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INITIAL DEVELOPMENT OF A METHOD OF SIGNIFICANT WAVEHEIGHT ESTIMATION FOR GEOS-III

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16. Abstract

Details are presented for the numerical method by which ocean surface significant waveheight estimates are produced from the GEOS-3 radar altimeter data. Four parameters characterizing the expected radar mean return waveform are determined from the altimeter's sixteen sample-and-hold waveform samplers through use of an iterative, least-squares approach. One of the four parameters is a risetime term which has contributions from both the transmitted radar pulse width and the rms ocean surface height; the estimated significant waveheight is extracted from this risetime. The noise character of the estimation process leads to occasional negative significant waveheight estimates, and the origin of these non-physical negative results is discussed. Possible modifications and areas for additional investigation are indicated.

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1.0 INTRODUCTION AND SUMMARY OF RESULTS

Work accomplished under this portion of Contract NAS6-2520 comprises derivation of an approximate numerical technique for use in estimating significant waveheight ($H_{1/3}$) from the GEOS-III radar altimeter waveform data.

The expression developed for $H_{1/3}$ has been incorporated into the GEOS-III data processing system at Wallops Fligh: Center, and was utilized in a near-realtime mode during the February 1976 North Atlantic High Sea State Exercise just concluded.

The algorithm currently in use represents only the result of an initial development designed for early computerization so that the data processing system would be complete. Work on refinement of this method as well as other approaches to $H_{1/3}$ estimation is still in progress under follow-on tasks.

2.0 SIGNIFICANT WAVEHEIGHT ESTIMATION

2.1 Background

The feasibility of estimating sea surface significant waveheight ($H_{1/3}$), using short pulse radar altimetry has previously been established [1]. Theoretical limits on $H_{1/3}$ determination resolution have also been established [2].

The algorithm developed in this task for $H_{1/3}$ estimation from GEOS-III waveform data is an extension of the analytical work previously done [3]. This prior work established a means of describing an altimeter average return waveform using the four parameters: A, t_o, β, and ξ, where

- A was a general waveform amplitude factor,
- t_o was a reference time,
- β was a pulsewidth-risetime parameter, and
- ξ was antenna off-nadir pointing angle.

The parameter β included effects of both the altimeter transmitted pulse width and the rms ocean surface roughness. The effect of changes in ocean surface roughness is primarily to alter the slope of the average ocean-scattered radar return's leading edge; the present $H_{1/3}$ estimation algorithm has been developed from this emphasis on slope of the rise-time portion of the return. For the GEOS-III altimeter, the pointing angle effects in the leading edge of the return can be ignored because of the relatively broad beamwidth of the antenna pattern; in this case the more complicated mean return waveform of Reference 3 and other work reduces to simply an integrated Gaussian (the Probability Integral). One additional parameter has been added, a factor to account for a possible non-zero baseline in the altimeter's waveform sampler-recorder average return signal. The $H_{1/3}$ problem has then been reduced to one of determining four parameters characterizing the mean return waveform using suitable time-averaging of the 16 waveform S&H gates on GEOS-III, followed by recovery of the sea-state-caused portion of the risetime parameter.

Since changes in surface roughness alter the slope of the average return leading edge, only the rise-time portion of the return need be considered in estimating surface roughness. The algorithm developed for this estimation, therefore, is derived on this basis. Thus it involves the four parameters a, b, c, and d, where:

a = return amplitude

b = time origin

c = return rise-time, and

d = return baseline.

For the GEOS-III altimeter, the pointing angle effects in the leading edge of the return can generally be ignored due to the relatively broad beamwidth of the radar antenna pattern. It is necessary to introduce a factor which accounts for a possible nonzero baseline in the sampler recorded average return - this is the parameter "d".

2.2 Development of $H_{1/3}$ Estimation Method

Based on an iterative, least squares approach [4], a means of estimating the parameter "c" (from which $H_{1/3}$ can be derived) is presented in the following; equations 1-6 and accompanying discussion are for the general 4-parameter least-squares problem.

