NOISE TRANSMISSION BY VISCOELASTIC SANDWICH PANELS

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16. Abstract

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SUMMARY

This report presents an analytical study on low frequency noise transmission into rectangular enclosures by viscoelastic sandwich panels. The dimensions of these panels are typical of aircraft fuselage skin constructions. The report demonstrates that these panels can effect significant noise reduction. The analysis considers two limiting cases of the core stiffness: (1) soft compressible cores with dilatational modes included and (2) hard incompressible cores with dilatational modes neglected. Sandwich panels with soft viscoelastic cores exhibit noise transmission characteristics similar to those of double wall elastic panels except in the frequency range where dilatational modes occur (300 to 600 Hz). At these frequencies noise can be reduced by as much as 30 dB more with viscoelastic panels. Sandwich constructions with hard cores show significant noise reduction advantage over the elastic panels for the whole frequency range considered in this study (0 to 1000 Hz). Numerical results contained in this report include response and noise transmission characteristics of elastic and viscoelastic panels for several geometric and material parameter configurations.

INTRODUCTION

The information available in the literature and from ongoing research programs on interior aircraft noise indicates that noise in many aircraft exceeds acceptable comfort limits. Propeller driven aircraft where maximum noise intensity occurs at low frequencies are especially blameworthy. Since acoustic absorption materials used in aircraft constructions are not very effective in reducing interior noise at low frequencies (ref. 1), new means of providing noise attenuation at low frequencies need to be established. Interior aircraft noise in the structural resonance range is strongly controlled by the vibrational characteristics of the fuselage skin panels. Past studies have demonstrated that a viscoelastic material sandwiched between two elastic plates is a very efficient way of dissipating vibrational energy (refs. 2 to 9). Replacing some of the elastic skin panels with viscoelastic sandwich constructions should achieve significant amounts of noise reduction. Since available information on noise transmission characteristics of viscoelastic sandwich panels which are suitable for aircraft constructions is very limited, an analytical study of this subject is undertaken in this paper. The actual aircraft fuselage construction is too complicated for a detailed mathematical treatment; therefore, a simplified analytical model has been constructed. The interior acoustic enclosure into which noise was transmitted is taken as a rectangular box. One wall is

flexible, and the remaining walls are rigid. (See fig. 1.) The dimensions of the flexible panel were chosen from typical aircraft skin panels, and the depth of the acoustic enclosure corresponds to about one-half of the aircraft cabin width.

In this paper the noise transmission into the enclosure is analyzed by solving the linearized acoustic wave equation for the interior noise field and the plate vibration equation for the viscoelastic sandwich panel vibrations. The acoustic equation is coupled to panel vibrations through the time-dependent boundary conditions. The solution to this system of equations is obtained by using modal expansions and a Galerkin-like procedure. Since the boundary conditions for these equations are time dependent, the commonly used method of separation of variables cannot be applied to this system (ref. 10). The time dependence, however, is removed by splitting the solution into two parts: a solution corresponding to a nonhomogeneous differential equation with homogeneous boundary conditions and a solution on the boundary. Following this procedure, a Fourier series solution is developed which converges rapidly not only in the interior acoustic space but also on the boundary. The solution possesses continuous derivatives up to the second order and satisfies the given partial differential equation and all the boundary conditions. It can be demonstrated that such a series solution is uniformly convergent (ref. 11). These series have a computational advantage over the nonuniformly convergent series which usually converge very slowly.

The governing differential equations for the vibration of the viscoelastic sandwich panels due to the prescribed external pressure are developed for two limiting cases of core stiffness. In both of these cases the core material is assumed to be isotropic and the elastic plates are perfectly bonded to the core so that no displacement discontinuities are present. In the first case, the viscoelastic material is taken to be very soft so that bending and shearing stresses can be neglected, and the core acts merely as a viscoelastic spring. For the sandwich construction with this soft core, the flexural vibration modes are governed by the stiffness of the two face plates, and the out-of-phase dilatational modes are controlled by the stiffness characteristics of the viscoelastic spring. In the second case, a stiff viscoelastic core material is used where the bending and shearing strains in the core are important. Because of the stiff character of this core material, dilatational modes occur at high frequencies. By limiting the analysis to low frequencies (below 1000 Hz), the core material can be assumed to be incompressible in this frequency range, and the out-of-phase motions of the two elastic face panels associated with the dilatational modes can be neglected. Furthermore, the stiff core model is simplified to satisfy the following assumptions: the shearing strains across the depth of the face plates are small, the stresses in the core material parallel to the plate surface are negligible, and the in-plane "rotary" inertia effects are not included. Since the analysis is limited to low frequencies and relatively thin viscoelastic cores, these assumptions are believed to be valid. For the stiff core panel, the governing equations of motion and the solution for panel vibrations are obtained from reference 2.

This report contains numerical results for a rectangular enclosure with acoustically hard walls. It is assumed that one face is covered with a flexible viscoelastic sandwich panel as shown in figure 1; otherwise, the enclosure is

2

rigid. Results include panel response spectral densities, noise transmission characteristics, and overall interior noise levels. A direct comparison of the results between elastic and viscoelastic sandwich panels is shown.

SYMBOLS

The units used for physical quantities defined in this paper are given in the International System (SI) of Units. Correlations between this system of units and U.S. Customary Units are given in reference 12.

A	conversion factor 0.000645, see reference 12
A_{ij}	variables defined in equation (11)
a	plate length, m
a_1, a_2	variables defined in equation (30)
B_F	flexible boundary
B_{ijk}	pressure modal coefficients, N/m^2
B_R	rigid boundary
b	plate width, m
C_{mn}	displacement modal coefficients of top face plate, soft core viscoelastic sandwich panel, m
$(C_p)_{ij}$	pressure coefficients, N/m^2
c	speed of sound in cavity, m/sec
c_1	damping coefficient of top elastic plate, $N\text{-}sec/m^3$
c_2	damping coefficient of bottom elastic plate, $N\text{-}sec/m^3$
D	$= Eh^3/12(1 - \nu^2)$, plate stiffness, N-m
D_{mn}	displacement modal coefficients of bottom face plate, soft core viscoelastic sandwich panel, m
D_1	$= E_1 h_1^3/12(1 - \nu_1^2)$, top face plate stiffness, N-m
D_2	$= E_2 h_2^3/12(1 - \nu_2^2)$, bottom face plate stiffness, N-m
d	cavity depth, m
E	modulus of elasticity of plate, N/m^2
E_1	modulus of elasticity of top face plate, N/m^2

3

E_2 modulus of elasticity of bottom face plate, N/m^2

E_3 modulus of elasticity of viscoelastic core, N/m^2

e_{ij} variables defined in equation (7)

e_k variables defined in equation (19)

F_{ijk} variables defined in equation (18)

f frequency, Hz

f_{mn} panel modal frequencies, Hz

G complex shear modulus defined in equation (36), N/m^2

G_{ij} variables defined in equation (6)

G_0 shear modulus of viscoelastic core, N/m^2

H_{ijk} variables defined in equation (54)

H_{mn} frequency response function of sandwich plate with hard viscoelastic core, m^3/N

H_{mn}^b frequency response function of bottom face plate in sandwich construction with soft core, m^3/N

H_{mn}^t frequency response function of top face plate in sandwich construction with soft core, m^3/N

h plate thickness, m

h_1, h_2, h_3 thickness of top face plate, bottom face plate, and viscoelastic core, respectively, m

$\left.\begin{array}{l} i,j,k,l,m, \\ n,o,q,r,s \end{array}\right\}$ indices

$\underset{\sim}{i}$ imaginary unit $(-1)^{1/2}$

K_{mn} variables defined in equation (40)

k_3 spring stiffness of soft viscoelastic core, N/m^3

L_{ijmn} variables defined in equation (55)

NT   noise transmission, dB; see equation (59)

OASPL   overall sound pressure level, dB

4

p	sound pressure in cavity, N/m^2
p_0	reference pressure, 0.00002 N/m^2
p^r	external random input pressure, N/m^2
Q_{mn}	generalized random forces, see equation (28)
R_x	spatial correlation coefficient corresponding to x-coordinate
R_y	spatial correlation coefficient corresponding to y-coordinate
r_1	variables defined in equation (44)
S_{mnrs}	cross-spectral densities of generalized random forces, $(N/m^2)^2/Hz$
S_p	spectral density of sound pressure in interior, $(N/m^2)^2/Hz$
S_p^r	spectral density of input pressure, $(N/m^2)^2/Hz$
S^r	cross-spectral density of input pressure, $(N/m^2)^2/Hz$
S_w	panel deflection response spectral density, m^2/Hz
t	time, sec
U_{mn}, V_{mn}, W_{mn}	modal displacement coefficients corresponding to x-, y-, and z-coordinates, respectively, m
u, v, w	displacement components corresponding to x-, y-, and z-coordinates, respectively, m
w_1	transverse displacement of top face plate, m
w_2	transverse displacement of bottom face plate, m
X_{ijk}	acoustic modes
x, y, z	spatial coordinates, m
Z_d	variables defined in equation (52)
Z_{ij}	variables defined in equation (13)
α_{ijk}	acoustic cavity modal damping coefficients
α_{mn}	plate modal damping coefficients
β	damping loss factor in viscoelastic core
γ_{mn}	variables defined in equation (29)

η_{mn} variables defined in equation (41)

θ_{ijk} variables defined in equations (53a) and (53b)

λ_{mn} variables defined in equation (29)

μ variable defined in equation (35)

ν Poisson's ratio

ν_1, ν_2, ν_3 Poisson's ratio of top face plate, bottom face plate, and viscoelastic core, respectively

ρ material density of elastic plate, kg/m^3

ρ_a air density in enclosure, kg/m^3

ρ_1, ρ_2, ρ_3 material densities of top face plate, bottom face plate, and viscoelastic core, respectively, kg/m^3

σ_1, σ_2 variables defined in equation (30)

τ variable defined in equation (43)

Φ_{ij} variables defined in equation (14)

ϕ_{mn} structural modes of panel

ψ_{mn} variables defined in equation (42)

ω frequency, rad/sec

ω_{ijk} cavity modal frequencies, rad/sec

ω_{mn} structural modal frequencies, rad/sec

Superscripts:

b   bottom

t   top

–   Fourier transform according to equations (20a) and (20b)

*   complex conjugate

    Dots over symbols denote time derivatives.

INTERIOR ACOUSTIC PRESSURE

The rectangular enclosure shown in figure 1 occupies a designated volume $V = abd$. The wall at $z = 0$ is flexible whereas the remaining walls are acoustically rigid. The pressure p inside the enclosure is determined from the linear acoustic wave equation

$$\nabla^2 p = \frac{1}{c^2} \ddot{p} \tag{1}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ and the boundary conditions to be satisfied are

$$\frac{\partial p}{\partial n} = 0 \tag{2}$$

on B_R and

$$\frac{\partial p}{\partial n} = -\rho_a \ddot{w} \tag{3}$$

on B_F at $z = 0$. Here B_R and B_F indicate rigid and flexible boundaries, respectively; $\partial p/\partial n$ is the pressure derivative normal to the wall surface; w is the displacement of the flexible wall in the z-direction; and a dot indicates a time derivative. For the analysis considered in this study, the wall displacement w is determined independently. The acoustic pressure in the enclosure is not assumed to affect the wall. Reference 13 has shown this approach to be valid for deep enclosures.

To solve equation (1) the pressure can be written

$$p = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (C_p)_{ij}(z,t)\, X_{ij0}(x,y) \tag{4}$$

where $X_{ij0} = \cos \dfrac{i\pi x}{a} \cos \dfrac{j\pi y}{b}$ are the modes of a cavity with hard walls at

$x = 0,a$, and $y = 0,b$. When the flexible wall motions in terms of these modes are expanded and orthogonality is used,

$$-\rho_a \ddot{w} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} G_{ij}(t)\, X_{ij0} \tag{5}$$

In this case,

$$G_{ij}(t) = -\frac{e_{ij}}{ab} \int_0^a \int_0^b \rho_a \ddot{w} X_{ij0}\; dx\; dy \tag{6}$$

7

and

$$e_{ij} = \begin{cases} 1 & (i = 0, \quad j = 0) \\ 2 & (\text{Either } i \neq 0 \text{ or } j \neq 0) \\ 4 & (i \neq 0, \quad j \neq 0) \end{cases} \qquad (7)$$

Equations (1) to (4) demonstrate that boundary conditions for the pressure coefficients $(C_p)_{ij}(z,t)$ are not homogeneous. A direct application of separation of variables technique for $(C_p)_{ij}(z,t)$ will not work, and a different method needs to be adopted. This solution method can be achieved by transforming a homogeneous differential equation with nonhomogeneous boundary conditions into a problem consisting of a nonhomogeneous differential equation with homogeneous boundary conditions (ref. 14). Using equations (1) to (6) and the expression

$$(C_p)_{ij} = \Phi_{ij} + Z(z,t) \qquad (8)$$

where Φ_{ij} are solutions of the associated homogeneous problem and $Z(z,t)$ is the solution on the boundary, gives

$$\frac{\partial^2 \Phi_{ij}}{\partial z^2} - \frac{1}{c^2} \ddot{\Phi}_{ij} - A_{ij}\Phi_{ij} = A_{ij}Z + \frac{1}{c^2} \ddot{Z} - \frac{\partial^2 Z}{\partial z^2} \qquad (9)$$

and

$$\frac{\partial \Phi_{ij}}{\partial z} + \frac{\partial Z}{\partial z} = G_{ij} \qquad (10)$$

at $z = 0$ where

$$A_{ij} = (i\pi/a)^2 + (j\pi/b)^2 \qquad (11)$$

Since Φ_{ij} are the solutions with homogeneous boundary conditions $\dfrac{\partial \Phi_{ij}}{\partial z} = 0$ at $z = 0$, equation (10) reduces to

$$\frac{\partial Z_{ij}}{\partial z} = G_{ij} \qquad (12)$$

at $z = 0$. Equation (12) is the boundary condition that must be satisfied by function Z_{ij}. Any continuous function which satisfies equation (12) is a suitable function for Z_{ij} (ref. 10). Such a function is a polynomial of the form

$$Z_{ij} = (z - z^2/2d)G_{ij} \qquad (13)$$

8

The associated homogeneous boundary value problem has a solution

$$\Phi_{ij} = \sum_{k=0}^{\infty} B_{ijk}(t) \cos \frac{k\pi z}{d} \tag{14}$$

Then the sound pressure distribution inside the enclosure may be determined by combining equations (4), (8), (13), and (14). The result is

$$p(x,y,z,t) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} B_{ijk}(t) X_{ijk} + \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} Z_{ij}(z,t) X_{ij0} \tag{15}$$

The series solution given in equation (15) is uniformly convergent everywhere including the flexible boundary (ref. 11). The equation of modal coefficients B_{ijk} can be obtained by substituting equation (15) into equation (9), multiplying by an orthogonal eigenfunction, and integrating over the volume. The result is

$$\ddot{B}_{ijk} + 2\alpha_{ijk}\omega_{ijk}\dot{B}_{ijk} + \omega_{ijk}^2 B_{ijk} = F_{ijk} \tag{16}$$

where the equivalent damping in the enclosure (due to wall absorption and viscous air damping) is included through the modal damping coefficient α_{ijk}. The modal frequencies in the enclosure ω_{ijk} can be obtained from

$$\omega_{ijk} = c\left[(i\pi/a)^2 + (j\pi/b)^2 + (k\pi/d)^2\right]^{1/2} \tag{17}$$

and

$$F_{ijk} = \frac{c^2 e_k}{d} \int_0^d \left(\frac{1}{c^2}\ddot{Z}_{ij} + A_{ij}Z_{ij} - \frac{\partial^2 Z_{ij}}{\partial z^2}\right) \cos \frac{k\pi z}{d}\, dz \tag{18}$$

where

$$e_k = \begin{cases} 1 & (k = 0) \\ 2 & (k \neq 0) \end{cases} \tag{19}$$

The solutions for the panel motions $\ddot{w}$ are included in equation (18) through the term Z_{ij}.

