NASA TECHNICAL MEMORANDUM NASA TM-73,217

(NASA-TM-73217) FLUT - A PROGRAM FOR AEROELASTIC STABILITY ANALYSIS (NASA) 57 p HC A04/MF A01 CSCL 01C

N77-28108

Unclas

G3/05 39261

NASA TM-73,217

FLUT – A PROGRAM FOR AEROELASTIC STABILITY ANALYSIS

Erwin H. Johnson
Ames Research Center
Moffett Field, California 94035

June 1977

NOMENCLATURE

[A]	aerodynamic influence coefficient matrix
AIC	abbreviation for aerodynamic influence coefficients
{a}	vector of modal participation factors
$\bar{c}$	reference chord length
g	scalar approximation of the structural damping of the system
[GA]	matrix of generalized aerodynamic forces
[I]	identity matrix
i	imaginary constant
[K]	stiffness matrix
$\lceil K \rfloor$	diagonal generalized stiffness matrix
k	reduced frequency parameter = $\omega\bar{c}/2V$
[M]	mass matrix
$\lceil M \rfloor$	diagonal generalized mass matrix
{u}	structural displacement vector
V	velocity
λ	eigenvalue of the complex eigenmatrix ($= \omega^2/(1+ig)$)
$[\Phi]$	matrix of normal modes from the vibration analysis
ω	circular frequency

Subscripts

r	refers to rigid body modes from the vibration analysis
f	refers to flexible modes from the vibration analysis
i	ith component of a vector or the ith column of a matrix

Superscripts

$(\ )^{-1}$	matrix inverse

Note: Additional notation is defined in the text.

FLUT — A PROGRAM FOR AEROELASTIC STABILITY ANALYSIS

Erwin H. Johnson*

Ames Research Center

SUMMARY

FLUT is a computer program that can be used to evaluate the aeroelastic stability of aircraft structures in subsonic flow. The algorithm synthesizes data from a structural vibration analysis with an unsteady aerodynamics analysis and then performs a complex eigenvalue analysis to assess the system stability. This document is divided into two main parts. The first of these describes the theoretical basis of the program. Special emphasis in this section is placed on some innovative techniques which improve the efficiency of the analysis.

The second section provides the user information needed to efficiently and successfully utilize the program. In addition to identifying the required input, this section summarizes the flow of the program execution and points out some possible sources of difficulty. The use of the program is demonstrated with a listing of the input and output for a simple example.

*National Research Council Postdoctoral Research Associate.

I. Introduction

The Advanced Vehicle Concept Branch at the NASA-Ames Research Center has been charged with responsibility of assessing new aircraft concepts and providing quantitative data on how these concepts might best be implemented. Many of these concepts were characterized by lightweight high-strength materials and high aspect ratio configurations. This naturally led to concern about the aeroelastic stability properties of the concepts and a need was recognized for a technique that would perform this stability analysis. In particular, there was great uncertainty as to the aeroelastic stability properties of a subsonic oblique wing transport concept under study by the Lockheed-Georgia Company. Under these circumstances, the FLUT program was developed with a special emphasis placed on using an aerodynamics package that did not require aircraft symmetry about the fuselage. It is felt that this program can be useful in a number of applications, which therefore motivates this documentation.

The first part of this document describes the analysis on which the program is based. Special emphasis is given to features of the program that may be new to workers in the field. It does not attempt to provide an in-depth description of flutter analysis techniques; a more complete description of such techniques can be found in any of several texts on aeroelasticity (e.g., Refs. 1 and 2).

The remainder of this report is divided into three sections. The first defines the input required by FLUT. The next provides some information on program execution, and describes its operation with the use of a flow chart. The last section presents a simple example.

Additional information on the program usage is contained in the source listing. The majority of the variables used are explicitly identified in comment cards and the program flow is outlined. The unsteady aerodynamics package contained in FLUT was developed by Giesing, Kalman and Rodden and is explained in detail in References 3 and 4.

It must be noted that much of the basis for the methods of analysis used in this program were obtained from a program by Crittenden and Weisshaar, (Ref. 5).

II. Basic Matrix Representation

The program FLUT is based on what has come to be known as the k method of flutter analysis (Ref. 1) which starts from the matrix equation of motion:

$$[(1+ig)[K] - \omega^2([M] + [A])] \{u\} = 0 \qquad (1)$$

This equation has degrees of freedom that are the actual structural displacements. The mass and stiffness representations are therefore the finite element representations developed by any computer code similar to one described in Reference 6.

The aerodynamic matrix contains the coupling between structural displacements and the aerodynamic forces they create. In FLUT, this matrix is calculated using doublet lattice methods contained in routines documented in References 3 and 4. This matrix is a function of the free stream Mach number and the reduced frequency parameter k. The point to be stressed here is that aerodynamic matrices of the form required by Eq. (1) are actually calculated during program execution. This contrasts with other programs which calculate generalized aerodynamic forces directly. More will be said on this in the "Efficiency Concepts" section.

The matrix eigenvalue problem given by Eq. (1) has a large number of degrees of freedom and is not in a form needed for a complex eigenvalue solution. A more convenient form is one that uses normal modes to represent the displacements. This is done by a transformation of the form

$$\{u\} = \sum_{i=1}^{mn} \{\Phi_i\} a_i \qquad (2)$$

Where mn is the number of normal modes retained in the analysis. If this transformation is placed in Eq. (1) and the result is premultiplied by the transpose of the matrix of the normal modes, the resulting equation is of the form

$$\left[[K] - \lambda([m] + [GA])\right] \{a\} = 0 \qquad (3)$$

Because normal modes were used, the generalized mass and stiffness matrices are diagonal. The generalized aerodynamics matrix is complex and nonhermitian. The complex eigenvalues of the matrix

$$[\mathcal{K}]^{-1}([\mathcal{M}] + [GA]) \tag{4}$$

are then the desired results.

When rigid body modes of the aircraft are included in the analysis, the generalized stiffness matrix has zeroes on the diagonal. This makes it impossible to perform the inversion called for by Eq. (4). The next subsection provides a means of dealing with this situation.

Reduction of Rigid Body Modes.- When rigid body modes are present, Eq. (3) can be subdivided into the form:

$$\left[\begin{bmatrix} 0 & 0 \\ 0 & \mathcal{K}_{ff} \end{bmatrix} - \lambda\left(\begin{bmatrix} \mathcal{M}_{rr} & 0 \\ 0 & \mathcal{M}_{ff} \end{bmatrix} + \begin{bmatrix} GA_{rr} & GA_{fr} \\ GA_{rf} & GA_{ff} \end{bmatrix}\right)\right]\begin{Bmatrix} a_r \\ a_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{5}$$

This gives two matrix equations

$$[\mathcal{M}_{rr} + GA_{rr}]\{a_r\} + [GA_{fr}]\{a_f\} = 0 \tag{6}$$

$$[\mathcal{K}_{ff}]\{a_f\} - \lambda\left([\mathcal{M}_{ff} + GA_{ff}]\{a_{ff}\} + [GA_{rf}]\{a_r\}\right) = 0 \tag{7}$$

A reduced problem is obtained by solving Eq. (6) for $\{a_r\}$ and placing the result in Eq. (7).

This equation is now well behaved and its solution can be obtained in the same way as one without rigid body modes. Although this technique for reducing rigid body modes is not generally known, it has been previously discussed by Cross and Albano in Reference 7.

Equation (4) or the system resulting from Eqs. (6) and (7) can now be evaluated to find the complex eigenvalues λ and their corresponding eigenvectors (which represent modal participation factors). The complex eigenvalue analysis is performed using routines extracted from Reference 8 which includes descriptions of the algorithms. Briefly, the routines transform the matrix to upper Hessenberg form and then use a "modified LR method" to extract the eigenvalues and eigenvectors.

With the solution obtained, the only remaining task is to get the results in a useful form for output. The eigenvalues contain the needed stability information. The λ's obtained represent $\omega^2/(1+ig)$;* therefore, the velocity, damping, and frequency information can be obtained from

$$
\begin{aligned}
g &= -\mathrm{Im}\lambda/\mathrm{Re}\lambda \\
\omega^2 &= (1+g^2)\mathrm{Re}\lambda \\
V &= \frac{\omega\bar{c}}{2k}
\end{aligned}
\tag{8}
$$

Section VI provides added detail on how these results are evaluated.

The eigenvectors from the eigenvalue analysis are the modal participation factors for the flexible models. It is possible to obtain the participation factors for the reduced rigid body modes through a relationship based on Eq. (6):

$$\{a_r\} = -[M_{rr} + GA_{rr}]^{-1}[GA_{fr}]\ \{a_f\} \tag{9}$$

If the actual structural deflections are required, they can, of course, be obtained from the superposition relationship of Eq. (2).

* The program actually solves for $1/\lambda$ and derives g, ω, and V from this variable.

III Efficiency Concepts

The utility of any analysis can be enhanced by making it as efficient as possible. This is particularly true for a flutter analysis; since, typically, many repetitive analyses are required. Steps have been taken in FLUT to perform these repetitive analyses with a minimum of additional effort. Two steps that are of sufficient general interest to be described here are (1) aerodynamic interpolation and (2) optimal use of aerodynamic influence coefficients.

Both of these concepts derive their importance from the fact that the time required to perform an unsteady aerodynamics analysis at a single set of k and M values can be as much as an order of magnitude higher than the time required to perform the complex eigenvalue analysis. It is therefore very important to keep the aerodynamics calculations to a minimum.

Interpolation of the aerodynamics can be done to give high quality results with a minimum of effort, thereby minimizing the aerodynamic calculations.

The interpolation is performed by fitting a quadratic in 1/k through generalized aerodynamics forces that were in turn calculated using aerodynamic influence coefficients (AIC's) computed at three reduced frequency values. The preceding sentence brings up several points:

(1) The AIC's are calculated using the routines of Reference 4.

(2) Generalized aerodynamics, rather than AIC's were used in the fit since the matrices involved are smaller and there is no loss in accuracy.

(3) The inverse of the reduced frequency was used to obtain the fit since it provided results vastly superior to those obtained using the reduced frequency directly. This is borne out by Table 1 which compares results for one branch of a V-g analysis obtained using either "exact" aerodynamics (aerodynamics computed directly without interpolation), aerodynamics interpolated using 1/k as the independent parameter and those obtained using k. It can be seen that the results obtained using 1/k are within one percent of the exact values, while the results using k are entirely unacceptable.

Table 1.- Comparison of Aerodynamic Interpolation Techniques

	"Exact"			Interpolation with k			Interpolation with 1/k		
	V	g	ω	V	g	ω	V	g	ω
.01*	678.5	.053	.6785	678.5	.0526	.6785	678.5	.0526	.6785
.03	702.0	-.113	2.11	336.1	.073	1.01	702.4	-.112	2.11
.05*	718.3	-.247	3.59	718.3	-.247	3.59	718.3	-.247	3.59
.10	720.4	-.378	7.20	141.0	.759	1.41	719.2	-.379	7.19
.15	656.8	-.380	9.85	86.4	.628	1.29	655.8	-.382	9.84
.20*	573.2	-.348	11.50	573.1	-.348	11.46	573.2	-.348	11.46
.30	426.2	-.278	12.79	12.53	+ .02	.375	428.2	-.269	12.85
.50	263.1	-.175	13.16	3.18	-.025	.159	265.7	-.152	13.29

* The aerodynamics used for the interpolation were computed at these points. Therefore, the interpolated results at these reduced frequencies are the exact results.

(4) Presently only interpolation in $1/k$ is performed. It may be desirable to include interpolation in Mach number for some applications.

Finally, it should be stressed that the interpolation factors that weight the contributions from each of the three basic generalized aerodynamic matrices are the same for all the mn^2 elements of the aerodynamics matrix, i.e.,

$$[GA_{int}] = f_1[GA_1] + f_2\,[GA_2] + f_3\,[GA_3] \tag{10}$$

Here, int refers to interpolation results, f_1, f_2, and f_3 are constant scalars which weight the three exact generalized aerodynamics matrices.

$$f_1 = \frac{(vk_i - vk_2)(vk_i - vk_3)}{(vk_1 - vk_2)(vk_1 - vk_3)}$$

$$f_2 = \frac{(vk_i - vk_3)(vk_i - vk_1)}{(vk_2 - vk_3)(vk_2 - vk_1)} \tag{11}$$

$$f_3 = \frac{(vk_i - vk_1)(vk_i - vk_2)}{(vk_3 - vk_1)(vk_3 - vk_2)}$$

where the convention $vk_j = 1/k_j$ is used.