Then, let there be a four-parameter theoretical function, y_t, which is a function of the independent variable t;

$$y_t = y(a,b,c,d,t) \quad . \tag{1}$$

Write the first-order Taylor series approximation to this (at a point a_o, b_o, c_o, and d_o in the parameter space) as follows;

$$y_t \doteq y_o + y_a\alpha + y_b\beta + y_c\gamma + y_d\delta \tag{2}$$

where

$$y_o = y(a_o, b_o, c_o, t) \quad ,$$

$$y_a = \left.\frac{dy}{da}\right|_{\substack{a_o, b_o, \\ c_o, d_o, \\ t}} \quad ,$$

$$y_b = \left.\frac{dy}{db}\right|_{\substack{a_o, b_o, \\ c_o, d_o, \\ t}} \quad ,$$

$$y_c = \left.\frac{dy}{dc}\right|_{\substack{a_o, b_o, \\ c_o, d_o, t}} \quad ,$$

$$y_d = \left.\frac{dy}{dd}\right|_{\substack{a_o, b_o, \\ c_o, d_o, t}}$$

and $\alpha = a - a_o$, $\beta = b - b_o$, $\gamma = c - c_o$ and $\delta = d - d_o$. Note that the factors y_i, $i = a, \ldots, d$, are functions of t.

Then form a sum over all N values (y_i, t_i) of the errors squared;

$$E = \sum_{i=1}^{N} (y_t - y_i)^2 = \sum_i (\alpha y_a + \beta y_b + \gamma y_c + \delta y_d + y_o - y_i)^2 \tag{3}$$

Minimizing E with respect to α ,

$$\frac{dE}{d\alpha} = 0 = 2 \sum_i (\alpha y_a + \beta y_b + \gamma y_c + \delta y_d + y_o - y_i) y_a \tag{4}$$

or

$$\alpha \sum_i y_a^2 + \beta \Sigma y_a y_b + \gamma \Sigma y_a y_c + \delta \Sigma y_a y_d = \Sigma y_a (y_i - y_o) \tag{5}$$

Similar equations follow from $\frac{dE}{d\beta}$, $\frac{dE}{d\gamma}$, $\frac{dE}{d\delta}$, and all four such equations rewritten in matrix form give:

$$\begin{pmatrix} \Sigma y_a^2 & \Sigma y_a y_b & \Sigma y_a y_c & \Sigma y_a y_d \\ \Sigma y_a y_b & \Sigma y_b^2 & \Sigma y_b y_c & \Sigma y_b y_d \\ \Sigma y_a y_c & \Sigma y_b y_c & \Sigma y_c & \Sigma y_c y_d \\ \Sigma y_a y_d & \Sigma y_b y_d & \Sigma y_c y_d & \Sigma y_d^2 \end{pmatrix} \times \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \Sigma y_a (y_i - y_o) \\ \Sigma y_b (y_i - y_o) \\ \Sigma y_c (y_i - y_o) \\ \Sigma y_d (y_i - y_o) \end{pmatrix} \tag{6}$$

Returning now to the specific $H_{1/3}$ estimation algorithm for which the specific y_t is taken as the Probability Integral on a baseline, let

$$y_t = a\, P\left(\frac{t-b}{c}\right) + d$$

where $P(\cdot)$ is the Probability Integral, $P(z) = \int_{-\infty}^{z} Z(q)dq$,

where $Z(q)$ is the Gaussian function, $Z(q) = \frac{1}{\sqrt{2\pi}} e^{-q^2/2}$

and the parameters a,b,c,d, have the physical meanings

a = amplitude

b = time origin

c = risetime

d = baseline .

In terms of Z, P as defined above,

$$y_o = a_o P\left(\frac{t-b_o}{c_o}\right) + d_o \qquad , \tag{7}$$

$$y_a = P\left(\frac{t-b_o}{c_o}\right) \qquad ,$$

$$y_b = \frac{-a_o}{c_o} Z\left(\frac{t-b_o}{c_o}\right) \qquad ,$$

$$y_c = \frac{-a_o}{c_o}\left(\frac{t-b_o}{c_o}\right) Z\left(\frac{t-b_o}{c_o}\right) ,$$

and $$y_d = 1 \qquad .$$

Rewriting the 4-matrix from the least-squares non-linear problem,

$$\begin{pmatrix} M_1 & M_2 & M_3 & M_4 \\ M_2 & M_5 & M_6 & M_7 \\ M_3 & M_6 & M_8 & M_9 \\ M_4 & M_7 & M_9 & M_{10} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} M_{11} \\ M_{12} \\ M_{13} \\ M_{14} \end{pmatrix} \tag{8}$$

