{LLW\,MAX}$

IF: $C^{*IGE}_{LRW} \geq C^{*}_{LRW\,MAX}$; SET $\Delta C^{*IGE}_{DRW} = 0.0$ & $C^{*IGE}_{LRW} = C^{*}_{LRW\,MAX}$

IF: $(ag/a) > 1.0$; SET $K_{99} = -1.0$

$(ag/a) \leq 1.0$; SET $K_{99} = +1.0$

CALCULATE

$$C''_{LLW} = C''^{IGE}_{LLW}$$

$$C''_{DLW} = C'''_{DLW} + \Delta C''^{IGE}_{DLW}$$

$$C''_{LRW} = C''^{IGE}_{LRW}$$

$$C''_{DRW} = C'''_{DRW} + \Delta C''^{IGE}_{DRW}$$

$$C^{*}_{LLW} = C^{*IGE}_{LLW}$$

$$C^{*}_{DLW} = C^{*\prime}_{DLW} + \Delta C^{*IGE}_{DLW}$$

$$C^{*}_{LRW} = C^{*IGE}_{LRW}$$

$$C^{*}_{DRW} = C^{*\prime}_{DRW} + \Delta C^{*IGE}_{DRW}$$

$$C_{LSLW} = K'_{A_L} \left\{ \left(\frac{S_i}{S}\right)_{LW} (C''_{LLW} \cos \varepsilon_{PLR} - C''_{DLW} \sin \varepsilon_{PLR}) + C^*_{LLW} (1 - C_{TSLR}) \left[1 - \left(\frac{S_i}{S}\right)_{LW} \right] \right\}$$

$$C_{LSRW} = K'_{A_R} \left\{ \left(\frac{S_i}{S}\right)_{RW} (C''_{LRW} \cos \varepsilon_{PRR} - C''_{DRW} \sin \varepsilon_{PRR}) + C^*_{LRW} (1 - C_{TSRR}) \left[1 - \left(\frac{S_i}{S}\right)_{RW} \right] \right\}$$

$$C_{DSLW} = K'_{A_L} \left\{ \left(\frac{S_i}{S}\right)_{LW} (C''_{LLW} \sin \varepsilon_{PLR} + C''_{DLW} \cos \varepsilon_{PLR}) + C^*_{DLW} (1 - C_{TSLR}) \left[1 - \left(\frac{S_i}{S}\right)_{LW} \right] \right\}$$

$$C_{DSRW} = K'_{A_R} \left\{ \left(\frac{S_i}{S}\right)_{RW} (C''_{LRW} \sin \varepsilon_{PRR} + C''_{DRW} \cos \varepsilon_{PRR}) + C^*_{DRW} (1 - C_{TSRR}) \left[1 - \left(\frac{S_i}{S}\right)_{RW} \right] \right\}$$

$$C_{MSLW} = K'_{A_L} \left\{ \left(\frac{S_i}{S}\right)_{LW} (C''_{MLW}) + C^*_{MLW} (1 - C_{TSLR}) \left[1 - \left(\frac{S_i}{S}\right)_{LW} \right] \right\}$$

$$C_{MSRW} = K'_{A_R} \left\{ \left(\frac{S_i}{S}\right)_{RW} (C''_{MRW}) + C^*_{MRW} (1 - C_{TSRR}) \left[1 - \left(\frac{S_i}{S}\right)_{RW} \right] \right\}$$

$$\Delta C_{\mathcal{L}S\ POWER} = 1/4 \left\{ [C_{LSLW} - (1 - \overline{C}_{TS})\ C^*_{LLW}]\ [1 - 1/2 (\frac{Si}{S})_{LW}] - [C_{LSRW} - (1 - \overline{C}_{TS})\ C^*_{LRW}]\ [1 - 1/2 (\frac{Si}{S})_{RW}] \right\}$$

$$\Delta C_{NS\ POWER} = 1/4 \left\{ [C_{DSRW} - (1 - \overline{C}_{TS})\ C^*_{DRW}]\ [1 - 1/2 (\frac{Si}{S})_{RW}] - [C_{DSLW} - (1 - \overline{C}_{TS})\ C^*_{DLW}]\ [1 - 1/2 (\frac{Si}{S})_{LW}] \right\}$$

$$C_{\mathcal{L}SW} = (K_{20} + K_{21}\ \overline{C}_L)\ (1 - \overline{C}_{TS})\ \beta_f + \left|\frac{1-\overline{C}_{TS}}{2b_W}\right|\ (K_{\mathcal{L}})\ (C^*_{LLW} - C^*_{LRW})\ \overline{Y}_{AC} + \Delta C_{\mathcal{L}S\ POWER}$$

$$C_{NSW} = (K_{22}\ \overline{C}_L^2)\ (1 - \overline{C}_{TS})\ \beta_f + \left|\frac{1-\overline{C}_{TS}}{2b_W}\right|\ (K_N) \left\{ (C^*_{DRW} - C^*_{DLW}) - [C^*_{LRW} \sin (\alpha_{RWO} - i_W) + C^*_{LLW} \sin (i_W - \alpha_{LWO})] \right\} \overline{Y}_{AC} + \Delta C_{NS\ POWER}$$

SPECIAL CONDITIONS (FOR UMBRELLAS OPEN)

IF: UMBRELLAS CLOSED; GO THROUGH WING EQUATIONS

IF: UMBRELLAS OPEN; CALCULATE THE WING FORCES AND MOMENTS AS FOLLOWS:

$$X^{LW}_{AERO} = fe_u \, q_{SLW} \, (1-C_{TSLR}) \left[\frac{-U_{LW}}{|U_{LW}|+.1} \right]$$

$$X^{RW}_{AERO} = fe_u \, q_{SRW} \, (1-C_{TSRR}) \left[\frac{-U_{RW}}{|U_{RW}|+.1} \right]$$

$$Y^{LW}_{AERO} = 0.0$$

$$Y^{RW}_{AERO} = 0.0$$

$$\left.\begin{aligned} Z^{LW'}_{AERO} &= T_L \, (D/T)_L \\ Z^{RW'}_{AERO} &= T_R \, (D/T)_R \end{aligned}\right\} \text{GO TO WING BENDING}$$

$$\left.\begin{aligned} M^{LW}_{AERO} &= -X_{\frac{C}{2}} \, Z^{LW}_{AERO} + (M/T)_L \, T_L \\ M^{RW}_{AERO} &= -X_{\frac{C}{2}} \, Z^{RW}_{AERO} + (M/T)_R \, T_R \end{aligned}\right\} Z^{LW}_{AERO} \text{ \& } Z^{RW}_{AERO} \text{ FROM WING BENDING}$$

$$\mathcal{L}^{W}_{AERO} = (b_w/2) \left\{ Z^{RW}_{AERO} \left[1 - \frac{(S_i/S)}{2}_{RW} \right] - Z^{LW}_{AERO} \left[1 - \frac{(S_i/S)_{LW}}{2} \right] \right\}$$

$$N^{W}_{AERO} = 0.0$$

IF: $[h/D]_{\substack{EFF \\ LR}} \leq 1.3$; $(D/T)_L = K_{\frac{D1}{T}} [h/D]^2_{\substack{EFF \\ LR}} + K_{\frac{D2}{T}} [h/D]_{\substack{EFF \\ LR}} + K_{\frac{D3}{T}}$;

$\&$ $(M/T)_R = K_{\frac{M1}{T}} [h/D]^2_{\substack{EFF \\ LR}} + K_{\frac{M2}{T}} [h/D]_{\substack{EFF \\ LR}} + K_{\frac{M3}{T}}$

IF: $[h/D]_{\substack{EFF\\LR}} > 1.3$; $(D/T)_L = K_{\frac{D4}{T}}$; & $(M/T)_L = K_{\frac{M4}{T}}$

IF: $[h/D]_{\substack{EFF\\RR}} \leq 1.3$; $(D/T)_R = K_{\frac{D1}{T}} [h/D]^2_{\substack{EFF\\RR}} + K_{\frac{D2}{T}} [h/D]_{\substack{EFF\\RR}} + K_{\frac{D3}{T}}$

& $(M/T)_R = K_{\frac{M1}{T}} [h/D]^2_{\substack{EFF\\RR}} + K_{\frac{M2}{T}} [h/D]_{\substack{EFF\\RR}} + K_{\frac{M3}{T}}$

IF: $[h/D]_{\substack{EFF\\RR}} > 1.3$; $(D/T)_R = K_{\frac{D4}{T}}$; & $(M/T)_R = K_{\frac{M4}{T}}$

FORCE AND MOMENT TRANSFORMATIONS FROM WING A. C. TO ELASTIC AXIS

PITCHING MOMENT

$$M^{RW}_{AERO} = C_{MSRW}\, q_{SRW}\, \frac{S_W}{2}\, c_W - X_{WAC}\, Z^{RW}_{AERO} + Z_{WAC}\, X^{RW}_{AERO}$$

$$M^{LW}_{AERO} = C_{MSLW}\, q_{SLW}\, \frac{S_W}{2}\, c_W - X_{WAC}\, Z^{LW}_{AERO} + Z_{WAC}\, X^{LW}_{AERO}$$

VERTICAL FORCES

$$Z^{RW'}_{AERO} = \left[-C_{LSRW} - C_{DSRW}\, \alpha'_{RWO}\right] q_{SRW}\, \frac{S_W}{2}$$

$$Z^{LW'}_{AERO} = \left[-C_{LSLW} - C_{DSLW}\, \alpha'_{LWO}\right] q_{SLW}\, \frac{S_W}{2}$$

NOTE: $Z^{RW'}_{AERO}$ & $Z^{LW'}_{AERO}$ ARE USED IN VERTICAL BENDING EQUATIONS.

WING FORCE & MOMENT RESOLUTION - BODY AXES @ C.G.

$$X^{LW}_{AERO} = \left[-C_{DSLW} + C_{LSLW}\,\alpha'_{LWO}\right] q_{SLW}\,\frac{S_W}{2}$$

$$X^{RW}_{AERO} = \left[-C_{DSRW} + C_{LSRW}\,\alpha'_{RWO}\right] q_{SRW}\,\frac{S_W}{2}$$

$$Y^{LW}_{AERO} = \left[-C_{DSLW}\,\beta_{LWO}\right] q_{SLW}\,\frac{S_W}{2}$$

$$Y^{RW}_{AERO} = \left[-C_{DSRW}\,\beta_{RWO}\right] q_{SRW}\,\frac{S_W}{2}$$

$$\left.\begin{array}{l} Z^{LW}_{AERO} \\ \\ Z^{RW}_{AERO} \end{array}\right\} \text{FROM VERTICAL BENDING}$$

$$\mathcal{L}^{W}_{AERO} = C_{\mathcal{L}SW}\,\bar{q}_S\,S_W\,b_W$$

$$M^{W}_{AERO} = M^{LW}_{AERO} + M^{RW}_{AERO} + X_{CG}\left(Z^{LW}_{AERO} + Z^{RW}_{AERO}\right) - Z_{CG}\left(X^{LW}_{AERO} + X^{RW}_{AERO}\right)$$

$$N^{W}_{AERO} = C_{NSW}\,\bar{q}_S\,S_W\,b_W$$

NOTE: OBSERVE WING SPECIAL CONDITIONS.

HORIZONTAL TAIL AERODYNAMICS

WING AND TAIL ALTITUDE - GROUND EFFECT

$$h_{Wc/4} = -Z_{DOWN} + (X_{WAC} - X_{CG}) \sin\theta + (Z_{CG} - Z_{WAC}) \cos\theta$$

$$h_{Tc/4} = -Z_{DOWN} + (X_{HT} - X_{CG}) \sin\theta + (Z_{CG} - Z_{HT}) \cos\theta$$

HORIZONTAL TAIL ANGLE OF ATTACK

$\ell_{AC} = X_{WAC} - X_{HT}$ (FROM PREPROCESSOR)

$$GEF = [b_W^2 + 4(h_{Tc/4} - h_{Wc/4})^2]/[b_W^2 + 4(h_{Tc/4} + h_{Wc/4})^2]$$

IF: $\bar{\varepsilon}_p \geq \varepsilon_o + d\varepsilon/d\alpha\,(\bar{\alpha}_W - \ell_{AC}\dot{W}/U^2)$

then $\varepsilon = \bar{\varepsilon}_p\,(1 - GEF)/\sqrt{1 - M^2}$

IF: $\bar{\varepsilon}_p < \varepsilon_o + d\varepsilon/d\alpha\,(\bar{\alpha}_W - \ell_{AC}\dot{W}/U^2)$

then $\varepsilon = \varepsilon_o + d\varepsilon/d\alpha\,(\bar{\alpha}_W - \ell_{AC}\dot{W}/U^2)\quad(1 - GEF)/\sqrt{1 - M^2}$

WHERE $\varepsilon_o = \varepsilon_o @ (\delta_{fL} + \delta_{fR})/2$, $d\varepsilon/d\alpha = d\varepsilon/d\alpha @ (\delta_{fL} + \delta_{fR})/2$

$$\alpha_{HT} = \mathrm{Tan}^{-1}(W_{HT}/U_{HT}) - \varepsilon + i_{HT} \quad (U<0)$$

$$= \mathrm{Tan}^{-1}(W_{HT}/U_{HT}) + i_{HT} \quad (U>0)$$

This form for α_{HT} is to be used for <u>resolution</u> of forces only.

IF: $|\alpha_{HT}| > 180°$ then calculate α_{HT} from

$$\alpha_{HT} = -(\mathrm{sign}\ \alpha_{HT})360° + \alpha_{HT}$$

and use this value to obtain the forces and moments.

HORIZONTAL TAIL LIFT AND DRAG

$$\alpha_{e_{HT}} = \alpha_{HT} + \tau_{HT}\,\delta_e$$

$$\hat{\alpha}_{HT_+} = (\alpha_{HT_{STALL}} - 2°) + \tau_{HT}\,\delta_e$$

$$\hat{\alpha}_{HT_-} = -(\alpha_{HT_{STALL}} - 2°) + \tau_{HT}\,\delta_e$$

$$C_{L\alpha} = C_{L\alpha_{HT}}\,(ag/a)_{HT}/\sqrt{1-M^2}$$

Where $(ag/a)_{HT} = f(h_{TC/4})$

IF: $\hat{\alpha}_{HT_-} \leq \alpha_{e_{HT}} \leq \hat{\alpha}_{HT_+}$

$$C_{L_{HT}} = C_{L\alpha}\,\alpha_{e_{HT}}$$

$$C_{D_{HT}} = C_{DO_{HT}} + C^2_{L_{HT}}/\pi AR_{HT} E_{HT}$$

IF: $\hat{\alpha}_{HT_+} < \alpha_{e_{HT}} \leq 90°$

$$C_{L_{HT}} = C_{L\alpha}\,\hat{\alpha}_{HT_+}\,(90° - \alpha_{e_{HT}})/(90° - \hat{\alpha}_{HT_+})$$

$$C_{L_{HT_{STALL}}} = C_{L\alpha}\,\hat{\alpha}_{HT_+}$$

$$C_{D_{HT_{STALL}}} = C_{DO_{HT}} + C^2_{L_{HT_{STALL}}}/\pi AR_{HT} E_{HT}$$

$$C_{D_{HT}} = C_{D_{HT_{STALL}}} + \frac{(\alpha_{e_{HT}} - \hat{\alpha}_{HT_+})(1.1 - C_{D_{HT_{STALL}}})}{90° - \hat{\alpha}_{HT_+}}$$

HORIZONTAL TAIL LIFT AND DRAG (CONTINUED)

IF: $90° < \alpha_{e_{HT}} \leq (180° - .5\ \hat{\alpha}_{HT_-})$

$$C_{L_{HT}} = .5\ C_{L\alpha}\ \hat{\alpha}_{HT_-}\ (\alpha_{e_{HT}} - 90°)/(90° - .5\ \alpha_{HT_-})$$

$$C_{L_{HT_{STALL}}} = .5\ C_{L\alpha}\ \hat{\alpha}_{HT_-}$$

$$C_{D_{HT_{STALL}}} = C^2_{L_{HT_{STALL}}}/\pi AR_{HT}E_{HT} + C_{DO_{HT}}$$

$$C_{D_{HT}} = C_{D_{HT_{STALL}}} + \frac{\left(\alpha_{e_{HT}} + .5\ \hat{\alpha}_{HT_-} - 180°\right)\left(1.1 - C_{D_{HT_{STALL}}}\right)}{(.5\ \hat{\alpha}_{HT_-} - 90°)}$$

IF: $(180° - .5\ \hat{\alpha}_{HT_-}) \leq \alpha_{e_{HT}} \leq 180°$

$$C_{L_{HT}} = C_{L\alpha}\ (\alpha_{e_{HT}} - 180°)$$

$$C_{D_{HT}} = C_{DO_{HT}} + C^2_{L_{HT}}/\pi AR_{HT}E_{HT}$$

IF: $-90 \leq \alpha_{e_{HT}} < \hat{\alpha}_{HT_-}$

$$C_{L_{HT}} = C_{L\alpha}\ \hat{\alpha}_{HT_-}\ (-90° - \alpha_{e_{HT}})/(-90° - \hat{\alpha}_{HT_-})$$

$$C_{L_{HT_{STALL}}} = C_{L\alpha}\ \hat{\alpha}_{HT_-}$$

$$C_{D_{HT_{STALL}}} = C_{DO_{HT}} + C^2_{L_{HT_{STALL}}}/\pi AR_{HT}E_{HT}$$

$$C_{D_{HT}} = C_{D_{HT_{STALL}}} + \frac{\left(\alpha_{e_{HT}} - \hat{\alpha}_{HT_-}\right)\left(1.1 - C_{D_{HT_{STALL}}}\right)}{(-90° - \hat{\alpha}_{HT_-})}$$

HORIZONTAL TAIL LIFT AND DRAG (CONTINUED)

