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PERFORMANCE OF BINARY FSK DATA TRANSMISSION SYSTEMS

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COMMUNICATIONS, POWER, AND DATA SYSTEMS BRANCH

National Aeronautics and Space Administration

LYNDON B. JOHNSON SPACE CENTER

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DATA TRANSMISSION SYSTEMS

Prepared by

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MATCHED-FILTER DETECTION OF BINARY SIGNALS

It is well known that matched-filter (or correlation) detection is the *best* means of detecting any class of binary signals, in the sense that the probability of bit error at the detector output is minimized. Although matched-filter detection is somewhat difficult to instrument because it is a *coherent* detection scheme and requires a knowledge of the RF phase of the signal, there are several good reasons for considering such schemes:

a. The matched-filter system is particularly easy to analyze.

b. Since matched-filter detection is the best detection technique, the performance of a matched filter sytem represents a *bound* which can only be approached by systems utilizing other detection schemes.

c. Using the bounds established by matched-filter detection and the results of typical non-matched-filter detectors, we can "guess" at the performance of systems which have not been analyzed in detail.

For matched-filter detection of binary signals, the probability of error is given by

$$P_e = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{E_b(1-\rho)}{N_o}}}^{\infty} e^{-\frac{x^2}{2}} dx$$

$$= \frac{1}{2} \frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b(1-\rho)}{2N_o}}}^{\infty} e^{-x^2} dx$$

$$= \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b(1-\rho)}{2N_o}} \qquad (1)$$

where E_b is the average energy per bit and ρ is the correlation coefficient between the two signal waveforms $S_1(t)$ and $S_2(t)$, or

$$\rho = \frac{1}{E_b} \int_o^T S_1(t) S_2(t) dt$$

where T is the bit duration.

Note that, for matched-filter detection of binary signals, P_e is a function of only two parameters — E_b/N_o and ρ. The only parameter that is a function of the particular signal set being transmitted is the correlation coefficient, ρ, which can assume values between -1 and +1. For any particular signal set, we need only to determine ρ in order to plot P_e as a function of E_b/N_o. This will now be accomplished for a few familiar cases.

Coherent PSK:

$$S_1(t) = A \sin \omega_c t \; ; \quad E_{b_1} = \frac{A^2T}{2}$$

$$S_2(t) = -A \sin \omega_c t \; ; \quad E_{b_2} = \frac{A^2T}{2}$$

$$E_b = \frac{E_{b_1} + E_{b_2}}{2} = \frac{A^2T}{2}$$

$$\rho = \frac{2}{A^2T} \int_0^T - \frac{A^2}{2} \left[1 - \cos(2\omega_c t)\right] dt = -1$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{A^2T}{2N_o}} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_o}}} \qquad (2)$$

Coherent FSK (Orthogonal):

$$S_1(t) = A \sin \omega_{c_1} t \; ; \quad E_{b_1} = \frac{A^2T}{2}$$

$$S_2(t) = A \sin \omega_{c_2} t \; ; \quad E_{b_2} = \frac{A^2T}{2}$$

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$$E_b = \frac{E_{b_1} + E_{b_2}}{2} = \frac{A^2 T}{2}$$

$\rho = 0$ if $\sin \omega_{c_1} t$ and $\sin \omega_{c_2} t$ are *orthogonal*

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{A^2 T}{4N_o}} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_o}}} \tag{3}$$

Coherent ASK (On-Off Keying):

$$S_1(t) = A \sin \omega_{c_1} t \; ; \quad E_{b_1} = \frac{A^2 T}{2}$$

$$S_2(t) = 0 \; ; \quad E_{b_2} = 0$$

$$E_b = \frac{E_{b_1} + E_{b_2}}{2} = \frac{A^2 T}{4}$$

$$\rho = 0$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{A^2 T}{8N_o}} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_o}}} \tag{4}$$

The results given by (2), (3), and (4) are plotted in figure 1.
Note that the performance of coherent orthogonal FSK is always 3 db

