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Surface Heat Flux Determination

An Analytical and Experimental Study Using a Single Embedded Thermocouple

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SUMMARY

A numerical method by which data from a single embedded thermocouple can be used to predict the transient thermal environment for both high- and low-conductivity materials is described. The results of an investigation performed to verify the method clearly demonstrate that accurate, transient, surface heating conditions can be obtained from a thermocouple 1.016 centimeters from the surface in a low-conductivity material. Space Shuttle Orbiter thermal protection system materials having temperature- and pressure-dependent properties and typical Orbiter entry heating conditions were used to verify the accuracy of the analytical procedure. Analytically generated, as well as experimental, data were used to compare predicted and measured surface temperatures.

INTRODUCTION

The design and development of a reusable thermal protection system (TPS) for the Space Shuttle is dependent on a detailed knowledge of the aerothermodynamic environment to which the TPS will be exposed. The TPS thermal performance is normally obtained from exhaustive plasma arc and radiant heating tests to establish reuse temperature and thermal response characteristics. In a previous study by Curry and Williams (ref. 1), a nonlinear least squares method was developed for the estimation of thermal property values from experimental in-depth temperature data. The current investigation represents the application and extension of this previously developed numerical method to the determination of surface heating rates and temperatures from measured in-depth temperatures.

The calculation of surface heat flux and surface temperature from an in-depth temperature history measurement is called the inverse heat conduction problem and has been discussed by numerous investigators (refs. 2 to 12). An excellent discussion of previous investigations (refs. 4 to 8) for solving the inverse problem can also be found in reference 2. In particular, Beck and Wolf (ref. 3) presented a method of solution using least squares and future temperatures.

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In a later publication, Beck (ref. 2) presented a technique using nonlinear estimation in the solution of the inverse problem. Howard (ref. 9) developed a numerical method for determining the heat flux to a thermally thick wall with variable thermal properties using a single embedded thermocouple. His best results were obtained for temperature measurements close to the heated surface in conjunction with a small computing interval.

Cornette (refs. 10 and 11), in analyzing the Project Fire calorimeter data, developed a transient inverse solution that required curve fitting of the basic temperature data. Cornette's solution accounted for variable material properties and yielded a closed-form analytical expression for the local surface heat flux at a given instant of time. The temperature-time data for several thermocouples embedded in a calorimeter plug were smoothed and the data replaced with a polynomial equation for temperature (at a particular thermocouple location) as a function of time. Imber and Khan (ref. 12) developed a closed-form inverse solution for constant properties and heat flux using two in-depth thermocouple readings. The solution was obtained by means of Laplace transform techniques in which the input thermocouple data were approximated by a temporal power series and a second series of error functions.

The analytical method discussed in this paper, using a single embedded thermocouple, accounts for variable thermal properties (as functions of temperature and pressure) as well as for the effect of radiation losses and in-depth conduction. In addition, the results can be obtained with approximately the same computational time required to solve the thermal model using a known heat rate.

The primary objectives of this paper are: (1) to present a recently developed numerical method by which data from a single embedded thermocouple can be used to predict the transient thermal environment for both high- and low-conductivity materials having temperature- and pressure-dependent properties; (2) to make a direct comparison between analytically predicted and experimentally measured surface temperatures; and (3) to compare the analytical procedures described in this paper with the methods of Beck (ref. 2).

As an aid to the reader, where necessary the original units of measure have been converted to the equivalent value in the Système International d'Unités (SI). The SI units are written first, and the original units are written parenthetically thereafter.

SYMBOLS

A,B,C	quadratic coefficients
a,b,c,d	coefficients of square temperature matrix
C_p	specific heat of material at constant pressure
E	thermocouple depth
f	function defined by equation (6)
h	function defined by equation (7)
i	individual measurements
k	thermal conductivity of material

L	thickness of material
ℓ	material designator
m	Beck's intermediate temperature data
P	point
$\dot{q}$	heat flux
$\dot{q}_{conv}$	convective heating rate
$\dot{q}_{net}$	net heat rate
r	Beck's future temperature data
T	temperature
T'	temperature at end of time step
T_r	calculated value of temperature at node r
TC	thermocouple
t	time
x	distance or function defined by equations (12) and (14)
y	function defined by equations (12) and (14)
α	thermal diffusivity
Δ	denotes change in quantity
Δt	computing time, interval
Δx	node thickness
$\Delta\tau$	dimensionless time step
δ	convergence tolerance
ε	emissivity
η	arbitrary value
ρ	density of material
σ	Stefan-Boltzmann constant

Subscripts:

i	location
j,n	thermocouple location
m,o,l,r	node identifiers
s,o	surface

Superscripts:

'	future time step
*	known or desired value

THEORETICAL FORMULATION

The heat conduction equation for a one-dimensional thermal model is

$$\rho \, C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x}\right) \tag{1}$$

A solution can be readily obtained if the boundary conditions and the initial temperature profile are known. However, when the thermophysical properties are temperature dependent, equation (1) is nonlinear and recourse is usually made to numerical methods. An implicit numerical solution (first order in time) has been used in this investigation.

It is assumed that a set of thermocouples TC_j has been placed at known locations x_j in the material. This assumption implies that a set of interior temperature-time histories $T(x_j,t)$ exists. If the temperature data are available at x_j, then by solving the boundary condition

$$k \frac{\partial T}{\partial x} \bigg|_{x=x_j} = -\dot{q} \tag{2}$$

for $\dot{q}$ together with the unknown temperatures, the heat rate at location x_j can be found directly (ref. 13).[1] If the temperature-time history data are available at the surface, then the convective heating rate can be found from

$$\dot{q}_{net} = \dot{q}_{conv} - \epsilon\sigma T_s{}^4 \tag{3}$$

where $\dot{q}_{net}$ is the total heat rate input, $\dot{q}_{conv}$ is the convective heating rate, ϵ is the total hemispherical emittance of the surface, σ is the Stefan-Boltzmann constant, and T_s is the temperature at the surface.

The problem in solving for the heating rate at the surface when the temperature at the surface is unknown is that one of the required boundary conditions is unavailable. There is no difficulty in solving for temperatures at any location between two thermocouples because both boundary conditions are known.

JSC Technique

A technique has been developed at the NASA Lyndon B. Johnson Space Center (JSC) to solve for the heat rate at the surface at each time step rather than to solve for the entire history. The technique is iterative; initially, a surface energy balance correction is used, followed by one step using the method of false position (regula falsi) (ref. 14) and then a quadratic fit. To derive the technique, it is necessary first to examine the finite difference approximation at the surface.

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[1] If the thermocouple is located on the surface, the $\dot{q}_{net}$ value can be calculated directly from the tridiagonal matrix.

$$\dot{q}_{net} - \frac{T_o' - T_1'}{\frac{\Delta x_\ell}{2\ {}_\ell k_o} + \frac{\Delta x_\ell}{2\ {}_\ell k_1}} = \frac{{}_\ell\rho_o\ {}_\ell C_{p_o}\ \Delta x_\ell}{2\ \Delta t}\ (T'_o - T_o) \tag{4}$$

where Δt is the interval of computing time, T' denotes the temperature value at time $t + \Delta t$, o and 1 are the node identifiers, and Δx is the node thickness. This form of the equation is for a composite material where ℓ is the material designator.

Expressions (1) and (4) can be rearranged into the general tridiagonal form

$$a_m T'_{m+1} + b_m T'_m + c_m T'_{m-1} + d_m = 0 \tag{5}$$

At the surface, one can define a function

$$f = a_o T_1' + b_o T_o' + \frac{{}_\ell\rho_o\ {}_\ell C_{p_o}\ \Delta x_\ell\ T_o}{2\ \Delta t} + \dot{q}_{net} = 0 \tag{6}$$

At each time step, it is desired that

$$h = T_1' - T_1'^* = 0 \tag{7}$$

where

$$f^* = a_o T_1'^* + b_o T_o'^* + \frac{{}_\ell\rho_o\ {}_\ell C_{p_o}\ \Delta x_\ell T_o}{2\ \Delta t} + \dot{q}_{net}^* = 0 \tag{8}$$

Thus, h may be rewritten as

$$h = \frac{1}{a_o}\ (f - f^*) - b_o(T_o' - T_o'^*) - (\dot{q}_{net} - \dot{q}_{net}^*) = 0 \tag{9}$$

Because both f and f* are zero, this results in

$$-b_o(T_o' - T_o'^*) - (\dot{q}_{net} - \dot{q}_{net}^*) = 0 \tag{10}$$

and the desired correction is

$$\dot{q}_{net}^{*} = \dot{q}_{net} + b_o(T_o' - T_o'^{*}) = \dot{q}_{net} + b_o\, \Delta T_o' \qquad (11)$$

(The quantity $\Delta T_o'$ may be approximated by the difference between the predicted temperature and the measured temperature response at the thermocouple location due to the next step in the algorithm.) The initial value of $\dot{q}_{net}$ is usually chosen to be the converged value for the previous time step. For the first time step, an arbitrary value such as 1 may be used.

Because the thermocouples generally are located internally rather than at the surface, application of the surface energy balance correction will result in a monotonic approach to the desired solution, but convergence is very slow and may require an excessive number of iterations. Therefore, as a practical solution, it is necessary to switch to an alternate technique. The method of false position can be used to obtain the next estimate of the solution from

$$y_{i+1} = (x_{i-1}y_i - x_i y_{i-1})/(x_{i-1} - x_i) \qquad (12)$$

where $y_i = (\dot{q}_{net})_i$, $x_i = [T_r' - T_n^{*}(t)]_i$, T_r' is the calculated value of the temperature at node r corresponding to the thermocouple location, and $T_n^{*}(t)$ is the temperature given by the thermocouple at that time.

The iterative process is halted when

$$|T_r' - T_n^{*}(t)| \leq \delta |T_n^{*}(t)| \qquad (13)$$

where δ is the relative convergence tolerance.

If convergence has not been achieved after using the regula falsi approximation, it can be obtained by using the quadratic fit. Assuming three points: $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, and $P_3(x_3,y_3)$, a quadratic equation can be formed to include all three points. The equation is

$$y = Ax^2 + Bx + C \qquad (14)$$

where A, B, and C are quadratic coefficients. By substituting each of the three points into equation (14), one has a system with three equations and three unknowns (the coefficients A, B, and C).