When initial conditions on the coefficients B_{ijk} are assumed, the solution to equation (16) can be obtained in the time domain. The linear character of the governing equations considered in this study, however, makes it more convenient to solve these equations in the frequency domain. Adopting the Fourier integral representation of $B_{ijk}(t)$ gives

$$B_{ijk}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{B}_{ijk}(\omega)\, e^{i\omega t}\, d\omega \qquad (20a)$$

and

$$\bar{B}_{ijk}(\omega) = \int_{-\infty}^{\infty} B_{ijk}(t)\, e^{-i\omega t}\, dt \qquad (20b)$$

Then the solution to equation (16) in the frequency domain is

$$\bar{B}_{ijk} = \frac{\bar{F}_{ijk}}{\omega^2_{ijk} - \omega^2 + 2i\alpha_{ijk}\omega_{ijk}\omega} \qquad (21)$$

where $\bar{F}_{ijk}(\omega)$ are Fourier transforms of $F_{ijk}(t)$. Since $\bar{F}_{ijk}$ are functions of panel response $\bar{w}$, where $\bar{w}$ is a Fourier transform of w, the response characteristics of viscoelastic sandwich panels are determined next.

RESPONSE OF THE VISCOELASTIC SANDWICH PANELS

The rectangular sandwich plates considered in this analysis are assumed to be flat and simply supported on all four edges. Acting on the top surface of the plate is a random noise pressure, and at the bottom face the plate is backed by an acoustic cavity as shown in figure 1. The sandwich plate consists of two isotropic elastic plates and an isotropic viscoelastic core. The response analysis of the sandwich panel is considered separately for the soft core (compressible) and for the hard core (incompressible).

Soft Core

When the viscoelastic core is very soft, Poisson's ratio of the material is nearly zero; consequently, such a material can be approximated by a viscoelastic spring. Assuming small-deflection theory, the governing equations of motion are (ref. 15)

$$D_1\nabla^4 w_1 + \left(\frac{1}{3}\rho_3 h_3 + \rho_1 h_1\right)\ddot{w}_1 + \frac{1}{6}\rho_3 h_3 \ddot{w}_2 + c_1\dot{w}_1 + k_3(w_1 - w_2) = p^r(x,y,t) \qquad (22)$$

and

$$D_2\nabla^4 w_2 + \left(\frac{1}{3}\rho_3 h_3 + \rho_2 h_2\right)\ddot{w}_2 + \frac{1}{6}\rho_3 h_3 \ddot{w}_1 + c_2\dot{w}_2 + k_3(w_2 - w_1) = 0 \qquad (23)$$

where $\nabla^4 = \partial^4/\partial x^4 + 2\,\partial^4/\partial x^4\,\partial y^2 + \partial^4/\partial y^4$ and the subscripts 1, 2, and 3 refer to the top plate, bottom plate, and the core, respectively. The terms

$\frac{1}{3}\rho_3 h_3$ and $\frac{1}{6}\rho_3 h_3$ represent the apportioned contributions of the mass of the viscoelastic core to the displacements w_1 and w_2. The viscoelastic spring constant of the core material is $k_3 = E_3(1 + i\beta)/h_3$ where E_3 is the stiffness modulus in compression and β is the loss factor in the core. In obtaining equation (23), the effect of cavity pressure on the bottom face plate motion has been neglected.

Solutions to equations (22) and (23) can be represented in terms of the simply supported plate modes

$$w_1(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn}(t) \; \phi_{mn}(x,y) \tag{24}$$

$$w_2(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} D_{mn}(t) \; \phi_{mn}(x,y) \tag{25}$$

where C_{mn} and D_{mn} are the generalized coordinates of the top and bottom plates, respectively, and $\phi_{mn} = \sin(m\pi x/a) \sin(n\pi y/b)$. Substitution of equations (24) and (25) into equations (22) and (23) and use of the orthogonality principle give a set of coupled differential equations in C_{mn} and D_{mn}. Taking the Fourier transform of these equations shows that

$$\bar{C}_{mn} = H_{mn}^t \bar{Q}_{mn} \tag{26a}$$

$$\bar{D}_{mn} = H_{mn}^b \bar{Q}_{mn} \tag{26b}$$

$$H_{mn}^t = \frac{\gamma_{mn}}{a_1(\lambda_{mn}\gamma_{mn} - \sigma_1\sigma_2)} \tag{27a}$$

$$H_{mn}^b = \frac{\sigma_2}{a_1(\lambda_{mn}\gamma_{mn} - \sigma_1\sigma_2)} \tag{27b}$$

in which $\bar{C}_{mn}$ and $\bar{D}_{mn}$ are the transformed deflection amplitudes of C_{mn} and D_{mn} which correspond to the top and the bottom plates, respectively, $\bar{Q}_{mn}$ are the generalized random forces

$$\bar{Q}_{mn} = \frac{4}{ab} \int_0^a \int_0^b \bar{p}^r(x,y,\omega) \; \phi_{mn} \; dx \; dy \tag{28}$$

11

in which $\bar{p}^r$ are the transformed pressures p^r, and

$$
\left.\begin{array}{l}
\lambda_{mn} = -\omega^2 + \underset{\sim}{i}\,\frac{c_1}{a_1}\,\omega + \frac{1}{a_1}\left\{D_1\Big[(m\pi/a)^2 + (n\pi/b)^2\Big]^2 + k_3\right\} \\[3mm]
\gamma_{mn} = -\omega^2 + \underset{\sim}{i}\,\frac{c_2}{a_2}\,\omega + \frac{1}{a_2}\left\{D_2\Big[(m\pi/2)^2 + (n\pi/b)^2\Big]^2 + k_3\right\}
\end{array}\right\} \qquad (29)
$$

$$
\left.\begin{array}{l}
\sigma_1 = \frac{1}{a_1}\left(\frac{1}{6}\,\rho_3 h_3 \omega^2 + k_3\right) \\[4mm]
\sigma_2 = \frac{a_1}{a_2}\,\sigma_1 \\[4mm]
a_1 = \frac{1}{3}\,\rho_3 h_3 + \rho_1 h_1 \\[4mm]
a_2 = \frac{1}{3}\,\rho_3 h_3 + \rho_2 h_2
\end{array}\right\} \qquad (30)
$$

The natural frequencies of the coupled system can be determined by setting $c_1 = c_2 = \beta = 0$ and using

$$
\lambda_{mn}\gamma_{mn} - \sigma_1\sigma_2 = 0 \qquad (31)
$$

Equation (31) gives two characteristic values for each set of modal indices (m,n). These roots are associated with in-phase flexural and out-of-phase dilatational vibration frequencies of the sandwich construction. The dilatational vibration frequencies are strongly dependent on the core stiffness $k_3 = E_3/h_3$. For large values of core stiffness k_3 these frequencies would become very large, and the approach developed in this study would lose practical significance.

The frequency response function for the bottom face plate given in equation (27b) can now be combined with equation (21) and subsequently with equations (6), (13), and (15) to obtain the pressure distribution inside the enclosure. Such an expression will be given in the later sections.

Hard Core

When the core material in a sandwich construction has a certain degree of stiffness in bending and extension, energy is dissipated in the core by the

flexural vibrations of the plate. The governing equations of motion for this case are taken from reference 2. It is assumed that the normal deflections of both face plates are always in phase, the shear strains across the depth of the face plate are small, the stresses in the core material parallel to the plate surface are negligible, and the in-plane inertia forces are small. The governing vibration equations for such a sandwich panel can then be written as (ref. 2)

$$D\nabla^4 w - \frac{G(h + h_3)}{2h_3}\left[(h + h_3)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - 2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] - \frac{\mu}{2}\frac{\partial^2 w}{\partial t^2} = \frac{1}{2}p^r(x,y,t) \qquad (32)$$

$$\frac{E}{1 - \nu^2}\frac{\partial^2 u}{\partial x^2} + \frac{E}{2(1 + \nu)}\frac{\partial^2 u}{\partial y^2} - \frac{2G}{hh_3}u + \frac{E(3\nu^2 + 1)}{2(1 - \nu^2)(1 - \nu)}\frac{\partial^2 v}{\partial x\,\partial y} + \frac{G}{hh_3}(h + h_3)\frac{\partial w}{\partial x} = 0$$

$$(33)$$

$$\frac{E}{1 - \nu^2}\frac{\partial^2 v}{\partial x^2} + \frac{E}{2(1 + \nu)}\frac{\partial^2 v}{\partial y^2} - \frac{2G}{hh_3}v + \frac{E(3\nu^2 + 1)}{2(1 - \nu^2)(1 - \nu)}\frac{\partial^2 u}{\partial x\,\partial y} + \frac{G}{hh_3}(h + h_3)\frac{\partial w}{\partial y} = 0$$

$$(34)$$

where

$$\mu = 2\rho h + \rho_3 h_3 \qquad (35)$$

$$G = G_0(1 + \underset{\sim}{i}\beta) \qquad (36)$$

The displacements u, v, and w correspond to the x-, y-, and z-coordinates, respectively, and G_0 is the shear modulus in the core. In this formulation it is assumed that the thickness h of both face plates is the same.

For simple support boundary conditions the modes of free vibration can be represented by

$$u = \sum_m \sum_n U_{mn} \cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad (37a)$$

$$v = \sum_m \sum_n V_{mn} \sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \qquad (37b)$$

$$w = \sum_m \sum_n W_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad (37c)$$

13

Substituting these equations into equations (32) to (34), utilizing the orthogonality principle, and taking the Fourier transformation, give (ref. 2)

$$\bar{W}_{mn} = H_{mn}\bar{Q}_{mn} \tag{38}$$

The generalized random forces $\bar{Q}_{mn}$ are defined in equation (28). $\bar{W}_{mn}$ are the Fourier transforms of W_{mn}, and

$$H_{mn} = \frac{1}{K_{mn}(1 + \underset{\sim}{i}\eta_{mn}) - \mu\omega^2} \tag{39}$$

$$K_{mn} = 2D\left(\frac{\pi}{b}\right)^4 n^4(r_1^2 + 1)^2\left[1 + \frac{3(1 + \tau)^2(1 + \psi_{mn} + \beta^2)}{(1 + \psi_{mn})^2 + \beta^2}\right] \tag{40}$$

$$\eta_{mn} = \frac{3\beta\psi_{mn}(1 + \tau)^2}{(1 + \psi_{mn})^2 + \beta^2 + 3(1 + \tau)^2(1 + \psi_{mn} + \beta^2)} \tag{41}$$

$$\psi_{mn} = \frac{r_1^2 + 1}{(1 - \nu)^2}\frac{\pi^2}{2}\frac{E}{G_0}\left(\frac{hn}{b}\right)^2\tau \tag{42}$$

$$\tau = \frac{h_3}{h} \tag{43}$$

$$r_1 = \frac{bm}{an} \tag{44}$$

The natural frequencies of the sandwich plate can be obtained from (ref. 2)

$$f_{mn} = \frac{1}{2\pi}\left[\frac{8D\pi^4}{\mu b^4} n^4(r_1^2 + 1)^2 K_{mn}\right]^{1/2} \tag{45}$$

The frequency response function for the vibration of a sandwich panel with a hard core given in equation (39) can be combined with equation (21) to determine noise pressure inside the enclosure in a fashion similar to that described earlier for a panel with soft core.

