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THE APPLICATION OF THE ROUTH APPROXIMATION METHOD TO TURBOFAN ENGINE MODELS

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Abstract

The Routh approximation technique is applied in the frequency domain to a 16th order state variable turbofan engine model. The results obtained motivate the extension of the frequency domain formulation of the Routh method to the time domain to handle the state variable formulation directly. The time domain formulation is derived and a new characterization, which specifies all possible Routh similarity transformations, is given. This characterization is computed by the solution of two eigenvalue-eigenvector problems. The application of the time domain Routh technique to the state variable engine model is described and some results are given. Additional computational problems are discussed including an optimization procedure which can improve the approximation accuracy by taking advantage of the transformation characterization.

INTRODUCTION

In control system design the system to be controlled is often represented by a complex mathematical model. Practically, this complex model may be difficult to use for design purposes. Additionally, the resultant control design may be too complex to implement. To eliminate these problems model reduction methods are often employed before designing the control system to reduce the complexity of the original model while maintaining the important characteristics.

One philosophy adopted in reduction methods is dominant mode approximation (ref. 1). In this paper one such dominant mode approximation technique, the Routh approximation technique (refs. 2 and 3), is applied in both the frequency and time domains to the simplification of a state variable turbofan engine model.

Hutton and Rabins (ref. 4) have shown the frequency domain formulation of the Routh approximation technique to be an excellent reduction method for many complex mechanical systems. This formulation is applicable to a turbofan engine model as well and is therefore a strong motivation for the work of this paper.

The objectives considered include the application of the Routh approximation method in the frequency domain to the F100 turbofan engine. This application demonstrates that adequate, reduced complexity, engine models can be determined via the Routh technique. Next, the frequency domain Routh formulation is extended to the time domain. This extension allows the usual engine time domain formulation to be handled directly. The time domain Routh approximation formulation also allows additional flexibility in the choice of the approximate system. Using the additional flexibility of the time domain formulation, optimized approximations may be readily generated. The time domain formulation is applied to the engine model and some results given. Approximant optimization results and the application of the Routh method for deriving fixed dimension realizations of the engine example are also discussed.

ROUTH APPROXIMATION

Modern jet engines can be represented as multiple input-multiple output (MIMO) dynamic systems using linear, constant coefficient differential equations to model small variations at various operating point conditions. A transfer function representation of these equations can be used as input information in a classical or multivariable frequency response control system design. The complexity of these transfer functions can cloud the design process with unnecessary detail or difficult computational problems. Additionally, control system complexity may be adversely influenced by unnecessary model complexity. Thus, there is a strong motivation to reduce the complexity of system and control models.

Frequency Domain

The Routh approximation method (refs. 2 and 3) is a dominant mode reduction technique. In the frequency domain the method incorporates parameters obtained from a Routh stability table analysis in the reduction process. These parameters, called alpha and beta parameters, are obtained from the coefficients of the MIMO transfer function

$$H(s) = \frac{B_1 s^{n-1} + \cdots + B_n}{a_0 s^n + a_1 s^{n-1} + \cdots + a_n} \tag{1}$$

where the a_i are scalars and the B_i are $\ell \times m$ matrices. Algorithmically, the n alpha parameters are computed from the denominator of (1) as (ref. 4)

$$\left.\begin{aligned} a_j^0 &= a_{n-j} \\ a_j^1 &= a_{n-j-1} \end{aligned}\right\} \quad j = 0,2,4,\ldots \le n \tag{2}$$

and

$$\left.\begin{aligned} \alpha_i &= a_0^{i-1}/a_0^i \\ a_j^{i+1} &= a_{j+2}^{i-1} - \alpha_i a_{j+2}^i \end{aligned}\right\} \begin{aligned} &i = 1,2,\ldots, n-1 \\ &j = 0,2,4,\ldots \le n-i-1 \end{aligned} \tag{3}$$

where

$$a_j^i = 0 \qquad \text{for } i + j > n$$

The coefficients a_j^i can be arranged as entries in a Routh table (ref. 2). This tabular construction suggests the name Routh approximation.