The second efficiency, the use of AIC's, allows a number of different vibration analyses to use the same aerodynamics results. This contrasts with unsteady aerodynamics programs that compute the generalized aerodynamic forces directly which require the calculation of new forces each time a different vibration analysis is made. Occasions where the AIC's can save computer resources are:

(1) When structural changes in the aircraft are being studied.

(2) When various boundary conditions are being evaluated, e.g., it is sometimes desirable to see what the effect of rigid body roll is on the flutter analysis. The AIC's remain the same while the mode shapes differ drastically from a clamped wing condition.

(3) When investigations into the sensitivity of the flutter speed to parameters such as static unbalance, attached masses, and fuselage inertias are being performed.

IV Program Input

The following is a list of inputs and their associated format statements. Where additional clarification is required, reference is made to notes contained at the end of this section. The input required for the aerodynamics package (Blocks C through O)was set by the algorithm's developers and is explained in Reference 4.

Dimensions used in the program are arbitrary, with the exception that they must be self-consistent; e.g., deflections for the mode shapes cannot be in inches while the chord length is in feet.

The format of this description of data is the following: A letter signifying the data block is given in the left hand column. The input is then listed and its associated format is given in parentheses in the right-hand column. A description of the data is then given before the next data block is listed.

A. TITLE (20A4)

Title is any eighty character title.

B. N,MN, NA, NR, NAR, NRD, NRFQ, NTRF, NREAD, NW, IPR1, IPR2, NAERO (20I3)

N - Total number of unconstrained degrees of freedom from the vibration analysis ($N \leq 90$)

MN - Number of normal modes retained from the vibration analysis ($MN \leq 20$)

NA - Number of aerodynamic degrees of freedom ($NA \leq 100$)

NR - Number of rigid body modes included in the vibration and flutter analysis ($NR \leq 3$)

NAR - Number of structural degrees of freedom that are not at an aerodynamic grid point and must therefore be eliminated. See Note a. ($NAR \leq 10$)

NRD - Should be zero. See program listing.

NRFQ - Number of reduced frequencies for which the aerodynamics are calculated explicitly ($NRFQ \leq 3$)

NTRF - Number of reduced frequencies at which the complex eigenvalue problem is to be solved. See Note b. ($NTRF \leq 20$).

NREAD - Number of the tape on which the vibration analysis results are written. This number must, of course, be the same as the tape number attached in the JCL.

NW - A tape number for writing plots data. It is recommended that NW = 0 be used so that no plots data will be generated. The plots information is unsatisfactory.

IPR1 - Print command. If it is greater than zero, the vibration analysis input, the generalized aerodynamics, the complex eigenmatrix and the modal participation factors are printed. IPR1 = 0 suppresses these prints. These are intermediate prints, intended for use during program debug.

IPR2 - A second print command. If it is greater than zero, the aerodynamic influence coefficients and the flutter mode shapes are printed. IPR2 = 0 suppresses these prints. The outputs from this parameter can be considerable and it is recommended that it usually be set to zero.

NAERO - Parameter indicating how the aerodynamics are to enter the formulation.

= 1 - Program computes the AIC's at the NRFQ reduced frequencies and saves results on tape 11. (Of course, the results are actually only saved when the appropriate cataloging card is included in the JCL.)

= 2 - Previously computed aerodynamics are read from tape 11.

If (NAERO = 2) go to Block P.

C. TITLE

This is a second title used for the aerodynamics package.

D. FMACH, ACAP, FL, B2, NDELT, NP, NB, NRF, NCORE, N1, N2, N3, N4 (4F10.0, 4I2, I6, 4I1).

FMACH - Free stream Mach number of the aerodynamics calculations. Must be the same as AMACH in Block R.

ACAP - Reference area, total area of the wing

FL - Reference chord. An average chord of the wing. Must agree with CBAR in Block R.

B2 - Wing semispan

NDELT - Symmetry flag for the ZY plane

= 1 Symmetric aerodynamics

= -1 Antisymmetric aerodynamics

= 0 No symmetry (this option is used for obliquely configured wings, for example.)

NP - Total number of aerodynamic panels. See Note c for a definition of the terms panel, strip and box.

NB - Total number of bodies. FLUT has never been used with bodies and would require some reprogramming to do so. Therefore, NB should be zero.

NRF - Number of reduced frequencies for which the aerodynamic analysis is to be performed. NRF must agree with the value of NRFQ in Block B.

NCORE - This parameter is a flag to determine which set of routines are to be used for solving linear systems of equations. See Note d. It should be set equal to the total number of boxes times the value of NA from Block B.

N1 - A flag which should always be zero for this configuration of the program.

N2 - A flag delineating whether polynomial modes on AIC's are being calculated. Should always be 1 for this configuration.

N3 - If N3 is >0, voluminous intermediate results are printed out. It is recommended N3=0 be used unless detailed debug is being done.

N4 - Print parameter for slender body data. N4 should always be zero for this configuration of the program.

E. (RFREQ(I), I = 1, NRF) (6F10.0)

RFREQ - Vector of reduced frequency values = $k = \frac{\omega\bar{c}}{2V_\infty}$. These numbers must be the same, and in the same order, as those read in by Block P.

Repeat input Blocks F through I, for each panel.

F. XCAP(1), XCAP(2), XCAP(3), XCAP(4), YCAP(1), YCAP(2) (6F10.0).

Blocks F and G read in panel coordinates as shown in Figure 1. The x coordinate is in the streamwise direction while the y coordinate is perpendicular to the flow. The streamwise edges of the panels are parallel to the flow. The origin of the coordinate system is arbitrary.

YCAP(1)< YCAP(2)
XCAP(1)< XCAP(2)
XCAP(3)< XCAP(4)

G. ZCAP(1), ZCAP(2), NS, NC, COEFF, (2F10.0, 1X, 2I3, 3X, F10.0).

ZCAP - Vertical locations of the streamwise panel edges. See Figure 1.

NS - Number of spanwise division boundaries for the panel. It is one greater than the number of strips.

NC - Number of chordwise division boundaries for the panel. It is one greater than the number of boxes per strip.

COEFF - Scale factor for body deflections. It should be set to 1.

H. (TH(I), I = 1, NC) (6F10.0)

TH(I) = θ_i - Chordwise divisions of the panel in fractions of the chord. Usually, θ_1 is 0.0 and θ_{NC} is 1.0.

I. (TAU(I), I = 1, NS) (6F10.0)

TAU(I) = τ_i - Spanwise divisions of the panel in fractions of the panel span. Usually, τ is 0.0 at the left edge of the panel and τ_{NS} is 1.0 at the right edge.

J. NSTRIP, NB1, NB2, NPR1, NPR2, NPR3, JOBNO, JOBNO2, NGUST, JSPECS, NPC, NSV, NBV, NYAW (1X, 3I3, 1X, 3I3, I15, I5, 1X, 5I3, 2I2)

NSTRIP - Total number of strips. See Note c.

NB1 - Sequence number of the first strip. For this configurations strip numbering starts from the left wing tip (or the inboard edge if only the right wing is being considered)

NB2 - Sequence number of the last strip. For this configuration, strip numbering ends at the right wing tip.

NPR1 - Print flag. If NPR1 = 1, intermediate results are presented. It is recommended that this be zero so that printing is suppressed.

NPR2 - Print flag. If NPR2 = 1, AIC's are printed. During early runs, these can be interesting, but it is recommended the NPR2 = 0 be used to suppress print and limit printout.

NPR3 - Print flag for stability derivatives

= 0-No stability derivative information is calculated

= 1 - dynamic stability derivatives are printed

= 3 - static and dynamic derivatives are printed. For this case, NRF must be $\geq$ 2 and RFREQ(1) must be 0.0.

It is recommended the NPR3 = 1 be used during early runs of a study since the derivative information provides a qualitative check of the program. During production runs, NPR3=0 should be used. NPR3 =3 cannot be used in a flutter study since the AIC's at RFREQ = 0 are not compatible with the aeroelastic stability algorithm. See Note e.

JOBNO - Tape number for AIC's. Use any four digit number. It appears that this input is never used.

JOBNO2, NGUST, JSPECS, NSV, NBV, and NYAW are not used for this configuration. Therefore their fields can be zero or blank. See Reference 2 for their definition and possible application.

NPC - Mode selector for AIC generation. NPC=0 gives plunge, pitch, control surface and tab degrees of freedom. NPC=1 gives "cambering" modes plus control surface and tab degrees of freedom. NPC = 0 should always be used for this configuration.

K. (LIM(I, 1), LIM(I,2), LIM(I,3), I = 1, NSTRIP) (6(IX, 3I3))

LIM(I, 1) - Number of the first box for strip I.

LIM(I, 2) - Number of the last box for strip I. (See Note c.)

LIM(I,3) - Probably for control surfaces. In any case, it is to be left blank in this configuration.

L. (NOP(I), (IS(I, J), J = 1, 7), I = I, NP) (6(1X, I2, 7I1))

NOP(I) - Number of the panel

IS (I, J) - Degree of freedom selector for panel. For this configuration, set IS(I, 1) = 1 for plunging. IS (I, 2) = 2 for pitching and leave the rest blank.

Repeat Blocks M and N for each panel.

M. XHI(I, 1) (F10.0)

The location of the reference axis (the line about which the aerodynamics are collocated) at the left edge of the panel as a fraction of the chord.

N. XHO(I, 1) (F10.0)

The location of the reference axis at the right edge of the panel as a fraction of the chord. NOTE: Other axes that could be read in by this card but are not relevant to this configuration, are not described here.

O. NMD, NTA, N5, N6, N7, NTPI, NTPM, NMTP, NMTB (5I2, 2I10, 2I2, 2I3)

NTA, N6, NTPM, NMTP and NMTB can be ignored for this configuration, i.e., their fields should be left blank.

NMD - Number of aerodynamic modes. This must agree with NA from Block B. This number is required only for NCORE $\geq$ 3700.

N5 - Flag to save intermediate results. Can be always set to zero for this configuration.

N7 - Flag for pressure forces and moments.

=1 calculate and print pressures

=0 suppress calculation

It is recommended N7 = 1 be used for early runs as a qualitative check of output, then set to zero to limit output.

NTPI - When NCORE is $>$ 3700, an arbitrary number (say 720) must be used here to save intermediate results. Otherwise, the field can be left blank.

This is the end of the aerodynamics package input.

P. (RFQ(I), I = 1, NRFQ) (6E12.4)

Values of the reduced frequencies at which the aerodynamics have been computed. These values must agree with the data in Block E.

Q (RFT(I), I = 1, NTRF) (6E12.4)

Values of the reduced frequency at which the complex eigenvalue problem is to be solved. See Note b.

R. NRHO, AMACH, CBAR, UC1, UC2 (I3, 4E12.4)

NRHO - Number of different densities at which the complex eigenvalue problem is to be solved.

AMACH - Mach number at which the aerodynamics are calculated. This must agree with FMACH read in by Block D.

CBAR - Reference chord length. Must agree with FL in Block D.

UC1 - Units conversion factor. UC1 is the factor required to change the input length dimension to feet (e.g., if input is in inches, UC1 = .083333)

UC2 - Units conversion factor UC2 is a factor required to change the mass dimension from slugs to the units of the input. The only situation where this should differ from one is if kilograms are used, in which case UC2 = 14.594.

S. IF(NAR.NE.0) READ (JRF(I),I = 1,NAR) (10I3)

JRF(I) = The integer number of the i^{th} degree of freedom that is not at an aerodynamic grid point. See Note a. JRF(I+1) > JRF(I).

T. (RHON(I), H(I), I= 1,NRHO) (2E12.4)

RHON(I) = ρ/ρ_0 - Ratio of the i^{th} atmospheric density at the altitude the flutter analysis is to be performed to the sea level density.

H(I) - Altitude corresponding to the i^{th} density (currently, this dimension is only in feet). This number is only used for output.

This completes the description of the data that is input from cards. Data from the vibration analysis and from the unsteady aerodynamic analysis is read from tapes. The aerodynamic calculations are all done within the program and are therefore self-consistent. The vibration results, however, must be input in a very specific way:

The vibration data is input in two steps, first the eigenvalues are read in and then the eigenvectors. The statements used for these reads are:

A1. (LBDA(I), I = 1, MN) (unformatted)

LBDA(I) - ith eigenvalue of the vibration analysis. This is the square of the ith natural frequency. Units are $(\text{rad/sec})^2$.