IF: $(-180° + .5\hat{\alpha}_{HT_+}) < \alpha_{e_{HT}} < -90°$

$$C_{L_{HT}} = .5\ C_{L\alpha}\ \hat{\alpha}_{HT_+}\ (\alpha_{e_{HT}} + 90°)/(-90° + .5\ \hat{\alpha}_{HT_+})$$

$$C_{L_{HT_{STALL}}} = .5\ C_{L\alpha}\ \hat{\alpha}_{HT_+}$$

$$C_{D_{HT_{STALL}}} = C_{DO_{HT}} + C^2_{L_{HT_{STALL}}} / \pi AR_{HT} E_{HT}$$

$$C_{D_{HT}} = C_{D_{HT\ STALL}} - \frac{(\alpha_{e_{HT}} + 180° - .5\ \hat{\alpha}_{HT_+})(1.1 - C_{D_{HT_{STALL}}})}{(.5\ \hat{\alpha}_{HT_+} - 90°)}$$

IF: $-180° \leq \alpha_{e_{HT}} < (-180° + .5\ \hat{\alpha}_{HT_+})$

$$C_{L_{HT}} = C_{L\alpha}\ (\alpha_{e_{HT}} + 180°)$$

$$C_{D_{HT}} = C_{DO_{HT}} + C^2_{L_{HT}} / \pi AR_{HT} E_{HT}$$

VERTICAL TAIL AERODYNAMICS

VERTICAL TAIL ANGLE OF ATTACK AND SIDESLIP

$$\beta_{VT} = \mathrm{Tan}^{-1}\left[V_{VT}/\sqrt{U_{VT}^2 + W_{VT}^2} \right]$$

$\alpha_{VT} = -\beta_{VT} + \beta_f \; (d\sigma/d\beta)$ {NOTE: THIS VALUE OF α_{VT} IS USED IN RESOLUTION OF FORCES AND MOMENTS}

IF: $|\alpha_{VT}| > 180°; \alpha_{VT} = \alpha_{VT} - (\text{sign } \alpha_{VT})(360°)$ {NOTE: THIS VALUE OF α_{VT} ONLY USED IN CALCULATION OF FORCE AND MOMENT COEFFICIENTS}

$$\alpha_{e_{VT}} = (\alpha_{VT} + \tau_{VT} \; \delta_{RUD})$$

$$\hat{\alpha}_{VT_+} = (\alpha_{VT_{STALL}} - 2°) + \tau_{VT} \; \delta_{RUD}$$

$$\hat{\alpha}_{VT_-} = -(\alpha_{VT_{STALL}} - 2°) + \tau_{VT} \; \delta_{RUD}$$

$$C_{Y\alpha} = C_{Y\alpha_{VT}}/\sqrt{1-M^2}$$

TAIL DYNAMIC PRESSURE AND SIDEWASH

$$\bar{q} = \rho/2 \; (u^2 + v^2 + w^2)$$

$$\sigma = (d\sigma/d\beta) \quad \beta_F$$

VERTICAL TAIL LIFT AND DRAG

IF: $\hat{\alpha}_{VT_-} \leq \alpha_{e_{VT}} < \hat{\alpha}_{VT_+}$

$$C_{Y_{VT}} = C_{Y\alpha} \; \alpha_{e_{VT}}$$

$$C_{D_{VT}} = C_{DO_{VT}} + C_{Y_{VT}}^2/\pi AR_{VT} E_{VT}$$

VERTICAL TAIL LIFT AND DRAG (CONTINUED)

IF: $\hat{\alpha}_{VT+} < \alpha_{e_{VT}} \leq 90°$

$$C_{Y_{VT}} = C_{Y\alpha}\,\hat{\alpha}_{VT_+}\,(90° - \alpha_{e_{VT}})/(90° - \hat{\alpha}_{VT_+})$$

$$C_{Y_{VT_{STALL}}} = C_{Y\alpha}\,\hat{\alpha}_{VT_+}$$

$$C_{D_{VT_{STALL}}} = C_{DO_{VT}} + C^2_{Y_{VT_{STALL}}}/\pi AR_{VT}E_{VT}$$

$$C_{D_{VT}} = C_{D_{VT_{STALL}}} + \frac{(\alpha_{e_{VT}} - \hat{\alpha}_{VT_+})(1.1 - C_{D_{VT_{STALL}}})}{(90° - \hat{\alpha}_{VT_+})}$$

IF: $90° < \alpha_{e_{VT}} \leq (180° - .5\,\hat{\alpha}_{VT_-})$

$$C_{Y_{VT}} = .5\,C_{Y\alpha}\,\hat{\alpha}_{VT_-}\,(\alpha_{e_{VT}} - 90°)/(90° - .5\,\hat{\alpha}_{VT_-})$$

$$C_{Y_{VT_{STALL}}} = .5\,C_{Y\alpha}\,\hat{\alpha}_{VT_-}$$

$$C_{Y_{VT_{STALL}}} = C_{DO_{VT}} + C^2_{Y_{VT_{STALL}}}/\pi AR_{VT}E_{VT}$$

$$C_{D_{VT}} = C_{D_{VT_{STALL}}} + \frac{(\alpha_{e_{VT}} + .5\,\hat{\alpha}_{VT_-} - 180°)(1.1 - C_{D_{VT_{STALL}}})}{(.5\,\hat{\alpha}_{VT_-} - 90°)}$$

IF: $(180° - .5\,\hat{\alpha}_{VT_-}) \leq \alpha_{e_{VT}} < 180°$

$$C_{Y_{VT}} = C_{Y\alpha}\,(\alpha_{e_{VT}} - 180°)$$

$$C_{D_{VT}} = C_{DO_{VT}} + C^2_{Y_{VT}}/\pi AR_{VT}E_{VT}$$

VERTICAL TAIL LIFT AND DRAG (CONTINUED)

IF: $-90° \leq \alpha_{e_{VT}} < \hat{\alpha}_{VT-}$

$$C_{Y_{VT}} = C_{Y\alpha}\, \hat{\alpha}_{VT_-}\, (-90° - \alpha_{e_{VT}})/(-90° - \hat{\alpha}_{VT_-})$$

$$C_{Y_{VT_{STALL}}} = C_{Y\alpha}\, \hat{\alpha}_{VT_-}$$

$$C_{D_{VT_{STALL}}} = C_{DO_{VT}} + C^2_{Y_{VT_{STALL}}}/\pi AR_{VT} E_{VT}$$

$$C_{D_{VT}} = C_{D_{VT_{STALL}}} + \frac{(\alpha_{e_{VT}} - \hat{\alpha}_{VT-})(1.1 - C_{D_{VT_{STALL}}})}{(-90° - \hat{\alpha}_{VT_-})}$$

IF: $(-180° + .5\, \hat{\alpha}_{VT_+}) < \alpha_{e_{VT}} < -90°$

$$C_{Y_{VT}} = .5\, C_{Y\alpha}\, \hat{\alpha}_{VT_+}\, (\alpha_{e_{VT}} + 90°)/(-90° + .5\, \hat{\alpha}_{VT_+})$$

$$C_{Y_{VT_{STALL}}} = .5\, C_{Y\alpha}\, \hat{\alpha}_{VT_+}$$

$$C_{D_{VT_{STALL}}} = C_{DO_{VT}} + C^2_{Y_{VT_{STALL}}}/\pi AR_{VT} E_{VT}$$

$$C_{D_{VT}} = C_{D_{VT_{STALL}}} - \frac{(\alpha_{e_{VT}} + 180° - .5\, \hat{\alpha}_{VT_+})(1.1 - C_{D_{VT_{STALL}}})}{(.5\, \hat{\alpha}_{VT_+} - 90°)}$$

IF: $-180° \leq \alpha_{e_{VT}} < (-180° + .5\, \hat{\alpha}_{VT_+})$

$$C_{Y_{VT}} = C_{Y\alpha}\, (\alpha_{e_{VT}} + 180°)$$

$$C_{D_{VT}} = C_{DO_{VT}} + C^2_{Y_{VT}}/\pi AR_{VT} E_{VT}$$

TAIL EQUATIONS LOGIC

HORIZONTAL TAIL

1. If $h_{W_{C/4}} > 100$ feet; set GEF = 0.0

2. If the umbrellas open; set $\varepsilon = \dfrac{\bar{\varepsilon}_P (1 - GEF)}{\sqrt{1 - M^2}}$

3. If $\alpha_{e_{HT}} > \hat{\alpha}_{HT+}$ print stall warning

4. If $\alpha_{e_{HT}} < \hat{\alpha}_{HT-}$ print stall warning

VERTICAL TAIL

1. If $\alpha_{e_{VT}} > \hat{\alpha}_{VT+}$ print stall warning

2. If $\alpha_{e_{VT}} < \hat{\alpha}_{VT-}$ print stall warning

TAIL FORCE AND MOMENT RESOLUTION TO C.G.

HORIZONTAL TAIL - NOTE: - IF UMBRELLAS OPEN AND U>0; SET $\eta_{HT} = .5\ \eta_{HT}$

$$X^{HT}_{AERO} = [-C_{DHT} \cos(\alpha_{HT} - i_{HT}) \cos(\beta_{VT} - \sigma) + C_{LHT} \sin(\alpha_{HT} - i_{HT})]\ \bar{q}\ S_{HT}\ \eta_{HT}$$

$$Y^{HT}_{AERO} = -C_{DHT} \sin(\beta_{VT} - \sigma)\ \bar{q}\ S_{HT}\ \eta_{HT}$$

$$Z^{HT}_{AERO} = \left[-C_{LHT} \cos(\alpha_{HT} - i_{HT}) - C_{DHT} \cos(\beta_{VT} - \sigma) \sin(\alpha_{HT} - i_{HT})\right] \bar{q}\ S_{HT}\eta_{HT}$$

$$\mathcal{L}^{HT}_{AERO} = -Y^{HT}_{AERO}\ (Z_{HT} - Z_{CG})$$

$$M^{HT}_{AERO} = Z^{HT}_{AERO}(X_{CG} - X_{HT}) + X^{HT}_{AERO}(Z_{HT} - Z_{CG})$$

$$N^{HT}_{AERO} = -Y^{HT}_{AERO}\ (X_{CG} - X_{HT})$$

VERTICAL TAIL

$$X^{VT}_{AERO} = \left[-C_{DVT} \cos(\beta_{VT} - \sigma)\cos(\alpha_{HT} - i_{HT}) - C_{YVT}\sin(\beta_{VT} - \sigma)\cos(\alpha_{HT} - i_{HT})\right] \bar{q}\ S_{VT}\ \eta_{VT}$$

$$Y^{VT}_{AERO} = \left[C_{YVT} \cos(\beta_{VT} - \sigma) - C_{DVT} \sin(\beta_{VT} - \sigma)\right] \bar{q}\ S_{VT}\ \eta_{VT}$$

$$Z^{VT}_{AERO} = \left[-C_{DVT} \cos(\beta_{VT} - \sigma) \sin(\alpha_{HT} - i_{HT}) - C_{YVT} \sin(\beta_{VT} - \sigma) \sin(\alpha_{HT} - i_{HT})\right] \bar{q}\ S_{VT}\ \eta_{VT}$$

VERTICAL TAIL (CONTINUED)

$$\mathcal{L}^{VT}_{AERO} = -Y^{VT}_{AERO} \left(Z_{VT} - Z_{CG}\right)$$

$$M^{VT}_{AERO} = Z^{VT}_{AERO} \left(X_{CG} - X_{VT}\right) + X^{VT}_{ZERO} \left(Z_{VT} - Z_{CG}\right)$$

$$N^{VT}_{AERO} = -Y^{VT}_{AERO} \left(X_{CG} - X_{VT}\right)$$

TOTAL TAIL CONTRIBUTION

$$X^{T}_{AERO} = X^{VT}_{AERO} + X^{HT}_{AERO}$$

$$Z^{T}_{AERO} = Z^{VT}_{AERO} + Z^{HT}_{AERO}$$

$$M^{T}_{AERO} = M^{VT}_{AERO} + M^{HT}_{AERO}$$

$$Y^{T}_{AERO} = Y^{VT}_{AERO} + Y^{HT}_{AERO}$$

$$\mathcal{L}^{T}_{AERO} = \mathcal{L}^{VT}_{AERO} + \mathcal{L}^{HT}_{AERO}$$

$$N^{T}_{AERO} = N^{T}_{AERO} + N^{HT}_{AERO}$$

NACELLE AERODYNAMICS

NACELLE ANGLE OF ATTACK AND SIDESLIP

$$\alpha_{RN} = \mathrm{Tan}^{-1}\left[W_{RR}/U_{RR}\right] \quad , \quad q_{RN} = 1/2\ \rho\ V_{RR}^2$$

$$\alpha_{LN} = \mathrm{Tan}^{-1}\left[W_{RL}/U_{RL}\right] \quad , \quad q_{LN} = 1/2\ \rho\ V_{LR}^2$$

$$\beta_{RN} = \mathrm{Tan}^{-1}\left[V_{RR}/\sqrt{U_{RR}^2 + W_{RR}^2}\right]$$

$$\beta_{LN} = \mathrm{Tan}^{-1}\left[V_{RL}/\sqrt{U_{RL}^2 + W_{RL}^2}\right]$$

NACELLE WIND AXIS FORCE & MOMENT COEFFICIENTS

$$C_{DRN} = C_{DON} + K_{30}|\alpha_{RN}| + K_{31}|\alpha_{RN}^2|$$

$$C_{DLN} = C_{DON} + K_{30}|\alpha_{LN}| + K_{31}|\alpha_{LN}^2|$$

NOTE: CHECK RANGE OF αRN & αLN TO DETERMINE VALUES FOR CONSTANTS.

$$C_{LRN} = K_{32}\ \mathrm{Sin}\ \alpha_{RN}\ \cos\ \alpha_{RN}$$

$$C_{LLN} = K_{32}\ \mathrm{Sin}\ \alpha_{LN}\ \cos\ \alpha_{LN}$$

$$C_{MRN} = C_{MON} + K_{34}\ \mathrm{Sin}\ \alpha_{RN}\ \cos\ \alpha_{RN} + K_{35}\ (\sin\ \alpha_{RN}\ \cos\ \alpha_{RN})|\sin\ \alpha_{RN}\ \cos\ \alpha_{RN}|$$

$$C_{MLN} = C_{MON} + K_{34}\ \sin\ \alpha_{LN}\ \cos\ \alpha_{LN} + K_{35}\ (\sin\ \alpha_{LN}\ \cos\ \alpha_{LN})|\sin\ \alpha_{LN}\ \cos\ \alpha_{LN}|$$

SPECIAL CONDITIONS

1. IF: $V_{RR}^2 \leq 1\,(FT/SEC)^2$; RIGHT NACELLE AERO $\equiv$ 0.0 & HOLD VALUE OF α_{RN} & β_{RN}
2. IF: $V_{LR}^2 \leq 1\,(FT/SEC)^2$; LEFT NACELLE AERO $\equiv$ 0.0 & HOLD VALUE OF α_{LN} & β_{LN}

$$C_{YRN} = K_{36} \text{ Sin } \beta_{RN} \text{ Cos } \beta_{RN} + K_{37}(\text{Sin } \beta_{RN} \text{ Cos } \beta_{RN})|\text{Sin } \beta_{RN} \text{ Cos } \beta_{RN}|$$

$$C_{YLN} = K'_{36} \text{ Sin } \beta_{LN} \text{ Cos } \beta_{LN} + K'_{37}(\text{Sin } \beta_{LN} \text{ Cos } \beta_{LN})|\text{Sin } \beta_{LN} \text{ Cos } \beta_{LN}|$$

$$C_{NRN} = C_{NORN} + K_{38}\text{Sin } \beta_{RN}\text{Cos } \beta_{RN} + K_{39}(\text{Sin } \beta_{RN}\text{Cos}\beta_{RN})|\text{Sin}\beta_{RN} \text{ Cos } \beta_{RN}|$$

$$C_{NLN} = C_{NOLN} + K_{40}\text{Sin } \beta_{LN}\text{Cos } \beta_{LN} + K_{41}(\text{Sin } \beta_{LN}\text{Cos}\beta_{LN})|\text{Sin}\beta_{LN} \text{ Cos } \beta_{LN}|$$

$$C_{\mathcal{L}RN} = C_{\mathcal{L}LN} = 0.0$$

NACELLE FORCES & MOMENTS - NACELLE AXES

RIGHT

$$\Delta X'_{RN} = q_{RN}S_W[-C_{DRN}\cos\alpha_{RN} + C_{LRN}\sin\alpha_{RN} - C_{YRN}\sin\beta_{RN}\cos\alpha_{RN}]1/2$$

$$\Delta Y'_{RN} = q_{RN}S_W[C_{YRN} \cos\beta_{RN} - C_{DRN}\sin\beta_{RN}]1/2$$

$$\Delta Z'_{RN} = q_{RN}S_W[-C_{LRN}\cos\alpha_{RN} - C_{DRN}\cos\beta_{RN} \sin\alpha_{RN} - C_{YRN}\sin\beta_{RN}\sin\alpha_{RN}]1/2$$

$$\Delta \mathcal{L}'_{RN} = q_{RN}S_Wb_W[\frac{c_W}{b_W} C_{MRN} \sin\beta_{RN} \cos\alpha_{RN} - C_{NRN} \sin\alpha_{RN}]1/2$$

$$\Delta M'_{RN} = q_{RN}S_Wc_W [C_{MRN} \cos \beta_{RN}]1/2$$

$$\Delta N'_{RN} = q_{RN}S_Wb_W [C_{NRN} \cos \alpha_{RN} - \frac{c_W}{b_W} C_{MRN} \sin \beta_{RN} \cos\alpha_{RN}]1/2$$

LEFT

$$\Delta X'_{LN} = q_{LN}S_W [-C_{DLN} \cos \alpha_{LN} + C_{LLN} \sin \alpha_{LN} - C_{YLN} \sin \beta_{LN} \cos \alpha_{LN}]1/2$$

$$\Delta Y'_{LN} = q_{LN}S_W [C_{YLN} \cos \beta_{LN} - C_{DLN} \sin \beta_{LN}]1/2$$

$$\Delta Z'_{LN} = q_{LN}S_W[-C_{LLN}\cos\alpha_{LN}-C_{DLN}\cos\beta_{LN}\sin\alpha_{LN}-C_{YLN}\sin\beta_{LN}\sin\alpha_{LN}]1/2$$

$$\Delta \mathcal{L}'_{LN} = q_{LN}S_Wb_W[-\frac{c_W}{b_W} C_{MLN} \sin \beta_{LN} \cos \alpha_{LN} - C_{NLN} \sin \alpha_{LN}]1/2$$

$$\Delta M'_{LN} = q_{LN}S_Wc_W[C_{MLN}\cos \beta_{LN}]1/2$$

$$\Delta N'_{LN} = q_{LN}S_Wb_W [C_{NLN} \cos \alpha_{LN} - \frac{c_W}{b_W} C_{MLN} \sin \beta_{LN} \cos\alpha_{LN}]1/2$$

LANDING GEAR EQUATIONS

PERFORM THE FOLLOWING CALCULATIONS FOR EACH WHEEL OF THE LANDING GEAR WHERE - n = 1 LEFT MAIN GEAR
n = 2 RIGHT MAIN GEAR
n = 3 NOSE GEAR

LANDING GEAR - A/C LOCATION

$$X_n = - X_{CG} + X_{Gn}$$

$$Y_n = Y_{Gn}$$

$$Z_n = - Z_{CG} + Z_{Gn}$$

STRUT DEFLECTION

$$h_{G\theta n} = X_n \sin \theta - Z_n \cos \theta - r_n$$

$$h_{G\phi n} = \left[Y_n \sin \phi + (Z_n + r_n) (\cos \phi - 1) \right] \cos \theta$$

$$h_{Tn} = (-Z_{DOWN} + h_{G\theta n} - h_{G\phi n})/(\cos \phi \cos \theta)$$

RATE OF STRUT DEFLECTION

$$\dot{h}_{Tn} = -\dot{Z}_{DOWN} / (\cos \phi \cos \theta) + X_n q - Y_n p$$

VERTICAL FORCE

$$F_{GZn} = K_{STn} h_{Tn} + D_{STn} \dot{h}_{Tn}$$

NOTE: COMPUTE F_{GZn} ONLY IF $h_{Tn} < 0$;

IF $h_{Tn} \geq 0$; $F_{GZn} = 0.0$ &

REMAINING CALCULATIONS MAY BE SET

TO ZERO.