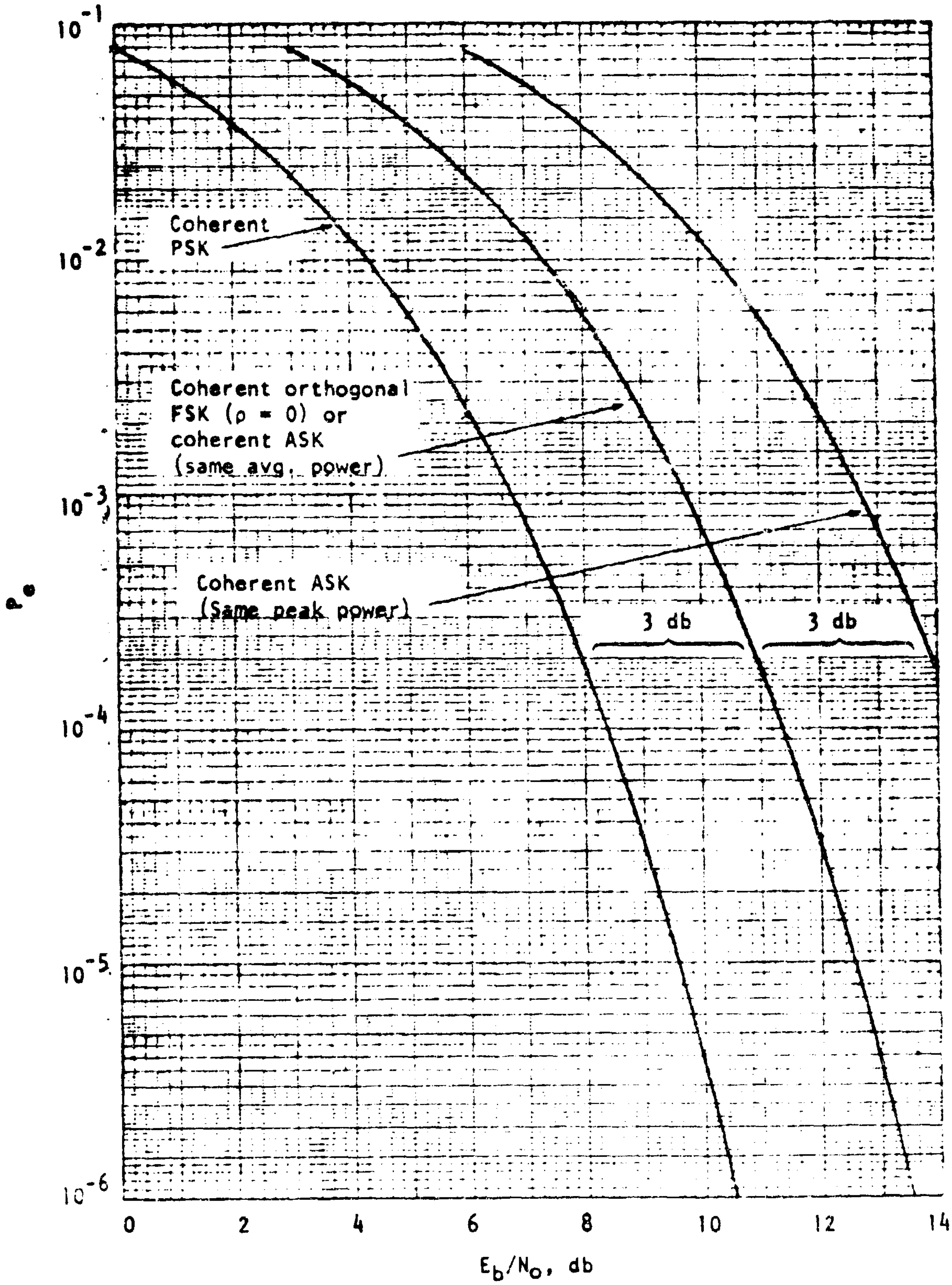


Figure 1.- Matched-filter detection of binary signals

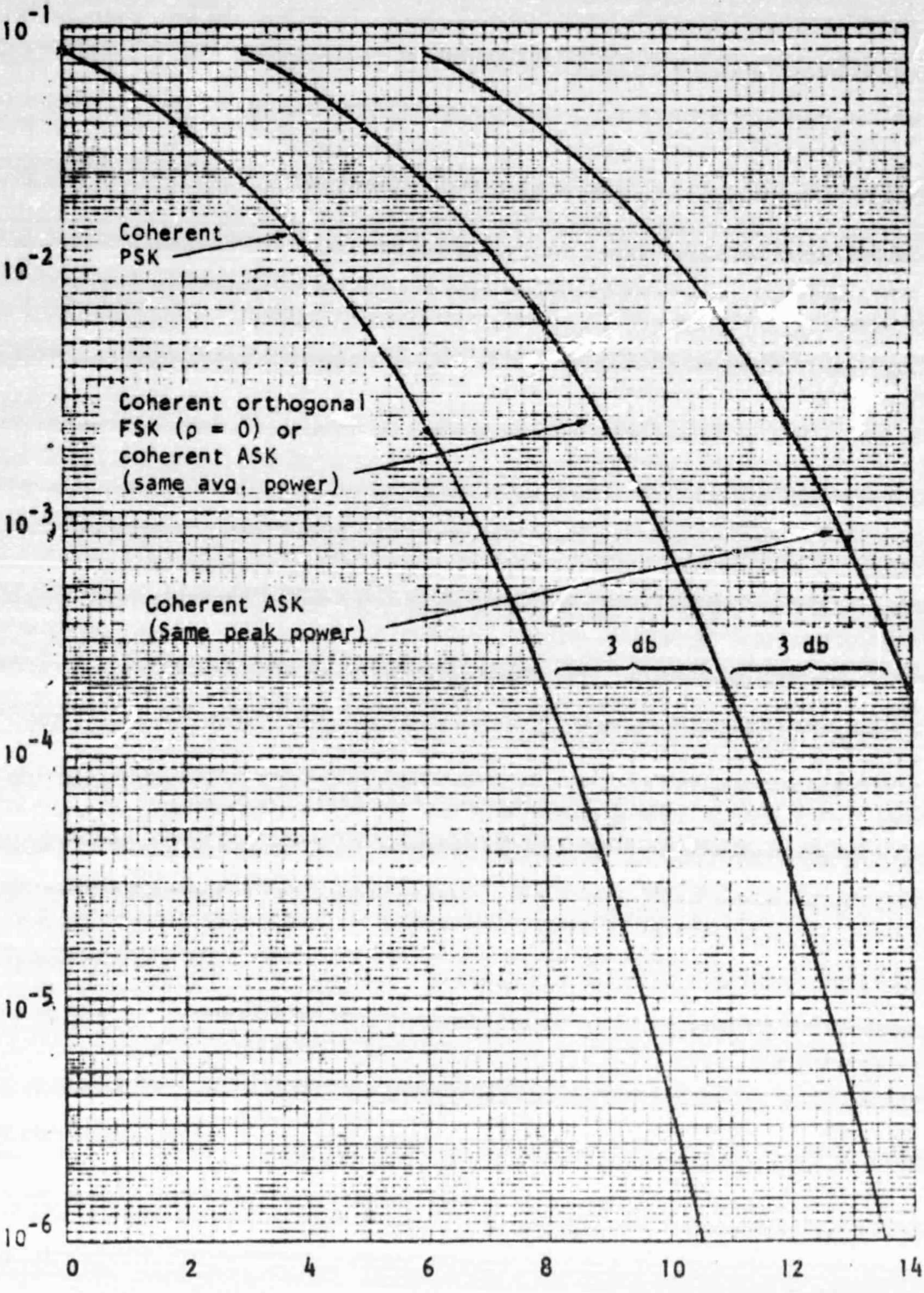


Figure 1.- Matched-filter detection of binary signals

worse than that of coherent PSK, while that of coherent ASK (on-off keying) is either 3 db worse than coherent PSK for equal *average* power (or equal E_b) or 6 db worse than coherent PSK for equal *peak* power (or equal signal amplitude, A).

MATCHED-FILTER DETECTION OF FSK SIGNALS

Figure 2 is a *functional* illustration of the generation of an FSK waveform. Here FSK is visualized as being the sum of two ASK (on-off keyed) waveforms, or as the switched outputs of two sinusoidal tone generators. An alternate means of obtaining FSK is to use the binary data sequence to control the frequency of a single oscillator. This could be done by using the binary sequence as the modulation input to an FM transmitter.

Regardless of the technique used to generate the FSK signal, the optimum detection scheme is the matched-filter which, for FSK, can consist of two coherent multipliers followed by low-pass filters (to reject unwanted products appearing at the multiplier outputs) and data matched-filters. The multiplier/LPF conbinations perform the coherent *demodulation* process and provide noisy baseband data which must subsequently be *detected* using appropriate binary decision devices. Figure 3 illustrates this process of coherent detection of FSK. Note that a phase-coherent reference for *each* of the two FSK tones is required, but that since a discrete spectral component is present at