After the coefficients have been determined, a point on the quadratic curve may be found. The points have been formed such that solution is at $y = C$ or at the point $P_4(0,C)$.

If, in evaluating the original function with C to obtain a new x_4, the solution is not within the desired tolerance as required by equation (13) (i.e., $|x_4| \leq \delta |T_n^*(t)|$, P_4 is substituted for one of the previous points (e.g., one with the largest $|x|$), and the coefficients are determined again. The process is repeated until the desired solution is obtained. Normally, this process requires only one quadratic iteration, since the surface energy balance correction and regula falsi techniques were converging to the desired solution.

Numerical difficulties arise in determing the surface heat rate or the surface temperature from data based on interior thermocouples. This difficulty is partly due to the timelag imposed on the system resulting from the finite distance between the surface and the thermocouple location. Another factor is the damping of surface changes at the thermocouple location. Other errors that may arise are due primarily to the method used for approximating the thermal model, the magnitude of $\dot{q}_{net}$, and the magnitude of the thermocouple temperatures. Of course, it is assumed that time steps compatible with the physical system would be used for the thermal model. It should be noted that both the net heating rate and the surface temperature have unique solutions, whereas the convective heating rate is dependent on the assumed value for emissivity.

Beck's Method

Beck's method (ref. 2) differs from the JSC method both in the convergence technique and the amount of temperature data that may be used to calculate the net heating rate. As with the JSC method, the thermal response of the material is calculated, from surface to backwall, using an assumed heating rate. Beck, however, allows for the use of measured temperatures at times other than the current time. That is, with Beck's method, one can use intermediate temperature data at $m - 1$ time steps, future temperature data at $r - 1$ time steps, or both.

Beck calculates $m \cdot r$ temperature differences, in a least-squares sense, to calculate a correction to the net heating rate. The iterative process is halted when the change in the net heating rate satisfies a prespecified convergence criterion. This is stated mathematically as

$$|\Delta \dot{q}_i| \leq \delta |\dot{q}_{i-1}| \tag{15}$$

where $\Delta \dot{q}_i$ is the change in the net heating rate at the i-th iteration, $\dot{q}_{i-1}$ is the net heating rate used at the iteration $i - 1$, and δ is the relative convergence criterion.

It should be pointed out that although the convergence criteria in equations (13) and (15) have the same effect, their relative magnitudes differ. A convergence criterion of 1×10^{-6} in equation (13) corresponded to a convergence criterion of 1×10^{-3} in equation (15) when comparing analytical test case results.

Efficiency Comparison

There is a significant difference between the computational cost in utilizing the JSC method and in utilizing Beck's method. Beck's method is designed to solve a linear problem in one iteration, whereas the JSC method is designed to solve a nonlinear problem in two iterations. As previously mentioned, the JSC method usually converges in three evaluations of the implicit tridiagonal solution at each time step, and, if further refinement is required, each additional iteration requires only one additional evaluation. On the other hand, Beck's method requires $4 \cdot m \cdot r$ evaluations of the implicit tridiagonal solution for the first iteration, and $2 \cdot m \cdot r$ evaluations for every iteration thereafter. Thus, the two methods require the same number of evaluations if and only if the JSC method converges at the end of three iterations and Beck's method with $m = 1$ and $r = 1$ can converge in one iteration.

ANALYTICAL VERIFICATION

The numerical methods discussed in the previous section have been evaluated for typical Space Shuttle Orbiter materials and environments. In general, the Space Shuttle Orbiter reusable TPS consists of reusable surface insulation (RSI) for areas with maximum surface temperatures of less than 1533 K (2759.4° R) and a reusable carbon-carbon (RCC) where surface temperature exceeds 1533 K (2759.4° R). The thermophysical properties used in this investigation are presented in table 1.

The analytical data were obtained by running an implicit, one-dimensional, thermal model using known boundary conditions.[2] The resulting temperature history from one of the internal nodes was then used by the inverse programs, JSC and Beck methods, as a boundary condition. This simulates the temperature history that would be provided from thermocouple data in an experimental case. The inverse programs, in turn, used this transient data to determine the now unknown surface conditions, the net heating rate and temperature. This allowed for a direct comparison between the known convective heating rate and that predicted by both Beck's method and the JSC method.

Comparisons were made for two cases. The first case consisted of a linear thermal model (constant thermal properties and a surface emittance of zero)

[2] Additional analytically generated data for the RCC are presented in reference 15.

subjected to a triangular heating rate. This case is similar to the one used by Beck in reference 2, and served to verify the correct implementation of his algorithm. The second case consisted of a nonlinear thermal model (temperature-dependent thermal properties and a nonzero surface emittance) subjected to a heating-rate history typical of a Space Shuttle Orbiter entry.

Linear Model

The thermal model consists of a 5.08 centimeters (2 inches) of aluminum with a thermocouple on the backwall. Since this is a linear problem, the surface emittance was set to zero and constant properties were used. The thermal properties were: density - 2851.29 kg/m^3 (178 lb/ft^3), specific heat - 836.80 J/kg-K (0.2 Btu/lb$_m$-°R), and conductivity - 145.28 W/m-K (84 Btu/ft-hr-°R). An initial temperature of 294.44 K (530° R) was used, and the backwall was insulated; i.e., adiabatic.

The heating rate, q, was given by

$$q = \begin{cases} (\text{Btu/ft}^2\text{-sec}) & (\text{W/m}^2) & \\ (\tfrac{1}{2}t)5 & , \; 1.1348931 \times 10^4(\tfrac{1}{2}t)5 & , \; 0 \le t < 24 \\ (24 - \tfrac{1}{2}t)5 & , \; 1.1348931 \times 10^4(24 - \tfrac{1}{2}t)5 & , \; 24 < t \le 48 \\ 0 & , \; 0 & , \; 48 < t \end{cases}$$

where t is the transient time in seconds. As with the case that Beck reported, it was assumed that the thermocouple was on the backwall of the aluminum. A time step of 2 seconds was used to correspond to the dimensionless time step of 0.05 used by Beck. The dimensionless time step is given by:

$$\Delta\tau = \alpha \, \Delta t/E^2$$

where $\Delta\tau$ is the dimensionless time step, α is the thermal diffusivity, Δt is the time step used in the thermal model, and E is the thermocouple depth.

The results of this investigation are summarized in figure 1 and table 2. From table 2, it can be seen that for Beck's method with $m = 1$ and $r = 1$, the average error[3] for a convergence criterion of 5×10^{-3} on the net heating rate falls somewhere between the errors calculated for the convergence criteria of

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[3]The average error equals $\sqrt{\frac{1}{n} \sum_{i=1}^{n} (\dot{q}_i - \dot{q}_i^*)^2}$, where $\dot{q}_i$ is the calculated convective heating rate, $\dot{q}_i^*$ is the actual convective heating rate, and n is the total number of individual measurements i taken.

10^{-6} and 10^{-7} on temperature using the JSC method. Table 2 and figure 1 show that with $m = 1$, the use of future times did not improve the accuracy. Only when the intermediate times were used ($m = 2$) did the accuracy improve with use of future times. For $m = 2$ and $r = 2$, the method became unstable, and none of the results using intermediate times ($m = 2$) were as accurate as when no intermediate times ($m = 1$) were used. The only methods that were able to handle the abrupt change in curvature in the heating were the $m = 1$ and $r = 1$ case for Beck's method and the JSC method.

Nonlinear Model

The RSI thermal model (fig. 2) consists of 5.08 centimeters (2 inches) of primary Shuttle insulation. The boundary conditions are assumed to be a heat rate on the surface and adiabatic on the backwall. The initial temperature is 294.44 K (530° R). An emissivity of 0.8 on the surface, radiation to space (0 K or 0° R heat sink), and a constant 101-kN/m^2 (1 atmosphere) pressure were used. The thermophysical properties used can be found in table 1. The reference heating rate used (fig. 3) is typical of that expected for Shuttle Orbiter entry. The effects of the convergence criterion and the use of future times for Beck's method can be seen in table 3.

From table 3, it can be seen that for Beck's method with no future times ($r = 1$) and a convergence criterion of 5×10^{-3} on the net heating rate, the average error falls between the errors calculated for the convergence criteria of 10^{-4} and 10^{-5} for the JSC model. Reducing the convergence criterion on Beck's method to either 1×10^{-3} or 5×10^{-4} requires the JSC convergence criterion to be reduced to either 10^{-6} or 10^{-7}. It was observed that very little advantage was gained in reducing the convergence criterion to these lower values (5×10^{-4} for Beck and 10^{-7} for JSC).

When future times ($r > 1$) were used for Beck's method, the average error for a given convergence criterion was two to three orders of magnitude greater than when no future times ($r = 1$) were used. This indicates that for analytical data, no advantage is gained by using future times.

Another objective in this analytical investigation was to determine the effects of thermocouple depth, x_j, and the convergence criterion, δ, on the accuracy in the calculation of the heating rate. Using the JSC method, the range of values for δ was 10^{-4}, 10^{-5}, 10^{-6}, and 10^{-7} with thermocouple depths of 0.254, 0.508, 0.762, and 1.016 centimeters (0.1, 0.2, 0.3, and 0.4 inch) from the heated surface. The effects of the convergence criterion and thermocouple depth on the average error can be seen in table 4. Basically, the results indicate that for each additional 0.254 centimeter (0.1 inch) in depth of the thermocouple, the convergence criterion must be decreased by a factor of 10 to maintain the same relative accuracy. (It should be noted that a thermocouple depth of 0.254 centimeter (0.1 inch) corresponds to a dimensionless time step of 0.4, whereas a thermocouple depth of 1.016 centimeters (0.4 inch) corresponds to a dimensionless time step of 0.02.)

EXPERIMENTAL DATA

Thermal evaluation tests have been conducted in the NASA/JSC Radiant Heat Test Facility (RHTF) on test models fabricated from RSI and covered with a high-emittance surface wash. The test specimen consisted of a set of staggered 33.02- by 33.02-centimeter (13 by 13 inch) tiles, 5.08 centimeters (2 inches) thick, bonded to a strain isolation pad (SIP), which, in turn, was bonded to an aluminum plate attached to T-bars. There was no spacing between the tiles; i.e., a no-gap configuration. The test environment was designed to simulate entry heating and pressure conditions.