The beta parameters are computed from both the numerator and the denominator of the given transfer function. Consider the single input-single output (SISO) transfer function from input q to output p implicitly given in (1), $H_{pq}(s)$. The beta parameters are found for this SISO transfer function as

$$\left.\begin{aligned} b_j^1 &= b_{n-j} \\ b_j^2 &= b_{n-j-1} \end{aligned}\right\} j = 0,2,4,\ldots \le n \tag{4}$$

and

$$\left.\begin{aligned} \beta_i &= b_0^i/a_0^i \\ b_j^{i+2} &= b_{j+2}^i - \beta_i a_{j+2}^i \end{aligned}\right\} \begin{aligned} &i = 1,2,\ldots,n \\ &j = 0,2,4,\ldots \le n-i-2 \end{aligned} \tag{5}$$

where

$$b_j^i = 0 \qquad \text{for } i + j > n$$

and b_r is the qp-th element of the matrix B_r, $r = 1, n$. In this way beta parameters for the $\ell \times m$ possible SISO transfer functions of (1) can be found. Again considering the SISO transfer function, $H_{pq}(s)$, the computation of the k-th approximant from the first k alpha and beta parameters is given recursively as

$$\left.\begin{aligned} a_0^k &= 1 \\ a_0^{i-1} &= \alpha_i a_0^i \\ b_0^i &= \beta_i a_0^i \\ a_j^{i-1} &= \alpha_i a_j^i + a_{j-2}^{i+1} \\ b_j^i &= \beta_i a_j^i + b_{j-1}^{i+2} \end{aligned}\right\} \begin{aligned} &i = k, k-1, \ldots, 1 \\ &j = 2,4,6,\ldots \le k-1 \end{aligned} \tag{6}$$

and

$$\left.\begin{aligned} c_{k-j} &= a_j^0 \\ c_{k-j-1} &= a_j^1 \end{aligned}\right\} j = 0,2,4,\ldots \le k \tag{7}$$

$$\left.\begin{aligned} d_{k-j} &= b_j^1 \\ d_{k-j-1} &= b_j^2 \end{aligned}\right\} j = 0,2,4,\ldots \le k \tag{8}$$

where

$$a_j^i = b_j^i = 0 \qquad \text{for } i + j > k$$

Note that for different output-input pairs different approximant orders, k, could be chosen. The k-th order approximant for the pq-th output-input pair is written as

$$H_{pq_k}(s) = \frac{d_1 s^{k-1} + d_2 s^{k-2} + \ldots + d_k}{c_0 s^k + c_1 s^{k-1} + \ldots + c_k} \tag{9}$$

Properties

The Routh approximation method exhibits several very useful properties. Each is important in the reduction problem and is described briefly.

Stability. If the original transfer function is asymptotically stable, the Routh approximant, of any order, will be asymptotically stable.

Pole-zero locations. The poles and zeros of the approximants approach the poles and zeros of the original function as the order of the approximation is increased.

Impulse response energy. If $h(t)$ is the impulse response of $H_{pq}(s)$, and if the "impulse response energy" is defined as

$$E = \int_0^\infty h^2(t)\,dt \tag{10}$$

then

$$E = \sum_{i=1}^{n} \frac{\beta_i^2}{2\alpha_i} \tag{11}$$

Also, if $h_k(t)$ is the impulse response of the k-th order approximant of $H_{pq}(s)$, then

$$E_{k+1} = E_k + \frac{\beta_{k+1}^2}{2\alpha_{k+1}} \tag{12}$$

Since $H_{pq}(s)$ is assumed asymptotically stable, $\alpha_i > 0$ for $i = 1, . . ., n$ and

$$0 < E_1 < E_2 < E_3 < . . . < E_n = E \tag{13}$$

The ratio E_k/E gives an indication of the approximation accuracy in terms of the percentage of total energy accounted for by the k-th approximation.

<u>Derivatives</u>. The k-th Routh approximant satisfies the following derivative condition.

$$\frac{d^i}{ds^i}\{H(s)\}\Big|_{s=0} = \frac{d^i}{ds^i}\{H_k(s)\}\Big|_{s=0} \qquad i = 0,1, . . ., k - 1 \tag{14}$$

<u>Initial and final values</u>. The initial and final values of the step response trajectories for the original system and its k-th order Routh approximant are the same. Note here that this is accomplished even though the approximant is a strictly proper transfer function for all p and q.

Time Domain

The Routh approximation method is being extended to the time domain to enable Routh approximants to be directly determined from a state-space model formulation. The alternative would be to determine the $p \times q$ transfer functions for the MIMO system and then reduce each function individually. For a typical engine model with $p = 16$ and $q = 5$ this alternative would involve the reduction of eighty transfer functions at each operating point. Computationally, this would be undesirable. Hutton (ref. 3) outlines a method for directly determining the alpha and beta parameters from the state-space formulation. However, computationally, this method failed for the 16-th order engine example of this paper. Thus an alternate computational procedure was devised that incorporates eigenvalue-eigenvector solutions which are well known and for which excellent computational solutions exist (ref. 8).

