OLD DOMINION UNIVERSITY RESEARCH FOUNDATION

SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

DYNAMIC CHARACTERISTICS OF ROTOR BLADES WITH PENDULUM ABSORBERS

(NASA-CR-153929) DYNAMIC CHARACTERISTICS OF ROTOR BLADES WITH PENDULUM ABSORBERS Progress Report, 1 Jan. - 30 Jun. 1977 (Old Dominion Univ. Research Foundation) 18 p HC A02/MF A01 CSCL 01C G3/05

N77-29138

Unclas
39211

By

V. R. Murthy

G. L. Goglia, Principal Investigator

Progress Report

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia

Under
NASA Grant NSG 1370
January 1, 1977 - June 30, 1977
Charles E. Hammond, Technical Monitor
Structures and Dynamics Division

July 1977

SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

DYNAMIC CHARACTERISTICS OF ROTOR BLADES WITH PENDULUM ABSORBERS

By

V. R. Murthy

G. L. Goglia, Principal Investigator

Progress Report

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665

Under
NASA Grant NSG 1370
January 1, 1977 - June 30, 1977
Charles E. Hammond, Technical Monitor
Structures and Dynamics Division

Submitted by the
Old Dominion University Research Foundation
Norfolk, Virginia 23508

July 1977

DYNAMIC CHARACTERISTICS OF ROTOR BLADES WITH PENDULUM ABSORBERS

By

V. R. Murthy[1]

DISCUSSION

The point transmission matrix for a vertical plane pendulum on a rotating blade undergoing combined flapwise bending, chordwise bending and torsion is derived. The gravitational effect on the pendulum is taken into account. The equilibrium equation of the pendulum is linearized for small oscillations about the steady state. The summary of equations obtained is given below.

Nonlinear Equilibrium Equation of the Pendulum

$$\ddot{\Theta} - \frac{\Omega^2}{\ell}(x_A + \ell \sin\Theta)\cos\Theta + \frac{\ddot{w}_A + y_A\ddot{\phi}}{\ell}\sin\Theta + \frac{g}{\ell}\sin\Theta = 0 \qquad (1)$$

Linearized Equation of the Pendulum About the Steady State

$$\ddot{\theta} + \frac{\Omega^2}{\ell}\left(\frac{x_A}{\sin\theta_0} + \ell\sin^2\theta_0\right)\theta + \frac{\ddot{w}_A + y_A\ddot{\phi}_A}{\ell}\sin\theta_0 = 0 \qquad (2)$$

Equation for Determination of the Steady State Angle

$$\Omega^2\cos\theta_0(x_A + \ell\sin\theta_0) = g\sin\theta_0 \qquad (3)$$

Uncoupled Pendulum Natural Frequency

$$\omega^2 = \left[\frac{\Omega^2}{\ell}\left(\frac{x_A}{\sin\theta_0} + \ell\sin^2\theta_0\right)\right] \qquad (4)$$

---

[1]Research Associate, Old Dominion University Research Foundation, Norfolk, Virginia 23508

Pendulum Deflection in a Natural Mode of the Blade

$$\theta = \frac{\omega^2(w_A + y_A\phi_A)\ \sin\theta_0}{\left[\Omega^2\left(\frac{x_A}{\sin\theta_0} + \ell\ \sin^2\theta_0\right) - \ell\omega^2\right]} \tag{5}$$

