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TABLES OF ASTRONOMICAL CONSTANTS AND LIBRATION-POINT PARAMETERS FOR SELECTED SOLAR SYSTEM OBJECTS

Prepared For
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Goddard Space Flight Center
Greenbelt, Maryland

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Prepared for

GODDARD SPACE FLIGHT CENTER

By

COMPUTER SCIENCES CORPORATION

Under

Contract NAS 5-11999
Task Assignment 570

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ABSTRACT

This report documents the currently-recognized most accurate values for the masses, radii, and certain mean orbital elements for most of the planets and many of the satellites of the Solar System. These values were used to produce computer-generated tables of libration-point parameters.

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Table

SECTION 1 - INTRODUCTION

Tables of the masses, radii, and certain orbital elements for selected Solar System bodies have been prepared for calculating improved libration point parameters as given in the tables of NASA Technical Report R-346, "The Control and Use of Libration-Point Satellites," by Robert W. Farquhar, Goddard Space Flight Center (Reference 41). These tables, presented in Section 2, include the most recent values, many derived from spacecraft data and from sophisticated ground-based observations of special astronomical phenomena during the past five years. The data in these tables were collected primarily for libration-point parameter calculations, but should have a wide range of other applications. Section 3 contains tables of libration-point parameters like those in Reference 41 using values for the constants given in Section 2. Definitions of all libration-point parameters are given in the reference.

SECTION 2 - TABLES OF ASTRONOMICAL CONSTANTS

Basic astronomical data about 8 planets and 14 satellites of the Solar System have been compiled from various sources and collected in three tables.

The planets and satellites selected are the same ones as those used for the tables in Reference 41 plus five satellites which are relatively close to their planet: Amalthea, Janus, Miranda, Ariel, and Titania. All the planets are included except Pluto, whose mass remains too poorly known for meaningful libration-point calculations. The additional satellites were selected to check the possibility that the collinear libration points might be below the surface of the satellite. This is not likely for any of the satellites, but is possible for Miranda (highest probability), Ariel, and Amalthea. The values for the masses and radii for these three satellites are very uncertain.

Mass ratios of the objects with respect to its primary (the Sun or the parent planet) are presented in Table 2-1, including error estimates, method of determination, and source references.

Mean radii are presented in Table 2-2, since these are used only for calculating mean shadow parameters. The mean radius is the cube root of the product of half of the lengths of the principal axes of the triaxial ellipsoid used to represent the actual surface of the body. Values for the lengths of the principal axes are available for many objects in the source references. Table 2-3 corresponds to Tables 2-7 and 2-8 in Reference 41.

The orbital elements listed in Table 2-3 are the semimajor axis, eccentricity, and the mean motion, which are useful for libration-point calculations.

Table 2-1. Values for Masses of Solar System Bodies

Objects	Mass (In Terms of Primary)	Standard Error	Low and High Values Used	Method of Determination	References and Notes
MERCURY	1/6023600	±600	1/6024600-1/6022600	Mariner 10	A-(3); B-(10), (28)
VENUS	1/408523.7	±1.0	1/408525.7-1/408521.7	Mariner 2, 5, 10	(3); Note b.
EARTH	1/332946.0	±0.2	1/332946.5-1/332945.5		(10); Note c.
EARTH + MOON	1/328900.53	±0.10	1/328900.78-1/328900.28	Lunar Laser Ranging	A-(31)
MARS	1/3098714	±5	1/3098724-1/3098704	Mariner 4	A-(3); B-(10), (28); Note d.
JUPITER	1/1047.3463	±0.004	1/1047.3483-1/1047.3443	Pioneer 10, 11	(23); Note e.
SATURN	1/3498.5	±0.4	1/3500.0-1/3497.0	Satellites and Perturbations on Planets	(10); C-(9); Note f.
URANUS	1/22945	±15	1/22995-1/22895	Satellites	A-(11); B-(10); Note g.
NEPTUNE	1/19332	±27	1/19382-1/19282	Triton and Uranus	(9); Note f.
MOON	1/81.30065	±0.00009	1/81.30085-1/81.30045	Lunar and Planetary Spacecraft; Lunar Laser Ranging	(42)
AMALTHEA	1.8×10^{-9}	Not Given	$0.9-2.7\times10^{-9}$	Magnitude-Density estimate	A-(38); B-(9)
IO	4.684×10^{-5}	$\pm.022\times10^{-5}$	$4.634-4.734\times10^{-5}$	Pioneer 10, 11	(23)
EUROPA	2.523×10^{-5}	$\pm0.025\times10^{-5}$	$2.473-2.573\times10^{-5}$	Pioneer 10, 11	(23)
GANYMEDE	7.803×10^{-5}	$\pm0.030\times10^{-5}$	$7.753-7.853\times10^{-5}$	Pioneer 10, 11	(23)
CALLISTO	5.661×10^{-5}	$\pm0.019\times10^{-5}$	$5.611-5.711\times10^{-5}$	Pioneer 10, 11	(23)
JANUS	5×10^{-9}	$\pm2\times10^{-9}$	$3-7\times10^{-9}$	Magnitude-Density estimate	Note h
MIMAS	6.59×10^{-8}	$\pm0.15\times10^{-8}$	$6.09-7.09\times10^{-8}$	Satellite Mutual Perturbations	A-(17); B-(9)
ENCELADUS	1.48×10^{-7}	$\pm0.61\times10^{-7}$	$0.87-2.09\times10^{-7}$	Satellite Mutual Perturbations	A-(17); B-(9)
RHEA	4×10^{-6}	$\pm2\times10^{-6}$	$2-6\times10^{-6}$	Satellite Mutual Perturbations	A-(29); B-(6)
TITAN	2.4619×10^{-4}	$\pm0.0029\times10^{-4}$	$2.4519-2.4719\times10^{-4}$	Hyperion	(9); Note i
MIRANDA	1.0×10^{-6}	$\pm0.7\times10^{-6}$	$0.3-1.7\times10^{-6}$	Dynamical & Magnitude estimate	Note j
ARIEL	1.7×10^{-5}	$\pm1.2\times10^{-5}$	$0.5-2.9\times10^{-5}$	Dynamical & Magnitude estimate	Note j
TITANIA	3×10^{-5}	$\pm2\times10^{-5}$	$1-5\times10^{-5}$	Dynamical & Magnitude estimate	Note j
TRITON	3.3×10^{-3}	$\pm2.0\times10^{-3}$	$1.3-5.3\times10^{-3}$	Neptune	(9); Note k

Table 2-2. Values for Radii of Solar System Bodies (1 of 2)

Object	Mean Radius, km	Standard Error, km	Low and High Values To Be Used	Method of Determination	References and Notes
SUN	696000	±250	695500-696500	Visual; Cinematography of Solar Eclipses	A (5); B (10), (32, p. 101); Note g.
MERCURY	2440.12	±0.11	None	Radar	(28); Note h.
VENUS	6110	±2	6100-6120	Radar to Surface, Venera Data for Cloud Heights	(28); (10)
EARTH	6371.004	±0.015	None	Artificial Satellites	(28); Note h.
MARS	3390.74	±0.04	None	Radar, with Mariner 9 Radio Occultation Data for the Figure	(28); (27); Note h.
JUPITER	70452	±2	70442-70462	Occultation of Beta Scorpii 1971	(30); Note e.
SATURN	57822	±300	57522-58322	Direct Visual Measurement	A (8); B (10); Note d.
URANUS	25150	±750	24150-26150	Direct Visual Measurement	A (8); B (10); Note d.
NEPTUNE	25092	±25	24992-25192	Occultation of B.D. $-17^{\circ}4388$, 1968	(30)

