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TR 7002

TECHNICAL REPORT

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TRASYS - CHECKOUT OF ACCURACY OF DIRECT IRRADIATION CALCULATIONS FOR DISCS, TRAPEZOIDS, CONES, AND CIRCULAR PARABOLOIDS

Job Order 81-067

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National Aeronautics and Space Administration
LYNDON B. JOHNSON SPACE CENTER
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CONTENTS

CONTENTS (CONTINUED)

TABLES

FIGURES

NOMENCLATURE

A_E	Earth albedo, average albedo of earth (.30)
F_p	Planetary view factor, form factor from surface element to earth's surface
I	Thermal energy radiated per average unit area and time
S	Solar constant, the irradiation from the sun intercepted by a plane surface normal to the sun's rays (443.7 Btu/hr-ft^2)
β	Orbit inclination, the angle between the earth-sun line and the orbit plane
δ	Angle between the sun's rays and the normal to the surface
θ	Orbit angle, angle between sun's rays and the surface
φ	Angle between a point on cone or circular paraboloid and the sun's rays
ψ	Angle between a point on cone or circular paraboloid and the earth-sun line

1. INTRODUCTION

A study is being made to evaluate the results of the direct irradiation link of the TRASYS program. Several surface configurations are being investigated. The accuracy of the results are being evaluated for simple cases where the answers are analytically known. Also by varying an accuracy factor in the program, the amount of computer time needed to achieve different degrees of accuracy is being determined.

The direct irradiation link of TRASYS calculates the incident heat on the external surfaces of a spacecraft due to the sun and the presence of a nearby planet. Solar, albedo, and planetary flux are calculated. Solar flux is the direct radiation from the sun. Albedo flux is the reflected radiation from the planet. Planetary flux is the radiation emitted from the planet.

This is a final report on the study. The data in this report is for four surface types: discs, trapezoids, cones, and circular paraboloids. Reference 1 contains the results for the other three surface types used in TRASYS, rectangles, cylinders, and spheres. For cases where the results are identical to those reported in reference 1, the listings of the actual values will not be recorded in this document.

2. DISCUSSION

The solar, albedo, and planetary flux was calculated for four surface types: discs, trapezoids, cones, and circular paraboloids. For both discs and trapezoids, two cases were run. One case was with the surface in earth orbit and the second case was with the surface along the earth-sun line. Two cases were run for both cones and circular paraboloids. One case has a half surface divided into 36 nodes; the other case uses a quarter surface for only one node.

The number of elements placed upon the node by TRASYS is determined by two variables; the percentage of the planet viewed by the node and the accuracy factor for shadowing calculations (DIACCS). The default value for DIACCS (0.1) was used for all cases. As the percentage of the planet viewed by the node increases, the number of surface elements also increases. For an unshaded surface, the program attempts to place less than 20 square elements upon the node as described in reference 3.

The number of planetary elements is determined by the user input accuracy factor for node to planet form factors (DIACC), and the percentage of the planet viewed by the node. As DIACC decreases, the number of planetary elements increases. As the percentage of the planet viewed by the node increases, the number of planetary elements increases. The minimum number of planetary elements is 52. The maximum number of planetary elements depends upon the altitude of the surface and cannot be greater than 400. The altitude used for all cases in this study was 100 nmi. At that altitude, the maximum number of planetary elements is 355.

The computer run time comparisons were made in two ways. For the 'TOTAL' category the Central Arithmetic Unit (CAU) time as printed on the tail sheet was used. The 'FLUX CALCULATION' comparison was based, however, on the number of storage accesses output by TRASYS.

The thermal energy radiated by earth conforms to the same laws as any other body; i.e., the amount depends upon the surface temperature and the emission properties. Since the temperature of earth does not vary greatly over a long period of time, it is reasonable to assume that the radiated energy is equal to the absorbed energy, and hence the average thermal radiation can be calculated from a simple heat balance. Using S as the solar heat flux per unit projected area, A_E as the earth albedo, R as the earth's radius, and I as the thermal energy radiated per average unit area and time, the energy balance is

$$\left(1-A_E\right) S R^2 = 4 R^2 I$$

or

$$I = \left(\frac{1-A_E}{4}\right) S$$

The planetary view factor (F_p) is the form factor from a surface element to the earth's surface. The form factor was calculated for a rectangular surface at an altitude of 100 nmi. The calculations were made from 0^o (facing the sun) to 180^o (facing the earth) in 0.5^o increments. In the calculations, the earth was divided into 115,200 elements to achieve very accurate form factors. This table of form factors was used to determine the "analytical" albedo and planetary fluxes to the surfaces.

2.1 DIRECT IRRADIATION TO DISC AND TRAPEZOID

Two cases were run for discs and trapezoids. The first case was for the surface rotating in earth orbit; the second case was for the surface along the earth-sun line. The flux was calculated for various orientations.