Transmission Matrix of the Pendulum

$$\begin{Bmatrix} w \\ v \\ \psi \\ \nu \\ \phi \\ M_x \\ M_z \\ M_y \\ -V_y \\ -V_z \end{Bmatrix}_{x = x_A^+} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ P_{61} & P_{62} & 0 & 0 & P_{65} & 1 & 0 & 0 & 0 & 0 \\ P_{71} & P_{72} & 0 & 0 & P_{75} & 0 & 1 & 0 & 0 & 0 \\ P_{81} & 0 & 0 & 0 & P_{85} & 0 & 0 & 1 & 0 & 0 \\ 0 & P_{92} & 0 & 0 & P_{95} & 0 & 0 & 0 & 1 & 0 \\ P_{101} & 0 & 0 & 0 & P_{105} & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} w \\ v \\ \psi \\ \nu \\ \phi \\ M_x \\ M_z \\ M_y \\ -V_y \\ -V_z \end{Bmatrix}_{x = x_A^-} \tag{6}$$

where

$$P_{61} = -\omega^2 y_A c_1 M$$

$$P_{62} = \left[(\omega^2 + \Omega^2)(z_A - \ell\ \cos\theta_0) + g\right]M$$

$$P_{65} = -\left[(\omega^2 + \Omega^2)z_A(z_A - \ell\ \cos\theta_0) + \omega^2 y_A^2 c_1 + g z_A\right]M$$

$$P_{71} = (\omega^2 + \Omega^2)\ell\ \cos\theta_0\ y_A c M$$

$$P_{72} = \Omega^2(x_A + \ell\ \sin\theta_0)M$$

$$P_{75} = \left[(\omega^2 + \Omega^2)\ell\ \cos\theta_0\ y_A^2 c - \Omega^2(x_A + \ell\ \sin\theta_0)z_A\right]M$$

$$P_{81} = c_2 M$$

$$P_{85} = c_2 y_A M$$

$$P_{92} = (\omega^2 + \Omega^2)M$$

$$P_{95} = -(\omega^2 + \Omega^2)z_A M$$

$$P_{101} = \omega^2 c_1 M$$

$$P_{105} = \omega^2 y_A c_1 M$$

$$a_1 = \ell \sin^2 \theta_0$$

$$a_2 = \Omega^2 (x_A / \sin \theta_0 + a_1)$$

$$c = \omega^2 \sin^2 \theta_0 / (a_2 - \ell\omega^2)$$

$$c_1 = 1 + \ell c$$

$$c_2 = \Omega^2 c\ (x_A + \ell \sin \theta_0) + (\omega^2 + \Omega^2)\ell \cos \theta_0 (z_A - \ell \cos \theta_0) c$$

Basic Transmission Matrix of the Blade

The differential equations of motion for a rotating blade derived in reference 1 can be arranged into a ten first-order differential equation as shown below.

$$\frac{d}{dx} \{z(x)\} = [A(x)]\{z(x)\} \tag{7a}$$

where

$$[A(x)] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{37} & A_{38} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{47} & A_{48} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{56} & 0 & 0 & 0 & 0 \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{74} & A_{75} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{83} & 0 & A_{85} & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{92} & 0 & 0 & A_{95} & 0 & 0 & 0 & 0 & 0 \\ A_{101} & 0 & 0 & 0 & A_{105} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and where

$$A_{37} = -(EI_2 - EI_1) \sin \beta \cos \beta / D$$

$$A_{38} = (EI_1 \sin^2 \beta + EI_2 \cos^2 \beta)/D$$

$$A_{47} = (EI_1 \cos^2 \beta + EI_2 \sin^2 \beta)/D$$

$$A_{48} = A_{37}$$

$$D = (EI_1 \cos^2 \beta + EI_2 \sin^2 \beta)(EI_1 \sin^2 \beta + EI_2 \cos^2 \beta) - (EI_2 - EI_1)^2 \cos^2 \beta \sin^2 \beta$$

$$A_{56} = 1/GJ$$

$$A_{61} = -\omega^2 me \cos \beta$$

$$A_{62} = (\omega^2 + \Omega^2) me \sin \beta$$

$$A_{63} = \Omega^2 mxe \cos \beta$$

$$A_{64} = -\Omega^2 mxe \cos \beta$$

$$A_{65} = \left[\Omega^2 m\left(k_{m_2}^2 - k_{m_1}^2\right) \cos 2\beta - \omega^2 m k_m^2\right]$$