Table 2-2. Values for Radii of Solar System Bodies (2 of 2)

Object	Mean Radius, km	Standard Error, km	Low and High Values To Be Used	Method of Determination	References and Notes
MOON	1738.065	±0.5	None	Occultations, Radar, Orbiters and Apollo	(10); Note b and e.
AMALTHEA	75	±50	25-125	Estimate from Magnitude	(20), (37), (39)
IO	1820	±5	1810-1830	Occultation of Beta Scorpii C, 1971	A-(24); B-(1)
EUROPA	1530	±25	1480-1580	Photoelectric Data of Mutual Phenomena, PIONEER 10, 11 Imaging	(1); (26); mean value
GANYMEDE	2640	±40	2590-2690	Photoelectric Data of Mutual Phenomena, Pioneer 10, 11 Imaging	(1); (26); weighted mean
CALLISTO	2410	±30	2360-2460	Pioneer 10, 11 Imaging	(26)
JANUS	110	Not Given	60-160	Estimate from magnitude using assumed albedo	(20)
MIMAS	200	Not Given	100-400	Estimate from magnitude using assumed albedo	(20)
ENCELADUS	275	±150	125-450	Visual, Diskmeter	(8)
RHEA	788	±43	738-838	Lunar Occultation	(14); Note f
TITAN	2916	±26	2966-2966	Lunar Occultation	(14)
MIRANDA	550	±250	300-800	Estimate from magnitude using assumed albedo	(37); C = (39)
ARIEL	1500	±800	700-2300	Estimate from magnitude using assumed albedo	(37); C = (39)
TITANIA	1800	±1000	800-2800	Estimate from magnitude using assumed albedo	(37); C = (39)
TRITON	1885	±650	1000-3000	Visual, Diskmeter	(8)

Table 2-3. Values for Orbital Elements of Solar System Bodies

Object	Semimajor Axis – a		Eccentricity	Mean Motion – n		References and Notes
	Source Form	km	e	Source Form	Radians/Sec	
MERCURY	0.387099A.U.(b)	579092×10^{7}	0.205623	5381016".309/J.Y.(c)	8.2667576×10^{-7}	A=(21); B=(32)
VENUS	0.723332	1.082089×10^{8}	0.006793	2106641".3832/J.Y	3.2363949×10^{-7}	A=(21); B=(32)
EARTH+MOON	1.000000	1.495979×10^{8}	0.016726	$2\pi/365.25636$Days	1.9909866×10^{-7}	A=(21); B=(32)
MARS	1.523691	2.279469×10^{8}	0.093368	689050".926/J.Y	1.0585764×10^{-7}	A=(21); B=(32)
JUPITER	5.202803	7.783283×10^{8}	0.048135	$2\pi/11.86223$T.Y. (c)	1.678490×10^{-8}	A=(16); B=(32)
SATURN	9.538843	1.426991×10^{9}	0.055682	$2\pi/29.45772$T.Y.	6.759056×10^{-9}	A=(16); B=(32)
URANUS	19.181951	2.869579×10^{9}	0.047209	$2\pi/81.01331$T.Y.	2.369938×10^{-9}	A=(21); B=(32)
NEPTUNE	30.057779A.U.	4.496580×10^{9}	0.008575	$2\pi/164.79345$T.Y.	1.208218×10^{-9}	A=(21); B=(32)
MOON		384399.061,04	0.0549005		$2.661699489 \times 10^{-6}$	A (12),(33); B=(15),(32); Note d
AMALTHEA	249".40(e)	1.8808×10^{5}	0	722°.63175/Day	1.4597573×10^{-4}	(10)
IO	$10^{9.5997740-10}$(f)	421761	0.00028	203°.4889516/Day	4.1105928×10^{-5}	(7); (1) for e
EUROPE	$10^{9.8014596-10}$(f)	671045	0.00006	101°.3747236/Day	2.0478272×10^{-5}	(7); (1) for e
GANYMEDE	$10^{0.0042524}$(f)	1070396	0.00001	50.3176081/Day	1.0164443×10^{-5}	(7); (1) for e
CALLISTO	$10^{0.2194696}$(f)	1882616	0.00007	21. 5710720/Day	4.3571795×10^{-6}	(7); (1) for e
JANUS	24".4(g)	1.69×10^{5}	0	$2\pi/19^{h}.6$	8.90×10^{-5}	(34)
MIMAS	26."826(h)	1.8559×10^{5}	0.0202	381°.994502/Day	7.7165066×10^{-5}	(17); (25) for n
ENCELADUS	34."416(h)	2.3810×10^{5}	0.00452	262.°731895/Day	5.3073340×10^{-5}	(17); (25) for n
RHEA	76."203(h)	5.2719×10^{5}	0.00106	79.°6900822/Day	1.6097850×10^{-5}	(17); (25) for n
TITAN	176."578(h)	1.22161×10^{6}	0.029192	22.°5770111/Day	4.5606849×10^{-6}	(17)
MIRANDA	178."96(e)	1.2979×10^{5}	0	254.°68983/Day	5.1448798×10^{-5}	(11)
ARIEL	263."09(e)	1.9081×10^{5}	0.0033	142.°835643/Day	2.8853614×10^{-5}	(11)
TITANIA	601."22(e)	4.3605×10^{5}	0.0018	41.°351422/Day	8.3532230×10^{-6}	(11)
TRITON	489."82(i)	3.5525×10^{5}	0.00	61.°2573679/Day	1.2374337×10^{-5}	A=(13); B=(32)

NOTES - TABLE 2-1

a. For all of these tables, if there is more than one reference, A = original; B = possibly more available secondary source; and C = source for error estimate if not A.

b. In last place, References 10 and 28 give .5 rather than .7.

c. Calculated from masses of Earth + Moon and Moon; Duncombe et al. (1976).

d. In last place, References 10 and 28 give 0 rather than 4.

e. References 10 and 28 give 1/1047.355 based mainly on studies of orbits of minor planets.

f. A weighted mean of many results.

g. The photographic satellite data probably give the strongest determination (formal error much smaller); fears about very large problems involving the photographic plate scale are believed to be unfounded.

h. The mass of Janus was calculated assuming that its density and albedo are the same as those for Mimas using the relation

$$\log (m/m_o) = -0.6\ (V-V_o),$$

where m = mass of Janus, m_o = mass of Mimas, V = stellar magnitude Janus (Ref. 20), and V_o = stellar magnitude of Mimas (Ref 6 and 20). The error is my estimate.

i. A weighted mean of various results, weighted in favor of a determination by Message (Reference 18) which had by far the smallest error.

j. Doing a calculation like the one in Note h above using V = 14.01 for Titania and V_o = 5.52 for Uranus (Ref. 36) yields $m/m_o = 8 \times 10^{-6}$ for Titania. However, the density of Titania is likely to be greater than that of Uranus, and the relative albedo lower. Both effects will raise Titania's mass, but not likely as great as $m/m_o = 10 \times 10^{-5}$, the very uncertain dynamical result. This is emphasized further in recent work by Greenberg (Ref. 35), who shows from the motion of Miranda the dynamically determined masses are most likely upper limits. Consequently, a reasonable mass for Titania would perhaps be $m/m_o = (3 \pm 2) \times 10^{-5}$, which has been used here. The other satellites of Uranus are probably physically rather similar to Titania, so the mass scaling factors based on magnitudes given in Reference 11 have been used.

k. A mean of determinations by Alden (Reference 2) and Nicholson (Reference 22).