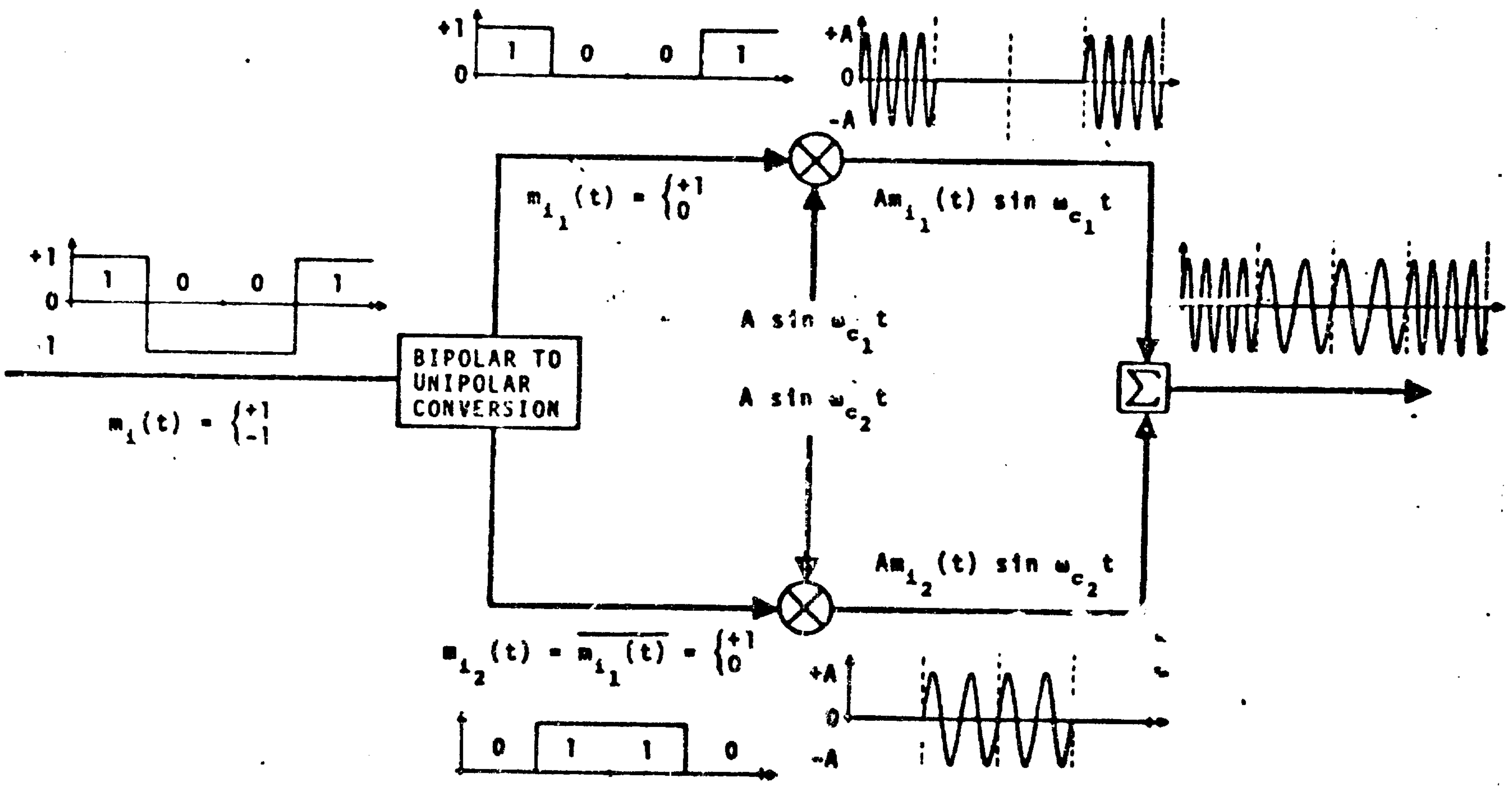


Figure 2.- Functional representation of FSK modulation

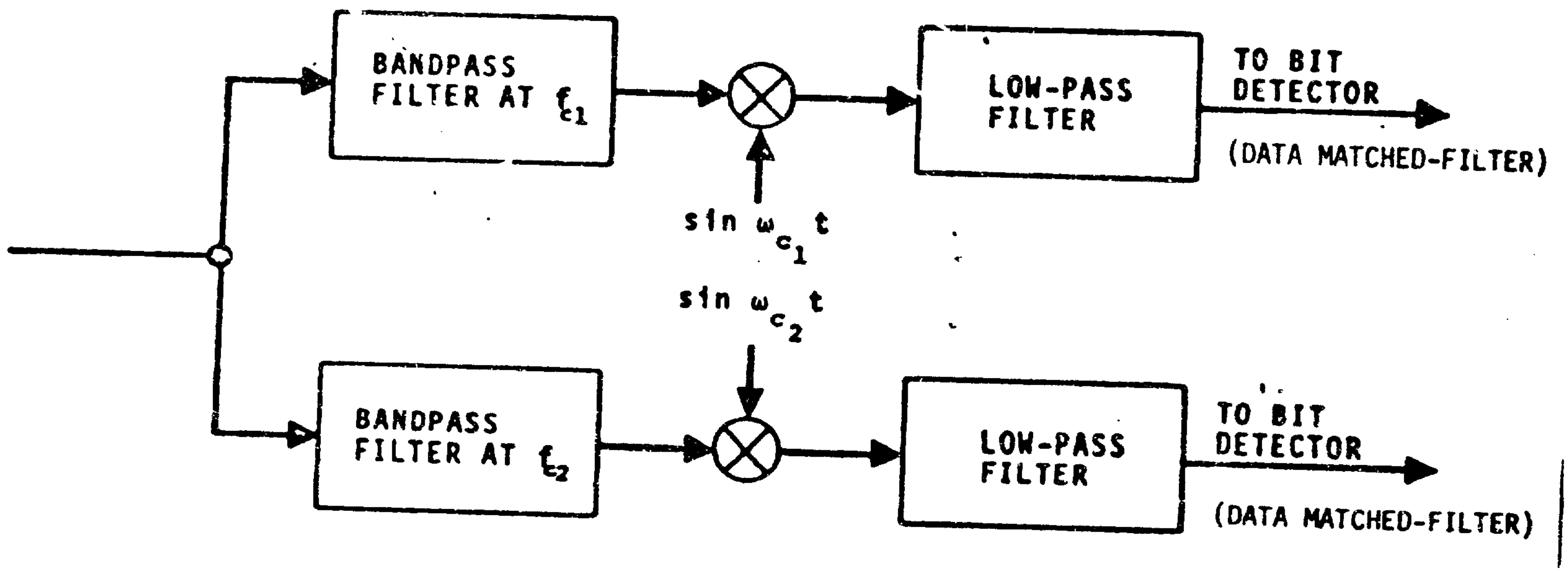


Figure 3.- Coherent detection of FSK.

each of the tone frequencies,[1] these coherent references can be readily obtained using phase-locked tracking filters.

The results summarized in the previous section for coherent detection of FSK assumed orthogonality ($\rho = 0$) between the two signaling waveforms (FSK tones) $S_1(t)$ and $S_2(t)$. In fact, however, it is not necessary that there be zero correlation between $S_1(t)$ and $S_2(t)$. In general, the correlation coefficient of two FSK tones is given by

$$\rho = \frac{1}{E_b}\int_0^T S_1(t)S_2(t)dt$$

$$= \frac{2}{A^2T}\int_0^T A^2\sin(\omega_{c_1}t)\sin(\omega_{c_2}t)dt$$

$$= \frac{2}{T}\int_0^T \sin(2\pi f_{c_1}t)\sin(2\pi f_{c_2}t)dt \qquad (5)$$

But f_{c_1} can be related to a center frequency f_c by, say,

$$f_{c_1} = f_c - \Delta f \qquad (6)$$

and f_{c_2} can be likewise expressed as

$$f_{c_2} = f_c + \Delta f \qquad (7)$$

where Δf is the instantaneous carrier frequency deviation caused by the modulating signal. Substituting (6) and (7) into (5) yields