2.1.1 DISC AND TRAPEZOID IN EARTH ORBIT

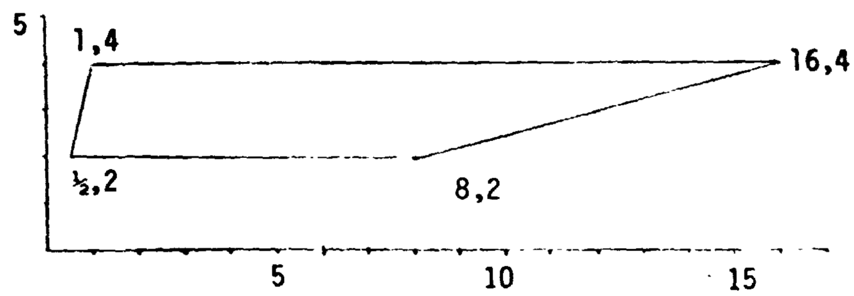
The solar, albedo, and planetary flux was calculated to a disk and a trapezoid rotating in earth orbit as shown in figure 1. The orbit angle ( ) ranged from 0^o to 60^o in 5^o increments. The disc used was a circle with a radius of 6 ft. The four corners of the trapezoid were

P1 = 1,4

P2 = 16,4

P3 = 8,2

P4 = 1/2,2



Both surfaces were at an altitude of 100 nmi.

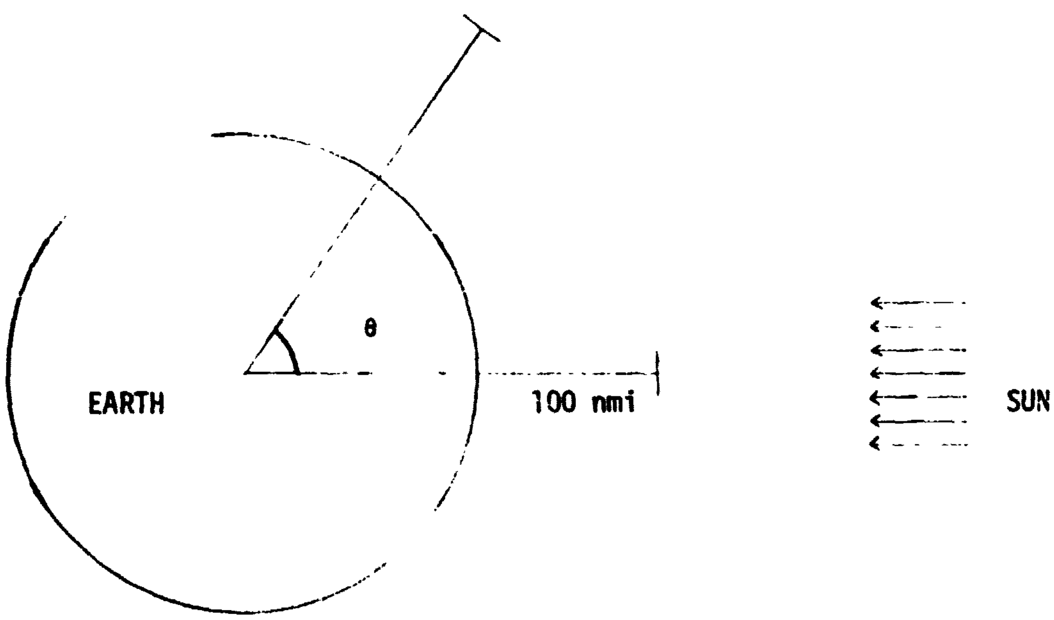


Figure 1. - Disk and trapezoid in earth orbit

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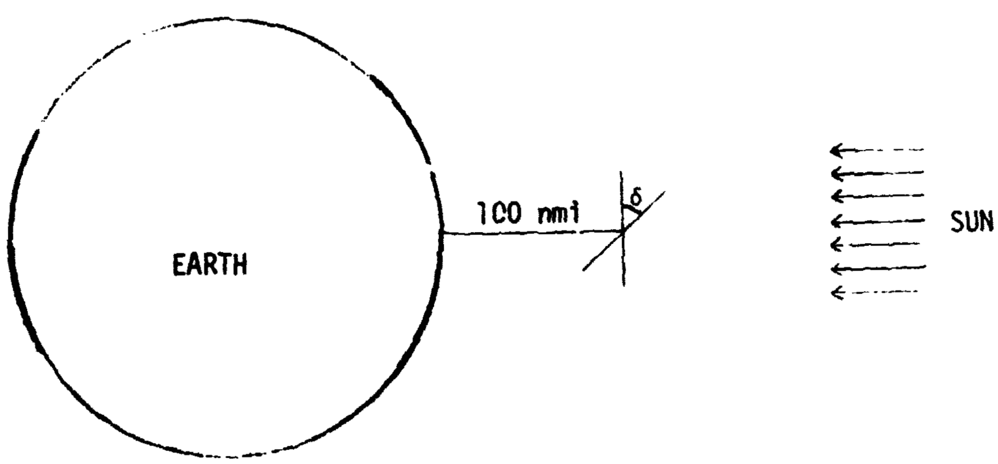

Figure 2. - Disc and trapezoid along earth-sun line

For this case, the flux to a disc or trapezoid should also be the same as the flux to a rectangle.

