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DISPLACEMENT COEFFICIENTS ALONG THE INNER BOUNDARIES OF RADIALLY CRACKED RING SEGMENTS SUBJECT TO FORCES AND COUPLES

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16. Abstract

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DISPLACEMENT COEFFICIENTS ALONG THE INNER BOUNDARIES OF RADIALLY CRACKED RING SEGMENTS SUBJECT TO FORCES AND COUPLES

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ABSTRACT

Displacement results of plane boundary collocation analysis are given for various locations on the inner boundaries of radially cracked ring segments (C-shaped specimens) subject to two complementary types of loading. Results are presented for ratios of outer to inner radius R_o/R_i in the range of 1.1 to 2.5, and ratios a/W in the range 0.1 to 0.8 where a is the crack length for a specimen of wall thickness W. By combination of these results the resultant displacement coefficient Δ or the corresponding influence coefficient $(E'Bv/P)$, can be obtained for any practical load line location of a pin loaded specimen.

SYMBOLS

a	crack length
B	specimen depth
E	Young's modulus
$E' = E/(1 - \nu^2)$	for plane strain conditions
$E' = E$	for plane stress conditions
H	distance between displacement gage points
K_I	Stress Intensity Factor
L	load line location
M	nominal net section bending moment

P	applied end force
R_i	ring segment inner radius
R_o	ring segment outer radius
$v = v_p + v_m$	total displacement across gage distance H
v_p	displacement due to nominal uniform net section tension
v_m	displacement due to nominal net section bending
W	curved beam wall thickness, $R_o - R_i$
$\Delta = E'v/(\sigma_p + \sigma_m)a$	resultant displacement coefficient
$\Delta_p = E'v_p/\sigma_p a$	displacement coefficient for ideal case of nominal uniform net section tension
$\Delta_m = E'v_m/\sigma_m a$	displacement coefficient for ideal case of nominal net section bending
ν	Poisson's ratio
σ_m	$6M/B(W - a)^2$ component of fictitious normal net stress due to moment M
σ_p	$P/B(W - a)$ component of fictitious normal net stress due to load P

Subscripts:

I	opening mode of crack tip deformation
m	value for net section bending
p	value for net section tension

INTRODUCTION

Kendall, Underwood et al. (Refs. 1 to 4) have proposed a new type of fracture toughness test specimen, namely a ring segment specimen (commonly referred to as C-shaped) which contains a radial crack. Figure 1 shows such a pin loaded specimen as equivalent to a combination of two complementary types of loading. This specimen is currently under consideration for standardization by ASTM Committee E-24 on Fracture Testing. A round robin program was initiated by ASTM Task Group E-24.01.12 for the purpose of evaluating this specimen. Nominal dimensions for the round robin specimen are: ratio of outer to inner radius, $R_o/R_i = 2.0$; ratio of crack length to wall thickness $a/W = 0.5$, where $W = R_o - R_i$; and load line locations $L/W = 0.5$ and $L/W = 0.0$.

Pursuant to this task, Buzzard and Fisher obtained displacement coefficients Δ (given in Table I) at various H/W ratios where H is the chord distance between symmetric pairs of gage points on the inner boundary. To augment and evaluate the results of Buzzard and Fisher, a boundary collocation analysis was undertaken to obtain comparable displacement coefficients. The boundary collocation method, using 60 boundary stations and an overdetermined system of equations is explained in a companion paper to this paper, Ref. 5. The advantages of the particular form of displacement coefficient, Δ, are discussed in Ref. 6.

The displacement coefficients Δ_p and Δ_m obtained here apply to two ideal, complementary types of specimen loading: coefficient Δ_p applies to a nominal uniform distribution of stress across the net section, so that the moment of the stress distribution at mid net section is zero; Δ_m applies to a nominal bending stress distribution, so that the net normal stress is zero (Fig. 1). While these two types of loading are impractical in themselves, the two coefficients can be combined to represent any practical case of loading of the specimen by a pair of equal and opposite forces normal to the crack (pin-loading). The appropriate combined displacement coefficient is then obtained from a simple relation which derives from superposition of the corresponding displacements. Alternatively, values of the dimensionless influence coefficient (E'Bv/P) can be obtained from another simple relation.