The Thermal Model

The thermal model consisted of 5.08 centimeters (2 inches) of RSI subdivided into approximately 20 nodes. This subdivision was accomplished such that node centers were forced at thermocouple locations. The boundary conditions were heating at the surface and adiabatic at the backwall[4]. The wash on the surface was ignored since both its thermal properties and depth of penetration were unknown[5]. The initial temperature profile on the interior nodes was determined from a linear interpolation of the initial interior thermocouple test data. It was assumed that the temperature from the last thermocouple remained constant to the backwall. The initial surface temperature and intervening node temperatures were obtained by linearly extrapolating the temperature from the first two interior thermocouples.

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[4]The use of the adiabatic boundary condition for this test environment and specimen has been justified by separate analysis. This analysis compared the surface temperatures predicted by using a measured temperature history boundary condition at the backwall to those predicted by using an adiabatic boundary condition. Temperature differences were not observed until cool-down had been initiated, where the model using the adiabatic boundary condition underpredicted the model using a temperature history boundary condition. The maximum temperature difference was less than 11 K (20° R).

[5]The surface wash was used to provide an emittance of approximately 0.9. It is assumed that the penetration is not deep and that the surface temperature is more dependent on the emittance of the surface than any material property characteristics exhibited by the wash.

Test Procedure and Identification

Thermocouple plugs were installed in the center of the end tile in each quadrant and in the center of the T-bar panel (approximately 127 by 152.4 centimeters (50 by 60 inches)), as shown in figure 4. Each thermocouple plug was 3.81 centimeters (1.5 inches) in diameter with five thermocouples. The thermocouples were located at 0.0, 0.381, 1.27, 2.286, and 3.81 centimeters (0.0, 0.15, 0.5, 0.9, and 1.5 inches) from the heated surface[6].

The T-bar panel was placed in the chamber with the RSI surface directed toward the radiant lamps. The lamps were graphite heater rods encapsulated in a nitrogen environment. Heating was controlled by monitoring a set of 13 control thermocouples lying midway between the thermocouple plugs in the first and fourth quadrants and the center thermocouple plug. During the test, the chamber pressure data and thermocouple data were recorded at 1-second intervals on magnetic tape. This magnetic tape constitutes the main data base used in this report to verify the numerical method.

Data Smoothing

The data that appear on the magnetic tape are generally too rough to be successfully used for determining the surface conditions. To smooth the data, recourse is made to a least squares procedure utilizing orthogonal polynomials. The polynomial is fitted over successively overlapping data sets to obtain a continuous temperature function. Finally, a standard deviation of temperature is taken of the measured and smoothed data for each thermocouple to ensure that the smoothed data conform to a desired tolerance. The data from a thermocouple plug were rejected if the standard deviation of temperature was greater than 11 K (20° R).

This data smoothing operation was performed for the surface data as well as the interior data to allow for a consistent comparison between measured and calculated values of the surface temperature. This permits a statistical evaluation of the test data prior to attempting to analyze test results, in addition to reducing computer time on worthless data.

Results

Only four of the five thermocouple plugs shown in figure 4 are presented since the center thermocouple plug, 5R, failed to meet the standard deviation requirement. The four in-depth thermocouples which provided the data used for temperature prediction were labeled 1R14, 2R21, 3R20, and 4R7. The corresponding surface thermocouples to which the predictions were compared were 1R13, 2R20, 3R19, and 4R6.

[6]The lead thermocouple was actually embedded slightly into the material and was below the surface. Its distance from the surface is unknown.

The surface temperatures predicted by the JSC method were based on smoothed data, whereas the surface temperatures predicted by Beck's method were based on the "raw," unsmoothed data using one ($r = 2$) and two ($r = 3$) future times. The surface temperature predictions by the JSC method and Beck's method (one future time) for thermocouple 1R14 can be seen in figure 5. During the first few steps, the JSC method oscillated and then the surface temperature predictions became smooth; whereas with Beck's method, the initial steps were smooth with oscillations occurring later throughout the data. When using two future times with Beck's method, these oscillations were still present (fig. 6) but the magnitude was reduced. The oscillations observed with Beck's method can be attributed to using raw, unsmoothed data. For thermocouple 2R21, no oscillations were observed for the JSC method. Beck's method using one future time (fig. 7) displayed oscillations at several times in the surface temperature predictions. These oscillations were damped considerably when two future times were used (fig. 8). Essentially, the same characteristics were observed for thermocouple 3R20 as for 2R21 (figs. 9 and 10).

The surface temperatures predicted by the JSC method and Beck's method for one and two future times for thermocouple 4R7 can be seen in figures 11 and 12, respectively. As with 1R14, the 4R7 curve for the JSC method exhibited initial oscillations and became very smooth; whereas with Beck's method, the initial phase was smooth, but large oscillations developed later. As with the previous cases, the use of temperatures at two future times damped the oscillations observed with one future time.

In comparing the predicted surface temperatures to those measured on the surface, it was observed that the predicted temperatures were consistently higher than those measured. This overprediction is attributed to the embedding of the lead thermocouple.

Overall, the agreement between the JSC method and Beck's method was good with only major discrepancies when oscillations were observed. With the JSC method, the only oscillations observed were during the initial startup phase. Beck's method did not display any oscillations during this phase, but oscillations did occur later in the data for all four tests. This was due to the thermocouple recording errors that are always present. These results demonstrate the desirability of smoothing the thermocouple data prior to predicting surface temperatures. With smooth data, no advantage is gained by using future times. This is illustrated in figure 13, where a comparison using Beck's method with two future times is made between smoothed and raw data during the first 200 seconds of the test for thermocouple plug 1R. As was mentioned, the computational cost in using the JSC method is much smaller than the cost of using Beck's method with future times (table 5). This holds true even though it is necessary to smooth the thermocouple data prior to using them for surface temperature predictions. Other than the problems with the initial oscillations that occurred with plugs 1R and 4R, there was no advantage in using future times. It should be mentioned that if the data were noisy at the initial time, Beck's method would have oscillated then also.

CONCLUDING REMARKS

An inverse solution technique using a single embedded thermocouple has been developed for predicting the transient thermal environment to which the Space Shuttle Orbiter thermal protection system is exposed during entry. The accuracy of the numerical method has been demonstrated for a low-conductivity material by comparison with experimental and analytical data.

A comparison was also made between the method developed by Beck and the method developed at JSC for solving the inverse problem using analytical and experimental data. The results of this investigation indicated that no advantage will be gained by using Beck's method with future temperatures. The procedure developed is quite general and has been incorporated into a previously developed program used to compute thermal conductivity values from experimental data. Thus, a capability now exists for computing surface conditions (heat flux and/or temperature) and thermal conductivity values using the data from a single experiment.

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TABLE 1.- THERMOPHYSICAL PROPERTIES OF RSI

[Density, 144 kg/m^3]

Temperature, K	Transverse thermal conductivity, W/(m·K), at a pressure, N/m^2, of -					Specific heat, J/(kg·K)
	10.13	101.3	1013	10 132	101 325	
117	0.009	0.013	0.026	0.038	0.040	293
172	.010	.014	.029	.040	.043	439
256	.013	.017	.032	.043	.048	628
394	.016	.022	.039	.055	.059	879
533	.022	.029	.048	.069	.075	1054
672	.030	.038	.056	.085	.092	1151
811	.040	.048	.068	.104	.114	1205
950	.053	.060	.085	.125	.135	1238
1089	.072	.079	.107	.151	.163	1256
1200	.092	.100	.127	.176	.189	1264
1228	.098	.105	.133	.183	.196	1268
1339	.121	.130	.156	.212	.226	1268
1367	.127	.135	.163	.219	.235	1268
1422	.140	.148	.174	.235	.252	1268
1450	.147	.156	.182	.244	.261	1268
1533	.167	.177	.200	.268	.288	1268

TABLE 2. - COMPARISON OF THE AVERAGE ERROR IN DETERMINING THE HEAT RATE FOR THE LINEAR PROBLEM

Method	Iteration Count	Convergence criterion (δ)	Average error	
			Btu/ft^2-sec	W/m^2
JSC	72	1×10^{-4}	2.405×10^{-3}	2.729×10^{1}
JSC	72	1×10^{-5}	2.405×10^{-3}	2.729×10^{1}
JSC	72	1×10^{-6}	2.405×10^{-3}	2.729×10^{1}
JSC	84	1×10^{-7}	4.642×10^{-4}	5.268×10^{0}
Beck (m = 1, r = 1)	98	5×10^{-3}	6.590×10^{-4}	7.479×10^{0}
Beck (m = 1, r = 2)	192	5×10^{-3}	1.319×10^{-1}	1.497×10^{3}
Beck (m = 1, r = 3)	288	5×10^{-3}	2.256×10^{-1}	2.560×10^{3}
Beck (m = 2, r = 1)	---	5×10^{-3}	Unstable	Unstable
Beck (m = 2, r = 2)	384	5×10^{-3}	2.802×10^{-1}	3.180×10^{3}
Beck (m = 2, r = 3)	576	5×10^{-3}	3.126×10^{-1}	3.548×10^{3}

TABLE 3. - COMPARISON OF THE AVERAGE ERROR IN DETERMINING THE HEAT RATE FOR THE ANALYTICAL RSI TRAJECTORY

Method	Iteration Count	Convergence criterion (δ)	Average error	
			Btu/ft^2-sec	W/m^2
JSC	545	1×10^{-4}	1.372×10^{-2}	1.557×10^{2}
	597	1×10^{-5}	6.954×10^{-4}	7.892×10^{0}
	600	1×10^{-6}	2.587×10^{-4}	2.936×10^{0}
	600	1×10^{-7}	2.587×10^{-4}	2.936×10^{0}
Beck (m=1, r=1)	780	5×10^{-3}	4.539×10^{-3}	5.151×10^{1}
	798	1×10^{-3}	2.618×10^{-4}	2.971×10^{0}
	802	5×10^{-4}	2.603×10^{-4}	2.954×10^{0}
Beck (m=1, r=2)	1572	5×10^{-3}	2.309×10^{-1}	2.620×10^{3}
	1600	5×10^{-4}	2.310×10^{-1}	2.622×10^{3}
Beck (m=1, r=3)	2340	5×10^{-3}	3.741×10^{-1}	4.246×10^{3}
	2400	5×10^{-4}	3.741×10^{-1}	4.246×10^{3}