For the surface shown in figure 2, the solar flux that is expected is

$$\dot{q}_s = S \cdot \cos \delta \tag{4}$$

The albedo flux that is expected is calculated by eq. (2).

$$\dot{q}_a = A_E \cdot S \cdot F_p(\delta) \cdot \cos \theta \cdot \cos \beta$$

The planetary flux that is expected is calculated by eq. (3).

$$\dot{q}_p = \left(\frac{1-A_E}{4}\right) \cdot S \cdot F_p(\delta)$$

The solar, albedo, and planetary fluxes calculated for the disc and trapezoid were identical to the fluxes calculated for the rectangle and summarized in tables 7 and 8 of reference 1.

2.2 DIRECT IRRADIATION TO CONE

The solar, albedo, and planetary flux was calculated for two different configurations of a cone. The first configuration has the cone divided into many small nodes. The second configuration has only one node which is a quarter of a cone.

2.2.1 CONFIGURATION 1

A half cone was divided into 36 equal nodes and was located along the earth-sun line as shown in figure 3. The cone was located at an altitude of 100 nmi. The radius of the base of the cone was 10 ft. and the altitude of the cone was 10 ft.

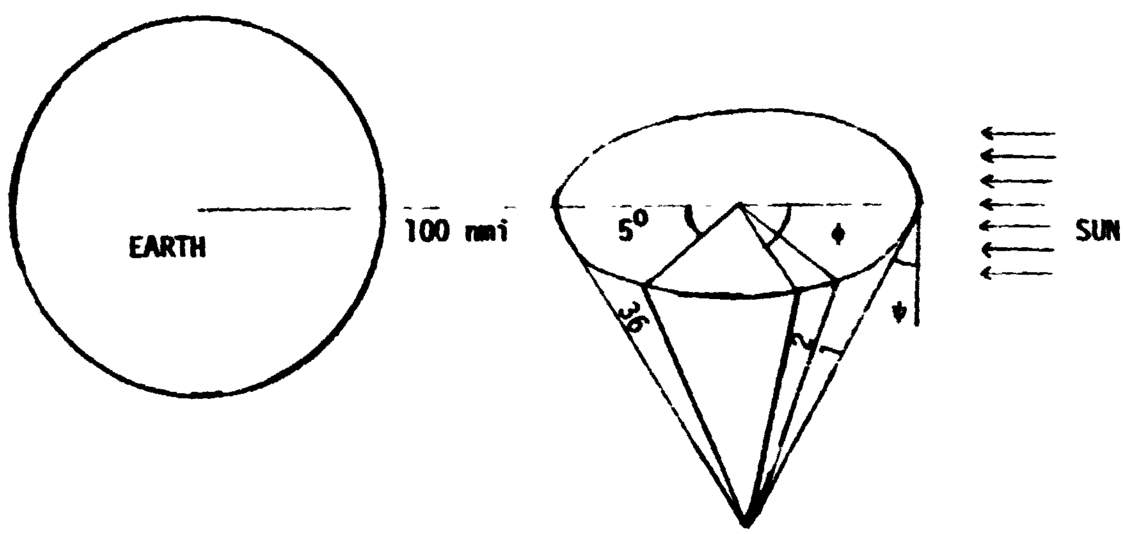

Figure 3. - Cone - small nodes

For all of the flux calculations, each node is divided into many small elements. The flux to each of these small elements is approximately equal to the flux to a rectangle tangent to the element at the midpoint. The flux to each node is calculated by integrating over its entire surface area.

The angle between the normal to each element and the sun's rays is

$$\delta = \cos^{-1} (\cos \phi \cos \psi)$$

The solar flux to each element from eq. (4) is

$$\dot{q}_s = S \cdot \cos \delta = S \cdot \cos \phi \cdot \cos \psi$$

The solar flux to each node is

$$\dot{q}_s = S \cdot \cos \psi \cdot \int_{\phi_1}^{\phi_2} \cos \phi \, d \phi \tag{5}$$

The albedo flux to each element from eq. (2) is

$$\dot{q}_a = A_E \cdot S \cdot F_p(\delta) \cdot \cos \theta \cdot \cos \beta$$

The albedo flux to each node is

$$\dot{q}_a = A_E \cdot S \cdot \cos \theta \cdot \cos \beta \int_{\phi_1}^{\phi_2} F_p(\delta) \, d\phi \tag{6}$$

The planetary flux to each element from eq. (3) is

$$\dot{q}_p = \frac{\left(1-A_E\right) \cdot S \cdot F_p(\delta)}{4}$$

The planetary flux to each node is

$$\dot{q}_p = \frac{\left(1-A_E\right) \cdot S}{4} \int_{\phi_1}^{\phi_2} F_p(\delta) \, d\phi \tag{7}$$

2.2.1.1 Solar Flux

The solar flux was calculated to the cone shown in figure 3. Fifteen elements were placed upon each of the nodes.

The solar fluxes calculated by the program were equal to the fluxes calculated analytically from eq. (5). Table I shows the analytical values and the calculated values.