NOTES - TABLE 2-2

a. Same as Note a, Table 1.

b. Further improvement of this value will be less than neglected topography and oblateness effects for the shadow calculations.

c. From Appendix 1: Astrometric analysis of the occultation of Beta Scorpii by Jupiter on 1971 May 13. This occultation result is preferred for optical shadow effects over Pioneer radio occultation data.

d. The standard error is calculated assuming 0".1 accuracy in the visual measurements.

e. Various analyses are consistent.

f. This is in agreement with observations of an occultation of a star in 1974, which can not be used since data were obtained from only one station (one chord observed).

g. The standard error is calculated assuming 0".3 at 1 A.U. from lunar radius uncertainty and Reference 19, which notes that the photosphere is not sharply defined, but has a maximum gradient of about 10 magnitudes/ 1".

NOTES - TABLE 2-3

a. The instantaneous osculating orbital elements depart considerably from the mean elements given here due mainly to third-body perturbations. These departures are considerably larger than the standard errors, which are therefore not given. Most important for libration-point calculations is the range of distances of the planet from the Sun (or of the satellites from the planet), which is adequately given in nearly all cases by the mean pericenter and apcenter distances, a (1-e) and a (1+e), respectively. In every case, the sidereal mean motion is the most accurately determined orbital element, introducing no significant error into libration-point calculations considering the larger errors introduced via errors in the other elements. Standard errors are not available for all mean elements, especially for those derived from the early investigations. Under semi-major axis and sidereal mean motion, values are listed as they are presented in the source, then converted to kilometers and radians per second, respectively.

b. The orbital elements for the planets are mean elements valid for long time spans based on comprehensive studies at the U.S. Naval Observatory

80 years ago. No improved values for mean elements have been derived since then, all modern ephemerides being computed by numerical integration, from which it is very difficult to derive mean elements. However, the scaling factor, 1 A.U. = 149 597 871.41 $\pm$ 0.03 km, is from the recent solution for JPL DE96 (Reference 26). The actual uncertainty in the scaling factor is $\pm$ 1 km due to uncertainty in c, the speed of light value, but this is removed if it is agreed to measure distances in light-time units with a fixed value of c (=299792.458 km/sec for DE96, the recently-adopted value).

c. J.Y. = Julian year = exactly 365.25 days; T.Y. = tropical year = 365.24220 days.

d. The semimajor axis of the Moon's orbit has been defined (Reference 15) to be the perturbed mean distance to the Moon = the semimajor axis of Hill's variational orbit = $F_2 (GE(1+\mu)/n^2)^{1/3}$. $F_2 = 0.999093142$ is the factor needed for modifying Kepler's third law. It depends only on the well-determined ratio of the mean motions of the Moon and the Sun. GE is the gravitational constant of the Earth = 398600.49 $\pm$ 0.10 km^3/sec^2 based on five years of lunar laser ranging data and using the recently-adopted speed of light (Reference 33). μ is the Earth-to-Moon mass ratio and n is the Moon's sidereal mean motion. Since all published analyses of lunar laser ranging data have used a numerically-integrated lunar ephemeris, it is not now possible to derive mean elements directly from this data. Abalakin (Institute of Theoretical Astronomy, Leningrad) reported, at the I.A.U. General Assembly in France in August 1976, that he is analyzing the laser ranging data using the j=2 new improved (Eckert-Brown) lunar ephemeris. Improved mean elements are expected from Abalakin's study.

e. Angular separation at unit distance, 1 A.U.

f. These are in units of a_o = 0.0070854378 A.U., log a_o = 7.85036669-10.

g. Angular separation at mean distance, 9.538843 A.U. Only a few observations of Janus were obtained when the rings were edge-on in 1966-1967, so the orbital elements are very uncertain. If more observations are obtained by Pioneer II or during the next edge-on view in 1979, the value for the orbital elements will be improved considerably. At present, a is uncertain by 0".8 and the period by $\pm$ 1^h.

h. Angular separation at mean distance, 9.538843 A.U.

i. Angular separation at unit distance, 1 A.U. A more recent study of Triton was completed by J. R. Gill and B. L. Gault in 1968 (Astron. J. 73, p. 595), but their orbital elements for Triton have not been published and an error is suspected. Alan Fiala at the U.S. Naval Observatory is examining their calculations.

SECTION 3 - TABLES OF LIBRATION-POINT PARAMETERS

Tables of libration-point parameters like those in Reference 41, but calculated using values for the constants given in the tables of Section 2, are presented in this section. Formulae for computing all libration-point parameters are given in the reference. The number of the corresponding table of Reference 41 is given in parentheses after each table heading in Section 3. Since the Section 3 tables were all generated using a computer, they can easily be updated as new values for the astronomical constants become available. Certain very large and very small numbers are expressed in the usual computer double precision exponent notation, such as "D 06" = 10^6 and "D-03" = 10^{-3}. For each libration-point parameter, three values were computed, corresponding to the nominal value for the mass and radius, and for upper and lower bounds for these values, as was done for Mercury in Reference 41. This will facilitate the calculation of new parameters when the mass or radius of an object is improved with future observations. The upper and lower bounds selected are generally a few times the formal standard error, to allow for the possibility that future values will be outside the standard error range due to currently unknown systematic errors. If the mass of a satellite or planet is larger than expected, it is probable that its radius will also be greater than its nominal value. However, when the low value of the mass is used for a shadow-table calculation, the high value for the radius is used, and vice versa, in order to compute minimum and maximum possible critical radii and shadow apex lengths. For the same reason, low and high values for the Solar radius were used with the low and high values of the mass of the body, respectively.