[1]This is because each of the tone frequencies is effectively modulated by a random binary sequence having a d.c. value of 1/2. Therefore, half of the total transmitted power is contained in the two discrete spectral components located at f_{c_1} and f_{c_2}.

$$\rho = \frac{2}{T}\int_0^T \sin\left[2\pi(f_c - \Delta f)t\right]\sin\left[2\pi(f_c + \Delta f)t\right]dt$$

$$= \frac{1}{T}\int_0^T \cos\left[2\pi(f_c + \Delta f)t - 2\pi(f_c - \Delta f)t\right]dt$$

$$- \frac{1}{T}\int_0^T \cos\left[2\pi(f_c + \Delta f)t + 2\pi(f_c - \Delta f)t\right]dt$$

$$= \frac{1}{T}\int_0^T \cos\left[2\pi(2\Delta f)t\right]dt - \frac{1}{T}\int_0^T \cos\left[2\pi(2f_c)t\right] \tag{8}$$

Assuming that an integral number of cycles of the center frequency f_c occurs in a bit period T, (8) becomes

$$\rho = \frac{1}{T}\int_0^T \cos\left[2\pi(2\Delta f)t\right]dt$$

$$= \frac{1}{T}\left.\frac{\sin\left[2\pi(2\Delta f)\right]t}{2\pi(2\Delta f)}\right|_{t=0}^{t=T}$$

$$= \frac{\sin\left[2\pi(2\Delta f)T\right]}{2\pi(2\Delta f)T} \tag{9}$$

Note that ρ is a function only of $(\Delta f)(T)$ and can assume either positive, negative, or zero values. We are interested in the maximum negative value of ρ, which can be found as follows:

Let $$2\pi(2\Delta f)T = x \tag{10}$$

Then $\rho = \dfrac{\sin x}{x}$ with maxima occuring when

$$\frac{d\rho}{dx} = \frac{x \cos x - \sin x}{x^2} = 0$$

or when

$$\frac{\sin x}{x} = \cos x \tag{11}$$

Equation (11) is satisfied for $x = 0$ (which corresponds to $\Delta f = 0$) but this is obviously not the point of interest. However, (11) is also satisfied for $x = 4.493$ and this corresponds to $(2\Delta f)T = 0.715$, or to

$$\Delta f = \left(\frac{0.715}{2}\right)\frac{1}{T}$$

$$\doteq 0.358 \ R \tag{12}$$

where $R = 1/T$ is the bit rate of the binary data being transmitted. For this value of Δf, the correlation coefficient given by (9) is

$$\rho = -0.22 \tag{13}$$

which is the maximum negative value of ρ achievable for FSK transmission. Substitution of (13) into (1) yields

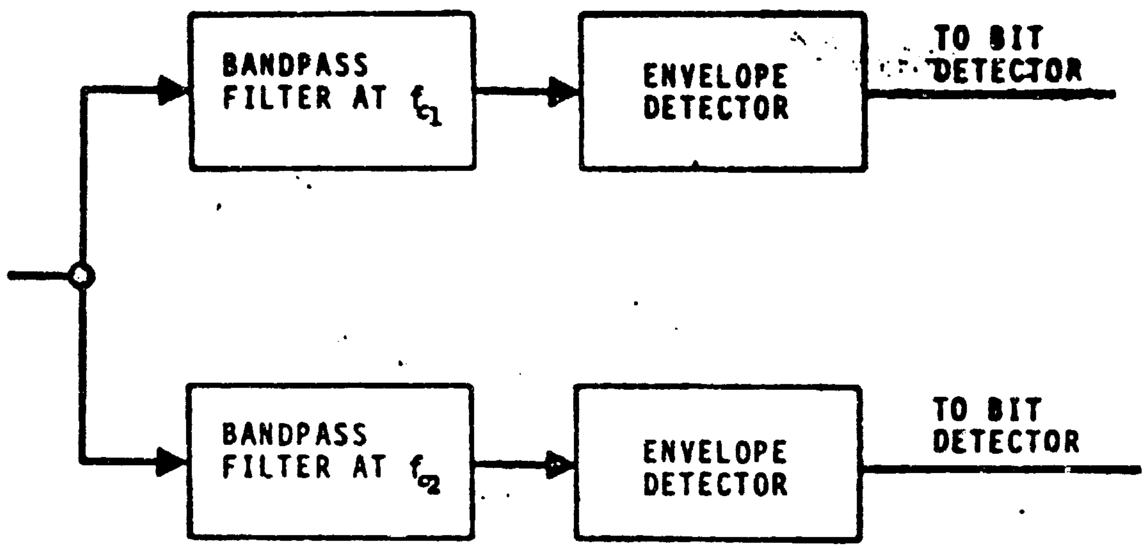
$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{0.61 \ E_b}{N_o}} \tag{14}$$

Equation (14) indicates that the best possible performance ($\rho = -0.22$) achievable using coherent detection of FSK is only 2.2 db worse than coherent PSK. This represents a 0.8-db improvement over the achievable performance using coherent detection of orthogonal FSK and constitutes a *bound* on the achievable performance of FSK systems utilizing suboptimum (non-matched-filter) detection schemes.