LONGITUDINAL FORCE:

$$F_{\mu n} = (\mu_0 + \mu_1 B_{Gn}) F_{GZn} \quad u/|u|$$

NOTE: B_{Gn} is percent brake pedal deflection.

SIDE FORCE:

$$F_{Sn} = \mu_S F_{GZn} \quad v/|v|$$

FORCE AND MOMENT CONTRIBUTION OF EACH WHEEL

$$\Delta X_n = F_{\mu n} - F_{GZn} \theta \quad (n = 1,2);$$

$$\Delta X_3 = F_{\mu 3} \cos \delta_{STEER} - F_{S3} \sin \delta_{STEER} - F_{GZ3} \theta$$

$$\Delta Y_n = F_{Sn} + F_{GZn} \phi \quad (n = 1,2);$$

$$\Delta Y_3 = F_{S3} \cos \delta_{STEER} + F_{\mu 3} \sin \delta_{STEER} + F_{GZ3} \phi$$

$$\Delta Z_n = F_{\mu n} \theta - F_{Sn} \phi + F_{GZn}$$

$$\Delta M_n = - \Delta Z_n X_n + \Delta X_n (Z_n + r_n + h_{Tn})$$

$$\Delta \mathcal{L}_n = \Delta Z_n Y_n - \Delta Y_n (Z_n + r_n + h_{Tn})$$

$$\Delta N_n = - \Delta X_n Y_n + X_n \Delta Y_n$$

$$\Delta X_{LG} = \sum_{1}^{3} \Delta X_n$$

$$\Delta Y_{LG} = \sum_{1}^{3} \Delta Y_n$$

$$\Delta Z_{LG} = \sum_{1}^{3} \Delta Z_n$$

$$\Delta \mathcal{L}_{LG} = \sum_{1}^{3} \Delta \mathcal{L}_n$$

$$\Delta M_{LG} = \sum_{1}^{3} \Delta M_n$$

$$\Delta N_{LG} = \sum_{1}^{3} \Delta N_n$$

FUSELAGE AERODYNAMICS

FUSELAGE INPUT EQUATIONS

$$\alpha_F = \mathrm{Tan}^{-1}\left|W/U\right| \qquad \beta_F = \mathrm{Tan}^{-1}\left[V/\sqrt{U^2 + W^2}\right]$$

$$\alpha_F' = \sin\alpha_F \cos\alpha_F \qquad \beta_F' = \mathrm{Sin}\,\beta_F \cos\beta_F$$

$$V_F = \sqrt{U^2 + V^2 + W^2}$$

$$q_F = 1/2\rho\, V_F^2$$

$$V_{FUS} = V_F\sqrt{\sigma_h}$$

FUSELAGE WIND AXIS COEFFICIENTS

$$C_{DF} = C_{DOF}\,(1 + K_0|\beta_F|^3) + K_2\,\alpha_F^2 + K_1|\alpha_F| + \Delta C_{DLG}$$

$$C_{LF} = K_3\,\alpha_F' + K_4\,\alpha_F'|\alpha_F'| + K_{42}$$

$$C_{YF} = K_7\,\beta_F' + K_8\,\beta_F'|\beta_F'|$$

$$C_{MF} = C_{MOF} + K_5\,\alpha_F' + K_6\,\alpha_F'|\alpha_F'| + \Delta C_{MLG}$$

$$C_{NF} = C_{NOF} + K_9\,\beta_F' + K_{10}\,\beta_F'|\beta_F'|$$

NOTE: IF GEAR IS UP; ΔC_{DLG} & $\Delta C_{MLG} \equiv 0.0$

SPECIAL CONDITIONS

1. IF $V_F^2 \leq 1\ (\mathrm{ft/sec})^2$ FUSELAGE AERO = 0.0 & HOLD VALUE OF α_F & β_F

FUSELAGE FORCES AND MOMENT ABOUT A/C C.G.

$$X^{F'}_{AERO} = [-C_{DF} \cos \alpha_F + C_{LF} \sin \alpha_F - C_{YF} \sin \beta_F \cos\alpha_F]\, q_F S_W$$

$$Y^{F'}_{AERO} = [C_{YF} \cos \beta_F - C_{DF} \sin \beta_F]\, q_F\, S_W$$

$$Z^{F'}_{AERO} = [-C_{LF} \cos \alpha_F - C_{DF} \cos \beta_F \sin \alpha_F$$

$$-C_{YF} \sin \beta_F \sin \alpha_F]\, q_F S_W$$

$$\mathcal{L}^{F'}_{AERO} = [-(c_w/b_w) C_{MF} \sin \beta_F \cos \alpha_F - C_{NF} \sin \alpha_F]\, q_F\, S_W\, b_W +$$

$$Y^{F'}_{AERO}\, [Z_{CG} - Z_{FAC}]$$

$$M^{F'}_{AERO} = [C_{MF} \cos \beta_F]\, q_F\, S_W\, c_W + Z^{F'}_{AERO}\, [X_{CG} - X_{FAC}]$$

$$-X^{F'}_{AERO}\, [Z_{CG} - Z_{FAC}]$$

$$N^{F'}_{AERO} = [C_{NF} \cos \alpha_F - (c_w/b_w)\, C_{MF} \sin \beta_F \sin \alpha_F]\, q_F\, S_W\, b_W$$

$$-\, Y^{F'}_{AERO}\, [X_{CG} - X_{FAC}]$$

$$X^{F}_{AERO} = X^{F'}_{AERO} + \Delta X_{LG}$$

$$Y^{F}_{AERO} = Y^{F'}_{AERO} + \Delta Y_{LG}$$

$$Z^{F}_{AERO} = Z^{F'}_{AERO} + \Delta Z_{LG}$$

$$\mathcal{L}^{F}_{AERO} = \mathcal{L}^{F'}_{AERO} + \Delta \mathcal{L}_{LG}$$

$$M^{F}_{AERO} = M^{F'}_{AERO} + \Delta M_{LG}$$

$$N^{F}_{AERO} = N^{F'}_{AERO} + \Delta N_{LG}$$

WING ON ROTOR INTERFERENCE

AVERAGE NACELLE INCIDENCE

$$\bar{i}_N = 0.5\ (i_{NL} + i_{NR})$$

AVERAGE LIFT COEFFICIENT

$$C_{LW} = 0.5\ \frac{(C_{LSRW} + C_{LSLW})}{(1 - \bar{C}_{TS})}$$

LOOK-UP: ε_{WRR} & ε_{WRL} @ $\bar{i}_N$ & C_{LW}

WING INTERFERENCE LOGIC

1. IF: Umbrellas open, set $C_{LW} = 0.0$ & $\varepsilon = \dfrac{\bar{\varepsilon}_P(1\text{-}GEF)}{\sqrt{1\text{-}M^2}}$

ROTOR/ROTOR INTERFERENCE

POSITIVE SIDESLIP, I.E., V > 0.0 (Logic Required)

$$\chi = 1.5708 - \varepsilon_{PRR}$$

$$\left(\frac{\delta v^*_{RL}}{v^*_{RR}}\right) = \left[T_1 + T_2\chi + T_3\chi^2\right]\chi$$

$$\delta v_{RL} = \left(\frac{\delta v^*_{RL}}{v^*_{RR}}\right) v_{*R}\sqrt{\frac{R_{RR}}{2\rho\pi R^2}}$$

$$\varepsilon'_{iRL} = -\tan^{-1}\left[\frac{\delta v_{RL}}{V_{LR} + 1.0}\right]$$

$$\varepsilon_{iRL} = (|\beta_F|)\ (.40528\ i_{NL})\ \varepsilon'_{iRL}$$

$$\varepsilon_{iLR} = 0.0$$

NEGATIVE SIDESLIP, I.E., V < 0.0

$$\chi = 1.5708 - \varepsilon_{PLR}$$

$$\left(\frac{\delta v^*_{LR}}{v^*_{LR}}\right) = \left[T_1 + T_2\chi + T_3\chi^2\right]\chi$$

$$\delta v_{LR} = \left(\frac{\delta v^*_{LR}}{v^*_{RR}}\right) v_{*L}\sqrt{\frac{R_{LR}}{2\rho\pi R^2}}$$

$$\varepsilon'_{iLR} = -\tan^{-1}\left[\frac{\delta v_{LR}}{V_{RR} + 1.0}\right]$$

$$\varepsilon_{iLR} = (|\beta_F|)\ (.40528\ i_{NR})\ \varepsilon'_{iLR}$$

$$\varepsilon_{iRL} = 0.0$$

NOTE: v_{*R} & v_{*L} FROM WING EQUATIONS.

ROTOR EQUATIONS

RIGHT ROTOR

$$\alpha_{RR} = \tan^{-1}\left\{\frac{\sqrt{V_{RR}^2 + (W_{RR} + U_{RR}\,\varepsilon_{WRR})^2}}{U_{RR}}\right\} + \varepsilon_{iLR}$$

$$V_{RR} = \sqrt{U_{RR}^2 + V_{RR}^2 + W_{RR}^2}\;;\quad \mu_{RR} = \frac{V_{RR}}{|\Omega_R|R}$$

LEFT ROTOR

$$\alpha_{LR} = \tan^{-1}\left\{\frac{\sqrt{V_{RL}^2 + (W_{RL} + U_{RL}\,\varepsilon_{WRL})^2}}{U_{RL}}\right\} + \varepsilon_{iRL}$$

$$V_{LR} = \sqrt{U_{RL}^2 + V_{RL}^2 + W_{RL}^2}\;;\quad \mu_{LR} = \frac{V_{LR}}{|\Omega_L|R}$$

ROTOR ANGULAR RATE TRANSFORMS

RIGHT-NACELLE AXES

$$P_{NR}^N = -p\cos i_{NR} + r\sin i_{NR}$$

$$Q_{NR}^N = q + \dot{i}_{NR}$$

$$R_{NR}^N = -r\cos i_{NR} - p\sin i_{NR}$$

LEFT-NACELLE AXES

$$P_{NL}^N = p\cos i_{NL} - r\sin i_{NL}$$

$$Q_{NL}^N = q + \dot{i}_{NL}$$

$$R_{NL}^N = r\cos i_{NL} + p\sin i_{NL}$$

RIGHT WIND AXES

$$P_{NR}^R = P_{NR}^N$$

$$Q_{NR}^R = Q_{NR}^N\cos\xi_{HR} + R_{NR}^N\sin\xi_{HR}$$

$$R_{NR}^R = R_{NR}^N\cos\xi_{HR} - Q_{NR}^N\sin\xi_{HR}$$

LEFT WIND AXES

$$P_{NL}^R = P_{NL}^N$$

$$Q_{NL}^R = Q_{NL}^N\cos\xi_{HL} - R_{NL}^N\sin\xi_{HL}$$

$$R_{NL}^R = R_{NL}^N\cos\xi_{HL} + Q_{NL}^N\sin\xi_{HL}$$

NOTE: USE WIND AXIS RATES IN ROTOR ROUTINE.

RIGHT ROTOR

THRUST

$$C'_{TRR} = \left[\frac{\tau_1 S+1}{\tau_2 S+1}\right]\left[C_{T_{ORR}} \text{ Cos } A_{1_{C_R}} \text{ Cos } B_{1_{C_R}}\right]$$

WHERE: $C_{T_O} = 0.000679\ \phi + 0.000015\ \phi^2$

$$+ 0.0022\ \mu\phi + 0.000211\ \mu^2\phi$$

and

$$\phi = \theta^\circ_{75} - \tan^{-1}\left[\frac{\mu \cos \alpha}{0.75}\right] - 6.3015\mu + 5.5816\mu^2$$

$$- 8\ \mu \sin \alpha + 1.115$$

GROUND EFFECT

$$h_{RR} = -Z_{DOWN} + (L_S \text{ Cos } i_{NR} - X_{CG}) \text{ Sin } \theta$$

$$+ \left[(L_S \text{ Sin } i_{NR} + Z_{CG}) \text{ Cos } \phi - Y_N \text{ Sin } \phi\right] \text{ Cos } \theta$$

$$\left(\frac{h}{D}\right)_{\substack{EFF\\RR}} = \frac{h_{RR}}{2R\left[\left|\text{Sin}(\theta + i_{NR}) \text{ Cos } \phi\right| + .0174\right]}$$

$$\left(\frac{T_{IGE}}{T_{OGE}}\right)_{RR} = \left[\left(\frac{h}{D}\right)^2_{\substack{EFF\\RR}} (.1741 - .6216\ \mu_{RR})\right.$$

$$+ \left(\frac{h}{D}\right)_{\substack{EFF\\RR}} (1.4779\ \mu_{RR} - .4143)$$

$$\left. + 1.2479 - .8806\ \mu_{RR}\right]$$

$$C_{T_{RR}} = C'_{T_{RR}} \left(\frac{T_{IGE}}{T_{OGE}}\right)_{RR}$$

SPECIAL CONDITIONS: IF $\mu_{RR} \geq 0.283$; $\left(\frac{T_{IGE}}{T_{OGE}}\right)_{RR} = 1.0$

or IF $\left(\frac{h}{D}\right)_{\substack{EFF\\RR}} \geq 1.3$; $\left(\frac{T_{IGE}}{T_{OGE}}\right)_{RR} = 1.0$

POWER

$$C_{P_{RR}} = C_{P_{ORR}}$$

$$= 0.00006 + 0.00057\,\mu + 0.000085\,\mu^2 + 1.12\,C_T{}^{3/2}$$

$$- 0.024075\,C_T + \mu C_T\,(0.53 + 0.456\,\mu - 39.937\,\mu C_T$$

$$+ 31.79\,C_T) + [0.0115\mu - 0.03\mu^2 - C_T(3.4\mu - 8\mu^2)]\,\frac{(\alpha_{RAD})}{\pi}$$

$$- 0.22064\mu\,(C_T + 0.001971)\,\sin\alpha$$

$$+ (0.3082\mu - 2.18\mu^2)\,C_T\,\sin\alpha$$

NORMAL FORCE

$$C_{NF_{RR}} = C_{NFO_{RR}} + \frac{dC_{NF_{RR}}}{dA_{1C_R}}\,A_{1CR} + \frac{dC_{NF_{RR}}}{dB_{1C_R}}\,B_{1C_R}$$

WHERE: $C_{NF_O} = C_{NF_1} = 0.068\mu^3\,\sin 2\alpha + [0.133695\mu C_T$

$$+ 73.444\mu\,C_T{}^2(1-\mu)]K \qquad 0 \leq \mu \leq 0.6$$

where $K = \sin\alpha$ for $\alpha > 20°$

and $K = \sin\alpha(10-0.45\alpha°)$ for $0 \leq \alpha \leq 20$

For $0.6 < \mu$

$$C_{NF} = (C_{NF_1})(1-0.8(\mu-0.6))$$

$$\frac{dC_{NF_{RR}}}{dA_{1CR}} = D_{NF_1}\,C_{T_{RR}} + D_{NF_2}\,\mu^2_{RR} + D_{NF_3}\,\mu_{RR} + D_{NF_4} + D_{NF_5}\,\mu_{RR}\,\sin 2\,\alpha_{RR}$$

$$\frac{dC_{NF_{RR}}}{dB_{1CR}} = E_{NF_1}\,C_{T_{RR}} + E_{NF_2}\,\mu^2_{RR} + E_{NF_3}\,\mu_{RR} + E_{NF_4} + E_{NF_5}\,\mu_{RR}\,\sin\alpha_{RR}$$