TABLE I. - SOLAR FLUX TO CONE (SMALL NODES)

Node	Analytical solar flux (Btu/hr-ft^2)	Calculated solar flux (Btu/hr-ft^2)
1	313.36	313.36
2	310.96	310.97
3	306.21	306.22
4	299.12	299.14
5	289.78	289.78
6	278.20	278.22
7	264.54	264.53
8	248.83	248.84
9	231.25	231.25
10	211.84	211.90
11	190.95	190.94
12	168.55	168.53
13	144.82	144.83
14	120.01	120.03
15	94.34	94.32
16	67.88	67.89
17	40.91	40.94
18	13.70	13.68
19	0.0	0.0
⋮	⋮	⋮
36	0.0	0.0

2.2.1.2 Albedo and Planetary Flux

The albedo and planetary flux was calculated for the cone shown in figure 3. Three cases were run with the accuracy factor for node to planetary form factor (DIACC) being equal to 0.25, 0.10 and 0.01. For the cone, the orbit angle (θ) and the orbit inclination (β) are both 0^0.

Table II shows a summary of the data for albedo and planetary flux for these three accuracy factors. For the albedo flux calculations, the analytical values, the calculated values, and the number of planetary elements used are shown in Table III. Table IV shows the analytical fluxes, the calculated fluxes, and the number of planetary elements used in the planetary flux calculations.

2.2.1.2.1 Case 1 DIACC = 0.25

The first case was run with DIACC set equal to the default value of 0.25. The number of elements placed upon the surface nodes was 15. The minimum number of planetary elements used in the program was 52 for nodes 1 through 18. The maximum number of planetary elements used was 171 for nodes 33 through 36.

The largest error in the albedo flux calculations was 2.7 Btu/hr-ft^2 at node 27. The analytical value was 79.1 Btu/hr-ft^2 and the calculated value was 81.8 Btu/hr-ft^2. The average error was 1.14 Btu/hr-ft^2.

The greatest error for planetary flux was 1.3 Btu/hr-ft^2 at node 13. The value calculated analytically using eq. (7) was 15.9 Btu/hr-ft^2 and the value calculated by the program was 14.6 Btu/hr-ft^2. The average error was 0.53 Btu/hr-ft^2.

TABLE II. - ALBEDO AND PLANETARY FLUX TO CONE (SMALL NODES)

	Elements		Albedo		Planetary		Computer time	
Accuracy factor	Planet	Node	Largest Error (Btu/hr-ft^2)	Average Error (Btu/hr-ft^2)	Largest Error (Btu/hr-ft^2)	Average Error (Btu/hr-ft^2)	Total (%)	Flux. Calc. (%)
0.25	52-171	15	2.7	1.14	1.3	0.53	-	-
0.10	52-355	15	1.0	0.64	1.3	0.29	+16.4	+34.7
0.01	95-355	15	1.0	0.41	.3	0.17	+38.4	+83.3

2.2.1.2.2 Case 2 DIACC = 0.10

The second case was run with DIACC = .10. The number of elements placed upon the surface nodes was 15. The number of planetary elements ranged from 52 for nodes 1 through 13 to 355 for nodes 28 through 36. The maximum number of planetary elements allowed by the program for this altitude is 355.

The largest error for the albedo flux was 1.0 Btu/hr-ft^2 at node 36. The flux calculated analytically using eq. (6) was 96.8 Btu/hr-ft^2 and the flux calculated by the program was 97.8 Btu/hr-ft^2. The average error was 0.64 Btu/hr-ft^2.

The largest error in the planetary flux calculations was 1.3 Btu/hr-ft^2 at node 13. The analytical value was 15.9 Btu/hr-ft^2 and the calculated value was 14.6 Btu/hr-ft^2. The average error was reduced to 0.29 Btu/hr-ft^2.

The computer time used in this run was 16.4 percent greater than the time used in the default case. The amount of computer time used in the albedo and planetary flux calculations was 34.7 percent greater than the time used in the default case.

2.2.1.2.3 Case 3 DIACC = 0.01

The next case was run with DIACC lowered to 0.01. The number of elements placed upon the surface nodes remained 15. The number of planetary elements ranged from 95 for nodes 1 and 2 to 355 for nodes 13 through 36.

The largest error in the albedo flux for the case remained 1.0 Btu/hr-ft^2 at node 36. The flux calculated analytically was 96.8 Btu/hr-ft^2 and the flux calculated by the program was 97.8 Btu/hr-ft^2. The average error was lowered to 0.41 Btu/hr-ft^2.

The greatest error for the planetary flux dropped to 0.3 Btu/hr-ft^2 for this accuracy factor. This error occurred at nodes 16, 18, 19, and 20. For node

16, the analytical value was 28.4 Btu/hr-ft^2 and the calculated value was 28.1 Btu/hr-ft^2. The average error was reduced to 0.17 Btu/hr-ft^2.

Lowering the accuracy factor to 0.01 increased the amount of computer time used in the run by 38.4 percent. The amount of computer time used in the albedo and planetary flux calculations was 83.3 percent greater than the time used in the first case.

