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MUTUAL ADMITTANCE BETWEEN CIRCULAR

WAVEGUIDE-FED APERTURES FOR LARGE SPACINGS

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SUMMARY

A closed-form asymptotic expression for the mutual admittance between two circular waveguide-fed apertures is developed for large element spacings. A comparison with calculations obtained by a numerical integration method indicates that the present analysis gives good results for spacings greater than two wavelengths.

INTRODUCTION

In the analysis of mutual coupling effects between radiating elements of a finite size antenna array, a calculation of the complex mutual admittance (or impedance) between each pair combination of the array is needed. For example, an array of N elements requires N^2 values of complex mutual admittance to form the complex admittance matrix from which the complex scattering matrix is obtained (see ref. 1).

A considerable reduction in the actual number of calculations can be achieved by taking advantage of the periodicity and symmetries of the array configuration; however, there still exists the need to calculate the mutual admittance between elements which are many wavelengths apart. An earlier analysis (ref. 1) requires the numerical evaluation of an integral for the calculation of mutual admittance between elements of the array. It has been found that the computer time required for the numerical integration increases as the distance between the pair of elements increases. The purpose of the present paper is to present an asymptotic evaluation of the integral for large element separations.

SYMBOLS

a	radius of aperture
$\bar{E}_1$	electric field vector
$\bar{E}_1(x,y)$	functional form of $\bar{E}_1$
$\bar{E}_1(k_x,k_y)$	Fourier transform of $\bar{E}_1(x,y)$
E_{1_x},E_{1_y}	x and y components of $\bar{E}_1$
$E_{1_x}(\alpha,\beta),E_{1_y}(\alpha,\beta)$	Fourier transforms of E_{1_x} and E_{1_y}
$f(\beta)$	functional notation
$f'(\beta)$	first derivative of $f(\beta)$ with respect to β
$f''(\beta)$	second derivative of $f(\beta)$ with respect tc β
$G_1(\beta)$	quantity defined by equation (7)
$G_2(\beta)$	quantity defined by equation (8)
$g(\beta)$	functional notation
$\bar{H}_2$	magnetic field vector
$\bar{H}_2(x,y)$	functional form of $\bar{H}_2$
$\bar{H}_2(k_x,k_y)$	Fourier transform of $\bar{H}_2(x,y)$
H_{2_x},H_{2_y}	x and y components of $\bar{H}_2$
$H_{2_x}(\alpha,\beta),H_{2_y}(\alpha,\beta)$	Fourier transforms of H_{2_x} and H_{2_y}
$J_n(w)$	Bessel function of the first kind of order n and argument w
$J_n'(w)$	first derivative of $J_n(w)$ with respect to w
$j = \sqrt{-1}$	
$k_o = 2\pi/\lambda$	plane wave propagation constant in free space
k_x,k_y,k_z	wave propagation constants in x,y,z direction
$\hat{n}_1$	unit outward normal to aperture 1
R	center-to-center spacing of apertures

r, θ, ϕ	spherical coordinate variables
S_1	area of aperture 1
$S_0(k_o \beta R)$	quantity defined by equation (11)
$S_2(k_o \beta R)$	quantity defined by equation (12)
V_1, V_2	modal voltages
x, y, z	variables in Cartesian coordinate system
$Y_{12} = G_{12} + jB_{12}$	complex mutual admittance
$\hat{z}$	unit vector in z direction
α	angular Fourier transform variable in cylindrical coordinate system
β	normalized radial Fourier transform variable in cylindrical coordinate system
δ	infinitesimal value
ε_o	permittivity of free space
λ	free space wave length
μ_o	permeability of free space

THEORY

The mutual admittance between two waveguide-fed apertures can be determined by the electromagnetic reaction between the equivalent electric and magnetic currents (ref. 1), i.e.

$$Y_{12} = \frac{1}{V_1 V_2} \iint_{S_1} [\bar{E}_1 \times \bar{H}_2] \cdot \hat{n}_1 \, dS_1 \tag{1}$$

where Y_{12} is the mutual admittance between apertures 1 and 2, V_1 and V_2 are the normalized modal voltages, S_1 is the area of aperture 1, $\hat{n}_1$ is the outward unit normal to S_1, $\bar{E}_1$ is the assumed electric field excitation of aperture 1, and $\bar{H}_2$ is the short circuit magnetic field due to an assumed excitation of aperture 2.