Given the state-space representation, Σ_1

$$\begin{aligned} \dot{\hat{x}} &= \hat{A}\hat{x} + \hat{B}u \\ y &= \hat{C}\hat{x} \end{aligned} \tag{15}$$

where $x \varepsilon R^n$, $u \varepsilon R^m$, and $y \varepsilon R^\ell$, define a reciprocal system, Σ_2 such that

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{16}$$

where

$$\begin{aligned} A &= \hat{A}^{-1} \\ B &= -A\hat{B} \\ C &= \hat{C} \end{aligned} \tag{17}$$

The system Σ_1 is assumed stable with distinct eigenvalues. For the jet engine systems studied here this is a realistic assumption. Note that the reciprocal transformation of Σ_2 is Σ_1. This transformation will always exist since A is assumed stable. The transformation is required to preserve the dominant or low frequency information of Σ_1 during the reduction process (ref. 3). Now consider a nonunique similarity transformation of Σ_2, called the Routh transformation, such that

$$AT_R = T_R R \tag{18}$$

where R is the Routh stability matrix. That is

$$R = \Lambda_R^{-1}\Gamma \tag{19}$$

and

$$\Gamma = \begin{bmatrix} -1 & -1 & & & & & \\ 1 & 0 & -1 & & & 0 & \\ & 1 & 0 & -1 & & & \\ & & & \ddots & & & \\ & 0 & & & & 0 & -1 \\ & & & & & 1 & 0 \end{bmatrix} \tag{20}$$

and

$$\Lambda_R = \mathrm{diag}[\alpha_i] \qquad i = 1, . . ., n \tag{21}$$

Also, consider the modal transformation of Σ_2 where

$$AT_mD_m = T_mD_m\Lambda \tag{22}$$

and

$$\Lambda = \mathrm{diag}[\lambda_i] \qquad i = 1, . . ., n \tag{23}$$

Here the λ_i are eigenvalues of A and the i-th column of T_m is the corresponding eigenvector. Note that T_m will be unique with respect to a scaling convention and a specific order of the λ_i's. The scaling convention is represented by D_m, an arbitrary, full rank, diagonal matrix with diagonal elements d_i that may be complex.

Since R is similar to A

$$RT_zD_z = T_zD_z\Lambda \tag{24}$$

Again T_z is a modal matrix for R and Λ, and it is unique with respect to a scaling convention represented by D_z. From (18), (22), (24), and the definition

$$D = D_n D_z^{-1} \tag{25}$$

all possible Routh transformations can be characterized as

$$T_R = T_m D T_z^{-1} \tag{26}$$

Recall that a fixed ordering of eigenvalues is given. Permutations of this ordering can be represented by a permutation matrix, E_P, where

$$T_{RP} = E_P T_R \tag{27}$$

and E_P is the identity matrix with appropriate columns interchanged. Although this permutation does not affect the input-output transfer relationship of the original system, it will affect the input-output characteristics of the approximate system. Undoubtedly, one of the $n!$ possible state permutations will yield a better approximation in some sense than another, but this problem is not considered here.

From (18) a Routh canonical system, Σ_3, can be written as

$$\begin{aligned} \dot{x}_R &= Rx_R + G_u \\ y &= Hx_R \end{aligned} \tag{28}$$

where

$$x = T_R x_R \tag{29}$$

The Routh approximation procedure starts by assuming that

$$x_{R_2} = 0 \tag{30}$$

where

$$x_R = \begin{bmatrix} x_{R_1} \\ \hline x_{R_2} \end{bmatrix} \tag{31}$$

and $x_{R_1} \epsilon R^k$, and $x_{R_2} \epsilon R^{n-k}$. The system Σ_2 incorporating the assumption of (30) becomes Σ_2^1 or

$$\begin{aligned} \dot{x}_1 &= T_{R_{11}} R_{11} T_{R_{11}}^{-1} x_1 + T_{R_{11}} G_1 u \\ y &= H_1 T_{R_{11}}^{-1} x_1 \end{aligned} \tag{32}$$

where $x = \begin{bmatrix} x_1 \\ \hline x_2 \end{bmatrix}$, and $x_1 \epsilon R^k$ and the subscripts indicate conformable partitioning of the appropriate matrices. The desired approximation then is the reciprocal transformation of Σ_2^1, called Σ_1^1.