$$A_{74} = T$$

$$A_{75} = -\Omega^2 mxe \sin \beta$$

$$A_{83} = T$$

$$A_{85} = \Omega^2 mxe \cos \beta$$

$$A_{92} = (\omega^2 + \Omega^2) m$$

$$A_{95} = -(\omega^2 + \Omega^2) me \sin \beta$$

$$A_{101} = \omega^2 m$$

$$A_{105} = \omega^2 me \cos \beta$$

$$T(x) = \int_x^R \Omega^2 m x_1 dx_1$$

The backward transmission matrix is given by (ref. 2):

$$\frac{d}{dx}[T(x)] = [A(x)] \; [T(x)] \tag{7b}$$

and

$$[T(0)] = [1] \quad , \quad \text{identity matrix}$$

Tension in the Blade When Pendulum is Attached

$$T = \int_x^R \Omega^2 m x_1 dx_1 + \Omega^2 M(x_A + \ell \sin \theta_0) \qquad x < x_A \tag{8a}$$

$$T = \int_x^R \Omega^2 m x_1 dx_1 \qquad x > x_A \tag{8b}$$

Transmission Matrix at Any Spanwise Location

Case 1. $x < x_A$

The transmission matrix is given directly by the solution of equation (7b).

Case 2. $x > x_A$

$$_x[T]_0 = {}_x[T]_{x_A} \; [P] \; {}_{x_A}[T]_0 \tag{9}$$

Frequency Equations and Mode Shapes

Collective Modes.

$$\begin{vmatrix} T_{64} & T_{66} + T_{65}/k\phi & T_{68} & T_{69} & T_{610} + T_{61}/D \\ T_{74} & T_{76} + T_{75}/k\phi & T_{78} & T_{79} & T_{710} + T_{71}/D \\ T_{84} & T_{86} + T_{85}/k\phi & T_{88} & T_{89} & T_{810} + T_{81}/D \\ T_{94} & T_{96} + T_{95}/k\phi & T_{98} & T_{99} & T_{910} + T_{91}/D \\ T_{104} & T_{106} + T_{105}/k\phi & T_{108} & T_{109} & T_{1010} + T_{101}/D \end{vmatrix} = 0$$

$$w(x) = x_1 T_{14}(x) + x_2[T_{16}(x) + T_{15}(x)/k\phi] + x_3 T_{18}(x) + x_4 T_{19}(x) + x_5[T_{110}(x) + T_{11}(x)/D]$$

$$v(x) = x_1 T_{24}(x) + x_2[T_{26}(x) + T_{25}(x)/k\phi] + x_3 T_{28}(x) + x_4 T_{29}(x) + x_5[T_{210}(x) + T_{21}(x)/D]$$

$$\phi(x) = x_1T_{54}(x) + x_2[T_{56}(x) + T_{55}(x)/k\phi] + x_3T_{58}(x)$$
$$+ x_4T_{59}(x) + x_5[T_{510}(x) + T_{51}(x)/D]$$

where

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{bmatrix} T_{14} & T_{16} + T_{15}/k\phi & T_{18} & T_{19} & T_{110} + T_{11}/D \\ T_{64} & T_{66} + T_{65}/k\phi & T_{68} & T_{69} & T_{610} + T_{61}/D \\ T_{74} & T_{76} + T_{75}/k\phi & T_{78} & T_{79} & T_{710} + T_{71}/D \\ T_{84} & T_{86} + T_{85}/k\phi & T_{88} & T_{89} & T_{810} + T_{81}/D \\ T_{94} & T_{96} + T_{95}/k\phi & T_{98} & T_{99} & T_{910} + T_{91}/D \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$D = \omega^2 M_V - k_V$$

Cyclic Modes.