If the two apertures lie in the x,y plane, as shown in figures 1 and 2, then the magnetic field outside aperture 1 due to the excitation of aperture 2 can be expressed in the form

$$\bar{H}_2 = \bar{H}_2(x,y)\, e^{-jk_z z} \tag{2}$$

where k_z is the wave propagation constant in the z direction. Then (1) can be written as

$$Y_{12} = \frac{1}{V_1 V_2} \iint_{S_1} \left[\bar{E}_1(x,y) \times \lim_{\delta \to 0} \left(\bar{H}_2(x,y) e^{-jk_z \delta} \right) \right] \cdot \hat{z} \, dx\, dy \tag{3}$$

The evaluation of (3) can be facilitated by letting $\delta = r \cos\theta$ and using a form of Parseval's theorem (ref. 2, eq. (C-15)). Then

$$Y_{12} = \frac{1}{V_1 V_2 (2\pi)^2} \lim_{\theta \to \pi/2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\bar{E}_1(k_x,k_y) \times \bar{H}_2(-k_x,-k_y)\, e^{-jk_z r \cos\theta} \right] \cdot \hat{z}\, dk_x\, dk_y \qquad (4)$$

where k_x and k_y are the wave propagation constants in the x and y directions.

The mutual admittance can also be expressed in terms of the two-dimensional Fourier transforms of the assumed aperture electric fields and the solutions to the wave equations for the region outside the aperture plane. The details are outlined in ref. 1; therefore (4) becomes

$$Y_{12} = \frac{k_o\sqrt{\frac{\varepsilon_o}{\mu_o}}}{V_1 V_2 (2\pi)^2} \lim_{\theta \to \pi/2} \int_{\beta=0}^{\infty}\int_{\alpha=0}^{2\pi} e^{-jk_o r\sqrt{1-\beta^2}\cos\theta}$$

$$\cdot \left[\frac{1}{\sqrt{1-\beta^2}} \left(E_{1_x}(\alpha,\beta)\cos\alpha + E_{1_y}(\alpha,\beta)\sin\alpha \right)\left(E_{2_x}(\alpha,-\beta)\cos\alpha + E_{2_y}(\alpha,-\beta)\sin\alpha \right) \right.$$

$$\left. + \sqrt{1-\beta^2} \left(E_{1_x}(\alpha,\beta)\sin\alpha - E_{1_y}(\alpha,\beta)\cos\alpha \right)\left(E_{2_x}(\alpha,-\beta)\sin\alpha - E_{2_y}(\alpha,-\beta)\cos\alpha \right) \right]$$

$$\cdot\ \beta\ d\beta\ d\alpha \qquad (5)$$

where a change of variables has been made in the transform domain such that $k_x = k_o\beta\cos\alpha$ and $k_y = k_o\beta\sin\alpha$.

If both apertures are excited by the circular waveguide TE_{11} mode, then the mutual admittance becomes (details are in ref. 1)

$$Y_{12} = 2\sqrt{\frac{\varepsilon_o}{\mu_o}} \left[\frac{1}{(1.841)^2 - 1}\right] \lim_{\theta \to \pi/2} \int_{\beta=0}^{\infty} e^{-jk_o r \sqrt{1-\beta^2}\cos\theta}$$

$$\cdot \Bigg[G_1(\beta)\Big(J_o(k_o\beta R) + J_2(k_o\beta R)\cos 2\phi\Big)$$

$$+ G_2(\beta)\Big(J_o(k_o\beta R) - J_2(k_o\beta R)\cos 2\phi\Big)\Bigg]\beta \, d\beta \tag{6}$$

where

$$G_1(\beta) = \frac{1}{\sqrt{1-\beta^2}} \left[\frac{J_1(k_o a\beta)}{\beta}\right]^2 \tag{7}$$

$$G_2(\beta) = \sqrt{1-\beta^2} \left[\frac{\left(\frac{1.841}{k_o a}\right)^2 J_1'(k_o a\beta)}{\left(\frac{1.841}{k_o a}\right)^2 - \beta^2}\right]^2 \tag{8}$$

