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SLENDER BODY TREATMENT OF SOME SPECIALIZED PROBLEMS ASSOCIATED WITH ELLIPTIC-CROSS-SECTION MISSILE CONFIGURATIONS

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16. Abstract

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SLENDER BODY TREATMENT OF SOME SPECIALIZED PROBLEMS ASSOCIATED

WITH ELLIPTIC-CROSS-SECTION MISSILE CONFIGURATIONS

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SUMMARY

Slender body methods have been applied to some specialized problems associated with missile configurations with elliptic cross sections. Expressions are derived for computing the velocity distribution on the nose section when the ellipse eccentricity is varying longitudinally on the missile. The cross-flow velocity on a triform fin section is also studied.

INTRODUCTION

In recent years, interest has grown in missiles with bodies having elliptic cross sections. One reason for this increased interest is the possibility of an aerodynamically efficient mounting of such a low-profile missile on the parent airplane. Another advantage of such missiles over conventional circular-cross-section configurations is the increase in directional stability that occurs with increasing ellipse eccentricity (ref. 1).

A primary tool for missile analysis is slender body theory (ref. 2). One of the advantages of this theory is that it permits the nose and finned sections of the configuration to be treated separately. Thus a missile with horizontal fins, constant-eccentricity elliptic cross sections in the nose section, and an elliptic cylinder body in the finned section could be analyzed by combining the results of reference 2 (sec. 4.3), 3, or 4 for the nose section with those of reference 5 for the finned section.

This report presents some further results for missiles with elliptic-cross-section bodies. First, velocity expressions are obtained for the nose section with ellipse eccentricity varying longitudinally along the missile (fig. 1). This result is important because the shape of the equipment that must be carried in the nose section closely controls the cross-section shape and, consequently, the permissible eccentricity. The analysis is based on a result derived in reference 4.

Second, expressions are obtained for the cross-flow velocities in the finned section for a triform configuration. This analysis utilizes, first, a conformal transformation of the ellipse to a circle, and then a series of transformations that has been previously used by Bryson (ref. 6).

The present work represents a supplement to the already substantial body of slender body results applicable to missile configurations. The use of the expressions obtained herein, together with those already available, permits the analysis of certain configurations not treated in previous work. A separate computer program has not been written for the calculation of these expressions. Rather, they are to be incorporated into already existing slender body programs to permit application of those programs to a larger class of missiles.

SYMBOLS

A	ellipse semimajor axis
B	ellipse semiminor axis
b_0	defined by equation (9)
c	$= \sqrt{A^2 - B^2}$
D	see equation (35)
$\mathcal{D}$	see equation (39)
d	y-coordinate of horizontal fin tip
$\mathcal{d}$	see equation (34)
F	complex force
L	lift force
M	free-stream Mach number
q	free-stream dynamic pressure
S	cross-sectional area of missile body
s	coordinate directed along missile axis
$\bar{s}$	coordinate in direction of undisturbed free stream
T	see equation (37)
t	z-coordinate of vertical fin tip
$\mathcal{t}$	see equation (36)
u,v,w	perturbation velocity components in s-, y-, and z-direction, respectively
W	complex velocity potential
x	complex variable in plane perpendicular to missile axis, $y + iz$

x_c complex coordinate of cross-section center of area for force calculation, $-i\alpha\bar{s}$

x_1, x_2, x_3, x_4 complex variables in successive transform planes

y, z real variables in missile cross plane

α angle of attack

β $= \sqrt{M^2 - 1}$

β_s sideslip angle

δ see equation (33)

η, ζ elliptic coordinates (eq. (1a))

η_0 value of η representing missile section contour

ξ $= \eta + i\zeta$

ξ_c defined by equation (27)

ρ defined by equation (2)

ρ_0 $= \rho(\eta_0)$

ϕ velocity potential, real part of W

Subscripts:

h associated with side flow

t associated with thickness

α associated with angle of attack (vertical component of free stream in cross plane)

ANALYSIS OF FLOW ON MISSILE NOSE SECTION WITH VARIABLE-ECCENTRICITY ELLIPTIC CROSS SECTIONS

In this section, expressions are derived for computing the velocity components due both to thickness and to angle of attack on a nose section having elliptic cross sections that vary in eccentricity longitudinally along the missile axis. It is also shown that, within the slender body approximation, the total force on the nose section depends only on the cross-section shape at the base of the nose section.

