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A STUDY OF THE LUMINOSITY FUNCTION FOR FIELD GALAXIES

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ABSTRACT

Nine determinations of the luminosity function (LF) for field galaxies are analyzed and compared. Corrections for differences in Hubble constants, magnitude systems, galactic absorption functions, and definitions of the LF are necessary prior to comparison. Errors in previous comparisons are pointed out. After these corrections, eight of the nine determinations are in fairly good agreement, contrary to some recent claims. The discrepancy in the ninth (by Arakelyan and Kalloglyan) appeats to be mainly an incompleteness effect. Van den Bergh's early work may contain a normalization error. Schechter's form for the LF is a good representation of the other data. The alternative form of Turner and Gott is based on a small-group sample which is biased toward bright galaxies; this form is not supported by earlier work of Holmberg. The LF data suggest that there is little if any distinction between "field" galaxies and those in small groups.

A new large-scale normalization of the LF is carried out, using inte-

gral counts in the Zwicky Catalogue to $m_{Z\ell} = 14.45$ in the galactic northeast and south, in the manner of Gott and Turner. A first-order K correction is developed, absorption $\propto \csc|b|$ is included, and a recalibration of the Zwicky scale by Kron and Shane is applied. The large-scale mean LF of galaxies (mostly field galaxies) is about a factor 2.3 below the "local" LF derived by Schechter. The nominal mean luminosity density arising within the B(0) isophotes of galaxies at the blue band is $\mathcal{L} \approx 8.6 \times 10^7$ (H/50) $L_\odot$ Mpc^{-3} for $\alpha_B = 0.25$ mag; dependence of $\mathcal{L}$ on input parameters is shown in a table. The true value of $\mathcal{L}$ is probably within a factor 1.6 of this. Larger variation in $\mathcal{L}$ is possible if the recalibrated Zwicky scale be seriously in error. More study of this scale is needed; Huchra's recalibration may be inadequate at the faint end.

I. INTRODUCTION

Luminosity functions (LF's) for galaxies are receiving increased attention, especially since Schechter's (1976) attempt to fit both field and cluster galaxies to a new analytical form. There are now in the English-language literature nine independent or quasi-independent modern determinations of the LF for field galaxies. All are strongly or entirely dependent on the local sample of prominent galaxies, and so there should be a great measure of agreement. Later authors, however, have not always paid close attention to the earlier work. This is understandable, for intercomparison of these papers is hampered in some cases by terse presentations, a variety of assumptions, and various ways of displaying the results. Such intercomparisons as have been made (e.g. Kiang 1976) have not always been successful, and give an impression of substantial discrepancies. Thus, theorists' estimates of such quantities as the mean luminosity density in space suffer from confusion. Some theorists compound this difficulty by trying, in effect, to integrate the LF and obtain some number of "galaxies per Mpc^3." Since present knowledge of the LF does not permit us to place any maximum on the number density of very faint galaxies, the LF cannot be characterized in this simple way.

Recognizing the confusion, Peebles (1971) presented a short review. My work here is much in the spirit of his, but is more detailed and extends the comparisons to recent results. The present time is apt for such comparisons. Most workers to date have used samples of roughly 200 galaxies – essentially the number in the de Vaucouleurs' Reference Catalogue (1963; "RC") down to magnitude $B(0) = 12$, certain areas of the sky being excluded. The appearance of the Second Reference Catalogue (de Vaucouleurs, de Vaucouleurs and Corwin 1976), the increasing use of the Zwicky Catalogue (Zwicky et al. 1961-68), and the rapid increase in number of measured redshifts insure that future work will be done with much larger samples. At such a juncture it is instructive to see what is known already.

Determining the shape of the LF and normalizing it can in principle be tackled as two separate problems. In the "standard" or "classical" method of determining the field LF, which I shall describe shortly, the two are done together. In the presence of inhomogeneities, however, there is some advantage in dealing with the two separately. This has been done in some papers. In Part VI I discuss the normalization problem, present a new normalization of the LF based on the work of Gott and Turner (1976), and give revised estimates of $\mathcal{L}$, the mean luminosity density in the universe.

II. STATEMENT OF THE PROBLEM

By a differential luminosity function ϕ(M) we mean a volume-averaged density of galaxies per (mag Mpc^3); i.e., on a Poissonian model, the function giving the probability ϕ(M)dMdV that a galaxy will be found in any small element of absolute magnitude dM and volume dV. The magnitude M of a spiral galaxy depends on the direction from which it is observed, but in the LF context M for a galaxy usually means the magnitude corresponding to its mean apparent luminosity averaged over all directions. Kiang (1961, "K61") took a different approach, and I discuss the matter further in Part IIIC.

The concept of a field-galaxy LF is more vague, since even the existence of a true "field" (unclustered) population is in doubt (Soneira and Peebles 1976, Huchra and Thuan 1977). In a way this doubt is strengthened by LF investigations, for as we shall see, two studies (Holmberg 1974, "Ho74"; Turner and Gott 1976b, "TG76") which attempted to single out galaxies belonging to small groups, and determine their LF, produced no strong evidence that their LF differs from that for the general "field." I shall therefore use the term "field galaxy" to denote any galaxy not in a rich cluster (Arakelyan and Kalloglyan 1970, "AK70"). If there are any systematic differences between "field" and "cluster" LF's, due to formation conditions or to galaxy-galaxy interactions, the effects should be most noticeable in rich clusters. Yet even in the richest clusters the existence of such effects is in some doubt (Schechter and Peebles 1976). It seems reasonable in the first instance to attempt a distinction between non-rich-cluster ("field") and rich-cluster galaxies.

It is of interest whether the LF for spirals has the same shape as those for ellipticals and irregulars. Several authors have addressed this point (van den Bergh 1961, "B61"; AK70; Shapiro 1971, "S71"; Ho74; Christensen 1975, "C75"; TG76), but the results are best summarized by saying that no significant differences have been found. The numbers of ellipticals and irregulars in these samples are too small to yield firm results. In this paper I shall lump all types together. Christensen (1975) discusses possible type-to-type differences in the small-sample statistics. Holmberg (1974, 1975), with a somewhat larger sample, suggests a difference between E-SO-Ir combined and Sa-Sb-Sc combined.

The field-galaxy LF has invariably been determined from a more or less local sample. There is a standard procedure for doing this, used in four of the nine papers discussed here. The procedure has been described clearly (C75; Schechter 1976, "Sc76"), but it is worthwhile to summarize it: A catalog is selected which is assumed to be complete to some chosen limiting apparent magnitude m_ℓ, at least in some chosen solid-angle region of the sky. A "magnitude-limited" sample is then obtained by throwing out all galaxies fainter than m_ℓ. For each absolute magnitude M, the volume V(M) effectively searched in this sample is calculated. This is done by writing down the formula for the maximum distance $r_m(M,\hat{r})$ which an object at M in the direction of unit vector $\hat{r}$ could have and still appear in the sample:

$$r_m(M,\hat{r}) = \mathrm{dex}\, 0.2\, [m_\ell - M - 25 - A(\hat{r})]\ \mathrm{Mpc}, \tag{1}$$

and integrating the volume out to this boundary over the solid angle included. $A(\hat{r})$ is the galactic absorption in direction $\hat{r}$. This function is controversial (Shane and Wirtanen 1967; Sandage 1973, 1975; Burstein and McDonald 1975; Heiles 1976), but most LF investigators have taken

$$A(\hat{r}) = a \csc |b|, \tag{2}$$

where a is some constant. When A is of this form, the volume searched at a given M in a whole-sky survey has an hourglass-like shape (Kiang 1976). If the solid angle is restricted, the volume is some portion of the hourglass. Sometimes the only restriction applied is that $|b|$ should be larger than some minimum value b_m; this avoids the large (and uncertain) absorption corrections near the plane. When this is the only restriction, and when Eq. 2 is assumed, the volume V(M) searched at absolute magnitude M, neglecting K corrections, is (Felten 1976)

$$V(M) = \frac{4}{3}\pi\, \mathrm{dex}\, [0.6(m_\ell - M - 25)]$$

$$\times \left[E_2(0.6\, a \ln 10) - \frac{E_2(0.6\, a \ln 10 \csc b_m)}{\csc b_m}\right], \tag{3}$$

where $E_2(x)$ is the second exponential integral (Abramowitz and Stegun 1964, Ch. 5).

The whole range of M is now divided into intervals (say, of 0.5 mag). Each galaxy in the sample is assigned an M, and a histogram is constructed in the usual way (Trumpler and Weaver 1962, Chap. 1.11 and 4.27); a division by the function V(M) must then be performed to convert the histogram to units of ϕ: $mag^{-1} Mpc^{-3}$. The assignment of M's is usually done by redshift z (though other methods are often used for a few members of the sample, especially nearby ones with small z). This requires an assumed value of the Hubble constant H, and it requires that z be known for essentially every galaxy in the magnitude-limited sample. This has limited total samples to a few hundred in the work to date. Some authors present the resulting histogrammatic points directly as a "luminosity function"; others (K61, Sc76) apply correction techniques discussed by Trumpler and Weaver to obtain a best fit to some analytical form.

This procedure suffers from a major handicap: It is sensitive to spatial inhomogeneities in the distribution of galaxies. If the mean density of galaxies at a distance r from the observer is, say, a decreasing function of r, the shape of the derived LF will be distorted from the true LF, because the volume V(M) sampled at M is a function of M. This is so even if the true LF ϕ(M) has the same shape everywhere in space. Everyone realizes this difficulty, and everyone excludes a region of solid angle containing the Virgo cluster. Most authors also exclude members of the Local group, and in some cases additional exclusions are made. As we shall see, it is not clear that these ad hoc measures can cope with the difficulty.

This "standard procedure" is used in four of the nine papers examined here. Of the remainder, one (Huchra and Sargent 1973, "HS73") uses a technique involving Schmidt's V_m estimator which, as applied to this problem, is almost equivalent to the standard procedure (Felten 1976). Four others (K61, AK70, Ho74, TG76) are unorthodox and will require special comment.

III. NINE DETERMINATIONS

A. Four Standard Determinations

I give brief summaries of and some immediate comments on the methods used in these nine papers, beginning with the four (B61, S71, C75, Sc76) which use the standard procedure. Van den Bergh (B61) was the usual reference for a decade. Like several others, it is confined to the northern sky. Galaxies down to $(m_{pg})_\ell = 12$ are included, the magnitudes and other data being taken mainly from the revised Shapley-Ames catalog of van den Bergh (1960). Galaxies with $m_{pg} > 12$ or $\delta < -27°$ are excluded, as are "Virgo cluster" galaxies. The remaining sample size N is 240. For galaxies in a recognized group, the M's are assigned by use of the mean redshift $\bar{z}$ of the group. For non-group galaxies, z is used when $v > 700$ km sec^{-1}; the luminosity classification is used when $v < 500$; and between 500 and 700 half weight is given to each method. A Hubble constant $H = 120$ km sec^{-1} Mpc^{-1} is assumed throughout. The galactic absorption is of the form (2) with $\alpha_{pg} = 0.24$ and $b = b^I$ (old galactic latitude). The paper is rather terse and makes no mention of how the function V(M) was calculated. It is not possible to reconstruct the details at this late date (S. van den Bergh, private communication).

Shapiro (S71) used the Reference Catalogue (RC) and its B(0) magnitudes, like many later authors. His sample is not confined to the north but is limited to $|b| > 25°$ (S. L. Shapiro, private communication), a fact not clear in the text. Galaxies down to $B(0)_\ell = 12$ are included. The z's are taken from the RC and a few from elsewhere. Galaxies with $v < v_0 = 100$ km sec^{-1} are excluded as being members of the Local group, and those in the region $255° < \ell < 300.6°$, $64.7° < b < 79.2°$ are excluded to suppress the effect of the Virgo cluster. Also excluded are an unstated number without known redshifts. The sample size N is 242. Magnitudes M are assigned strictly according to redshifts, with H = 100. The absorption is Eq. 2 with $\alpha_B = 0.24$. The paper is noteworthy for its internal check on completeness of the RC at $B(0)_\ell = 12$, and for its investigation of how the results change when $B(0)_\ell$ or v_0 is changed, or when the Local group is included. Shapiro, like B61, fails to discuss V(M) and its computation, but in fact he was aware of the problem and did a computer calculation.