It should be appreciated that the accuracy with which the present results will apply to actual pin-loaded C-shaped specimens will depend on the proximity of gage points to the loading pin holes: the closer together these are, the less the accuracy. The reason is that the boundary conditions on the actual specimens are concentrated distributions of stress around the bearing surfaces of the pin holes, whereas the boundary conditions assumed in the present analysis, are statically equivalent stress distributions across the entire cross-section of the specimen. Estimation of the applicable accuracy requires comparison of the present results with experimental results, as given in Table I.

APPROACH

The results presented herein were obtained by boundary collocation analysis of a homogeneous isotropic specimen under plane elasto-static conditions. All plane elasto-static problems reduce to finding a stress function that satisfies the biharmonic equation and associated boundary conditions. A stress function in the form of an infinite series satisfying the biharmonic equation and boundary conditions on the crack surface was employed (Ref. 7). The coefficient of each term of the stress function is initially unknown, but the postulated boundary conditions determine the value of the stress function and its normal derivative at any point on the boundary. The series is truncated and the coefficients of the remaining terms can then be determined by satisfying the boundary conditions at a finite number of selected boundary stations. Such boundary points, equally distributed were used to determine the stress function coefficients of the truncated series.

The stress intensity factor K_I is directly proportional to the coefficient of the first term of the series, which is singular and dominates the stress field near the crack tip. However, the number of terms in the stress function series required to obtain the displacements depend on the relative depth of the crack. In computing the displacements, all coefficients of the truncated stress function series were used, regardless of the crack depth.

RESULTS AND DISCUSSION

Solutions were obtained for Mode I deformation for each of the two types of loading shown in Fig. 1. The stress function boundary conditions appropriate to each case are given in Ref. 5. Displacement coefficients in Table II were obtained for the proportions of the ASTM round-robin radially cracked ring segment specimens, namely: $R_o/R_i = 2$; a/W ratios from 0.40 to 0.60; and H/W locations from 0.0 to 1.8. In addition, displacement coefficients in Tables III to V were computed for all combinations of $a/W = 0.1$ to 0.8 with three R_o/R_i ratios: 2.5, 1.5, and 1.1 at H/W locations varying from 0.0 to 1.0. The results presented in Tables II through V are independent of the load line location L/W.

A comparison of the experimentally obtained displacement coefficients, for two nominal load line locations, $L/W = 0.5$ and 0.0, and nominal H/W values of 0.0, 1.0, and 1.8, with the boundary collocation results is given in Table I. Very good agreement is obtained with one exception: H/W = 1.8. As discussed in the INTRODUCTION one may explain this disagreement of analytical results with experimental results by noting that near the loading pin hole region, where the measurements were made, the analytical model does not accurately simulate the actual complex boundary load conditions. Furthermore, experimental measurements are liable to be less accurate at this location.

Values of Δ were obtained for two complementary, ideal types of specimen loading: Δ_p from a nominal uniform net section stress and Δ_m from a nominal linear bending stress distribution, as shown in Fig. 1. The two coefficients can then be combined to represent any practical case of loading of the specimen by a pair of equal and opposite pin loaded forces normal to the crack line.