For the analysis presented in this paper, it is assumed that the input pressure spectral density is specified. Thus, the response (panel deflection and noise pressure inside the enclosure) needs to be expressed in the form of a spectral density. Following the procedure given in reference 16 shows that the spectral density of vertical deflections w can be determined from

$$S_w(x,y,\omega) = \sum_m \sum_n \sum_r \sum_s H_{mn} \overset{*}{H}_{rs} \phi_{mn} \phi_{rs} S_{mnrs} \tag{46}$$

where H_{mn} are given in equations (27a), (27b), and (39), S_{mnrs} are the cross-spectral densities of the generalized random forces $\bar{Q}_{mn}$, and a star indicates conjugation. If the input pressure p^r is assumed to be a stationary random process, using equation (28) and reference 16 gives

$$S_{mnrs} = \frac{16}{a^2 b^2} \int_0^a \int_0^b \int_0^a \int_0^b S^r(\xi,\eta,\omega)\, \phi_{mn}(x_1,y_1)\, \phi_{rs}(x_2,y_2)\, dx_1\, dx_2\, dy_1\, dy_2 \tag{47}$$

where $S^r(\xi,\eta,\omega)$ are the cross-spectral densities of the input pressure p^r, and $\xi = x_2 - x_1$, $\eta = y_2 - y_1$ are the spatial pressure separations. For the cases where the random process p^r has similar spatial correlation properties at all delays, the input pressure cross-spectral density can be written as

$$S^r(\xi,\eta,\omega) = R_x(\xi,\omega)\, R_y(\eta,\omega)\, S_p^r(\omega) \tag{48}$$

in which R_x and R_y are the narrow band spatial correlation coefficients corresponding to the x- and y-coordinates, respectively, and S_p^r is the specified input pressure spectral density.

The analysis developed in this paper for sandwich panel vibrations considered separately a soft core and a hard core. A general procedure which would not limit the analysis to a particular core stiffness can be developed using Lagrange's equation and the expressions for strain and kinetic energies. An attempt to solve such a problem has been presented in references 8 and 9. However, as pointed out in reference 7, there remain some significant uncertainties in these formulations and solutions. Thus, an in-depth study on the vibration characteristics of viscoelastic sandwich panels with no limitations on core stiffness would be a useful extension of the work presented in this paper.

ACOUSTIC-STRUCTURAL MODEL

The equations developed in previous sections can be combined to construct a noise transmission model for a viscoelastic sandwich panel. Applying a Fourier transform to equation (15) and using equations (13) and (21) give

$$\bar{p}(x,y,z,\omega) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{\bar{F}_{ijk} X_{ijk}}{\omega_{ijk}^2 - \omega^2 + 2i\alpha_{ijk}\omega_{ijk}\omega} + \frac{z - z^2}{2d} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \bar{G}_{ij} X_{ij0} \tag{49}$$

Equations (6), (25), and (26b) demonstrate that

$$\bar{G}_{ij} = \frac{e_{ij}\omega^2 \rho_a}{ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\int_0^a \int_0^b X_{ij0}\phi_{mn}\, dx\, dy \right) H_{mn}^b \bar{Q}_{mn} \tag{50}$$

15

When equations (18), (49), and (50) are combined, the pressure inside the enclosure caused by the vibration of a viscoelastic sandwich panel with a soft core can be written as

$$\bar{p}(x,y,z,\omega) = \frac{\rho_a \omega^2 c}{ab} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} e_{ij}\left[\theta_{ijk}H_{ijk} + e_k Z_d(z)\right] \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} L_{ijmn}H_{mn}^b \bar{Q}_{mn}X_{ijk}$$

(51)

where

$$Z_d(z) = \frac{z - z^2}{2d}$$

(52)

$$\theta_{ij0} = \left(-\frac{\omega^2}{c^2} + \frac{3}{d^2} + A_{ij}\right) \qquad (k = 0) \qquad (53a)$$

$$\theta_{ijk} = -\frac{d}{2(\pi k)^2}\left(-\frac{\omega^2}{c^2} + A_{ij}\right) \qquad (k \neq 0) \qquad (53b)$$

$$H_{ijk} = \frac{1}{\omega_{ijk}^2 - \omega^2 + 2i\alpha_{ijk}\omega_{ijk}\omega}$$

(54)

$$L_{ijmn} = \int_0^a \int_0^b X_{ij0}(x,y)\ \phi_{mn}(x,y)\ dx\ dy$$

(55)

Noise pressure inside the enclosure caused by the vibration of a viscoelastic sandwich panel with a hard core can be obtained from equation (51) by replacing H_{mn}^b with H_{mn} where the latter expression is given in equation (39).

The spectral density of the interior pressure $\bar{p}$ can be obtained by taking the mathematical expectation of equation (51) and then following the procedure presented in reference 16. The result is

$$S_p(x,y,z,\omega) = \left(\frac{\rho_a \omega^2 c}{ab}\right)^2 \sum_{i,j,k,r,s,q} e_{ij}e_{rs}\left[\theta_{ijk}H_{ijk} + e_k Z_d(z)\right]\left[\theta_{rsq}H_{rsq}\right.$$

$$\left. + e_q Z_d(z)\right]^* \sum_{m,n,l,o} L_{ijmn}H_{mn}^b X_{ijk}S_{mnlo}(L_{rslo}H_{lo}^b X_{rsq})^* \qquad (56)$$

16

where each index indicates a separate summation and a star denotes conjugation. For the cases where acoustic damping in the enclosure is small and the acoustic modal frequencies are well separated, the contribution to equation (56) from the cross terms $(\theta_{ijk}H_{ijk} + e_k Z_d)L_{ijm}X_{ijk}(\theta_{rsq}H_{rsq} + e_q Z_d)^*L_{rslo}X_{rsq}$ is relatively small. It can therefore be neglected. When the structural damping is small,

similar restrictions can be imposed on the cross terms $L_{ijmn}H^b_{mn}S_{mnlo}L_{rslo}(H^b_{lo})^*$. However, for viscoelastic sandwich panels, damping is usually large and no such simplifying assumption is valid.

RESULTS AND DISCUSSION

The numerical results presented in this paper correspond to the acoustic cavity and the sandwich panel shown in figure 1. The cavity beneath the panel is assumed to be acoustically sealed. For the calculations made, the top and bottom face plates are assumed to be aluminum alloy whereas the core is assumed to be a lightweight low modulus viscoelastic material. The physical data used are given in table 1. The input random pressure p^r acting on the top face of the flexible sandwich panel is taken to be that of truncated Gaussian white noise for which the spectral density is

$$S_p^r = \begin{cases} 0.00058 \ (N/m^2)^2/Hz & (0 \leqq f \leqq 1000 \text{ Hz}) \\ 0 & (\text{Otherwise}) \end{cases} \tag{57}$$

The spatial pressure distribution is assumed to be uniform over the panel surface. The spectra given in equation (57) correspond to a 100-dB level at the specified frequency range and to about a 130-dB level overall.

The modal damping in the enclosure was taken as

$$\alpha_{ijk} = \alpha_{001}(\omega_{001}/\omega_{ijk})^2 \tag{58}$$

where the fundamental modal damping coefficient is assumed $\alpha_{001} = 0.01$ (hard wall cavity) or $\alpha_{001} = 0.10$ (cavity with absorbing walls). The acoustic modal damping behavior given in equation (58) was observed experimentally in reference 14 for a hard wall cavity. The effect of acoustic wall absorption on the interior noise in a cavity is strongly influenced by frequency, wall geometry, and the type of absorbing material. However, for small acoustic absorption, equation (58) is a useful approximation of modal damping for low frequencies.

The deflection response spectral density at the middle of a sandwich panel with a soft core is presented in figure 2 for several values of core loss factor β. The peaks in the spectral density correspond to flexural and dilatational modes. The fundamental dilatational mode frequency is 401 Hz. Figure 2 shows how the low frequency is dominated by flexural modes, whereas at mid-range frequencies (400 to 600 Hz), dilatational modes are also excited for low values of core loss factor β. Since damping in the face plates and in the core material was taken to be constant at all frequencies, the response levels at frequencies

17

above 650 Hz are very low and no distinct peaks are observed. For the case
where the core damping factor β is zero, the core acts as an elastic material
with no damping and the response at the first dilatational mode is very large.
The noise transmission characteristics for the same panel are shown in figure 3.
The noise transmission curve was obtained from

$$NT(f) = 10 \log \frac{S_p(f)}{S_p{}^r(f)} \tag{59}$$

where S_p is sound pressure spectral density in the interior calculated from
equation (56). The input pressure spectral density $S_p{}^r$ is defined in equa-
tion (57). The significant structural panel modes and the acoustic cavity modes
are indicated in figure 3. These modes were calculated from equation (31). As
can be observed from figure 3, the noise transmission characteristics of an
elastic panel and a viscoelastic panel with soft core are similar for frequen-
cies up to about 250 Hz and above 600 Hz. In the intermediate frequency range,
structural modes corresponding to dilatational vibrations have a significant
effect on noise transmission. At the frequency of the first dilatational mode
(401 Hz), a panel with a viscoelastic core can achieve about 30 dB more noise
reduction. These results indicate that noise transmission into the enclosure is
dominated by flexural modes for frequencies below 250 Hz, by dilatational and
acoustic modes in the frequency region 250 to 500 Hz, and by acoustic modes
above 500 Hz.

For the viscoelastic sandwich panel with a hard core, damping in the face
plates was assumed to be negligible in comparison to the damping in the core.
The thickness of each metallic face plate was taken to be equal to 0.00051 m.
Viscoelastic cores with two different values of shear modulus G_0 were selected
for the study. For each of these cases, the core loss factor β and the core
thickness h_3 were varied. The displacement response spectral densities at the
center of the panel are shown in figures 4 and 5 for several values of core
damping factor β. For low values of core loss factor β, distinct peaks can be
observed at the natural frequencies of the panel. The natural panel frequencies
were calculated from equation (45). Since the core stiffness shown in figure 5
is much larger than that shown in figure 4, the peaks associated with modal fre-
quencies are shifted to the right in figure 5. The deflection response is lower
for the panel with a stiffer core. These calculations were based on core thick-
ness $h_3 = 0.00635$ m. Similar results are presented in figures 6 and 7 for dif-
ferent core thicknesses. These figures illustrate that response spectral densi-
ties are significantly affected by change in core thickness and core stiffness.
Viscoelastic panels with thin but stiff cores respond more than panels with less
stiff cores. However, when the core thickness is about 0.008 m or larger, panels
with stiff cores respond less than panels with not very stiff cores. Noise
transmission characteristics for these panels are given in figures 8 to 13 for
an enclosure with low ($\alpha_{001} = 0.01$) and high ($\alpha_{001} = 0.1$) interior acoustic
damping. When the acoustic damping in the enclosure is high, the peaks at the
cavity modal frequencies are suppressed. The two acoustic damping coefficients
chosen could be representatives of a cavity with hard or absorbing walls,
respectively. For low acoustic damping and large core loss factor β, noise
inside the enclosure is dominated by the acoustic cavity modes. Approximately
20 dB more noise reduction can be achieved at the first panel mode by increasing

18

the core loss factor from 0.1 to 1.0. Furthermore, by increasing cavity modal damping from 0.01 to 0.1, about 10 dB more noise reduction can be gained at most acoustic modal frequencies. Figures 12 and 13 show that a significant amount of noise reduction can be achieved by increasing the core thickness. For example, by increasing the core thickness 10 times, about 20 and 15 dB more noise reduction is obtained at the first panel mode for a core with $G_0 = 68\ 900\ 000$ and $G_0 = 2\ 760\ 000$ N/m^2, respectively. In figure 14 the overall sound pressure levels in the interior are plotted against the core thickness. The increase in total panel mass with the increase in core thickness is also shown in figure 14. The mass curve is obtained under the condition that the density of the viscoelastic core material is one-fifth the density of the face plates. The overall sound pressure was obtained from

$$OASPL = 10 \log (\sigma^2/p_0{}^2) \tag{60}$$

where σ^2 is the variance of the interior pressure and p_0 is the reference pressure $p_0 = 0.00002$ N/m^2. These results indicate that the overall noise levels are about 5 dB higher for a panel with a stiff core ($G_0 = 68\ 900\ 000$ N/m^2). By increasing the core thickness 10 times, the overall noise level was reduced by about 12 dB.

A direct comparison between elastic and viscoelastic panel response is shown in figure 15. The curves corresponding to the viscoelastic sandwich panel were obtained for $\beta = 1.0$ and $h_3 = 0.00635$ m. The thickness of the elastic panel was taken to be equal to 0.00102 m, and the structural damping characteristics were represented by

$$\alpha_{mn} = \alpha_{11}(\omega_{11}/\omega_{mn}) \tag{61}$$

where α_{mn} are the modal coefficients and ω_{mn} are the natural frequencies of the elastic plate. The results shown in figure 15 correspond to $\alpha_{11} = 0.02$. The mass of the elastic panel was adjusted to be equivalent to the mass of the viscoelastic sandwich panel. A comparison of noise transmission by elastic and viscoelastic panels is shown in figure 16. As can be observed from figures 15 and 16, panel response and noise transmission for an elastic panel is strongly dominated by structural vibrations while the noise transmission by a viscoelastic sandwich panel is dominated by acoustic cavity modes. At low frequencies (below 200 Hz) noise reductions of about 50 dB or more can be achieved by viscoelastic sandwich panels. However, at frequencies where the acoustic modes are dominant, no significant noise reduction is achieved by viscoelastic panels. The overall sound pressures in the interior are plotted against core thickness for equivalent (mass) elastic and viscoelastic panels. (See fig. 17.) A direct comparison of these results indicates that for a viscoelastic panel with a thin core ($h_3 = 0.0025$ m), the overall noise in the cavity is about 5 dB ($G_0 = 68\ 900\ 000$ N/m^2) and 10 dB ($G_0 = 2\ 760\ 000$ N/m^2) lower than the overall noise for an equivalent elastic panel. For a thick core ($h_3 = 0.025$ m), these noise reduction values are 13 and 17 dB, respectively. The higher noise reduction values for a thicker core can be attributed to an increase in the sandwich construction stiffness and a larger capacity to dissipate vibrational energy.

CONCLUDING REMARKS

An analytical study was conducted to determine noise transmission characteristics of viscoelastic sandwich panels. The results indicate that noise transmission by sandwich panels is strongly dependent on thickness, damping, and material properties of the viscoelastic core.

Sandwich panels with very soft viscoelastic cores transmit noise much like elastic panels except in the frequency range where dilatational (out-of-phase) modes are excited. About 30 dB more noise reduction can be achieved by viscoelastic sandwich panels in this frequency range (300 to 600 Hz).