LBDA(I + 1) $\geq$ LBDA(I)

B1. ((PH(I,J), J = MN), I = 1, N) (unformatted)

PH(I,J) - The ith component of the j^{th} normal mode.

Note a. contains information on what the structural components are and how they relate to the aerodynamic degrees of freedom as well as how to modify the program to accept other forms of the eigenvalues and eigenvectors.

V. Notes

These notes are intended to clarify and amplify on the descriptions of the input given above. Most of the notes describe quirks of the program that could prove troublesome to a new user.

(a) Structural and aerodynamic degrees of freedom.-

Figure 2 shows a representation of an oblique wing planform with a set of grid points and aerodynamic boxes. Points 1 through 9 are structural grid points while points 2, 3, 4, 6, 7 and 8 are aerodynamic grid points as well. When a vibration analysis is being run to determine the natural frequencies and the normal modes, consideration must be taken of the aerodynamic representation that is to be used so that the requisite degrees of freedom are obtained. This is because structural nodes must be placed at the centers of the aerodynamic boxes. Each of these grid points must include three, and only three, degrees of freedom: (1) deflection in the z direction, U_Z , (2) rotation about the global x axis, θ_x, and (3) rotation about the global y axis, θ_y. The dof's must be in the order given in the preceding sentence and must start from the left grid point and proceed to the right until the last grid point is reached. The eigenvectors must be normalized to give a unit mass matrix.

Assume in Figure 2 that the structural model is represented by the nine nodes shown in the figure connected by eight beam elements. Each of the nodes has the three dof's listed above except node 5 which, in this example, has freedom only in the Z direction (allowing for rigid body plunge. The structural dof's are then:

Structural dof	Node	Direction
1	1	U_z
2	1	θ_x
3	1	θ_y
4	2	U_z
5	2	θ_x
.	.	.
.	.	.
.	.	.
.	.	.
11	4	θ_x
12	4	θ_y
13	5	U_z

Structural dof	Node	Direction
14	6	U_z
.	.	.
.	.	.
22	8	θ_y
23	9	U_z
24	9	θ_x
25	9	θ_y

There are, therefore, 25 structural degrees of freedom whose deflections must be passed to FLUT in the order listed above. Of these dof's, several (numbers 1, 2, 3, 13, 23, 24, and 25) are not at aerodynamic grid points. These are the JRF's given in Block S.

As a further note on degrees of freedom, the aerodynamics package calculates forces due to pitch and plunge at the center of each strip. No forces are calculated due to θ_x deflections and the plunging degree of freedom is in the -z direction. Internal to the program, the θ_x degrees of freedom are deleted and the sign is changed on the U_z dof's.

A potential user may wish to input the eigenvectors and eigenvalues using formatted data from their vibration package. If these data are in the order given above in this note, it will be necessary for the user to modify only the read statements for the A1 and B1 data blocks to reflect the desired formats. These read statements are near the 80th line of the FLUT subroutine of the computer program. It is more likely that the user's data are in a form other than that given above. In this case, it will be necessary to alter either the data or the program.

(b) Calculated versus interpolated aerodynamics.-

Data Blocks B, P, and Q differentiate between the reduced frequencies used for aerodynamic calculations (RFQ) and the reduced frequencies used for flutter analyses (RFT). The motivation for these two sets is explained in Section III and has mainly to do with minimizing the utilization of computer resources. Typically, the aerodynamics are computed explicitly at three reduced frequency values and these results can be used to interpolate to many reduced frequencies. The interpolation has worked well on several examples, with the following conditions:

(i) It is advised that extrapolation of the aerodynamics be kept to a minimum. The program will produce answers, but their quality may be suspect.

(ii) Interpolation using k values that differ by an order of magnitude should always be acceptable (unless extremely accurate results are needed such as, for example, those required for flutter derivative information). Ranges larger than an order of magnitude should be used with caution.

(iii) If an RFT equals an RFQ, no interpolation is performed. Likewise, if NRF < 3, no interpolation can be performed.

(iv) An instance where interpolation can be extremely valuable is when added definition is needed for a particular reduced frequency range. If the aerodynamics from the RFQ values have been saved on tape, they can be read off the tape and the flutter analysis done at the added reduced frequencies without a new aerodynamic analysis.

(c) Panels, strips and boxes.-

The terminology used in the definitions of the aerodynamic representation is sufficiently confusing to warrant some clarification. Figure 3, which is another representation of the wing of Figure 2, is used for illustrative purposes.

A panel is a large aerodynamic surface which can be represented by a trapezoid. The edges of the panel must be parallel to the free stream direction. There are two panels in Figure 3.

A strip is a chordwise segment of a panel. It extends from the leading to the trailing edge. There are six strips in Figure 3.

Each strip can be further subdivided into boxes. In Figure 3 each strip has four boxes.

Reference 2 has a number of rules for setting up the aerodynamic model. Among these are:

(i) The aspect ratio of the boxes should be unity or less.

(ii) There should be a minimum of four boxes per strip, with even more at high reduced frequencies.

Both of these"rules" have been violated by the author while performing analyses with little adverse effect. A brief study to determine how refined the aerodynamic grid had to be also indicated that the rules were conservative, particularly at low k's.

The use of AIC's involves collocating the aerodynamics from each strip at a chordwise location given by data Blocks M and N and a spanwise location at the center of each strip.

(d) The variable NCORE.-

The size of the problems under study is given by NCORE. This parameter dictates which of two subroutines is to be used to solve the linear systems of equations in the aerodynamics package. If the value of NCORE is less than 3700, all the calculations can be performed in core. If it is greater than 3700, auxiliary storage is required and a separate subroutine handles this case. Unfortunately, the equations are solved with a change in sign between the two subroutines. Therefore, <u>if a case is run with NCORE greater than 3700, the sign of the AIC's must be changed</u> before they are used in FLUT. A simple way of doing this is to input values of RHON in data Block T that are the negative of the actual density ratios.

(e) Very low reduced frequencies.-

If static divergence types of instability are being investigated, very low values of k (reduced frequency) should be used. In fact, static divergence only occurs, by definition, at $k = 0$. Unfortunately, the aerodynamics package is configured in a way that gives different types of AIC's at zero k than it does at nonzero values. These static AIC's are not compatible with FLUT. Therefore static divergence must be investigated in the limit as $k \to 0$. The crossover from dynamic to static AIC's occurs at $k = 1\times10^{-4}$, representing a lower limit for k's in this program. A secondary crossover occurs when k/FL (see Block D) is 5×10^{-5}. For values of k less than this value, only steady state aerodynamics are calculated with no increment due to oscillations. This should not cause any problems, but users should be aware of this transition when they are interpolating across this value.

VI. Program Execution

If the input described in the previous section is correctly entered, results will be printed out at the values of reduced frequencies and atmospheric densities specified. It is the intention of this section to explain briefly the algorithm used to achieve these results and to then provide some information that could aid in the interpretation of the results.

(a) Algorithm for FLUT

Figure 4 shows a simple block diagram for the algorithm. The diagram indicates that the program is initiated by reading the basic parameters of Block B. The broken line to the box "calculate AIC's" indicates that this calculation, which uses input from Blocks C through O, is performed only if the results have not been saved on tape during a previous run. After the remaining input is read from cards, the vibration and aerodynamic data is read from tape and the generalized aerodynamic forces at the RFQ reduced frequencies are calculated. The complex eigenvalue analysis is then performed for all the RFT reduced frequencies and RHON density values, with density on the inner loop and reduced frequency on the outer loop.

The program is contained in two overlays plus a zero overlay that controls the program execution.

Overlay one contains the flutter analysis and has five subroutines of its own. Four of these are a "black box" that performs the complex eigenvalue analysis.

Overlay two contains the unsteady aerodynamics analysis. With slight modification,this is the program developed by Giesing, Kalman and Rodden (Ref. 4).

(b) Interpretation of the Results

The aeroelastic stability analysis contained in this program is based on the V-g method of flutter analysis. Any standard reference text (e.g. Ref. 1) on aeroelasticity contains a description of this method. The principle features of this analysis are shown in the sketch of Figure 5 which depicts two branches of a V-g curve at a constant atmospheric density. FLUT solves the complex eigenvalue problem at each reduced frequency and density. It is the user's task to take the output, which gives values of velocity, V, artificial damping, g, and frequency and plot curves like the one shown in Figure 5. The plotting of these curves is something of an art with some helpful tips listed here:

(1) There is a branch on the V-g curve for each of the normal modes included in the stability analysis. Typically, only a handful are in the velocity range of interest with the remainder at physically unrealistic velocities.

(2) For a given branch, high k values correspond to low velocities and vice versa; i.e., in Figure 5, $k_i < k_{i+1}$. There can be exceptions to this where a branch doubles back on itself slightly.

(3) The frequency results serve as an aid in tracking a branch through a range of k values. At very high values of k, the velocities approach zero and the frequencies approach the natural vibration frequencies. Frequently, it is possible to start from a high k value, identify the branches and follow them by assigning results at lower k's to branches that have a corresponding frequency.

(4) When in doubt, results at intermediate k values should be obtained. If the aerodynamics at the RFQ values have been saved on tape, it is simple and inexpensive to run more RFT's. With enough data points, the correct branch assignments should be obvious. Once the branches have been drawn, the only remaining task is an assessment of the stability of the system. There are two types of instability that the wing can exhibit. The first type is depicted

in Figure 5 by one of the branches crossing the V-g axis. In this case, at g = 0 the system is said to be in neutral equilibrium at the corresponding velocity. Given a disturbance, the system will oscillate harmonically at the frequency that corresponds to this point. At higher velocities g is positive and the system is unstable with a response that is characterized by divergent oscillations.

A second type of instability that sometimes occurs is manifested by a V-g branch that approaches g = 0 from below, but never crosses into positive g. In the limiting case as $k \to 0$, this phenomenon corresponds to static aeroelastic divergence. The response of the wing in this case is characterized by a nonoscillatory divergence.

As mentioned, the V-g curves are drawn at a constant atmospheric density. The effect of altitude on the flutter velocity can be studied by plotting V-g curves at a series of RHON's.

FLUT is programmed so that all results of a given run are obtained at a constant Mach number. This means that, strictly speaking, the results are valid only at one velocity on the V-g curve. It is extremely unlikely that this velocity will correspond to the speed of instability so that the results are inaccurate. If greater precision is desired, it is necessary to run the program at several Mach numbers and then interpolate to find the point where Mach number and the flutter or divergence velocity correspond. An example of this is shown in Figure 6, which has been taken from Reference 9. In this figure, flutter velocities calculated at three altitudes and Mach numbers are given by the square data points. At a given altitude, the flutter point is seen to decrease slightly with Mach number. The three lines emanating from the origin are simply lines where the velocity and Mach number agree for each of the altitudes. The circled data points are therefore matched flutter points where the flutter velocity and Mach number agree for the given altitude. It should be noted that, in this case, the matched flutter point for the two highest altitudes are at Mach numbers that are outside the range where the extrapolation from calculated results is quantitatively meaningful.

VII. Example: Goland's wing

Reference 10 is a journal article that presents an early technique used for flutter analysis. In the article, the author, Martin Goland, presents a simple example that can be used to verify the FLUT program. Figure 7 depicts the cantilevered wing he used in his example and lists the relevant structural parameters.

A vibration analysis of this model was performed using the PASS computer program (Ref. 9). The 11 structural grid points used in the analysis were arranged along the elastic axis at the following intervals: (given as a fraction of the semi-span): 0.0, .0625, .1875, .3125, .4375, .5625, .6825, .8125, .9167, .979, 1.00. These points, except for the first and the last, are at the center of the aerodynamic strips selected for the unsteady aerodynamic analysis. It should be noted that the vibration analysis was done in inches so that the mode shapes are in the units of inches and radians.

The input deck used to obtain the output listed for this example is listed below. The input is identified by reference to the data blocks listed in Section IV. In order to limit output, only one reduced frequency value was used.