Christensen (C75), like S71, used the RC and tested and verified completeness at his chosen $B(0)_\ell = 11.85$. Some z's are taken from other sources. He excludes the region $|b| < 20°$ and a Virgo-cluster region $252° < \ell < 303°$, $62° < b < 82°$, but includes the Local group. Nine galaxies with no distance estimates are excluded. N is 222. For $v > 400$, the M's are generally assigned by z (with H = 50), but two are done in other ways. For $v < 400$, other methods are used, and this includes 62 galaxies out of the sample of 222. All M determinations are adjusted to H = 50. This paper is the only one to use a true Sandage-type absorption, with "holes" at the poles:

$$\begin{aligned} A_B &= 0 & |b| &> 50° \\ &= 0.132(\csc|b| - 1) & |b| &< 40°, \end{aligned} \qquad (4)$$

A_B being connected smoothly between 40° and 50°. Christensen also has results (obtainable privately) for the cosecant law (2) with $\alpha_B = 0.25$. Another unique feature of C75 is that ϕ is derived for the two galactic hemispheres separately. A difference is found which suggests motion of the Local group towards the galactic north. It is not clear that the effect is statistically significant. The discussion in C75 is notably fuller and more informative than that in some earlier papers, though it contains two errors which I shall point out later.

Schechter's paper (Sc76) has attracted much attention. The RC is used, to $B(0)_{\ell} = 11.75$, with a few z's from elsewhere. Galaxies with $|b| < 30°$ are excluded, as are five galaxies with blueshifts (local?) and all galaxies within 6° of the point $\alpha = 12^h 27^m$, $\delta = 13.5°$ (the Virgo cluster center). Two others with no distance estimates are excluded. The sample size is 185, and the sample is apparently identical to that used earlier by HS73. Magnitudes are assigned by z (with H = 50) whenever z is available; six must be assigned instead by membership in recognized groups. The absorption is Eq. 2 with a fairly small $\alpha_B = 0.12$. Rather than simply presenting a histogrammatic graph, Sc76 uses sophisticated curve-fitting techniques, including an Eddington-type correction for standard error (similar to that used by K61 fifteen years earlier), to fit an analytical form to the data. He then fits this same form to LF's for clusters. Like C75, the paper is noteworthy for careful presentation. Schechter points out that using z to determine distances whenever possible confers an advantage: The standard error in the distances can be estimated plausibly, so that the Eddington correction can be applied systematically.

B. Huchra and Sargent (1973, HS73)

The sample criteria in HS73 are apparently identical to those of Sc76, and the sample size is identical at 185. Magnitudes M are apparently assigned as in Sc76, except that H = 75 is used and the absorption in Eq. 2 is $\alpha_B = 0.24$. Huchra and Sargent perform a V/V_m test for completeness, and test the effects of including Virgo-cluster and Local-group galaxies; these effects are mostly small. They calculate ϕ by an unusual method not involving direct use of V(M): the use of the "V_m estimator," first applied by Kafka (1967) and Schmidt (1968) to quasar problems. I show elsewhere (Felten 1976) that in the present context this technique is almost equivalent to the standard procedure. The results of HS73 should then differ from those of Sc76 only because of differing H and $A(\hat{r})$ and resulting differences in binning.

C. Kiang (1961, K61)

Kiang's classic but unusual paper, though the earliest, is among the most sophisticated. It is perhaps a little oversophisticated in some respects and must be read with care. Kiang realizes that one need not have a complete magnitude-limited sample to derive a LF. It is sufficient to have a magnitude-selected sample, i.e. one in which the degree of incompleteness is a function of apparent magnitude m but not of M (and therefore not of any related property such as galaxy type or surface brightness).[1] Now this is difficult, for it is certain that surface brightness is a strong determinant of whether a galaxy is noticed. Nevertheless Kiang assumes that the Humason, Mayall and Sandage (1956) redshift catalog is a magnitude-selected sample and unfolds the shape of the LF. He takes magnitudes m from several sources, but essentially they are on Holmberg's (1958) photographic system m_{pg}. Galaxies with $m_{pg} > 15$, $\delta < -30°$, or $v < 100$ km sec^{-1} are excluded, along with galaxies in the (Virgo) region $12^h8^m < \alpha < 12^h48^m$, $2.5° < \delta < 18.5°$. Galaxies at low $|b|$ are not excluded. Magnitudes are assigned strictly by z (with H = 100). The absorption is Eq. 2 with $a_{pg} = 0.25$, except at $|b| < 20°$, where it is slightly smaller.

Tests within the sample now indicate a deficiency of low-luminosity objects, so Kiang makes a debatable addition to the sample from a list of faint companions. Holmberg (1974) has criticized Kiang's data in the faint luminosity classes. Kiang makes an

[1]The same idea had occurred to van den Bergh (1960), who used it with luminosity classifications to derive a rudimentary and non-normalized LF.

Eddington-type correction to his histogram to compensate for errors in the M-determinations, then fits the histogram to a cubic at the bright end and to Zwicky's form $\phi \propto$ dex 0.2M at the faint end.

The advantage of Kiang's procedure is that by abandoning the completeness requirement, he obtains a much larger sample of redshifts; N is effectively about 600, compared to ~200 in the magnitude-limited samples of others. A more accurate determination of the shape of ϕ should then be possible. On the other hand, this procedure does not yield directly a normalization of ϕ, because the completeness as a function of m is not known. It is necessary to normalize in a separate calculation with a complete sample of galaxies at brighter m. Kiang's normalization[2] uses only 119 galaxies and is therefore somewhat less accurate than the standard procedure as applied by others.

Another unusual feature of Kiang's work is that he applies an inclination correction to the magnitudes of spiral galaxies, to undo their differential internal obscuration. This correction is typically ~ 0.1 to 0.3 mag. Its effect is that the LF he derives, insofar as it refers to spirals, refers to their absolute magnitudes as viewed pole-on. The LF's of other authors refer to the galaxies' magnitudes as viewed from the direction of Earth, and for a large sample with random orientations their work should give the direction-averaged LF. It is this latter LF which yields a cosmologically interesting quantity, the mean luminous emission $\mathcal{L}$ per Mpc^3 of space (B61, Felten 1966). This difference between the meaning of Kiang's LF and those of others has not always been noticed, though Schectman (1973) perceived it. Kiang's inclination correction introduces

[2]Careful readers will note that in normalizing he changes his assumption about $A(\hat{\underset{\sim}{r}})$. The difference is not great enough to deserve further comment.

an additional function (with attached uncertainty) into the analysis, and it is not likely that future authors will adopt it.

There is another subtle point arising from Kiang's inclination correction. The magnitude-selected analysis requires (cf. Eq. 5 of K61) that the sample incompleteness be dependent only upon the corrected apparent magnitude. But this cannot be true after inclination corrections have been applied. There will be spiral galaxies missing from the sample, although their corrected apparent magnitudes would be bright enough to get them into it, because they are highly inclined and their observed (uncorrected) magnitudes are too faint. There is no similar effect for ellipticals and irregulars. The sample is thus biased against spirals. The resulting LF will be too low in any luminosity range which contains relatively more spirals than another range. This effect is interesting in principle but probably rather small in practice.

D. Arakelyan and Kalloglyan (1970, AK70)

This work is unusual in several respects. We must pay close attention to the sources of error present, because as we shall see, the results of AK70 do not agree well with others. Arakelyan and Kalloglyan exclude the region $\delta < -20°$, the region $|b| < 20°$, and a Virgo-cluster zone to be discussed shortly. Apart from these exclusions, their sample includes all galaxies in the RC with listed $v > 700$ km sec^{-1}, and the M's for these are obtained from the redshifts (with H = 100) with an absorption $a_B = 0.25$. The B(0) system is used. The sample also includes all galaxies in the RC with $v \leq 700$ for which photometric distances are given by Holmberg (1965). For these, the M's are derived from the photometric distances.[3]

[3]Presumably corrected from Holmberg's H = 80 to AK70's H = 100.

There is a certain degree of incompleteness among these nearby galaxies, since not all RC galaxies with $v \leq 700$ appear in Holmberg's list. Arakelyan and Kalloglyan estimate that this should not cause the LF to be in error by more than 25%. This may be an underestimate of the error in the faint magnitude classes. In Eq. 1, let $M = -20$, and let $m_\ell = 12$; Shapiro (1971) showed that substantial incompleteness in the RC is encountered when m_ℓ is taken much fainter than this. Let A be 0.25 mag in the polar directions. Then r_m for the complete search at $M = -20$ is ≈ 22 Mpc in the polar directions, and smaller in other directions. But if $H = 50$, galaxies within about 14 Mpc have $v \leq 700$ km sec^{-1}, so this portion of the volume searched is represented in AK70 only by galaxies in Holmberg's photometric catalog, and there must be appreciable incompleteness here. From Eq. 3 we find that this portion of the volume is only about 30% of the total, so the AK70 estimate of < 25% error is probably correct for $M = 20$, even at $H = 50$. But for $M = -19$, we have $r_m \approx$ 14 Mpc, so that at this M, essentially the <u>entire</u> sample is drawn from the Holmberg list, and there must be substantial incompleteness. At even fainter M we are dealing with galaxies even nearer, and here, as AK70 points out, Holmberg's list is likely to be fairly complete. But there remains the likelihood of a sizable incompleteness effect around $M = -19$. It seems unwise to exclude RC galaxies with low v in this way, because the sample is already too limited. They can be included, and the errors in magnitude assignments introduced by peculiar velocities can be corrected, if one wishes, by the Eddington technique (K61, Sc76).

Another kind of incompleteness is present in the work of AK70, since their sample is not in fact magnitude-limited (no m_ℓ has been imposed). To deal with this, AK70 use an interesting method of "partial space densities" which has been applied by Arakelyan (1973, 1975) in other contexts. When a distance r and an absolute magnitude

M have been assigned to each sample galaxy, those galaxies in a given narrow interval (M, M + dM) are arranged in a sequence k = 1, 2, 3, . . . , in order of increasing r. The kth partial density is calculated as

$$\phi_k(M)dM \equiv k\left(\frac{\Omega}{4\pi}\frac{4}{3}\pi\, r_k^3\right)^{-1}, \qquad (4)$$

where Ω is the solid angle included in the survey. Thus the density is estimated over larger and larger volumes. At sufficiently small k (small distances) the sample at this M is assumed to be essentially complete, but the sample is small. At some large k, the partial densities will drop off rapidly due to the onset of incompleteness. The true density (the LF) should then be estimated from the ϕ's just prior to the dropoff. If a sample size k large enough for statistical reliability is thereby obtained, the determination should be a good one.

There is a pitfall in this method which is most easily made clear by use of a figure. Figure 1 shows the positions of galaxies in a hypothetical sample obtained in one galactic hemisphere in a "small" magnitude interval (M, M + dM). To obtain a plane figure I have rotated the sample about the galactic polar axis, projecting the galaxies onto one quadrant of a meridional plane. (Note that this projection produces a surface density of galaxies which increases with distance from the polar axis.) The straight line shows the restriction $|b| > 20°$ imposed by AK70. Curve a is the outer boundary r_m of the hourglass volume V(M) given by Eq. 3 when m_ℓ is set equal to 12.0, a nominal limit of completeness for the RC(S71). Within this boundary the sample should then be nearly complete; outside it (since AK70 impose no magnitude limit m_ℓ) some outlying galaxies in the RC are present in the sample, but their space density decreases. If AK70 form their sequence and calculate a partial density stopping at galaxy A, an estimate of the true LF will be obtained. But consider the volume generated by rotating the shaded area about the polar axis. This volume lies within the sphere of radius r_A and is therefore included in Eq. 4. But it lies outside curve a and therefore is in a sense missing from the survey, for the RC will have substantial incompleteness here. The estimate will therefore drop below the true LF. If instead the sphere (and estimate) are extended, say to galaxy B, this effect becomes more pronounced. By this point in the sequence, the dropoff will be noticed, and the sequence will be truncated at a nearer point. Indeed, the incompleteness begins at galaxy C! But because of the shape of curve a, its onset is gradual, and with the presence of statistical fluctuations in a sample of moderate size it is doubtful whether the incompleteness can be recognized even by the time galaxy A is reached. Thus there is a tendency to underestimate the true LF. This kind of error is not large when $|b| > 20°$; the fractional error depends upon how the truncation is decided but should not exceed $N^{-1/2}$, where N is the number of sample galaxies interior to galaxy C. The method of AK70 could be refined to eliminate this error by using the function r/r_m in arranging the sequence of galaxies and using $(r/r_m)^3$ V(M) as the volume divisor in Eq. 4. I have discussed this source of error at some length because it is interesting in principle. The incompleteness error due to the condition $v > 700$ is likely to be much larger in practice.