The vibration response and noise transmission of sandwich panels with hard cores are low when compared with equivalent elastic panels. As much as 50 dB more noise reduction can be achieved by viscoelastic panels at some frequencies in the low frequency range (below 200 Hz). At frequencies above 200 Hz, acoustic modes dominate interior noise for the acoustic enclosure chosen in this study. About 10 dB more overall noise reduction can be gained by increasing the core thickness 10 times (from 0.0025 to 0.025 m). With increasing core stiffness (from a shear modulus of 2 760 000 N/m^2 to a shear modulus of 68 900 000 N/m^2), noise reduction decreased by about 5 dB overall.

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TABLE 1.- PHYSICAL DATA USED IN THE STUDY

Enclosure (fig. 1)	$a = 0.250$ m, $b = 0.508$ m, $d = 0.762$ m $\rho_a = 1.225$ kg/m^3, $c = 330$ m/sec	
Location in the cavity where interior noise was computed	$x = 0.102$ m, $y = 0.152$ m, $z = 0.254$ m	
	Soft core	Hard core
Elastic face plates (aluminum)	$E_1 = E_2 = 72\ 400\ 000\ 000$ N/m^2 $c_1 = c_2 = 27.13$ N-sec^2/m^3 $h_1 = h_2 = 0.00051$ m $\nu_1 = \nu_2 = 0.3$ $\rho_1 = \rho_2 = 2770$ kg/m^3	$E = 72\ 400\ 000\ 000$ N/m^2 $c_1 = c_2 = 0$ $h = 0.00051$ m $\nu = 0.3$ $\rho = 2770$ kg/m^3
Viscoelastic core	$E_3 = 34\ 500$ N/m^2 $h_3 = 0.00635$ m $\nu_3 = 0$ $\rho_3 = 0.1\rho$ $\beta = 0, 0.1, 1.0$	$G_0 = 2\ 760\ 000$ N/m^2 $G_0 = 68\ 900\ 000$ N/m^2 $h_3 = $ Varied $\nu_3 = 0.17$ $\rho_3 = 0.2\rho$ $\beta = 0.1, 0.5, 1.0$

(a) Enclosure.