Examination of (32) shows that the reduced order system matrix, $T_{R_{11}} R_{11} T_{R_{11}}^{-1}$, is simply a similarity transformation of the truncated Routh stability matrix. A truncation of an n-th order Routh matrix (R) yields another Routh matrix of k-th order (R_{11}). Since the original system is stable, the alpha parameters of R, α_i, $i = 1, n$ are positive. Therefore, R_{11} is also stable. Additionally, the reduction process implied in (32) exhibits the same pole properties as the frequency response approach.

Hutton (ref. 3) has shown that if the system given by (16) is a single input-multiple output system and T_R is selected such that

$$T_R^{-1} B = \begin{bmatrix} -1/\alpha_1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{33}$$

then the elements of the rows of H, h_i, become the beta parameters for the output considered

$$h_i = (\beta_{1i}, \beta_{2i}, \ldots, \beta_{ni}) \tag{34}$$

and the properties of the approximation described previously hold. It can be shown that if D is selected as

$$D = \mathrm{diag}[\theta_i]$$

where

$$\theta = T_m^{-1} B$$

then (33) is satisfied.

COMPUTATIONAL ASPECTS OF ROUTH APPROXIMATION

Computational aspects of both the frequency and time domain formulations are discussed in this section.

Frequency Domain

In the frequency domain the Routh approximation problem is solved by the calculation of the alpha and beta parameters for a given transfer function. Straightforward programming of equations (3), (5), and (6) gives these parameters and the Routh approximants. This Routh table procedure is computationally efficient and accurate. For example, it has been employed directly to reduce a 43rd order line dynamics problem (ref. 9). Also, the frequency domain engine results were obtained using this procedure with no computational difficulties. More complete engine results using the frequency domain formulation are given in ref. 9.

Time Domain

Computationally, given the state-space formulation of (15), time domain Routh approximation requires (a) the alpha coefficients and (b) the Routh transformation matrix. The alpha coefficients can be found readily from (3) if the system characteristic equation coefficients can be found. A program that incorporates the method of Danielevski (ref. 6) to find the system characteristic equation of the system matrix, $\hat{A}$, was used in this report.

A method for the computation of the Routh transformation matrix of (18) has been proposed by Datta (ref. 7). This method was programmed and applied to the 16th order jet engine model. Numerical results were unacceptable due to large computational errors and another technique was sought. Since accurate and efficient eigenvalue-eigenvector techniques are well known (for the case of distinct eigenvalues), the computation of the Routh transformation matrix was reformulated as the solution of two eigenvalue-eigenvector problems. These two problems are represented by (22) and (24) and solved using the computational methods of reference 5. Since A and R are similar, they have the same eigenvalues. The Routh matrix R is also known from (19) since the alpha parameters have been found.

The transformation matrices of (22) and (24) may have elements in the complex field. Thus from (2) to (6), T_R can in general be complex. However, a real T_R is desired to yield a physically realizable approximation and to facilitate computer computations. The matrix, T_R, can be constrained to be real by the proper selection of the elements d_i of D. To determine these elements consider the eigenvector solution of (24) using real number computer operations. Given A and its eigenvalues, the modal matrix of A can be written as

$$T_{mm} = T_m V_m \tag{35}$$

where V_m is block diagonal and T_{mm}, the modified modal matrix of A, is a matrix of real numbers. If the eigenvalues of A were all real, then $V_m = I$. However, if λ_i and λ_{i+1} are a complex conjugate pair, then the i-th block of V_m, V_{m_i} is defined as

$$V_{m_i} = \frac{1}{2}\begin{pmatrix} 1 - j & 1 + j \\ 1 + j & 1 - j \end{pmatrix} \tag{36}$$

Likewise for (24)

$$T_{zm} = T_z V_m \tag{37}$$

Now from (26), (33), and (35) and the definition

$$D_B = V_m^{-1} D V_m \tag{38}$$

T_R can be written

$$T_R = T_{mm} D_B T_{zm}^{-1} \tag{39}$$

Now if T_R is to be real, then D_B, a block diagonal matrix, must be real. This can be assured by selecting the elements of D, d_i, such that:

(1) d_i is real if λ_i is real

(2) d_i, d_{i+1} are a complex conjugate pair if λ_i, λ_{i+1} are a complex conjugate pair.

It is remarked that the reciprocal transformation of (17) does not represent a significant increase in computations since the modal transformation is the same for A and $\hat{A}$ and the eigenvalues are simply reciprocals.