SIDE FORCE

$$C_{SF_{RR}} = C_{SF_{ORR}} + \frac{dC_{SF_{RR}}}{dA_{1CR}} A_{1CR} + \frac{dC_{SF_{RR}}}{dB_{1CR}} B_{1CR}$$

WHERE: $C_{SF_O} = 0.00430 \mu \sin \alpha - 0.0028827 \mu (\alpha_{RAD})^2$

$$+ 0.012 \mu \sin \alpha \, C_T (90-\psi°) + 2.19\mu^3 \sin \alpha \, C_T$$

where $\psi° = \tan^{-1} \left[\frac{\mu-\mu_i \cos \alpha}{\mu_i \sin \alpha} \right]$

and $\mu_i = \left[(\mu^4 + C_T^2)^{1/2} - \mu^2 \,/2 \right]^{1/2}$

$$\frac{dC_{SF_{RR}}}{dA_{1CR}} = D_{SF_1} C_{T_{RR}} + D_{SF_2} \mu_{RR}^2 + D_{SF_3} \mu_{RR} + D_{SF_4}$$

$$+ D_{SF_5} \mu_{RR} \sin \alpha_{RR}$$

$$\frac{dC_{SF_{RR}}}{dB_{1CR}} = E_{SF_1} C_{T_{RR}} + E_{SF_2} \mu_{RR}^2 + E_{SF_3} \mu_{RR} + E_{SF_4}$$

$$+ D_{SF_5} \mu_{RR} \sin 2 \alpha_{RR}$$

HUB PITCHING MOMENT

$$C_{PM_{RR}} = C_{PM_{ORR}} + \frac{dC_{PM_{RR}}}{dA_{1CR}} A_{1CR} + \frac{dC_{PM_{RR}}}{dB_{1CR}} B_{1CR} + \frac{dC_{PM_{RR}}}{dQ} Q_{NR}^{R}$$

WHERE:

$$C_{PM_O} = 0.009950\ \mu \sin \alpha - 0.010960 \mu^2 \sin \alpha$$

$$+ 0.0028126\ \mu \sin 2\alpha - 0.0057743\ \mu \sin \alpha \left[\frac{RPM}{298}\right]$$

$$+ (1.802\ \mu \sin \alpha - 7.56\ (\mu \sin\alpha)^2\ C_T$$

$$1000\ \frac{dC_{PM}}{dQ} = 1.5 + \mu \qquad 0 \le \mu \le .2$$

$$= 0.25 + 7.26\ \mu \qquad .2 < \mu \le .39$$

$$= 4.1681 - 2.79\ \mu \qquad \mu > .39$$

$$\frac{dC_{PM_{RR}}}{dA_{1CR}} = D_{PM_1} C_{T_{RR}} + D_{PM_2}\ \bar{\lambda}_{RR} + D_{PM_3}\ \mu_{RR} + D_{PM_4}$$

$$+ D_{PM_5}\ \mu_{RR} \sin 2\ \alpha_{RR} + D_{PM_6}\ \mu_{RR}\ (|\Omega_R| - \Omega_O)$$

HUB PITCHING MOMENT (CONTINUED)

$$\frac{d\ C_{PMRR}}{d\ B_{1CR}} = E_{PM_1} C_{TRR} + E_{PM_2} \mu_{RR}^2 + E_{PM_3} \mu + E_{PM_4}$$

$$+ E_{PM_5} \mu_{RR} \sin \alpha_{RR} + E_{PM_6} \mu_{RR} (|\Omega_R| - \Omega_o)$$

HUB YAWING MOMENT

$$C_{YM_{RR}} = C_{YM_{ORR}} + \frac{dC_{YM_{RR}}}{dA_{1CR}} A_{1CR} + \frac{dC_{YM_{RR}}}{dB_{1CR}} B_{1CR} + \frac{dC_{YM_{RR}}}{dR} R_{NR}^{R}$$

Where:

For $0 \le \mu \le 0.37$

$$C_{YM} = (0.018369\ \mu - 0.0007)\mu \sin\alpha - 1.2\ \mu^2\ C_T \sin \alpha$$

$$+ \left[0.00631 - 0.002604\mu - 0.004877 \left(\frac{RPM}{298} - 1\right)\right]\left(\frac{RPM}{298} - 1\right) \sin \alpha$$

and for $\mu > 0.37$

$$C_{YM} = (0.01916 - 0.15321\ (\mu - 0.5435)^2) \sin \alpha$$

$$- 1.2\ \mu^2\ C_T \sin \alpha$$

$$\frac{dC_{YM}}{dR} = - \frac{dC_{PM}}{dQ}$$

3

HUB YAWING MOMENT (CONTINUED)

$$\frac{dC_{YM_{RR}}}{dA_{1CR}} = D_{YM_1} C_{T_{RR}} + D_{YM_2} \mu_{RR}^2 + D_{YM_3} \mu_{RR} + D_{YM_4}$$

$$+ D_{YM_5} \mu_{RR} \sin \alpha_{RR} + E_{YM_6} \mu_{RR} (|\Omega_R| - \Omega_o)$$

$$\frac{dC_{YM_{RR}}}{dB_{1CR}} = E_{YM_1} C_{T_{RR}} + E_{YM_2} \mu_{RR}^2 + E_{YM_3} \mu_{RR} + E_{YM_4}$$

$$+ E_{YM_5} \mu_{RR} \sin 2\alpha_{RR} + E_{YM_6} \mu_{RR} (|\Omega_R| - \Omega_o)$$

ROTOR FORCE & MOMENT CALCULATION

$$T_R = f_{T_R} C_{TRR} \rho \pi R^4 \Omega_R^2$$

$$NF_R = f_{NF_R} C_{NFRR} \rho \pi R^4 \Omega_R^2$$

$$SF_R = f_{SF_R} C_{SFRR} \rho \pi R^4 \Omega_R^2$$

$$M_R = f_{PM_R} C_{PMRR} \rho \pi R^5 \Omega_R^2$$

$$N_R = f_{YM_R} C_{YMRR} \rho \pi R^5 \Omega_R^2$$

$$Q_{RREQ} = f_{Q_R} C_{PRR} \rho \pi R^5 \Omega_R^2$$

$$RHP_{RR} = \left| Q_{RREQ} \frac{\Omega_R}{550} \right|$$

LEFT ROTOR FOLLOWS SIMILAR FORMAT WITH SUBSCRIPTS CHANGED.

THE LEFT ROTOR ALTITUDE EQUATION IS AS FOLLOWS:

$$h_{LR} = -Z_{DOWN} + (L_S \cos i_{NL} - X_{CG}) \sin \theta$$
$$+ [(L_S \sin i_{NL} + Z_{CG}) \cos \phi + Y_N \sin \phi] \cos \theta$$

or;

$$h_{LR} = h_{RR} + 2 Y_N \sin \phi \cos \theta$$

ROTOR FORCE & MOMENT RESOLUTION

HUB MOMENTS - NACELLE AXES

LEFT

$$\mathcal{L}_{LRH} = - Q_{LREQ} - I_P \dot{\Omega}_L \kappa$$

$$M_{LRH} = M_L \cos \xi_{HL} - N_L \sin \xi_{HL}$$

$$- (p \sin i_{NL} + r \cos i_{NL}) (KI_p\Omega_L + N_{EL} K_1 I_E \Omega_{EL})$$

$$N_{LRH} = -N_L \cos \xi_{HL} - M_L \sin \xi_{HL} + (KI_p\Omega_L + N_{EL} K_1 I_E \Omega_{EL})(q+\dot{i}_{NL})$$

RIGHT

$$\mathcal{L}_{RRH} = Q_{RREQ} + I_P \dot{\Omega}_R \kappa$$

$$M_{RRH} = M_R \cos \xi_{HR} + N_R \sin \xi_{HR}$$

$$+ (p \sin i_{NR} + r \cos i_{NR})(KI_p\Omega_R - N_{ER} K_1 I_E \Omega_{ER})$$

$$N_{RRH} = N_R \cos \xi_{HR} - M_R \sin \xi_{HR} - (KI_p\Omega_R - N_{ER} K_1 I_E \Omega_{ER})(q + \dot{i}_{NR})$$

<u>NOTE</u>: NACELLE AXES ARE RIGHT HANDED SYSTEMS

K_1 = 0 if non-tilting engines

= 1 if tilting engines

RESOLUTION OF ROTOR/NACELLE FORCES TO BODY AXES AT PIVOTS

LEFT ROTOR

$$X^{NL}_{AERO} = (T_L + \Delta X'_{LN})\cos i_{NL} - \sin i_{NL}(NF_L \cos \xi_{HL} + SF_L \sin \xi_{HL} - \Delta Z'_{LN})$$

$$Y^{NL}_{AERO} = SF_L \cos \xi_{HL} - NF_L \sin \xi_{HL} + \Delta Y'_{LN}$$

$$Z^{NL'}_{AERO} = -(T_L + \Delta X'_{LN}) \sin i_{NL} - \cos i_{NL} (NF_L \cos \xi_{HL} + SF_L \sin \xi_{HL} - \Delta Z'_{LN})$$

$$\mathcal{L}^{NL'}_{AERO} = (\mathcal{L}_{LRH} + \Delta\mathcal{L}'_{LN}) \cos i_{NL} + \sin i_{NL} (N_{LRH} + \Delta N'_{LN} + L_s Y^{NL}_{AERO})$$

$$M^{NL}_{AERO} = M_{LRH} + \Delta M'_{LN} + NF_L L_s \cos \xi_{HL} + SF_L L_s \sin \xi_{HL} - L_s \Delta Z'_{LN} - I_E \Omega_{EL} r N_{EL} K_2$$

$$N^{NL}_{AERO} = \cos i_{NL} (N_{LRH} + \Delta N'_{LN} + L_s Y^{NL}_{AERO}) - \sin i_{NL}(\mathcal{L}_{LRH} + \Delta\mathcal{L}'_{LN}) + I_E \Omega_{EL} q N_{EL} K_2$$

NACELLE EQUATION INPUT - LEFT

$$M_{NLAERO} = M^{NL}_{AERO} + I_E \Omega_{EL} r N_{EL} K_2$$

GLAS INPUTS - LEFT

$$M_{\substack{NLAERO\\GLAS}} = M_{LRH} + L_s (NF_L \cos \xi_{HL} + SF_L \sin \xi_{HL})$$

$$N_{\substack{NLAERO\\GLAS}} = N_{LRH} + L_s (SF_L \cos \xi_{HL} - NF_L \sin \xi_{HL})$$

K_2 = 0 if tilting engines

= 1 if non-tilting engines

RIGHT ROTOR

$$X^{NR}_{AERO} = (T_R + \Delta X'_{RN}) \cos i_{NR} + \sin i_{NR} (-NF_R \cos \xi_{HR} + SF_R \sin \xi_{HR} + \Delta Z'_{RN})$$

$$Y^{NR}_{AERO} = -SF_R \cos \xi_{HR} - NF_R \sin \xi_{HR} + \Delta Y'_{RN}$$

$$Z^{NR'}_{AERO} = -(T_R + \Delta X'_{RN}) \sin i_{NR} + \cos i_{NR} (-NF_R \cos \xi_{HR} + SF_R \sin \xi_{HR} + \Delta Z'_{RN})$$

$$\mathcal{L}^{NR'}_{AERO} = (\mathcal{L}_{RRH} + \Delta \mathcal{L}'_{RN}) \cos i_{NR} + \sin i_{NR} (N_{RRH} + L_s Y^{NR}_{AERO} + \Delta N'_{RN})$$

$$M^{NR}_{AERO} = M_{RRH} + \Delta M'_{RN} + NF_R L_s \cos \xi_{HR} - SF_R L_s \sin \xi_{HR} - L_s \Delta Z'_{RN} - I_E \Omega_{ER} r N_{ER} K_2$$

$$N^{NR}_{AERO} = \cos i_{NR} (N_{RRH} + \Delta N'_{RN} + L_s Y^{NR}_{AERO}) - \sin i_{NR} (\mathcal{L}_{RRH} + \Delta \mathcal{L}'_{RN}) + I_E \Omega_{ER} q N_{ER} K_2$$

NACELLE EQUATION INPUT - RIGHT

$$M_{NRAERO} = M^{NR}_{AERO} + I_E \Omega_{ER} r N_{ER} K_2$$

GLAS INPUTS - RIGHT

$$M_{\substack{NRAERO \\ GLAS}} = M_{RRH} + L_s (NF_R \cos \xi_{HR} - SF_R \sin \xi_{HR})$$

$$N_{\substack{NRAERO \\ GLAS}} = N_{RRH} - L_s (SF_R \cos \xi_{HR} + NF \sin \xi_{HR})$$

WING VERTICAL BENDING

RIGHT WING TIP DEFLECTION

$$\bar{a}_{RT} = \frac{Z_{AERO}}{m} + Y_N \dot{p}$$

$$\bar{a}_{RWAC} = \frac{Z_{AERO}}{m} + Y_{WAC} \dot{p}$$

$$h_{1_R} = K_{W_1} Z^{NR'}_{AERO} + K_{W_2} Z^{RW'}_{AERO} + K_{W_3} \mathcal{L}^{NR'}_{AERO} - K_{W_4} \bar{a}_{RT} - K_{W_5} \bar{a}_{RWAC}$$

$$\dot{h}_{1_R} = \Delta h_{1_R}/\Delta t$$

Where Δh_{1_R} is the difference of h_{1_R} between time frames and Δt is the time frame.

RIGHT WING A.C. DEFLECTION

$$h_{1_{RWAC}} = K_{W_6} Z^{NR'}_{AERO} + K_{W_7} Z^{RW'}_{AERO} + K_{W_8} \mathcal{L}^{NR'}_{AERO} - K_{W_9} \bar{a}_{RT} - K_{W_{10}} \bar{a}_{RWAC}$$

$$\dot{h}_{1_{RWAC}} = \Delta h_{1_{RWAC}}/\Delta t$$

Where: $\Delta h_{1_{RWAC}}$ is the difference of $h_{1_{RWAC}}$ between time frames and Δt is the time frame.

FORCE AND MOMENT EFFECTS

$$\ddot{Z}^{NR}_{AERO} = -2\xi_{W1}\,\omega_{W1}\,\dot{Z}^{NR}_{AERO} - \omega^2_{W1} Z^{NR}_{AERO} + \omega^2_{W1} Z^{NR'}_{AERO}$$

$$\ddot{Z}^{RW}_{AERO} = -2\xi_{W2}\,\omega_{W2}\,\dot{Z}^{RW}_{AERO} - \omega^2_{W2} Z^{RW}_{AERO} + \omega^2_{W2} Z^{RW'}_{AERO}$$

$$\ddot{\mathcal{L}}^{NR}_{AERO} = -2\xi_{W3}\,\omega_{W3}\,\dot{\mathcal{L}}^{NR}_{AERO} - \omega^2_{W3} \mathcal{L}^{NR}_{AERO} + \omega^2_{W3} \mathcal{L}^{NR'}_{AERO}$$

LEFT WING DEFLECTION

$$\bar{a}_{LT} = \frac{Z_{AERO}}{m} - Y_N \dot{p}$$

$$\bar{a}_{LWAC} = \frac{Z_{AERO}}{m} - Y_{WAC} \dot{p}$$

$$h_{1L} = K_{W_1} Z^{NL'}_{AERO} + K_{W_2} \dot{Z}^{LW'}_{AERO} - K_{W_3} \mathcal{L}^{NL'}_{AERO} - K_{W_4} \bar{a}_{LT} - K_{W_5} \bar{a}_{LWAC}$$

$$\dot{h}_{1L} = \Delta h_{1L}/\Delta t$$

Where: Δh_{1L} is the difference of h_{1L} between time frames and Δt is the time frame.

LEFT WING A.C. DEFLECTION

$$h_{1L_{WAC}} = K_{W_6} Z^{NL'}_{AERO} + K_{W_7} Z^{LW'}_{AERO} - K_{W_8} \mathcal{L}^{NL'}_{AERO} - K_{W_9} \bar{a}_{LT} - K_{W_{10}} \bar{a}_{LWAC}$$

$$\dot{h}_{1L} = \Delta h_{1L_{WAC}}/\Delta t$$

Where: $\Delta h_{1L_{WAC}}$ is the difference of $h_{1L_{WAC}}$ between time frames and Δt is the time frame.

FORCE AND MOMENT EFFECTS

$$\ddot{Z}^{NL}_{AERO} = -2\xi_{W1}\, \omega_{W1}\, \dot{Z}^{NL}_{AERO} - \omega^2_{W1}\, Z^{NL}_{AERO} + \omega^2_{W1}\, Z^{NL'}_{AERO}$$

$$\ddot{Z}^{LW}_{AERO} = -2\xi_{W2}\, \omega_{W2}\, \dot{Z}^{LW}_{AERO} - \omega^2_{W2}\, Z^{LW}_{AERO} + \omega^2_{W2}\, Z^{LW'}_{AERO}$$

$$\ddot{\mathcal{L}}^{NL}_{AERO} = -2\xi_{W3}\, \omega_{W3}\, \dot{\mathcal{L}}^{NL}_{AERO} - \omega^2_{W3}\, \mathcal{L}^{NL}_{AERO} + \omega^2_{W3}\, \mathcal{L}^{NL'}_{AERO}$$

FORM Z^{NL}_{AERO}, Z^{LW}_{AERO}, and $\mathcal{L}^{NL}_{AERO}$

WING TORSION

LEFT WING TWIST AT TIP

$$K_{\theta t}\ \theta_{tLW} = M_{NLACT} - I_E\ \Omega_{EL}\ r + q_{SLW}\ \frac{c_w^2 b_w}{2}\ C_{MO}\ (1 - C_{TSLR}) + (1 - C_{TSLR})\ q_{SLW}\ c_w^2 \left(\frac{dC_{MWc/4}}{dC_L} + \frac{X_{WAC}}{c_w}\right)\left(\frac{C_{L\alpha} b_W}{6\Pi}\right)\left(4\theta_{tLW} + 3\Pi\alpha_{LWRIGID}\right)$$

RIGHT WING TWIST AT TIP

$$K_{\theta t}\ \theta_{tRW} = M_{NRACT} - I_E\ \Omega_{ER}\ r + q_{SRW}\ \frac{c_w^2 b_w}{2}\ C_{MO}\ (1 - C_{TSRR}) + (1 - C_{TSRR})\ q_{SRW}\ c_w^2 \left(\frac{dC_{MWC/4}}{dC_L} + \frac{X_{WAC}}{c_W}\right)\left(\frac{C_{L\alpha} b_W}{6\Pi}\right)\left(4\theta_{tRW} + 3\Pi\alpha_{RWRIGID}\right)$$

WHERE: $C_{MO} = C_1 + C_2\ \delta_F + C_3\ \delta_F^2$

$$\theta_{tLWAC} = \frac{Y_{WAC}}{Y_N}\ \theta_{tLW}$$

$$\theta_{tRWAC} = \frac{Y_{WAC}}{Y_N}\ \theta_{tRW}$$

NOTE: If umbrellas are open; set terms containing $q_S\ (1 - C_{TS})$ equal to zero.