Basic Relations

The coordinate system is taken along the missile axis (s-axis) and perpendicular to it (x- and y-axes) (fig. 2). The velocity distribution on the missile is more conveniently obtained with the use of elliptic coordinates η,ζ defined by the transformation (see ref. 7, pp. 157-159),

$$y + iz = x = c \cosh \xi = c \cosh (\eta + i\zeta) \tag{1a}$$

or

$$\left.\begin{aligned} y &= c \cosh \eta \sin \zeta \\ z &= c \sinh \eta \sin \zeta \end{aligned}\right\} \tag{1b}$$

From equation (1a), one obtains the relation,

$$\sqrt{x^2 - c^2} = c \sinh \xi$$

Another useful variable is defined by

$$\rho = \frac{ce^{\eta}}{2} \tag{2}$$

On the surface $\eta = \eta_0$,

$$\left.\begin{aligned} A &= c \cosh \eta_0 \\ B &= c \sinh \eta_0 \end{aligned}\right\} \tag{3}$$

and ρ has the value

$$\rho_0 = \frac{ce^{\eta_0}}{2} = \frac{A + B}{2} \tag{4}$$

Thus the ellipse $\eta = \eta_0$ maps into the circle $\rho = \rho_0$. Then the transformation (eqs. (1b)) can be written in terms of ρ and ζ as follows:

$$\left.\begin{aligned} y &= \frac{1}{2}\left(ce^{\eta} + \frac{c^2}{ce^{\eta}}\right)\cos\zeta = \left(\rho + \frac{c^2}{4\rho}\right)\cos\zeta \\ z &= \frac{1}{2}\left(ce^{\eta} - \frac{c^2}{ce^{\eta}}\right)\sin\zeta = \left(\rho - \frac{c^2}{4\rho}\right)\sin\zeta \end{aligned}\right\} \qquad (5)$$

The surface values of $d\rho/ds$ and $d\zeta/ds$ will be needed in the analysis. To obtain them, first differentiate equations (5) with respect to s:

$$\left.\begin{aligned} \left(1 - \frac{c^2}{4\rho^2}\right)\cos\zeta\,\frac{d\rho}{ds} - \left(\rho + \frac{c^2}{4\rho}\right)\sin\zeta\,\frac{d\zeta}{ds} &= -\frac{c}{2\rho}\cos\zeta\,\frac{dc}{ds} \\ \left(1 + \frac{c^2}{4\rho^2}\right)\sin\zeta\,\frac{d\rho}{ds} + \left(\rho - \frac{c^2}{4\rho}\right)\cos\zeta\,\frac{d\zeta}{ds} &= \frac{c}{2\rho}\sin\zeta\,\frac{dc}{ds} \end{aligned}\right\} \qquad (6)$$

The coefficients simplify considerably on the missile surface because of the relations

$$1 - \frac{c^2}{4\rho_0^2} = \frac{2B}{A + B}$$

$$1 + \frac{c^2}{4\rho_0^2} = \frac{2A}{A + B}$$

$$\rho_0 + \frac{c^2}{4\rho_0} = A$$

$$\rho_0 - \frac{c^2}{4\rho_0} = B$$

Making these substitutions in equations (6) and solving for the derivatives yields, on the missile surface,

$$\left.\begin{aligned} \frac{d\zeta}{ds} &= \frac{\sin\zeta\cos\zeta}{2(A^2\sin^2\zeta + B^2\cos^2\zeta)}\frac{d(A^2 - B^2)}{ds} \\ \frac{d\rho}{ds} &= \frac{A\sin^2\zeta - B\cos^2\zeta}{4(A^2\sin^2\zeta + B^2\cos^2\zeta)}\frac{d(A^2 - B^2)}{ds} \end{aligned}\right\} \tag{7}$$

Velocity Components on Nose Section Due to Thickness

According to slender body theory, the velocity components due to thickness can be determined separately from those due to angle of attack. The scalar potential function representing the effect of thickness is derived in reference 4 (eq. (23)) to be (in the notation of this report)

$$\phi_t(\eta,\zeta) = \frac{1}{2\pi}\frac{dS(s)}{ds}\left(\eta + \ln\frac{c}{2}\right) + b_0 + \frac{A^2}{4}\frac{d}{ds}\left(\frac{B}{A}\right)e^{-2(\eta-\eta_0)}\cos 2\zeta \tag{8}$$

where

$$b_0 = \frac{1}{2\pi}\left[\frac{dS(s)}{ds}\ln\frac{\beta}{2} - \int_0^s \ln(s-\sigma)\frac{d^2S(\sigma)}{d\sigma^2}d\sigma\right] \tag{9}$$

and unit undisturbed free-stream velocity is assumed. From equation (2),

$$\eta + \ln\frac{c}{2} = \ln\rho$$

and on the surface

$$\begin{aligned} \frac{A^2}{4}\frac{d}{ds}\left(\frac{B}{A}\right)e^{2\eta_0} &= \frac{c^2\cosh^2\eta_0}{4}e^{2\eta_0}\frac{d}{ds}(\tanh\eta_0) \\ &= \frac{c^2}{4}e^{2\eta_0}\frac{d\eta_0}{ds} \\ &= \rho_0^2\frac{d\eta_0}{ds} \end{aligned}$$