Now using the semiconvergent series of Hankel (see pages 137 and 138 of ref. 3), $J_0(k_o\beta R)$ and $J_2(k_o\beta R)$ can be expressed as

$$J_o(k_o\beta R) = \frac{1}{\sqrt{2\pi}} e^{-j\frac{\pi}{4}} \left[\frac{e^{jk_o\beta R}}{\sqrt{k_o\beta R}} S_o(k_o\beta R) + j\frac{e^{-jk_o\beta R}}{\sqrt{k_o\beta R}} S_o(-k_o\beta R)\right] \tag{9}$$

$$J_2(k_o\beta R) = \frac{1}{\sqrt{2\pi}} e^{-j\frac{5\pi}{4}} \left[\frac{e^{jk_o\beta R}}{\sqrt{k_o\beta R}} S_2(k_o\beta R) + j\frac{e^{-jk_o\beta R}}{\sqrt{k_o\beta R}} S_2(-k_o\beta R)\right] \tag{10}$$

where

$$S_o(k_o\beta R) = 1 - j\left(\frac{1}{k_o\beta R}\right) - \frac{9}{128}\left(\frac{1}{k_o\beta R}\right)^2 + j\frac{75}{1024}\left(\frac{1}{k_o\beta R}\right)^3 + \ldots \tag{11}$$

$$S_2(k_o\beta R) = 1 + j\frac{15}{8}\left(\frac{1}{k_o\beta R}\right) - \frac{105}{128}\left(\frac{1}{k_o\beta R}\right)^2 + j\frac{315}{1024}\left(\frac{1}{k_o\beta R}\right)^3 + \ldots \tag{12}$$

The mutual admittance then becomes

$$Y_{12} = 2\sqrt{\frac{\varepsilon_o}{\mu_o}}\ \frac{1}{\sqrt{2\pi}}\left[\frac{e^{-j\frac{\pi}{4}}}{(1.841)^2 - 1}\right]\lim_{\theta\to\pi/2}\int_0^\infty e^{-jk_o r\sqrt{1-\beta^2}\cos\theta}$$

$$\cdot\left\{\frac{e^{jk_o\beta R}}{\sqrt{k_o\beta R}}\left[\Big(G_2(\beta) - G_1(\beta)\Big)S_2(k_o\beta R)\cos 2\phi + \Big(G_2(\beta) + G_1(\beta)\Big)S_o(k_o\beta R)\right]\right.$$

$$\left. - \frac{e^{-jk_o\beta R}}{\sqrt{-k_o\beta R}}\left[\Big(G_2(\beta) - G_1(\beta)\Big)S_2(-k_o\beta R)\cos 2\phi + \Big(G_2(\beta) + G_1(\beta)\Big)S_o(-k_o\beta R)\right]\right\}$$

$$\cdot\ \beta\, d\beta \qquad (13)$$

If β is substituted for $-\beta$ in terms which contain $e^{-jk_o\beta R}$ and we let $R = r\sin\theta$, (13) can be expressed as

$$Y_{12} = 2\sqrt{\frac{\varepsilon_o}{\mu_o}}\ \frac{1}{\sqrt{2\pi}}\left[\frac{e^{-j\frac{\pi}{4}}}{(1.841)^2 - 1}\right]\lim_{\theta\to\pi/2}\int_{-\infty}^\infty e^{-jr\left(k_o\beta\sin\theta - k_o\sqrt{1-\beta^2}\cos\theta\right)}$$

$$\cdot\ \frac{1}{\sqrt{k_o\beta r\sin\theta}}\left[\Big(G_2(\beta) - G_1(\beta)\Big)S_2(k_o\beta r\sin\theta)\cos 2\phi\right.$$

$$\left. + \Big(G_2(\beta) + G_1(\beta)\Big)S_o(k_o\beta r\sin\theta)\right]\beta\, d\beta \qquad (14)$$