The exclusion applied by AK70 for the Virgo cluster is an unusually large one. They exclude the entire slice of right ascension $11^h < \alpha < 13^h.5$, which contains not only the Virgo cluster, but also the U Ma, C Vn and Coma groups (de Vaucouleurs 1975) and indeed the entire supergalactic equator in the northern hemisphere (Abell 1975; RC; de Vaucouleurs 1976). Exclusions of this kind are explicitly intended to cut out known inhomogeneities, but the exclusion of every area of the sky where clumping is perceived would certainly produce an underestimate of the mean density of galaxies if they were Poisson-distributed. It is not clear whether this choice by AK70 is better or worse than the smaller exclusions applied in other papers. We must be aware of the difference.

E. Holmberg (1974, Ho74)

This is the most unorthodox paper. The date is deceptive, for the work was completed in 1969. Holmberg's approach is to seek a LF for small groups by counting faint neighbors of 160 prominent spirals on Sky Survey plates. No new redshifts are obtained, and so the satellites cannot be identified individually; they are only counted statistically by comparison with nearby control fields. (This comparison constitutes a subtractive correction for optical pairs.) For lack of photometry of the satellites, their absolute magnitudes M must be obtained indirectly: Their major diameters are derived from their angular sizes on the plates and the distance moduli of their prominent spiral neighbors (for H = 80). The known correlation between diameter and luminosity then gives approximate M's. The corrections for optical pairs are large at faint M. Magnitudes are on Holmberg's system, and an absorption $a_{pg} = 0.25$ is assumed.

This procedure yields a relative LF for galaxies in small groups. Holmberg omits the prominent spirals themselves and includes only the companions in deriving the LF; since the groups are discovered through the presence of a bright spiral in each, including them would obviously bias the sample toward bright galaxies. The resulting sample size is 274. The work is especially valuable in that the LF derived extends to absolute magnitudes as faint as $M \geq -12$. A separate normalization is performed, using bright nearby galaxies as in K61, to get an absolute LF for "field" galaxies. The method implies, and the results suggest, that the LF for small groups is identical to that for the field. The sample size for Holmberg's normalization is only 83, so that a fractional error $\sim (83)^{-1/2}$ may be present.

Now simply omitting the prominent spirals cannot yield a perfectly unbiased sample. Though each small group can then be regarded as a random sample from the LF, each one of these samples is subject to the condition that no galaxy brighter than its omitted spiral be present. The omitted galaxies are heavily concentrated toward the bright end ($M \lesssim -20$ for $H = 50$), so the resulting LF is bound to be relatively too low in this domain. Holmberg's omission of the bright spirals is thus in fact a bit of an overcorrection. He did not discuss this effect. But the sample contains too few bright galaxies for a good determination at the bright end anyway, and the overcorrection does not affect the faint end, where the method provides valuable information to supplement that given by the standard procedure.

F. Turner and Gott (1976b, TG76)

In this paper too the object is to extract a LF for small groups. Turner and Gott (1976a) study galaxies in the Zwicky Catalogue brighter than $m_Z = 14$, excluding the regions $\delta < 0°$ and $b < +40°$. They devise an algorithm for identifying 103 clumps of galaxies on the sky, which they then treat as real physical groups. In the present paper (1976b) they discard those groups having no members with measured z. For the remaining 63 groups, they derive group distances from the mean redshifts for members, the redshifts being taken from Nilson (1973). A Hubble constant $H = 50$ is assumed, and galactic absorption is neglected (at $b > +40°$). A relative LF (a histogram) for these assumed group members can then be drawn. The sample size is 642, but because of the peculiar weighting system adopted, the statistical errors are larger than this number would suggest; the sample includes the large Virgo cluster (238 of 642 members), which for this purpose is classified by the authors as a small group, but this cluster receives only a small statistical weight. The authors realize that "unknown biases" may be introduced in excluding 40 groups without measured z's. It appears, in particular, that this should bias the remaining sample in favor of highly luminous galaxies. Another problem in this work is that, unlike Holmberg, the authors make no corrections for foreground and background galaxies; they realize that the results are therefore unreliable at the extreme bright and faint ends.

In a third paper (Gott and Turner 1976) the authors normalize this relative LF to obtain an absolute LF for field galaxies. Since their normalization is explicitly non-local, I defer discussion of it to Part VI.

IV. CORRECTIONS FOR INTERCOMPARISON

It is tempting to make an immediate graphical comparison of these nine LF's. Such a naive comparison is possible but not very enlightening, because the discrepancies are large. Before a useful comparison can be made, it is necessary to correct the results for the variety of assumptions introduced by the several authors. Four principal corrections must be made:

1. For differing values of the Hubble constant H;
2. For differing magnitude systems;
3. For differing absorption functions $A(\hat{\underset{\sim}{r}})$;
4. For inclination effects in the analysis (K61 only).

Fortunately these corrections can all be made, at least to first order, by horizontal and vertical shifts on the usual logarithmic graph. It will be useful to think of them in terms of a set of "standard assumptions." For example, as regards correction (1), let us adopt $H = 50$ km sec^{-1} Mpc^{-1} as a standard value of H. As shown explicitly by K61, correction (1) to any other value of H is easy:[4]

$$\log_{10}\phi_{50}\left(M - 5\log_{10}\frac{H}{50}\right) = \log_{10}\phi_H(M) - 3\log_{10}\frac{H}{50} \quad (5)$$

Thus if we have a LF ϕ_H derived with $H \neq 50$, and we plot the right-hand side of Eq. 5 against the quantity $M - 5 \log_{10} H/50$, we obtain a graph of $\log_{10} \phi_{50}$ (M) versus its argument M. This is just a horizontal and vertical displacement on the logarithmic graph of ϕ_H(M) versus M. The correction is exact insofar as the assigned luminosities of the sample galaxies are $\propto H^{-2}$. This is true of practically all the galaxies used in these papers.

As regards magnitude systems, most workers have used the B(0) system of the RC, which I adopt as standard. Three papers (B61, K61, Ho74) use magnitudes which are essentially or entirely on Holmberg's photographic system m_{pg}. The difference has sometimes been ignored in LF comparisons but is substantial, mainly because m_{pg} is effectively a large-aperture magnitude. A recent study by Huchra (1976) shows that

$$B(0) = m_{pg} + 0.34 \quad (6)$$

is a good approximation. This is roughly what would be expected from the discussion in the RC. An exact relation[5] would of course contain additional terms, including a small color term, for the photographic and blue bands are not identical. Huchra found the scale difference between B(0) and m_{pg} to be negligible. Equation 6 corresponds to a horizontal (rightward) displacement of a LF curve, and I adopt this correction.

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[4]Relation (5) is misprinted in C75.

[5]The relation given by Sandage and Tammann (1974) omits the aperture correction and should be used only in conjunction with an aperture-correction curve such as that in the RC; otherwise it clashes with Eq. 6.

The remaining paper (TG76) uses the Zwicky magnitudes m_z. The Zwicky magnitude scale has been studied by Kron and Shane (1976), Rubin et al. (1976), and Huchra (1976), and the results are in only rough agreement. The situation is complicated, for there appears to be some scale error. Huchra finds a mean relation

$$B(0) \approx 0.886 m_z + 1.315 \tag{7}$$

for his sample, but with large scatter.[6] Kron and Shane find that there are large differences between the magnitude corrections applicable to Volume 1 and to Volumes 2-6 of the Zwicky Catalogue. A more extensive investigation is necessary, for much use is being made of the Zwicky galaxy counts. In the meantime we may use Eq. 7 to transpose the TG76 histogram roughly to the B(0) system. To first order I simply choose a characteristic mean value $\overline{m}_z$ for the galaxies in the histogram. For a complete magnitude-limited sample, this value, for every class in M, would be near the limit $(m_z)_\ell = 14$. But with the TG76 selection criteria it will be somewhat brighter. I choose $\overline{m}_z \approx 13$ and find, from Eq. 7,

$$M_{B(0)} \approx M_z - 0.17. \tag{8}$$

This is a leftward transposition of the LF, and the rough correction is not more accurate than ±0.1 mag. Qualitatively we can say from Eq. 7 that the leftward motion of the points should be somewhat less at the bright-M end, and somewhat more at the faint end; i.e., in the second approximation the correction for Zwicky scale error would steepen the TG76 LF slightly in transposing it to B(0) magnitudes.

[6] The rms scatter is $\sigma = 0.44$ mag. The constant 1.315 was supplied by Huchra (private communication).

Theorists should note that none of these magnitudes necessarily corresponds to the asymptotic, or total, magnitude of a galaxy, which is still a matter of controversy. I offer some further comments on this in Part VI.

The third correction, for differing absorption functions $A(\hat{\underset{\sim}{r}})$, is also made (approximately) by a horizontal displacement. If we have a LF $\phi(M)$ derived for an absorption $A(\hat{\underset{\sim}{r}})$ and we wish to obtain the LF $\phi_s(M)$ corresponding to a "standard" absorption $A_s(\hat{\underset{\sim}{r}})$, the relationship is

$$\phi_s(M + \langle A - A_s \rangle) \approx \phi(M), \tag{9}$$

where $\langle A - A_s \rangle$ is a weighted mean difference between the two functions. Thus a graph of the given $\phi(M)$ versus the quantity $M + \langle A - A_s \rangle$ is a graph of $\phi_s(M)$ versus its argument M. This is a horizontal displacement. Since it is often asserted (C75, Kiang 1976) that this displacement should be vertical, I give a rigorous proof of Eq. 9 in the Appendix. The result is approximate and is only valid when the sample is large and the function $A(\hat{\underset{\sim}{r}}) - A_s(\hat{\underset{\sim}{r}})$ remains appropriately close to its mean value. That the result is physically correct may easily be seen from the following simple argument. Let a function $A(\hat{\underset{\sim}{r}})$ be assumed, and suppose that the absolute magnitudes M of a very large sample of galaxies have been determined therefrom. Consider the subset of sample galaxies lying in a narrow interval dM around a given value M. These galaxies are sprinkled more or less uniformly through an hourglass volume. Now suppose a new function $A_s(\hat{\underset{\sim}{r}})$ is adopted. For simplicity let $A - A_s = 0.1$ mag, independent of direction. These galaxies now all shift faintward in M by 0.1 mag, but the volume associated with them does not change, because its bounding surface is fixed by the redshift of the most distant galaxy in this subset in each direction. The histogrammatic point given by this subset of galaxies then shifts horizontally (faintward) but not vertically on the graph, because neither N nor V changes. This is the physical content of Eq. 9.

As standard absorption function I choose

$$A_{Bs}(\hat{\underset{\sim}{r}}) = 0.25 \csc |b| \tag{10}$$

in the B system. From Eqs. A13 and A10, and the absorptions used by the various authors, I estimate the following corrections $\langle A_B - A_{Bs} \rangle$, in magnitudes: B61 0.00, S71 -0.01, C75 0.00, Sc76 -0.18, HS73 -0.01, K61 0.02, AK70 0.00, Ho74 0.02, TG76 -0.31. Christensen's (C75) LF calculated directly for the absorption in Eq. 10 is obtainable from him privately, and I have used this, so that the correction becomes zero, as shown. These corrections are all very small except for TG76 and Sc76, who assumed considerably less absorption.

As pointed out earlier, Kiang (K61) introduced an inclination term into his absolute magnitudes. To undo the effect of this, a fourth correction is necessary for his LF only. I content myself with the simplest appropriate correction, namely a horizontal (rightward) translation of his points. An average of the corrections used by Kiang (Holmberg 1958) over galaxy types and inclinations, weighted by the numbers of galaxies of various types in his sample (Humason, Mayall and Sandage 1956), gives $\Delta M \approx 0.18 \approx 0.2$ mag. The correction is probably not more accurate than this. Shectman (1973) used $\Delta M = 0.22$ mag.