The matrix inversion of (26) can be eliminated by noting that

$$\Gamma = \tilde{T}\Gamma^T\tilde{T} \tag{40}$$

where

$$\tilde{T} = \mathrm{diag}[(-1)^{i-1}] \tag{41}$$

From (19) then

$$\Lambda_R \tilde{T} R = R^T \Lambda_R \tilde{T} \tag{42}$$

Now from (24), (26), and (40)

$$T_R = T_m D_m T_z^T \tilde{T} \Lambda_R \tag{43}$$

APPLICATION TO ENGINE MODEL

The frequency and time domain Routh approximation formulations were applied to a 16th order linear state variable representation of engine dynamics for small deviations about an operation point. The actual engine has five inputs and sixteen outputs when the states are taken as outputs. However, for the purposes of this paper only two inputs and two outputs were considered. The inputs are

$$u_1 = w_f = \text{Engine fuel flow}$$

$$u_2 = A_N = \text{Engine exhaust nozzle area}$$

and the outputs are

$$y_1 = N_c = \text{Engine compressor rotational speed}$$

$$y_2 = T_T = \text{Engine turbine inlet temperature}$$

These variables were selected because of their importance in engine control applications and because of the wide range of dynamic content of N_c and T_T. In an engine the speed, N_c, is one of the slowest responding variables while the temperature, T_T, is one of the fastest. The input data for this engine are given in Table V.

Frequency Domain Application

As an initial experiment the frequency domain formulation was applied to two 16th order SISO transfer functions. These transfer functions represent dynamics of the turbofan engine from the input, w_f, to the outputs, N_c and T_T. The coefficients for these transfer function were calculated using the programs of reference 9 and are given in Table I. Table II shows the impulse response ratios for Routh approximations of increasing order for the two transfer functions. These ratios were calculated from the alpha and beta coefficients as outlined in (12). From these ratios an estimate of an acceptable approximation order can be obtained.

The approximant order was selected by first choosing a minimum acceptable level of accuracy defined by the impulse response energy ratio. The level chosen corresponds to a ratio of 0.81. This level of accuracy was assumed adequate for the purpose of this paper. Next the order that most nearly corresponds to the selected ratio was chosen as the order of the Routh approximant. For the energy ratios given in Table II the Routh approximant orders as found by the criterion described above are

$$k_1 = 3 \qquad (y_1 = N_c)$$

$$k_1 = 9 \qquad (y_2 = T_T)$$

The Routh approximants for N_c and T_T were calculated using the approximant orders k_1 and k_2 and the algorithm given in (6) to (9). The results are summarized in Table III. Comparisons of pole locations and step responses for given and approximate transfer functions were made to evaluate the adequacy of the approximations. Table IV gives the pole locations and the step response comparisons are given in Figure 1. Note that the pole comparison shows a good correlation between actual and approximate system poles. Likewise the step responses of Figure 1 indicate excellent agreement between actual and approximate model representations. Based on these results it is concluded that the Routh approximation technique is a viable approach to the reduction of complexity in frequency domain models of jet engine dynamics. Also, the accuracy level selected in this initial example may be too stringent based on the step response comparison. Further results are summarized in reference 9.

Time Domain Application

The time domain calculation procedures outlined above were applied to the engine model assuming that a 5th order approximation for each input was adequate. A comparison of alpha parameters calculated by the frequency and time domain approaches summarized in Table VI shows the accuracy of the time domain approach formulated in this paper. Step response trajectories for the original and approximate systems are shown in Figure 2 and indicate, in the case of T_T, the tradeoff between accuracy and complexity.

DISCUSSION

Note that for a five input system approximated by 5th-order models, a total reduced system realization may require a state vector dimension of 25. In fact the realization dimension will probably vary for different numbers of inputs and orders of approximation. Often, however, the allowable dimension of the total reduced approximation is fixed by some engineering or economic constraint.

Constructing a fixed dimension realization of the total system from the Routh approximation can be a very difficult computational problem especially if the numbers of inputs and outputs are large. Thus the technique as outlined does not directly handle the fixed dimension problem. However, if the same T_R matrix is used in the time domain reduction process for each input, the total approximation can be realized by a system of k-th order where k is the number of alpha parameters retained and the fixed dimension approximation problem is solved. The selection of the T_R matrix which will give the best k-th order approximation is therefore an interesting problem.

In this regard the somewhat arbitrary selection of D can be used to good advantage. Many different transformations can be found very quickly once the original two eigenvector problems have been solved. Indeed, the selection process could be automated by performing an optimization of some function of error between the original and total approximant system over the n parameter space of D. Such an optimization scheme was tried on the example given in this paper. Two different error functions were used for comparison. The first was a weighted sum of the differences in system and approximant step response energy. The second was a weighted sum of squares of the differences in system and approximant steady-state values. Significant minimization of each error function was easily achieved using a conjugate direction optimization scheme in relatively small amounts of computer time. Thus, the general optimization procedure appears to be a good approach to improve the accuracy of approximants for linear systems while maintaining a fixed order of realization. However, the time domain Routh approximation procedure when constrained to yield a fixed dimension realization in the multiple input case does not exhibit the final value property of the single input case. For the engine example posed, the significant improvement in the approximation gained via function optimization was overshadowed by these final value errors for the multi-input case.