TOTAL FORCE AND MOMENT SUMMATION ABOUT C.G.

$$X_{AERO} = X^{NL}_{AERO} + X^{NR}_{AERO} + X^{F}_{AERO} + X^{LW}_{AERO} + X^{RW}_{AERO} + X^{T}_{AERO}$$

$$Y_{AERO} = Y^{NL}_{AERO} + Y^{NR}_{AERO} + Y^{F}_{AERO} + Y^{LW}_{AERO} + Y^{RW}_{AERO} + Y^{T}_{AERO}$$

$$Z_{AERO} = Z^{NL}_{AERO} + Z^{NR}_{AERO} + Z^{F}_{AERO} + Z^{LW}_{AERO} + Z^{RW}_{AERO} + Z^{T}_{AERO}$$

$$\mathcal{L}_{AERO} = \mathcal{L}^{NL}_{AERO} + \mathcal{L}^{NR}_{AERO} + \mathcal{L}^{F}_{AERO} + \mathcal{L}^{W}_{AERO} + \mathcal{L}^{T}_{AERO}$$

$$+ Y_N \left(Z^{NR}_{AERO} - Z^{NL}_{AERO}\right) + Z_{CG} \left(Y^{NL}_{AERO} + Y^{NR}_{AERO}\right)$$

$$M_{AERO} = M^{NL}_{AERO} + M^{NR}_{AERO} + M^{F}_{AERO} + M^{W}_{AERO} + M^{T}_{AERO}$$

$$+ X_{CG} \left(Z^{NL}_{AERO} + Z^{NR}_{AERO}\right) - Z_{CG} \left(X^{NL}_{AERO} + X^{NR}_{AERO}\right)$$

$$N_{AERO} = N^{NL}_{AERO} + N^{NR}_{AERO} + N^{F}_{AERO} + N^{W}_{AERO} + N^{T}_{AERO}$$

$$+ Y_N \left(X^{NL}_{AERC} - X^{NR}_{AERO}\right) - X_{CG} \left(Y^{NL}_{AERO} + Y^{NR}_{AERO}\right)$$

BASIC EQUATIONS OF MOTION

PRELIMINARY CALCULATIONS

FUSE[illegible] W.R.T. A/C C.G.

X_f [illegible] $_{CG}$

$$Z_f = h_f - Z_{CG}$$

WING C.G. W.R.T. A/C C.G.

$$X_w = \ell_w - X_{CG}$$

$$Z_w = h_w - Z_{CG}$$

NACELLE C.G.'s W.R.T. A/C C.G.

$$X_R = \ell \cos (i_{NR} - \lambda) - X_{CG}$$

$$Z_R = -\ell \sin (i_{NR} - \lambda) - Z_{CG}$$

$$X_L = \ell \cos (i_{NL} - \lambda) - X_{CG}$$

$$Z_L = -\ell \sin (i_{NL} - \lambda) - Z_{CG}$$

PRELIMINARY CALCULATIONS

INERTIA TERMS

$$\sum_k I_{ij}^{(k)} = I_{ij}^{(f)} + I_{ij}^{(w)} + 2I'_{ij}$$

$$I_{xx} = \sum_k I_{xx}^{(k)} + (I'_{zz} - I'_{xx})(\sin^2 i_{NR} + \sin^2 i_{NL})$$

$$-I'_{xz}\ (\sin 2\, i_{NR} + \sin 2\, i_{NL}) + 2\, m_N\, Y_N^2$$

$$+ m_f\, h_f\, Z_f + m_w\, h_w\, Z_w$$

$$- \ell m_N [Z_R \sin (i_{NR} - \lambda) + Z_L \sin (i_{NL} - \lambda)]$$

$$J_{xx} = I_{zz} - I_{yy}$$

$$I_{xz} = I_{xz}^{(f)} + I_{xz}^{(w)} + 1/2\ (I'_{xx} - I'_{zz})(\sin 2\, i_{NR} + \sin 2\, i_{NL})$$

$$+ I'_{xz}\ (\cos 2\, i_{NR} + \cos 2\, i_{NL}) + (m_f\, \ell_f\, Z_f + m_w\, \ell_w\, Z_w)$$

$$+ m_N \ell\ [Z_R \cos (i_{NR} - \lambda) + Z_L \cos (i_{NL} - \lambda)]$$

INERTIA TERMS

$$I_{yy} = \sum_k I_{yy}^{(k)} + m_f (\ell_f X_f + h_f Z_f) + m_w (\ell_w X_w + h_w Z_w)$$

$$+ m_N \ell [X_R \cos (i_{NR} - \lambda) - Z_R \sin (i_{NR} - \lambda)]$$

$$+ m_N \ell [X_L \cos (i_{NL} - \lambda) - Z_L \sin (i_{NL} - \lambda)]$$

$$J_{yy} = I_{xx} - I_{zz}$$

INERTIA TERMS

$$I_{zz} = \sum_k I_{zz}^{(k)} + (I'_{xx} - I'_{zz}) (\sin^2 i_{NR} + \sin^2 i_{NL})$$

$$+ I'_{xz} (\sin 2 i_{NR} + \sin 2 i_{NL}) + 2 m_N Y_N^2$$

$$+ m_f \ell_f X_f + m_W \ell_W X_W$$

$$+ m_N \ell [X_R \cos (i_{NR} - \lambda) + X_L \cos (i_{NL} - \lambda)]$$

$$J_{zz} = I_{yy} - I_{xx}$$

ROLL EQUATION

$$I_{xx}\dot{p} = -J_{xx}\,rq + I_{xz}\,(\dot{r} + pq)$$
$$+ \ell m_N\, Y_N\left\{\ddot{i}_{NR}\cos(i_{NR} - \lambda) - \ddot{i}_{NL}\cos(i_{NL} - \lambda)\right\}$$
$$+ \mathcal{L}_{AERO}$$

PITCH EQUATION

$$I_{yy}\dot{q} = -J_{yy}pr - I_{xz}\,(p^2 - r^2)$$
$$- \ddot{i}_{NR}\left\{I'_{yy} + \ell m_N[-Z_R\sin(i_{NR} - \lambda) + X_R\cos(i_{NR} - \lambda)]\right\}$$
$$- \ddot{i}_{NL}\left\{I'_{yy} + \ell m_N[-Z_L\sin(i_{NL} - \lambda) + X_L\cos(i_{NL} - \lambda)]\right\}$$
$$+ M_{AERO}$$

YAW EQUATION

$$I_{zz}\dot{r} = -J_{zz}\,pq - (rq - \dot{p})\,I_{xz}$$
$$- \ell m_N\, Y_N\left\{\ddot{i}_{NR}\sin(i_{NR} - \lambda) - \ddot{i}_{NL}\sin(i_{NL} - \lambda)\right\}$$
$$+ N_{AERO}$$

RIGHT NACELLE ACTUATOR PITCHING MOMENT EQUATION

$$
\begin{aligned}
M_{NRACT} = &- \ddot{i}_{NR} \left[I'_{yy} + \ell^2 m_N \left(1 - \frac{m_N}{m}\right) \right] \\
&- \ell^2 m_N \left(1 - \frac{m_N}{m}\right) \Big[- pr \cos 2 (i_{NR} - \lambda) + \dot{q} \\
&+ (r^2 - p^2) \sin (i_{NR} - \lambda) \cos (i_{NR} - \lambda) \Big] \\
&- (r^2 - p^2) \left[I'_{zz} \sin i_{NR} \cos i_{NR} \right] - I'_{yy} \dot{q} \\
&+ \ell \frac{m_N}{m} \left[X_{AERO} \sin (i_{NR} - \lambda) + Z_{AERO} \cos (i_{NR} - \lambda) \right] \\
&- \ell m_N Y_N \Big\{ (\dot{r} - pq) \left[\sin (i_{NR} - \lambda) \right] \\
&\qquad - (\dot{p} + rq) \left[\cos (i_{NR} - \lambda) \right] \Big\} \\
&+ M_{NRAERO}
\end{aligned}
$$

LEFT NACELLE PITCHING MOMENT EQUATION OBTAINED BY CHANGING SIGN OF Y_N AND CHANGING SUBSCRIPT FROM R TO L.

NOTE: THE ABOVE EQUATION MUST BE CALCULATED FOR WING TORSION CALCULATION ONLY.

MOTION OF A.C. MASS CENTER

$$\dot{U} = \frac{X_{AERO}}{m} - g \sin \theta - qW + rV$$

$$\dot{V} = \frac{Y_{AERO}}{m} + g \cos \theta \sin \phi - rU + pW$$

$$\dot{W} = \frac{Z_{AERO}}{m} + g \cos \theta \cos \phi + qU - pV$$

EULER ANGLE CALCULATION

$$\dot{\psi} = (r \cos \phi + q \sin \phi)/\cos \theta$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\phi} = p + \dot{\psi} \sin \theta$$

AIRCRAFT CONDITION CALCULATIONS

GROUND TRACK

NORTHWARD VELOCITY

$$\dot{X}_{NORTH} = U \cos\theta \cos\psi + V (\sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi) + W (\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi)$$

EASTWARD VELOCITY

$$\dot{Y}_{EAST} = U \cos\theta \sin\psi + V (\sin\phi \sin\theta \sin\psi + \cos\phi \cos\psi) + W (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi)$$

DOWNWARD VELOCITY

$$\dot{Z}_{DOWN} = - U \sin\theta + V \sin\phi \cos\theta + W \cos\phi \cos\theta$$

PILOT STATION ACCELERATIONS (BODY AXES)

$$a_{XPA} = \frac{X_{AERO}}{m} + (\dot{q} + pr)\ (Z_{PA} - Z_{CG})$$

$$+ (q^2 + r^2)\ (X_{CG} - \ell_{PA}) + Y_{PA}\ (pq - \dot{r})$$

$$- 2\ q\ \dot{Z}_{CG} - \ddot{X}_{CG}$$

$$a_{YPA} = \frac{Y_{AERO}}{m} + (\dot{p} - qr)\ (Z_{CG} - Z_{PA}) + (\dot{r} + pq)(\ell_{PA} - X_{CG})$$

$$- Y_{PA}\ (r^2 + p^2) + 2\ (p\dot{Z}_{CG} - r\dot{X}_{CG})$$

$$a_{ZPA} = \frac{Z_{AERO}}{m} + (\dot{q} - pr)\ (X_{CG} - \ell_{PA}) + (p^2 + q^2)\ (Z_{CG} - Z_{PA})$$

$$+ Y_{PA}\ (\dot{p} + qr) + 2q\dot{X}_{CG} - \ddot{Z}_{CG}$$

PILOT STATION VELOCITIES (BODY AXES)

$$U_{PA} = U_P + qZ_{PA} - rY_{PA}$$

$$V_{PA} = V_P + r\ell_{PA} - pZ_{PA}$$

$$W_{PA} = W_P + pY_{PA} - q\ell_{PA}$$

GUST MODEL

The gust model will be that represented by NASA-AMES program NAPS-80. The output of this program, in the form of gust velocity components Ug, Vg, Wg, pg, qg, rg will be added to the aircraft velocity components in clear air as follows:

$$U = U' + U_g \qquad p = p' + p_g$$

$$V = V' + V_g \qquad q = q' + q_g$$

$$W = W' + W_g \qquad r = r' + r_g$$

PRELIMINARY CALCULATIONS PREPROCESSOR

$$g = 32.174$$

$$m = [w'_f + w'_{NT} + w'_{HT} + w'_{VT} + w'_W + w'_{NF} + w'_{CR} + w'_{FUEL} + w'_C]/g$$

$$m_N = w'_{NT}\ /2g$$

$$m_f = [w'_f + w'_{HT} + w'_{VT} + w'_{CR} + w'_C]/g$$

$$m_W = [w'_W + w'_{FUEL} + w'_{NF}]/g$$

$$\ell'_f = [(FS)_P - (FS)_{f'}]/12$$

$$\ell'_{NT} = [(FS)_P - (FS)_{HTCG}]/12$$

$$\ell'_{VT} = [(FS)_P - (FS)_{VTCG}]/12$$

$$\ell'_{PA} = [(FS)_P - (FS)_{PA}]/12$$

$$\ell'_C = [(FS)_P - (FS)_C]/12$$

$$\ell'_f = [w'_f \ell'_f + w'_{HT} \ell'_{HT} + w'_{VT} \ell'_{VT} + w'_{CR} \ell'_{PA} + w'_C \ell'_C]/(m_f g)$$

$$\ell'_W = [(FS)_P - (FS)_W]/12$$

$$\ell'_{FUEL} = [(FS)_P - (FS)_{FUEL}]/12$$

$$\ell'_{NF} = [(FS)_P - (FS)_{NF}]/12$$

$$\ell_W = [w'_W\ \ell'_W + w'_{FUEL}\ \ell'_{FUEL} + w'_{NF}\ \ell'_{NF}]/(m_W g)$$

$z'_f = [(WL)_P - (WL)_{f'}]/12$

$z'_{HT} = [(WL)_P - (WL)_{HTCG}]/12$

$z'_{VT} = [(WL)_P - (WL)_{VTCG}]/12$

$Z_{PA} = [(WL)_P - (WL)_{PA}]/12$

$z'_c = [(WL)_P - (WL)_C]/12$

$h_f = 1/(32.174\ m_f)\ [w'_f z'_f + w'_{HT} z'_{HT} + w'_{VT} z'_{VT} + w'_{CR} Z_{PA} + w'_C z'_C]$

$z'_W = [(WL)_P - (WL)_W]/12$

$z'_{FUEL} = [(WL)_P - (WL)_{FUEL}]/12$

$z'_{NF} = [(WL)_P - (WL)_{NF}]/12$

$h_W = 1/(32.174\ m_W)\ [w'_W z'_W + w'_{FUEL} z'_{FUEL} + w'_{NF} z'_{NF}]$

$X_{WAC} = [(FS)_P - (FS)_{WAC}]/12$

$Y_{WAC} = [(BL)_{WAC}]/12$

$Z_{WAC} = [(WL)_P - (WL)_{WAC}]/12$

$Y_N = [(BL)_N]/12$

$$X_{HT} = [\ (FS)_P - (FS)_{HT}]\ 1/12$$

$$Z_{HT} = [\ (WL)_P - (WL)_{HT}]\ 1/12$$

$$X_{VT} = [\ (FS)_P - (FS)_{VT}]\ 1/12$$

$$Z_{VT} = [\ (WL)_P - (WL)_{VT}]\ 1/12$$

$$A = 3.14159\ R^2$$

$$\overline{Y}_{WAC} = [\ (\overline{BL})_{WAC}]\ 1/12$$

$$X_{G2} = X_{G1} = [\ (FS)_P - (FS)_{G2}]\ 1/12$$

$$Z_{G2} = Z_{G1} = [\ (WL)_P - (WL)_{G2}]\ 1/12$$

$$Y_{G2} = [\ (BL)_{G2}]\ 1/12$$

$$Y_{G1} = -Y_{G2}$$

$$Y_{G3} = 0$$

$Y_{PA} = [\ (BL)_{PA}]\ 1/12$; POSITIVE FOR PILOT IN RIGHT SEAT

$$X_{fAC} = [\ (FS)_P - (FS)_{fAC}]\ 1/12$$

$$Z_{fAC} = [\ (WL)_P - (WL)_{fAC}]\ 1/12$$

$X_{c/2} = [(FS)_P - (FS)_{c/2}]/12$

$Z_{G3} = [(WL)_P - (WL)_{G3}]/12$

$X_{G3} = [(FS)_P - (FS)_{G3}]/12$

$Y_{NF} = [(BL)_{NFCG}]/12$

$Y_{HT} = [(BL)_{HTCG}]/12$

$Y_W = [(BL)_{WCG}]/12$

$Y_{FUEL} = [(BL)_{FUELCG}]/12$

INERTIA CALCULATIONS

$\eta'_{f'} = \ell_f - \ell'_f$

$\delta'_{f'} = h_f - z'_f$

$\eta'_{HT} = \ell_f - \ell'_{HT}$

$\delta'_{HT} = h_f - z'_{HT}$

$\eta'_{VT} = \ell_f - \ell'_{VT}$

$\delta'_{VT} = h_f - z'_{VT}$

$\eta'_{CR} = \ell_f - \ell_{PA}$

$\delta'_{CR} = h_f - Z_{PA}$

$$\eta'_c = \ell_f - \ell'_c$$

$$\delta'_c = h_f - z'_c$$

$$I^{(f)}_{yy} = I^{(w_{f'})}_{yyo} + I^{(HT)}_{yyo} + I^{(VT)}_{yyo} + I^{(CR)}_{yyo} + I^{(C)}_{yyo} + W'_f/g$$

$$(\eta'^2_{f'} + \delta'^2_{f'}) + W'_{HT}/g\ (\eta'^2_{HT} + \delta'^2_{HT}) + W'_{VT}/g\ (\eta'^2_{VT} + \delta'^2_{VT})$$