V. COMPARISON OF LUMINOSITY FUNCTIONS

The nine determinations have been corrected according to the precepts above, and the results are shown[7] in Figure 2. Histogrammatic bin widths are 0.5 or 1 mag in

[7] The histogrammatic points of K61 were not given explicitly but can be obtained from his Figures 2 and 3 and Eq. 30. Shapiro (1971) kindly supplied privately a table of values, so that it was not necessary to read his small graph.

most cases and are shown for a few points, but most are omitted to preserve clarity in the figure. Error flags represent factors $1 \pm N_i^{-1/2}$, where N_i is the number of galaxies in the ith bin, and are shown if the N_i are reported. These statistical error flags are appropriate for comparing an observed histogram with an analytical fit, but the comparison between two histograms should be closer than these flags would suggest, because there is great overlap in the samples, and remaining statistical differences should arise mainly from different binning.

Two analytical fits are shown, those by K61 and Sc76. These have been corrected to the standard assumptions in the same way as the histogrammatic points. Each of these fits included an Eddington correction, and each used three free parameters. Kiang used a rather ill-determined maximum luminosity as one of his three parameters; the vertical dotted asymptote shows its chosen value. He did not seek a best-fit analytical form in the faint-magnitude range, preferring to adopt Zwicky's form $\phi \propto \text{dex}\ 0.2M$ there. This gives a straight line on the figure, which joins (with discontinuous slope) to Kiang's cubic at $x = -20.45$.

In magnitude units (usual for ϕ), the function proposed by Schechter (1976) in his Eq. 1 takes the form

$$\phi(M)dM = \frac{2}{5}\phi^* \ln 10 \left[\text{dex}\,\frac{2}{5}(M^* - M)\right]^{a+1} \exp\left[-\text{dex}\,\frac{2}{5}(M^* - M)\right] dM, \qquad (11)$$

and Schechter finds $\phi^* = 0.005$, $M^*_{B(O)} = -20.6$, $a = -5/4$. Conversion to our standard assumptions changes $M^*_{B(O)}$ to -20.78 and yields the solid curve in Figure 2, which has the following equation for ordinate y vs. abscissa x:

$$y = \log_{10}\phi(x) = \log_{10}\left\{0.002 \ln 10 \left[\text{dex}\,\frac{2}{5}(-20.78 - x)\right]^{-1/4} \exp\left[-\text{dex}\,\frac{2}{5}(-20.78 - x)\right]\right\}. \qquad (12)$$

This form of $\phi(x)$ is, curiously, a special case of a form once suggested by Kiang (1966). At the bright end it is essentially a double exponential. At the faint end the logarithmic curve has asymptotic slope $(-2/5)(\underline{a}+1)$, which for Schechter's $\underline{a} = -5/4$ is 0.1, i.e. $\phi \propto$ dex 0.1M. The parameter $\underline{a}$ is thus a measure of this slope. Schechter's other two parameters ϕ^* and M^* give the normalization and the position of the exponential dropoff, respectively.

The agreement between these two analytical fits is rather good considering that the papers are separated by fifteen years and use different methods. In fact either curve lies close to most of the recent histogrammatic points. This shows immediately that Kiang's own (1976) recent comparison must be in error. Kiang performed our correction 3 improperly, having introduced a renormalization (i.e. a vertical displacement of his curve), whereas I have shown above that the correct displacement is horizontal. Far more important, he omitted corrections 2 and 4, which in this case give together a horizontal displacement of 0.54 mag. This immediately brings Kiang's 1961 curve into good agreement with the recent results, as shown in Figure 2.

My approach will be to adopt Schechter's convenient result as a standard and compare the others to it. Note first of all the relation between Schechter's curve and his own points , whose consequence it is. The curve lies above the points near the knee, where the error flags are small, but below the points at both extremes of the magnitude range, where the error flags are large. This is expected by users of the Eddington correction; the correction undoes the effect of errors in determination of the magnitudes, which errors tend to remove galaxies from the well-populated bins and add too many galaxies to the end bins where the expected numbers are smaller. We should remember this in judging the fit of a curve like Schechter's to any set of histogrammatic points.

The sample of HS73 is identical to Schechter's, and that of C75 is rather similar. These points agree rather well with his and are therefore fitted by his analytical curve, especially in the region where the error flags are small. Shapiro's (S71) points scatter more (two at M = -20.52 and -20.02 lie surprisingly low) and appear a little lower on average; Shapiro's sample is a little deeper [$B(0)_{\ell} = 12$], and his analysis suggests that an incompleteness effect of $\lesssim$20% (0.1 in the logarithm) might be present. Still, his results are not too far from Schechter's.

Van den Bergh's (B61) points lie systematically low by a factor of 1.5 to 2, which indicates a normalization error in his much-quoted paper. This too may be an incompleteness effect. His sample extended to $m_{pg} = 12$ [$B(0) \approx 12.34$]. Shapiro's work suggests an incompleteness effect of $\approx$40% if the RC is used to this level, and van den Bergh's catalog is older than the RC. On the other hand, van den Bergh's calculation of effective volume may have been faulty. After a renormalization the B61 points would apparently be in good agreement with Schechter's.

Kiang's (K61) points are in good agreement with Schechter's for M > -21, but are significantly lower for M < -21, i.e. Kiang's results indicate fewer bright galaxies. (Note the discrepancy at M = -21.6, and the small error flags.) This steep part of the LF is where the shortcomings of our first-order correction for the differing absorptions, and especially of our attempt to undo Kiang's inclination corrections, will be most apparent. But there may be other reasons for this difference. Recall that Kiang's sample is less complete but deeper. The surface-brightness effect on completeness will tend, if anything, to favor discovery of galaxies of high luminosity (though this may be cancelled to some extent by the bias against spirals discussed in Part IIIC). We seek a more convincing reason for the relative paucity of bright galaxies in Kiang's sample.

Consider the relative depths of the two surveys, as given by Eq. 1. Kiang took $(m_{pg})_\ell = 15$. For $M_{B(0)} = -22.6$, i.e. $M_{pg} \approx -22.94$, and taking $A_{pg} = 0.24$, we have $r_m \approx 350$ Mpc. Thus Kiang's survey reaches deeper at the bright end than any known scale of inhomogeneity in the universe (Peebles and Hauser 1974, Abell 1975). Schechter's survey, like most others, stops at $B(0)_\ell \approx 11.75$ and reaches only $r_m \approx 70$ Mpc. At $M_{B(0)} = -21.6$, the respective depths are 220 Mpc and 45 Mpc. If Kiang's points fall below Schechter's at the bright end, it may indicate (i) that the mean density of bright galaxies is less than that in the local inhomogeneity, or even (ii) that the entire LF should be normalized downward to get the mean over large regions. The Schechter LF is, on the latter interpretation, too high because of local enhancement. The points between M = -21 and -23 suggest that this factor could be ~2.

If we believe that the field LF also applies to clusters, then we might prefer the former interpretation (i) because of Schechter's finding that his LF predicts a non-existent correlation between the richnesses of clusters and the luminosities of their brightest galaxies. A steeper slope at the bright end would ease this discrepancy. But a re-examination of the problem by Geller and Peebles (1976) indicates that Schechter's function is, after all, steep enough to fit the data.

The following conclusions may be drawn: (i) For M > -21, Kiang's data are consistent with the Schechter LF. (ii) For M < -21, Kiang's data provide weak evidence that either (a) the true LF on a scale >100 Mpc falls off more rapidly than Schechter's, or (b) the true LF on a scale >100 Mpc should be normalized about a factor of two lower than Schechter's. This evidence is not strong because of the various corrections involved and the difficulties in Kiang's sophisticated procedure, and the LF must be regarded as quite uncertain at M < -21. Little further progress can be made until the sample is much enlarged. It would be interesting to obtain a complete sample of non-local field galaxies at some distance. Such a project is afoot (R. P. Kirshner, A. Oemler, Jr., and P. L. Schechter, private communication).

The Holmberg (Ho74) points are seen to be in good agreement with Schechter's. Holmberg's two brightest points should probably be raised somewhat to undo the bias against bright galaxies discussed in Part IIIE, and this would then imply a small downward renormalization of all his points. The agreement with Schechter's curve, which is good, might even be improved a little by this change. Note that at the faint end ($M \gtrsim -16$; note inset of Fig. 2) the Holmberg points provide unique information, and favor Schechter's form $\phi \propto$ dex 0.1M over the Zwicky form dex 0.2M adopted by Kiang. Christensen's points shown in this domain represent galaxies of the Local group and give no information about the large-scale LF, as he realized.

In the range $-20.5 < M < -18.5$, the data of AK70 are substantially discrepant from all other results. Several sources of error in this work were discussed in Part IIID. It was pointed out that incompleteness in the local sample as selected could depress the curve in the intermediate-luminosity range.

We should note that AK's exclusion of the supergalactic equator will also act in the same way. Inspection of the maps prepared by van den Bergh (1960) shows that the concentration of neighboring galaxies toward the supergalactic equator is strong only for galaxies in a certain distance range (roughly D = 15-25 Mpc for H = 50); nearer and more distant galaxies are more isotropically distributed. In this situation, exclusion of this region of solid angle will depress the derived LF in the intermediate range of luminosities relative to what it would be were the region included. The truth of this is grasped readily by noting that if a region of enhanced density at a distance of ~ 20 Mpc is excluded in determining a LF, the very faint region of the derived LF will not be affected, because faint galaxies at a distance as great as 20 Mpc do not appear in a magnitude-selected sample anyway. The very bright region will also be unaffected, because bright galaxies are seen in the sample to distances $\gg 20$ Mpc and their number is affected little by a local perturbation. The intermediate region, however, will be depressed, in a manner depending upon the completeness of the sample. Qualitatively this is the effect observed in the AK70 LF.

From this argument we cannot say, however, that exclusion of the supergalactic equator is erroneous. We can only say that exclusion and inclusion give different results for the shape of the LF. If the true LF has a uniform shape everywhere in space, this shape can be unfolded from a sample by the standard procedure provided we choose a part of the sky where the normalization parameter of the LF (ϕ^* in Eq. 11) has the same value at all distances. This would suggest avoidance of the supergalactic equator. But we should also avoid any "holes," and this is nontrivial. The problem of the local LF and that of the local spatial distribution of galaxies are intertwined in the local sample, and await successful disentanglement. Another approach to this is discussed in Part VI.

All these effects in the work of AK should be smaller if a deeper sample be used, extending beyond local inhomogeneities. Arakelyan (private communication) has recently repeated the analysis of AK70, using a new sample: all galaxies of known redshift in the Zwicky Catalogue. The histogrammatic points obtained by him are shown in Figure 2, after adjustments. These results are closer to those of other investigators than to the points of AK70, though there is still a small depression in the curve between M = -19 and -20. Details of this work are not yet published. Kiang's deep sample does not show the dip, and it must be regarded as doubtful, though it should be looked for in future samples.

The last set of data shown in Figure 2, displaced arbitrarily along the vertical axis, is the relative (non-normalized) LF for small groups derived by TG76 and discussed in Part IIIF. The points in parentheses are regarded by the authors as "not trustworthy." The solid line is their best renormalization of the Schechter function (Eq. 12) to these data. Turner and Gott found the fit unsatisfactory, and proposed a flatter function (broken line), obtained by setting $\underline{a} = -1$ and $M_Z^* = -20.85$ in Schechter's general form (Eq. 11).

The question arises whether the departure of these data from Schechter's own function is significant. The groups studied by Ho74 should be rather similar to those of TG76, but we have seen that the LF of Ho74 is well fitted by the Schechter function. As pointed out in Part IIIF, it appears that Holmberg's corrections for bias and for foreground and background galaxies give his work some advantages. The TG76 sample is quite likely biased toward bright galaxies, and this is seen to be the sense in which the results depart from the Schechter function. From this comparison it seems that the evidence for a departure from the Schechter LF (Eq. 12) in small groups, or for any difference between LF's in small groups and in the general "field," is very weak.

VI. A NEW NORMALIZATION

We have seen that, while spatial inhomogeneities may complicate determination of the shape of the field-galaxy LF, several methods seem to converge on a shape not very different from the convenient analytical form proposed by Schechter (1976). The proper large-scale normalization of this function is, however, still in doubt. It appears that a normalization based on prominent (RC) galaxies may give a value too high by roughly a factor of two due to the influence of the local supercluster.