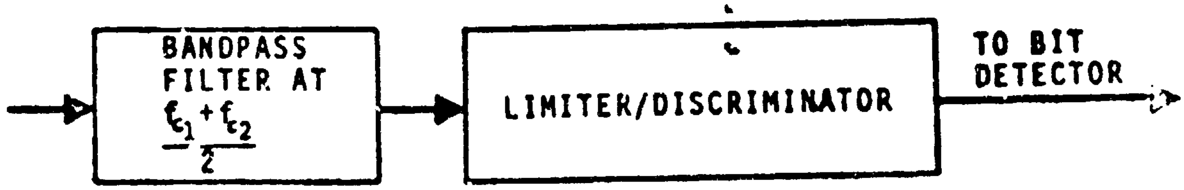
SUBOPTIMUM (NONCOHERENT) DETECTION OF FSK SIGNALS

Since systems employing coherent detection of FSK have about the same complexity as coherent PSK systems and, at best, perform about 2.2 db worse than coherent PSK systems, it is difficult to conceive of an application in which coherent FSK would be preferred. Coherent detection of FSK is, in fact, rarely (if ever) used in practical systems. The primary attractiveness of FSK arises from the relative simplicity associated with the various noncoherent (and, therefore, suboptimum) detection techniques which can be employed. Figure 4 illustrat : two noncoherent demodulation approaches that can be utilized, one approach being based on the functional structure of the FSK signal as the sum of two amplitude-modulated (ASK) signals which are subject to envelope detection, and the other approach being based on use of a frequency discriminator. The frequency discriminator approach is probably of more general interest and will be discussed here because the same modulation/demodulation equipment used for transmission of binary FSK data can then be used for transmission of information in analog form. Thus a system employing discriminator detection of FSK is by nature a somewhat versatile system. In addition, discriminator detection of FSK is of considerable intere t because it has been shown to perform almost as well as coherent detection of optimum FSK..

The analysis of systems employing discriminator detection of FSK is complicated by (1) the fact that it is very difficult to account for the effects of signal distortion due to bandpass filtering and by



(a) Technique #1
(envelope detection)



(b) Technique #2
(discriminator detection)

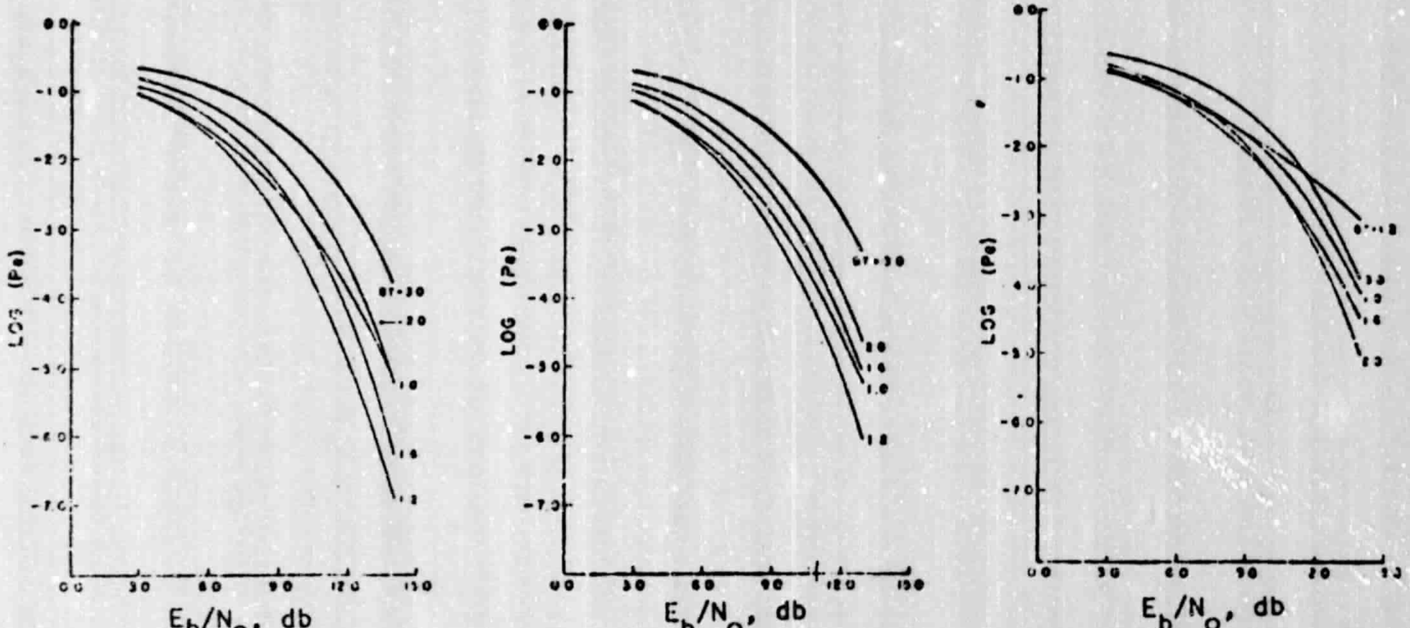
Figure 4.- Noncoherent detection of FSK

(2) the presence of non-Gaussian noise at the discriminator output and the resulting difficulties associated with computation of error probabilities.

Several recent studies of error probabilities in noncoherent FSK systems have been performed. Klapper (ref. 1), Mazo and Salz (ref. 2), and Schilling, *et. al.*, (ref. 3) evaluated FSK error probabilities based on Rice's (ref. 4) click theory of noise in FM. However, these papers assumed a sufficiently broad bandpass filter in the system for negligible distortion of the FSK signal. In fact, it is possible to make a favorable tradeoff between signal distortion and input noise reduction, so these results do not indicate error rate performance of the "optimum" FSK system employing discriminator detection.