Thus when the eccentricity is constant, this quantity is zero. With these substitutions, equation (8) becomes

$$\phi_t(\rho,\zeta) = \frac{1}{2\pi}\frac{dS(s)}{ds}\ln\rho + b_0 + \rho_0^2\frac{d\eta_0}{ds}e^{-2\eta}\cos 2\zeta \tag{10}$$

Then the axial perturbation velocity u_t is determined on the surface by differentiating equation (10):

$$u_t = \frac{d\phi}{ds}$$

$$= \frac{1}{2\pi}\frac{d^2S(s)}{ds^2}\ln\rho_0 + \frac{1}{2\pi}\frac{dS(s)}{ds}\frac{1}{\rho_0}\left.\frac{d\rho}{ds}\right|_{\eta=\eta_0} + \frac{db_0}{ds} + 2\rho_0\frac{d\rho_0}{ds}e^{-2\eta_0}\cos 2\zeta\frac{d\eta_0}{ds}$$

$$+ \rho_0^2\frac{d^2\eta_0}{ds^2}e^{-2\eta_0}\cos 2\zeta - 2\rho_0^2 e^{-2\eta_0}\cos 2\zeta\frac{d\eta_0}{ds}\left.\frac{d\eta}{ds}\right|_{\eta=\eta_0}$$

$$- 2\rho_0^2 e^{-2\eta_0}\sin 2\zeta\frac{d\eta_0}{ds}\left.\frac{d\zeta}{ds}\right|_{\eta=\eta_0}$$

where from equation (2),

$$\left.\frac{d\eta}{ds}\right|_{\eta=\eta_0} = \frac{1}{\rho_0}\left.\frac{d\rho}{ds}\right|_{\eta=\eta_0} - \frac{1}{c}\frac{dc}{ds} \tag{11}$$

and from equation (9),

$$\frac{db_0}{ds} = \frac{1}{2\pi}\left[\frac{d^2S(s)}{ds^2}\ln\frac{\beta}{2} - \frac{d^2S(0)}{ds^2}\ln s - \int_0^s \ln(s-\sigma)\frac{d^3S(\sigma)}{d\sigma^3}d\sigma\right]$$

Combining terms and substituting $e^{-2\eta_0} = \dfrac{A - B}{A + B}$ yields

$$u_t = \frac{1}{2\pi} \frac{d^2S(s)}{ds^2} \ln \frac{\beta\rho_0}{2} + \frac{1}{2\pi} \frac{dS(s)}{ds} \frac{1}{\rho_0} \frac{d\rho}{ds}\bigg|_{\eta=\eta_0} - \frac{1}{2\pi} \frac{dS^2(0)}{ds^2} \ln s$$

$$- \frac{1}{2\pi} \int_0^s \ln (s - \sigma) \frac{d^3S(\sigma)}{d\sigma^3} d\sigma + (A - B) \cos 2\zeta \frac{d\rho_0}{ds} \frac{d\eta_0}{ds} + \frac{c^2}{4} \cos 2\zeta \frac{d^2\eta_0}{ds^2}$$

$$- \frac{c^2}{2} \cos 2\zeta \frac{d\eta_0}{ds} \frac{d\eta}{ds}\bigg|_{\eta=\eta_0} - \frac{c^2}{2} \sin 2\zeta \frac{d\eta_0}{ds} \frac{d\zeta}{ds}\bigg|_{\eta=\eta_0} \tag{12}$$

where $d\eta/ds$ is given by equation (11) with $d\rho/ds$ and $d\zeta/ds$ computed from equations (7).

For the special case of constant ellipse eccentricity, the last term in equation (8) is zero. Consequently, only the first four terms of equation (12) are retained for this case which is treated, for example, in reference 3.