An integral of the form in (14) can be evaluated asymptotically for large values of r (ref. 4), i.e.

$$\int_{-\infty}^\infty g(\beta)e^{jrf(\beta)}\, d\beta \sim e^{jrf(\beta_o)} g(\beta_o)\sqrt{\frac{2\pi j}{rf''(\beta_o)}} \qquad (15)$$

where

$$f'(\beta)\Big|_{\beta=\beta_o} = 0 \tag{16}$$

$$f''(\beta_o) = f''(\beta)\Big|_{\beta=\beta_o} \neq 0 \tag{17}$$

From (15), we have

$$f(\beta) = k_o\left(\beta \sin \theta - \sqrt{1 - \beta^2} \cos \theta\right) \tag{18}$$

therefore,

$$\beta_o = -\sin \theta \tag{19}$$

$$f(\beta_o) = - k_o \tag{20}$$

$$f''(\beta_o) = \frac{k_o}{\cos^2 \theta} \tag{21}$$

and the asymptotic evaluation of the mutual admittance for large spacing becomes

$$\begin{aligned} Y_{12} \approx 2j \sqrt{\frac{\varepsilon_o}{\mu_o}} &\left[\frac{1}{(1.841)^2 - 1}\right] \left[J_1(k_o a)\right]^2 \left[\frac{e^{-jk_oR}}{k_oR}\right] \\ &\cdot \left\{ \left[2 \sin^2\phi - \frac{3}{128}\left(3 - 35 \cos 2\phi\right)\left(\frac{1}{k_oR}\right)^2\right] \right. \\ &\left. + j\left(\frac{1}{k_oR}\right)\left[\frac{1}{8}\left(1 + 15 \cos 2\phi\right) - \frac{15}{1024}\left(5 - 21 \cos 2\phi\right)\left(\frac{1}{k_oR}\right)^2\right] \right\} \end{aligned} \tag{22}$$

RESULTS

Calculations are given in figures 3 and 4 for the mutual admittance of circular waveguide-fed apertures in both E-plane ($\phi = \pi/2$) and H-plane ($\phi = 0$) coupling configurations. A comparison is made between the asymptotic evaluation (equation 22) and the numerical evaluation (ref. 1) as a function of aperture spacing. Good agreement with the mutual admittance obtained by numerical integration is indicated for element spacings greater than two wavelengths.

CONCLUSION

The present analysis has been shown to give good results for elements spaced greater than two wavelengths. The judicious use of the asymptotic expression for mutual admittance together with the numerical integration method could result in a considerable savings in computer time when calculating the effects of mutual coupling in a large planar array.

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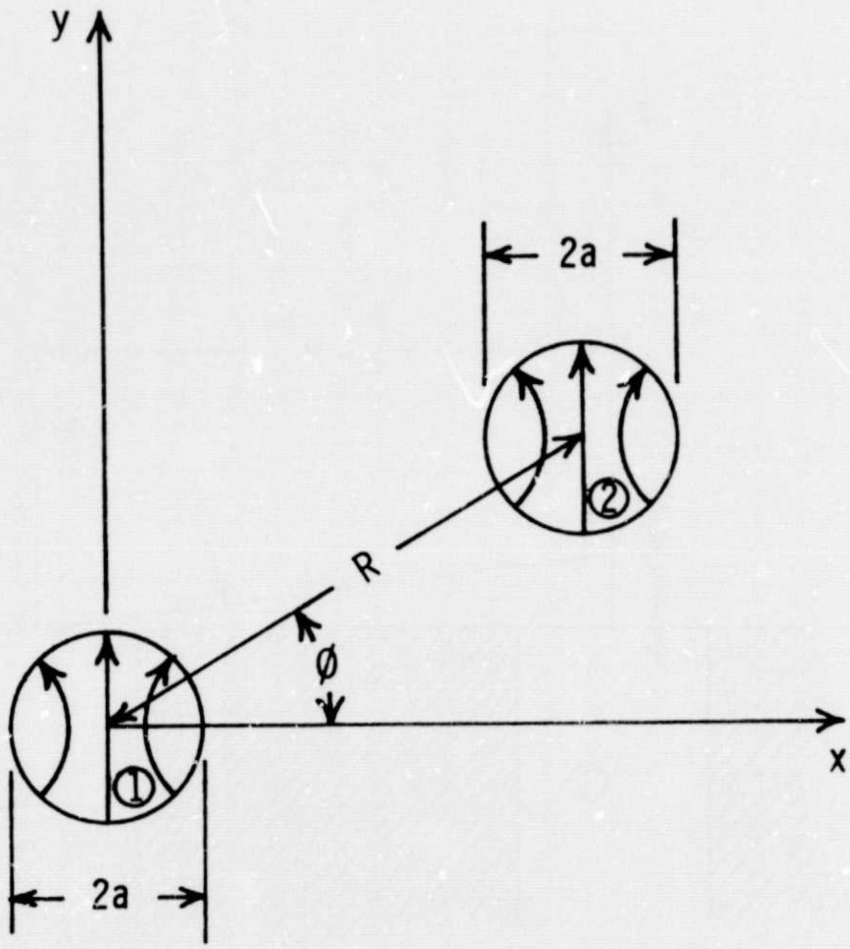