APPLICATION OF RESULTS

The displacement coefficients are defined as follows:

$$\Delta_p = E'v_p/\sigma_p a \tag{1}$$

$$\Delta_m = E'v_m/\sigma_m a \tag{2}$$

$$\Delta = E'v/(\sigma_p + \sigma_m)a \tag{3}$$

By the superposition principle, $v = v_p + v_m$ and through algebraic manipulation we obtain

$$\Delta = [\sigma_p/(\sigma_p + \sigma_m)]\Delta_p + [\sigma_m/(\sigma_p + \sigma_m)]\Delta_m \tag{4}$$

Since $\sigma_m = 6M/B(W - a)^2$, $M = P[L + (W + a)/2]$ and $\sigma_p = P/B(W - a)$ where B = specimen thickness, substitution into Eq. (4), and some algebraic manipulation gives:

$$\Delta = \frac{(1 - a/W)\Delta_p + 3(2L/W + a/W + 1)\Delta_m}{2(3L/W + a/W + 2)} \tag{5}$$

Alternatively, the dimensionless influence coefficient (E'Bv/P) can be obtained directly. From Eqs. (3) and (4):

$$\frac{E'Bv}{P} = \frac{Ba(\sigma_p \Delta_p + \sigma_m \Delta_m)}{P} \tag{6}$$

which, on substitution of σ_p and σ_m becomes

$$(E'Bv/P) = (a/W)\left[\frac{(1 - a/W)\Delta_p + 3(2L/W + a/W + 1)\Delta_m}{(1 - a/W)^2}\right] \tag{7}$$

An alternative form is obtained from Eq. (3) on substituting the values of σ_p and σ_m

$$\frac{E'Bv}{P} = 2(a/W)\,\frac{(3L/W + a/W + 2)}{(1 - a/W)^2}\,\Delta \tag{8}$$

As an example, let us determine the resultant displacement coefficient Δ, for a pin loaded specimen with load line $L/W = 0.5$, $R_o/R_i = 2$, and $a/W = 0.5$.

From Eq. (5),

$$\Delta = 0.0625\ \Delta_p + 0.9375\ \Delta_m$$

For $H/W = 1.0$ from Table II, $\Delta_p = -1.476$ and $\Delta_m = 4.366$, hence we obtain $\Delta = 4.001$.

As a second example, let us determine the resultant displacement coefficient Δ at the crack mouth of a pin loaded specimen with load line at $L/W = 0.5$, $R_o/R_i = 2$, and $a/W = 0.5$. Once again from Eq. (5)

$$\Delta = 0.0625\ \Delta_p + 0.9375\ \Delta_m$$

From Table II, $\Delta_p = -0.295$ and $\Delta_m = 3.438$, hence $\Delta = 3.205$.

CONCLUSIONS

The accuracy with which the present results will apply to actual pin loaded C-shaped specimens will depend on the proximity of the gage points to the loading pin holes: the closer together these are the less the accuracy. Indeed, very good agreement is obtained on comparison of the present results with the experimental results (Table I) with one exception: the displacement along the load line.

This disagreement can be explained as follows:

(a) The analytical model does not accurately simulate the actual complex boundary load conditions near the loading pin hole.

(b) The experimental measurements are liable to be less accurate at this location.

For $H/W < 1.1$, on comparison with the analytical results, 13 experimental specimens had displacement coefficient (Δ) variations of less than 2 percent and 4 specimens were within 2.5 and 3.7 percent. For $H/W = 1.8$, where the measurements were made along the load line, on comparison with the analytical results a significantly greater variation occurred. Four specimens had variations under 5 percent and the remaining four had variations between 5.3 and 6.4 percent.

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TABLE I. - COMPARISON OF ANALYTICALLY AND EXPERIMENTALLY DETERMINED DIMENSIONLESS DISPLACEMENT COEFFICIENTS FOR RADIALLY CRACKED RING SEGMENTS