The calculation of the cross-flow velocities resulting from the thickness effect is simplified somewhat by the use of the complex potential W_t. A formula for W_t can be obtained from equation (10) for the scalar potential by the method of reference 7 (p. 126). The result is

$$W_t(\xi) = \frac{1}{2\pi} \frac{dS(s)}{ds} \left(\xi + \ln \frac{c}{2}\right) + b_0 + \rho_0^2 \frac{d\eta_0}{ds} e^{-2\xi} \tag{13}$$

Then v_t and w_t are determined by

$$v_t - iw_t = \frac{dW_t}{d\xi} \frac{d\xi}{dx} \tag{14}$$

Here

$$\frac{d\xi}{dx} = \frac{1}{dx/d\xi} = \frac{1}{c \sinh \xi} \frac{c \sinh \bar{\xi}}{c \sinh \bar{\xi}} = \frac{c \sinh \eta \cos \zeta - ic \cosh \eta \sin \zeta}{c^2(\sinh^2 \eta + \sin^2 \zeta)} \tag{15}$$

where the bar indicates complex conjugate. On the surface, equations (3) apply. By making these substitutions and replacing c^2 by $A^2 - B^2$, one obtains

$$\left.\frac{d\xi}{dx}\right|_{\eta=\eta_0} = \frac{B \cos \zeta - iA \sin \zeta}{A^2 \sin^2 \zeta + B^2 \cos^2 \zeta} \tag{16}$$

The velocity in the ξ-plane is

$$\frac{dW_t}{d\xi} = \frac{1}{2\pi} \frac{dS(s)}{ds} - 2\rho_0{}^2 \frac{d\eta_0}{ds} e^{-2\xi}$$

On the surface

$$\left.\frac{dW_t}{d\xi}\right|_{\eta=\eta_0} = \frac{1}{2\pi} \frac{dS(s)}{ds} - 2\rho_0{}^2 e^{-2\eta_0} \frac{d\eta_0}{ds}(\cos 2\zeta - i \sin 2\zeta) \tag{17}$$

Substituting equations (16) and (17) in equation (14) yields

$$\left.\frac{dW_t}{dx}\right|_{\eta=\eta_0} = \left[\frac{1}{2\pi} \frac{dS(s)}{ds} - 2\rho_0{}^2 e^{-2\eta_0} \frac{d\eta_0}{ds}(\cos 2\zeta - i \sin 2\zeta)\right]\left[\frac{B \cos \zeta - iA \sin \zeta}{A^2 \sin^2 \zeta + B^2 \cos^2 \zeta}\right]$$

Thus,

$$v_t\Big|_{\eta=\eta_0} = \frac{\dfrac{B \cos \zeta}{2\pi} \dfrac{dS(s)}{ds} - \dfrac{c^2}{2} \dfrac{d\eta_0}{ds}(B \cos 2\zeta \cos \zeta - A \sin 2\zeta \sin \zeta)}{A^2 \sin^2 \zeta + B^2 \cos^2 \zeta} \tag{18}$$

$$w_t\Big|_{\eta=\eta_0} = \frac{\dfrac{A \sin \zeta}{2\pi} \dfrac{dS(s)}{ds} - \dfrac{c^2}{2} \dfrac{d\eta_0}{ds}(B \sin 2\zeta \cos \zeta + A \cos 2\zeta \sin \zeta)}{A^2 \sin^2 \zeta + B^2 \cos^2 \zeta} \tag{19}$$

For the special case of constant ellipse eccentricity, only the first term in the numerator remains for both components.

Velocity Components on Nose Section Due to Angle of Attack

The perturbation velocity potential for the cross flow due to angle of attack is (see ref. 7, p. 159), in the notation of this report,

$$W_{\alpha} = -i\alpha(A + B) \sinh (\xi - \eta_0) \tag{20}$$

The scalar potential, which is the real part of W, is

$$\phi_{\alpha} = \alpha \sin \zeta \left(\rho + \frac{\rho_0{}^2}{\rho}\right)$$

and its derivative with respective to s is

$$\frac{d\phi_{\alpha}}{ds} = \frac{\partial\phi}{\partial\rho_0}\frac{d\rho_0}{ds} + \frac{\partial\phi}{\partial\zeta}\frac{d\zeta}{ds} + \frac{\partial\phi}{\partial\rho}\frac{d\rho}{ds}$$

Thus,

$$u_{\alpha} = \alpha\left[\frac{2\rho_0}{\rho} \sin \zeta \frac{d\rho_0}{ds} + \cos \zeta \left(\rho + \frac{\rho_0{}^2}{\rho}\right)\frac{d\zeta}{ds} + \sin \zeta \left(1 - \frac{\rho_0{}^2}{\rho^2}\right)\frac{d\rho}{ds}\right] \tag{21}$$