Substituting in the values of y_a, y_b, y_c, y_d etc.,

$$M_1 = \sum_{i=1}^{N} \left[P\left(\frac{t_i-b_o}{c_o}\right)\right]^2$$

$$M_2 = -\frac{a_o}{c_o}\sum_{i=1}^{N} P\left(\frac{t_i - b_o}{c_o}\right) Z\left(\frac{t_i - b_o}{c_o}\right) ,$$

$$M_3 = -\frac{a_o}{c_o}\sum_{i=1}^{N} \left(\frac{t_i - b_o}{c_o}\right) P\left(\frac{t_i - b_o}{c_o}\right) Z\left(\frac{t_i - b_o}{c_o}\right) ,$$

$$M_4 = \sum_{i=1}^{N} P\left(\frac{t_i - b_o}{c_o}\right) ,$$

$$M_5 = +\frac{a_o^2}{c_o^2}\sum_{i=1}^{N} \left[Z\left(\frac{t_i - b_o}{c_o}\right)\right]^2 ,$$

$$M_6 = +\frac{a_o^2}{c_o^2}\sum_{i=1}^{N} \left(\frac{t_i - b_o}{c_o}\right)\left[Z\left(\frac{t_i - b_o}{c_o}\right)\right]^2 ,$$

$$M_7 = -\frac{a_o}{c_o}\sum_{i=1}^{N} Z\left(\frac{t_i - b_o}{c_o}\right) ,$$

$$M_8 = +\frac{a_o^2}{c_o^2}\sum_{i=1}^{N} \left[\left(\frac{t_i - b_o}{c_o}\right) Z\left(\frac{t_i - b_o}{c_o}\right)\right]^2 ,$$

$$M_9 = \frac{a_o}{c_o}\sum_{i=1}^{N} \left(\frac{t_i - b_o}{c_o}\right) Z\left(\frac{t_i - b_o}{c_o}\right) ,$$

$$M_{10} = N ,$$

$$M_{11} = \sum_{i=1}^{N} P\left(\frac{t_i - b_o}{c_o}\right)\left[-a_o P\left(\frac{t_i - b_o}{c_o}\right) - d_o + y_i\right] = \sum_{i=1}^{N} y_i P\left(\frac{t_i - b_o}{c_o}\right) - a_o M_1 - d_o M_4 ,$$

$$M_{12} = -\frac{a_o}{c_o}\sum_{i=1}^{N} Z\left(\frac{t_i-b_o}{c_o}\right)\left[-a_oP\left(\frac{t_i-b_o}{c_o}\right)-d_o+y_i\right] = -\frac{a_o}{c_o}\sum_{i=1}^{N} y_iZ\left(\frac{t_i-b_o}{c_o}\right)-a_oM_2-d_oM_7 ,$$

$$M_{13} = -\frac{a_o}{c_o}\sum_{i=1}^{N}\left(\frac{t_i-b_o}{c_o}\right) Z\left(\frac{t_i-b_o}{c_o}\right)\left[-a_oP\left(\frac{t_i-b_o}{c_o}\right)-d_o+y_i\right]$$

$$= -\frac{a_o}{c_o}\sum_{i=1}^{N} y_i\left(\frac{t_i-b_o}{c_o}\right) Z\left(\frac{t_i-b_o}{c_o}\right) - a_oM_3 - d_oM_9$$

$$M_{14} = 1\sum_{i=1}^{N}\left[-a_oP\left(\frac{t_i-b_o}{c_o}\right)-d_o+y_i\right] = +\sum_{i=1}^{N} y_i - a_oM_4 - d_oM_{10}$$

Completion of the problem requires finding the inverse of the 4 by 4 matrix on the left side of (8) above in order to solve for α, β, γ, and δ. The initial guesses a_o, b_o, c_o, and d_o are corrected to produced revised estimates $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$ since $\hat{a} \doteq a_o+\alpha$, $\hat{b} \doteq b_o+\beta$, $\hat{c} \doteq c_o+\gamma$, and $\hat{d} \doteq d_o+\delta$. The "$\doteq$" sign is used to emphasize that $\hat{a} \neq a$; since only the first order terms were retained in the original Taylor series at equation (2), and a by definition was $a = a_o + \delta$, thus $\hat{a}$ is only an approximate estimate of a. Similarly $\hat{b}$ is an estimate of b, $\hat{c}$ of c, and $\hat{d}$ of d.