In this respect the work of Gott and Turner (1976) is of interest (see Figure 3). Examining the Zwicky Catalogue in the east and west halves of a large portion of the north galactic cap ($b > +40^0$, $\delta > 0^0$) and in a smaller portion of the south galactic cap ($b < -40^0$, $\delta > 0^0$), they noted the following: If galaxies are distributed uniformly in Euclidean space, and if magnitude corrections such as scale correction, the K correction and evolutionary corrections can be neglected, we expect the number of galaxies in the field brighter than m to follow the law

$$N(<m) \propto \text{dex } 0.6m \quad (13)$$

The counts in the Zwicky Catalogue in east and south obey this law, and the numbers of galaxies per steradian N/Ω are rather similar, implying a uniform density. The counts in the west are much higher and depart from law (13) in the way expected for a local enhancement. This region contains the Virgo cluster and the supergalactic equator. It is clear that the west suffers from local contamination, but it seems that the east and south may give a fairly good sample of a uniform population. Gott and Turner therefore use east and south (with peculiar weights) to normalize their luminosity function to $N(<m_Z)$, picking $m_Z = 14$.

This procedure is obviously ad hoc, but no alternative systematic method of evaluating the large-scale average density has been proposed. It might seem that the differential galaxy count $dN/(dmd\Omega)$ evaluated at rather faint m_Z, say ≈ 14, should give a good measure of the space density of distant galaxies independent of the local supercluster. Unfortunately $dN/(dmd\Omega)$, like N itself, shows a sizable asymmetry between east and west in the Zwicky galaxy counts as analyzed by Gott and Turner (1976, and E. L. Turner, private communication; cf. Soneira and Peebles 1976, Table 1). Apparently the differential count, even at $m_Z \approx 14$, is still perturbed by local galaxies. Another objection to the differential procedure is that taking a differential degrades the statistics.

I adopt the method of Gott and Turner, using their integral counts $N(< m_Z)$ in east and south to normalize the LF, but I drop their peculiar weighting scheme and add some refinements. It has been pointed out by R. M. Soneira (private communication) that all such counts should be adjusted for errors in the Zwicky magnitude scale. This subject has already arisen in Part IV, where the approximate relations (7) and (8) were used to transpose the shape of the Turner and Gott LF. The corrections are perhaps not well known, but as they are large ($\sim$ 0.5 mag in some cases), it is essential to make a stab at least. In recalibrating the integral counts we can apply conveniently the more detailed corrections derived by Kron and Shane (1976) rather than Eqs. 7 and 8 (from Huchra 1976). Huchra's corrections agree with those of Kron and Shane in a general way, but not in detail. In general the corrections Δm tabulated by Kron and Shane depend upon m_Z but also differ between Vol. 1 and Vols. 2-6 of the Zwicky Catalogue. For this reason, Δm for a given m_Z is different in different regions of the sky. Things are simpler if we choose $m_Z \approx 14.45$, for here $\Delta m \approx -0.17$ in all volumes. Then we have

$$m_{pe} \approx 14.45 - 0.17 \approx 14.28 \quad , \tag{14}$$

where m_{pe} is a corrected mean photoelectric magnitude derived by Kron and Shane. It differs from the Holmberg magnitude m_{pg} by an offset $\approx$ 0.12 mag (Kron and Shane's aperture corrections to m_{pg} are negligible), and so, using Eq. 6, we have

$$B(0) \approx 14.45 - 0.17 - 0.12 + 0.34 = 14.50 \tag{15}$$

Then

$$N_{E+S}\,[B(0) < 14.50] = N_{E+S}\,(m_Z < 14.45) = 786 \quad , \tag{16}$$

where the last number comes from the galaxy counts (E. L. Turner, private communication). A different relationship will hold at any other value of m_Z. Furthermore, a different result would be obtained from Huchra's relationship, as I shall show later. A more extensive investigation of the Zwicky magnitudes is badly needed.

We may use Eq. 16 to normalize the LF by assuming Schechter's general form (Eq. 11), predicting the expected count $E(N_{E+S})$, and matching it to Eq. 16 to evaluate ϕ^*. This is similar to the approach of Gott and Turner, who however neglected both the K correction and galactic absorption. The K correction is surprisingly large at $m_Z \gtrsim 14$, and it is useful to evaluate the correction to first order at least because it helps us judge the size of similar effects such as the evolutionary correction. I neglect space curvature, which is a smaller effect (Brown and Tinsley 1974).

The calculation is most easily made by casting Eq. 11 into the alternative guise

$$\phi(L/L^*)\, d(L/L^*) = \phi^*(L/L^*)^a \exp(-L/L^*)\, d(L/L^*) \tag{17}$$

and generalizing the integrals of Gott and Turner. With a K correction Δm_K, the K-corrected form r' of the quantity r_m in Eq. 1 becomes

$$\begin{aligned} r'(M,\hat{r}) &= \text{dex}\,(-0.2\Delta m_K')\ \text{dex}\,[0.2(m_\ell - M - 25 - A(\hat{r})] \\ &= \text{dex}\,(-0.2\Delta m_K')\ r_m\,(M,\hat{r}) \end{aligned} \tag{18}$$

if $\Delta m_K'$ is the correction for a galaxy at distance r'. We have (cf. Gott and Turner 1976)

$$E[N(m < m_\ell)] = \int d\Omega \int_0^\infty \phi\left[\frac{L}{L^*}\right] \int_0^{r'} r^2\, dr\, d\left[\frac{L}{L^*}\right] \quad , \tag{19}$$

where Ω is solid angle. For an approximate K correction in the blue, I set

$$\Delta m_K = K_B z \quad , \tag{20}$$

where z is redshift and K_B is some constant. Then the quantity r' can be written

$$r'(M,\hat{r}) = \text{dex}\,[-0.2\,K_B z'(M,\hat{r})]\,(L/L^*)^{1/2}\,r_m(M^*,\hat{r}) \quad , \tag{21}$$

where

$$z'(M,\hat{r}) \equiv Hc^{-1} r'(M,\hat{r}) \tag{22}$$

Let

$$\xi \equiv 0.2(\ell n\ 10)\,K_B H c^{-1} = 0.2(\ell n\ 10) K_B z'(r')^{-1} \tag{23}$$

Now the redshifts of galaxies counted in a survey are typically small. The redshift z' in Eq. 22 cannot be small for all values of M, but perhaps we could neglect the contribution to the galaxy counts from galaxies with M so bright that $z' \not< 1$. If so, then for K_B not too large we can assume $\xi r' \ll 1$, and the exponential in Eq. 21 can be expanded. Imposing the somewhat stronger condition that

$$4\xi r_m(M,\hat{r}) \ll 1 \tag{24}$$

for the galaxies of importance in the counts, expanding Eq. 21, integrating the Schechter form of ϕ (Eq. 17) in Eq. 19, and dropping terms of higher than first order in ξr_m, we find

$$E[N(m < m_\ell)] \approx \phi^*[\tfrac{1}{3}\Gamma(a+\tfrac{5}{2}) \int r_m{}^3(M^*,\hat{r})\,d\Omega - \xi\Gamma(a+3) \int r_m{}^4(M^*,\hat{r})\,d\Omega] \quad , \tag{25}$$

where r_m is the uncorrected form in Eq. 1 and depends on m_ℓ and $A(\hat{r})$ as well as M*. The second term on the right-hand side of Eq. 25 contains the small parameter ξ and therefore is the first-order K correction.

The integrals in Eq. 25 can be put into more transparent forms:

$$\int r_m{}^3(M^*,\hat{r})\,d\Omega = \text{dex}\,0.6(m_\ell - M^* - 25) \int \text{dex}\,[-0.6A(\hat{r})]\,d\Omega \quad ; \tag{26}$$

$$\int r_m{}^4(M^*,\hat{r})\,d\Omega = \text{dex}\,0.8(m_\ell - M^* - 25) \int \text{dex}\,[-0.8A(\hat{r})]\,d\Omega \quad . \tag{27}$$

The coefficients are known when m_ℓ and M* are given. The integrals remaining on the right-hand sides, which we may call I_3 and I_4, are numbers generally of order unity which may be computed from the absorption law and the solid angle. For example, with the cosecant law of Eq. 2, integral I_3 takes the following form for the Gott and Turner "east" region (in 1950 coordinates):

$$I_3 = \int_{2\pi/9}^{\tan^{-1}C^{-1}} \mathrm{dex}(-0.6\alpha \csc x)\,[\pi + \cos^{-1}(C \tan x)] \cos x \, dx + \int_{\tan^{-1}C^{-1}}^{\pi/2} \mathrm{dex}(-0.6\alpha \csc x)\,\pi \cos x \, dx \quad , \qquad (28)$$

where $C \equiv \tan 27^{\circ}24.0'$. I_3 and I_4 have both been computed for the "east" and "south" regions for several values of α, and the results are shown in Table I. These values and their sums for east plus south are shown in Figure 4, and smooth curves are drawn through the points.

To proceed with evaluation of ϕ^* from Eqs. 16 and 25, we should set

$$K_B \simeq 3 \qquad (29)$$

in Eq. 20 (Pence 1976); K_B for giant ellipticals is about 5, but for the spirals which dominate galaxy counts a smaller mean value is appropriate. Then

$$\xi \simeq 2.304 \times 10^{-4}\ (H/50)\ \mathrm{Mpc}^{-1} \qquad (30)$$

For Schechter's value $a = -\frac{5}{4}$, we have $\Gamma(a+\frac{5}{2}) \simeq 0.9064$ and $\Gamma(a+3) \simeq 0.9191$ (Abramowitz and Stegun 1964, Ch. 6). Setting the absorption equal to our standard value $\alpha = 0.25$, we have $B(0)_\ell = 14.50$ and $M_{B(0)}{}^* = -20.78$ in Eqs. 26 and 27. Equations 16 and 25 then yield

$$\phi^* \simeq 2.20 \times 10^{-3} (H/50)^3 \quad \mathrm{Mpc}^{-3} \qquad (31)$$

for Schechter's own choice of the form of $\phi(M)$.

Schechter's own normalization, based on the local (RC) sample, was $\phi^* \simeq 5 \times 10^{-3}$ for H = 50. Thus the factor of local enhancement in space density of galaxies is about[8] 2.3. Gunn (1974) inferred a factor 2.5, also

[8]Note that this agrees roughly with the factor of two inferred in Part V from comparison of Kiang's and Schechter's LF's.

from the Zwicky galaxy counts; the agreement is somewhat fortuitous, as Gunn neglected scale error and the K correction and used E + W rather than E + S. Gott et al. (1974) also inferred a factor 2.5, this time from the work of Sandage, Tammann and Hardy (1972), essentially in the region E + W + S. Again this agreement is somewhat fortuitous, as Gott et al. made no hypothesis about the Zwicky zero-point error and scale error suggested by Sandage et al. and omitted the K correction. The work of Sandage et al. is interesting in itself, for it shows that the Zwicky counts in north and south tend to merge at the faint end of the catalogue, which strengthens further the impression of a local enhancement mainly in the northwest.

Another interesting question raised by the work of Sandage et al. is that of the influence of rich clusters on such a normalization. The LF shape used has been derived from non-rich-cluster galaxies, but the Zwicky Catalogue includes rich-cluster galaxies; in particular, the Coma cluster lies in our "east" region. Our normalization (Eq. 31) then includes all galaxies, and strictly speaking is logically inconsistent unless the cluster LF is identical in shape to the field LF. Sandage et al., however, showed that inclusion or exclusion of the Coma and Ursa Major clouds affects the Zwicky counts per unit area by only a factor $\sim$ dex 0.1 or 26%. This suggests that field galaxies dominate the counts, so that differences between field and cluster LF's should not distort the results unless the differences are dramatic. The normalization parameter ϕ^* in Eq. 31 should nevertheless be thought of as including both field and cluster galaxies.

From Eq. 31 we may evaluate the large-scale luminosity density in space, $\mathcal{L}$:

$$\mathcal{L} = \Gamma(a + 2)\, \phi^* L^* = \Gamma(a + 2)\, \phi^* L_\odot \text{ dex } 0.4(M_\odot - M^*) \qquad (32)$$

(Sc76). Setting $M_{B\odot} = 5.48$ (Allen 1973), and taking $a = -\frac{5}{4}$ (Schechter's function again) and $M_{B(0)}{}^* = -20.78$ (having transposed to our standard absorption), we find

$$\mathcal{L} \approx 8.60 \times 10^7 \, (H/50) L_\odot \quad \text{Mpc}^{-3} \tag{33}$$

Theorists should note that this is the luminosity density arising from sources within the B(0) isophotes of galaxies. The luminosities corresponding to the "large-aperture" magnitudes m_{pg} of Holmberg are brighter by $\approx$ 0.34 mag (a factor 1.37) on average (Huchra 1976). In the Second Reference Catalogue, de Vaucouleurs, de Vaucouleurs and Corwin (1976) derive "asymptotic" or "total" magnitudes B_T for many galaxies, and they find that these are very close to m_{pg}, within 0.02 mag in the mean and within $\sim$ 0.1 mag in almost all cases. Graham's (1976) conclusions are similar. This extrapolation, however, is still controversial. Kron and Shane (1974), relying partly on their own unpublished data at very large apertures, conclude that the correct extrapolation to infinite aperture is uncertain and could be as large as a factor of three. Because of this problem, the "total" luminosity density is still doubtful.