On the surface,

$$\rho = \rho_0$$

$$\frac{d\rho_0}{ds} = \frac{1}{2}\left(\frac{dA}{ds} + \frac{dB}{ds}\right)$$

and $d\zeta/ds$ and $d\rho/ds$ are obtained from equations (7). Substituting these quantities into equation (21) and simplifying yields

$$u_{\alpha} = \frac{\alpha \sin \zeta \left[A^2 \frac{dA}{ds} + B(A + B) \cos^2 \zeta \frac{dA}{ds} + A(A \sin^2 \zeta - B \cos^2 \zeta)\frac{dB}{ds}\right]}{A^2 \sin^2 \zeta + B^2 \cos^2 \zeta} \tag{22}$$

For a body whose cross sections have the same eccentricity,

$$\frac{dB}{ds} = \frac{B}{A}\frac{dA}{ds}$$

and equation (22) reduces to

$$u_\alpha = \frac{\alpha A(A + B) \sin \zeta \dfrac{dA}{ds}}{A^2 \sin^2 \zeta + B^2 \cos^2 \zeta}$$

which is the known result for this case. (See ref. 2, p. 78, eq. (4-43).)

The cross-flow velocity components due to angle of attack are obtained from the relation

$$v_\alpha - iw_\alpha = \frac{dW_\alpha}{dx} = \frac{dW_\alpha}{d\xi}\frac{d\xi}{dx}$$

$$= -i\alpha(A + B) \cosh (\xi - \eta_0)\frac{d\xi}{dx}$$

Evaluating this expression on the surface with the use of equation (15) yields

$$v_\alpha - iw_\alpha = \frac{-i\alpha(A + B) \left[\cosh (\eta - \eta_0) \cos \zeta - i \sinh (\eta - \eta_0) \sin \zeta\right](B \cos \zeta - iA \sin \zeta)}{A^2 \sin^2 \zeta + B^2 \cos^2 \zeta}$$

$$v_\alpha - iw_\alpha\Big|_{\eta=\eta_0} = \frac{-i\alpha(A + B) \cos \zeta\ (B \cos \zeta - iA \sin \zeta)}{A^2 \sin^2 \zeta + B^2 \cos^2 \zeta}$$

Thus,

$$v_\alpha = \frac{-\alpha A(A + B) \sin \zeta \cos \zeta}{A^2 \sin^2 \zeta + B^2 \cos^2 \zeta} \qquad (23)$$

$$w_\alpha = \frac{\alpha B(A + B)\cos^2\zeta}{A^2\sin^2\zeta + B^2\cos^2\zeta} \tag{24}$$

These, of course, are perturbation velocity components. The total vertical velocity due to angle of attack is

$$\alpha\left[\frac{B(A + B)\cos^2\zeta}{A^2\sin^2\zeta + B^2\cos^2\zeta} - 1\right]$$

Total Force on Nose Section

For a missile at angle of attack, the complex force from 0 to $\bar{s}$ is given in reference 2 (eq. (3-62), p. 50) and, in the notation of the present report, is

$$\frac{F}{q} = 4\pi a_1 + 2\frac{d}{d\bar{s}}\left[S(\bar{s})\ x_c(\bar{s})\right] \tag{25}$$

where a_1 is the coefficient of x^{-1} in the Laurent expansion of $W(x)$ and where the coordinates are referred to the $\bar{s}$-axis parallel to the free-stream direction. Thus, in equations (13) and (20) for the complex thickness and lift (angle-of-attack) potentials, x is replaced by $x - x_c$, where

$$x_c = -i\alpha\bar{s}$$

Then in the equation for the total potential,

$$W = W_\alpha + W_t$$

the thickness component is written

$$W_t = b_0(s) + \frac{1}{2\pi}\frac{dS(s)}{ds}\ln\frac{x - x_c + \sqrt{(x - x_c)^2 - c^2}}{2} + \rho_0{}^2\frac{d\eta_0}{ds}e^{-2\xi_c} \tag{26}$$

where

$$e^{-\xi_c} = \frac{1}{c}\left[x - x_c - \sqrt{(x - x_c)^2 - c^2}\right] \tag{27}$$

Similarly, with the appropriate substitutions, equation (20) for the complex potential due to lift can be written

$$W_\alpha = -\frac{i\alpha}{2}\left\{x - x_c + \sqrt{(x - x_c)^2 - c^2} - \frac{(A + B)^2}{c^2}\left[x - x_c - \sqrt{(x - x_c)^2 - c^2}\right]\right\} \qquad (28)$$