TABLE 4. - COMPARISON OF THE AVERAGE ERROR IN DETERMINING THE HEAT RATE FOR THE RSI TRAJECTORY AT DIFFERENT THERMOCOUPLE DEPTHS USING THE JSC METHOD

Lead thermocouple depth, cm	Lead thermocouple depth, in.	Convergence criterion (δ)	Average error, Btu/ft^2-sec	Average error, W/m^2
0.254	0.1	1×10^{-4}	1.372×10^{-2}	1.557×10^{2}
		1×10^{-5}	6.954×10^{-4}	7.892×10^{0}
		1×10^{-6}	2.587×10^{-4}	2.936×10^{0}
		1×10^{-7}	2.587×10^{-4}	2.936×10^{0}
.508	.2	1×10^{-4}	5.502×10^{-2}	6.244×10^{2}
		1×10^{-5}	7.228×10^{-4}	8.203×10^{0}
		1×10^{-6}	2.823×10^{-4}	3.204×10^{0}
		1×10^{-7}	2.564×10^{-4}	2.910×10^{0}
.762	.3	1×10^{-4}	3.484×10^{-1}	3.954×10^{3}
		1×10^{-5}	1.752×10^{-2}	1.988×10^{2}
		1×10^{-6}	2.053×10^{-3}	2.330×10^{1}
		1×10^{-7}	4.107×10^{-4}	4.661×10^{0}
1.016	.4	1×10^{-4}	1.442×10^{0}	1.637×10^{4}
		1×10^{-5}	1.396×10^{-1}	1.584×10^{3}
		1×10^{-6}	2.354×10^{-2}	2.672×10^{2}
		1×10^{-7}	1.523×10^{-3}	1.728×10^{1}

TABLE 5. - COMPARISON OF THE ITERATION COUNT IN DETERMINING THE HEAT RATE FOR THE EXPERIMENTAL TRAJECTORY USING THERMOCOUPLE 1R14

Time, sec	Method			
	JSC[a]	Beck[a] m = 1, r = 1	Beck[b] m = 1, r = 2	Beck[b] m = 1, r = 3
4100	15	20	44	66
4200	75	98	228	318
4300	135	174	392	546
4400	195	250	548	786
4500	255	328	708	1026
4600	314	404	860	1266
4700	374	478	1020	1506
4800	432	554	1180	1734
4900	492	640	1340	1974

[a]Used smoothed data.
[b]Used raw data.

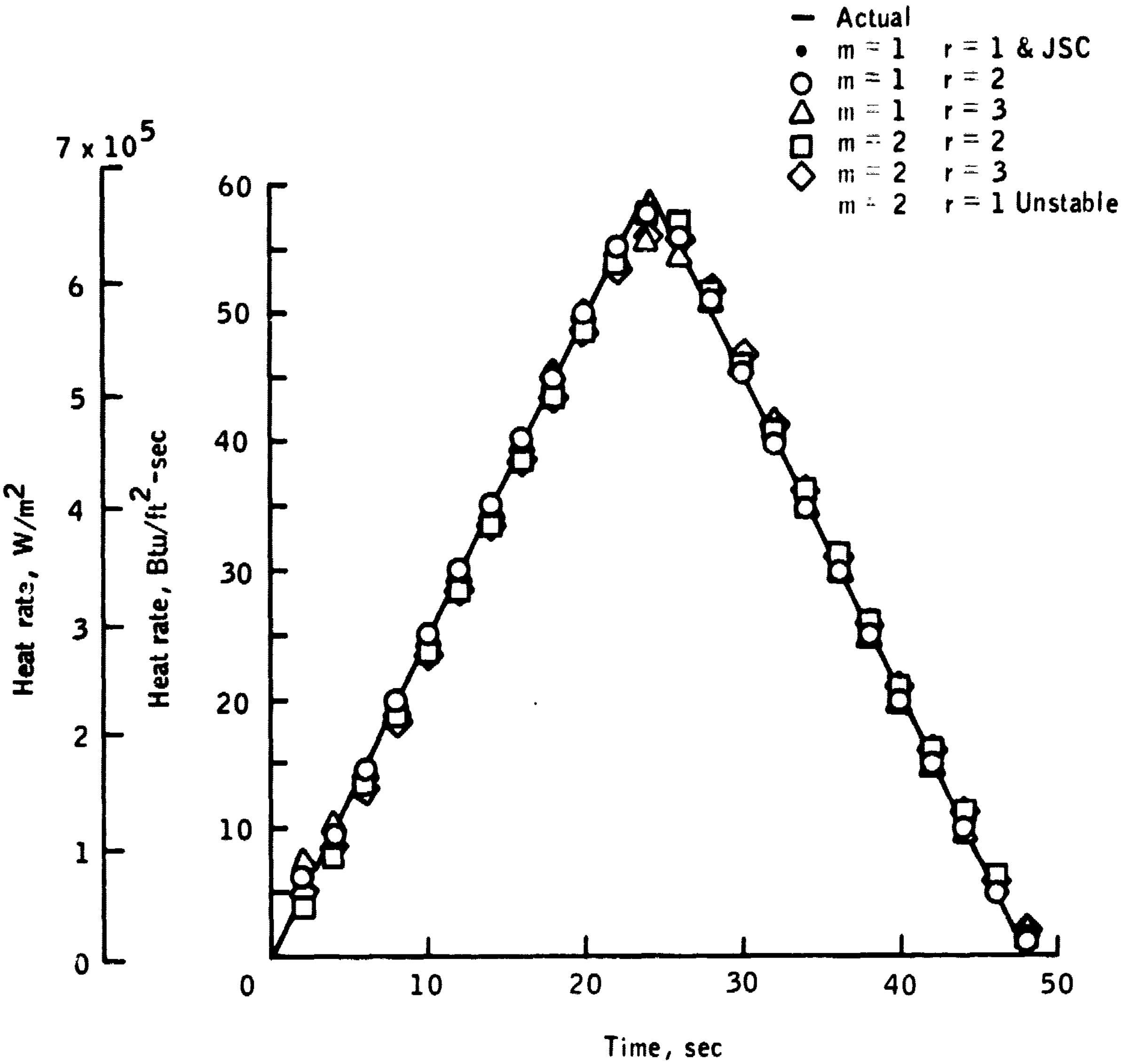

Figure 1.- Calculated heat flux for a linear problem.

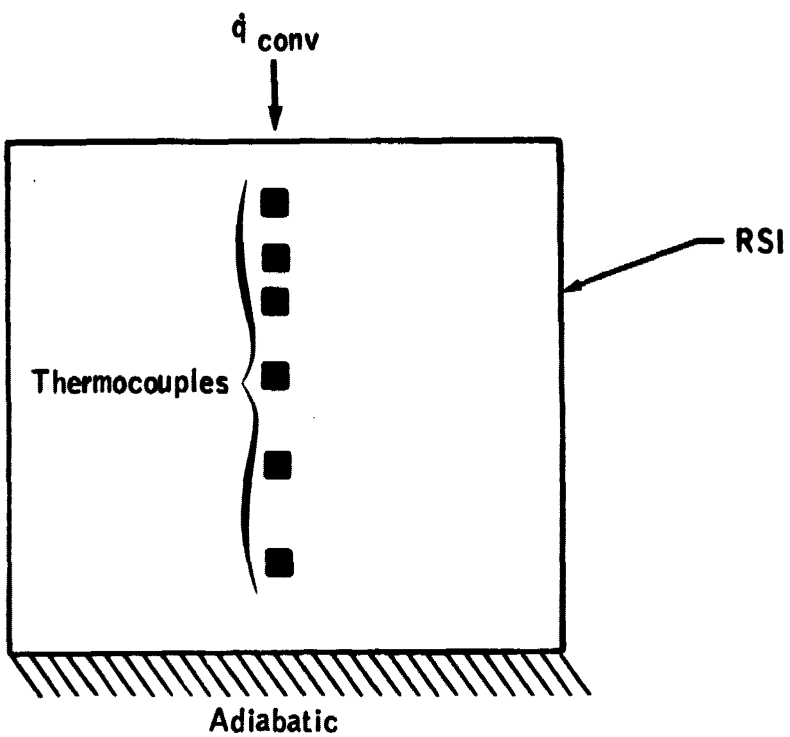

Figure 2.- The RSI thermal model.

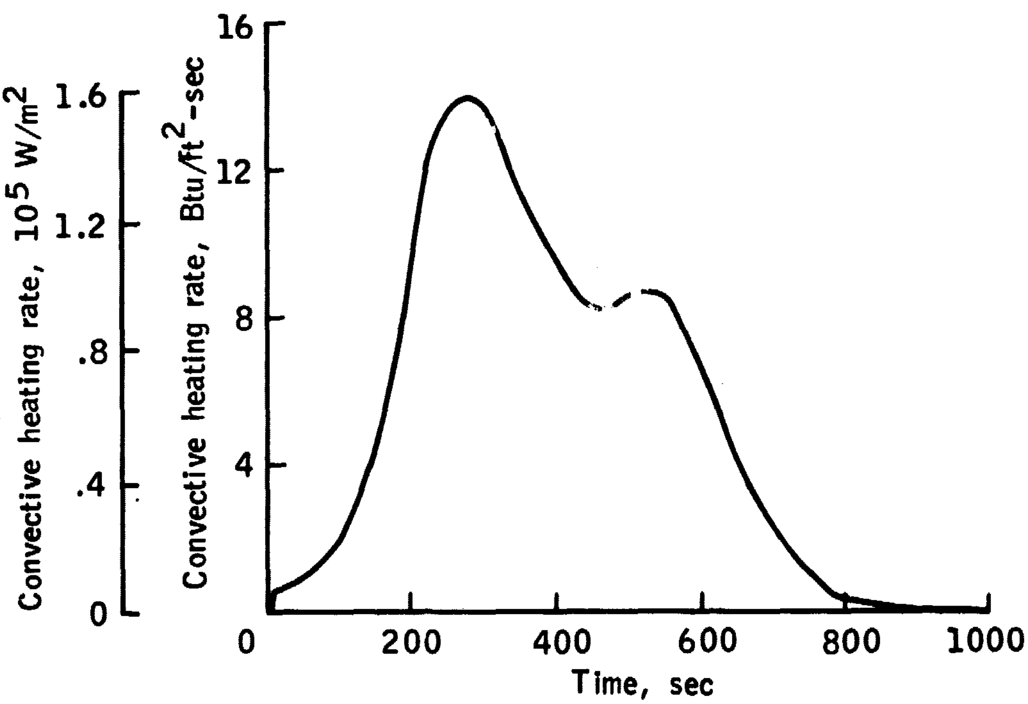

Figure 3.- Heating rate versus time for RSI trajectory.