Figure 1. Circular waveguide-fed apertures in x,y plane.

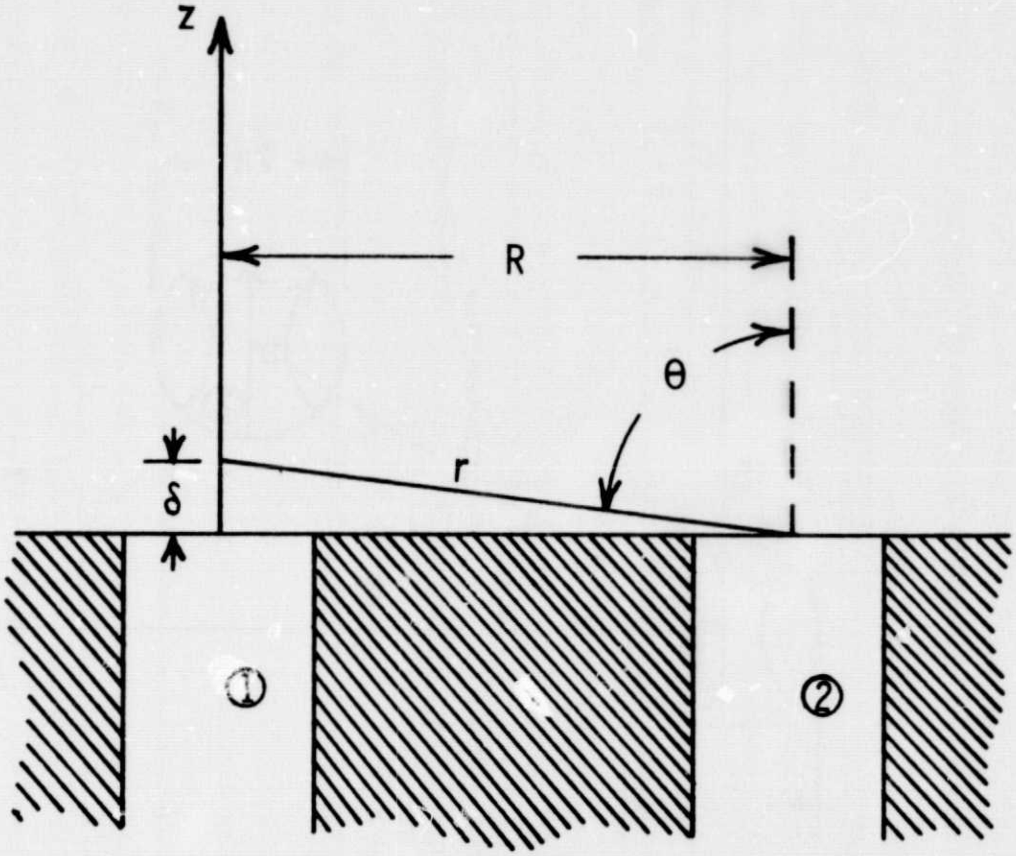

Figure 2. Cross sectional drawing of circular waveguide-fed apertures.