$$+ W'_{CR}/g\ (\eta'^2_{CR} + \delta'^2_{CR}) + W'_c/g\ (\eta'^2_c + \delta'^2_c)$$

$$I^{(f)}_{xx} = I^{(w_{f'})}_{xxo} + I^{(HT)}_{xxo} + I^{(VT)}_{xxo} + I^{(CR)}_{xxo} + I^{(C)}_{xxo} + W'_f/g\ \delta'^2_{f'}$$

$$+ W'_{HT}/g\ (\delta'^2_{HT} + Y^2_{HT}) + W'_{VT}/g\ \delta'^2_{VT} + W'_{CR}/g\ \delta'^2_{CR}$$

$$+ W'_c/g\ \delta'^2_c$$

$$I^{(f)}_{zz} = I^{(W_{f'})}_{zzo} + I^{(HT)}_{zzo} + I^{(VT)}_{zzo} + I^{(CR)}_{zzo} + I^{(C)}_{zzo} + W'_f/g\ \eta'^2_{f'}$$

$$+ W'_{HT}/g\ (\eta'^2_{HT} + Y^2_{HT}) + W'_{VT}/g\ \eta'^2_{VT} + W'_{CR}/g\ \eta'^2_{CR} + W'_c\ /g\ \eta'^2_c$$

$$I^{(f)}_{xz} = I^{(W_{f'})}_{xzo} + I^{(HT)}_{xzo} + I^{(VT)}_{xzo} + I^{(CR)}_{xzo} + I^{(C)}_{xzo} + W'_f/g\ \eta'_{f'}\ \delta'_{f'}$$

$$+ W'_{HT}/g\ \eta'_{HT}\ \delta'_{HT} + W'_{VT}/g\ \eta'_{VT}\ \delta'_{VT} + W'_{CR}/g\ \eta'_{CR}\ \delta'_{CR}$$

$$+ W'_c/g\ \eta'_c\ \delta'_c$$

$$H'_{w'w} = \ell_w - \ell'_w$$

$$\Delta'_{w'w} = h_w - z'_w$$

$$H'_{w'FUEL} = \ell_w - \ell'_{FUEL}$$

$$\Delta'_{w'FUEL} = h_w - z'_{FUEL}$$

$$H'_{w'NF} = \ell_w - \ell'_{NF}$$

$$\Delta'_{w'NF} = h_w - z'_{NF}$$

$$I_{yy}^{(w)} = I_{yyo}^{(w'w)} + I_{yyo}^{(w'FUEL)} + I_{yyo}^{(w'NP)} + w'_w/g\ (H'^2_{w'w} + \Delta'^2_{w'w})$$

$$+ w'FUEL/g\ (H'^2_{w'FUEL} + \Delta'^2_{w'FUEL}) + w'_{NF}/g\ (H'^2_{w'NF} + \Delta'^2_{w'NF})$$

$$I_{xx}^{(w)} = I_{xxo}^{(w'w)} + I_{xxo}^{(w'FUEL)} + I_{xxo}^{(w'NF)} + w'_w/g\ (\Delta'^2_{w'w} + Y_w^2)$$

$$+ w'FUEL/g\ (\Delta'^2_{w'FUEL} + Y^2_{FUEL}) + w'_{NF}/g\ (\Delta'^2_{w'NF} + Y^2_{NF})$$

$$I_{zz}^{(w)} = I_{zzo}^{(w'w)} + I_{zzo}^{(w'FUEL)} + I_{zzo}^{(w'NF)} + w'_w/g\ (H'^2_{w'w} + Y_w^2)$$

$$+ w'FUEL/g\ (H'^2_{w'FUEL} + Y^2_{FUEL}) + w'_{NF}/g\ (H'^2_{w'NF} + Y^2_{NF})$$

$$I_{xz}^{(w)} = I_{xzo}^{(w'w)} + I_{xzo}^{(w'FUEL)} + I_{xzo}^{(w'NF)} + w'_w/g\ \ H'_{w'w}\ \Delta_{w'w}$$

$$+ w'FUEL/g\ H'_{w'FUEL}\ \ \Delta'_{w'FUEL} + w'_{NF}/g\ \ H'_{w'NF}\ \Delta'_{w'NF}$$

APPENDIX F

This appendix contains the numerical constants and functions required by the equations presented in the preceding pages. The data is listed by reference to the page number in Appendix E where the numerical constant or function first appears.

INPUT DATA

PAGE NO.	QUANTITY	VALUE	UNITS
E-7	$K_{\delta_{STEER}}$	20.0	degrees/inch
	$K_{\delta_{RUD}}$	-8.0	degrees/inch
	K_{δ_r}	1.0	--
	ω_A	20.0	rad/sec
	ζ	1.0	--
	K_{δ_s}	1.0	--
	ω_L	35.5	rad/sec
	ζ'	0.18	--
	K_{δ_B}	1.0	--
	$K_{\delta_s'}$	0.0	--
	K_{δ_e}	-3.33	deg/inch
	Schedule A	to be determined	
	" B	"	
	" C	"	
	" D	"	
	" E	"	
	" F	"	
	" H	"	
	" I	"	
	" J	"	
E-8	Schedule A	to be determined	
	" B	"	
	" C	"	
	" D	"	
E-9	Gains and Schedules to be determined		

PAGE NO.	QUANTITY	VALUE	UNITS
E-9	Gains and Schedules to be determined		
E-13	Engine Data	See page A-17	---
	WDTIND	0.0	--
	SHP*	4120.0	HP
	$\dot{w}_{MAX}/\dot{w}^*$	1.0	--
	N1IND	0.0	--
	$N_{I_{MAX}}/N_I^*$	1.0	--
	N1θIND	1.0	--
	$(N1/\sqrt{\theta}_1/N_I^*)_{MAX}$	1.115	--
	QIND	1.0	--
	QMAX/Q*	1.0	--
E-15	$N_{II_{MAX}}/N_{II}^*$	1.214	--
	N_I^*	27932	RPM
E-16	$(N_{II}/N_{II_{MAX}})_{REF}$	.7547	--
	Ω_{REF}	27.436	rad/sec
	G_1	2.5	deg/sec/rad/sec
	G_2	2.66	deg/rad/sec
	G_3	0.05	deg/sec/deg
	I_p	150378.0	slug-ft^2
	κ	-1.0	--
	η_{tr}	1.0	--

PAGE NO.	QUANTITY	VALUE	UNITS
	Schedule A	See pages A-19 to A-22	
	" B	"	
	" C	"	
	" D	"	
	" E	"	
	" F	"	
	" G	"	
	" H	"	
E-17	ϕ_p	-12.0	degrees
	m_f	1397.5	slugs
	ℓ_f	0.091288	ft
	m_w	560.82	slugs
	ℓ_w	-0.83235	ft
	m	2323.27	slugs
	ℓ	7.4	ft
	m_N	182.48	slugs
	λ	0.0	deg
E-18	h_f	7.0896	ft
	h_w	0.10271	ft
E-19	Z_{WAC}	0.54	ft
	Y_{WAC}	17.51	ft
	X_{WAC}	1.4167	ft
	Y_N	36.55	ft
	L_S	8.5	ft
E-20	i_W	2.0	degrees

PAGE NO.	QUANTITY	VALUE	UNITS
E-22	Z_{HT}	-15.583	ft
	X_{HT}	-49.33	ft
	Z_{VT}	-5.25	ft
	X_{VT}	-40.4167	ft
E-23	Solutions of Quartic	See Page A-23	
	A	2485.0	ft^2
E-24	ε_{WRR}	See page A-24	rad
	ε_{WLR}	" A-24	rad
	PC	4.0	ft
	h_p	0.54	ft
	D	56.25	ft
	F_{iN}	75.0	deg
E-25	c_w	10.23	ft
	S_w	747.5	ft^2
	$C_{L\alpha W}$	4.1832	rad^{-1}
E-28	a_7	0.0269	deg^{-1}
	δ_2	22.22906	deg
	a_8	-2.437137	---
	a_9	0.20607	deg^{-1}
	a_{10}	-0.003128	deg^{-2}
	δ_3	29.786	deg
	a_{11}	0.442188	---

PAGE NO.	QUANTITY	VALUE	UNITS
	a_{12}	0.0263	deg^{-1}
	a_{13}	-0.000338	deg^{-2}
	a_{29}	0.000011	deg^{-1}
	a_{30}	0.00003	deg^{-2}
	δ_5	30.0	deg
	a_{31}	-0.03105	---
	a_{32}	0.001946	deg^{-1}
E-29	a_{14}	-0.00911	deg^{-1}
	δ_{SP1}	30.0	deg
	a_{15}	0.066537	---
	a_{16}	-0.016342	deg^{-1}
	a_{17}	0.00014	deg^{-2}
	b_o	-0.000088247	deg^{-1}
	b_1	0.000008596	deg^{-2}
	F_1	1.003412	---
	F_2	0.011163	deg^{-1}
	F_3	0.002168	deg^{-2}
	δ_4	20.1665	deg
	F_4	-0.756323	---
	F_5	0.185684	deg^{-1}
	F_6	-0.002159	deg^{-2}
	a_o	14.0	deg
	a_1	-0.07	---
	δ_1	30.0	deg

PAGE NO.	QUANTITY	VALUE	UNITS
	a_2	11.9	deg
	a_3	-15.0	deg
	a_4	-0.06	---
	a_5	-16.8	deg
	a_6	0.0888	---
	$C'_{L_{\alpha_w}}$	0.073	deg^{-1}
	a_{23}	-2.051491	---
	a_{24}	0.215913	deg^{-1}
	a_{25}	-0.005276	deg^{-2}
	C_{DO_w}	0.0152	---
	a_{26}	0.0036	---
	a_{27}	0.054313	---
	a_{28}	0.0269	---
E-30	a_{18}	11.32	deg
	a_{20}	-1.910958	---
	a_{21}	0.211969	deg^{-1}
	a_{22}	-0.005391	deg^{-2}
E-31	a_{19}	10.92	deg
E-33	b_2	-0.035178	---
	b_3	-0.000244	deg^{-1}
	b_4	0.0	---
	b_5	-0.010154	deg^{-1}
	b_6	0.000081	deg^{-2}

PAGE NO.	QUANTITY	VALUE	UNITS
E-33	$C_{L_{MAX}}$	1.2335	---
	$(a_g/a)_w$	See page A-25	---
E-37	K_{20}	-0.0908	rad^{-1}
	K_{21}	-0.04	rad^{-1}
	b_w	73.1	ft
	K_{ℓ}	1.0	---
	$\overline{Y}_{AC}$	17.51	ft
	K_{22}	0.02	rad^{-1}
	K_N	1.0	---
E-38	f_{e_u}	220.0	ft^2
	$X_{c/2}$	-1.1408	ft
	$K_{\frac{D1}{T}}$	0.0	---
	$K_{\frac{D2}{T}}$	0.0	---
	$K_{\frac{D3}{T}}$	0.05	---
	$K_{\frac{M1}{T}}$	0.0	---
	$K_{\frac{M2}{T}}$	0.0	---
	$K_{\frac{M3}{T}}$	0.0	---
E-39	$K_{\frac{D4}{T}}$	0.05	---
	$K_{\frac{M4}{T}}$	0.0	---

PAGE NO.	QUANTITY	VALUE	UNITS
E-42	ε_o	See page A-26	deg
	$d\varepsilon/d\alpha$	See page A-26	deg/deg
	i_{HT}	0	deg
E-43	τ_{HT}	0.52	---
	$\alpha_{HT_{STALL}}$	17.15	deg
	$C_{L_{\alpha_{HT}}}$	0.0694	deg^{-1}
	$(a_g/a)_{HT}$	See page A-25	---
	$C_{DO_{HT}}$	0.00734	---
	AR_{HT}	5.384	---
	E_{HT}	0.8	---
E-46	$d\sigma/d\beta$	-0.557	---
	τ_{VT}	0.46	---
	$\alpha_{VT_{STALL}}$	26.9	deg
	$C_{Y_{\alpha_{VT}}}$	0.0613	deg^{-1}
	E_{VT}	0.88	---
	$C_{DO_{VT}}$	0.00615	---
	AR_{VT}	1.536	
E-50	η_{HT}	1.0	---
	i_{HT}	0.0	deg
	S_{HT}	227.5	ft^2
	η_{VT}	1.0	---
	S_{VT}	278.0	ft^2

PAGE NO.	QUANTITY	VALUE	UNITS
E-52	for $\alpha_N \leq .5236$ rad		
	C_{DON}	0.00268	---
	K_{30}	-0.001908	---
	K_{31}	0.061849	---
	for $\alpha_N > .5236$ rad		
	C_{DON}	-.102244	---
	K_{30}	0.271593	---
	K_{31}	-0.077763	---
	K_{32}	0.0546	---
	C_{MON}	0.0	---
	K_{34}	0.02532	---
	K_{35}	-0.00206	---
E-53	K_{36}	-0.0546	---
	K_{37}	0.0	---
	K'_{36}	-0.0546	---
	K'_{37}	0.0	---
	C_{NORN}	0.0	---
	K_{38}	-0.001206	---
	K_{39}	0.0	---
	C_{NOLN}	0.0	---
	K_{40}	0.001206	---
	K_{41}	0.0	---

PAGE NO.	QUANTITY	VALUE	UNITS
E-54	X_{G1}	-7.41	ft
	X_{G2}	-7.41	ft
	X_{G3}	33.33	ft
	Y_{G1}	-7.33	ft
	Y_{G2}	7.33	ft
	Y_{G3}	0.0	ft
	Z_{G1}	13.5	ft
	Z_{G2}	13.5	ft
	Z_{G3}	14.0	ft
	r_1	1.375	ft
	r_2	1.375	ft
	r_3	1.0	ft
	K_{ST1}	20000	lb/ft
	K_{ST2}	20000	lb/ft
	K_{ST3}	20000	lb/ft
	D_{ST1}	1200	lb/ft/sec
	d_{ST2}	1200	lb/ft/sec
	D_{ST3}	1200	lb/ft/sec
E-55	μ_o	0.03	---
	μ_1	0.005	---
	μ_s	0.5	---

PAGE NO.	QUANTITY	VALUE	UNITS
E-57	C_{DOF}	0.01288	---
	K_o	65.14	rad^{-3}
	K_2	0.4	rad^{-2}
	K_1	-0.00553	rad^{-1}
	ΔC_{DLG}	0.0401	---
	K_3	1.26	rad^{-1}
	K_4	0.0	rad^{-2}
	K_{42}	0.0261	---
	K_7	-0.361	rad^{-1}
	K_8	-0.0987	rad^{-2}
	C_{MOF}	-0.00455	---
	K_5	1.432	rad^{-1}
	K_6	-0.494	rad^{-2}
	ΔC_{MLG}	-0.0568	---
	C_{NOF}	0.0	---
	K_9	-0.0051	rad^{-1}
	K_{10}	0.0	rad^{-2}
E-58	Z_{FAC}	5.75	ft
	X_{FAC}	1.4167	ft
E-60	T_1	0.2434	rad^{-1}
	T_2	-0.483	rad^{-2}
	T_3	0.5208	rad^{-3}
	R	28.125	ft

PAGE NO.	QUANTITY	VALUE	UNITS
E-62	τ_1	0.1	sec
	τ_2	0.1	sec
E-63	D_{NF_1}	0.00425	deg^{-1}
	D_{NF_2}	0.0014483	deg^{-1}
	D_{NF_3}	-0.0000734	deg^{-1}
	D_{NF_4}	0.00002175	deg^{-1}
	D_{NF_5}	-0.0006	deg^{-1}
	E_{NF_1}	-0.0245	deg^{-1}
	E_{NF_2}	-0.0017028	deg^{-1}
	E_{NF_3}	-0.0010492	deg^{-1}
	E_{NF_4}	0.0000425	deg^{-1}
	E_{NF_5}	0.0017892	deg^{-1}
E-64	D_{SF_1}	0.0245	deg^{-1}
	D_{SF_2}	0.0017028	deg^{-1}
	D_{SF_3}	0.0010492	deg^{-1}
	D_{SF_4}	-0.0000425	deg^{-1}
	D_{SF_5}	-0.001735	deg^{-1}

PAGE NO.	QUANTITY	VALUE	UNITS
E-65	D_{PM_1}	.002	deg^{-1}
	D_{PM_2}	-.00072556	deg^{-1}
	D_{PM_3}	.00111967	deg^{-1}
	D_{PM_4}	.0002094	deg^{-1}
	D_{PM_5}	.00036524	deg^{-1}
	D_{PM_6}	-.00007296	deg^{-1}/rad/sec
	Ω_o	19.1637	rad/sec
E-66	E_{PM_1}	-.0025	deg^{-1}
	E_{PM_2}	0.0004375	deg^{-1}
	E_{PM_3}	0.0000729	deg^{-1}
	E_{PM_4}	-0.000111245	deg^{-1}
	E_{PM_5}	0.00063045	deg^{-1}
	E_{PM_6}	-.00006809	deg/rad/sec
E-67	D_{YM_1}	-.0025	deg^{-1}
	D_{YM_2}	.0004375	deg^{-1}
	D_{YM_3}	.0000792	deg^{-1}
	D_{YM_4}	-0.000111245	deg^{-1}
	D_{YM_5}	0.0005	deg^{-1}
	D_{YM_6}	-0.00007296	deg^{-1}/rad/sec