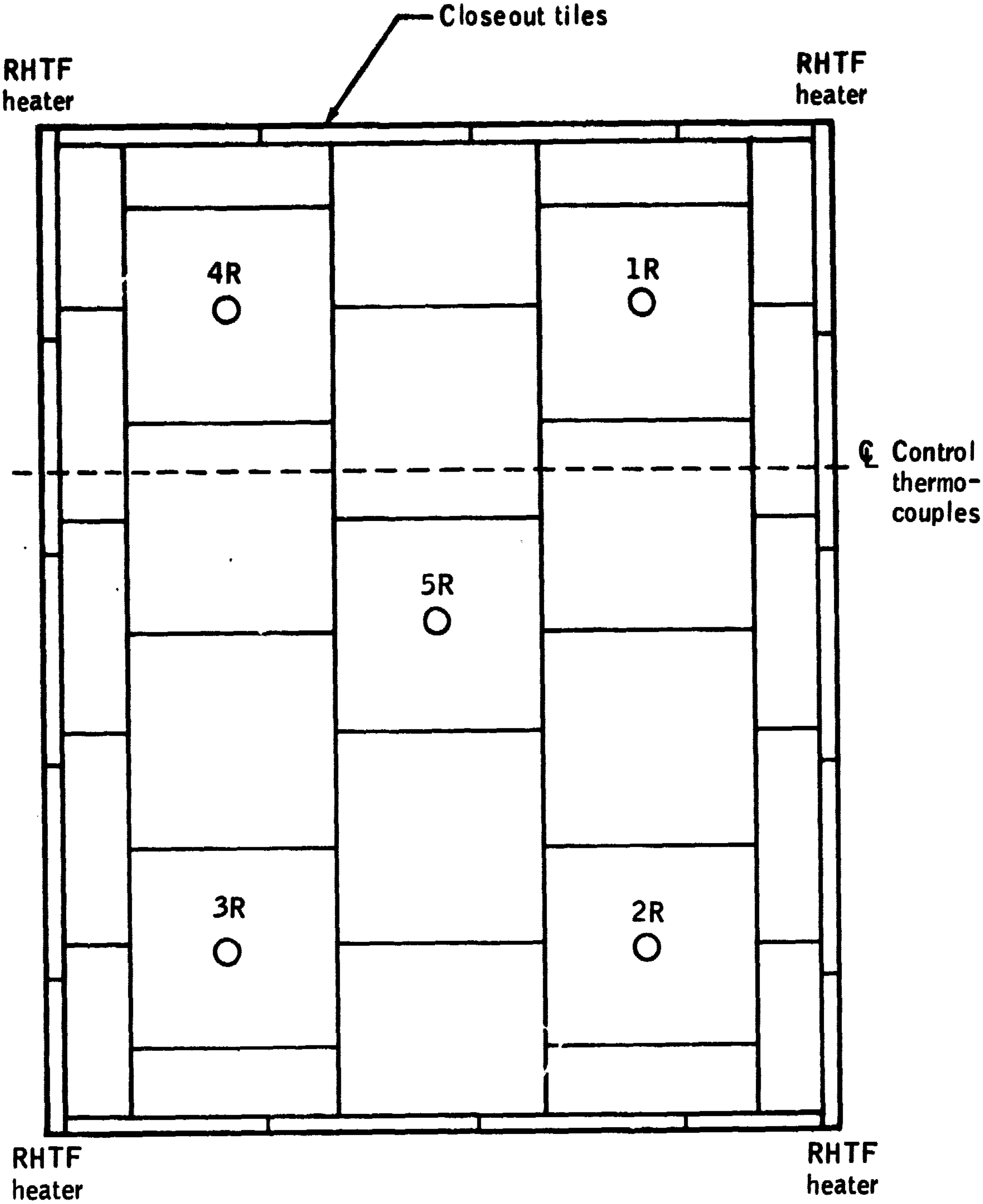

Figure 4.- Thermal protection system radiant heatsink test.

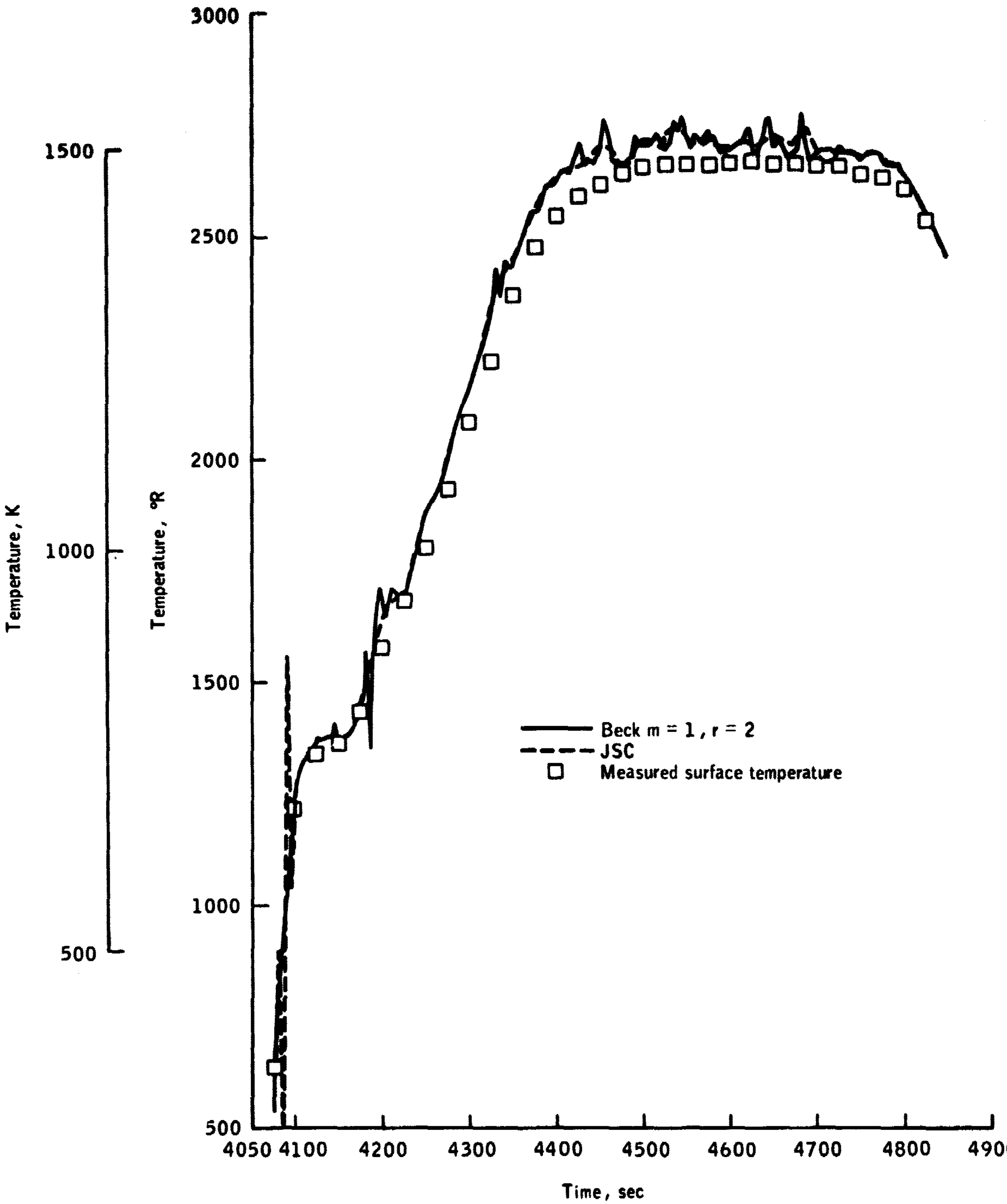

Figure 5.- Predicted values of surface temperature using the JSC method and Beck's method (one future time) from thermocouple 1R14.