Theorists should also note that $\mathcal{L}$ is not a bolometric luminosity, but a spectral luminosity at the B band, relative to the sun. The reference figure (Allen 1973) is

$$L_\odot \approx 5.08 \times 10^{29} \text{ erg sec}^{-1} \text{ Å}^{-1} \text{ around } 4400 \text{ Å} \tag{34}$$

The quantity $\mathcal{L}$ is often used as a bookkeeping device for calculating the cosmic mass density, and in that context it does not matter much what precise meaning is attached to $\mathcal{L}$, provided it is used consistently. This cosmic bookkeeping is discussed by Gunn (1974) and Gott et al. (1974).

In the calculation leading to Eqs. 31 and 33, the second term of Eq. 25 (i.e. the K correction) is about 7% of the leading term. This validates the first-order treatment, and it means that the values of ϕ^* and $\mathcal{L}$ derived are about 7% larger than they would be without a K correction. This figure of 7% is larger than might have been expected at $B(0)_\ell = 14.5$. From Eqs. 26 and 27 we see that the fractional correction is $\propto$ dex $0.2B(0)_\ell$, so that at $B(0)_\ell = 16$

it would be 14%. The correction also becomes larger if a flatter $\underline{a} > -\frac{5}{4}$ be chosen (more bright galaxies), as in the LF of TG76. The correction, is, however, independent of H.

The evolutionary correction in the blue (Tinsley 1977, and private communication) is not well determined but is apparently about $\Delta m_E \approx -1z$ for ellipticals and may be larger for spirals. Thus it is opposite to the K correction and possibly smaller. It will then not play a major role in normalization of the LF provided we stay at fairly bright m_ℓ, where the K correction is small.

An interesting feature of this work is that the expected numbers of galaxies in east and south separately can be calculated from Eq. 25, and the ratio can be taken. The result, for our standard cosecant absorption ($\alpha = 0.25$), is $E(N_E)/E(N_S) = 2.33$. The observed ratio (E. L. Turner, private communication) is 551/235 = 2.34. Thus the observed ratio agrees remarkably well with the hypothesis of a rather heavy cosecant absorption. But the statistics are such that we cannot exclude an expected ratio of 2.23. This is just the ratio of the solid angles (given in Table I), and it would be the expected ratio for the hypothesis of zero absorption. A more careful study of the Zwicky scale will be necessary before we can pursue this calculation to fainter magnitudes and learn about the absorption in this way.

The number given in Eq. 33 is not well determined because of uncertainties in the inputs. Let us compare it with earlier determinations and consider the uncertainties. Shapiro (1971) evaluated $\mathcal{L}$; had he applied a K correction and assumed H = 50, his value would have been $\approx 13 \times 10^7$. His determination was local; had he divided by 2.3 for local enhancement, he would have obtained $\approx 6 \times 10^7$ (cf. my 8.6). I noted in Part V that Shapiro's points are a little low compared with other determinations on Figure 2. This is still a bit of a puzzle, though it may be partly an incompleteness effect (cf. Part V).

Gott and Turner (1976) obtained $\mathcal{L} = 4.7 \times 10^7$, substantially smaller than

Eq. 33. Here the situation is clearer. They chose considerably different values of α, $\underline{a}$ and M*. It is useful to construct a table showing what happens to the coefficient in Eq. 33 for various input values of α, $\underline{a}$ and $M_{B(0)}^*$. The central entry 8.6 in Table II is marked $\underline{s}$ and corresponds to the calculation already made. The two other entries marked $\underline{s}$ also correspond in essence to Schechter's own fit but for two other (extreme) choices of α_B. When α_B is changed, M* for the Schechter fit must be adjusted as in Eq. 9. In choosing values of M* in the table I have done this according to

$$\Delta M^* \approx - 1.55 \ (\alpha - 0.25) \qquad ; \qquad (35)$$

this relationship arises from Eq. A14, which is roughly characteristic of the LF determinations in Figure 2.

The remaining entries in the table correspond to extreme variations from Schechter's choice of fit to the data in Figure 2. I allow M* to change by $\pm$ 0.5 mag and $\underline{a}$ to change by $\pm$ ¼. The entries in parentheses are really poor combinations for fitting most of the data and should probably be rejected. One of these (4.6) in fact corresponds closely to the fit chosen by TG76 and the number therefore agrees well with their 4.7. This fit agrees only with their own LF and should probably be rejected for reasons discussed in Part V. The flatter curve ($\underline{a}$ = - 1) tends to give a smaller value of $\mathcal{L}$. The K correction, by the way, is about 9% in the TG76 case.

Note that $\underline{a} = - \frac{3}{2}$ corresponds to the Zwicky shape $\phi \propto$ dex 0.2M at the faint end. It gives large values of $\mathcal{L}$ because faint galaxies make a large contribution. Christensen (1975) assumed an even steeper shape at the faint end and obtained an even larger $\mathcal{L}$, which should be rejected in light of the work of Ho74 and others.

The results in Table II suggest that plausible values of $\mathcal{L}$ are within a factor 1.6 of the value given in Eq. 33. Studies of the LF shape with larger samples could reduce this uncertainty. On the other hand, an even larger variation is possible if my corrections (Kron and Shane 1976) to the Zwicky magnitudes at

$m_Z \leq 14.45$ be in error. If the Huchra relationship (Eq. 7) be used instead, the result for Schechter's fit, with $\alpha_B = 0.25$, becomes $\mathcal{L} = 15$ instead of 8.6. If Eq. 7 be used again with $m_{Z\ell}$ shifted from 14.45 to 15.45, $\mathcal{L}$ balloons to 26, suggesting an increase in $\mathcal{L}$ with distance, which is unpalatable. Thus the Huchra correction at faint magnitudes aggravates (rather than correcting) a problem with the Zwicky scale pointed out by Sandage, Tammann and Hardy (1972). Possibly the Huchra relationship is too much of a simplification to be reliable at $m_Z > 14$. More study of the Zwicky scale is essential.

VII. SUMMARY

Nine determinations of the luminosity function $\phi(M)$ of field galaxies must be adjusted for differences in definitions, and in assumptions regarding magnitude systems, the Hubble constant and absorption, before they can be compared. The adjustments for magnitude systems and for galactic absorption require particular attention; the latter is performed to first order by a horizontal displacement of the LF curve, not by a vertical displacement as sometimes claimed.

After appropriate corrections, the data are in fair agreement, except for those of Arakelyan and Kalloglyan (1970). There are several possible reasons for this anomaly, chief among which is the likelihood of an incomplete sample at values of M around -19. Recent unpublished results from Arakelyan are closer to the consensus of other workers. Van den Bergh's (1961) work is affected by incompleteness or by a normalization error.

The consensus LF is well fitted by the analytical form of Schechter (1976) (my Eq. 11). An alternative form suggested by Turner and Gott (1976b) for small groups is probably biased toward bright galaxies, and is unreliable at the bright and faint ends because of a lack of foreground and background corrections. The work of Holmberg (1974) suggests that $\phi(M)$ at the faint end is closer to Schechter's $\phi \propto$ dex 0.1M than to Zwicky's form $\phi \propto$ dex 0.2M or Turner and Gott's ϕ = constant.

The evidence is very weak for any distinction based on LF shape between general "field" galaxies and galaxies explicitly in small groups.

Schechter's normalization, being local, is probably too high for a large-scale mean. The results of Kiang (1961) suggest that the factor of local (super-cluster) enhancement in Schechter's sample may be ~2. A further investigation of the large-scale normalization, using integral counts in the Zwicky Catalogue in the galactic northeast and south, where Gott and Turner (1976) suggest that the distribution of galaxies is approximately homogeneous, implies that this factor is about 2.3. This normalization is carried out at Zwicky magnitude $m_Z = 14.45$ and incorporates a galactic absorption 0.25 csc|b|, a first-order K correction, and a recalibration of the Zwicky scale given by Kron and Shane (1976). The K correction is of order 5% to 10%. (Evolutionary corrections are opposite in sign to the K correction and possibly somewhat smaller.) The resulting nominal value of $\mathcal{L}$, the mean luminosity density in the universe, is $8.6 \times 10^7 (H/50) L_\odot \mathrm{Mpc}^{-3}$. This is a spectral luminosity at the blue band (not a bolometric luminosity) and refers to the luminosity arising within the B(0) isophotes of galaxies. The total blue emission from galaxies, including the outer parts, is greater by $\gtrsim$0.36 mag if conservative extrapolations are correct, and could be even greater. The quantity $\mathcal{L}$ includes rich-cluster galaxies as well as field galaxies, with field galaxies apparently making the major contribution.

The value of $\mathcal{L}$ depends upon the galactic absorption coefficient α_B and upon the values of the Schechter parameters a and M* chosen to fit the LF. Calculations for reasonable input parameters show that $\mathcal{L}$ is probably within a factor 1.6 of the nominal value above. After correction for local enhancement, Shapiro's (1971) $\mathcal{L}$ agrees tolerably well with the nominal value. Gott and Turner (1976) obtained a smaller $\mathcal{L}$ because they assumed an LF which fits their own data but is too flat to fit the consensus. Christensen (1975) obtained a larger $\mathcal{L}$ by assuming an LF too steep at the faint end to fit Holmberg's results.

A greater change in $\mathcal{L}$ could occur if the Kron and Shane recalibration of the Zwicky scale were seriously in error. The galaxy counts suggest that the alternative recalibration by Huchra (1976) is not reliable at $m_Z > 14$. The homogeneity postulated in northeast and south cannot be verified until the Zwicky scale is well understood.

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APPENDIX: CORRECTING A DERIVED LF FOR A CHANGE IN THE ABSORPTION FUNCTION

Let the correct galactic absorption be $A(\hat{\underset{\sim}{r}})$. I assume that galaxies are distributed homogeneously according to some true LF $\phi(M)$, but that a very large sample (I ignore statistical effects) has been analyzed with an assumed (but incorrect) absorption $A_1(\hat{\underset{\sim}{r}})$, yielding an incorrect LF $\phi_1(M)$. Given ϕ_1, A_1 and A, can we find ϕ?

Let $\phi_{0i}(M)$ be the LF that would be derived from galaxies in solid-angle element $\Delta\Omega_i$ only, on the assumption that $A(\hat{\underset{\sim}{r}}_i) = 0$. This ϕ_{0i} is derived from a count of the number of sample galaxies per unit M in $\Delta\Omega_i$ having magnitudes near M and a calculation of the volume searched at M in $\Delta\Omega_i$, always assuming $A(\hat{\underset{\sim}{r}}_i) = 0$:

$$\phi_{0i}(M) = \frac{N_{0i}(M)}{V_{0i}(M)} \quad . \tag{A1}$$

Now from the physical argument in the main text it follows that the <u>true</u> LF ϕ is given by

$$\phi(M) = \phi_{0i}\,[M + A(\hat{\underset{\sim}{r}}_i)] \quad ; \tag{A2}$$

the absorption $A(\hat{\underset{\sim}{r}}_i)$ is by hypothesis the correct one. The homogeneity of ϕ then guarantees that the same ϕ will be obtained on the LHS for every direction i. The function ϕ_1 corresponding to the <u>assumed</u> absorption A_1 will be

$$\begin{aligned}\phi_1(M) &= \frac{N_1(M)}{V_1(M)} = \frac{\sum_i N_{0i}\,[M + A_1(\hat{\underset{\sim}{r}}_i)]}{\sum_i V_{0i}\,[M + A_1(\hat{\underset{\sim}{r}}_i)]} \\ &= \frac{\sum_i \phi_{0i}\,[M + A_1(\hat{\underset{\sim}{r}}_i)]\,V_{0i}[M + A_1(\hat{\underset{\sim}{r}}_i)]}{\sum_i V_{0i}\,[M + A_1(\hat{\underset{\sim}{r}}_i)]} \\ &= \frac{\sum_i \phi\,[M + A_1(\hat{\underset{\sim}{r}}_i) - A(\hat{\underset{\sim}{r}}_i)]\,V_{0i}[M + A_1(\hat{\underset{\sim}{r}}_i)]}{\sum_i V_{0i}\,[M + A_1(\hat{\underset{\sim}{r}}_i)]} \quad ,\end{aligned} \tag{A3}$$

or, passing to the integral,

$$\phi_1(M) = \frac{\int \phi\,[M + A_1(\hat{r}) - A(\hat{r})]\,dV_1(M)}{\int dV_1(M)} \quad . \tag{A4}$$

The integral is over solid angle, and the volume element has been written $dV_1(M)$ because, as may be seen, it is in fact the volume element searched at M if the absorption be $A_1(\hat{r})$.