In this iterative procedure, the next step then is to use $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$ in place of the original a_o, b_o, c_o, d_o, and repeat the entire cycle to produce (presumably) improved estimates of a, b, c, d. The elements M_1 through M_{14} above are of course re-evaluated each time through the cycle; the indicated sums are for the N=16 separate values from the 16 GEOS-III waveform samplers. The iterative procedure is repeated until a defined relative error criterion is satisfied (unless the matrix inversion fails first and causes an error exit); a typical loop exit criterion is to require that the total sum of errors squared decrease by less than 0.1% of the sum of errors squared in the last previous time through the loop. For a reasonable first guess a_o, b_o, c_o, d_o (and "reasonable" first guesses can be as much as a factor of two away from the final answer in our experience with the particular functional form in this $H_{1/3}$-related problem), the problem will converge in typically two to four iterations to reasonable final answers for a,b,c, and d. Given the risetime parameter, c, determined by this iterative least-squares procedure above, the estimation of $H_{1/3}$ is as follows:

The convolutional model for the mean sea-scattered pulse return assumes that the risetime parameter c is the composite result from the calm sea (assumed Gaussian) equivalent radar pulsewidth σ_c ("equivalent" implies inclusion of transmitter and receiver bandwidths, etc.) and the rough-sea rms heights σ_s.* Because of the convolution of Gaussians of widths σ_s and σ_c,

$$c^2 = \sigma_s^2 + \sigma_c^2 \tag{9}$$

or

$$\sigma_s = \sqrt{c^2 - \sigma_c^2}$$

Here σ_s is two-way ranging time in units of nanoseconds. Conversion of ranging times (nanoseconds) to surface height (meters) is accomplished by multiplying by the two-way speed of light = .15 m/ns. Since $H_{1/3}$ is four times the rms surface height relative to mean sea level, σ_s must also be multiplied by four;

$$\begin{aligned} H_{1/3} &= 4(.15)\sqrt{c^2 - \sigma_c^2} \qquad (10) \\ &= 0.6\sqrt{c^2 - \sigma_c^2}\ , \ (H_{1/3} \text{ in m, c and } \sigma_c \text{ in ns}). \end{aligned}$$

Expression (10) is true only in a limiting sense. Actually the c in (10) is an experimental estimate based on a finite data span, and as such, c is noisy and fluctuating. The quantity under the radical in (10) is consequently capable of going negative in some data spans and thus ending the problem since any practical computer program will terminate upon trying to take the square root of a negative number. To avoid this, (10) is modified to

$$H_{1/3} = 0.6\sqrt{|c^2 - \sigma_c^2|} \times \mathrm{Sign}(|c^2 - \sigma_c^2|) \quad . \tag{11}$$

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*Tracking loop jitter has been ignored.

Our bringing out the sign from within the radical was done in order to allow recovery of the quantity c^2 (knowing already the value of σ_c) for any possible c value o to ∞. The actual value of σ_c used in expression (11) was based upon early observations of several passes over known calm seas; the value currently used is $\sigma_c = 8.55$ ns, but this should be reexamined now that considerably more data are available.

3.0 CONCLUSIONS, RECOMMENDATIONS, AND FUTURE PLANS

The $H_{1/3}$ algorithm just described has been utilized in the GEOS-III data processing programs at Wallops Flight Center; specifically it has been installed in the computer program GAP more fully described in Leitao et al. [5]. Work now underway is comparing this $H_{1/3}$ product with other ground truth efforts particularly for the February 1976 North Atlantic data.

Additional work is now being done on the $H_{1/3}$ algorithm itself in two areas. First, the $H_{1/3}$ results can be smoothed over a several data frame period by either averaging the individual-frame $H_{1/3}$ results or by averaging together the waveform sample-and-hold gate results over the several-frame interval and using these to produce a single $H_{1/3}$ value representing the several-frame period. Averaging the single-frame $H_{1/3}$ results is simpler to do but we are now examining carefully the question of whether this estimate converges to that from the waveform averaging approach.

Second, the effects of tracker jitter are now being examined; these were left out of the above-described first version of a $H_{1/3}$ procedure. The outcome of this additional work will be reported later.

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