(b) Viscoelastic sandwich panel.
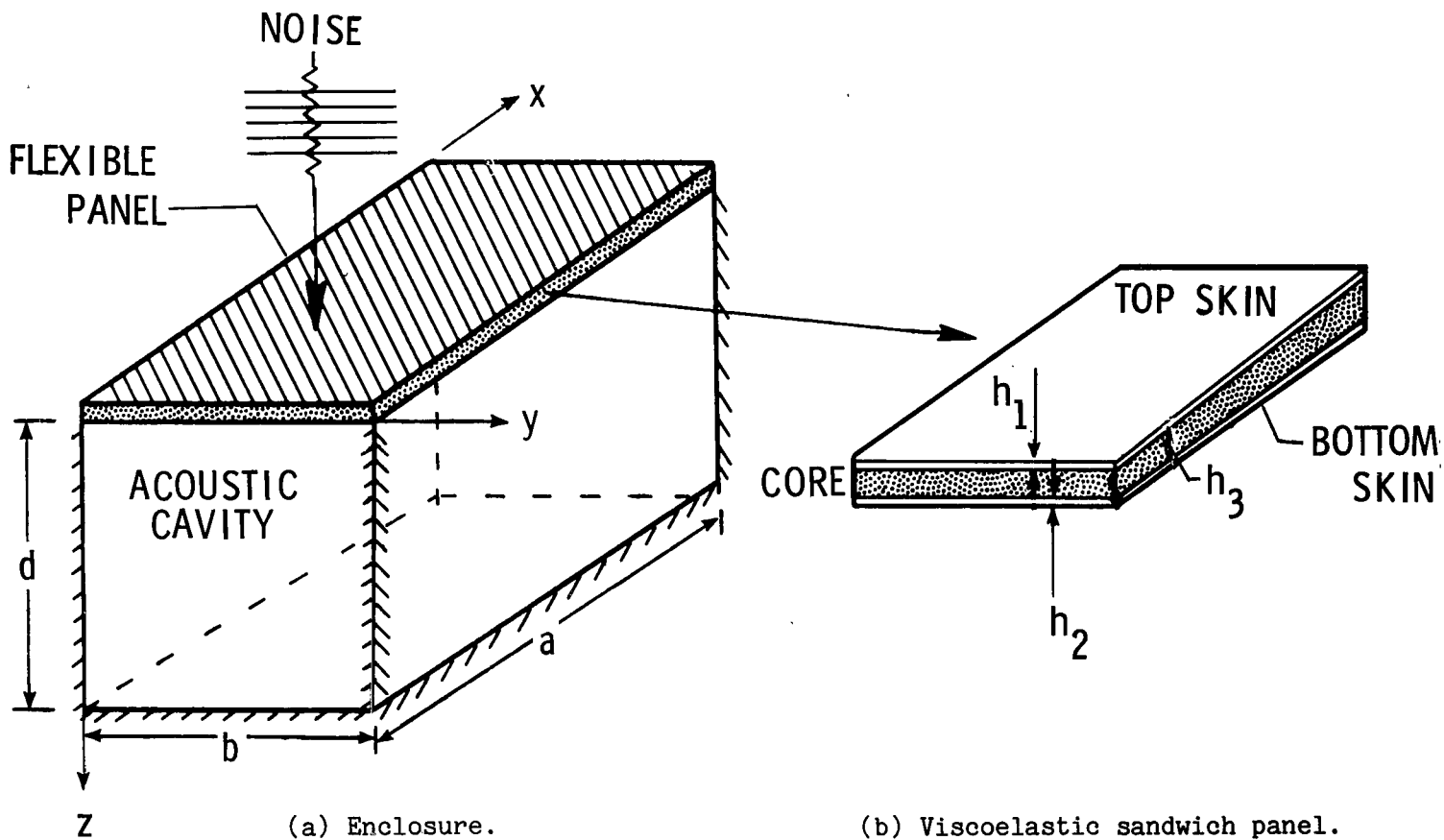
Figure 1.- Geometry of enclosure and viscoelastic sandwich panel.
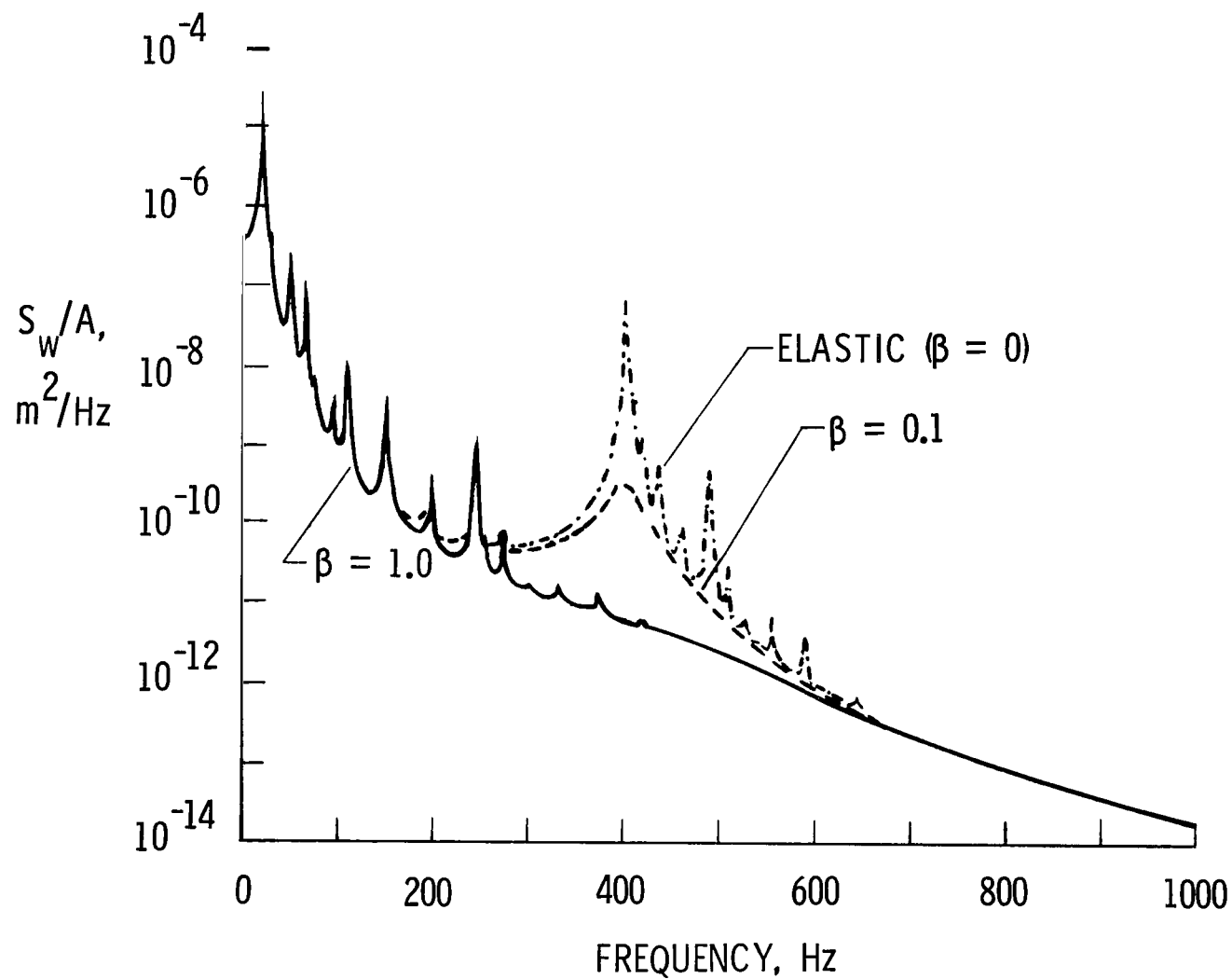
Figure 2.- Displacement response spectral density for panel with soft core.
$h_3 = 0.00635$ m; $A = 0.000645$.
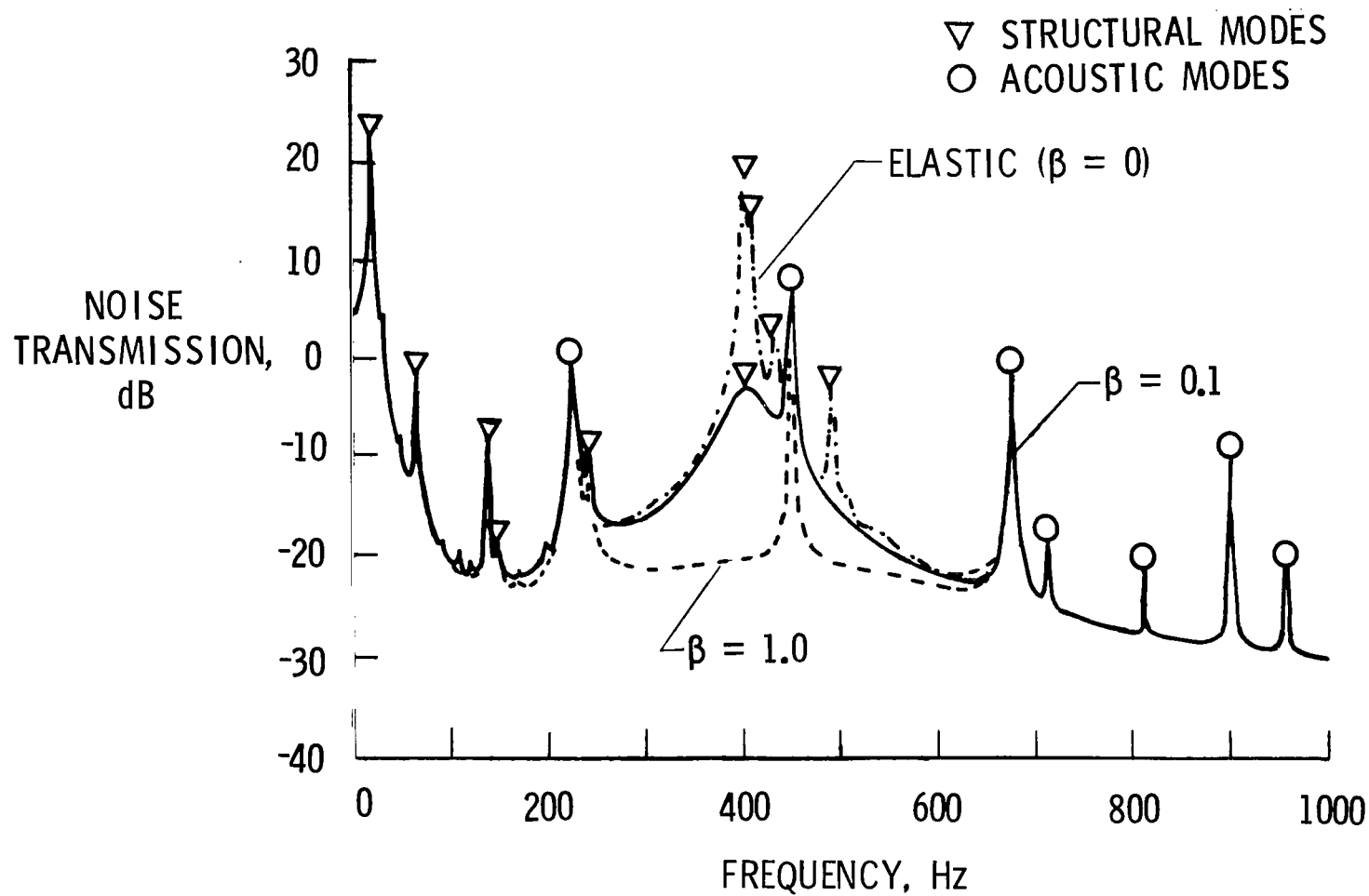
Figure 3.- Noise transmission by viscoelastic panel with soft core.
$h_3 = 0.00635$ m; $\alpha_{001} = 0.01$.
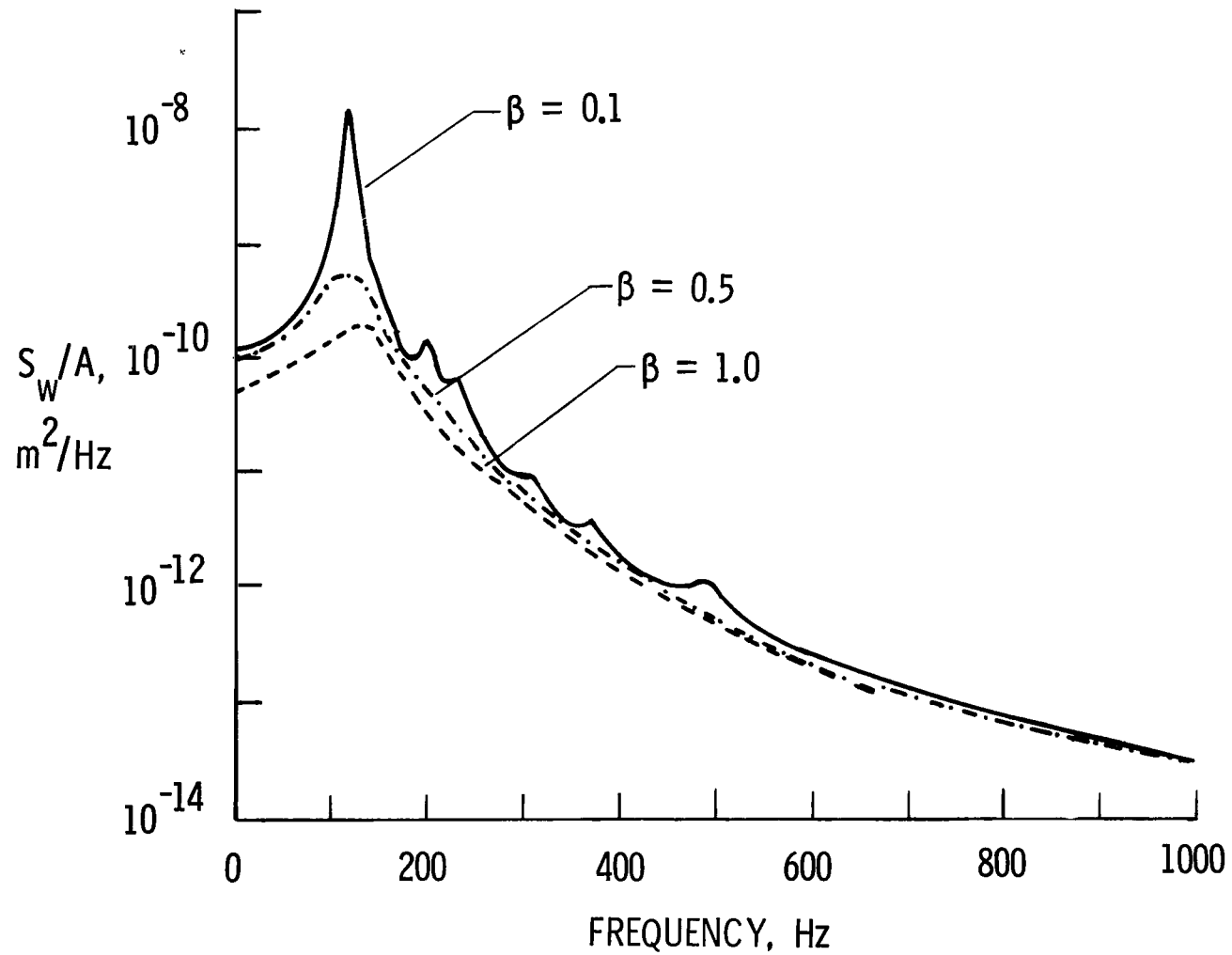
Figure 4.- Displacement response spectral density for panel with hard core and low shear modulus. A = 0.000645;   h₃ = 0.00635 m;   G₀ = 2 760 000 N/m².