Based upon this observation the time domain formulation was modified to insure that the final value property would be met by a fixed dimension, multi-input Routh approximant. In the original formulation the difficulty with final values in the multi-input, fixed dimension problem can be traced to the original assumption of the reduction process (30). If the initial assumption were changed to

$$\dot{x}_{R_2} = 0 \tag{44}$$

then the approximation would force the final value property. Unfortunately, the assumption of (44) when applied to the original system may yield an unstable approximant for a given, stable system. This was the case for the engine example and, thus, the modification of the Routh procedure was rejected.

CONCLUSIONS

In this paper the Routh approximation process was reformulated in the time domain. A new characterization of the nonunique Routh similarity transformation was derived which describes all possible Routh transformations. This characterization casts the computation of the Routh transformation into two eigenvector-eigenvalue problems which are easily solved. The application of the time domain formulation to a 16th order state variable description of a turbofan engine was described and results given. These results indicate that the time domain Routh approximation technique can be a valuable technique for reducing engine model complexity when dealing with the model on a single input basis. An optimization procedure was discussed that can significantly improve the approximation in a computationally efficient manner by taking advantage of the new time domain characterization.

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TABLE I. COEFFICIENTS FOR TWO 16TH ORDER ENGINE TRANSFER FUNCTIONS WITH FUEL FLOW INPUT (w_f)

i	a_i	N_c(p = 1, q = 1) b_i	T_4(p = 2, q = 1) b_i
0	1.0000		
1	1.0638E03	0.1140	5.7270
2	3.7805E06	4.2916E01	5.8161E03
3	6.6915E08	-4.2414E03	1.8835E06
4	7.0216E10	-2.2082E06	2.9209E08
5	4.7772E12	-8.2163E07	2.6079E10
6	2.2150E14	2.2542E10	1.4714E12
7	7.1949E15	2.8702E12	5.5192E13
8	1.6584E17	1.5789E14	1.4142E15
9	2.7151E18	5.0230E15	2.5027E16
10	3.1255E19	1.0028E17	3.0507E17
11	2.4739E20	1.2850E18	2.5208E18
12	1.2973E21	1.0446E19	1.3685E19
13	4.2578E21	5.1525E19	4.6358E19
14	7.9983E21	1.4079E20	9.0249E19
15	7.4304E21	1.7841E20	8.7170E19
16	2.4119E21	7.4230E19	2.9263E19

TABLE II. IMPULSE RESPONSE ENERGY RATIOS FOR INCREASING APPROXIMANT ORDER (Fuel Flow Input)

Routh approximant order (k)	N_C (p = 1, q = 1)- impulse response energy ratio	T_T (p = 2, q = 1)- impulse response energy ratio
1	0.41	0.007
2	.72	.03
3	.89	.07
4	.94	.14
5	.941	.23
6	.95	.36
7	.96	.51
8	.98	.67
9	.99	.81
10	.997	.91
11	.998	.97
12	.999	.99
13	.9992	.998
14	.9998	.999
15	.9999	.9999
16	1.0000	1.0000

TABLE III. ROUTH APPROXIMANT COEFFICIENTS FOR TWO TRANSFER FUNCTIONS

i	N_c (p = 1, q = 1) c_i	N_c (p = 1, q = 1) d_i	T_T (p = 2, q = 1) c_i	T_T (p = 2, q = 1) d_i
0	1.0000		1.0000	
1	2.6132	0.43978	4.7092	3.6223
2	2.5702	.61711	1.0446E03	9.3469E01
3	.8343	.25676	1.3858E04	1.3400E03
4			1.1722E05	1.1906E04
5			6.3304E05	6.6700E04
6			2.1019E06	2.2877E05
7			3.9634E06	4.4718E05
8			3.6848E06	4.3229E05
9			1.1961E06	1.4512E05