(a) Input

BLOCK

A. TITLE
FLUTTER ANALYSIS OF THE GOLAND WING

B. N MN NA NR NAR NRD NRFQ NTRF NREAD NW IPR1 IPR2 NAERO
30 10 18 0 3 0 1 1 8 0 1 0 1

C. TITLE FOR AERODYNAMICS
AERODYNAMICS FOR THE GOLAND WING

	FMACH	ACAP	FL	B2	NDELT	NP	NB	NRF	NCORE	N	N	N	N
D.	0.0	34560.0	72.0	240.0	1	1	0	1	3240	1	0	0	

	RFREQ(1)
E.	.40

	XCAP(1)	XCAP(2)	XCAP(3)	XCAP(4)	YCAP(1)	YCAP(2)
F.	0.0	72.0	0.0	72.0	0.0	240.0

	ZCAP(1)	ZCAP(2)	NS	NC	COEFF
G.	0.0	0.0	10	3	1.0

	TH(1)	TH(2)	TH(3)
H.	0.0	0.5	1.0

27

I.

TAU(1)	TAU(2)	TAU(3)	TAU(4)	TAU(5)	TAU(6)
0.0	0.125	0.25	0.375	0.5	0.625

TAU(7)	TAU(8)	TAU(9)	TAU(10)
0.75	0.875	0.958333	1.0

J.

NSTRIP	NB1	NB2	NPR1	NPR2	NPR3	JOBNO	JOBNO2	NGUST	JSPECS	NPC	NSV	NBV	NYAW
9	1	9	0	0	1	1001		0	0	0	0	0	0

K.

LIM(1,1)	LIM(1,2)	LIM(2,1)	LIM(2,2)								
1	2	3	4	5	6	7	8	9	10	11	12

				LIM(9,1)	LIM(9,2)
13	14	15	16	17	18

L. 112 ← NOP(1) IS(1,1) IS(1,2)

M.

XHI(1,1)
0.33

N.

XHO(1,1)
0.33

O. N5 N7
0 1

P. RFQ(1)
0.40

Q. RFT(1)
0.40

R. NRHO AMACH CBAR UC1 UC2
1 0.0 72.0 0.08333 1.0

S. JRF(1) JRF(2) JRF(3)
28 29 30

T. RHON(1) H(1)
1.0 0.0

(b) Output

The output obtained for a single reduced frequency for this example is given here. For the sake of brevity, some of the less meaningful output has been deleted.

Figure 8 shows the complete V-g diagram obtained using this model. The figure indicates a flutter type of instability at 450 ft/sec. and a divergence instability at 1000 ft/sec.

```
* * * * * * * * * * * * * * * * * * * * * * * * * *

                PROGRAM [illegible]

       PROGRAM FOR AEROELASTIC STABILITY
         ANALYSIS USING NORMAL MODES

    BY ERWIN H. JOHNSON              JULY 1976

  NASA AMES RESEARCH CENTER, MOFFETT FIELD, CALIF.

* * * * * * * * * * * * * * * * * * * * * * * * * *
```

GOLAND WING FLUTTER ANALYSIS

** ARRAY OF REDUCED FREQUENCIES **

.400000

REF. CHORD = 72.00000 REF. SEMI-SPAN = 240.00000 REF. AREA = 34560.00000

MACH NO. = 0.00000 BETA = 1.00000

*** PANEL NO. 1 INPUT VALUES ***

X1 = 0.000000 X2 = 72.000000 Y1 = 0.000000 Z1 = 0.000000

X3 = 0.000000 X4 = 72.000000 Y2 = 240.000000 Z2 = 0.000000

NC = 3 NS = 10 NDELT = 1 NO. OF PANELS = 1

3 CHORDWISE DIVISIONS FOR PANEL 1

0.	.50000000E+00	.10000000E+01

10 SPANWISE DIVISIONS FOR PANEL 1

0.	.12500000E+00	.25000000E+00	.37500000E+00	.50000000E+00	.62500000E+00
.75000000E+00	.87500000E+00	.95833300E+00	.10000000E+01		

30 *** XI ELEMENTS FOR PANEL 1***

0.	.36000000E+02	.72000000E+02	0.	.36000000E+02	.72000000E+02
0.	.36000000E+02	.72000000E+02	0.	.36000000E+02	.72000000E+02
0.	.36000000E+02	.72000000E+02	0.	.36000000E+02	.72000000E+02
0.	.36000000E+02	.72000000E+02	0.	.36000000E+02	.72000000E+02
0.	.36000000E+02	.72000000E+02	0.	.36000000E+02	.72000000E+02

30 *** ETA ELEMENTS FOR PANEL 1***

0.	0.	0.	.30000000E+02	.30000000E+02	.30000000E+02
.60000000E+02	.60000000E+02	.60000000E+02	.90000000E+02	.90000000E+02	.90000000E+02
.12000000E+03	.12000000E+03	.12000000E+03	.15000000E+03	.15000000E+03	.15000000E+03
.18000000E+03	.18000000E+03	.18000000E+03	.21000000E+03	.21000000E+03	.21000000E+03
.22999992E+03	.22999992E+03	.22999992E+03	.24000000E+03	.24000000E+03	.24000000E+03

30 ZEE ELEMENTS FOR PANEL NO. 1

ADDITIONAL OUTPUT OMITTED

STABILITY DERIVATIVES FOR THE 9 STRIPS

PLUNGING DERIVATIVES

STRIP	YOS	NORMAL SPAN LOAD		NORMAL SPAN MOMENT		NORMAL AILERON HINGE MOMENT		NORMAL TAB HINGE MOMENT	
1	.0625	-3.84669	-.41768	.32095	-.17183	0.00000	0.00000	0.00000	0.0000
2	.1875	-3.83344	-.42880	.32064	-.17091	0.00000	0.00000	0.00000	0.0000
3	.3125	-3.80285	-.45162	.31986	-.16897	0.00000	0.00000	0.00000	0.0000
4	.4375	-3.74522	-.48699	.31828	-.16576	0.00000	0.00000	0.00000	0.0000
5	.5625	-3.64117	-.53482	.31519	-.16087	0.00000	0.00000	0.00000	0.0000
6	.6875	-3.45222	-.59094	.30900	-.15355	0.00000	0.00000	0.00000	0.0000
7	.8125	-3.09784	-.63654	.29520	-.14196	0.00000	0.00000	0.00000	0.0000
8	.9167	-2.46180	-.60351	.25906	-.12211	0.00000	0.00000	0.00000	0.0000
9	.9792	-1.57848	-.43311	.18039	-.08393	0.00000	0.00000	0.00000	0.0000

C-SUB-L OR Y	-3.44834896	-.51176219	0.00000000	0.00000000
C-SUB-M OR N	-.83396067	-.32428550	0.00000000	0.00000000
C-SUB-M OR N-DEL-A	0.00000000	0.00000000	0.00300000	0.00000000
C-SUM-M OR N-DEL-T	0.00000000	0.00000000	0.00000000	0.00000000
C-SUB-SCRIPT-L	0.00000000	0.00000000		

PITCHING DERIVATIVES

STRIP	YOS	NORMAL SPAN LOAD		NORMAL SPAN MOMENT		NORMAL AILERON HINGE MOMENT		NORMAL TAB HINGE MOMENT	
1	.0625	-3.87086	-1.72064	.37339	-.29289	0.00000	0.00000	0.00000	0.00000
2	.1875	-3.85387	-1.72792	.37279	-.29203	0.00000	0.00000	0.00000	0.00000
3	.3125	-3.81555	-1.74184	.37141	-.29024	0.00000	0.00000	0.00000	0.00000
4	.4375	-3.74576	-1.76032	.36884	-.28734	0.00000	0.00000	0.00000	0.00000
5	.5625	-3.62478	-1.77746	.36426	-.28298	0.00000	0.00000	0.00000	0.00000
6	.6875	-3.41457	-1.77733	.35592	-.27628	0.00000	0.00000	0.00000	0.00000
7	.8125	-3.03805	-1.71551	.33884	-.26406	0.00000	0.00000	0.00000	0.00000
8	.9167	-2.39369	-1.48026	.29703	-.23435	0.00000	0.00000	0.00000	0.00000
9	.9792	-1.52595	-1.00642	.20688	-.16444	0.00000	0.00000	0.00000	0.00000

C-SUB-L OR Y	-3.43348492	-1.69291530	0.00000000	0.00000000
C-SUB-M OR N	-.78149973	-.83326968	0.00000000	0.00000000
C-SUB-M OR N-DEL-A	0.00000000	0.00000000	0.00000000	0.00000000
C-SUM-M OR N-DEL-T	0.00000000	0.00000000	0.00000000	0.00000000
C-SUB-SCRIPT-L	0.00000000	0.00000000		

PRESSURE COEFFICIENTS FOR MODE NO. 1

STRIP NO.	XOC	C-SUB-P	
1	.1250	-.00216055	-.06469893
1	.6250	.01144241	-.02078304
2	.1250	-.00197410	-.06451118
2	.6250	.01150291	-.02067634
3	.1250	-.00158839	-.06407571
3	.6250	.01162444	-.02043216
4	.1250	-.00098209	-.06324980
4	.6250	.01180402	-.01997735
5	.1250	-.00013784	-.06174005
5	.6250	.01202273	-.01916688
6	.1250	.00092340	-.05899584
6	.6250	.01220850	-.01772024
7	.1250	.00203641	-.05373628
7	.6250	.01210897	-.01510458
8	.1250	.00248564	-.04379054
8	.6250	.01092577	-.01091695
9	.1250	.00194846	-.02871292
9	.6250	.00767611	-.00636440

PRESSURE COEFFICIENTS FOR MODE NO. 2

[illegible]P NO.	XOC	C-SUB-P	
1	.1250	-6.06118109	-.85881988
1	.6250	-1.68053809	-2.59246845
2	.1250	-6.03872976	-.87081777
2	.6250	-1.66981027	-2.58501397
3	.1250	-5.98799639	-.89440152
3	.6250	-1.64310504	-2.58927363
4	.1250	-5.89536097	-.92781627
4	.6250	-1.59615691	-2.59282140
5	.1250	-5.73429186	-.96548933
5	.6250	-1.51526464	-2.58943133
6	.1250	-5.45205670	-.99213525
6	.6250	-1.37628043	-2.56251550
7	.1250	-4.94024193	-.96804869
7	.6250	-1.13585651	-2.46297629
8	.1250	-4.01267299	-.80931309
8	.6250	-.77470418	-2.15119798
9	.1250	-2.62781055	-.52981680
9	.6250	-.42409557	-1.46302268

EIGENVALUES

2322.
9163.
.6034E+05
.1226E+06
.2087E+06
.4084E+06
.7456E+06
.9209E+06
.1214E+07
.1762E+07

EIGENVECTORS

DOF	1	2	3	4	5	6	7	8
1	-.1125E-01	.5258E-02	.3227E-01	.5578E-01	.4878E-01	-.7901E-02	.2040E-01	.1912
2	-.1477E-02	.6963E-03	.4058E-02	.7077E-02	.5991E-0[illegible]	-.1165E-02	.21[illegible]9E-02	.2705E-01
3	.8841E-03	.[illegible]961E-02	-.1388E-01	.7[illegible]7E-0[illegible]	-.2645E-01	-.4783E-01	-.3[illegible]40E-01	[illegible]E-01
4	-.9570E-01	.4598E-01	.2254	.4076	.3034	-.8773E-01	.7264E-[illegible]	1.0[illegible]
5	-.4031E-02	.1907E-02	.7893E-02	.1462E-01	.9223E-0[illegible]	[illegible]	.7134E-03	[illegible]