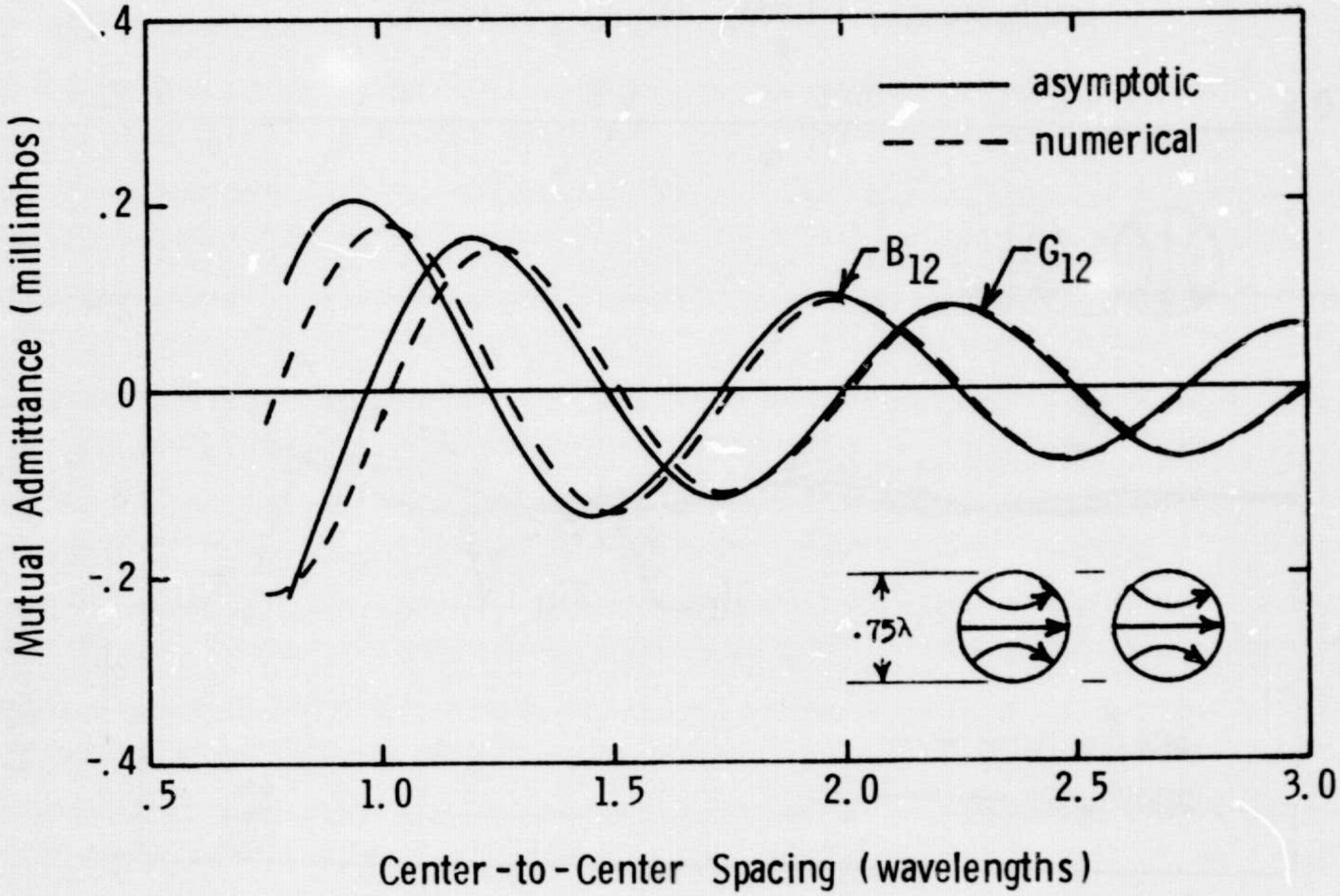

Figure 3. Calculated mutual admittance of two circular waveguide-fed apertures for E-plane coupling.