The contribution of the first term of W_t (eq. (26)) to a_1 is zero. The contribution of the second term is known to be $-\frac{x_c}{2\pi}\frac{dS(s)}{ds}$. To find the contribution of the third term, which accounts for the variable eccentricity of the cross section, one must expand $e^{-2\xi_c}$ as follows:

$$e^{-2\xi_c} = \frac{1}{c^2}\left[(x - x_c)^2 - 2(x - x_c)\sqrt{(x - x_c)^2 - c^2} + (x - x_c)^2 - c^2\right]$$

$$= \frac{2(x - x_c)^2}{c^2} - \frac{2(x - x_c)}{c^2}\left[(x - x_c) - \frac{c^2}{2(x - x_c)} - \frac{c^4}{8(x - x_c)^3} + \ldots\right] - 1$$

$$= 0 + \frac{c^2}{4(x - x_c)^2} + \ldots$$

Thus there can be no x^{-1} term in the Laurent expansion of $e^{-2\xi_c}$. Consequently the contribution of the last term in equation (26) to a_1 is zero, and the effect of the variable eccentricity on the total complex force is zero.

By a similar expansion of the right side of equation (28), the contribution of W_α to a_1 is found to be

$$a_{1,\alpha} = \frac{i\alpha A(A + B)}{2}$$

Summing all the contributions to a_1 and substituting in equation (25) yields the force, which in this case is lift,

$$\frac{L}{q} = 4\pi(a_{1,t} + a_{1,\alpha}) + 2\pi\left[x_c \frac{d(AB)}{d\bar{s}} + AB \frac{dx_c}{d\bar{s}}\right]$$

$$= -2\pi x_c \frac{d(AB)}{d\bar{s}} + 2\pi i\alpha A(A + B) + 2\pi x_c \frac{d(AB)}{d\bar{s}} + 2\pi AB \frac{dx_c}{d\bar{s}}$$

Substituting $dx_c/d\bar{s} = -i\alpha$ into this expression yields for the lift

$$\frac{L}{q} = 2\pi\alpha A^2$$

CROSS-FLOW VELOCITY IN FINNED SECTION FOR TRIFORM CONFIGURATION

If the missile has only a horizontal set of fins, the velocity distribution in the region of the fins can be computed by the method of reference 5. If the missile also possesses a vertical set of fins of equal length, the vertical onset cross flow is not affected by the vertical fins because of the symmetry of the configuration. Similarly, the streamlines in the presence of a horizontal onset flow are not affected by the presence of the horizontal fins. Consequently, the effects of this horizontal component of the onset flow can also be computed by the method of reference 5 by switching the appropriate variables in that solution, in a manner somewhat similar to that of reference 8 or reference 2 (p. 123).

A third case of considerable interest is a triform configuration with a single vertical fin. For vertical cross flow (fig. 3), the potential is the same as if the configuration had only horizontal fins because of the symmetry of the flow. For horizontal cross flow, on the other hand, the flow is not symmetric. For this case the potential function can be found by first mapping the elliptical cross section into a circle and subsequently mapping the resulting circle trifin configuration into a flat plate. The latter mapping can be accomplished by a series of transformations used by Bryson (ref. 6).

The transformations and the corresponding contours are outlined in figure 4. The mapping

$$x_2 = \frac{1}{2}\left(x_1 \pm \sqrt{x_1^2 - c^2}\right) \tag{29}$$

maps the ellipse into a circle. In terms of elliptic coordinates $\xi = \eta + i\zeta$,

$$x_2 = \frac{c}{2} e^{\xi} \tag{30}$$

The circle is flattened by the transformation,

$$x_3 = x_2 + \frac{\rho_0^2}{x_2} = (A + B) \cosh (\xi - \eta_0) \tag{31}$$

where ρ_0 is defined by equation (4). Finally the resulting perpendicular segment is mapped into a single vertical segment by the transformation,

$$x_4 = \pm \sqrt{x_3^2 - \mathcal{d}^2} - i\delta \tag{32}$$

where

$$\delta = \frac{1}{2}\left(\sqrt{\mathcal{t}^2 + \mathcal{d}^2} - \mathcal{d}\right) \tag{33}$$

$$\mathcal{d} = D + \frac{\rho_0^2}{D} \tag{34}$$

$$D = \frac{1}{2}\left(d + \sqrt{d^2 - c^2}\right) \tag{35}$$

$$\mathcal{t} = T - \frac{\rho_0^2}{T} \tag{36}$$

$$T = \frac{1}{2}\left(t + \sqrt{t^2 + c^2}\right) \tag{37}$$

The complex potential function for the horizontal cross flow in the x_4-plane is

$$W = \beta_s \sqrt{x_4^2 + \mathcal{D}^2} \tag{38}$$

where

$$\mathcal{D} = \frac{1}{2}\left(\mathcal{d} + \sqrt{\mathcal{t}^2 + \mathcal{d}^2}\right) \tag{39}$$