L/W	R_o/R_i	H/W	Machined notch a/W	Machined notch displacement coefficient, Δ^* experimental	Machined notch displacement coefficient, Δ analytic	Fatigue crack a/W	Fatigue crack displacement coefficient, Δ^* experimental	Fatigue crack displacement coefficient, Δ analytic
0.502	1.953	0.100	0.478	3.36	3.26	0.525	3.20	3.17
.502		1.056		4.31	4.22		4.06	4.01
.502		1.792		6.53	6.87		6.08	6.32
0.499	1.988	0.106	0.456	3.28	3.31	0.512	3.17	3.19
.499		1.022		4.20	4.24		3.99	3.99
.499		1.759		6.78	6.93		6.28	6.29
0.498	1.949	0.088	0.478	3.30	3.26	0.536	3.07	3.15
.498		1.066		4.26	4.22		3.90	3.98
.498		1.796		6.50	6.87		5.86	6.26
0.499	1.949	0.099	0.478	3.38	3.26	0.543	3.09	3.14
.499		1.060		4.30	4.23		3.85	3.95
.499		1.790		6.49	6.85		5.78	6.17
0.0005	1.946	0.104	0.474	3.16	3.11			

*In house experimental results from R. J. Buzzard and D. M. Fisher.

TABLE II. - DISPLACEMENT COEFFICIENTS Δ_p AND Δ_m FOR $R_o/R_i = 2/1$ AND RATIOS a/W FROM 0.40 TO 0.60 AND H/W FROM 0.00 TO 1.80

a/W	0.40	0.45	0.50	0.55	0.60
H/W			Δ_p		
0.00	0.477	0.078	-0.295	-0.648	-0.984
.20	.461	.060	-0.315	-0.669	-1.006
.40	.388	-0.016	-0.391	-0.746	-1.083
.60	.183	-0.218	-0.588	-0.936	-1.265
.80	-0.199	-0.585	-0.938	-1.268	-1.579
1.00	-0.811	-1.159	-1.476	-1.772	-2.050
1.20	-1.765	-2.034	-2.281	-2.514	-2.732
1.40	-3.315	-3.428	-3.541	-3.655	-3.766
1.60	-5.998	-5.808	-5.662	-5.550	-5.459
1.73	-8.911	-8.369	-7.924	-7.551	-7.229
1.80	-10.989	-10.177	-9.506	-8.940	-8.447
			Δ_m		
0.00	3.680	3.543	3.438	3.327	3.233
.20	3.714	3.578	3.468	3.354	3.257
.40	3.833	3.681	3.559	3.436	3.331
.60	4.069	3.882	3.733	3.589	3.467
.80	4.441	4.195	3.999	3.822	3.672
1.00	4.951	4.625	4.366	4.141	3.952
1.20	5.619	5.191	4.845	4.560	4.320
1.40	6.514	5.948	5.481	5.116	4.806
1.60	7.784	7.013	6.378	5.891	5.479
1.73	8.940	7.979	7.207	6.594	6.088
1.80	9.665	8.588	7.725	7.037	6.471

TABLE III. - DISPLACEMENT COEFFICIENTS Δ_p AND Δ_m FOR $R_o/R_i = 5/2$ AND RATIOS a/W FROM 0.10 TO 0.80 AND H/W FROM 0.00 TO 1.00

Δ_p

a/W H/W	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.00	4.422	2.579	1.318	0.365	-0.432	-1.095	-1.721	-2.315
.10	4.454	2.578	1.311	.356	-0.443	-1.105	-1.732	-2.325
.20	4.603	2.595	1.309	.344	-0.458	-1.127	-1.754	-2.349
.30	4.894	2.605	1.281	.307	-0.499	-1.173	-1.800	-2.393
.40	5.236	2.587	1.212	.227	-0.577	-1.252	-1.875	-2.463
.50	5.515	2.511	1.085	.095	-0.705	-1.371	-1.986	-2.563
.60	5.622	2.346	.886	-0.102	-0.890	-1.537	-2.135	-2.695
.70	5.465	2.056	.597	-0.373	-1.141	-1.757	-2.331	-2.865
.80	4.985	1.589	.190	-0.734	-1.466	-2.044	-2.582	-3.079
.90	4.126	.868	-0.381	-1.221	-1.885	-2.421	-2.907	-3.352
1.00	2.731	-0.235	-1.210	-1.900	-2.442	-2.935	-3.340	-3.708