TABLE IV. COMPARISON OF POLE LOCATIONS FOR THE EXACT (16TH ORDER) AND APPROXIMATE (3RD AND 9TH ORDER) TRANSFER FUNCTIONS

	Poles of 16th order transfer functions	Poles of 3rd order Routh approximant	Poles of 9th order Routh approximant
1	-0.648	-0.635	-0.648
2	-1.91	-0.989±0.580j	-1.91
3	-2.62		-2.62
4	-6.71±1.31j		-6.50±1.16j
5	-17.8±4.80j		-7.64±3.29j
6	-18.2		-6.82±8.73j
7	-21.6±1.55j		
8	-38.7		
9	-47.1		
10	-50.6		
11	-59.2		
12	-175.		
13	-577.		
14			
15			
16			

TABLE V. ENGINE MODEL DATA (n = 16, p = 2, q = 2)

The A Matrix

-4.32800	0.218859	0.200384	3.95571	-2.98299	-0.144833	0.769294-01	-0.144819	0.252614
-0.344744	-5.64300	3.72188	-1.80121	-1.40020	1.56028	.120495	-0.299362	2.24174
27.8479	208.042	-165.000	-1.18466	115.858	-165.738	-10.7525	20.1663	-0.636953-01
53.8484	-14.0784	496.865	-578.300	42.6308	-70.2873	-8.77498	-36.0530	-79.0958
2.05891	-4.84795	.507217	3.76362	-10.0500	3.55273	-0.161294	-0.755865	.128376
11.1442	-0.212853	-0.177448-01	-0.798490-01	2.06863	-19.7900	-0.182496	-0.582995-01	-0.659112-01
8.97363	-0.213997	-0.478545-01	-0.465845-01	4.37978	-0.816499-01	-20.4700	-0.768059-01	.629645-01
-0.657472	7.50337	4.26387	.701969-01	-2.85571	15.0693	0.303936	-19.9700	.386312-01
-0.347415-01	-0.826121-01	.272257-01	.844885-02	-0.137080	.207504	.168996-01	3.86258	-49.9900
-0.461982-02	-0.110436-01	.360582-02	.112675-02	-0.182567-01	.277078-01	.225353-02	.515019	-5.99940
-4.09217	-29.0787	3.31853	.417099	-17.7117	26.1649	1.91267	18.4196	.244677
-0.143460	-3.04565	-10.3264	10.2900	-0.607676	.883145	.711020-01	7.55591	-12.7924
-0.570369-01	-1.21806	-4.13034	4.11573	-0.242984	.353474	.287463-01	3.02213	-5.11614
-0.908447	-1.77558	-5.04877	1.29700	3.39206	.691218	.588312-01	3.72598	-6.28627
-0.163698	.507438	-0.587604-01	-0.624872-02	.307427	-0.446147	19.6607	.242382	-0.375007-02
-11.9372	16.8839	-1.02879	8.06630	10.1496	-14.7933	-1.11110	-0.656673	.913777

0.427346-01	-1.19810	1.82058	0.213133	-0.106835	-0.997036-02	0.132963-01
.239841	-0.940377	-0.855236-02	-0.100398-01	-0.104123-01	-0.353279-01	-0.104131-01
.101911	44.6143	.114667	.101938	.114658	2.24267	.101904
-12.9188	170.896	-4.53242	-4.53157	-4.43444	-3.61533	-4.53105
-0.358717	-2.92106	-0.424440	-0.420034	1.47850	.922059-01	-0.422479
-0.608226-01	-0.253476-01	-0.595834-01	-0.621119-01	-0.595750-01	-0.380239-02	-0.621158-01
-0.793110-01	-0.365148-01	-0.793274-01	-0.818338-01	-0.793363-01	.894036-01	-0.818498-01
.341217-01	.167423-01	.341306-01	.367042-01	.341328-01	-0.193187-01	.341318-01
.751111-02	46.0276	.375602-02	.751138-02	.375590-02	.000000	.751331-02
-0.665700	6.13815	.450678-03	.101370-02	.450645-03	.000000	.101380-02
.222139	-47.6500	.219747	.219719	.218157	-0.111896	.219763
-1.41104	59.6425	-50.0100	.112333-01	.112245-01	-0.187131-01	.124796-01
-0.564075	23.8539	-18.0000	-1.99600	.449175-02	-0.718453-02	.494061-02
-0.688976	29.3366	-3.33640	-0.359398	-19.7700	-0.213957-01	.117686-01
-0.249931-02	.712218-01	-0.249937-02	-0.374785-02	-0.250001-02	-20.0000	-0.374824-02
-0.291566-01	-5.14614	-0.147456	-0.164604	39.2746	10.9155	-50.1600