Then the conjugate of the complex velocity is obtained by differentiating equation (38) by the chain rule and using the transformation equations (29), (31), and (32):

$$\frac{dW}{dx_1} = \frac{dW}{dx_4}\frac{dx_4}{dx_3}\frac{dx_3}{dx_2}\frac{dx_2}{dx_1}$$

$$= \left(\frac{\beta_s x_4}{\sqrt{x_4^2 + \mathcal{D}^2}}\right)\left(\frac{x_3}{\sqrt{x_3^2 - d^2}}\right)\left(1 - \frac{\rho_0^2}{x_2^2}\right)\left(\frac{1}{2} + \frac{1}{2}\frac{x_1}{\sqrt{x_1^2 - c^2}}\right)$$

Consequently,

$$v_h - iw_h = \frac{\beta_s}{2}\left(\frac{x_4}{\sqrt{x_4^2 + \mathcal{D}^2}}\right)\left(\frac{x_3}{\sqrt{x_3^2 - d^2}}\right)\left(1 - \frac{\rho_0^2}{x_2^2}\right)\left(1 + \frac{x_1}{\sqrt{x_1^2 - c^2}}\right) \tag{40}$$

Resolving the right side of equation (40) analytically into its real and imaginary parts is a tedious procedure resulting in rather lengthy expressions. Calculation of the complex velocity is considerably simplified by retaining the complex variables in the programming code for the calculation of dW/dx_1 and then taking the real and imaginary parts of the calculated value at each point.

CONCLUDING REMARKS

Some special problems relating to missiles with elliptic-cross-section body shapes have been treated by slender body methods. Expressions have been derived for the velocity distribution in the nose section when the ellipse eccentricity is varying longitudinally on the missile. The cross-flow velocity on a triform fin section is also studied.

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April 29, 1977

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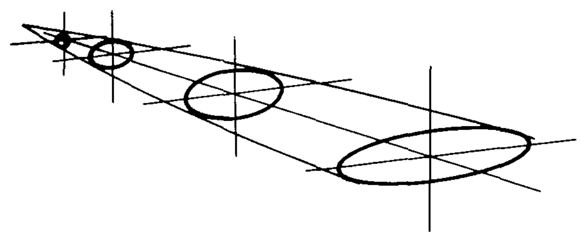

Figure 1.- Nose section geometry with longitudinally varying cross-section eccentricity.

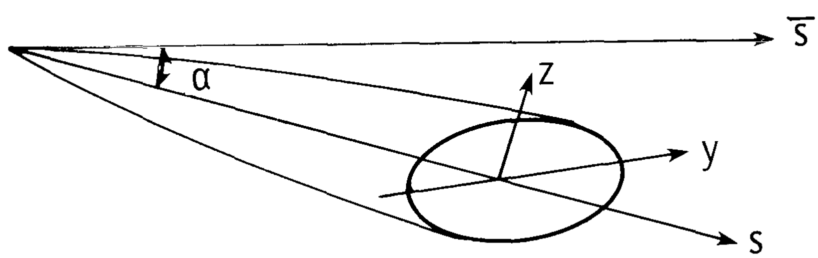

Figure 2.- System of axes.