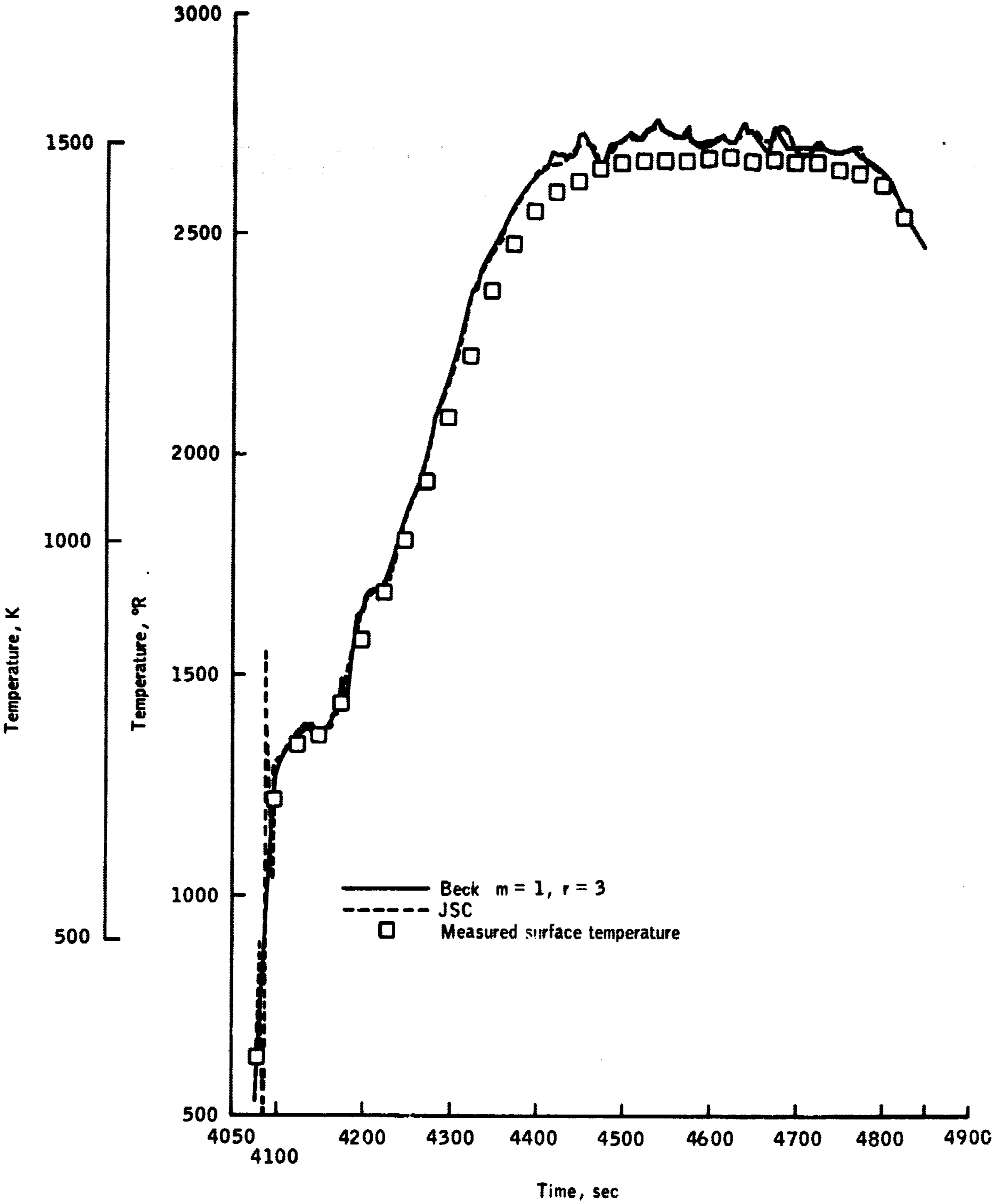

Figure 6.- Predicted values of surface temperature using the JSC method and Beck's method (two future times) from thermocouple 1R14.

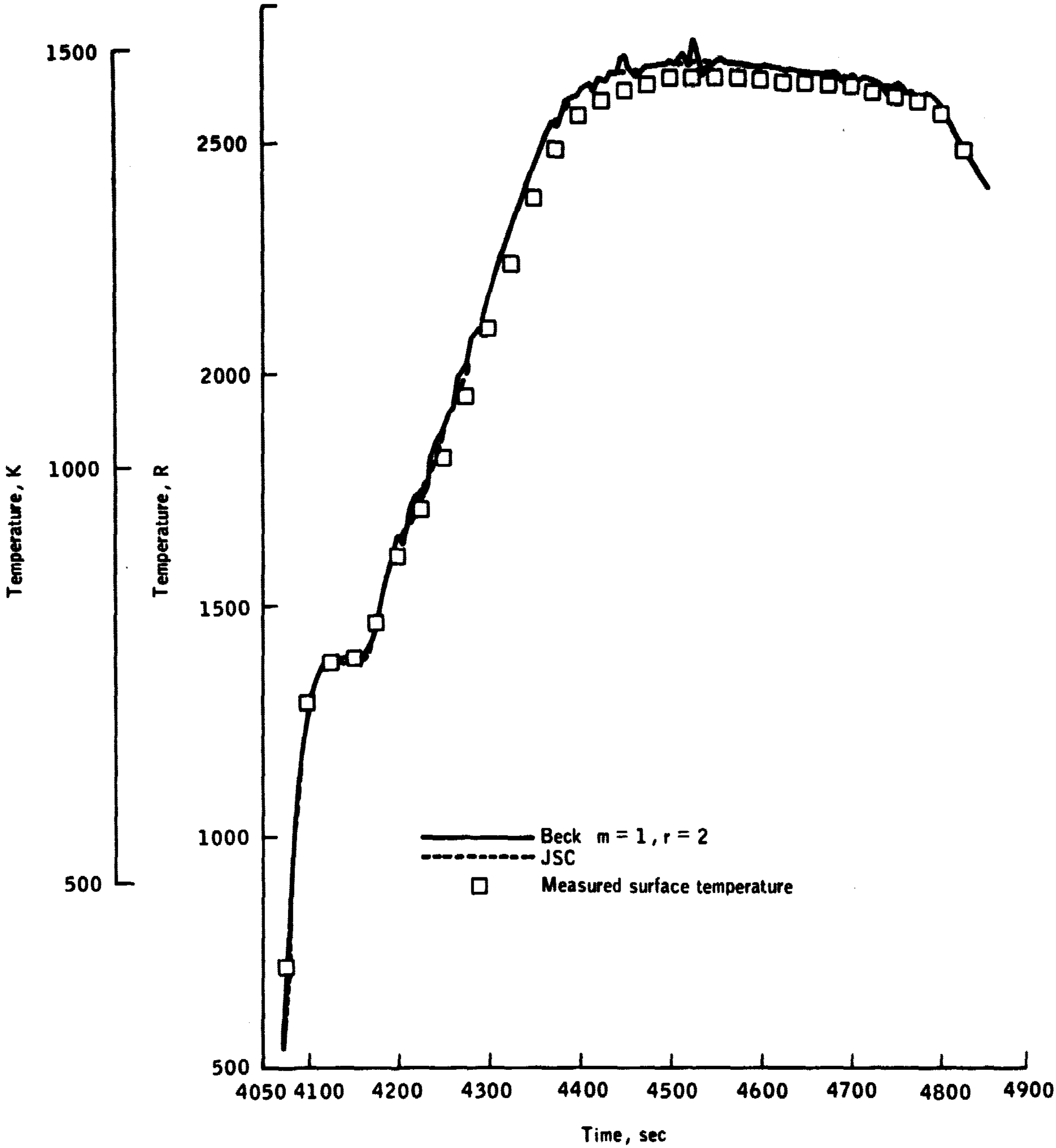

Figure 7.- Predicted values of surface temperature using the JSC method and Beck's method (one future time) from thermocouple 2R21.

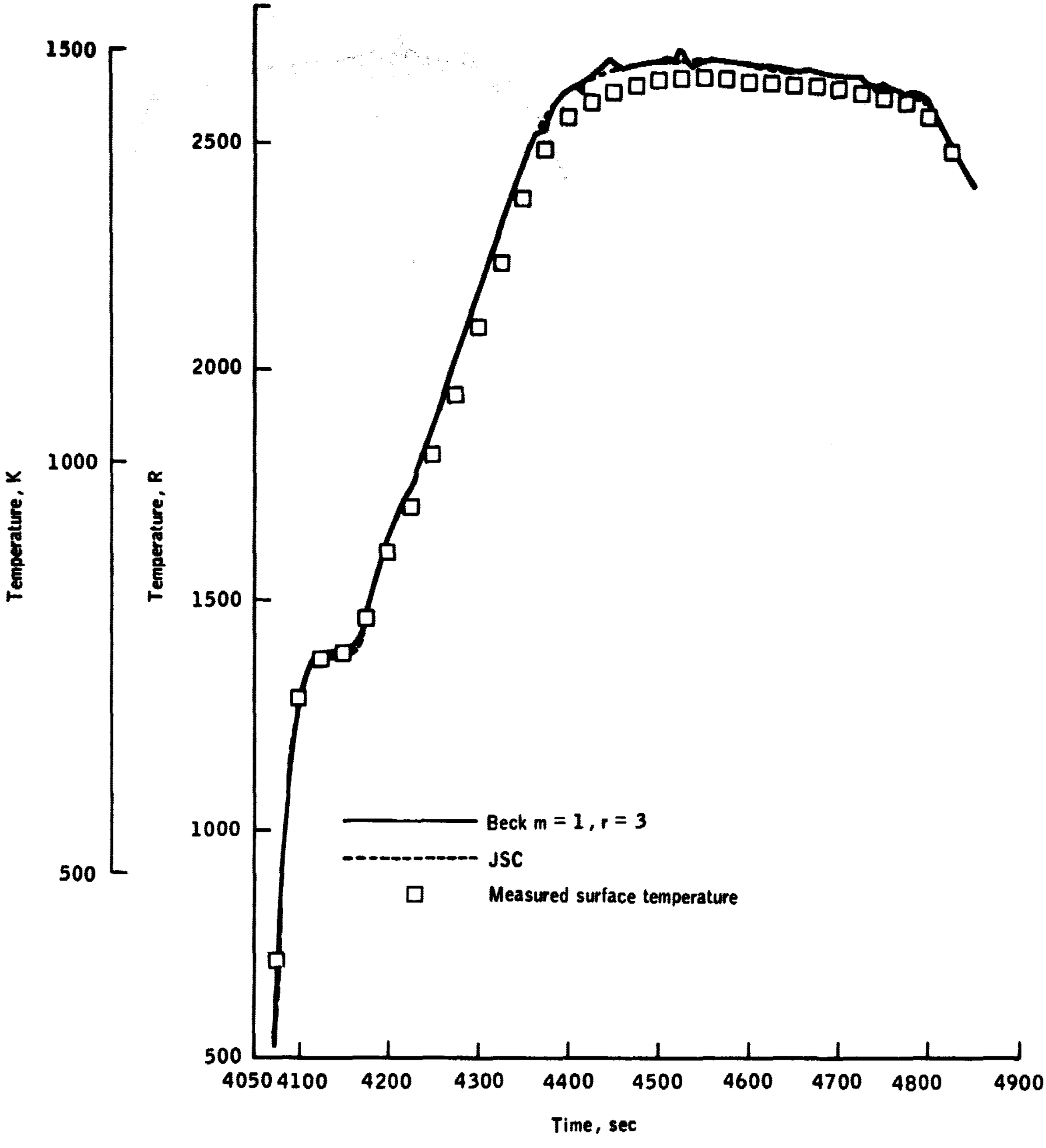

Figure 8.- Predicted values of surface temperature using the JSC method and Beck's method (two future times) from thermocouple 2R21.