TABLE III. - ALBEDO FLUX TO CONE (SMALL NODES)

Node	Analytical flux (Btu/hr-ft^2)	Calculated flux		
		Case 1 (Planetary elements) (Btu/hr-ft^2)	Case 2 (Planetary elements) (Btu/hr-ft^2)	Case 3 (Planetary elements) (Btu/hr-ft^2)
36	96.8	99.1 (171)	97.8 (355)	97.8 (355)
35	96.4	98.6 (171)	97.3 (355)	97.3 (355)
34	95.5	97.6 (171)	96.4 (355)	96.4 (355)
33	94.2	96.3 (171)	95.1 (355)	95.1 (355)
32	92.5	94.5 (170)	93.4 (355)	93.4 (355)
31	90.5	92.0 (144)	91.3 (355)	91.3 (355)
30	88.1	89.5 (139)	88.9 (355)	88.9 (355)
29	85.4	86.7 (136)	86.1 (355)	86.1 (355)
28	82.4	83.6 (134)	83.0 (355)	83.0 (355)
27	79.1	81.8 (114)	79.5 (305)	79.6 (355)
26	75.7	78.0 (108)	76.1 (288)	76.1 (355)
25	72.1	73.9 (105)	72.7 (233)	72.5 (355)
24	68.3	69.6 (98)	68.8 (223)	68.7 (355)
23	64.5	65.3 (95)	64.8 (210)	64.8 (355)
22	60.6	61.0 (93)	60.5 (206)	60.7 (355)
21	56.6	57.9 (74)	56.6 (179)	56.5 (355)
20	52.7	53.6 (71)	52.3 (144)	52.4 (355)
19	48.7	49.5 (66)	48.1 (136)	48.5 (355)
18	44.8	44.8 (52)	44.4 (108)	44.6 (355)
17	41.1	40.4 (52)	40.2 (98)	40.8 (355)
16	37.4	36.3 (52)	36.5 (95)	37.1 (355)
15	33.9	32.5 (52)	32.8 (72)	33.6 (355)
14	30.5	28.6 (52)	28.8 (63)	30.4 (355)
13	27.3	25.1 (52)	25.1 (52)	27.2 (355)
12	24.3	22.4 (52)	22.4 (52)	24.2 (338)
11	21.6	20.1 (52)	20.1 (52)	21.4 (288)
10	19.1	18.3 (52)	18.3 (52)	18.8 (223)
9	16.8	16.6 (52)	16.6 (52)	16.5 (210)
8	14.9	14.8 (52)	14.8 (52)	14.6 (180)
7	13.1	13.1 (52)	13.1 (52)	12.7 (144)
6	11.6	11.7 (52)	11.7 (52)	11.3 (136)
5	10.3	10.6 (52)	10.6 (52)	10.1 (112)
4	9.4	9.60 (52)	9.60 (52)	9.19 (105)
3	8.7	8.85 (52)	8.85 (52)	8.38 (98)
2	8.3	8.30 (52)	8.30 (52)	7.90 (95)
1	8.0	7.97 (52)	7.97 (52)	7.76 (95)

TABLE IV. - PLANETARY FLUX TO CONE (SMALL NODES)

Node	Analytical flux (Btu/hr-ft^2)	Calculated flux					
		Case 1 (Planetary elements) (Btu/hr-ft^2)		Case 2 (Planetary elements) (Btu/hr-ft^2)		Case 3 (Planetary elements) (Btu/hr-ft^2)	
36	56.5	57.4	(171)	56.6	(355)	56.6	(355)
35	56.2	57.1	(171)	56.4	(355)	56.4	(355)
34	55.7	56.6	(171)	55.8	(355)	55.8	(355)
33	55.0	55.8	(171)	55.1	(355)	55.1	(355)
32	54.0	54.8	(170)	54.1	(355)	54.1	(355)
31	52.8	53.3	(144)	52.9	(355)	52.9	(355)
30	51.4	51.8	(139)	51.5	(355)	51.5	(355)
29	49.8	50.3	(136)	49.9	(355)	49.9	(355)
28	48.0	48.5	(134)	48.1	(355)	48.1	(355)
27	46.2	47.4	(114)	46.1	(305)	46.1	(355)
26	44.2	45.2	(108)	44.1	(288)	44.1	(355)
25	42.1	42.8	(105)	42.1	(233)	42.0	(355)
24	39.9	40.3	(98)	39.9	(223)	39.8	(355)
23	37.6	37.8	(95)	37.5	(210)	37.5	(355)
22	35.3	35.4	(93)	35.1	(206)	35.2	(355)
21	33.0	33.6	(74)	32.8	(179)	32.8	(355)
20	30.7	31.1	(71)	30.3	(144)	30.4	(355)
19	28.4	28.7	(66)	27.9	(136)	28.1	(355)
18	26.2	26.0	(52)	25.8	(108)	25.9	(355)
17	23.9	23.4	(52)	23.3	(98)	23.7	(355)
16	21.8	21.1	(52)	21.2	(95)	21.5	(355)
15	19.7	18.8	(52)	19.0	(72)	19.5	(355)
14	17.8	16.6	(52)	16.7	(63)	17.6	(355)
13	15.9	14.6	(52)	14.6	(52)	15.8	(355)
12	14.2	13.0	(52)	13.0	(52)	14.1	(333)
11	12.6	11.7	(52)	11.7	(52)	12.4	(288)
10	11.1	10.7	(52)	10.7	(52)	10.9	(223)
9	9.8	9.62	(52)	9.62	(52)	9.61	(210)
8	8.7	8.59	(52)	8.59	(52)	8.47	(180)
7	7.6	7.61	(52)	7.61	(52)	7.40	(144)
6	6.8	6.77	(52)	6.77	(52)	6.57	(136)
5	6.1	6.15	(52)	6.15	(52)	5.89	(112)
4	5.5	5.58	(52)	5.58	(52)	5.34	(105)
3	5.1	5.15	(52)	5.15	(52)	4.87	(98)
2	4.8	4.83	(52)	4.83	(52)	4.59	(95)
1	4.7	4.64	(52)	4.64	(52)	4.51	(95)