PAGE NO.	QUANTITY	VALUE	UNITS
	E_{YM_1}	-0.002	deg^{-1}
	E_{YM_2}	0.00072556	deg^{-1}
	E_{YM_3}	-0.00111967	deg^{-1}
	E_{YM_4}	-0.0002094	deg^{-1}
	E_{YM_5}	-0.0004702	deg^{-1}
	E_{YM_6}	0.00007296	deg^{-1}/rad/sec
E-68	f_{T_R}	1.0	---
	f_{T_L}	1.0	---
	f_{NF_R}	1.0	---
	f_{NF_L}	1.0	---
	f_{SF_R}	-1.0	---
	f_{SF_L}	-1.0	---
	f_{PM_R}	1.0	---
	f_{PM_L}	1.0	---
	f_{YM_R}	-1.0	---
	f_{YM_L}	-1.0	---
	f_{QR}	-1.0	---
	f_{QL}	-1.0	---
E-69	I_E	0.40	slug-ft^2
E-72	K_{W1}	3.707×10^{-5}	ft/lb
	K_{W2}	1.023×10^{-5}	ft/lb
	K_{W3}	1.22×10^{-4}	ft/lb

PAGE NO.	QUANTITY	VALUE	UNITS
	K_{W_4}	$1.09\text{x}10^{-2}$	slugs ft/lb
	K_{W_5}	$4.66\text{x}10^{-3}$	slugs ft/lb
	K_{W_6}	$1.004\text{x}10^{-5}$	ft/lb
	K_{W_7}	$3.49\text{x}10^{-5}$	ft/lb
	K_{W_8}	$2.37\text{x}10^{-5}$	ft/lb
	K_{W_9}	$2.94\text{x}10^{-3}$	slugs ft/lb
	$K_{W_{10}}$	$1.52\text{x}10^{-3}$	slugs ft/lb
	ξ_{W1}	0.5	---
	ω_{W1}	16.33	rad/sec
	ξ_{W2}	0.5	---
	ω_{W2}	16.33	rad/sec
	ξ_{W3}	0.5	---
	ω_{W3}	16.33	rad/sec
E-74	K_{θ_t}	$4.90\text{x}10^{6}$	ft lb/rad
	$\dfrac{dC_{MW_{c/4}}}{dC_L}$	-0.002	---
	C_{L_α}	4.1832	rad^{-1}
	C_1	-0.0352	---
	C_2	-0.010154	deg^{-1}
	C_3	0.000081	deg^{-2}

REFERRED POWER TURBINE SPEED $N_{II}/\sqrt{\theta}/N_{II}^{*}$

M \ T/θ	1625	1700	1800	1900	2000	2100	2200	2300	2400	2600	2800	3000	3200
0.	.287	.418	.527	.610	.685	.751	.808	.856	.899	.975	1.040	1.095	1.142
.2	.292	.425	.534	.618	.691	.758	.814	.864	.909	.984	1.049	1.105	1.154
.4	.306	.432	.542	.625	.700	.768	.825	.874	.918	.993	1.059	1.116	1.163
.6	.339	.452	.559	.643	.717	.786	.843	.893	.936	1.013	1.080	1.136	1.184
.7	.345	.459	.564	.650	.726	.798	.853	.901	.946	1.022	1.092	1.150	1.198

REFERRED GAS GENERATOR SPEED $N_{I}/\sqrt{\theta}/N_{I}^{*}$

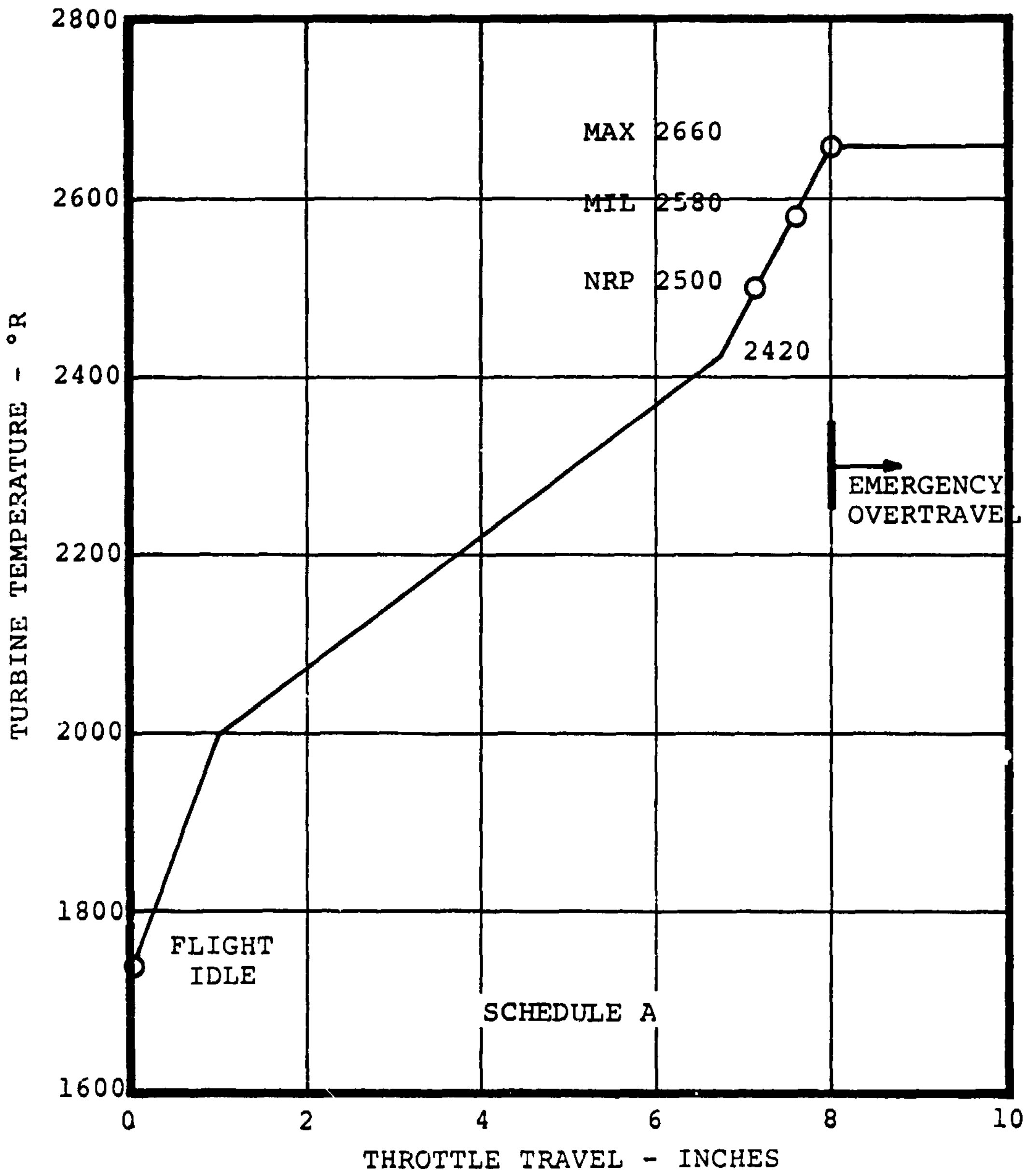
M \ T/θ	1625	1700	1800	1900	2000	2200	2400	2600	2800	3000	3200
0.	.654	.700	.750	.790	.825	.883	.938	.985	1.031	1.076	1.118
.2	.654	.700	.750	.790	.825	.883	.938	.985	1.031	1.076	1.118
.4	.654	.700	.750	.790	.825	.883	.938	.985	1.031	1.076	1.118
.6	.654	.700	.750	.790	.825	.883	.938	.985	1.031	1.076	1.118
.7	.654	.700	.750	.790	.825	.883	.938	.985	1.031	1.076	1.118

REFERRED FUEL FLOW $wf/\delta\sqrt{\theta}/SHP^*$

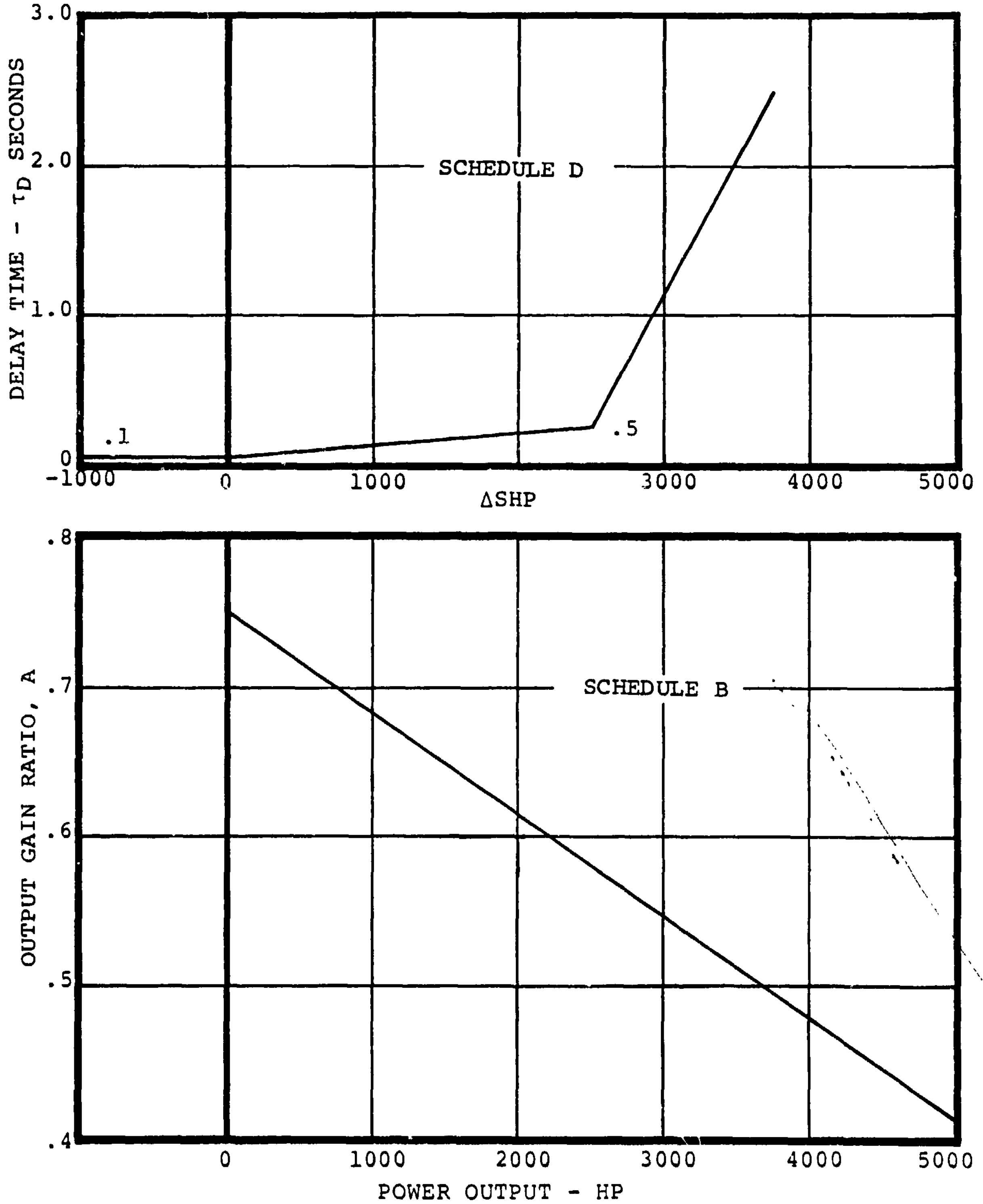
M \ T/θ	1625	1800	2000	2200	2400	2600	2800	3000	3200
0	.048	.105	.172	.242	.324	.402	.471	.531	.588
.2	.048	.106	.175	.248	.334	.415	.488	.558	.618
.4	.048	.107	.179	.254	.343	.426	.503	.582	.650
.6	.048	.108	.185	.268	.357	.454	.546	.630	.708
.7	.048	.109	.188	.275	.368	.468	.565	.655	.740

REFERRED POWER $SHP/\sqrt{\theta}/SHP^*$

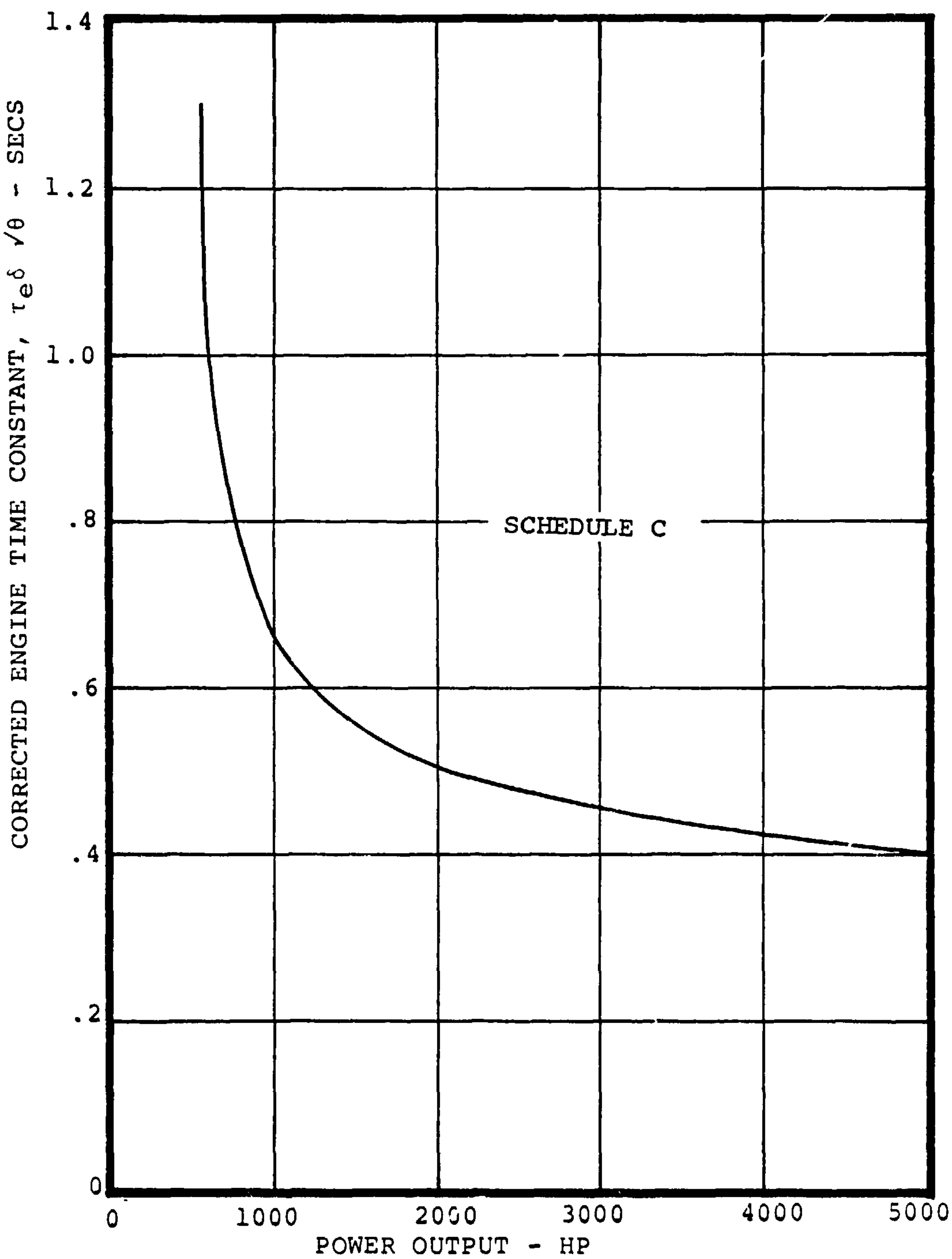
M \ T/θ	1625	1800	2000	2200	2400	2600	2800	3000	3200
0	0.	.156	.322	.500	.740	.941	1.125	1.325	1.488
.2	0.	.162	.340	.535	.778	.996	1.200	1.416	1.591
.4	0.	.170	.364	.570	.816	1.043	1.261	1.498	1.689
.6	0.	.202	.421	.636	.908	1.180	1.425	1.662	1.881
.7	0.	.214	.446	.672	.959	1.248	1.503	1.740	1.976