$$\begin{vmatrix} T_{63} + T_{69}k\psi & T_{66} + T_{65}/k\phi & T_{67} & T_{69} + T_{62}/D_1 & T_{610} \\ T_{73} + T_{78}k\psi & T_{76} + T_{75}/k\phi & T_{77} & T_{79} + T_{72}/D_1 & T_{710} \\ T_{83} + T_{88}k\psi & T_{86} + T_{85}/k\phi & T_{87} & T_{89} + T_{82}/D_1 & T_{810} \\ T_{93} + T_{98}k\psi & T_{96} + T_{95}/k\phi & T_{97} & T_{99} + T_{92}/D_1 & T_{910} \\ T_{103} + T_{108}k\psi & T_{106} + T_{105}/k\phi & T_{107} & T_{109} + T_{102}/D_1 & T_{1010} \end{vmatrix} = 0$$

$$w(x) = x_1[T_{13}(x) + T_{18}(x)\ k\psi] + x_2[T_{16}(x) + T_{15}(x)/k\phi]$$
$$+ x_3T_{17}(x) + x_4[T_{19}(x) + T_{12}(x)/D_1]$$
$$+ x_5T_{110}(x)$$

$$v(x) = x_1[T_{23}(x) + T_{28}(x)k\psi] + x_2[T_{26}(x) + T_{25}(x)/k\phi]$$
$$+ x_3T_{27}(x) + x_4[T_{29}(x) + T_{22}(x)/D_1]$$
$$+ X_5T_{210}(x)$$

$$\phi(x) = x_1[T_{53}(x) + T_{58}(x)k\psi] + x_2[T_{56}(x) + T_{55}(x)/k\phi]$$
$$+ x_3T_{57}(x) + x_4[T_{59}(x) + T_{52}(x)/D_1]$$
$$+ x_5T_{510}(x)$$

where

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{bmatrix} T_{13} + T_{18}k\psi & T_{16} + T_{15}/k\phi & T_{17} & T_{19} + T_{11}/D_1 & T_{110} \\ T_{63} + T_{68}k\psi & T_{66} + T_{65}/k\phi & T_{67} & T_{69} + T_{61}/D_1 & T_{610} \\ T_{73} + T_{78}k\psi & T_{76} + T_{75}/k\phi & T_{77} & T_{79} + T_{71}/D_1 & T_{710} \\ T_{83} + T_{88}k\psi & T_{86} + T_{85}/k\phi & T_{87} & T_{89} + T_{81}/D_1 & T_{810} \\ T_{93} + T_{98}k\psi & T_{96} + T_{95}/k\phi & T_{97} & T_{99} + T_{91}/D_1 & T_{910} \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$D_1 = \omega^2 M_H - k_H$$

Scissor Modes.

$$\begin{vmatrix} T_{66} + T_{65}/k\phi & T_{67} & T_{68} & T_{69} & T_{610} \\ T_{76} + T_{75}/k\phi & T_{77} & T_{78} & T_{79} & T_{710} \\ T_{86} + T_{85}/k\phi & T_{87} & T_{88} & T_{89} & T_{810} \\ T_{96} + T_{95}/k\phi & T_{97} & T_{98} & T_{99} & T_{910} \\ T_{106} + T_{105}/k\phi & T_{107} & T_{108} & T_{109} & T_{1010} \end{vmatrix} = 0$$

$$w(x) = x_1[T_{16}(x) + T_{13}(x)/k\phi] + x_2T_{17}(x) + x_3T_{18}(x) + x_4T_{19}(x) + x_5T_{110}(x)$$

$$v(x) = x_1[T_{26}(x) + T_{25}(x)/k\phi] + x_2T_{27}(x) + x_3T_{28}(x) + x_4T_{29}(x) + x_5T_{210}(x)$$

$$\phi(x) = x_1[T_{56}(x) + T_{55}(x)/k\phi] + x_2T_{57}(x) + x_3T_{58}(x) + x_4T_{59}(x) + x_5T_{510}(x)$$

where

$$
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{bmatrix} T_{16} + T_{15}/k\phi & T_{17} & T_{18} & T_{19} & T_{110} \\ T_{66} + T_{65}/k\phi & T_{67} & T_{68} & T_{69} & T_{610} \\ T_{76} + T_{75}/k\phi & T_{77} & T_{78} & T_{79} & T_{710} \\ T_{86} + T_{85}/k\phi & T_{87} & T_{88} & T_{89} & T_{810} \\ T_{96} + T_{95}/k\phi & T_{97} & T_{98} & T_{99} & T_{910} \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}
$$