7	-.2476	.1236	.4557	.8439	.5206	-.1988	.8263E-01	1.399
8	-.6556E-02	.3150E-02	.6739E-02	.1330E-01	.4387E-02	-.2409E-02	.6866E-03	-.794[illegible]E-02
9	.4338E-02	.3327E-01	-.4857E-01	.5777E-01	-.2794E-01	.1321E-01	.7879E-01	.2000E-01
10	-.4535	.2331	.5955	1.124	.5448	-.1800	.1199	.6260
11	-.7572E-02	.4114E-02	.2184E-02	.4329E-02	-.2686E-02	.4011E-02	.1063E-02	-.3[illegible]55E-0[illegible]
12	.5929E-02	.4454E-01	-.4312E-01	.4017E-01	.3234E-01	.7053E-01	.1925E-01	.3292E-01
13	-.6974	.3682	.5737	1.059	.3822	.1493E-01	.8724E-01	-.6143
14	-.8616E-02	.4852E-02	-.3677E-02	-.9052E-02	-.7605E-02	.7800E-02	-.3872E-02	-.3647E-01
15	.7348E-02	.5395E-01	-.2180E-01	.1171E-01	.7257E-01	.1407E-01	-.7998E-01	-.1134E-01
16	-.9663	.5217	.3821	.5828	.1203	.1995	-.7961E-01	-1.227
17	-.9246E-02	.5343E-02	-.8838E-02	-.2229E-01	-.9431E-02	.3139E-02	-.5604E-02	-.8179E-03
18	.8521E-02	.6120E-01	.9029E-02	-.1227E-01	.5376E-01	-.6370E-01	.1007E-01	-.5395E-01
19	-1.249	.6864	.6375E-01	-.2346	-.1675	.1611	-.1463	-.6073
20	-.9547E-02	.5600E-02	-.1201E-01	-.3126E-01	-.9637E-02	-.5587E-02	.2213E-02	.4641E-01
21	.9375E-02	.6608E-01	.3930E-01	-.2332E-01	-.1154E-01	-.3779E-01	.7355E-01	-.1100E-01
22	-1.489	.8276	-.2514	-1.062	-.4081	-.4035E-01	-.4796E-02	.6749
23	-.9628E-02	.5674E-02	-.1299E-01	-.3429E-01	-.9617E-02	-.9768E-02	.8231E-02	.5888E-01
24	.9790E-02	.6827E-01	.5633E-01	-.2410E-01	-.6233E-01	.3780E-01	-.1966E-01	.2919E-01
25	-1.633	.9127	-.4474	-1.580	-.5523	-.1921	.1274	1.583
26	-.9635E-02	.5681E-02	-.1310E-01	-.3463E-01	-.9621E-02	-.1029E-01	.9108E-02	.6133E-01
27	.9891E-02	.6877E-01	.6068E-01	-.2340E-01	-.7640E-01	.6532E-01	-.6822E-01	.3459E-01
28	-1.601	.9411	-.5129	-1.753	-.6005	-.2436	.1730	1.889
29	-.9636E-02	.5681E-02	-.1310E-01	-.3464E-01	-.9622E-02	-.1030E-01	.9123E-02	.6138E-01
30	.9897E-02	.6880E-01	.6098E-01	-.2332E-01	-.7738E-01	.6738E-01	-.7212E-01	.3461E-01

DOF	9	10
1	-.4077E-01	.8834E-02
2	-.4477E-02	.1541E-02
3	.7749E-01	.9847E-01
4	-.1708	.6176E-01
5	-.3123E-02	.1231E-02
6	-.6336E-02	-.5747E-01
7	-.2199	.2167E-01
8	-.1513E-03	-.4248E-02
9	-.9038E-01	-.2087E-02
10	-.1363	-.6564E-01
11	.9674E-02	-.2805E-03
12	.7809E-01	.6212E-01
13	.1370	-.3502E-01
14	.9032E-02	.5171E-03
15	-.805[illegible]E-02	-.1007
16	.2541	-.2799E-01
17	-.2504E-02	.1726E-02
18	-.6252E-01	.9576E-01
19	.4458E-01	.6248E-01
20	-.8934E-02	.2043E-02
21	.[illegible]640E-01	-.5332E-01
22	-.1523	.3365E-01
23	-.6658E-02	-.4167E-02
24	-.1476E-02	-.2303E-01
25	-.2462	-.4264E-01
26	-.6049E-02	-.5573E-02
27	-.7495E-01	.5570E-01
28	-.2764	-.7061E-01
29	-.6038E-02	-.5602E-02
30	-.8131E-01	.6343E-01

1	.1125E-01	-0.	-.5258E-02	-0.	-.3227E-01	-0.	-.5578E-01	-0.
2	.984'E-03	0.	.6961E-02	0.	-.1388E-01	0.	.2087E-01	0.
3	.9520E-01	-0.	-.4598E-01	-0.	-.2254	-0.	-.4036	-0.
4	.2638E-02	0.	.2057E-01	0.	-.3732E-01	0.	.5171E-01	0.
5	.2478	-0.	-.1236	-0.	-.4557	-0.	-.8439	-0.
6	.4338E-02	0.	.3327E-01	0.	-.4857E-01	0.	.5777E-01	0.
7	.4535	-0.	-.2331	-0.	-.5955	-0.	-1.124	-0.
8	.5929E-02	0.	.4454E-01	0.	-.4312E-01	0.	.4017E-01	0.
9	.6974	-0.	-.3682	-0.	-.5737	-0.	-1.059	-0.
10	.7348E-02	0.	.5395E-01	0.	-.2180E-01	0.	.1171E-01	0.
11	.9663	-0.	-.5217	-0.	-.[illegible]	-0.	-.5828	-0.

12	.8821E-02	0.	.6120E-01	0.	.5328E-01	0.	-.1227E-01	
13	1.249	-0.	-.6864	-0.	-.6375E-01	-0.	.2246	
14	.9375E-02	0.	.6608E-01	0.	.3930E-01	0.	-.2332E-01	0.
15	1.489	-0.	-.8276	-0.	.2514	-0.	1.062	-0.
16	.9790E-02	0.	.6827E-01	0.	.5633E-01	0.	-.2410E-01	0.
17	1.633	-0.	-.9127	-0.	.4474	-0.	1.580	-0.
18	.9891E-02	0.	.6877E-01	0.	.6068E-01	0.	-.2340E-01	0.

GENERALIZED AERODYNAMICS

-.6353E-01	-.2263	-.8143	-.4788	-.6236E-01	-.8100E-01	-.5620E-01	.7814E-01
-.9359E-01	-.4750E-01	.1201	.5772E-01	-.7050E-01	-.8063E-01	.5178E-01	.3086E-01
-.1265	-.7426E-01	.2753E-01	.5405E-01	.1358	.1447	.9083	-.1502
.2142E-01	.3122E-01	.8131E-01	-.1340	.1037	-.3072E-01	-.1243	.1439E-01
.7795E-01	.1561E-01	-.3746E-01	-.1394E-01	.1304	-.6771E-02	-.3401E-01	-.1803E-01
.4439E-01	.7737E-01	.3233	.1311	.4627E-01	-.3202	.3932E-01	.2470
.9698E-01	.1185	-.5190E-01	.4742E-02	.2517E-01	-.2529E-01	-.2414E-01	.6684E-01
.5260E-01	-.3152E-01	-.4612E-02	.3493E-01	.7047E-01	.5679E-01	.4995	.1056
-.6346	-.2179	.6598	-.8008E-01	.1869	.1802	-.7343E-01	.4659E-01
-.8681E-01	-.1152E-01	.1456	-.1324	.9993E-02	-.2126E-01	.1139	.7063E-02
.3609E-01	.2509E-01	.2362	.2934E-01	-.2578	-.1684	.1921	.8355E-01
.3831	-.2306	.4024E-01	-.3400E-01	-.5436E-01	.1352E-01	-.3226E-01	.1373
-.2926E-01	.5527E-03	.4671E-01	.1171E-01	-.1205E-01	.4602E-02	-.4633E-01	.4337E-01
.6270E-01	.4600E-01	-.8821E-01	-.1935E-01	.9677E-01	-.1498E-01	.3217	-.3054
.7183E-01	-.4700E-01	.1125E-01	-.2759E-01	-.7888E-02	.1192E-01	-.6512E-01	.4379E-01
.8589E-02	.2045E-02	.3902E-01	-.1811E-01	-.1321E-01	-.5969E-01	.2796E-01	.3913E-01
-.2204E-01	.1005E-01	.7238E-01	-.2616E-01	.3760	-.3329	.4681E-02	.2420E-01
-.5662E-01	.3938E-01	.3500E-02	-.6973E-03	.1706E-01	-.1762E-01	.6234E-01	-.1223E-01
-.4013	-.1774	.5147	.1711	-.5056	-.2731	.8140E-01	.6371E-01
-.5437E-01	.2255E-01	.4676	-.6802E-02	.8200E-01	-.5610E-01	.8510E-01	-.1040E-02
.7247E-02	.3862E-02	.3058E-01	-.3847E-01	.6904E-01	.1723E-01	-.7128E-01	-.3420E-01
.4677E-01	.8613E-01	-.2010E-01	-.1295E-01	-.5378E-01	.3385E-01	.2497E-02	-.7019E-01
.4476	-.3761	.5602E-01	-.4911E-01	-.1949E-02	-.2200E-02	-.5783E-02	.6748E-02
-.3435E-01	.2689E-01	.3213E-01	-.2197E-01	-.2584E-01	-.1920E-01	-.7000E-01	.2455E-01
-.4764E-02	.5393E-02	.7553E-02	-.6802E-03	.6766E-01	-.4595E-01	.5532	-.4559

EIGENMATRIX

.4034E-03	-.9749E-04	-.3508E-03	-.2062E-03	-.2686E-04	-.3489E-04	-.2421E-04	.3366E-04
-.4247E-04	-.2046E-04	.5174E-04	.2486E-04	-.3037E-04	-.3473E-04	.2230E-04	.1321E-04
-.5448E-04	-.3290E-04	.1186E-04	.2328E-04	.1482E-04	.1579E-04	.2083E-03	-.1726E-04
.2338E-05	.3408E-05	.8874E-05	-.1463E-04	.1132E-04	-.3352E-05	-.1357E-04	.1571E-05
.8507E-05	.1704E-05	-.4083E-05	-.1521E-05	.1423E-04	-.7390E-06	-.3712E-05	-.1068E-05
.7356E-06	.1282E-05	.5358E-05	.2172E-05	.1734E-04	-.5307E-05	.6516E-06	.4093E-05
.1607E-05	.1964E-05	-.8601E-06	.7959E-07	.4171E-06	-.4191E-06	-.4000E-06	.1108E-05
.8717E-06	-.5224E-06	-.7643E-07	.5789E-06	.5750E-06	.4633E-06	.4075E-05	.8617E-06
-.5178E-05	-.1778E-05	.1354E-04	-.6534E-06	.1525E-05	.1470E-05	-.5991E-06	.3802E-06
-.7083E-06	-.9393E-07	.1188E-05	-.1081E-05	.8154E-07	-.1734E-06	.9297E-06	.5763E-07
.1730E-06	.1202E-06	.1132E-05	.1406E-06	-.1235E-05	-.8072E-06	.9207E-06	.4004E-06
.6628E-05	-.1105E-05	.1929E-06	-.1629E-06	-.2605E-06	.6081E-07	-.1546E-06	.6578E-06
-.1402E-06	.2649E-08	.2239E-06	.5610E-07	-.2951E-07	.1127E-07	-.1134E-06	.1062E-06
.1535E-06	.1128E-06	-.2160E-06	-.4739E-07	.2370E-06	-.3669E-07	.3237E-05	-.7479E-06
.1759E-06	-.1151E-06	.2754E-07	-.2143E-06	-.1932E-07	.2920E-07	-.1595E-06	.1072E-06
.1152E-07	.3816E-08	.5233E-07	-.2428E-07	-.1771E-07	-.8005E-07	.3750E-07	.5255E-07
-.2957E-07	.1348E-07	.9707E-07	-.3508E-07	.1845E-05	-.4465E-06	.6278E-08	.5246E-07
-.7594E-07	.5281E-07	.4694E-08	-.9352E-09	.1853E-07	-.1913E-07	.6768E-07	-.1328E-07
-.4358E-06	-.1926E-06	.5589E-06	.1258E-06	-.5490E-08	-.2965E-06	.8840E-07	.6919E-07
-.5904E-07	.2448E-07	.1594E-05	-.7451E-08	.8905E-07	-.6092E-07	.7063E-07	-.1129E-08
.5970E-08	.3182E-08	.2519E-07	-.3169E-07	.5687E-07	.1426E-07	-.5072E-07	-.2817E-07
.3853E-07	.7095E-07	-.1656E-07	-.1067E-07	-.4430E-07	.2789E-07	.2057E-08	-.5782E-07
.1188E-05	-.3099E-06	.4615E-07	-.4045E-07	-.1106E-06	-.1248E-08	-.3282E-08	.4965E-08
-.1949E-07	.1526E-07	.1823E-07	-.1247E-07	-.1467E-07	-.1089E-07	-.3373E-07	.1393E-07
-.2704E-08	.3061E-08	.4287E-08	-.3860E-09	.3040E-07	-.2608E-07	.8815E-06	-.2588E-06

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8	-.6579E-01	-.2161E-01	[illegible]E-02	.1064E-01	-.1577E-01	-.4337E-02	[illegible]	[illegible]
9	.5988E-02	.6003E-02	-.3391E-05	-.1545E-02	-.9077E-02	[illegible]02E-02	-.2170[illegible]	[illegible]
10	-.2179E-02	-.3150E-03	-.1964E-02	-.7814E-03	.5255E-05	.1176E-03	-.2251E-02	-.2234E-03

VIB. MODE	9		10	
1	.8034E-02	.1162E-01	.2548E-02	.1439E-02
2	-.1061E-01	-.5790E-02	-.1990E-02	.5279E-03
3	-.6194E-02	.2568E-02	-.2213E-04	.1182E-02
4	-.4703E-02	-.2301E-02	.3741E-02	.8245E-04
5	.8389E-02	.5772E-02	.1894E-02	-.9568E-03
6	-.2796E-02	-.3896E-02	-.2995E-02	.3339E-02
7	.2492E-01	.9686E-03	.7179E-03	-.1836E-02
8	-.8828E-02	.3612E-01	.3371E-02	-.1914E-02
9	.1436	.5824E-01	.7065E-02	-.7474E-02
10	.2247E-01	-.1698E-02	-.4978E-01	.1647E-01

XFLUTGL.$001765 LINES PRINTED.   (REMOTE RM)

$$$

VIII. Concluding Remarks

The preceding has pointed out the main features of the FLUT program. It has been successfully applied to several example problems and to the analysis of the Lockheed Subsonic Oblique Wing Transport Concept (Ref. 9). Recommendations for enhancements and conclusions reached during its use can therefore be made here. Experience with the use of this program has shown that, in performing a flutter analysis, it is extremely helpful to use two separate programs to do the same analysis. While it may seem like a luxury to do the same thing twice, in practice one program serves as a check on the other, thereby pointing out errors in input and finally adding confidence to the results when matched results are obtained from the two programs. For instance, while using FLUT, it was found useful to perform the same analysis using NASTRAN (Ref. 11).