Δ_m

a/W H/W	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.00	5.630	4.616	4.144	3.770	3.502	3.290	3.102	2.932
.10	5.721	4.666	4.169	3.789	3.515	3.300	3.110	2.938
.20	5.997	4.753	4.222	3.829	3.547	3.325	3.132	2.958
.30	6.613	4.932	4.321	3.900	3.603	3.371	3.170	2.991
.40	7.536	5.231	4.484	4.012	3.689	3.441	3.227	3.040
.50	8.695	5.658	4.716	4.171	3.808	3.534	3.304	3.103
.60	10.033	6.209	5.026	4.381	3.963	3.655	3.402	3.185
.70	11.562	6.880	5.413	4.645	4.157	3.805	3.523	3.285
.80	13.399	7.682	5.880	4.969	4.396	3.900	3.670	3.408
.90	15.793	8.658	6.440	5.366	4.687	4.213	3.851	3.556
1.00	19.031	9.896	7.126	5.860	5.049	4.489	4.070	3.741

TABLE IV. - DISPLACEMENT COEFFICIENTS Δ_p AND Δ_m FOR $R_o/R_i = 3/2$ AND RATIOS a/W FROM 0.10 TO 0.80 AND H/W FROM 0.00 TO 1.00

a/W	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
H/W				Δ_p				
0.00	3.743	2.569	1.560	0.699	-0.057	-0.747	-1.401	-2.044
.10	3.811	2.580	1.560	.696	-0.061	-0.751	-1.404	-2.047
.20	3.896	2.569	1.549	.688	-0.068	-0.759	-1.412	-2.055
.30	4.119	2.549	1.523	.667	-0.087	-0.775	-1.427	-2.069
.40	4.463	2.537	1.484	.631	-0.119	-0.804	-1.452	-2.092
.50	4.865	2.535	1.432	.579	-0.166	-0.845	-1.488	-2.124
.60	5.265	2.532	1.368	.510	-0.229	-0.901	-1.536	-2.166
.70	5.632	2.518	1.288	.423	-0.309	-0.973	-1.599	-2.219
.80	5.957	2.486	1.189	.317	-0.408	-1.061	-1.676	-2.284
.90	6.235	2.433	1.070	.189	-0.527	-1.167	-1.767	-2.359
1.00	6.451	2.353	.926	.037	-0.668	-1.292	-1.875	-2.447
				Δ_m				
0.00	4.804	4.249	3.884	3.575	3.332	3.154	3.001	2.868
.10	4.890	4.279	3.892	3.580	3.336	3.157	3.004	2.870
.20	5.048	4.309	3.904	3.590	3.347	3.166	3.011	2.876
.30	5.434	4.374	3.929	3.609	3.365	3.180	3.023	2.887
.40	6.032	4.499	3.976	3.640	3.391	3.201	3.041	2.902
.50	6.776	4.692	4.053	3.687	3.429	3.230	3.065	2.922
.60	7.615	4.948	4.163	3.753	3.477	3.267	3.095	2.947
.70	8.536	5.258	4.304	3.838	3.538	3.312	3.131	2.976
.80	9.540	5.613	4.482	3.946	3.613	3.367	3.173	3.011
.90	10.633	6.012	4.697	4.076	3.700	3.432	3.222	3.051
1.00	11.814	6.457	4.943	4.226	3.800	3.504	3.279	3.096