2.2.2 CONFIGURATION 2

The second configuration was for a node which was one-quarter of a cone. This node was located along the earth-sun line as shown in figure 4. The angle to the midpoint of the node (ϕ) defines the orientation. The node was rotated from $\phi = 0^o$ (facing the sun) to $\phi = 180^o$ (facing the earth) in 5^o increments. The cone was located at an altitude of 100 nmi. The radius of the base of the cone was 10 ft. and the altitude of the cone was 10 ft.

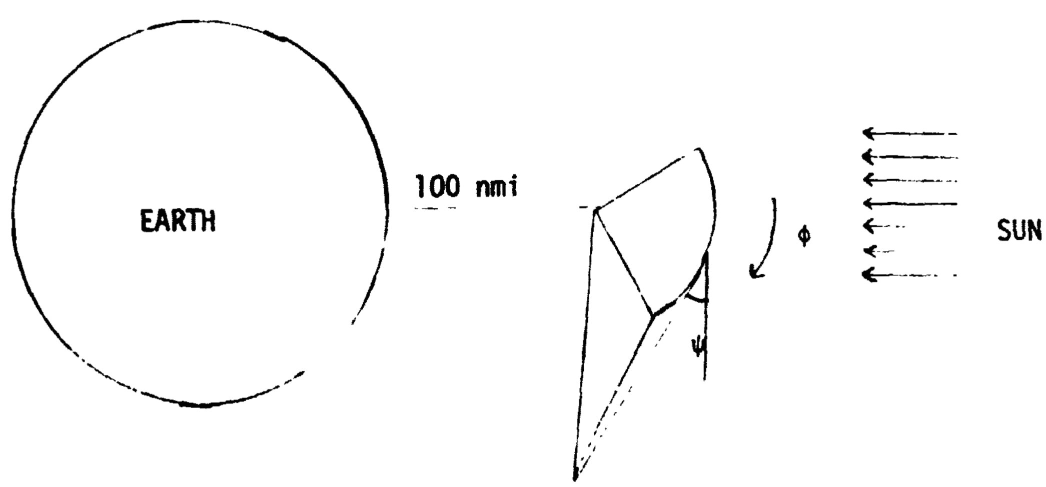

Figure 4. - Cone - large node

The solar flux to the node is calculated from eq. (5)

$$\dot{q}_s = S \cdot \cos \psi \cdot \int_{\phi_1}^{\phi_2} \cos \phi \, d\phi$$

For this configuration

$$\phi_1 = \phi - 45^\circ, \ \phi_2 = \phi + 45^\circ$$

$$\dot{q}_s = S \cdot \cos \psi \cdot \int_{\phi-45^\circ}^{\phi+45^\circ} \cos \phi \, d\phi \tag{8}$$

The albedo flux to the node from eq. (6) is

$$\dot{q}_a = A_E \cdot S \cdot \cos \theta \cdot \cos \beta \cdot \int_{\phi-45^\circ}^{\phi+45^\circ} F_p (\delta) \, d\phi \tag{9}$$

The planetary flux to the node from eq. (7) is

$$\dot{q}_p = \frac{(1-A_E) \cdot S}{4} \int_{\phi-45^\circ}^{\phi+45^\circ} F_p(\delta) \, d\phi \tag{10}$$

..2.1 <u>Solar Flux</u>

The solar flux was calculated to the node shown in figure 4. Twelve elements were placed upon each orientation of the node.

The largest error was 6.81 Btu/hr-ft^2 at ϕ=120°. The flux calculated analytically using eq. (8) was 6.81 Btu/hr-ft^2 and the flux calculated by the program was 0.0 Btu/hr-ft^2. The average error was 2.41 Btu/hr-ft^2. The analytical fluxes and the calculated fluxes are shown in table V.