Equation A4 is an inconvenient integral equation for finding the true LF ϕ when A, A_1 and the "false" LF ϕ_1 are given. Things are simple, however, if $A_1(\hat{r})$ be assumed close to the correct $A(\hat{r})$, or more generally if their difference be assumed everywhere close to some suitable mean value:

$$\Delta A_1(\hat{r}) \equiv A_1(\hat{r}) - A(r) \approx \langle A_1 - A\rangle \equiv \langle \Delta A_1\rangle \quad , \tag{A5}$$

i.e. $|\Delta A_1(\hat{r}) - \langle \Delta A_1\rangle| \ll 1$. Then ϕ inside the integral can be expanded in a Taylor series, and we have

$$\phi_1(M) = \phi(M + \langle \Delta A_1\rangle) + \left.\frac{d\phi}{dM}\right|_{M + \langle \Delta A_1\rangle} \frac{\int [\Delta A_1(\hat{r}) - \langle \Delta A_1\rangle]\,dV_1(M)}{\int dV_1(M)} + \dots , \tag{A6}$$

and the integral term on the RHS will vanish if we take

$$\langle A_1 - A\rangle = \frac{\int [A_1(\hat{r}) - A(\hat{r})]\,dV_1(M)}{\int dV_1(M)} \quad , \tag{A7}$$

i.e. if we weight the mean of ΔA_1 by the volumes searched at absorption $A_1(\hat{r})$ in various parts of the solid angle.

To avoid too much complication, I restrict myself to the usual magnitude-limited survey, in which (cf. Eq. 1)

$$dV_1(M) = \frac{1}{3}\, r_m{}^3 d\Omega = \frac{1}{3}\, d\Omega \;\text{dex}\; 0.6[m_\ell - M - 25 - A_1(\hat{r})]\ \text{Mpc}^3 \quad . \tag{A8}$$

In this case the volume elements searched at two different M's are proportional (i.e. the volumes are geometrically similar), so that the ratio in Eq. A7 is in fact independent of M. Then we can change M to $M - \langle \Delta A_1\rangle$ in Eq. A6 without changing Eq. A7, and so, when the mean value is chosen according to Eq. A7, we find

$$\phi(M) \approx \phi_1(M - \langle \Delta A_1 \rangle) \equiv \phi_1(M - \langle A_1 - A \rangle) \quad . \tag{A9}$$

The quantity $\langle A_1 - A \rangle$ is a constant independent of M.

This shows how to make a transformation, correct to first order, from an incorrect absorption A_1 and its resulting LF ϕ_1 to the correct LF ϕ when the correct absorption A is known. The transformation does not contain the vertical component which might have been expected, but is purely a horizontal translation. The condition for validity is that the difference $\Delta A_1(\hat{r})$ should everywhere be close enough to its mean (Eq. A7) so that the higher terms of Eq. A6 can be neglected. For a common case, viz. $A = \alpha \csc|b|$ and $A_1 = \alpha_1 \csc|b|$, ΔA_1 is unbounded as $|b| \to 0$. Because of Eq. A8, however, these regions of large A_1 contribute little to the ratios of integrals in the second and higher terms of Eq. A6, and so the error in our transformation should not be large.

Since no one knows which $A(\hat{r})$ is correct, the above is purely a theoretical exercise. The reader may, however, suspect that a formula like Eq. A9 will hold for the transformation between two "incorrect" (hypothetical) absorption functions A_1 and A_2. I sketch the proof of this. Assume that ΔA_1 satisfies Eqs. A5 and A7, and that similarly $\Delta A_2(\hat{r})$ is close to $\langle \Delta A_2 \rangle$. Write Eq. A9 again, replacing 1 by 2, and combine this with Eq. A9. Define

$$\langle A_1 - A_2 \rangle \equiv \frac{\int [A_1(\hat{r}) - A_2(\hat{r})] dV_2(M)}{\int dV_2(M)} \tag{A10}$$

and use the fact that

$$\frac{\int \Delta A_1 dV_2(M)}{\int dV_2(M)} \approx \frac{\int \Delta A_1 dV_1(M)}{\int dV_1(M)} = \langle \Delta A_1 \rangle \quad , \tag{A11}$$

because of Eqs. A8 and A5. It follows that

$$\phi_1(M) \approx \phi_2 (M + \langle A_1 - A_2 \rangle) \quad . \tag{A12}$$

The transformation rule in Eqs. A10 and A12 is the same as that in Eqs. A7 and A9. So this procedure can be used to transform ϕ_2 to ϕ_1, provided that both A_1 and A_2

are (a) close to the true $A(\hat{\underset{\sim}{r}})$, or (b) at worst, offset from it by amounts ΔA_1, ΔA_2 which are close to their respective mean values everywhere in that domain of Ω which contributes significantly to the third and higher terms in expansion A6. I have used Eq. A12 in labelling the abscissa of Figure 2, taking $A_2 = 0.25 \csc|b|$.

If $A_1 = \alpha_1 \csc|b|$ and $A_2 = \alpha_2 \csc|b|$, and if the survey covers the whole celestial sphere except for an equatorial zone $|b| \leq b_m$, then Eq. A10 becomes (cf. Felten 1976)

$$<A_1 - A_2> = (\alpha_1 - \alpha_2)\,\frac{E_1(0.6\alpha_2 \ln 10) - E_1(0.6\alpha_2 \ln 10 \csc b_m)}{E_2(0.6\alpha_2 \ln 10) - \dfrac{E_2(0.6\alpha_2 \ln 10 \csc b_m)}{\csc b_m}} \quad . \qquad \text{(A13)}$$

The functions E_1 and E_2 are tabulated (Abramowitz and Stegun 1964, Chap. 5). The second terms in numerator and denominator both approach zero as $b_m \to 0$. Taking $\alpha_2 = 0.25$, and using $b_m \approx 20^0$ as roughly typical of the LF investigations, we find

$$<A_1 - A_2> \approx 1.55\,(\alpha_1 - \alpha_2) \quad . \qquad \text{(A14)}$$

When the sky coverage and/or the forms of A_1 and A_2 are different, Eq. A13 or inspection of Eq. A10 would still provide the basis for an adequate estimate of $<A_1 - A_2>$ in many cases.

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TABLE I. Integrals I_3 and I_4 for Gott and Turner's "east" and "south" regions in 1950 coordinates, with cosecant absorption.

Absorption half-thickness of disk, α (mag)	East I_3	East I_4	South I_3	South I_4
0	0.9163	0.9163	0.4118	0.4118
0.1	0.7752	0.7333	0.3418	0.3212
0.2	0.6562	0.5873	0.2838	0.2507
0.25	0.6038	0.5257	0.2586	0.2215
0.3	0.5556	0.4707	0.2356	0.1957
0.4	0.4707	0.3776	0.1957	0.1529
0.5	0.3990	0.3031	0.1626	0.1195

TABLE II. Values of the coefficient in the luminosity density $\mathcal{L}$ (Eq. 33) for various values of parameters α_B, a and $M_{B(0)}^*$. Values marked "s" all correspond to Schechter's own choice for the shape of the LF.

Absorption half-thickness, α_B(mag)	Characteristic Magnitude $M_{B(0)}^*$	Schechter index a: -1	-5/4	-3/2
0	-19.89	7.0	8.3	(10.8)
0	-20.39	5.6	6.7s	8.7
0	-20.89	(4.6)	5.4	7.0
0.25	-20.28	9.0	10.7	(13.9)
0.25	-20.78	7.2	8.6s	11.2
0.25	-21.28	(5.9)	7.0	9.0
0.5	-20.67	11.5	13.7	(17.9)
0.5	-21.17	9.3	11.1s	14.4
0.5	-21.67	(7.6)	9.0	11.6

FIGURE CAPTIONS

Fig. 1. Spatial distribution of a hypothetical sample of galaxies obtained in one galactic hemisphere in a catalog like the RC. The galaxies shown are those which lie in an interval dM around M = - 20. The three-dimensional sample has been projected onto a meridional plane by rotation about the galactic polar axis. Curve a is r_m from Eq. 1 for $m_\ell = 12$, $M = -20$ and $A(\hat{r}) = 0.25 \csc|b|$.

Fig. 2. A comparison of nine published determinations of the differential luminosity function $\phi(M)$ for field galaxies. A recent unpublished determination by Arakelyan is also shown. Approximate corrections for the authors' various choices of Hubble constant and galactic absorption have been performed implicitly in the choice of ordinate and abscissa scales. Additional corrections for differing magnitude systems, and for an inclination effect in Kiang's definition of ϕ, have been made as described in the text. The points and curves of Turner and Gott are normalized arbitrarily. Points in parentheses are unreliable.

Fig. 3. The "east," "west," and "south" regions of Gott and Turner (1976). The galactic (G) and celestial (C) poles are shown and are in the plane of the figure. The "east" and "west" are symmetric (back and front) halves of the north cap. The "west" contains the Virgo cluster and supergalactic equator.

Fig. 4. Integrals I_3 and I_4 (in Eqs. 26 and 27 respectively) for $A(\hat{r}) = \alpha \csc|b|$, evaluated in the Gott and Turner east, south, and east + south regions, and shown as functions of α.

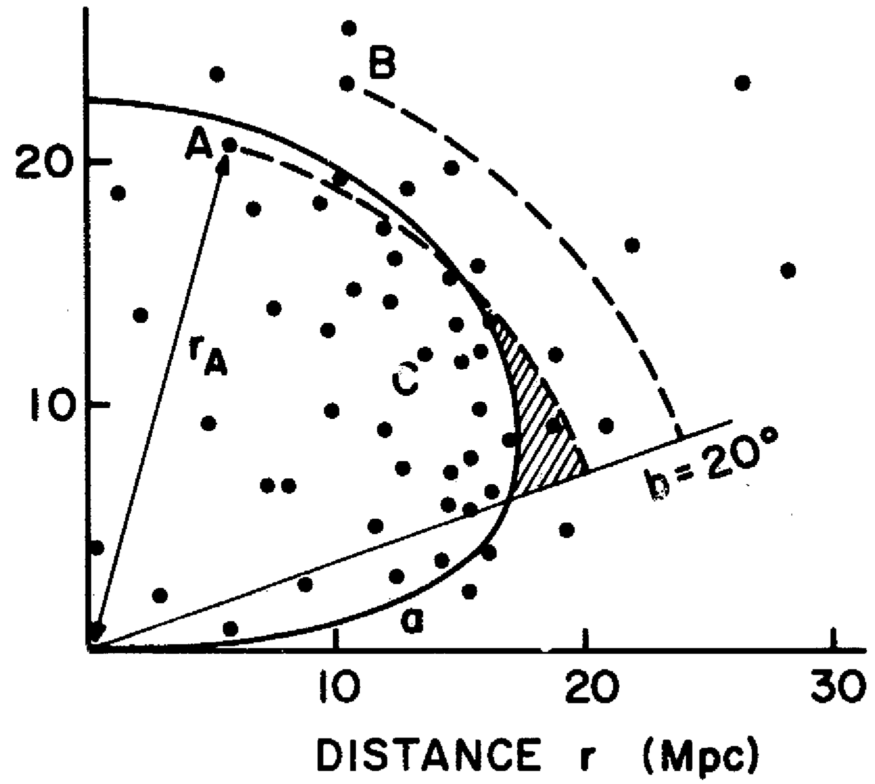

B
A
20
r_A
C
10
b=20°
a
10
20
30
DISTANCE r (Mpc)