Bennett and Salz (ref. 5) determined error rates for a binary FSK system, taking into account the effects of distortion due to a bandpass filter. However, their receiver model did not include a data matched filter after the discriminator.

Tjhung and Wittke (ref. 6) evaluated error probabilities for a binary FSK system (utilizing discriminator detection) taking into account the effects of both a bandpass filter and a data matched filter. In order to account for the FM signal distortion due to bandpass filtering, a periodic modulating signal (a 30-bit pseudo random sequence) was used. The particular sequence used was {11000 00101 10111 00111 11010 01000} and it was determined that the FM spectrum for this signal was a good approximation to the spectrum for FM by a random binary signal. The predetection bandpass filter was assumed to have a symmetrical passband and a linear phase characteristic. Results were obtained for two filter models: rectangular passband and Gaussian passband. Using Rice's click theory of FM noise, Tjhung and Wittke computed overall error probabilities by taking the average of the error probabilities for the individual bits. A number of error-rate curves were calculated as functions of E_b/N_o (for the unfiltered FM signal), with $2\Delta f$ and BT (the product of the filter bandwidth and the bit period or, alternately, the ratio of the filter bandwidth to the bit rate) as parameters. These curves are shown in figure 5 (for rectangular bandpass filter) and in figure 6 (for Gaussian bandpass filter). Figure 7 contains the data shown in figure 6, plotted in a way that allows an interesting comparison of the effects of the various parameters. The various sets of curves indicate that, for a given filter type and bit rate, there is a bandwidth B and a frequency deviation Δf that minimize the probability of error. Tables I and II were provided by Tjhung and Wittke to allow some degree of precision in determining the optimum values of these parameters for an error probability of 10^{-4}. It can be seen from these tables that for both the Gaussian and the rectangular bandpass filters, a value of $2\Delta f = 0.7R$ is best in that it requires the smallest value of E_b/N_o to achieve a 10^{-4} bit error probability. The optimum IF bandwidth for $P_e = 10^{-4}$ is seen to be 1.2 times the bit rate for the rectangular bandpass filter and 1.0 times the bit rate for the Gaussian filter. Optimum parameter values for error probabilities other than 10^{-4} can