<u>Articulated Modes.</u>

$$
\begin{vmatrix} T_{63} & T_{64} & T_{66} + T_{65}/k\phi & T_{69} & T_{610} \\ T_{73} & T_{74} & T_{76} + T_{75}/k\phi & T_{79} & T_{710} \\ T_{83} & T_{84} & T_{86} + T_{85}/k\phi & T_{89} & T_{810} \\ T_{93} & T_{94} & T_{96} + T_{95}/k\phi & T_{99} & T_{910} \\ T_{103} & T_{104} & T_{106} + T_{105}/k\phi & T_{109} & T_{1010} \end{vmatrix} = 0
$$

$$
\begin{aligned} w(x) = {} & x_1 T_{13}(x) + x_2 T_{14}(x) + x_3 [T_{16}(x) + T_{15}(x)/k\phi] \\ & + x_4 T_{19}(x) + x_5 T_{110}(x) \end{aligned}
$$

$$
\begin{aligned} v(x) = {} & x_1 T_{23}(x) + x_2 T_{24}(x) + x_3 [T_{26}(x) + T_{25}(x)/k\phi] \\ & + x_4 T_{29}(x) + x_5 T_{210}(x) \end{aligned}
$$

$$
\begin{aligned} \phi(x) = {} & x_1 T_{53}(x) + x_2 T_{54}(x) + x_3 [T_{56}(x) + T_{55}(x)/k\phi] \\ & + x_4 T_{59}(x) + x_5 T_{510}(x) \end{aligned}
$$

where

$$
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{bmatrix} T_{13} & T_{14} & T_{16} + T_{15}/k\phi & T_{19} & T_{110} \\ T_{63} & T_{64} & T_{66} + T_{65}/k\phi & T_{69} & T_{610} \\ T_{73} & T_{74} & T_{76} + T_{75}/k\phi & T_{79} & T_{710} \\ T_{83} & T_{84} & T_{86} + T_{85}/k\phi & T_{89} & T_{810} \\ T_{93} & T_{94} & T_{96} + T_{95}/k\phi & T_{99} & T_{910} \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}
$$

RESULTS

A Fortran program was written for a special subcase of the above formulation, ViZ, a vertical plane pendulum attached to a uniform blade with flapwise bending degree of freedom for cantilever boundary conditions. The results obtained are shown in table 1. The frequency has a singular value right at the uncoupled pendulum natural frequency and thus introduces two frequencies corresponding to the nearest natural frequency of the blade without pendulum. This behavior is shown in figures 1 and 2. In both of these modes it was observed that the pendulum deflection is large. One frequency can be thought of as a coupled pendulum frequency and the other as a coupled bending and pendulum frequency. The data of the system considered is given below.

span = 260 in.
weight per unit length = 0.69552 lb/in.
bending stiffness = 0.4×10^8 lb/in^2.
simichord = 10.5 in.
pendulum weight = 15 lb.
pendulum length = 26 in.
spanwise location of the pendulum = 130 in.

Table 1. Frequencies (rad/sec).