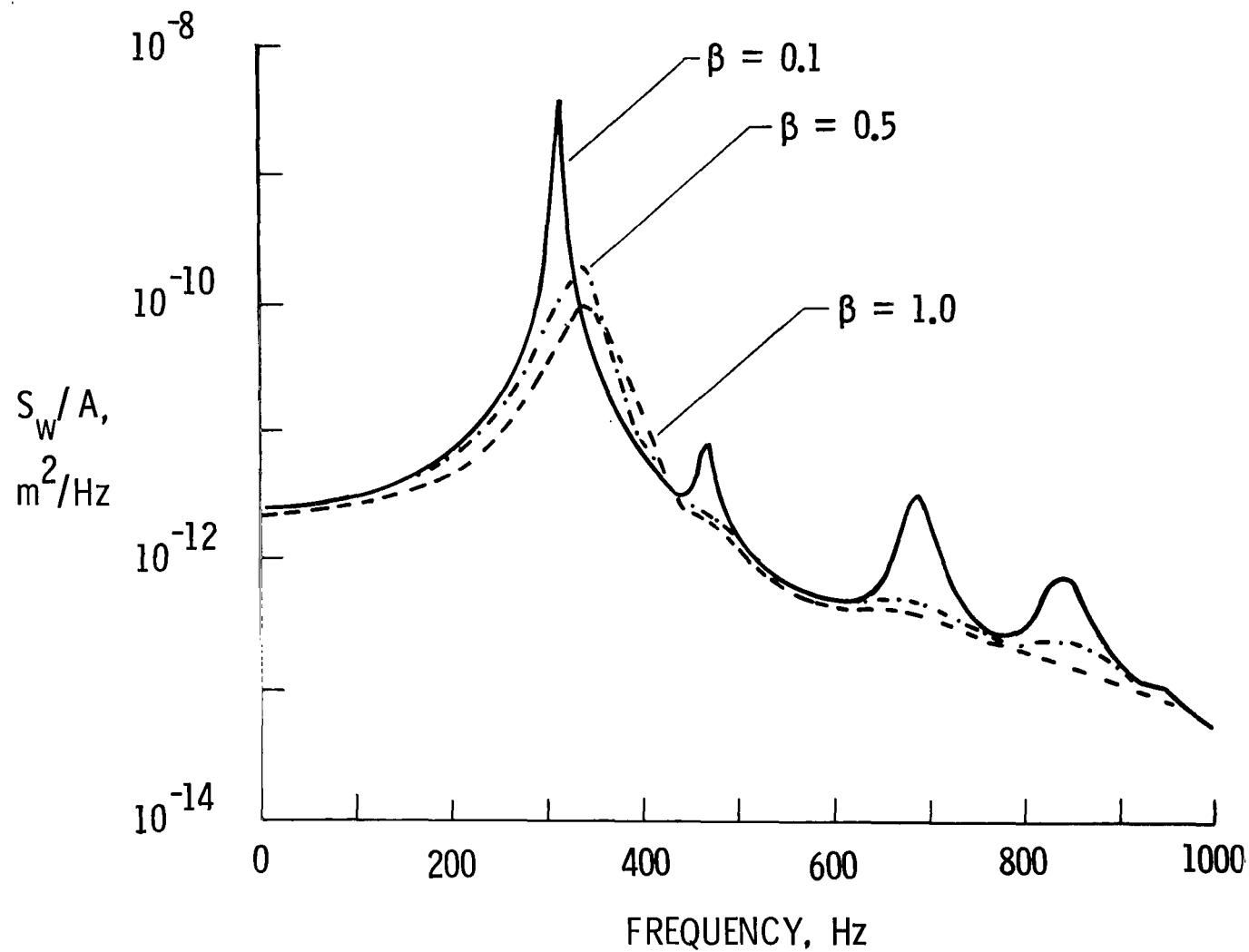
Figure 5.- Displacement response spectral density for panel with hard core and large shear modulus. A = 0.000645; h_3 = 0.00635 m; G_0 = 68 900 000 N/m^2.
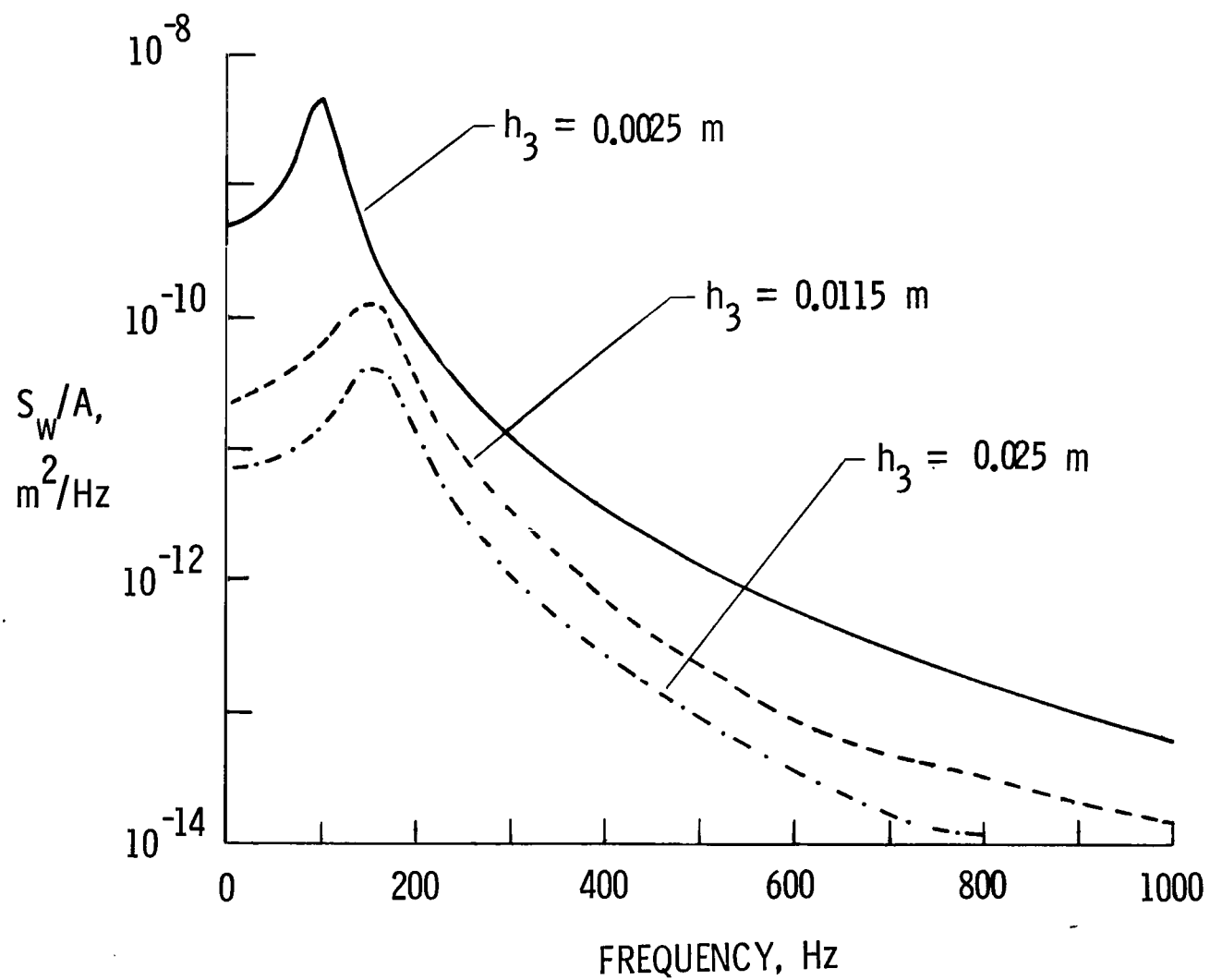
Figure 6.- Displacement response spectral density for different hard core thicknesses. Small shear modulus. $\beta = 0.5$; $A = 0.000645$; $G_0 = 2\ 760\ 000\ N/m^2$.
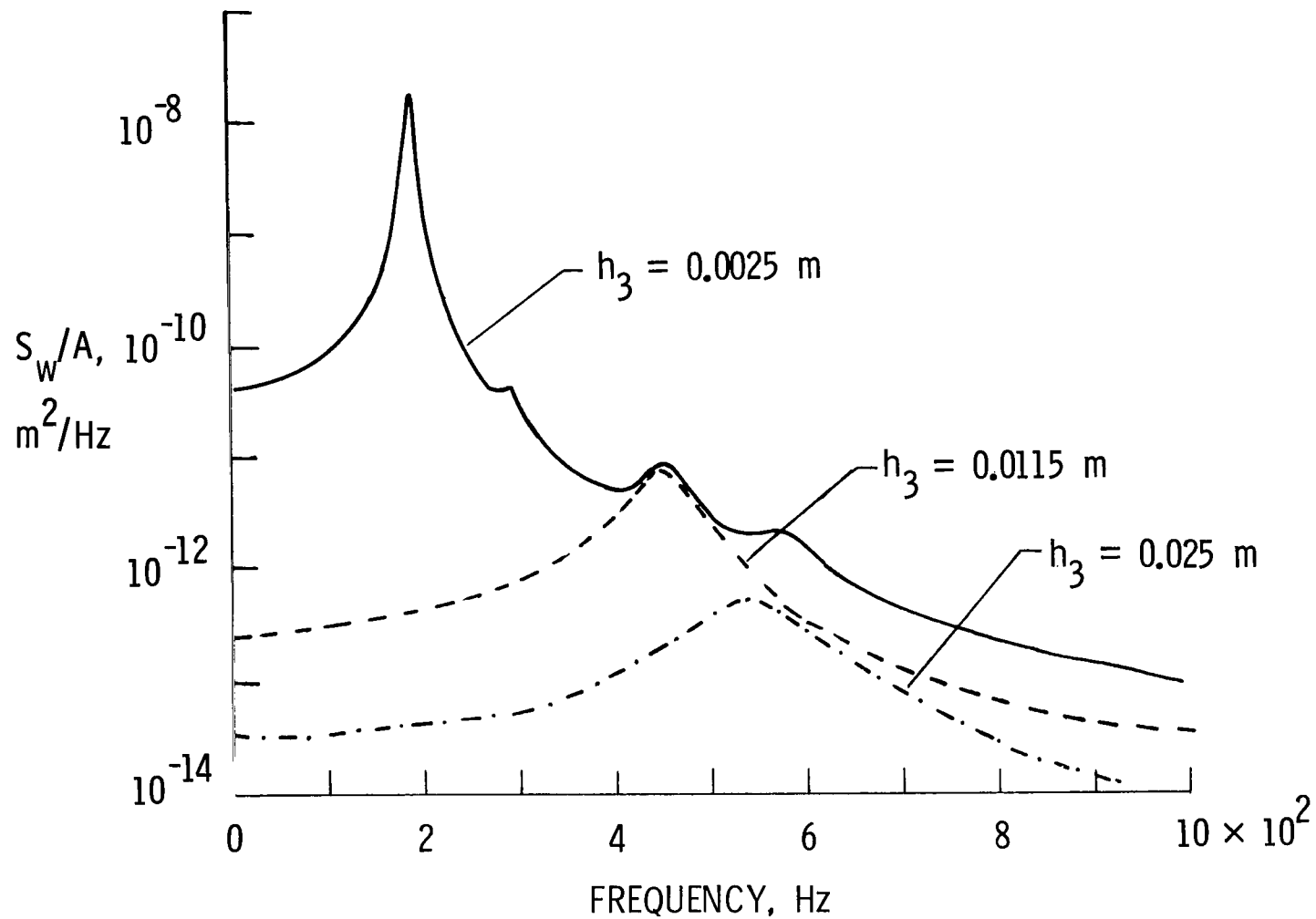
Figure 7.- Displacement response spectral density for different hard core thicknesses. Large shear modulus. $\beta = 0.5$; $A = 0.000645$; $G_0 = 68\ 900\ 000\ N/m^2$.