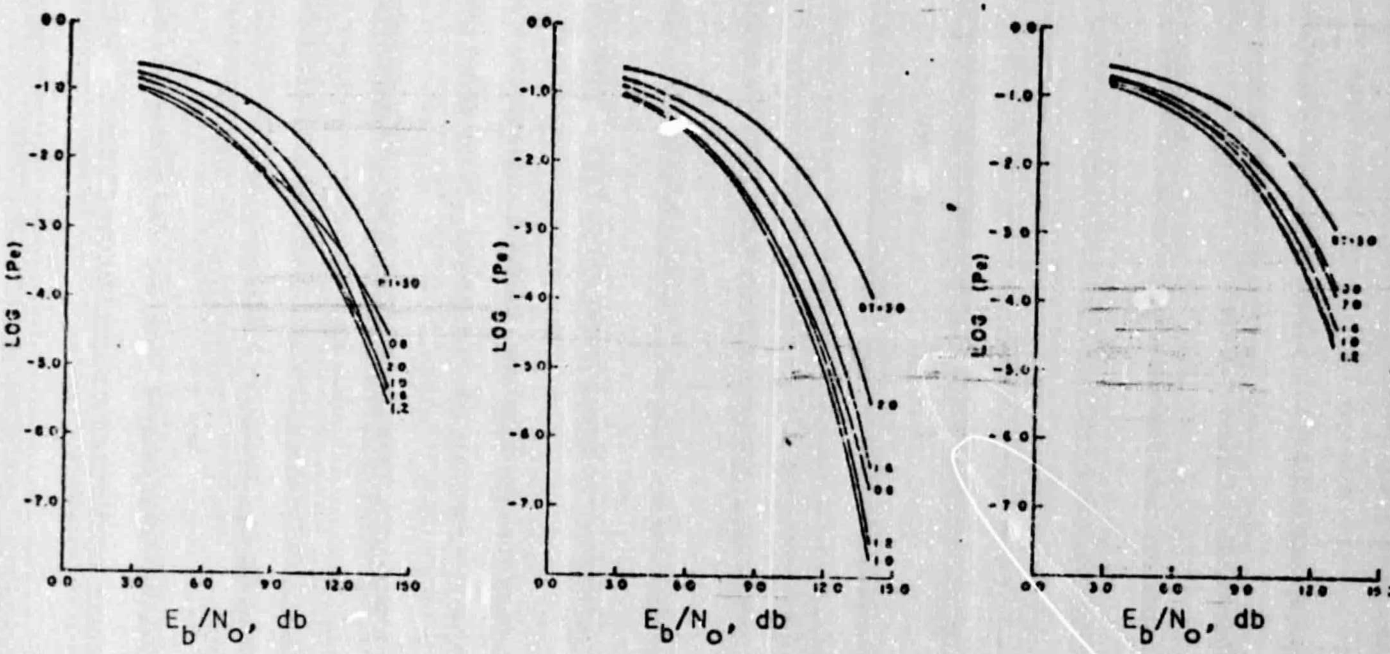


(a) 2Δf = 0.5 R

(b) 2Δf = 0.7 R

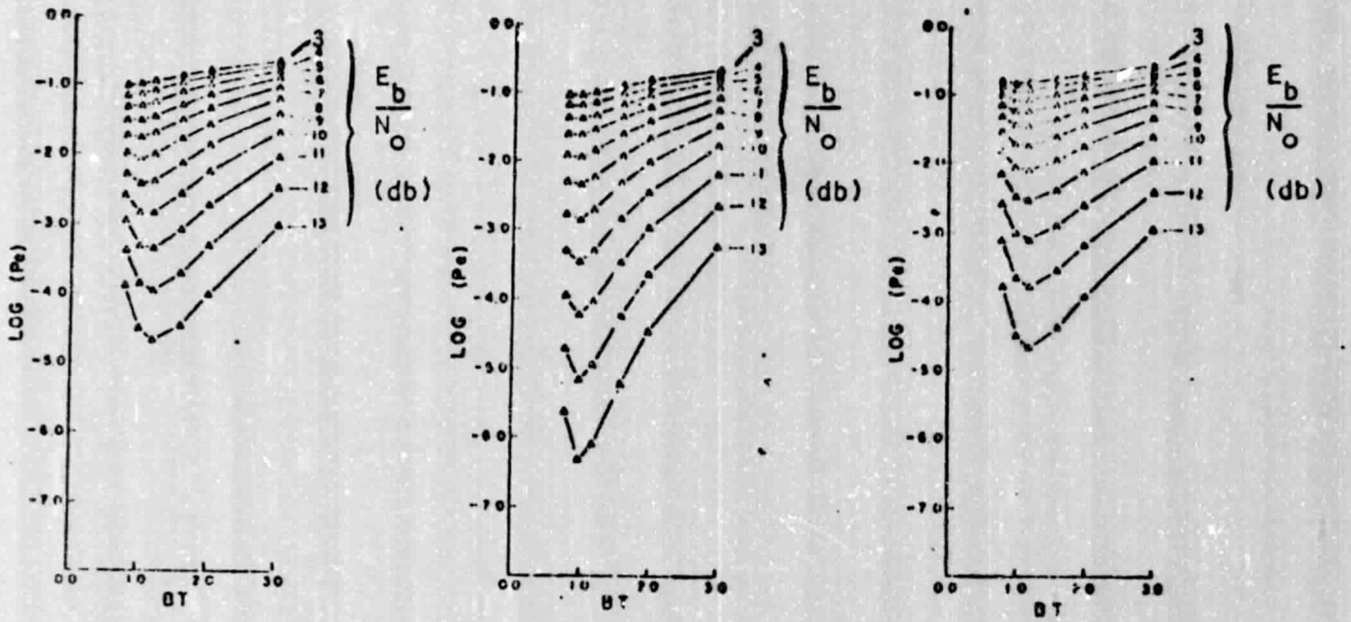
(c) 2Δf = 1.0 R

Figure 5.- Error rate curves for discriminator detection of binary FSK (rectangular bandpass filter).



(a) $2\Delta f = 0.5R$ (b) $2\Delta f = 0.7R$ (c) $2\Delta f = 1.0R$

Figure 6.- Error rate curves for discrimination detection of binary FSK (Gaussian bandpass filter).



(a) $2\Delta f = 0.5$ R (b) $2\Delta f = 0.7$ R (c) $2\Delta f = 1.0$ R

Figure 7.- Error rate curves for discrimination detection of binary FSK (Gaussian bandpass filter).

be obtained (with less precision) from the curves shown in figures 5 and 6. In general, it appears that a value of about 0.7R for $2\Delta f$ and a value of about 1.0 (or slightly greater) for BT will minimize the error probability for binary FSK systems employing discriminator detection.

TABLE I.- E_b/N_o IN DB REQUIRED TO ACHIEVE A 10^{-4} BIT ERROR PROBABILITY IN BINARY FSK SYSTEMS EMPLOYING DISCRIMINATOR DETECTION (RECTANGULAR BANDPASS FILTER)

$2\Delta f$	BT				
	1.0	1.2	1.6	2.0	3.0
0.5R	12.27	10.95	11.7	12.63	
0.7R	11.28	10.65	11.7	12.23	
1.0R	13.8		13.25	12.8	

TABLE II.- E_b/N_o IN DB REQUIRED TO ACHIEVE A 10^{-4} BIT ERROR PROBABILITY IN BINARY FSK SYSTEMS EMPLOYING DISCRIMINATOR DETECTION (GAUSSIAN BANDPASS FILTER)

$2\Delta f$	BT					
	0.8	1.0	1.2	1.6	2.0	3.0
0.5R	13.2	12.26	12.08	12.42	13.0	
0.7R	11.09	10.74	11.0	11.73	12.45	14.06
1.0R		12.38	12.23	12.53		

It is very significant that (from table I), using discriminator detection of binary FSK, it is possible to achieve an error probability of 10^{-4} for $E_b/N_o = 10.65$ db. This is only 2.25 db more than is required for coherent PSK and is within 0.1 db of the best performance achievable using coherent detection of FSK. Thus the results of Tjhung and Wittke indicate that the performance bound represented by coherent FSK is almost achievable using discriminator detection, given that some discretion is exercised in choice of frequency deviation and IF filter bandwidth.

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