Normalized mass ratios μ and distances of collinear libration points from the respective planets, expressed in units of the semimajor axis of the planet's orbit, are given in Table 3-1. Similar data for the satellites are given in Table 3-2. Constants for the equations of motion of an unpowered spacecraft

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Table 3-1. Mass Parameters and Distance Ratios for Sun-Planet Systems (2.1)

LIBRATION POINT PARAMETERS

PLANET	VALUE	MASS	MU	GAMMA L1	GAMMA L2
MERCURY	LOW	1/6.0246000D 06	1.65986100-07	3.8C5667JD-03	3.81534730-03
MERCURY	NOM.	1/6.0236000D 06	1.66013650-07	3.8058776D-03	3.8155587D-03
MERCURY	HIGH	1/6.0226000D 06	1.66041220-07	3.8060888D-03	3.8157701D-03
VENUS	LOW	1/4.0852570D 05	2.44782040-06	9.3152376D-03	9.3734491D-03
VENUS	NOM.	1/4.0852370D 05	2.44783240-06	9.3152526D-03	9.3734645D-03
VENUS	HIGH	1/4.0852170D 05	2.44784440-06	9.3152679D-03	9.3734798D-03
EARTH	LOW	1/3.3294650D 05	3.00347650-06	9.9703982D-03	1.0037115D-02
EARTH	NOM.	1/3.3294600D 05	3.00348100-06	9.9704031D-03	1.0037120D-02
EARTH	HIGH	1/3.3294550D 05	3.00348550-06	9.9704081D-03	1.0037125D-02
EARTH+MOON	LOW	1/3.2890078D 05	3.04042140-06	1.0010975D-02	1.0078238D-02
EARTH+MOON	NOM.	1/3.2890053D 05	3.04042370-06	1.0010978D-02	1.0078241D-02
EARTH+MOON	HIGH	1/3.2890028D 05	3.04042600-06	1.0010980D-02	1.0078243D-02
MARS	LOW	1/3.0987240D 06	3.22713370-07	4.7483400D-03	4.7634190D-03
MARS	NOM.	1/3.0987140D 06	3.22714420-07	4.7483451D-03	4.7634241D-03
MARS	HIGH	1/3.0987040D 06	3.22715460-07	4.7483502D-03	4.7634292D-03
JUPITER	LOW	1/1.0473483D 03	9.53881450-04	6.6680676D-02	6.9784548D-02
JUPITER	NOM.	1/1.0473463D 03	9.53883270-04	6.6680717D-02	6.9784594D-02
JUPITER	HIGH	1/1.0473443D 03	9.53885090-04	6.6680759D-02	6.9784639D-02
SATURN	LOW	1/3.5000000D 03	2.85632680-04	4.4960258D-02	4.6349931D-02
SATURN	NOM.	1/3.4985000D 03	2.85755110-04	4.4966580D-02	4.6356650D-02
SATURN	HIGH	1/3.4970000D 03	2.85877640-04	4.4972905D-02	4.6363373D-02
URANUS	LOW	1/2.2995000D 04	4.34858240-05	2.4182821D-02	2.4579121D-02
URANUS	NOM.	1/2.2945000D 04	4.35805800-05	2.4200228D-02	2.4597103D-02
URANUS	HIGH	1/2.2895000D 04	4.36757510-05	2.4217685D-02	2.4615137D-02
NEPTUNE	LOW	1/1.9382000D 04	5.15916010-05	2.5588179D-02	2.6032306D-02
NEPTUNE	NOM.	1/1.9332000D 04	5.17250300-05	2.5610026D-02	2.6054918D-02
NEPTUNE	HIGH	1/1.9282000D 04	5.18591510-05	2.5631947D-02	2.6077808D-02

Table 3-2. Mass Parameters and Distance Ratios for Planet-Satellite Systems (2.2)

SATELLITE	VALUE	MASS	MU	GAMMA L1	GAMMA L2
MOON	LOW	1/8.13008500 01	1.2150543D-02	0.1509341	0.1678325
MOON	NOM.	1/8.13006500 01	1.2150572D-02	0.1509342	0.1678327
MOON	HIGH	1/8.13004500 01	1.2150602D-02	0.1509344	0.1678328
AMALTHEA	LOW	9.0000000D-10	9.0000000D-10	0.0006693	0.0006696
AMALTHEA	NOM.	1.8000000D-09	1.8000000D-09	0.0008432	0.0008437
AMALTHEA	HIGH	2.7000000D-09	2.7000000D-09	0.0009652	0.0009658
IO	LOW	4.6340000D-05	4.6337853D-05	0.0246960	0.0251094
IO	NOM.	4.6840000D-05	4.6837606D-05	0.0247837	0.0252001
IO	HIGH	4.7340000D-05	4.7337759D-05	0.0248708	0.0252902
EUROPA	LOW	2.4730000D-05	2.4729388D-05	0.0200698	0.0203353
EUROPA	NOM.	2.5230000D-05	2.5229363D-05	0.0201972	0.0204728
EUROPA	HIGH	2.5730000D-05	2.5729338D-05	0.0203288	0.0206081
GANYMEDE	LOW	7.7530000D-05	7.7523990D-05	0.0292709	0.0298530
GANYMEDE	NOM.	7.8030000D-05	7.8023912D-05	0.0293950	0.0299182
GANYMEDE	HIGH	7.8530000D-05	7.8523834D-05	0.0293949	0.0299825
CALLISTO	LOW	5.6110000D-05	5.6106852D-05	0.0263073	0.0267770
CALLISTO	NOM.	5.6610000D-05	5.6606795D-05	0.0263845	0.0268570
CALLISTO	HIGH	5.7110000D-05	5.7106739D-05	0.0264613	0.0269365
JANUS	LOW	3.0000000D-09	3.0000000D-09	0.0009997	0.0010003
JANUS	NOM.	5.0000000D-09	5.0000000D-09	0.0011852	0.0011861
JANUS	HIGH	7.0000000D-09	7.0000000D-09	0.0013258	0.0013269
MIMAS	LOW	6.0900000D-08	6.0899996D-08	0.0027254	0.0027304
MIMAS	NOM.	6.5900000D-08	6.5899996D-08	0.0027980	0.0028032
MIMAS	HIGH	7.0900000D-08	7.0899995D-08	0.0028670	0.0028725
ENCELADUS	LOW	8.7000000D-08	8.6999992D-08	0.0030692	0.0030755
ENCELADUS	NOM.	1.4800000D-07	1.4799998D-07	0.0036631	0.0036721
ENCELADUS	HIGH	2.0900000D-07	2.0899996D-07	0.0041091	0.0041204
RHEA	LOW	2.0000000D-06	1.9999960D-06	0.0087103	0.0087612
RHEA	NOM.	4.0000000D-06	3.9999840D-06	0.0109659	0.0110467
RHEA	HIGH	6.0000000D-06	5.9999640D-06	0.0125461	0.0126519
TITAN	LOW	2.4519000D-04	2.4512990D-04	0.0427593	0.0440143
TITAN	NOM.	2.4619000D-04	2.4612941D-04	0.0428165	0.0440749
TITAN	HIGH	2.4719000D-04	2.4712891D-04	0.0428735	0.0441353
MIRANDA	LOW	3.0000000D-07	2.9999991D-07	0.0046344	0.0046488
MIRANDA	NOM.	1.0000000D-06	9.9999900D-07	0.0069176	0.0069496
MIRANDA	HIGH	1.7000000D-06	1.6999971D-06	0.0082523	0.0082979
ARIEL	LOW	5.0000000D-06	4.9999750D-06	0.0118093	0.0119030
ARIEL	NOM.	1.7000000D-05	1.6999711D-05	0.0177217	0.0179336
ARIEL	HIGH	2.9000000D-05	2.8999159D-05	0.0211499	0.0214524
TITANIA	LOW	1.0000000D-05	9.9999000D-06	0.0148633	0.0150120
TITANIA	NOM.	3.0000000D-05	2.9999100D-05	0.0213885	0.0216979
TITANIA	HIGH	5.0000000D-05	4.9997500D-05	0.0253242	0.0257591
TRITON	LOW	1.3000000D-03	1.2983122D-03	0.0737082	0.0775158
TRITON	NOM.	3.3000000D-03	3.2891458D-03	0.0955267	0.1066085
TRITON	HIGH	5.3000000D-03	5.2720581D-03	0.1157758	0.1254690