TABLE V. - DISPLACEMENT COEFFICIENTS Δ_p AND Δ_m FOR

R_o/R_i = 11/10 AND RATIOS a/W FROM 0.10 TO 0.80

AND H/W FROM 0.00 TO 1.00

a/W	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
H/W				Δ_p				
0.00	3.593	2.681	1.780	0.967	0.234	-0.456	-1.124	-1.813
.10	3.659	2.681	1.780	.965	.232	-0.457	-1.125	-1.814
.20	3.747	2.670	1.769	.958	.229	-0.460	-1.129	-1.816
.30	3.945	2.637	1.736	.942	.218	-0.467	-1.134	-1.820
.40	4.264	2.615	1.703	.916	.200	-0.480	-1.145	-1.827
.50	4.681	2.626	1.659	.881	.174	-0.500	-1.160	-1.838
.60	5.143	2.648	1.615	.840	.138	-0.527	-1.182	-1.854
.70	5.648	2.703	1.582	.792	.096	-0.562	-1.210	-1.874
.80	6.154	2.758	1.549	.741	.047	-0.603	-1.243	-1.899
.90	6.681	2.835	1.527	.688	-0.008	-0.652	-1.281	-1.929
1.00	7.198	2.923	1.505	.632	-0.068	-0.707	-1.326	-1.963
				Δ_m				
0.00	4.460	4.041	3.718	3.434	3.221	3.059	2.927	2.816
.10	4.529	4.059	3.719	3.434	3.222	3.059	2.927	2.816
.20	4.643	4.064	3.713	3.433	3.223	3.060	2.930	2.819
.30	4.934	4.087	3.710	3.433	3.225	3.063	2.932	2.820
.40	5.393	4.153	3.716	3.436	3.229	3.067	2.935	2.823
.50	5.977	4.264	3.743	3.448	3.237	3.074	2.941	2.827
.60	6.646	4.419	3.792	3.470	3.251	3.082	2.948	2.833
.70	7.374	4.613	3.868	3.505	3.270	3.096	2.957	2.840
.80	8.145	4.847	3.969	3.553	3.297	3.112	2.968	2.847
.90	8.953	5.118	4.089	3.613	3.330	3.133	2.982	2.857
1.00	9.793	5.420	4.224	3.684	3.370	3.158	2.998	2.868

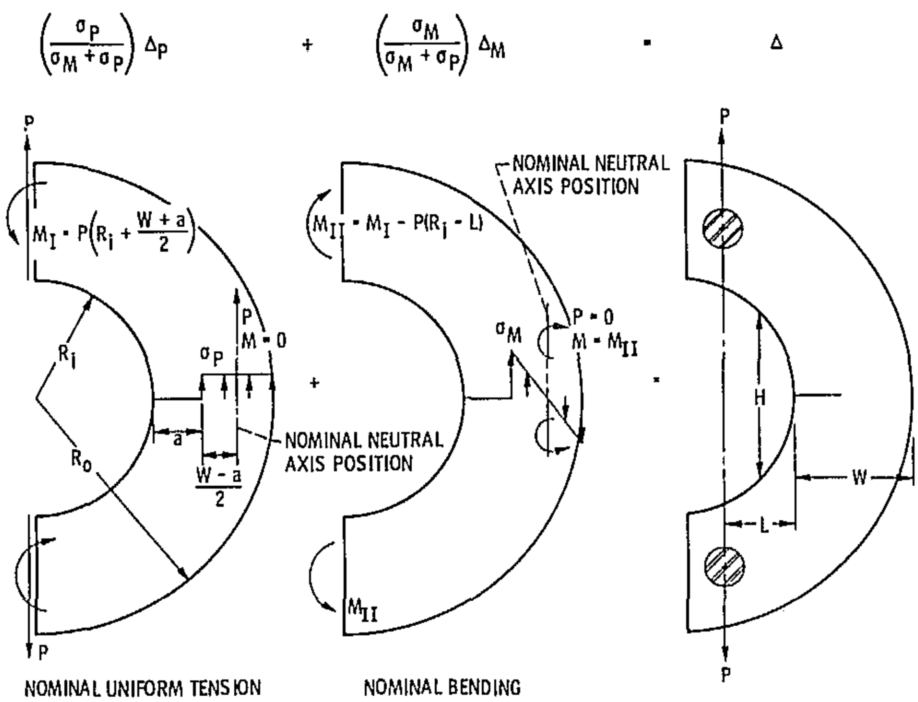

Figure 1. - Application of superposition of displacements to specimen loaded through pins at selected distance L from crack mouth.