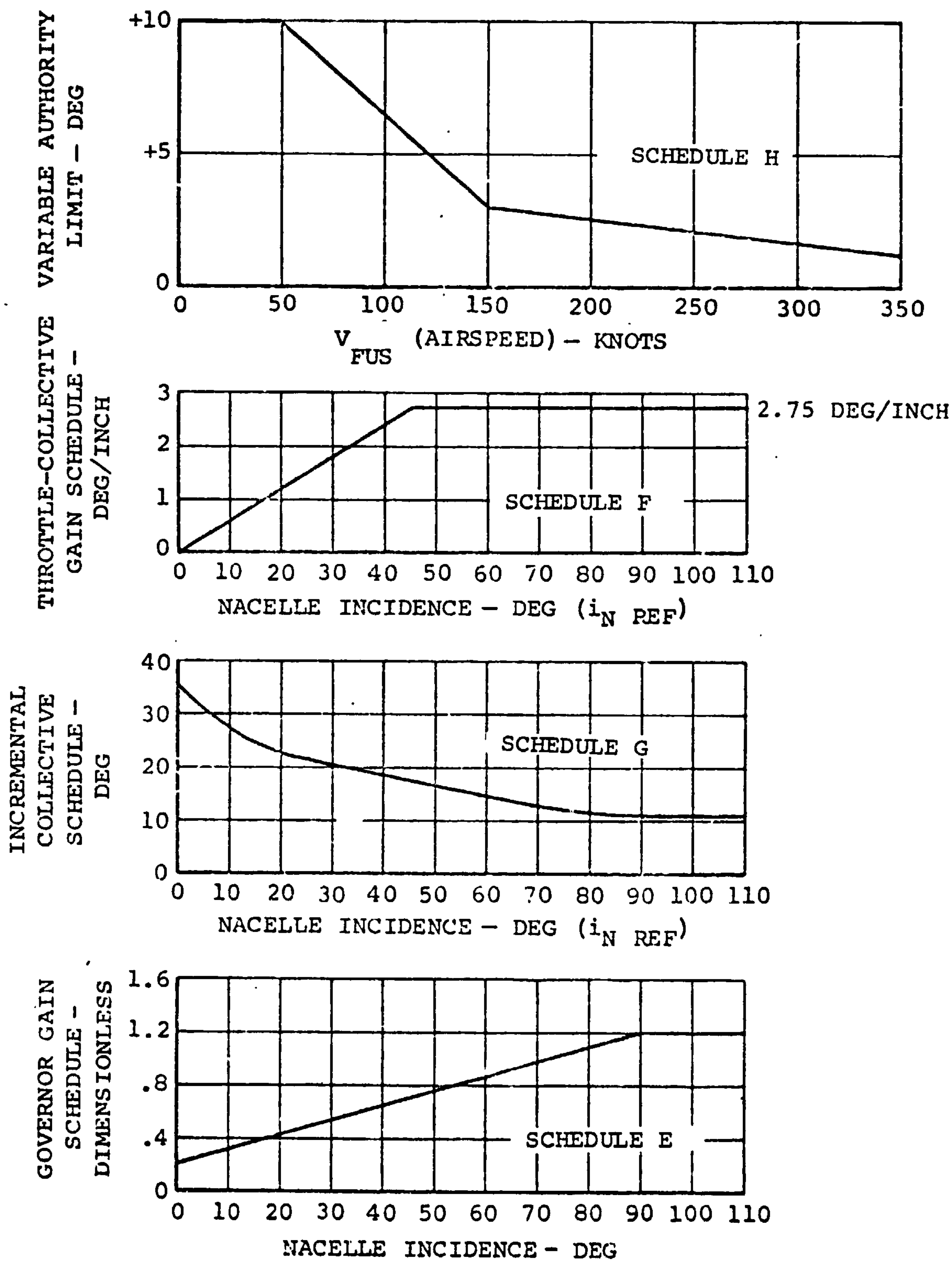
THRUST MANAGEMENT SYSTEM SCHEDULES



THRUST MANAGEMENT SYSTEM SCHEDULES



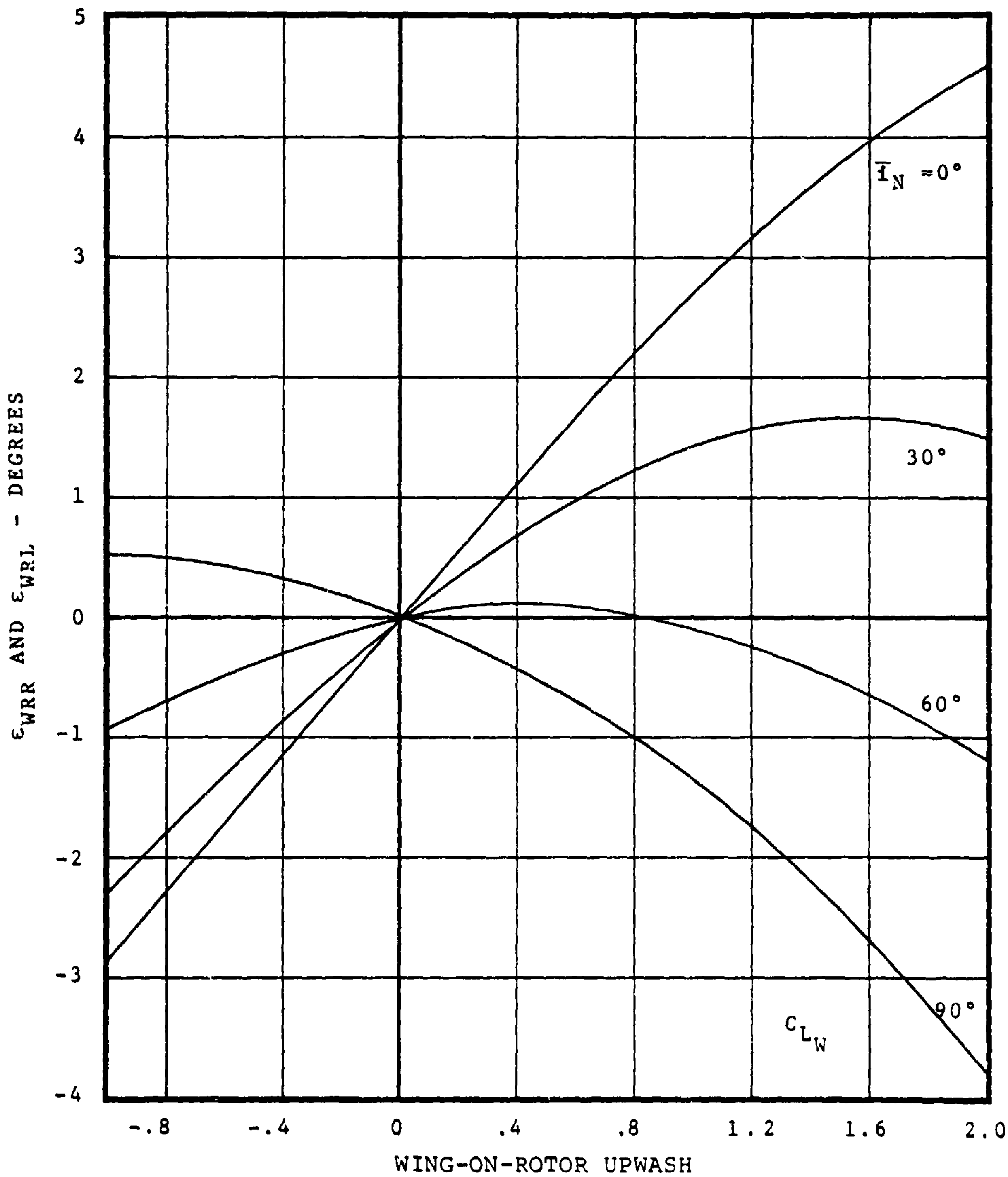
THRUST MANAGEMENT SYSTEM SCHEDULES

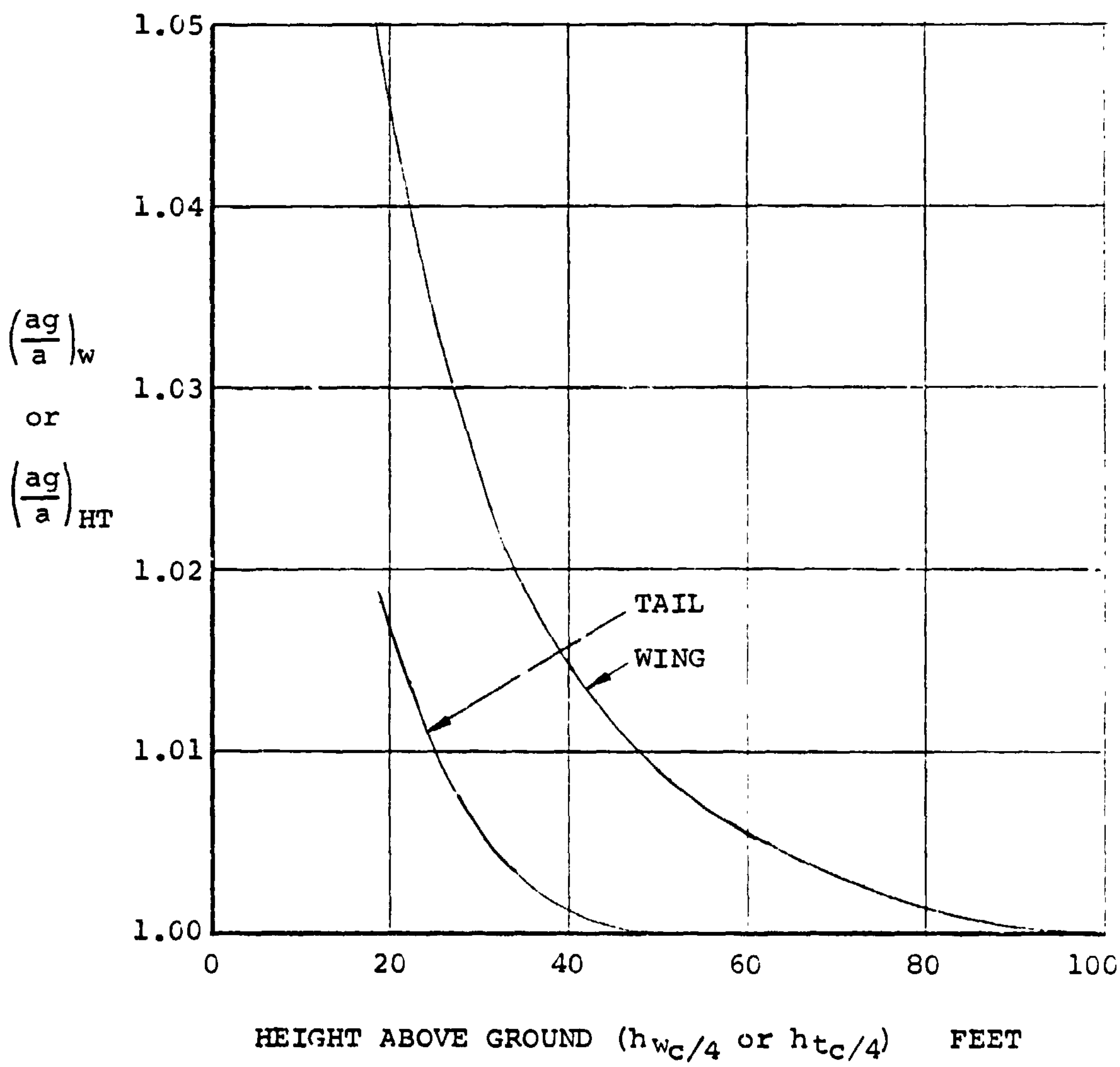


THRUST MANAGEMENT SYSTEM SCHEDULES

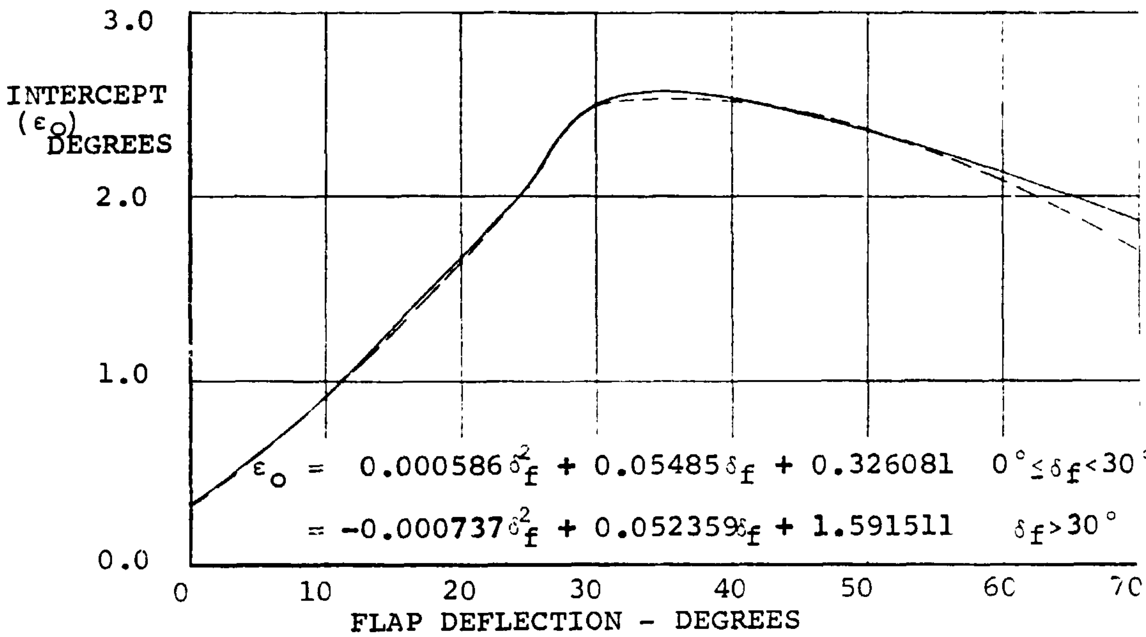
V* \ τ	0°	±15°	±30°	±45°	±60°	±75°	±90°	105°	120°
50	.01999	.01999	.01999	.01999	.02000	.02000	.02000	.02000	.02000
40	.02498	.02498	.02499	.02499	.02499	.02500	.02500	.02500	.02501
30	.03330	.03330	.03330	.03331	.03331	.03332	.03333	.03334	.03335
20	.04988	.04988	.04989	.04991	.04994	.04997	.05000	.05003	.05006
15	.06637	.06638	.06641	.06646	.06652	.06659	.06667	.06674	.06681
10	.09902	.09905	.09915	.09930	.09950	.09974	.10000	.10026	.10050
9	.10977	.10982	.10995	.11015	.11043	.11075	.11117	.11146	.11180
8	.12311	.12317	.12335	.12364	.12403	.12448	.12498	.12550	.12598
7	.14005	.14015	.14041	.14084	.14141	.14208	.14283	.14359	.14432
6	.16228	.16242	.16283	.16349	.16437	.16543	.16660	.16782	.16900
5	.19258	.19281	.19348	.19457	.19605	.19783	.19984	.20196	.20403
4	.23607	.23647	.23767	.23964	.24234	.24567	.24952	.25368	.25790
3.5	.26556	.26612	.26779	.27054	.27435	.27914	.28477	.29102	.29752
3.0	.30278	.30357	.30595	.30992	.31550	.32266	.33132	.34126	.35207
2.5	.35078	.35194	.35544	.36136	.36983	.38102	.39510	.41217	.43208
2.0	.41421	.41594	.42121	.43025	.44352	.46170	.48587	.51764	.55954
1.5	.5000	.50258	.51051	.52434	.54515	.57480	.61660	.67688	.76971
1.0	.61803	.62167	.63287	.65252	.68233	.72522	.78615	.87344	1.0000
.9	.64659	.65041	.66217	.68276	.71389	.75842	.82091	.90836	1.02956
.8	.67703	.68101	.69320	.71449	.74650	.79183	.85437	.93933	1.05150
.7	.70948	.71355	.72599	.74763	.77989	.82494	.88576	.96560	1.06595
.6	.74403	.74810	.76055	.78203	.81370	.85718	.91437	.98670	1.07333
.5	.78078	.78475	.79682	.81749	.84755	.88796	.93956	1.00236	1.07414
.4	.81980	.82352	.83475	.85377	.88097	.91667	.96083	1.01249	1.06897
.35	.84020	.84371	.85429	.87212	.89736	.93007	.96986	1.01549	1.06433
.30	.86119	.86444	.87421	.89055	.91346	.94274	.97776	1.01713	1.05840
.25	.88278	.88571	.89447	.90902	.92920	.95462	.98450	1.01744	1.05127
.20	.90499	.90751	.91505	.92747	.94450	.96564	.99005	1.01644	1.0430
.15	.92781	.92985	.93592	.94585	.95930	.97574	.99439	1.01416	1.03367
.10	.95125	.95272	.95706	.96411	.97353	.98487	.99750	1.01063	1.02334
.05	.97531	.97610	.97843	.98218	.98712	.99298	.99938	1.00590	1.01210
0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

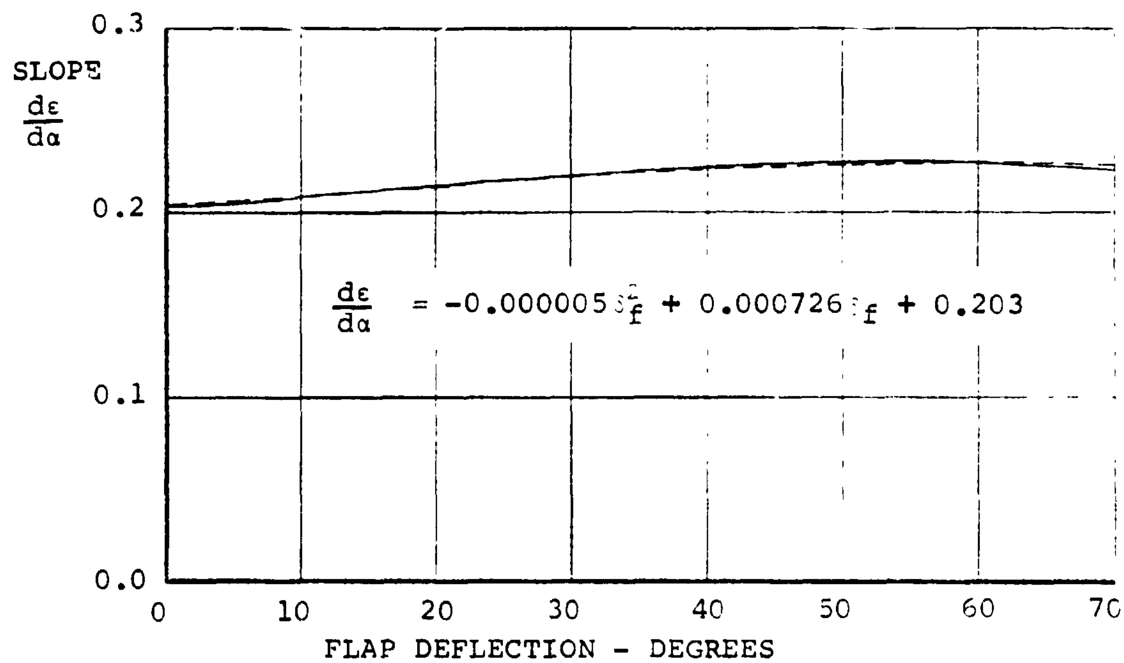
SOLUTIONS TO INDUCED VELOCITY QUARTIC Values of v* at V* and τ


5
4
3
2
1
0
-1
-2
-3
-4
ε_{WRR} AND ε_{WRL} - DEGREES
$\bar{i}_N$ =0°
30°
60°
90°
C_{L_W}
-.8
-.4
0
.4
.8
1.2
1.6
2.0
WING-ON-ROTOR UPWASH



VARIATION OF LIFT CURVE SLOPE WITH HEIGHT ABOVE GROUND





DOWNWASH FUNCTIONS @ $C_T = 0$, $i_w = 2$