near the collinear libration points are presented in Tables 3-3 and 3-4. Roots of the characteristic equation for estimating stability of orbits near the libration points are given in Tables 3-5 and 3-6. Tables 3-7 and 3-8 list the factors for normalizing units of time, velocity, and acceleration to the non-dimensional units of the restricted three body problem. Distances between the planets and their collinear libration points are given in Table 3-9. Similar data for the satellites are listed in Table 3-10. Distance variations between some planets and their collinear libration points, based on their perihelion and aphelion distances, are presented in Table 3-11. Libration point distances and planetary shadow lengths are compared in Table 3-12. The "critical radius" is the radius a planet would have to have in order for the apex of its shadow to just reach L2. A table of critical radii and shadow lengths was computed for the satellites, similar to the one for planets, but is not included here. The critical radii were all much smaller than the actual radii, ranging from 300 kilometers for the Moon to 0.1 kilometers for Janus, so a total solar eclipse by the satellite will occur at L2 during each satellite's revolution about the planet.

Table 3-3. Constants for Equations of Motion for Sun-Planet Systems (2.3)

PLANET	VALUE	B L1	B L2	C L1	C L2	D L1	D L2
MERCURY	LOW	4.0229792	3.9772522	790.29785	784.29799	207931.233	205306.828
MERCURY	NOM.	4.0229804	3.9772509	790.28429	784.28443	207908.297	205284.037
MERCURY	HIGH	4.0229817	3.9772497	790.21072	784.21087	207885.359	205261.244
VENUS	LOW	4.0567640	3.9446185	324.05201	318.05286	34899.797	33829.429
VENUS	NOM.	4.0567641	3.9446184	324.05148	318.05233	34899.684	33829.318
VENUS	HIGH	4.0567642	3.9446183	324.05095	318.05181	34899.571	33829.207
EARTH	LOW	4.0608224	3.9407610	302.88957	296.89055	30484.334	29464.494
EARTH	NOM.	4.0608224	3.9407609	302.88942	296.89040	30484.304	29464.464
EARTH	HIGH	4.0608224	3.9407609	302.88927	296.89025	30484.273	29484.434
EARTH+MOON	LOW	4.0610740	3.9405222	301.66958	295.67097	30238.570	29243.194
EARTH+MOON	NOM.	4.0610740	3.9405222	301.66991	295.67090	30238.555	29243.179
EARTH+MOON	HIGH	4.0610740	3.9405222	301.66983	295.67082	30238.540	29243.164
MARS	LOW	4.0287161	3.9716438	633.79949	627.79971	133693.790	131591.026
MARS	NOM.	4.0287162	3.9716438	633.79861	627.79903	133693.503	131590.742
MARS	HIGH	4.0287162	3.9716438	633.79813	627.79835	133693.217	131590.457
JUPITER	LOW	4.4461596	3.6228451	46.93305	40.98419	725.004	577.079
JUPITER	NOM.	4.4461599	3.6228449	46.93302	40.98417	725.003	577.078
JUPITER	HIGH	4.4461602	3.6228447	46.93299	40.98414	725.002	577.077
SATURN	LOW	4.2904902	3.7411980	68.70082	62.72273	1556.020	1336.047
SATURN	NOM.	4.2905340	3.7411630	68.69143	62.71334	1555.594	1335.651
SATURN	HIGH	4.2905779	3.7411280	68.68204	62.70396	1555.167	1335.256
URANUS	LOW	4.1510275	3.8582270	126.04819	120.05418	5259.037	4848.365
URANUS	NOM.	4.1511406	3.8581272	125.95895	119.96495	5251.571	4841.196
URANUS	HIGH	4.1512540	3.8580271	125.86958	119.87559	5244.099	4834.022
NEPTUNE	LOW	4.1601742	3.8501806	119.23398	113.24072	4704.215	4316.227
NEPTUNE	NOM.	4.1603167	3.8500557	119.13395	113.14070	4696.302	4308.646
NEPTUNE	HIGH	4.1604597	3.8499303	119.03376	113.04062	4688.381	4301.059

Table 3-4. Constants for Equations of Motion for Planet-Satellite Systems (2.4)

SATELLITE	VALUE	B L1	B L2	C L1	C L2	D L1	D L2
MOON	LOW	5.1475950	3.1904260	21.51165	15.84517	157.355	91.701
MOON	NOM.	5.1475941	3.1904255	21.51163	15.84516	157.355	91.701
MOON	HIGH	5.1475951	3.1904249	21.51162	15.84514	157.355	91.701
AMALTHEA	LOW	4.0040202	3.9959870	4484.40519	4478.40519	6701866.180	6686868.149
AMALTHEA	NOM.	4.0056563	3.9949451	3559.89396	3553.89387	4223099.155	4211242.825
AMALTHEA	HIGH	4.0036004	3.9942145	3110.23314	3104.23315	3223464.343	3213126.830
IO	LOW	4.1543625	3.8552867	123.47024	117.47650	5045.496	4643.407
IO	NOM.	4.1549333	3.8547842	123.04010	117.04641	5010.297	4609.640
IO	HIGH	4.1555002	3.8542853	122.61603	116.62239	4975.716	4575.470
EUROPA	LOW	4.1244556	3.8819267	151.51859	145.52267	7607.005	7111.521
EUROPA	NOM.	4.1253105	3.8811565	150.53103	144.53517	7507.908	7015.715
EUROPA	HIGH	4.1261546	3.8803967	149.56923	143.57343	7412.020	6923.028
GANYMEDE	LOW	4.1843386	3.8291887	104.48078	98.48569	3609.074	3270.184
GANYMEDE	NOM.	4.1847466	3.8288356	104.20369	98.27253	3594.036	3256.698
GANYMEDE	HIGH	4.1851573	3.8284841	104.04824	98.05722	3579.156	3241.735
CALLISTO	LOW	4.1648707	3.8460708	110.02857	110.03570	4453.934	4075.615
CALLISTO	NOM.	4.1653755	3.8456299	110.09425	109.70203	4428.271	4052.063
CALLISTO	HIGH	4.1658774	3.8451917	110.36504	109.37227	4402.981	4027.871
JANUS	LOW	4.0060080	3.9940080	3003.00066	2997.00067	3005007.674	2995007.651
JANUS	NOM.	4.0071250	3.9928974	2833.29877	2827.29879	2138360.822	2129926.458
JANUS	HIGH	4.0079722	3.9920559	2264.84319	2258.84321	1709087.646	1701543.141
MIMAS	LOW	4.0164270	3.9836917	1102.73977	1096.73984	404981.826	401315.570
MIMAS	NOM.	4.0166605	3.9832586	1074.19185	1068.19193	384275.721	380705.033
MIMAS	HIGH	4.0172843	3.9826471	1048.39485	1042.39493	366032.196	362547.487
ENCELADUS	LOW	4.0135094	3.9816411	979.46375	973.46384	319461.005	316206.062
ENCELADUS	NOM.	4.0221130	3.9761013	820.97933	814.97946	224399.769	221673.115
ENCELADUS	HIGH	4.0248238	3.9754459	732.08995	726.09011	178412.310	175981.924
RHEA	LOW	4.0530244	3.9481847	346.41933	340.42008	39891.210	38746.298
RHEA	NOM.	4.0670057	3.9349087	275.57421	269.57540	25226.484	24317.674
RHEA	HIGH	4.0768621	3.9256419	241.11696	235.11852	19303.428	18509.442
TITAN	LOW	4.2752836	3.7834069	72.13786	66.15755	1716.158	1484.778
TITAN	NOM.	4.2756772	3.7830893	72.04411	66.06387	1711.686	1480.617
TITAN	HIGH	4.2760699	3.7827725	71.95088	65.97069	1707.244	1476.484
MIRANDA	LOW	4.0280217	3.9723212	649.33329	643.33350	140332.584	138173.043
MIRANDA	NOM.	4.0419858	3.9567774	435.67902	429.67948	63131.358	61684.951
MIRANDA	HIGH	4.0501979	3.9508876	365.53601	359.53668	44421.567	43212.941
ARIEL	LOW	4.0722599	3.9299594	256.03629	250.03768	21770.832	20927.131
ARIEL	NOM.	4.1095033	3.8954812	171.28072	165.28389	9726.751	9165.443
ARIEL	HIGH	4.1314278	3.8756607	143.83971	137.84426	6853.590	6533.676
TITANIA	LOW	4.0914080	3.9121027	203.83759	197.83960	13766.710	13116.940
TITANIA	NOM.	4.1329627	3.8742858	142.25727	136.26193	6703.214	6233.572
TITANIA	HIGH	4.1534529	3.8516906	120.45632	114.46291	4801.461	4409.404
TRITON	LOW	4.4987238	3.5853128	42.62963	36.69326	558.224	464.471
TRITON	NOM.	4.7011807	3.4501162	32.00298	26.12798	338.460	239.450
TRITON	HIGH	4.8360987	3.3666933	27.71604	21.89325	255.290	170.101