▽ STRUCTURAL MODES
○ ACOUSTIC MODES
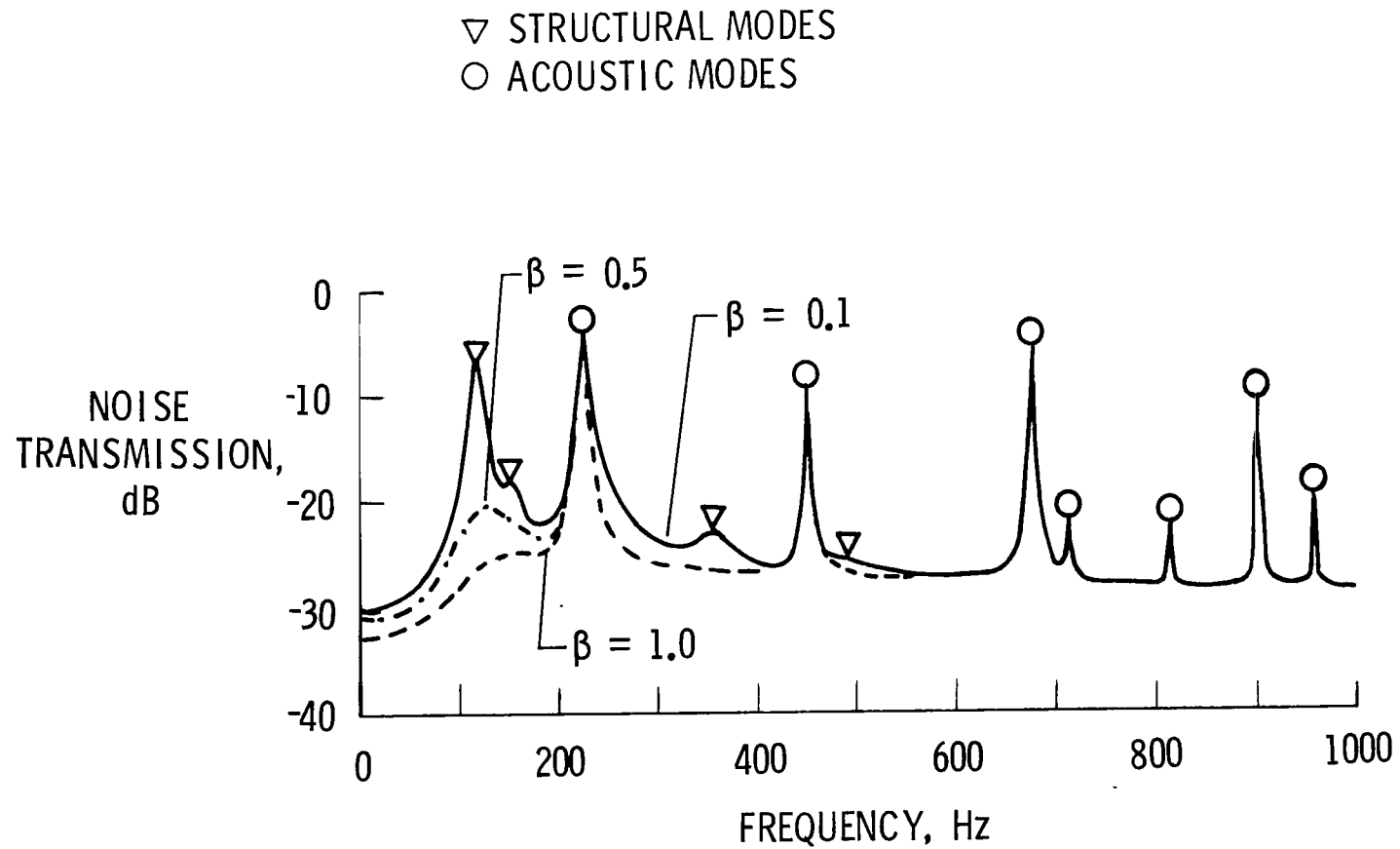
Figure 8.- Noise transmission by viscoelastic panel with hard core and small shear modulus.
$\alpha_{001} = 0.01$; $h_3 = 0.00635$ m; $G_0 = 2\ 760\ 000$ N/m^2.
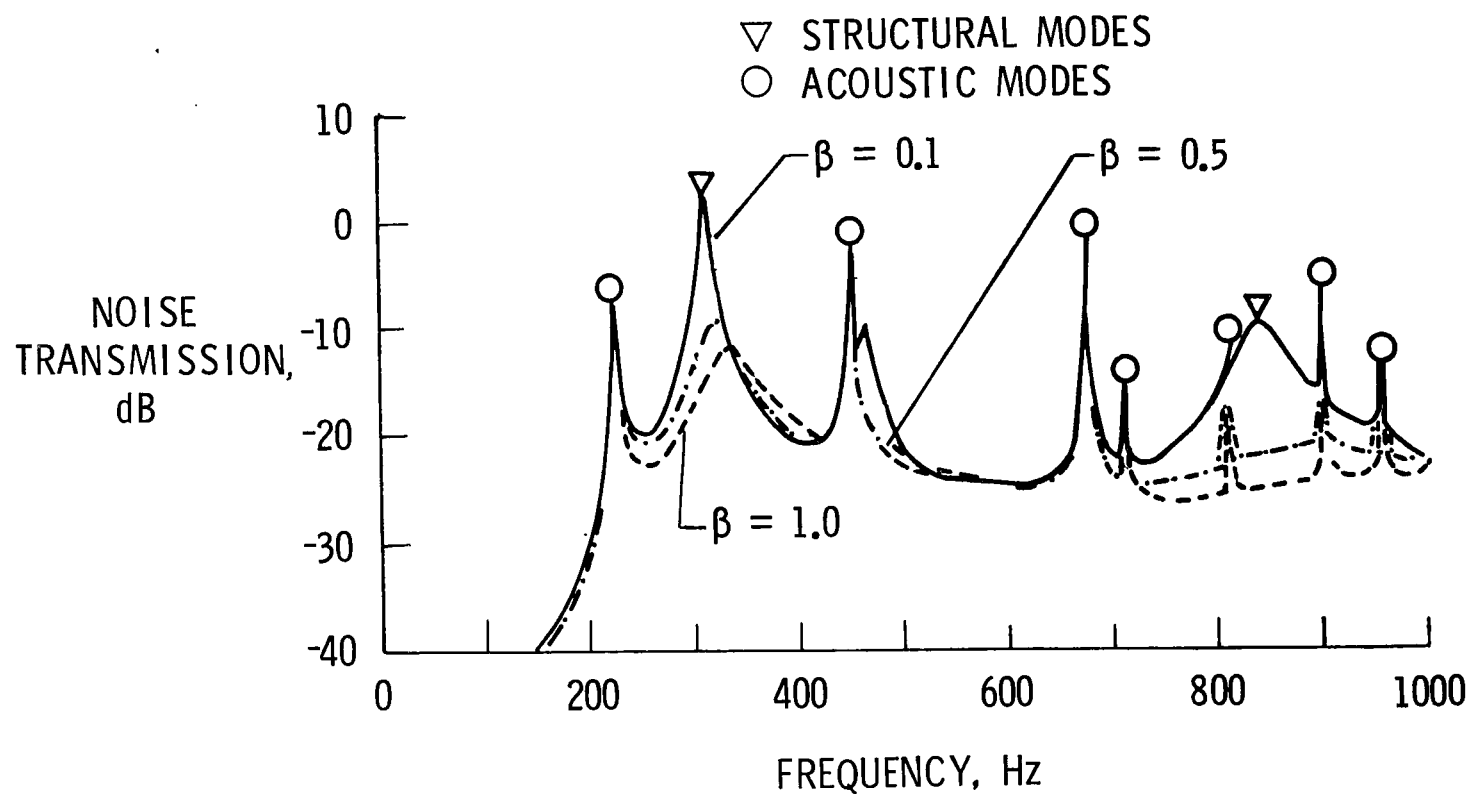
Figure 9.- Noise transmission by viscoelastic panel with hard core and large shear modulus. $\alpha_{001} = 0.01$; $h_3 = 0.00635$ m; $G_0 = 68\ 900\ 000$ N/m^2.
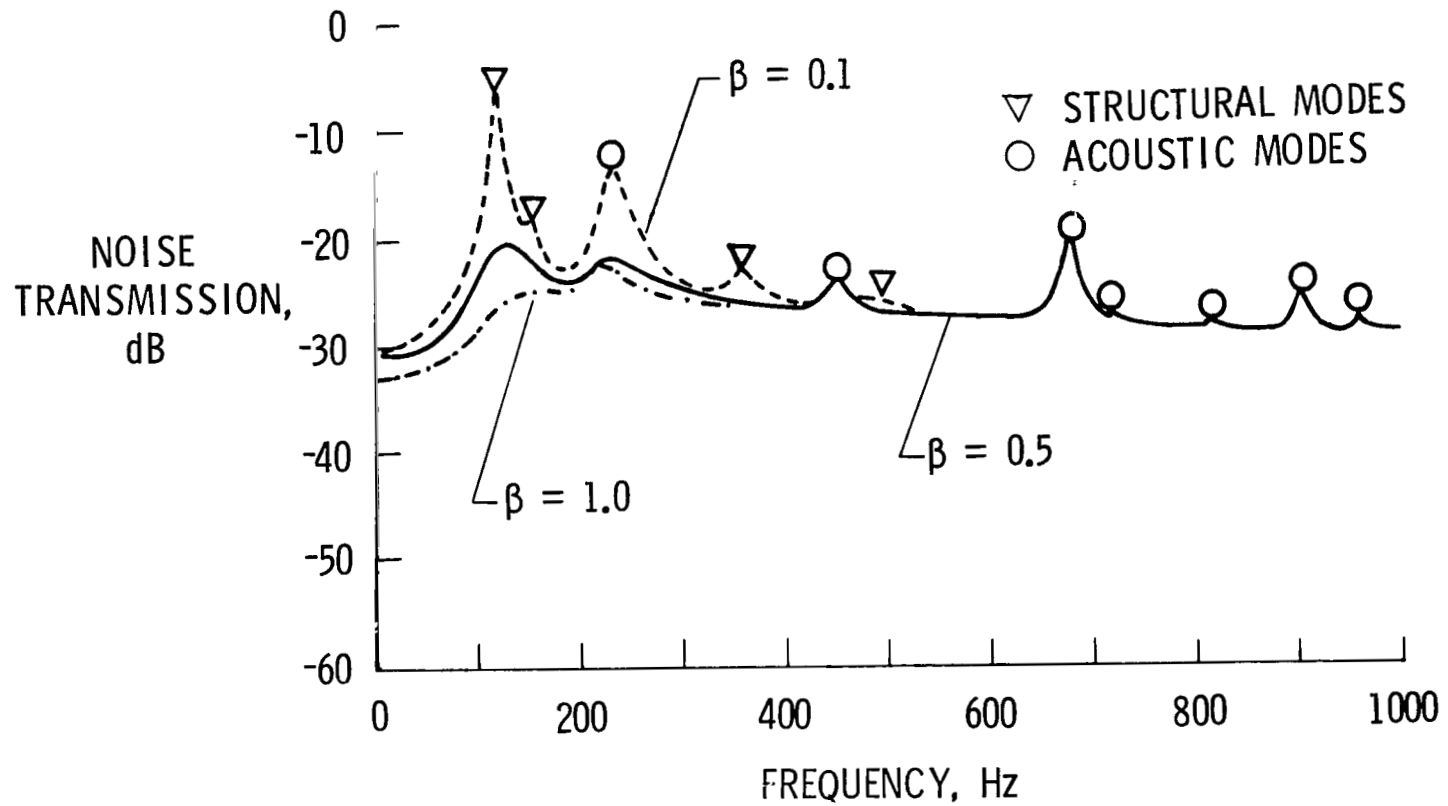
Figure 10.- Noise transmission by panel with hard core into cavity with large acoustic damping. Small shear modulus. $h_3 = 0.00635$ m; $G_0 = 2\ 760\ 000$ N/m^2; $\alpha_{001} = 0.1$.
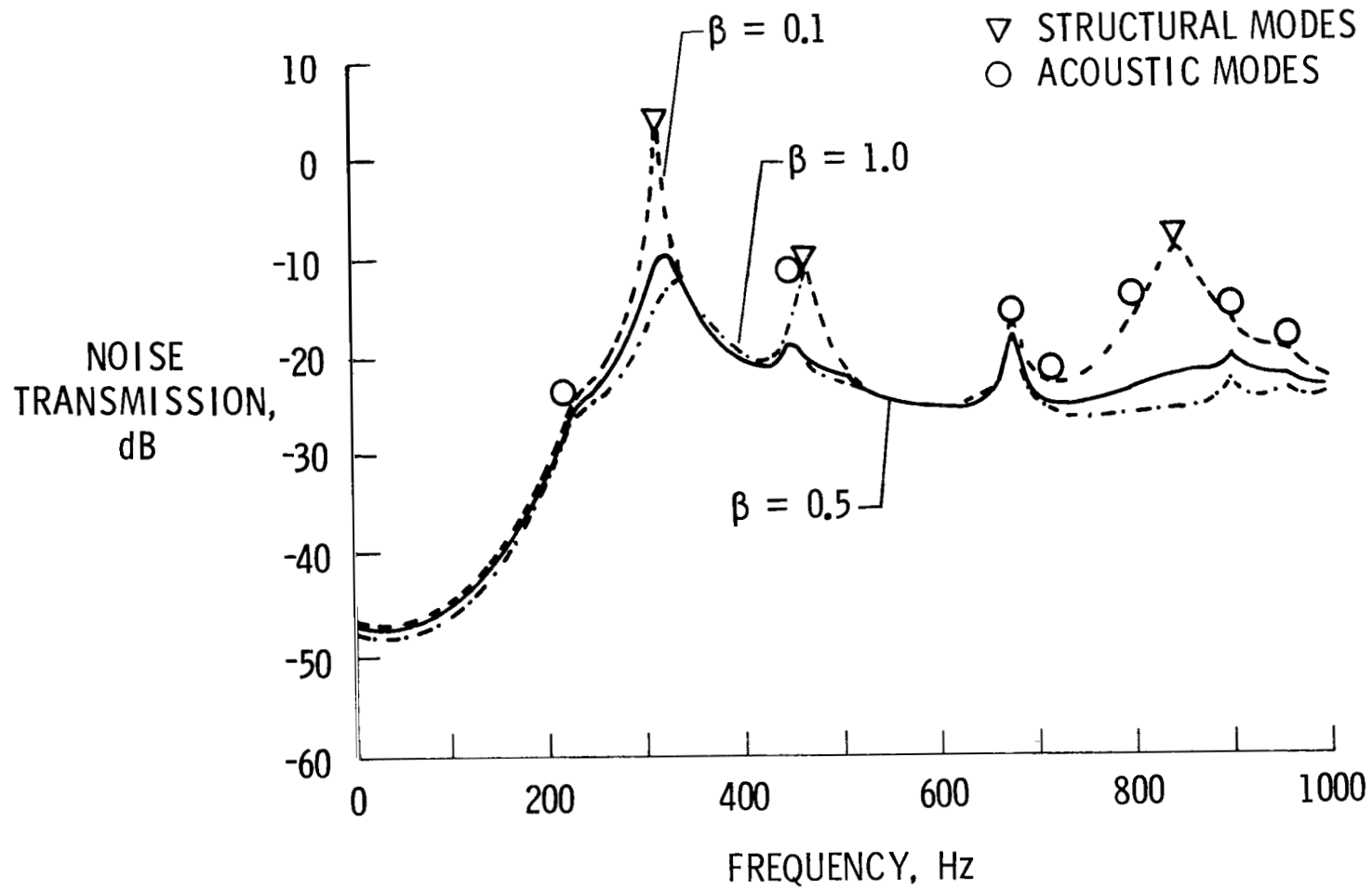
Figure 11.- Noise transmission by panel with hard core into cavity with large acoustic damping. Large shear modulus. $h_3 = 0.00635$ m; $G_0 = 68\ 900\ 000$ N/m^2; $\alpha_{001} = 0.1$.
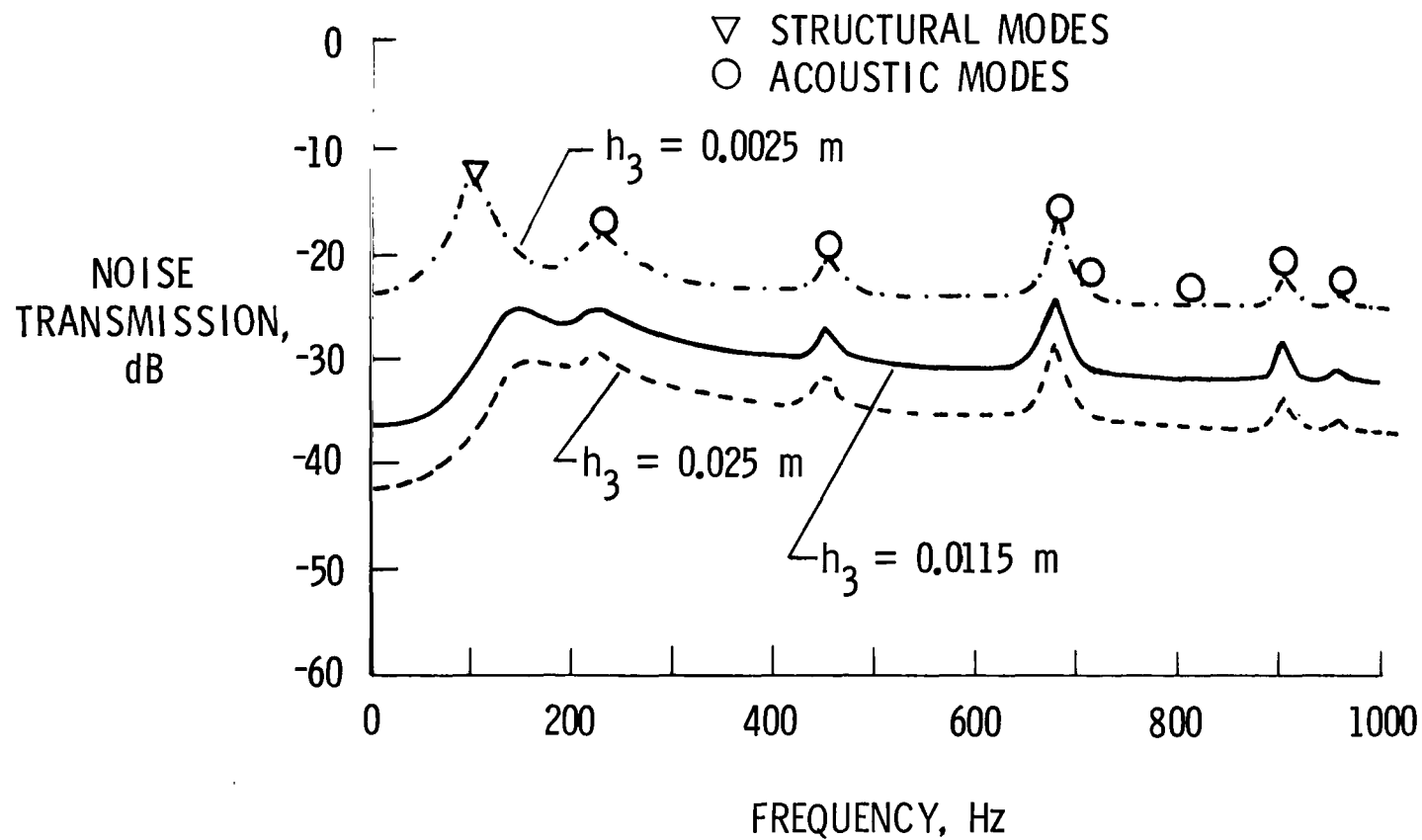
Figure 12.- Noise transmission by viscoelastic panels with different hard core thicknesses into cavity with large acoustic damping. Small shear modulus. $\alpha_{001} = 0.1$; $\beta = 0.5$; $G_0 = 2\ 760\ 000\ N/m^2$.