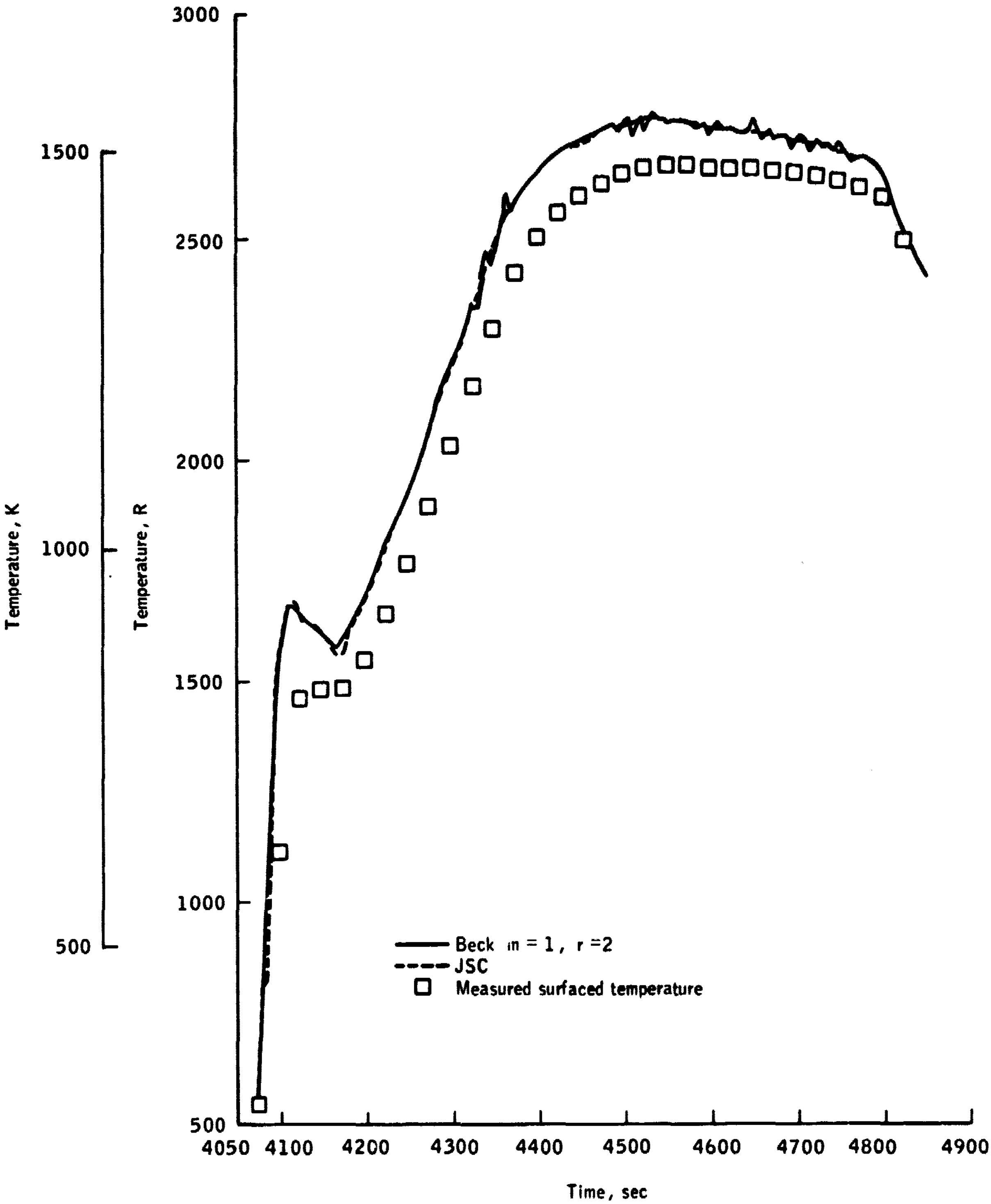

Figure 9.- Predicted values of surface temperature using the JSC method and Beck's method (one future time) from thermocouple 3R20.

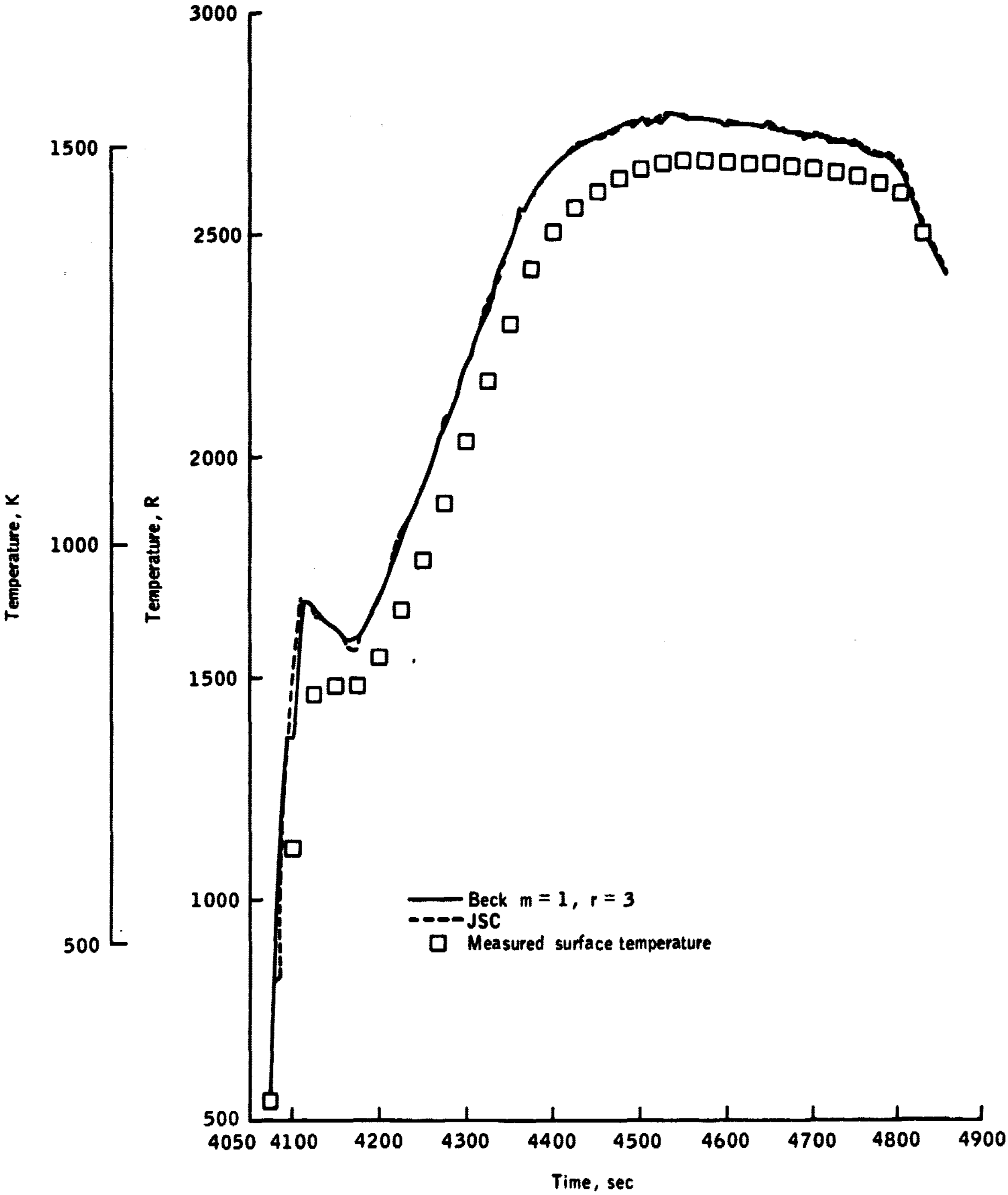

Figure 10.- Predicted values of surface temperature using the JSC method and Beck's method (two future times) from thermocouple 3R20.