A minor recommendation is that the unsteady aerodynamics package is several years old and could probably be improved upon. There may, in fact, be newer packages that are more suitable. Alternatively, it may be worthwhile to perform some re-programming of the algorithm. The package was designed for a machine with limited core space and makes liberal use of tapes for intermediate storage. This results in awkward data handling that could certainly be streamlined. The recommendation is minor in the sense that it is not clear that the benefits would justify the expense. FLUT has already been developed so that it minimizes the calculations in this most inefficient part.

A more ambitious recommendation is that this flutter analysis program has many features which make it attractive for use as a design tool. With the major addition of flutter derivative calculations, the program could be used in optimization studies with constraints on the aeroelastic stability. References 12 and 13 present various ways that the program could be used to achieve this.

Conclusions regarding the use of aerodynamic interpolation and the reduction of rigid body modes as well as other concepts are made throughout the text. Aerodynamic influence coefficients are felt to be of sufficient merit to reiterate their usefulness a final time. Finally, the program has been shown to be efficient and reliable in its execution.

ACKNOWLEDGMENTS

The research documented in this report was accomplished while the author held a National Research Council Postdoctoral Resident Research Associateship supported by the Ames Research Center, Moffett Field, California. The author wishes to express his appreciation to M. J. Rutkowski, who carried the burden of preparing the report for publication and to G. N. Vanderplaats who served as the author's research advisor.

References

1. Bisplinghoff, R.L., and Ashley, H.: Principles of Aeroelasticity, Dover Publications, New York, 1969.

2. Fung, Y.C.: An Introduction to the Theory of Aeroelasticity, Dover Publications, New York, 1969.

3. Giesing, J.P., Kalman, T.P. and Rodden, W.P.: Subsonic Unsteady Aerodynamics for General Configurations, Part 1, Vol. I - Direct Application of the Nonplanar Doublet-Lattice Method. AFFDL-TR-71-5, Part 1, Vol.I,Nov.1971.

4. Ibid: Subsonic Unsteady Aerodynamics for General Configuration, Part 1, Vol. II - Computer Program H7WC. AFFDL-TR-71-5, Part 1, Vol. II, Nov.1971.

5. Crittenden, J.B. and Weisshaar, T.A.: The AFLUT Computer Code - Flutter Analysis of Asymmetric and Bilaterally Symmetric Wings. Virginia Polytechnic Institute and State University, September 1975.

6. Johnson, E. H.: PASS - A Computer Program for Preliminary Aircraft Structural Synthesis. NASA TM-73,216, June 1977.

7. Cross, A.K. and Albano, E.A.: Computer Techniques for the Rapid Flutter Clearance of Aircraft Carrying External Stores, Part 1. Perturbation Theory and Applications. AFFDL-TR-72-114, Part 1, February 1973.

8. Garbow, B.S. and Dongarra, J.T.: Path Chart and Documentation for the EISPACK Package of Matrix Eigensystem Routines. TM 250, Applied Mathematics Division, Argonne National Laboratory, Argonne, Illinois, August 1975, (updated).

9. Bradley, E.S., et. al.: An Analytical Study for Subsonic Oblique Wing Transport Concept: NASA CR-137896 prepared by Lockheed Georgia Company, June 1976.

10. Goland, M.: The Flutter of a Uniform Cantilever Wing, Journal of Applied Mechanics, December 1945, pp A197-A208.

11. Rodden, W.P., Harder, R.L., Bellinger, E.D.: Aeroelastic Addition to NASTRAN, Second Annual NASTRAN Users Conference, Pasadena, CA, March 1976.

12. O'Connell, R.F., Hassig, H.T. and Radovich, N.A.: Study of Flutter Related Computational Procedures for Minimum Weight Structural Sizing of Advanced Aircraft. NASA CR-2607, March 1976.

13. Rizzi, P.: Optimization of Structures with Complex Constraints via a General Optimality Criteria Method: PhD dissertation, Dept. of Aeronautics and Astronautics, Stanford University, Stanford, CA, July 1976.