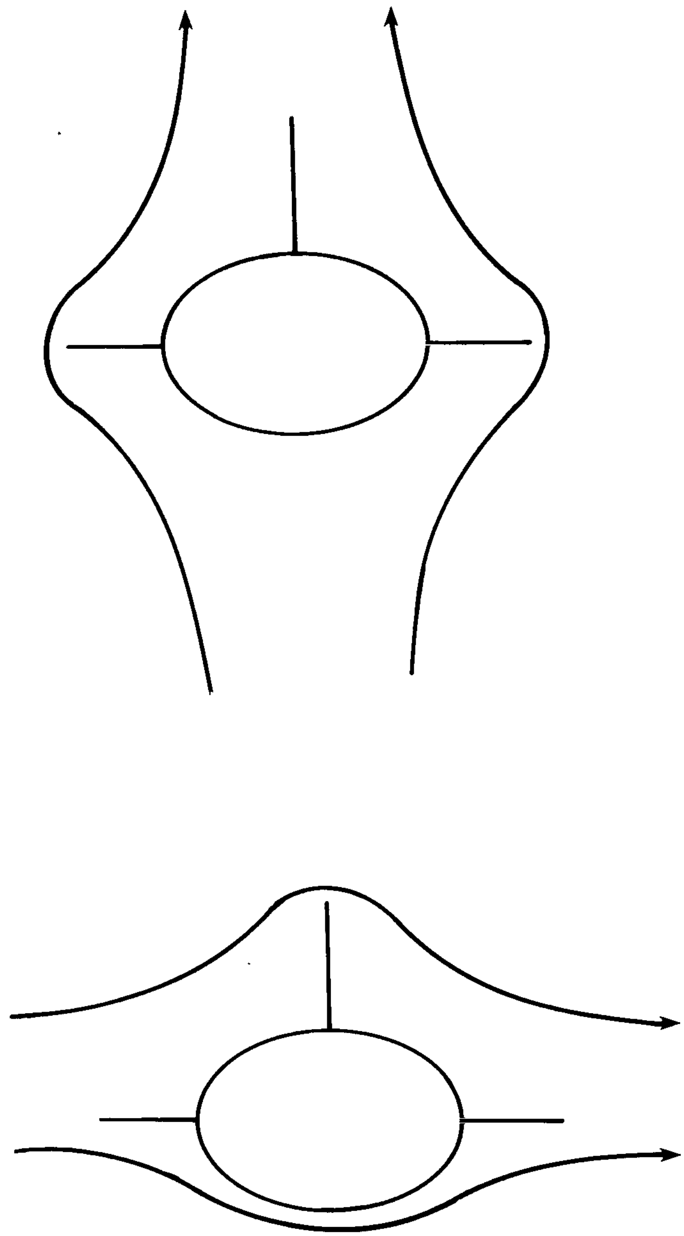

Figure 3.- Cross-flow diagrams for triform fin section indicating symmetry of vertical component and asymmetry of horizontal component.

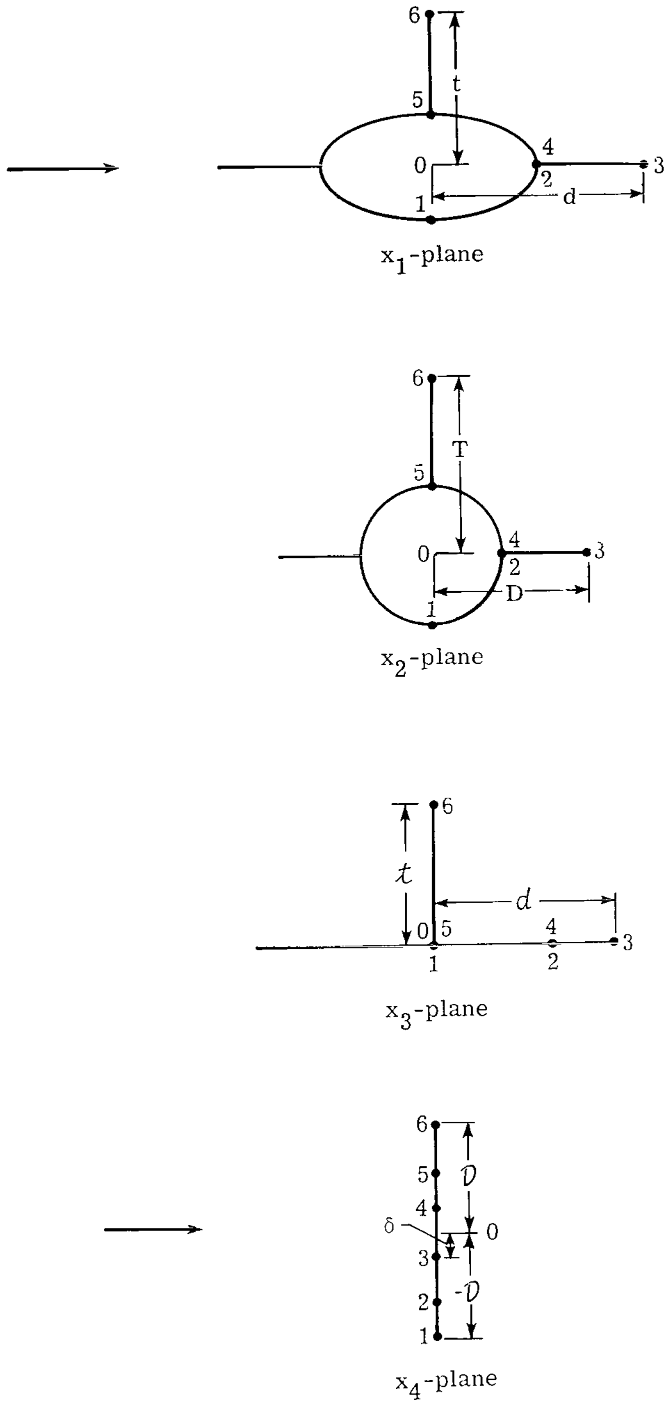

Figure 4.- Sequence of contours and parameters for mapping missile side flow into flow over a flat plate. Numerals denote corresponding points.

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