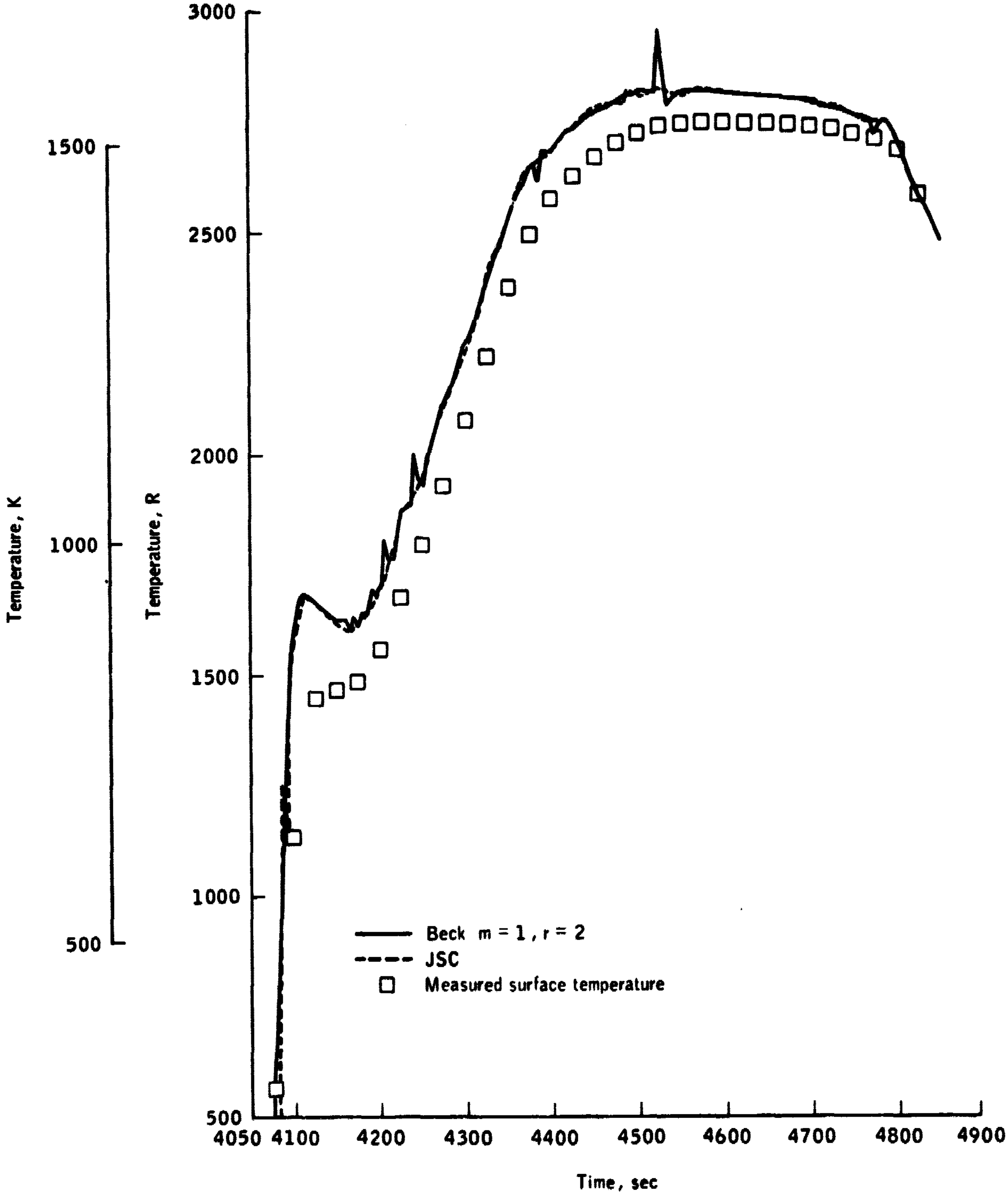

Figure 11.- Predicted values of surface temperature using the JSC method and Beck's method (one future time) from thermocouple 4R7.

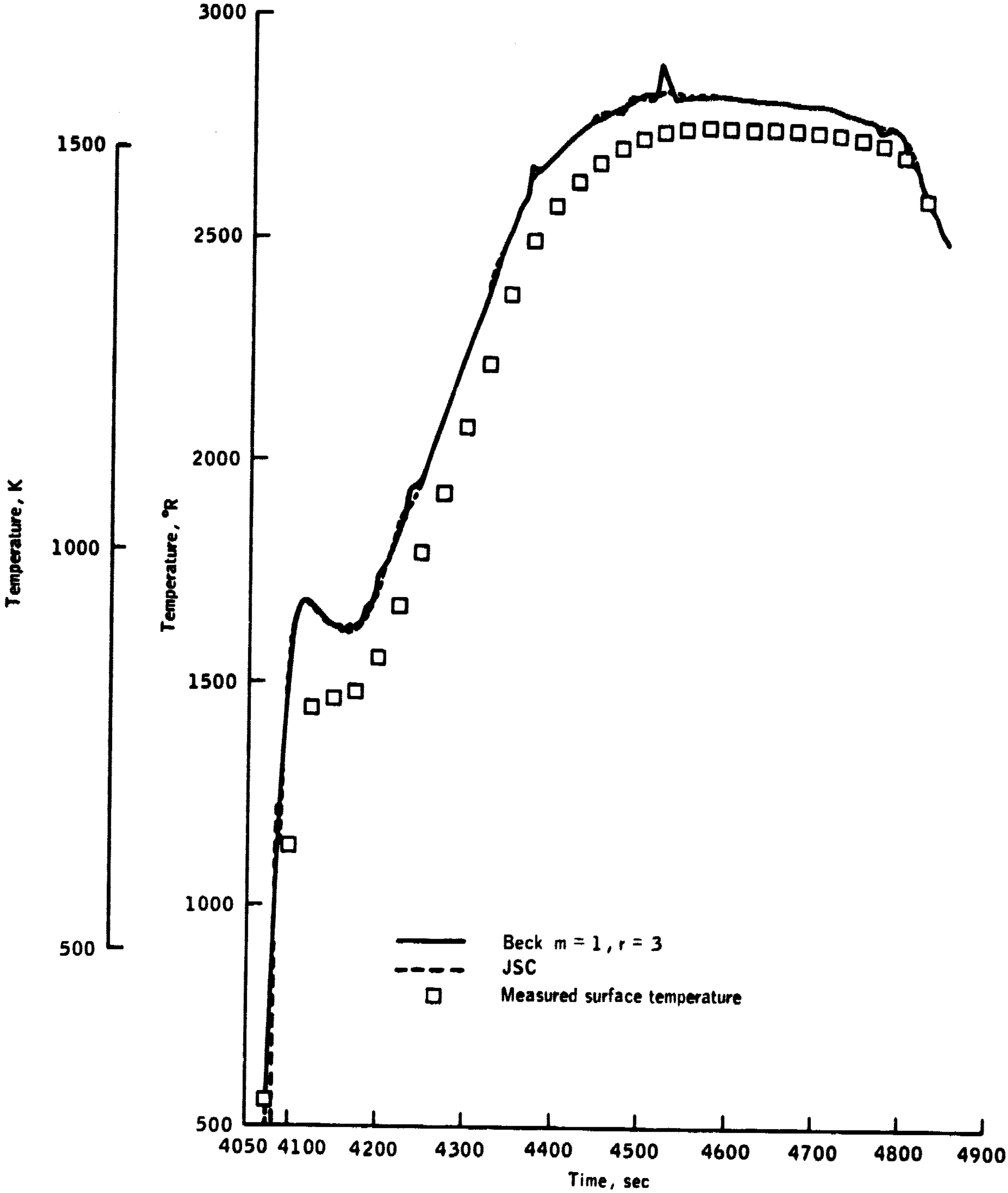

Figure 12.- Predicted values of surface temperature using the JSC method and Beck's method (two future times) from thermocouple 4R7.

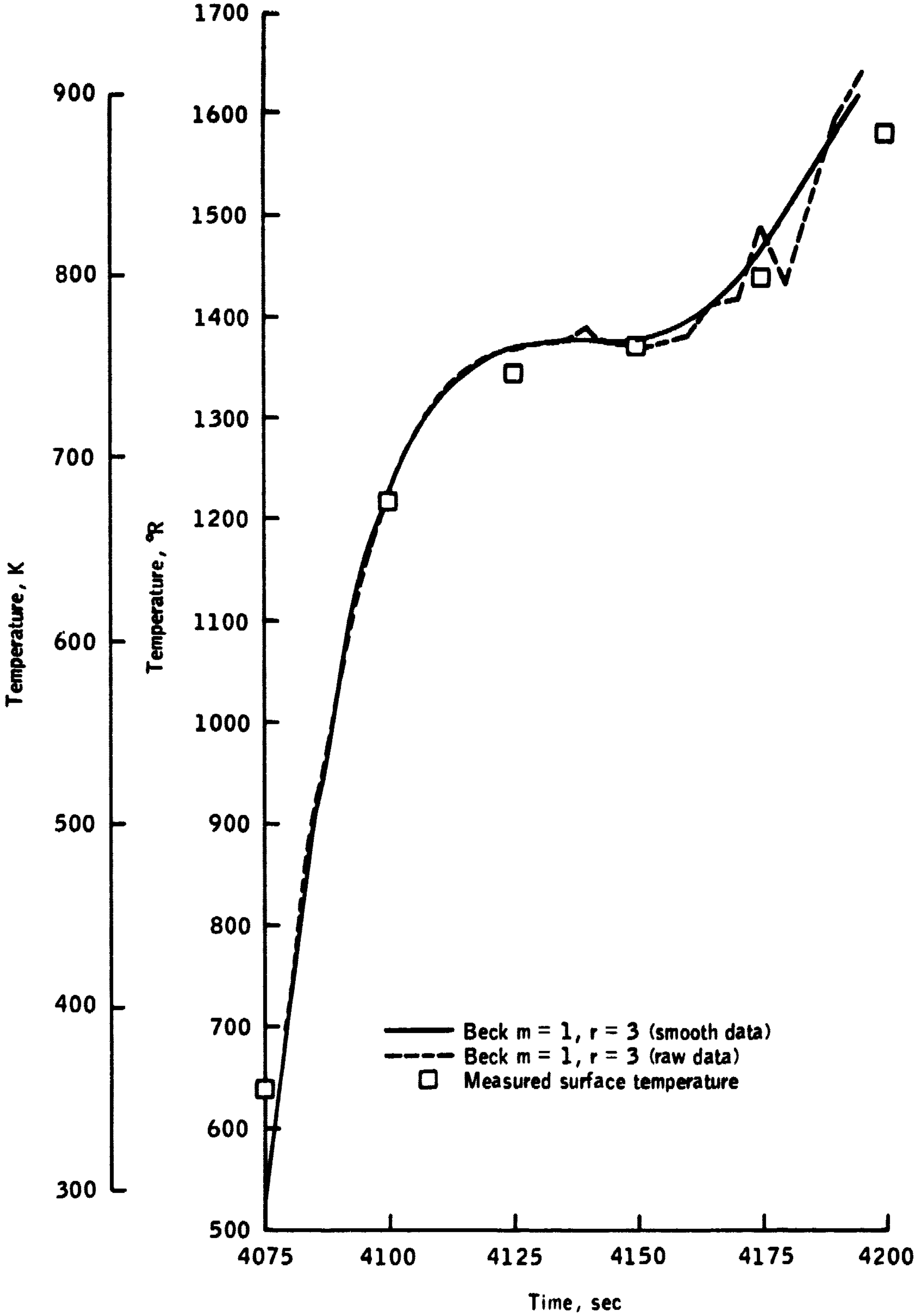

Figure 13.- Predicted surface temperature using Beck's method (two future times) with smoothed data and raw data from thermocouple 1R14.

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15. Supplementary Notes

16. Abstract

An inverse solution technique developed for predicting the transient heat flux to a material using a single embedded thermocouple is presented. The accuracy and efficiency of the numerical method is demonstrated by comparison with experimental and analytical data.

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