No.	$\Omega = 0$ No Mass	$\Omega = 360$ RPM No Mass	$\Omega = 0$ Just Mass Effect	$\Omega = 360$ RPM Just Mass Effect	$\Omega = 360$ RPM Pendulum Effect
1	7.74	40.19	7.60	40.14	40.10
					84.40
2	48.58	107.07	45.04	101.56	
					121.54
3	136.05	207.81	136.05	210.18	210.97
4	266.62	348.15	249.56	329.33	355.00
5	440.77	528.64	440.77	531.56	533.88

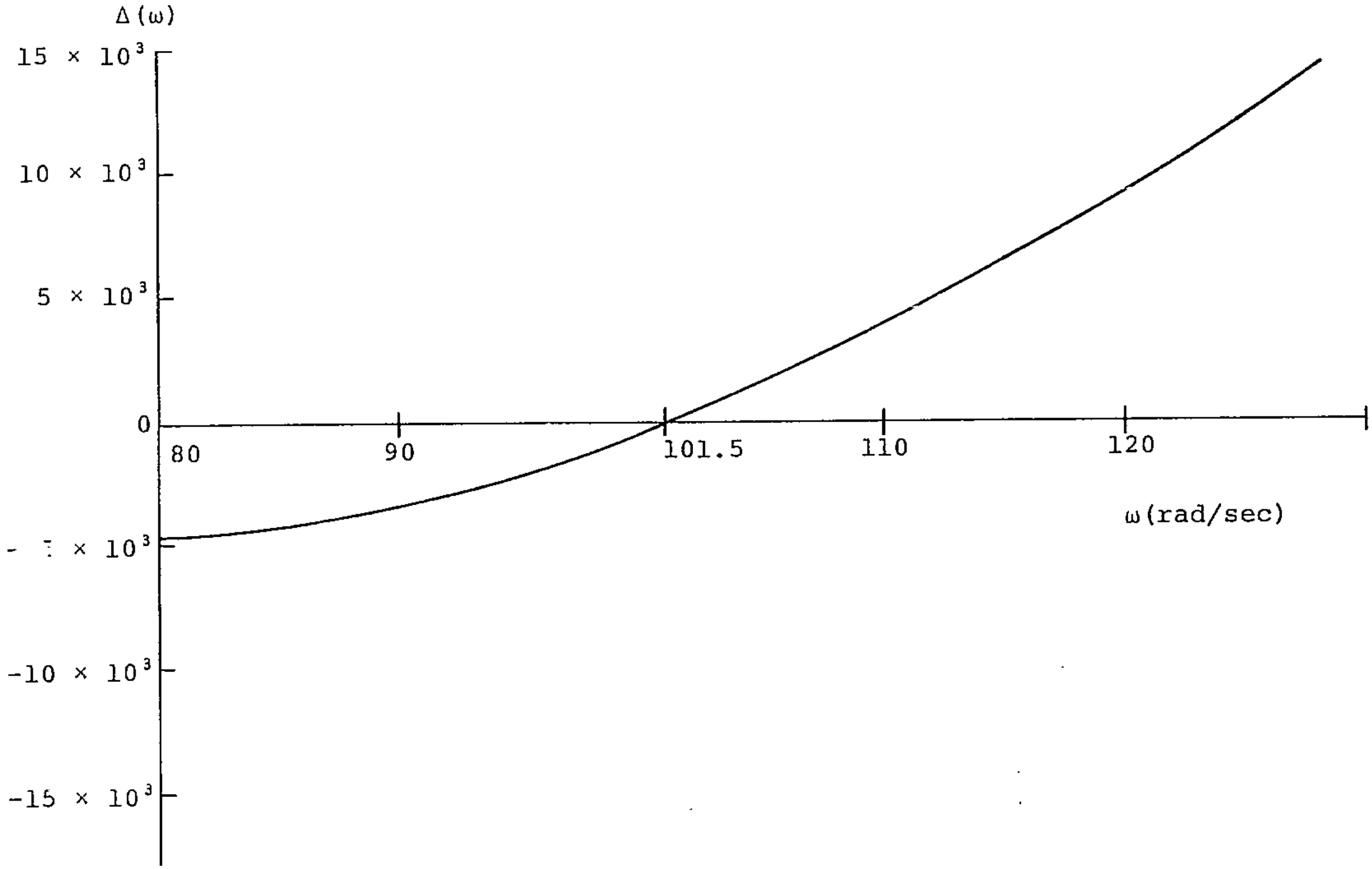

Figure 1. Frequency plot, mass effect.