The B Matrix

-0.469024-01	-0.124675
.895387-01	-0.118071
5.92813	10.0878
33.9885	5.82984
2.48005	-6.59813
.373669	.263890
.233509	.262489
-0.538716	-0.264328
.156463	-0.841755-02
.208467-01	-0.112409-02
19.9275	-1.63330
.750732	-0.352531-01
.299580	-0.141273-01
.391725	.258106-01
.253898-01	.332296-01
1.24292	.868843

The C^T Matrix

1.00000	0.000000
.000000	.000000
.000000	.000000
.000000	.000000
.000000	.000000
.000000	.000000
.000000	.000000
.000000	.000000
.000000	.000000
.000000	.000000
.000000	1.000000
.000000	.000000
.000000	.000000
.000000	.000000
.000000	.000000
.000000	.000000

TABLE VI. COMPARISON OF COMPUTATIONAL ACCURACY OF FREQUENCY AND TIME DOMAIN FORMULATIONS

Alpha parameters	
From Routh table	From transformation $T_R^{-1} A T_R$
0.32460	0.32460
1.1230	1.1231
2.[illegible]886	2.2886
3.9662	3.9662
6.2954	6.2954
9.3914	9.3914
13.377	13.378
18.425	18.425
24.806	24.806
33.033	33.033
44.121	44.121
60.072	60.072
85.236	85.236
131.79	131.79
254.39	254.39
806.00	806.00

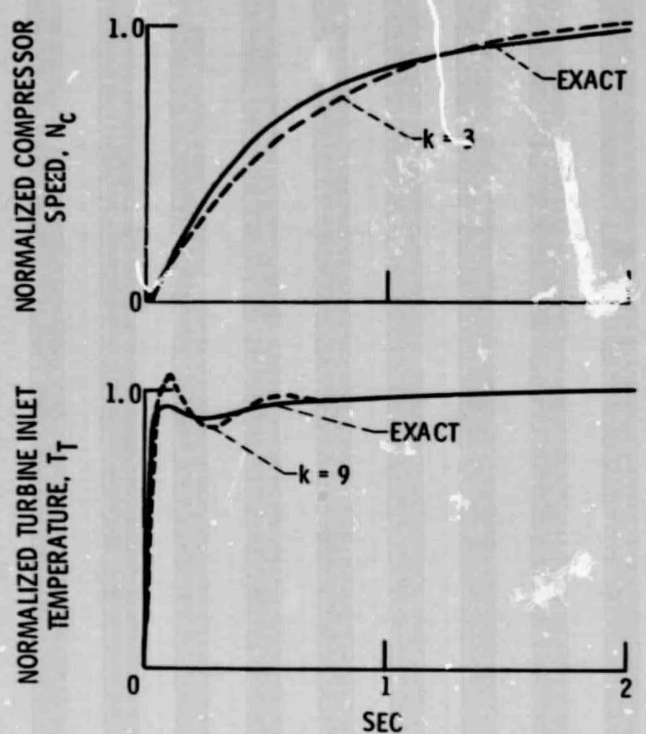

Figure 1. - Step response comparisons for outputs N_C and T_T for fuel flow, w_f input.

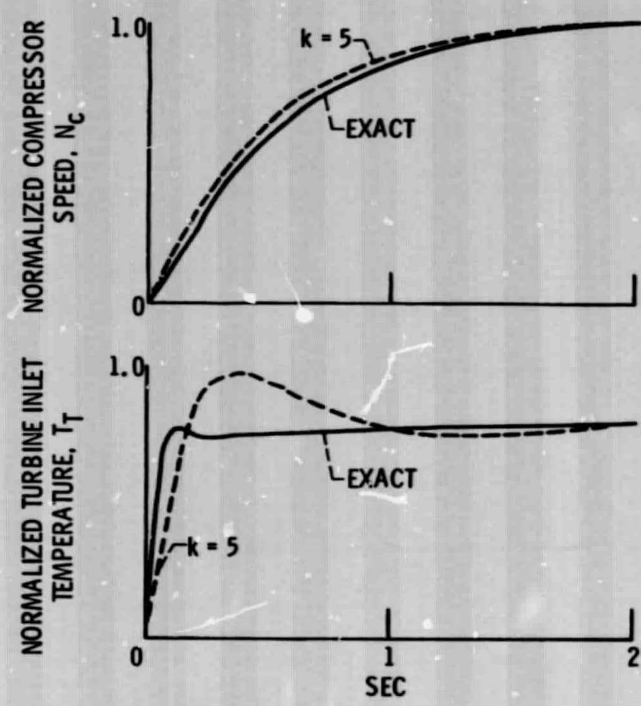

Figure 2. - Step response comparisons for outputs N_C and T_T for fuel flow input (time domain approach.

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