TABLE V. - SOLAR FLUX TO CONE (LARGE NODE)

Angle (ϕ)	Analytical solar flux (Btu/hr-ft^2)	Calculated solar flux (Btu/hr-ft^2)
0°	282.47	285.72
5	281.39	284.63
10	278.18	281.38
15	272.84	275.99
20	265.43	268.49
25	256.00	258.95
30	244.62	247.44
35	231.38	234.05
40	216.38	218.87
45	199.74	202.04
50	182.33	183.66
55	165.05	163.88
60	148.04	142.86
65	131.42	129.87
70	115.32	115.88
75	99.87	101.02
80	85.17	85.384
85	71.35	69.100
90	58.50	52.291
95	46.73	44.198
100	36.12	35.764
105	26.76	27.068
110	18.71	18.160
115	12.05	9.1148
120	6.81	.00
125	3.03	.00
130	.86	.00
135	.00	.00
⋮	⋮	⋮
180	.00	.00

2.2.2.2 Albedo and Planetary Flux

The albedo and planetary flux was calculated for the cone shown in figure 4. Three cases were run with DIACC set equal to 0.25, 0.10 and 0.01 for these cases. For this cone, the orbit angle (θ) and the orbit inclination (β) are both 0^{0}. Table VI gives a summary of the data for albedo and planetary flux for these three accuracy factors. The analytical values, the calculated values, and the number of planetary elements used in the calculations are given in table VII for albedo flux and in table VIII for planetary flux.

TABLE VI. - ALBEDO AND PLANETARY FLUX TO CONE (LARGE NODE)

Accuracy factor	Elements		Albedo		Planetary		Computer time	
	Planet	Node	Largest error ($Btu/hr\text{-}ft^2$)	Average error ($Btu/hr\text{-}ft^2$)	Largest error ($Btu/hr\text{-}ft^2$)	Average error ($Btu/hr\text{-}ft^2$)	Total(%)	Calc(%)
0.25	52-144	9-12	2.4	1.15	1.1	0.56	---	---
0.10	52-355	9-12	1.3	0.66	0.9	0.33	+5.3	+34.7
0.01	112-355	12	1.3	0.59	0.7	0.28	+12.9	+83.3

2.2.2.2.1 Case 1 DIACC = 0.25

The first case was run with DIACC set equal to the default value of 0.25. The number of elements placed upon the surface nodes was either 9 or 12. The number of planetary elements ranged from 52 for $\phi = 0^{0}$ through $\phi = 85^{0}$ to 144 for $\phi = 165^{0}$ through $\phi = 180^{0}$.

The largest error for albedo flux was 2.4 Btu/hr-ft^{2} for $\phi = 135^{0}$. The flux calculated analytically was 77.8 Btu/hr-ft^{2} and the flux calculated by the program was 80.2 Btu/hr-ft^{2}. The average error was 1.15 Btu/hr-ft^{2}.

The greatest error in the planetary flux calculations was 1.1 Btu/hr-ft^{2} for $\phi = 135^{0}$. The analytical flux was 45.4 Btu/hr-ft^{2} and the calculated flux was 46.5 Btu/hr-ft^{2}. The average error was 0.56 Btu/hr-ft^{2}.

2.2.2.2.2 Case 2 DIACC = 0.10

The second case was run with the accuracy factor lowered to 0.10. The number of elements placed upon the surface nodes was either 9 or 12. The minimum number of planetary elements used was 52 for $\phi = 0^{0}$ through $\phi = 55^{0}$ and the maximum number was 355 for $\phi = 150^{0}$ through $\phi = 180^{0}$.

The greatest error for the albedo flux was 1.3 Btu/hr-ft^{2} at $\phi = 50^{0}$, $\phi = 175^{0}$, and $\phi = 180^{0}$. At $\phi = 50^{0}$, the flux calculated analytically was 23.8 Btu/hr-ft^{2} and the flux calculated by the program was 22.5 Btu/hr-ft^{2}. The average error was reduced to 0.66 Btu/hr-ft^{2}.

The largest error for planetary flux dropped to 0.9 Btu/hr-ft^{2} and occurred at $\phi = 40^{0}$. The analytical value was 11.5 Btu/hr-ft^{2} while the calculated value was 10.6 Btu/hr-ft^{2}. The average error improved to 0.33 Btu/hr-ft^{2}.

The computer time used in this run was 5.3 percent greater than the time used in the previous case. The amount of computer time used in the albedo and planetary flux calculations increased by 34.7 percent over the default case.

2.2.2.2.3 Case 3 DIACC = 0.01

For the third case DIACC was lowered to 0.01. The number of elements placed upon the surface nodes was 12 for all orientations. The number of planetary elements ranged from 112 for $\phi = 0^0$ to 355 at $\phi = 45^0$ through $\phi = 180^0$.

The largest error for albedo flux remained at 1.3 Btu/hr-ft^2 at $\phi = 175^0$ and $\phi = 180^0$. At $\phi = 175^0$, the analytical flux was 91.2 Btu/hr-ft^2 and the calculated flux was 92.5 Btu/hr-ft^2. The average error was reduced to 0.59 Btu/hr-ft^2.

For planetary flux, the maximum error occurred at $\phi = 40^0$ and was reduced to 0.7 Btu/hr-ft^2. The flux calculated using eq. (10) was 11.5 Btu/hr-ft^2 while the flux calculated by the program was 10.8 Btu/hr-ft^2. A small reduction was seen in the average error which dropped to 0.28 Btu/hr-ft^2.