FIG. 1

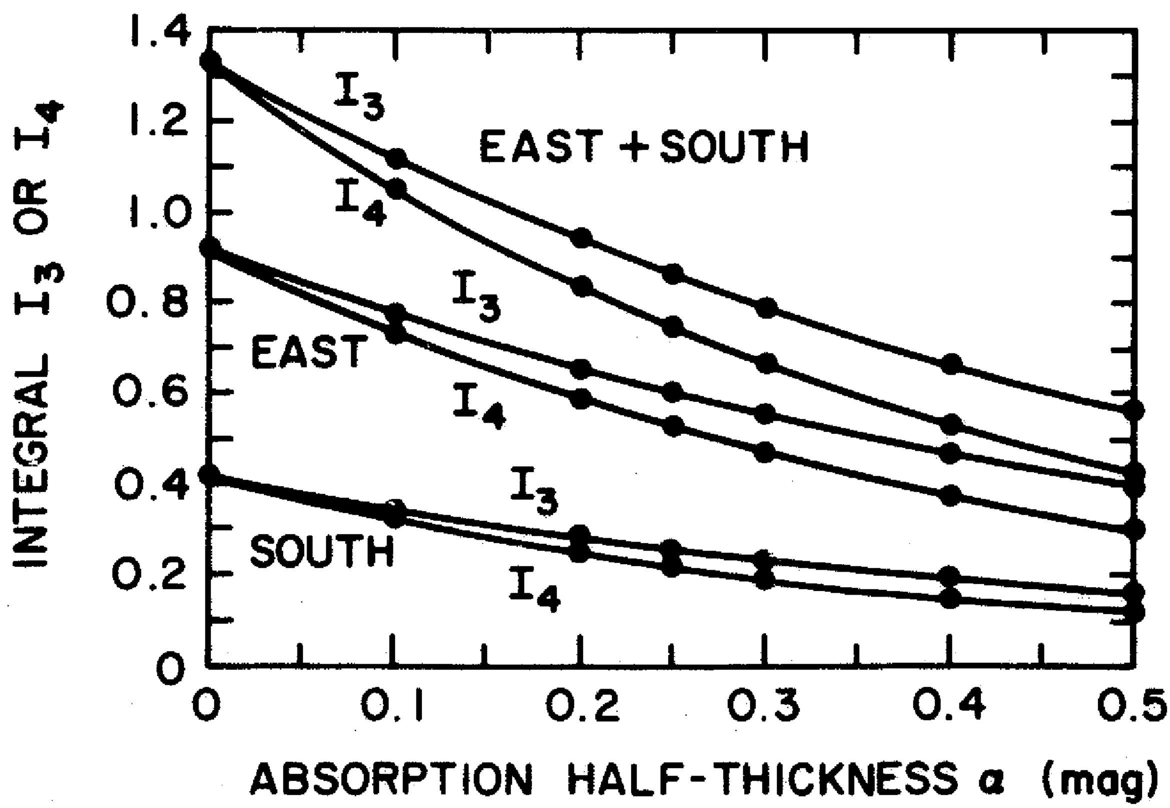
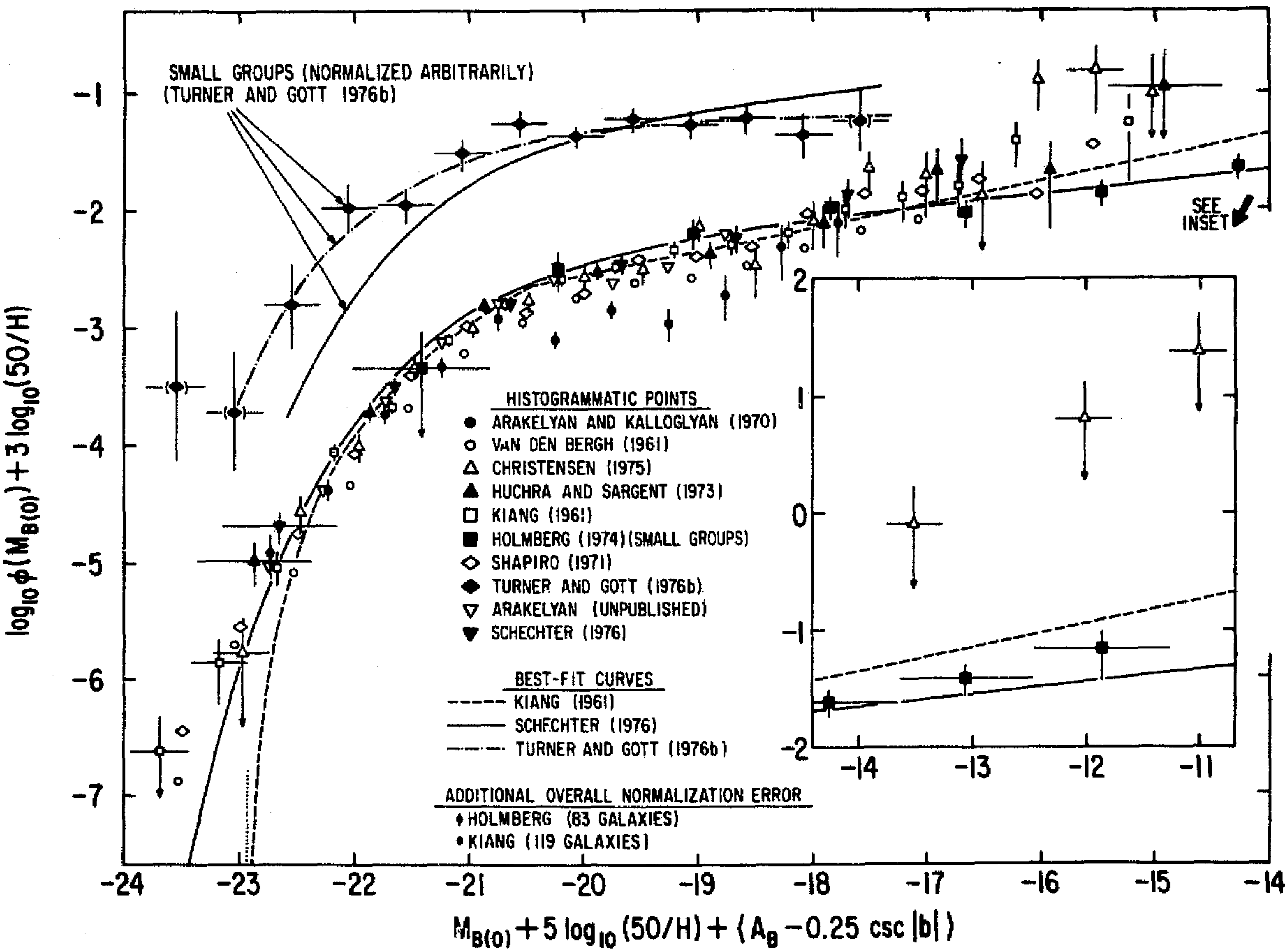

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
INTEGRAL I_3 OR I_4
I_3
I_4
EAST + SOUTH
I_3
EAST
I_4
I_3
SOUTH
I_4
0
0.1
0.2
0.3
0.4
0.5
ABSORPTION HALF-THICKNESS a (mag)

FIG. 4



SMALL GROUPS (NORMALIZED ARBITRARILY)
(TURNER AND GOTT 1976b)
SEE INSET
HISTOGRAMMATIC POINTS
ARAKELYAN AND KALLOGLYAN (1970)
VAN DEN BERGH (1961)
CHRISTENSEN (1975)
HUCHRA AND SARGENT (1973)
KIANG (1961)
HOLMBERG (1974)(SMALL GROUPS)
SHAPIRO (1971)
TURNER AND GOTT (1976b)
ARAKELYAN (UNPUBLISHED)
SCHECHTER (1976)
BEST-FIT CURVES
KIANG (1961)
SCHECHTER (1976)
TURNER AND GOTT (1976b)
ADDITIONAL OVERALL NORMALIZATION ERROR
HOLMBERG (83 GALAXIES)
KIANG (119 GALAXIES)
$\log_{10}\phi(M_{B(0)}) + 3\log_{10}(50/H)$
$M_{B(0)} + 5\log_{10}(50/H) + (A_B - 0.25 \csc|b|)$
-1
-2
-3
-4
-5
-6
-7
-24
-23
-22
-21
-20
-19
-18
-17
-16
-15
-14
2
1
0
-1
-2
-14
-13
-12
-11

FIG. 2

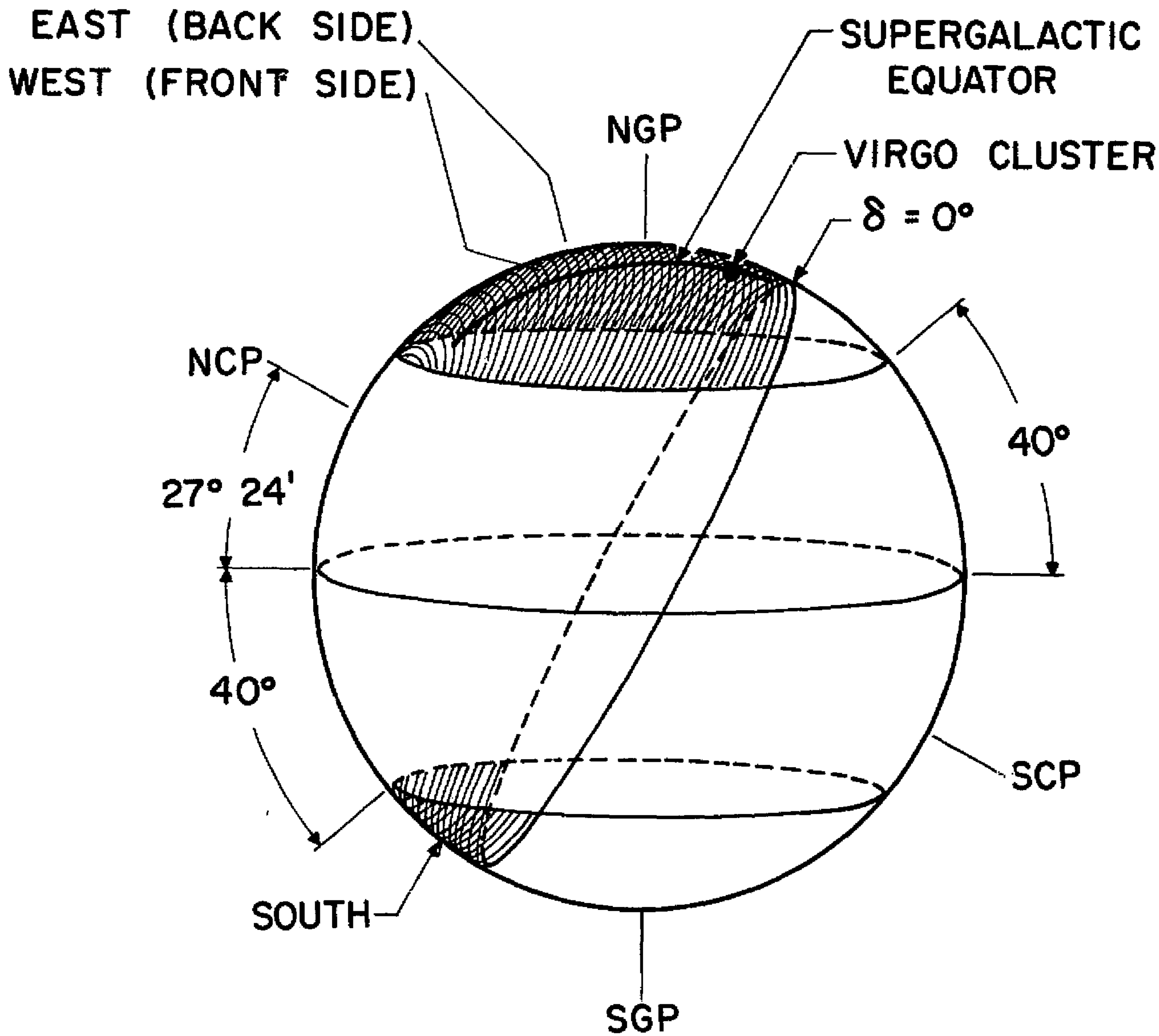

EAST (BACK SIDE)
WEST (FRONT SIDE)
NGP
SUPERGALACTIC EQUATOR
VIRGO CLUSTER
δ = 0°
NCP
27° 24'
40°
40°
SCP
SOUTH
SGP

FIG. 3