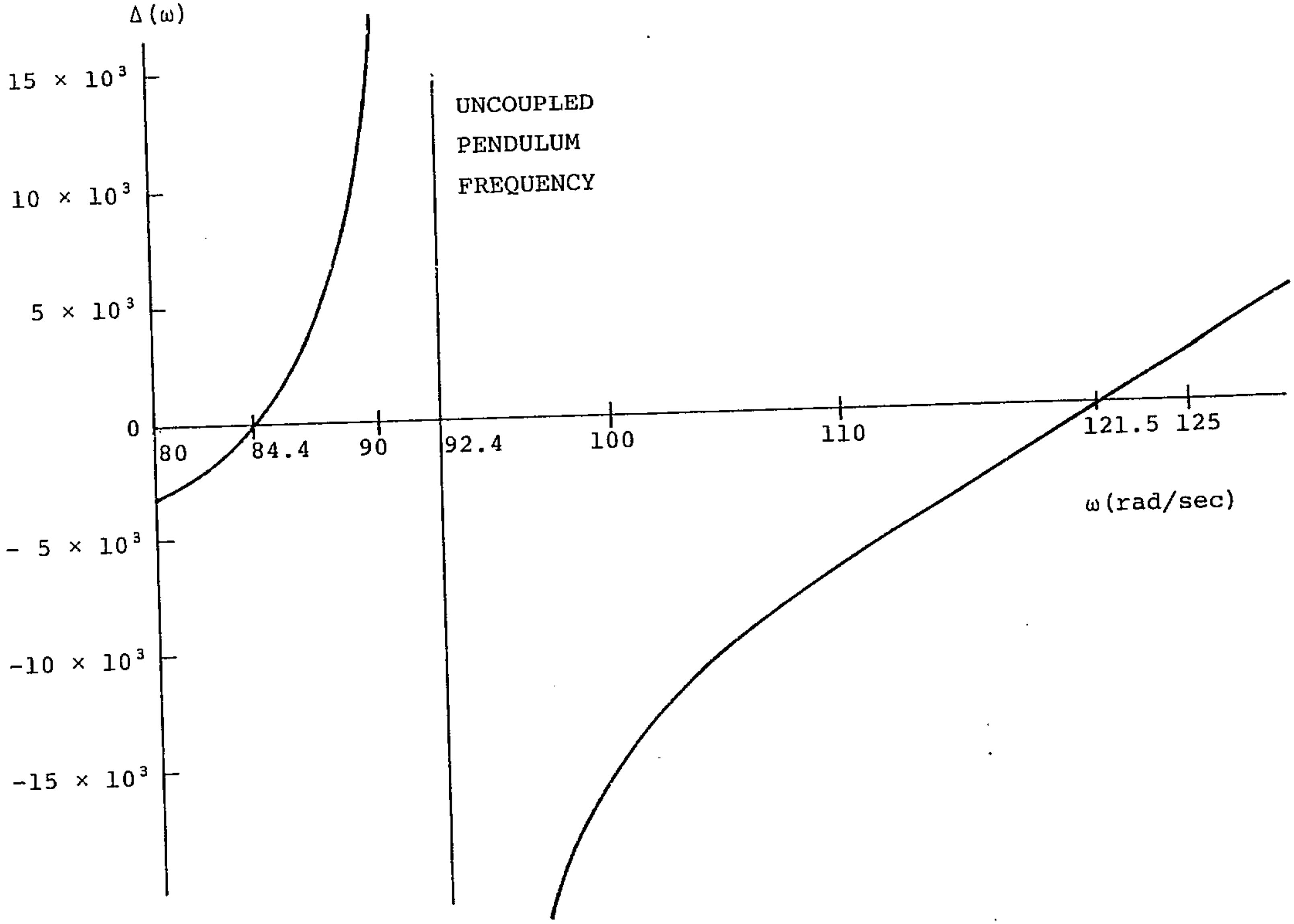

Figure 2. Frequency plot, pendulum effect.