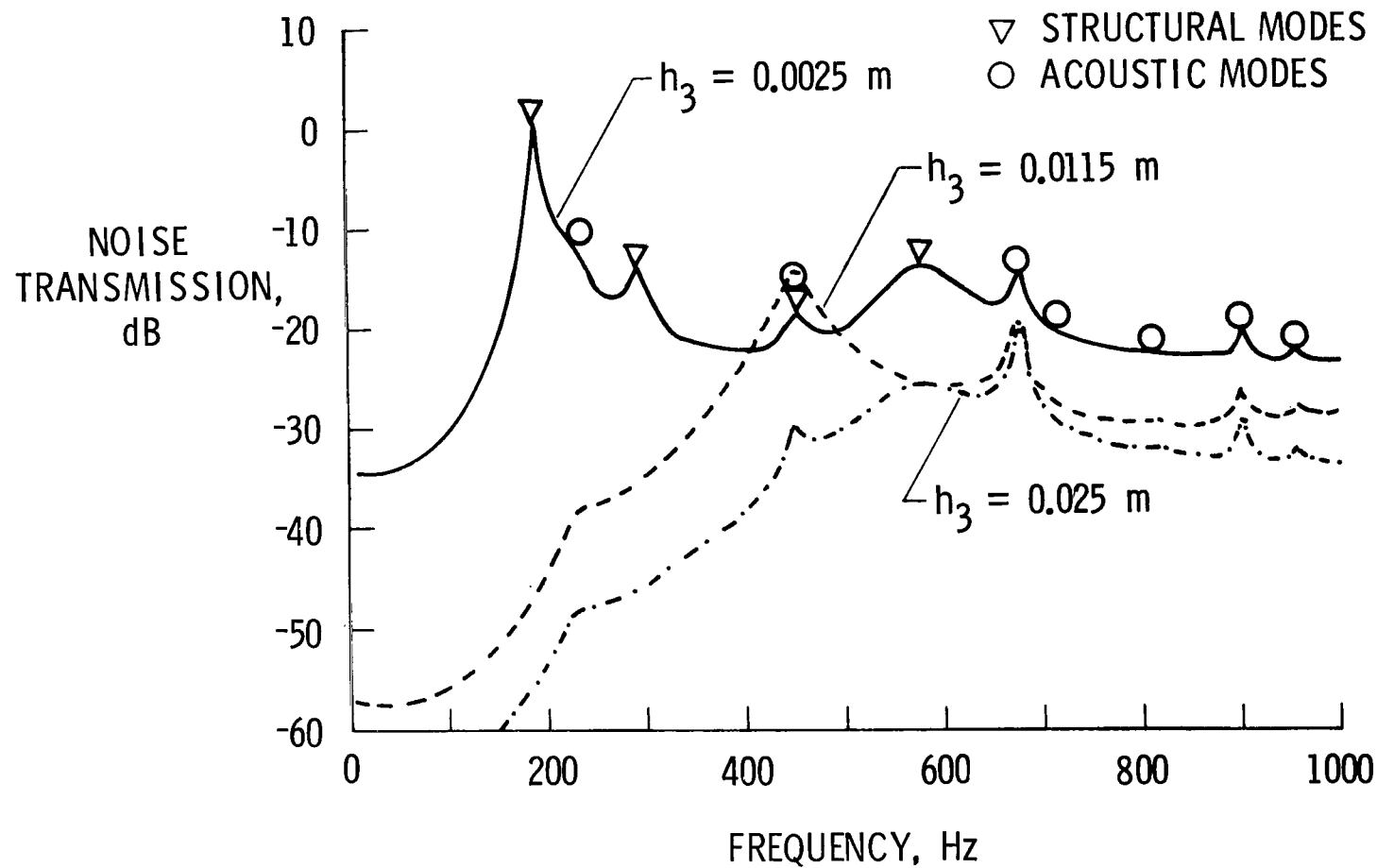
Figure 13.- Noise transmission by viscoelastic panels with different hard core thicknesses into cavity with large acoustic damping. Large shear modulus. G_0 = 68 900 000 N/m^2; β = 0.5; α_{001} = 0.1.
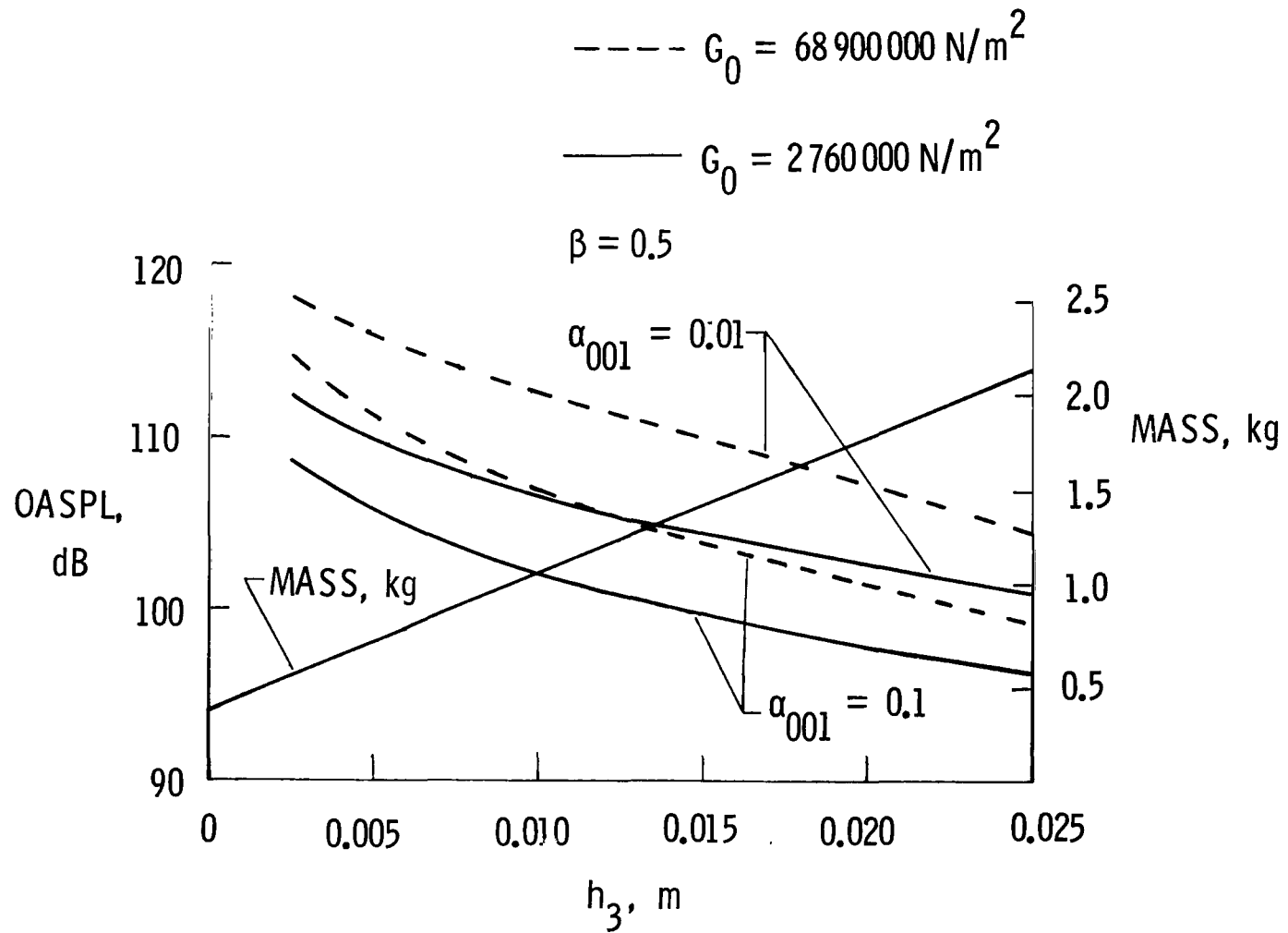
Figure 14.- Overall sound pressure level and panel mass with increasing hard core thickness.
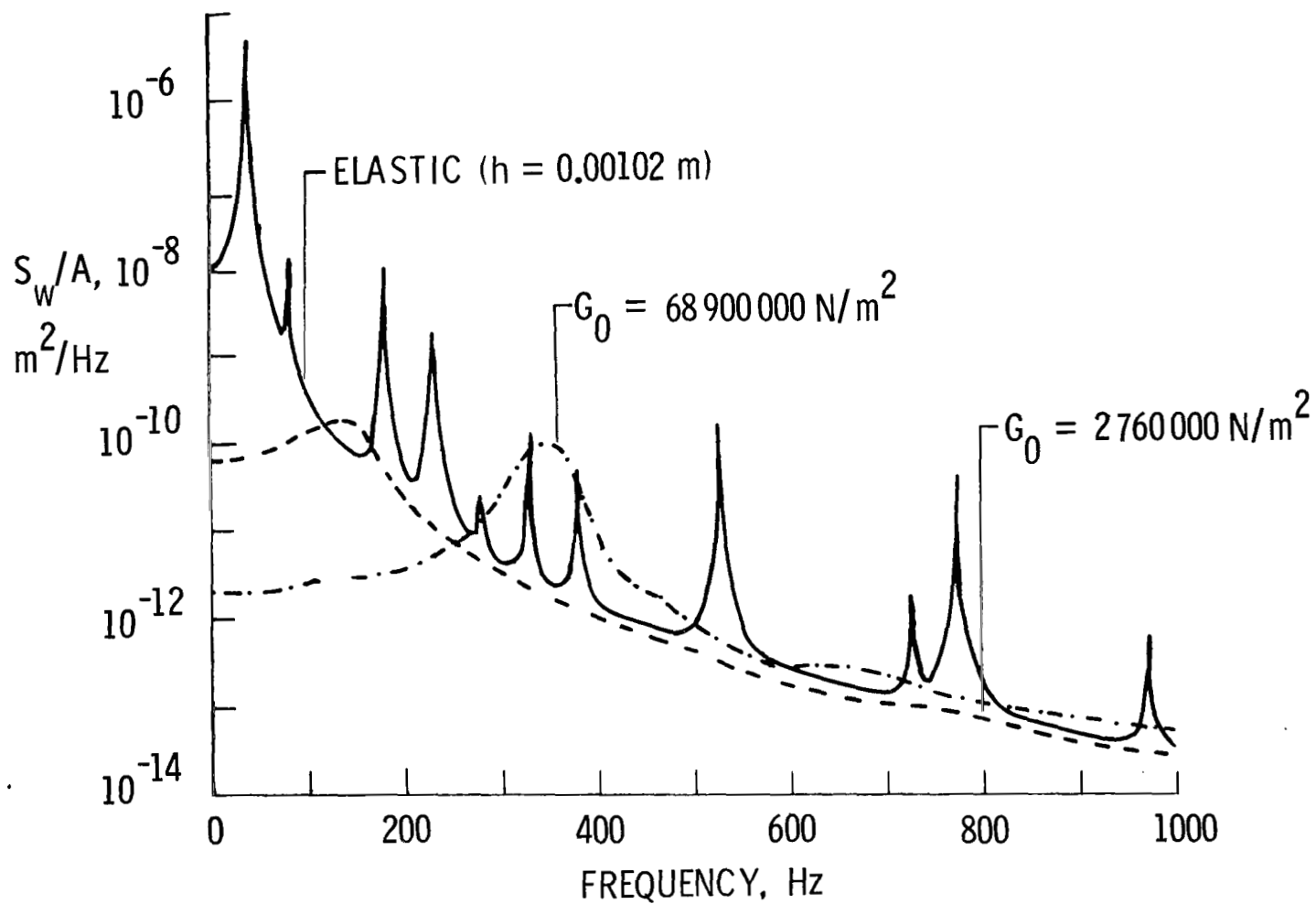
Figure 15.— Displacement response spectral density for elastic and viscoelastic panels with hard cores. $\alpha_{11} = 0.02$; $\beta = 1.0$; $A = 0.000645$; $h_3 = 0.00635$ m.
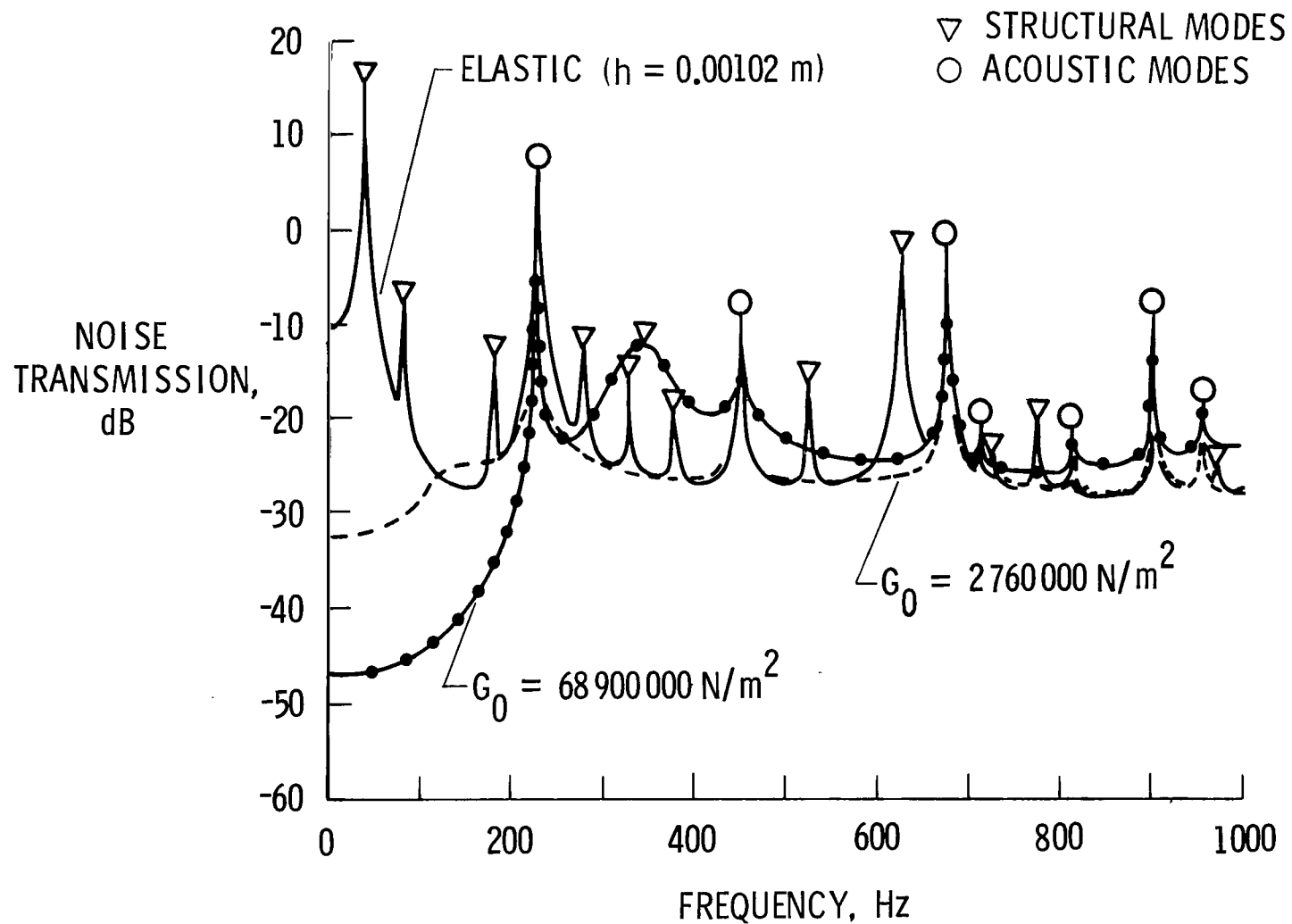
Figure 16.- Noise transmission by elastic and viscoelastic panels with hard cores. $h_3 = 0.00635$ m; $\beta = 1.0$.
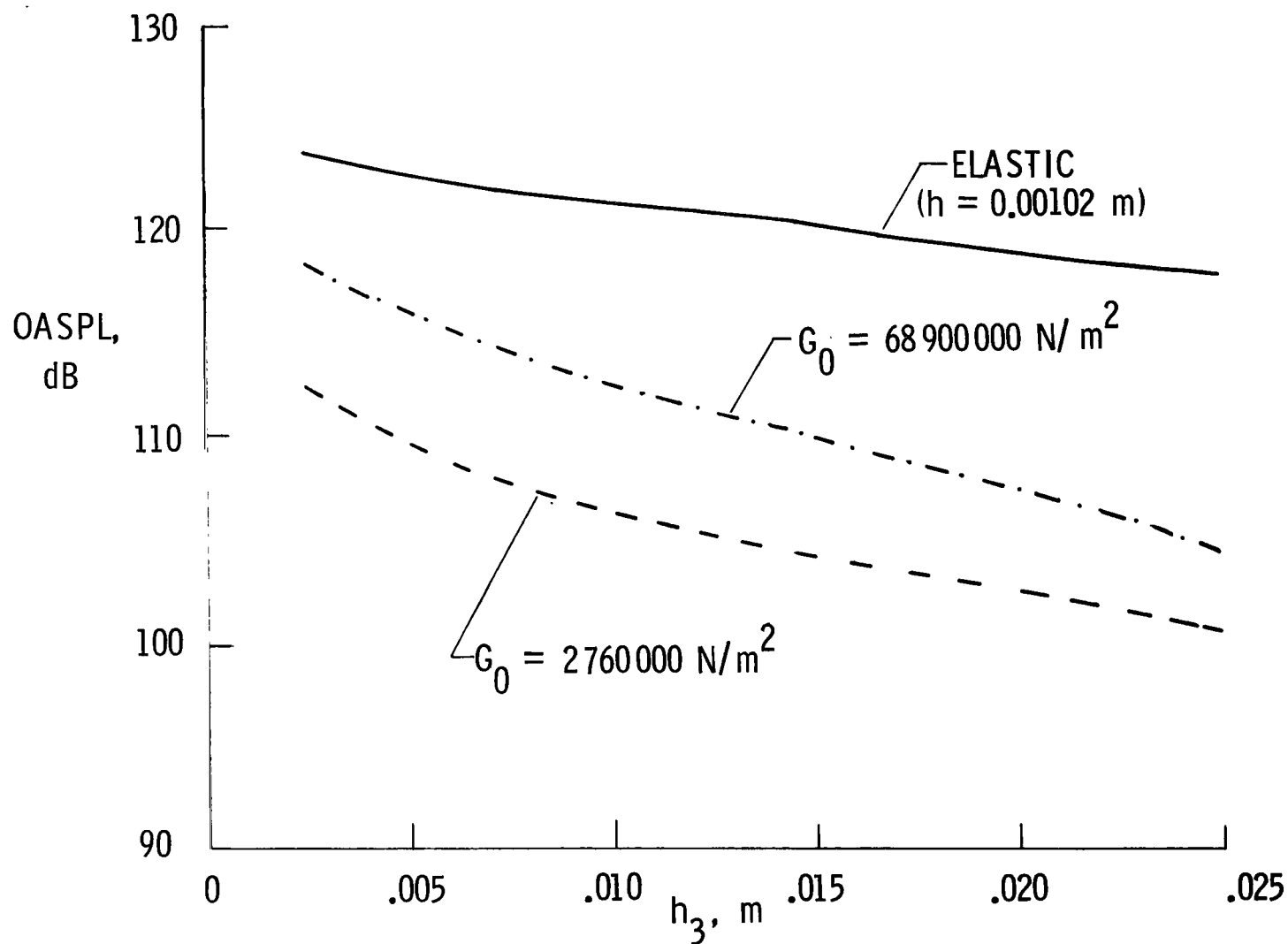
Figure 17.- Overall sound pressure levels for elastic and viscoelastic panels with varying hard core thicknesses. $\alpha_{11} = 0.02$; $\beta = 1.0$.

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