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Table 3-5. Roots of Characteristic Equation 2.49 (2.5)

ROOTS OF CHARACTERISTIC EQUATION (2.49)

PLANET	VALUE	ROOTS FOR L1 REAL	ROOTS FOR L1 IMAGINARY	ROOTS FOR L2 REAL	ROOTS FOR L2 IMAGINARY
MERCURY	LOW	2.5174852	2.0771982	2.4991409	2.0660303
MERCURY	NOM.	2.5174857	2.0771985	2.4991464	2.0660305
MERCURY	HIGH	2.5174862	2.0771989	2.4991459	2.0660302
VENUS	LOW	2.5309470	2.0854086	2.4859751	2.0580218
VENUS	NOM.	2.5309471	2.0854086	2.4859751	2.0580213
VENUS	HIGH	2.5309471	2.0854086	2.4859751	2.0580213
EARTH	LOW	2.5325592	2.0863926	2.4844134	2.0570729
EARTH	NOM.	2.5325593	2.0863926	2.4844134	2.0570729
EARTH	HIGH	2.5325593	2.0863926	2.4844134	2.0570729
EARTH+MOON	LOW	2.5326592	2.0864536	2.4843167	2.0570142
EARTH+MOON	NOM.	2.5326592	2.0864536	2.4843167	2.0570142
EARTH+MOON	HIGH	2.5326592	2.0864536	2.4843167	2.0570142
MARS	LOW	2.5197763	2.0785948	2.4968883	2.0646567
MARS	NOM.	2.5197763	2.0785949	2.4968883	2.0646567
MARS	HIGH	2.5197763	2.0785949	2.4968883	2.0646567
JUPITER	LOW	2.6811409	2.1776953	2.3520592	1.9772045
JUPITER	NOM.	2.6811410	2.1776954	2.3520591	1.9772044
JUPITER	HIGH	2.6811411	2.1776955	2.3520590	1.9772043
SATURN	LOW	2.6221489	2.1413020	2.4022044	2.0073335
SATURN	NOM.	2.6221657	2.1413124	2.4021897	2.0073246
SATURN	HIGH	2.6221825	2.1413227	2.4021750	2.0073158
URANUS	LOW	2.5681267	2.1061383	2.4507560	2.0366585
URANUS	NOM.	2.5681710	2.1061654	2.4507150	2.0366337
URANUS	HIGH	2.5682153	2.1061926	2.4506739	2.0366088
NEPTUNE	LOW	2.5717052	2.1103302	2.4474493	2.0346566
NEPTUNE	NOM.	2.5717609	2.1103643	2.4473979	2.0346255
NEPTUNE	HIGH	2.5718168	2.1103985	2.4473464	2.0345943

Table 3-6. Roots of Characteristic Equation 2.49 (2.6)

ROOTS OF CHARACTERISTIC EQUATION (2.49)