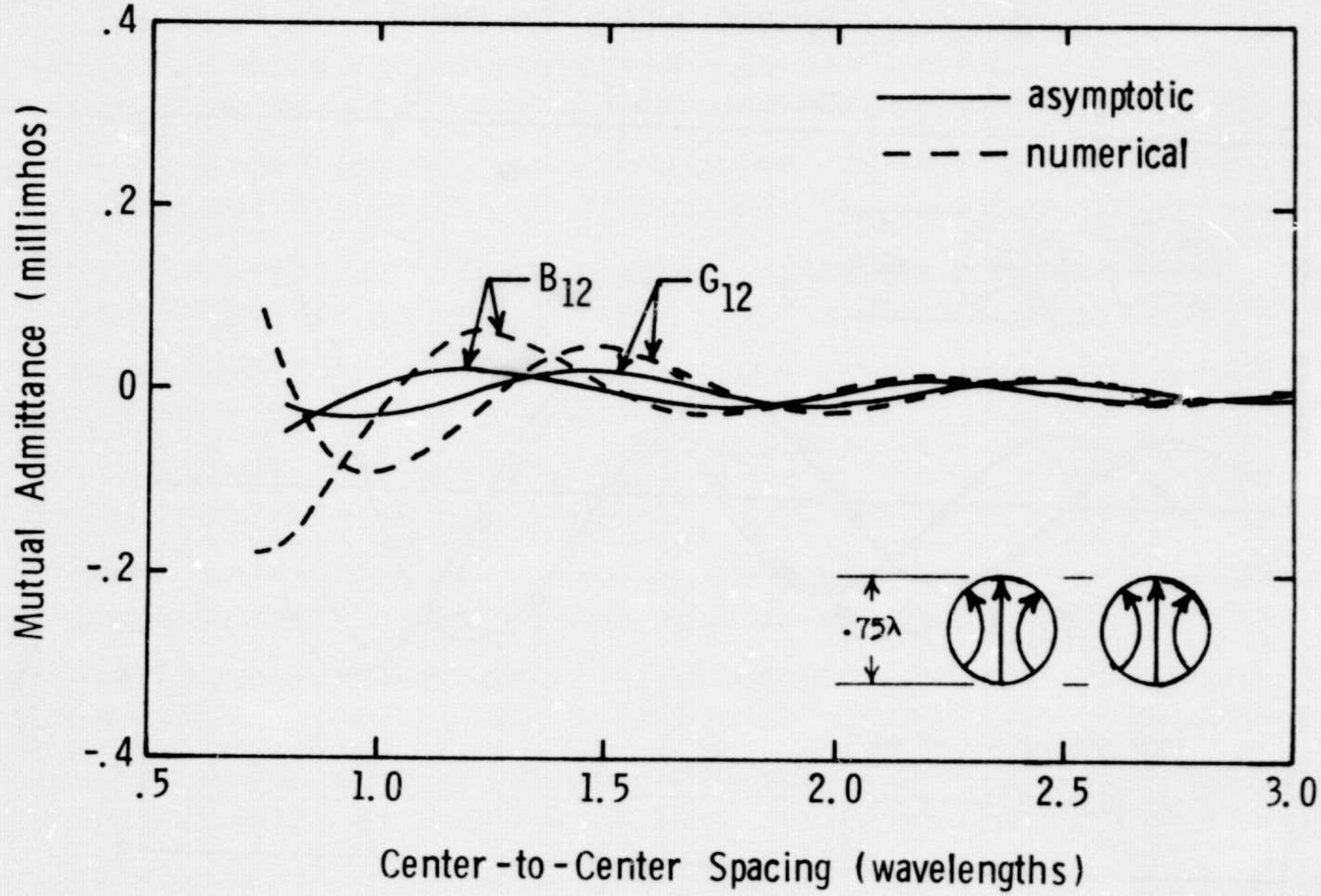

Figure 4. Calculated mutual admittance of two circular waveguide-fed apertures for H-plane coupling.

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