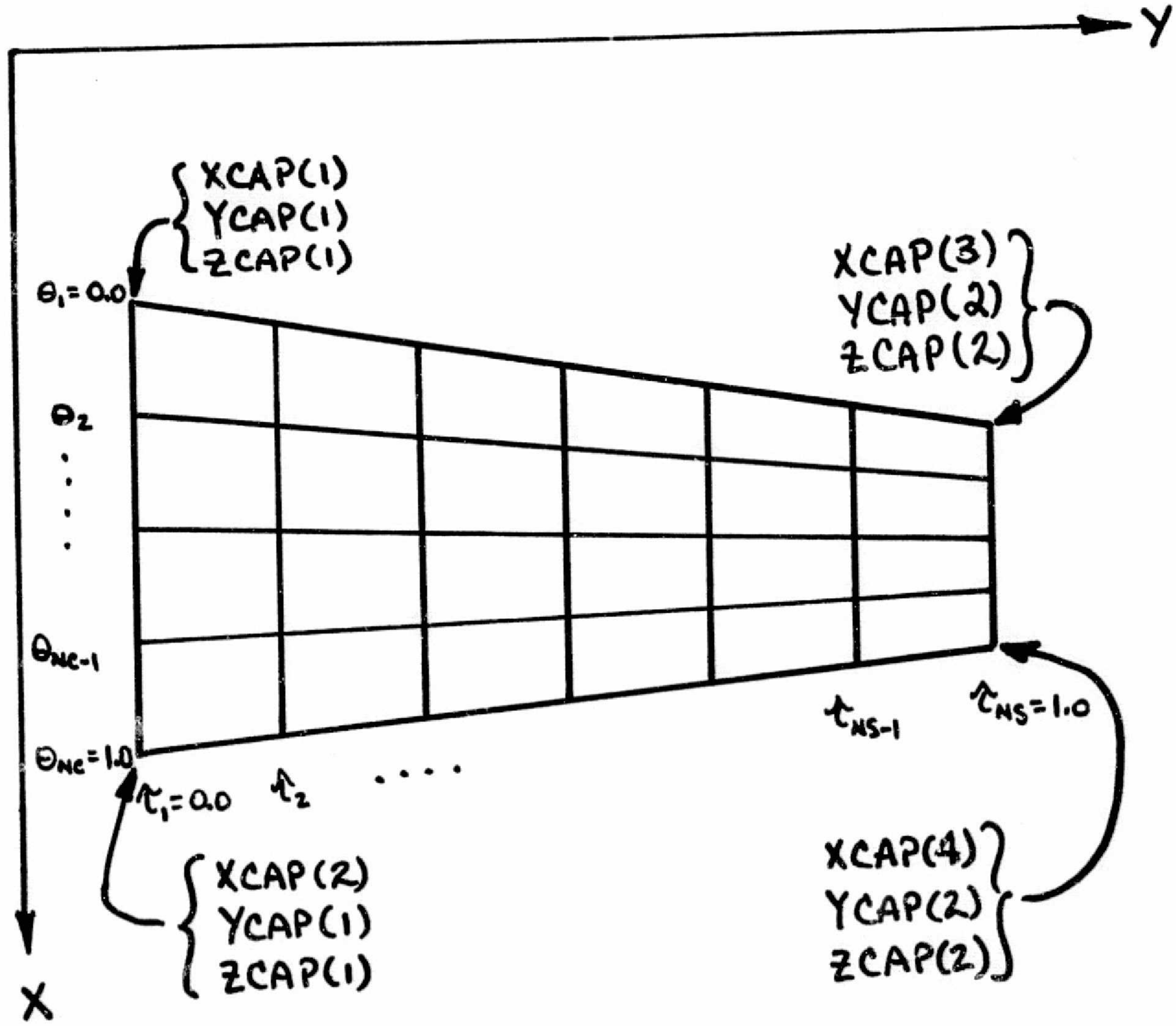


FIGURE 1.– WING PLANFORM NOTATION

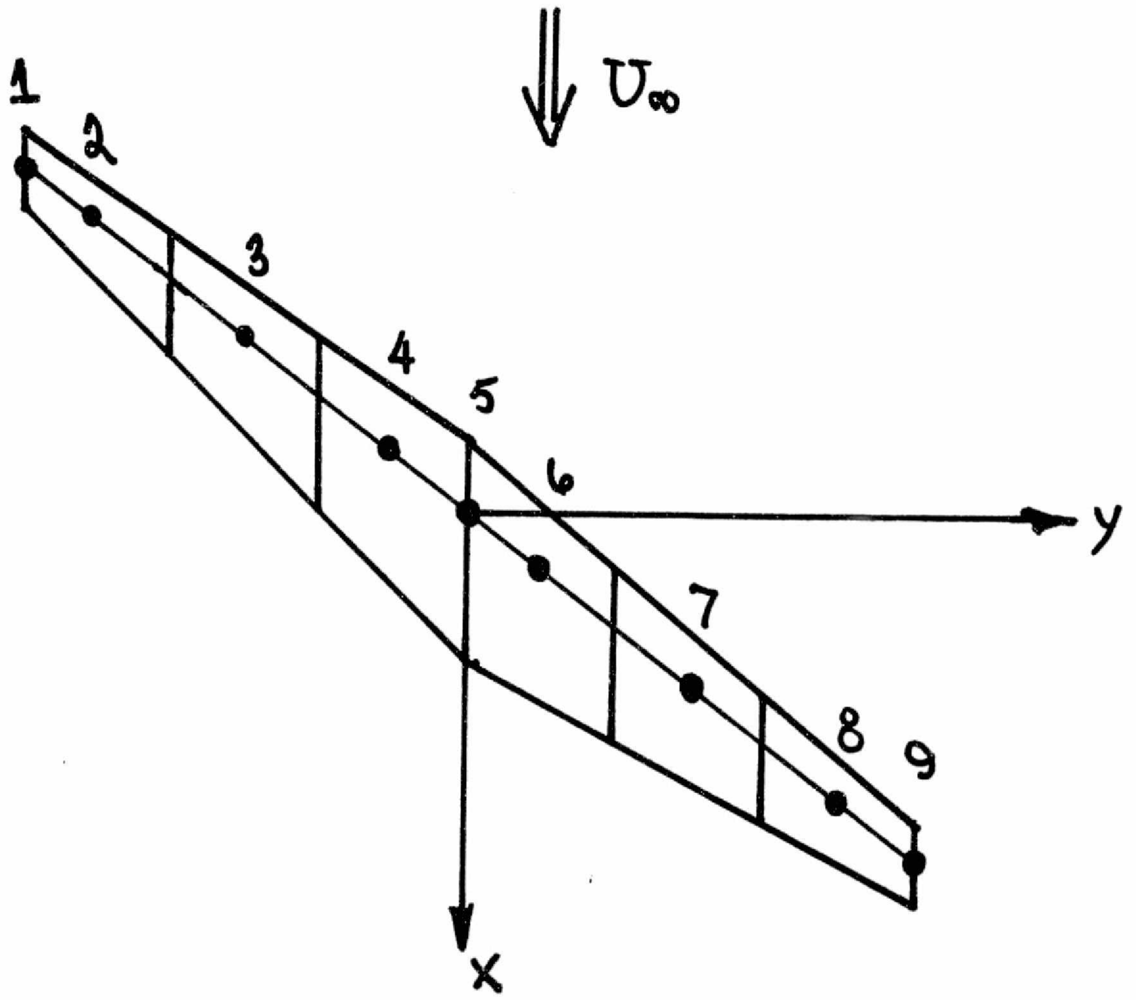


FIGURE 2. - SIMPLIFIED MODEL OF AN OBLIQUE WING

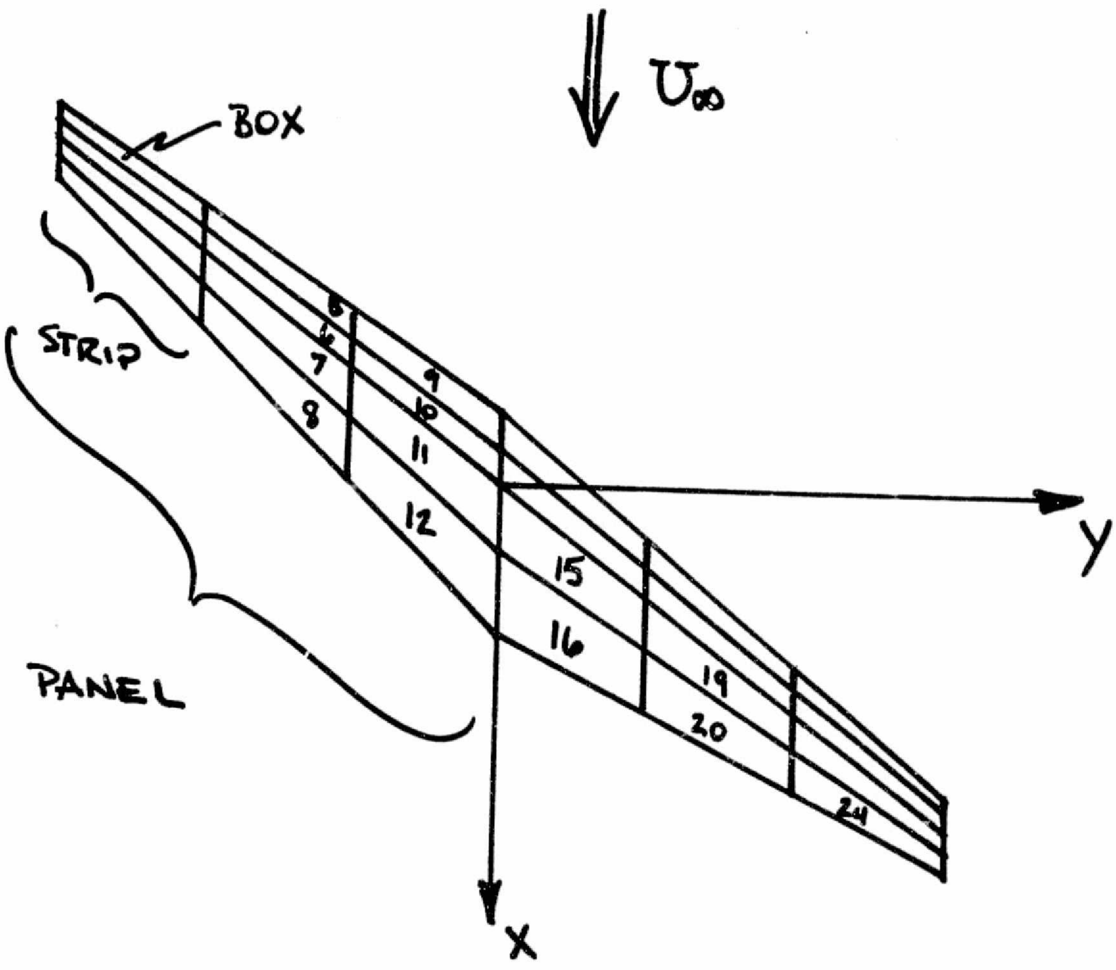


FIGURE 3.– AERODYNAMIC MODELLING

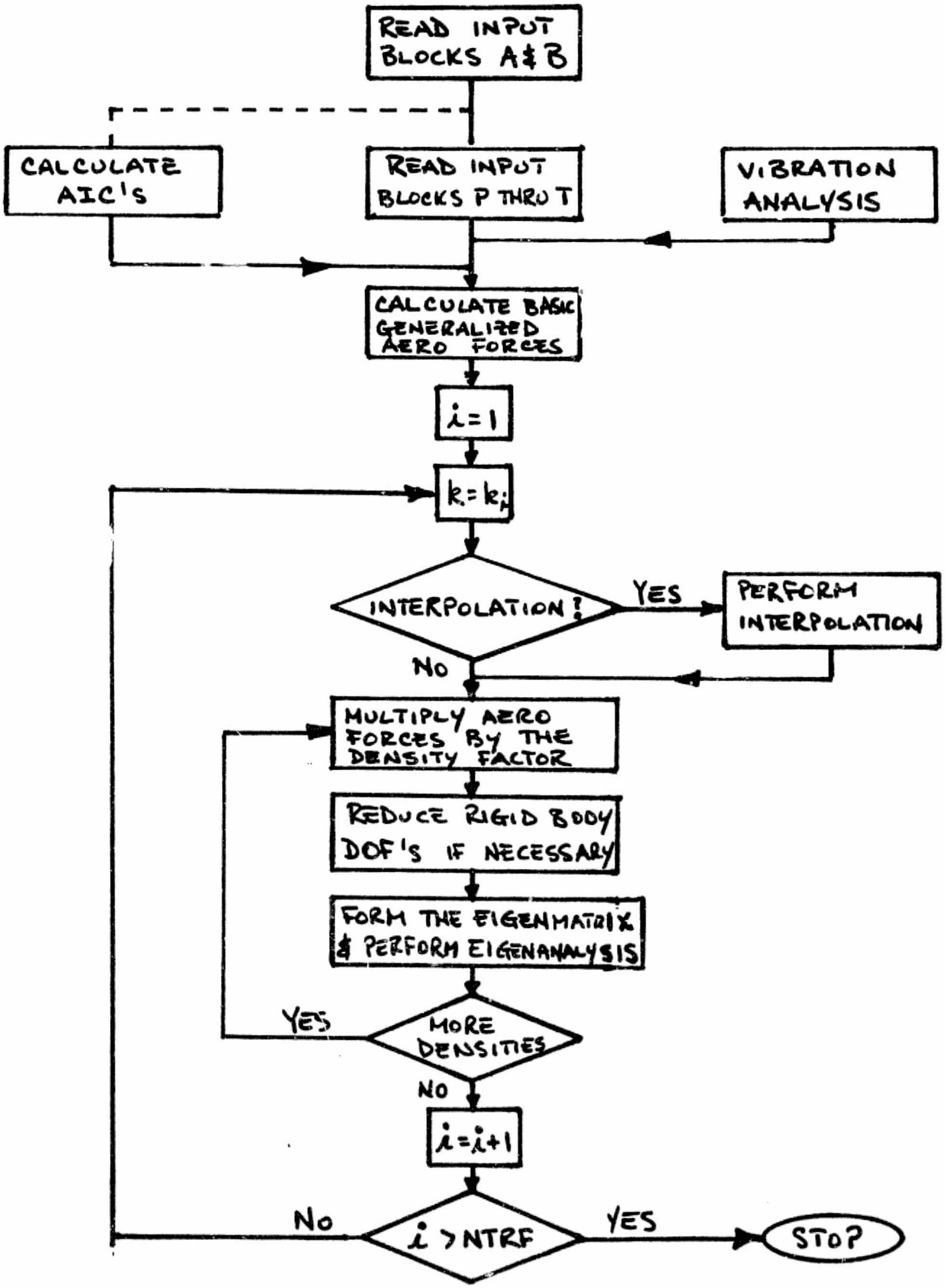


FIGURE 4.- BLOCK DIAGRAM FOR FLUT

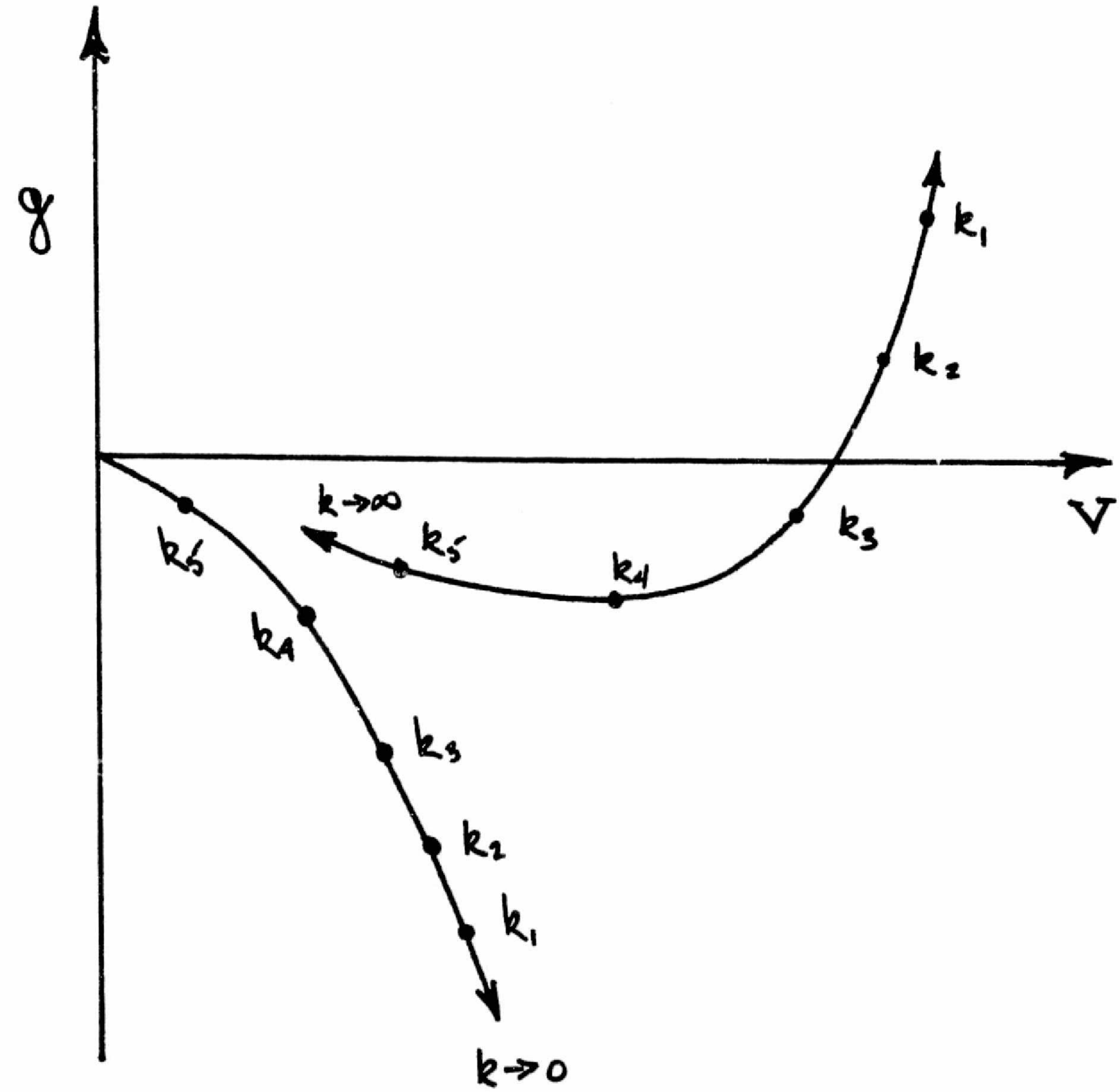


FIGURE 5.— REPRESENTATIVE V-g DIAGRAM

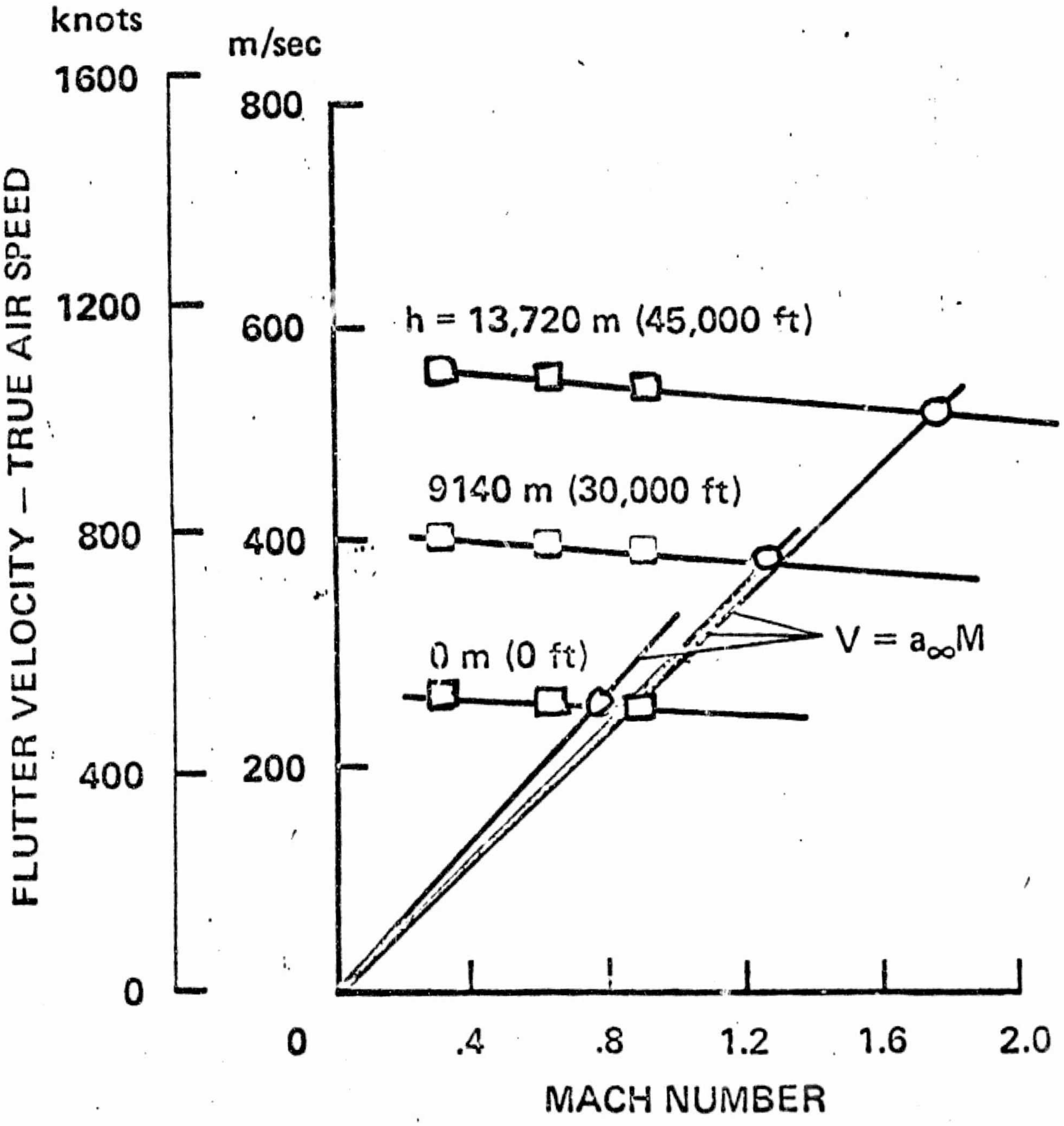
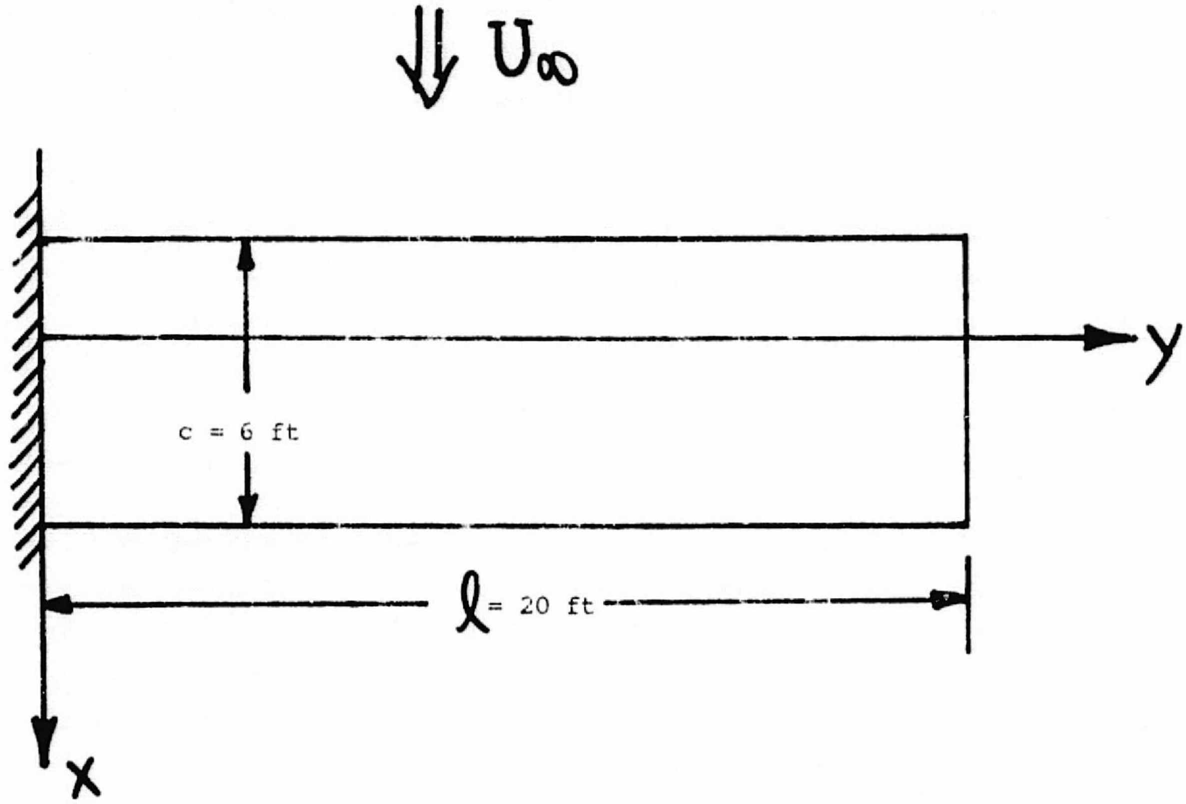


FIGURE 6.— MATCHED FLUTTER POINT AT THREE ALTITUDES



mass/unit length = 746 slugs/ft

mass moment of inertia/unit length* = 1.678 slug-ft^2/ft

flexural stiffness = 2.368 x 10^7 lb/ft^2

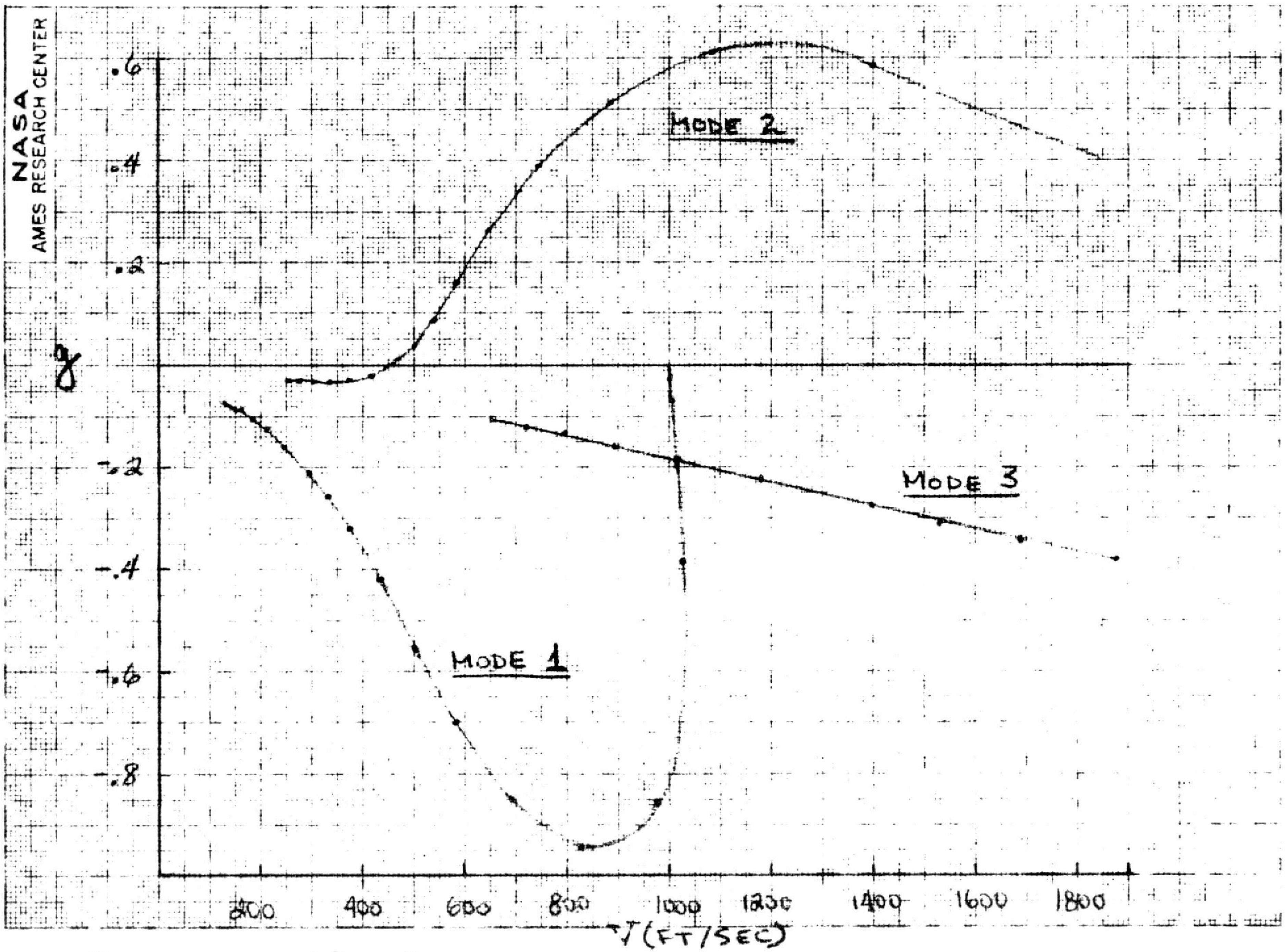
torsional stiffness = 2.389 x 10^6 lb/ft^2

The elastic axis is 2 feet aft of the leading edge.

The center of gravity is 2.6 feet aft of the leading edge.

*At the chordwise location of the center of gravity.

Figure 7.- Goland's Uniform Cantilever Wing.



54

FIGURE 8.— V-g DIAGRAM RESULTS FOR GOLAND WING USING 12 BOXES

1. Report No. NASA TM-73,217	2. Government Accession No.	3. Recipient's Catalog No
4. Title and Subtitle FLUT — A PROGRAM FOR AEROELASTIC STABILITY ANALYSIS		5. Report Date
		6. Performing Organization Code
7. Author(s) Erwin H. Johnson*		8. Performing Organization Report No.
		10. Work Unit No. A-6955
9. Performing Organization Name and Address Ames Research Center Moffett Field, California 94035		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

*National Research Council Postdoctoral Research Associate

16. Abstract

FLUT is a computer program that can be used to evaluate the aeroelastic stability of aircraft structures in subsonic flow. The algorithm synthesizes data from a structural vibration analysis with an unsteady aerodynamics analysis and then performs a complex eigenvalue analysis to assess the system stability. This document is divided into two main parts. The first of these describes the theoretical basis of the program. Special emphasis in this section is placed on some innovative techniques which improve the efficiency of the analysis.

The second section provides the user information needed to efficiently and successfully utilize the program. In addition to identifying the required input, this section summarizes the flow of the program execution and points out some possible sources of difficulty. The use of the program is demonstrated with a listing of the input and output for a simple example.

17. Key Words (Suggested by Author(s)) Flutter analysis Aeroelastic stability	18. Distribution Statement Unlimited STAR Category — 05

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	66	$4.25

*For sale by the National Technical Information Service, Springfield, Virginia 22161