SATELLITE	VALUE	ROOTS FOR L1 REAL	ROOTS FOR L1 IMAGINARY	ROOTS FOR L2 REAL	ROOTS FOR L2 IMAGINARY
MOON	LOW	2.9320554	2.3343856	2.1586747	1.8626401
MOON	NOM.	2.9320556	2.3343858	2.1586744	1.8626403
MOON	HIGH	2.9320561	2.3343860	2.1586742	1.8626405
AMALTHEA	LOW	2.5098985	2.0725758	2.5066769	2.0706139
AMALTHEA	NOM.	2.5103177	2.0728311	2.5062527	2.0703593
AMALTHEA	HIGH	2.5106119	2.0730103	2.5059655	2.0701808
IO	LOW	2.5694320	2.1069378	2.4495482	2.0359272
IO	NOM.	2.5696554	2.1070746	2.4493417	2.0358022
IO	HIGH	2.5698772	2.1072104	2.4491367	2.0356781
EUROPA	LOW	2.5577019	2.1017572	2.4604689	2.0425427
EUROPA	NOM.	2.5580363	2.1019629	2.4601535	2.0423517
EUROPA	HIGH	2.5583698	2.1021659	2.4598430	2.0421633
GANYMEDE	LOW	2.5811548	2.1161092	2.4388012	2.0294247
GANYMEDE	NOM.	2.5812945	2.1162072	2.4386654	2.0293386
GANYMEDE	HIGH	2.5814537	2.1163047	2.4385103	2.0292483
CALLISTO	LOW	2.5735406	2.1114546	2.4457586	2.0336333
CALLISTO	NOM.	2.5737378	2.1115755	2.4455772	2.0335235
CALLISTO	HIGH	2.5739339	2.1116956	2.4453969	2.0334144
JANUS	LOW	2.5106951	2.0730610	2.5058826	2.0701303
JANUS	NOM.	2.5111426	2.0733335	2.5054367	2.0698585
JANUS	HIGH	2.5114819	2.0735402	2.5050988	2.0696531
MIMAS	LOW	2.5146659	2.0756020	2.5017377	2.0676073
MIMAS	NOM.	2.5150417	2.0757091	2.5015636	2.0675013
MIMAS	HIGH	2.5152088	2.0758109	2.5013981	2.0674006
ENCELADUS	LOW	2.5156987	2.0761094	2.5009130	2.0671054
ENCELADUS	NOM.	2.5171391	2.0769873	2.4994887	2.0662384
ENCELADUS	HIGH	2.5182221	2.0776474	2.4984197	2.0655884
RHEA	LOW	2.5294606	2.0845015	2.4874180	2.0588587
RHEA	NOM.	2.5350135	2.0878906	2.4820422	2.0556325
RHEA	HIGH	2.5389208	2.0902767	2.4782828	2.0533494
TITAN	LOW	2.6163136	2.1377122	2.4073163	2.0104141
TITAN	NOM.	2.6164648	2.1378052	2.4071834	2.0103340
TITAN	HIGH	2.6166157	2.1378980	2.4070505	2.0102541
MIRANDA	LOW	2.5194991	2.0764259	2.4971612	2.0648227
MIRANDA	NOM.	2.5250676	2.0818214	2.4916588	2.0615008
MIRANDA	HIGH	2.5283365	2.0838156	2.4886110	2.0596630
ARIEL	LOW	2.5370972	2.0891630	2.4800351	2.0544134
ARIEL	NOM.	2.5518167	2.0981576	2.4660064	2.0458599
ARIEL	HIGH	2.5604415	2.1034335	2.4579047	2.0409887
TITANIA	LOW	2.5446758	2.0937925	2.4727756	2.0500085
TITANIA	NOM.	2.5610442	2.1038024	2.4573417	2.0406475
TITANIA	HIGH	2.5710321	2.1099179	2.4480702	2.0350324
TRITON	LOW	2.7007646	2.1858414	2.3359265	1.9675466
TRITON	NOM.	2.7750308	2.2359820	2.2768407	1.9323270
TRITON	HIGH	2.8234217	2.2661887	2.2396715	1.9102976

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Table 3-7. Conversion Factors for Normalized Units for Sun-Planet Systems (2.9)

CONVERSION FACTORS FOR NORMALIZED UNITS FOR SUN-PLANET SYSTEMS.

PLANET	TIME (DAYS)	VELOCITY (METERS/SEC.)	ACCELERATION (M/SEC**2)	ACCELERATION (G=9.7643125)
MERCURY	14.000742	4.7872132D 04	3.9574731D-02	4.0529972D-03
VENUS	35.762243	3.5020673D 04	1.1334073D-02	1.1607651D-03
EARTH	58.132355	2.9784741D 04	5.9301021D-03	6.0732408D-04
EARTH+MOON	58.132355	2.9784741D 04	5.9301021D-03	6.0732408D-04
MARS	109.536219	2.4129286D 04	2.5542692D-03	2.6159233D-04
JUPITER	689.552757	1.3084163D 04	2.1928066D-04	2.2457358D-05
SATURN	1712.560261	9.0451121D 03	6.5191853D-05	6.6765430D-06
URANUS	4583.703516	5.8007243D 03	1.6117295D-05	1.6506329D-06
NEPTUNE	9579.458464	5.4328489D 03	5.5640658D-06	5.7225069D-07

Table 3-8. Conversion Factors for Normalized Units for Planet-Satellite Systems (2.10)

CONVERSION FACTORS FOR NORMALIZED UNITS FOR PLANET-SATELLITE SYSTEMS.

SATELLITE	TIME (DAYS)	VELOCITY (METERS/SEC.)	ACCELERATION (M/SEC**2)	ACCELERATION (G=9.7643125)
MOON	4.348377	1.0231548D 03	2.7233366D-03	2.7890653D-04
AMALTHEA	0.079266	2.6404090D 04	3.8543563D 00	3.9473914D-01
IO	0.281507	1.7336877D 04	7.1264843D-01	7.2985038D-02
EUROPA	0.505188	1.3741842D 04	2.8140918D-01	2.8820173D-02
GANYMEDE	1.138033	1.0879979D 04	1.1058893D-01	1.1325828D-02
CALLISTO	2.656140	8.2034606D 03	3.5746412D-02	3.6609246D-03
JANUS	0.130046	1.5041000D 04	1.3386496D 00	1.3709608D-01
MIMAS	0.149991	1.4321065D 04	1.1050859D 00	1.1317601D-01
ENCELADUS	0.218077	1.2636762D 04	6.7067518D-01	6.8686370D-02
RHEA	0.718983	8.4866256D 03	1.3061642D-01	1.3991402D-02
TITAN	2.537793	5.5713783D 03	2.5409301D-02	2.6022621D-03
MIRANDA	0.224903	8.6775395D 03	3.4355138D-01	3.5184390D-02
ARIEL	0.401131	5.5055581D 03	1.5885525D-01	1.6268964D-02
TITANIA	1.385562	3.6424229D 03	3.0425971D-02	3.1160382D-03
TRITON	0.935529	4.3959832D 03	5.4397378D-02	5.5710402D-03

Table 3-9. Distance Between the Smaller Finite Body and a Collinear Point for Sun-Planet Systems (2.11)

PLANET		MEAN DISTANCE BETWEEN PLANET AND L1 (AU)	(KM)	MEAN DISTANCE BETWEEN PLANET AND L2 (AU)	(KM)
MERCURY	LOW	0.0014732	220383	0.0014769	220944
MERCURY	NOM.	0.0014733	220395	0.0014770	220956
MERCURY	HIGH	0.0014733	220408	0.0014771	220968
VENUS	LOW	0.0067380	1007991.6	0.0067801	1014290.6
VENUS	NOM.	0.0067380	1007993.3	0.0067801	1014292.3
VENUS	HIGH	0.0067380	1007994.9	0.0067801	1014293.9
EARTH	LOW	0.0099704	1491550.6	0.0100371	1501531.4
EARTH	NOM.	0.0099704	1491551.4	0.0100371	1501532.1
EARTH	HIGH	0.0099704	1491552.1	0.0100371	1501532.9
EARTH+MOON	LOW	0.0100110	1497620.8	0.0100782	1507683.3
EARTH+MOON	NOM.	0.0100110	1497621.2	0.0100782	1507683.7
EARTH+MOON	HIGH	0.0100110	1497621.6	0.0100782	1507684.0
MARS	LOW	0.0072350	1082340.9	0.0072580	1085778.0
MARS	NOM.	0.0072350	1082342.1	0.0072580	1085779.2
MARS	HIGH	0.0072350	1082343.2	0.0072580	1085780.3
JUPITER	LOW	0.3469264	51899457	0.3630753	54315289
JUPITER	NOM.	0.3469267	51899489	0.3630755	54315324
JUPITER	HIGH	0.3469269	51899522	0.3630758	54315359
SATURN	LOW	0.4288690	64157883	0.4421248	66140935
SATURN	NOM.	0.4289293	64166904	0.4421889	66150522
SATURN	HIGH	0.4289896	64175931	0.4422531	66160116
URANUS	LOW	0.4638737	69394516	0.4714755	70531729
URANUS	NOM.	0.4642076	69444466	0.4718204	70583330
URANUS	HIGH	0.4645424	69494560	0.4721664	70635081
NEPTUNE	LOW	0.7691239	115059296	0.7824733	117056347
NEPTUNE	NOM.	0.7697805	115157529	0.7831530	117156022
NEPTUNE	HIGH	0.7704394	115256100	0.7838350	117260049