APPENDIX

NOMENCLATURE

C H D = distance of the lead-lag hinge from the axis of rotation

E = Young's modulus of elasticity

e = distance between mass and elastic axes, positive when mass axis lies ahead of shear center

F H D = distance of the flapping hinge from the axis of rotation

G = shear modulus of elasticity

g = acceleration due to gravity acting on the pendulum

I_1, I_2 = bending moments of inertia about major and minor axes, respectively (both pass through centroid of cross-sectional area effective in carrying tensions

J = torsional stiffness constant

k_H = horizontal spring rate of the horizontal restraint of the blade at the root

k_m = polar radius of gyration of cross-sectional mass about elastic axis, $k_m^2 = k_{m_1}^2 + k_{m_2}^2$

k_{m_1}, k_{m_2} = mass radii of gyration about major neutral axis and about an axis perpendicular to chord through the elastic axis, respectively

k_V = vertical spring rate of the vertical restraint of the blade at the root

k_ϕ = control system spring rate

k_ψ = blade flapping spring rate

ℓ = length of the pendulum

M = mass of the pendulum

M_H = effective horizontal hub mass per blade

M_V = effective vertical hub mass per blade

M_x, M_y, M_z = resultant cross-sectional moments about x, y, z directions, respectively

m	=	mass per unit length of the blade
$[P]$	=	transmission matrix
P H D	=	distance of the pitch hinge from the axis of rotation
R	=	span of the blade
T	=	tension in the blade
${}_b[T(x)]_a$	=	transmission matrix of the blade obtained by integrating equation (7b) from $x = a$ to $x = b$
$[T]$	=	transmission matrix of the blade obtained by integrating equation (7b) from $x = x_0$ to $x = R$
T_{ij}	=	(i,j)th element of $[T]$
$T_{ij}(x)$	=	(i,j)th element of ${}_x[T(x)]_{x_0}$
V_y, V_z	=	shear in y and z directions, respectively
v	=	amplitude of simple harmonic in-plane displacement, positive towards leading edge
w	=	amplitude of simple harmonic out-of-plane displacement, positive vertically upwards
w_A	=	out-of-plane displacement at the pendulum attachment point in equations (1) and (2)
w_A	=	amplitude of simple harmonic out-of-plane displacement at the pendulum attachment point in equation (5)
x, y, z	=	right-handed Cartesian coordinate system which rotates with the blade such that x-axis falls along undeformed elastic axis, positive towards tip, y-axis positive towards leading edge, z-axis is vertical axis, positive vertically upwards
x_0	=	defined by the columns of transmission matrix x_0 = C H D for II, IV, VII, IX columns x_0 = F H D for I, II, VIII, X columns x_0 = P H D for V, VI columns
x_A	=	spanwise location of the pendulum attachment point
y_A, z_A	=	undeformed cross-sectional coordinates of the pendulum hinge

β = blade twist prior to deformation

Θ = pendulum deflection from vertical position

θ = amplitude of simple harmonic pendulum deflection

θ_0 = steady state deflection of the pendulum

ν = slope of deflection curve in the plane of rotation

ϕ = torsional deflection, positive leading edge upwards

ϕ_A = torsional deflection at the pendulum attachment point in equations (1) and (2)

ϕ_A = amplitude of simple harmonic torsional deflection in equation (5)

ψ = slope of deflection curve normal to plane of rotation

Ω = angular velocity of rotation

ω = frequency of vibration

. denotes differentiation with respect to time
/ denotes differentiation with respect to x

REFERENCES

1. John C. Houbolt and George W. Brooks, Differential Equations of Motion for Combined Flapwise Bending, Chordwise Bending, and Torsion of Twisted Nonuniform Rotor Blades, NACA Report 1346, 1958.

2. V. R. Murthy, Dynamic Characteristics of Rotor Blades, Vol. 49, No. 4, pp. 483-500, J. Sound and Vibration, 1976.