Table 3-10. Distance Between the Smaller Finite Body and a Collinear Point for Planet-Satellite Systems (2.12)

SATELLITE		MEAN DISTANCE BETWEEN SATELLITE AND L1 (KM)	MEAN DISTANCE BETWEEN SATELLITE AND L2 (KM)	RADIUS OF SATELLITE (KM)
MOON	LOW	58018.9	64514.7	1738.1
MOON	NOM.	58019.0	64514.7	1738.1
MOON	HIGH	58019.0	64514.8	1738.1
AMALTHEA	LOW	121	121	125
AMALTHEA	NOM.	153	153	75
AMALTHEA	HIGH	175	175	25
IO	LOW	10416	10590	1830
IO	NOM.	10453	10628	1820
IO	HIGH	10490	10666	1810
EUROPA	LOW	13464	13646	1580
EUROPA	NOM.	13553	13738	1530
EUROPA	HIGH	13642	13829	1480
GANYMEDE	LOW	31331	31965	2690
GANYMEDE	NOM.	31598	32024	2640
GANYMEDE	HIGH	31864	32093	2590
CALLISTO	LOW	49527	50411	2460
CALLISTO	NOM.	49672	50561	2410
CALLISTO	HIGH	49816	50711	2360
JANUS	LOW	169	169	160
JANUS	NOM.	200	200	110
JANUS	HIGH	224	224	60
MIMAS	LOW	506	507	400
MIMAS	NOM.	519	520	200
MIMAS	HIGH	532	533	100
ENCELADUS	LOW	731	732	450
ENCELADUS	NOM.	872	874	275
ENCELADUS	HIGH	978	981	125
RHEA	LOW	4592	4619	838
RHEA	NOM.	5761	5824	788
RHEA	HIGH	6614	6670	738
TITAN	LOW	52235	53768	2966
TITAN	NOM.	52305	53842	2916
TITAN	HIGH	52375	53916	2866
MIRANDA	LOW	601	603	800
MIRANDA	NOM.	898	902	550
MIRANDA	HIGH	1071	1077	300
ARIEL	LOW	2253	2271	2300
ARIEL	NOM.	3381	3422	1500
ARIEL	HIGH	4036	4093	700
TITANIA	LOW	6481	6546	2800
TITANIA	NOM.	9326	9461	1800
TITANIA	HIGH	11043	11232	800
TRITON	LOW	26185	27539	3000
TRITON	NOM.	35358	37873	1885
TRITON	HIGH	41129	44573	1000

Table 3-11. Distance Variation Between the Smaller Finite Body and a Collinear Point (2.13)

DISTANCE VARIATION BETWEEN THE SMALLER FINITE BODY AND A COLLINEAR POINT

BODY		MIN. DISTANCE BETWEEN BODY AND L1 (KM)	MAX. DISTANCE BETWEEN BODY AND L1 (KM)	MIN. DISTANCE BETWEEN BODY AND L2 (KM)	MAX. DISTANCE BETWEEN BODY AND L2 (KM)
MERCURY	LOW	175066.4	265699.9	175611.7	266375.7
MERCURY	NOM.	175076.1	265714.6	175621.4	266390.5
MERCURY	HIGH	175085.8	265729.2	175631.2	266405.2
EARTH+MOON	LOW	1472571.6	1522670.0	1482465.8	1532900.8
EARTH+MOON	NOM.	1472572.0	1522670.4	1482466.1	1532901.2
EARTH+MOON	HIGH	1472572.4	1522670.8	1482466.5	1532901.6
MARS	LOW	961284.9	1183396.9	964401.1	1187154.9
MARS	NOM.	961285.9	1183398.2	964402.1	1187156.2
MARS	HIGH	961287.0	1183399.4	964403.2	1187157.5
JUPITER	LOW	49385707.0	54413207.4	51684527.9	56946049.9
JUPITER	NOM.	49385737.6	54413241.2	51684561.5	56946086.9
JUPITER	HIGH	49385768.3	54413274.9	51684595.1	56946123.9
MOON	LOW	54833.7	61204.2	60972.8	68056.6
MOON	NOM.	54833.7	61204.2	60972.8	68056.6
MOON	HIGH	54833.7	61204.3	60972.9	68056.7

Table 3-12. L2 Distance and Length of Planetary Shadow (2.14)

L2 DISTANCE AND LENGTH OF PLANETARY SHADOW

PLANET		MEAN RADIUS OF PLANET (KM)	CRITICAL RADIUS OF PLANET (KM)	MEAN DISTANCE BETWEEN PLANET AND L2 (KM)	MEAN DISTANCE BETWEEN PLANET AND SHADOW APEX (KM)
MERCURY	LOW	2440.1	2643.49	220943.7	203886
MERCURY	NOM.	2440.1	2643.53	220956.0	203739
MERCURY	HIGH	2440.1	2647.58	220968.2	203593
VENUS	LOW	6120.0	6458.69	1014290.0	966629
VENUS	NOM.	6110.0	6463.35	1014292.3	958350
VENUS	HIGH	6100.0	6468.00	1014295.9	950075
EARTH	LOW	6371.0	6911.44	1501531.4	1383034
EARTH	NOM.	6371.0	6916.41	1501532.1	1332031
EARTH	HIGH	6371.0	6921.39	1501532.9	1381030
EARTH+MOON	LOW	6371.0	6939.48	1507683.3	1383034
EARTH+MOON	NOM.	6371.0	6944.47	1507683.7	1382031
EARTH+MOON	HIGH	6371.0	6949.46	1507684.0	1381030
MARS	LOW	3390.7	3297.25	1085778.0	1116714
MARS	NOM.	3390.7	3299.63	1085779.2	1115908
MARS	HIGH	3390.7	3302.00	1085780.3	1115103
JUPITER	LOW	70462.0	45369.09	54315288.9	87742775
JUPITER	NOM.	70452.0	45401.74	54315324.2	87658797
JUPITER	HIGH	70442.0	45434.38	54315359.5	87574956
SATURN	LOW	58322.0	30808.41	66140934.7	130614944
SATURN	NOM.	57822.0	30834.83	66150522.4	129292256
SATURN	HIGH	57322.0	30861.26	66160115.7	127973707
URANUS	LOW	26150.0	16684.68	70531728.9	112108002
URANUS	NOM.	25150.0	16708.60	70583329.7	107579605
URANUS	HIGH	24150.0	16732.57	70635081.2	103071812
NEPTUNE	LOW	25192.0	17646.10	117056346.8	168993721
NEPTUNE	NOM.	25092.0	17673.73	117156021.6	166172365
NEPTUNE	HIGH	24992.0	17701.44	117260049.1	167352477

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