The amount of computer time used in this run increased by 12.9 percent over the default case. The amount of computer time used in the albedo and planetary flux calculations was 83.3 percent greater than the time used in the default case.

TABLE VII. - ALBEDO FLUX TO CONE (LARGE NODE)

Angle(ϕ)	Analytical flux (Btu/hr-ft^2)	Calculated flux		
		Case 1 (Planetary elements) (Btu/hr-ft^2)	Case 2 (Planetary elements) (Btu/hr-ft^2)	Case 3 (Planetary elements) (Btu/hr-ft^2)
180^o	91.4	93.5 (144)	92.7 (355)	92.7 (355)
175	91.2	93.4 (144)	92.5 (355)	92.5 (355)
170	90.7	92.8 (144)	91.9 (355)	91.9 (355)
165	89.8	91.9 (144)	91.0 (355)	91.0 (355)
160	88.5	90.6 (139)	89.7 (355)	89.7 (355)
155	87.0	88.9 (137)	88.1 (355)	88.1 (355)
150	85.1	86.9 (136)	86.2 (355)	86.2 (355)
145	83.0	84.5 (136)	84.0 (351)	84.0 (355)
140	80.5	81.9 (133)	81.4 (345)	81.4 (355)
135	77.8	80.2 (114)	78.5 (303)	78.6 (355)
130	74.9	77.1 (112)	75.5 (293)	75.6 (355)
125	71.8	73.8 (105)	72.3 (288)	72.5 (355)
120	68.6	70.2 (105)	69.1 (233)	69.1 (355)
115	65.2	66.4 (98)	65.6 (220)	65.7 (355)
110	61.8	62.8 (95)	62.0 (210)	62.1 (355)
105	58.3	59.1 (93)	58.3 (206)	58.4 (355)
100	54.7	56.4 (74)	55.0 (179)	54.8 (355)
95	51.2	52.3 (71)	51.2 (144)	51.2 (355)
90	47.6	48.2 (66)	47.6 (136)	47.6 (355)
85	44.2	44.4 (52)	44.0 (112)	44.1 (355)
80	40.8	40.7 (52)	40.5 (105)	40.6 (355)
75	37.6	37.4 (52)	37.1 (98)	37.2 (355)
70	34.4	34.2 (52)	33.8 (95)	34.0 (355)
65	31.4	30.8 (52)	31.0 (72)	31.0 (355)
60	28.5	27.7 (52)	28.2 (69)	28.2 (355)
55	25.9	24.9 (52)	24.9 (52)	25.5 (355)
50	23.8	22.5 (52)	22.5 (52)	22.9 (355)
45	21.1	20.4 (52)	20.4 (52)	20.6 (355)
40	19.1	18.2 (52)	18.2 (52)	18.6 (293)
35	17.4	16.3 (52)	16.3 (52)	16.8 (220)
30	15.6	14.6 (52)	14.6 (52)	15.2 (206)
25	14.3	13.5 (52)	13.5 (52)	13.9 (171)
20	13.3	12.6 (52)	12.6 (52)	12.8 (139)
15	12.2	11.9 (52)	11.9 (52)	11.9 (136)
10	11.7	11.4 (52)	11.4 (52)	11.2 (134)
5	11.3	11.1 (52)	11.1 (52)	10.8 (114)
0	11.2	10.8 (52)	10.8 (52)	10.6 (112)

TABLE VIII. - PLANETARY FLUX TO CONE (LARGE NODE)

Angle(Φ)	Analytical flux ($Btu/hr-ft^2$)	Calculated flux: Case 1 (Planetary elements) ($Btu/hr-ft^2$)		Case 2 (Planetary elements) ($Btu/hr-ft^2$)		Case 3 (Planetary elements) ($Btu/hr-ft^2$)	
180^0	53.3	54.2	(144)	53.7	(355)	53.7	(355)
175	53.2	54.1	(144)	53.6	(355)	53.6	(355)
170	52.9	53.8	(144)	53.3	(355)	53.3	(355)
165	52.4	53.2	(144)	52.7	(355)	52.7	(355)
160	51.7	52.5	(139)	52.0	(355)	52.0	(355)
155	50.7	51.5	(137)	51.1	(355)	51.1	(355)
150	49.7	50.3	(136)	50.0	(355)	50.0	(355)
145	48.4	49.0	(136)	48.7	(351)	48.7	(355)
140	47.0	47.5	(133)	47.2	(345)	47.2	(355)
135	45.4	46.5	(114)	45.5	(303)	45.5	(355)
130	43.7	44.7	(112)	43.7	(293)	43.8	(355)
125	41.9	42.7	(105)	41.9	(288)	42.0	(355)
120	40.0	40.7	(105)	40.0	(233)	40.1	(355)
115	38.1	38.5	(98)	38.0	(220)	38.1	(355)
110	36.1	36.4	(95)	35.9	(210)	36.0	(355)
105	34.0	34.3	(93)	33.8	(206)	33.9	(355)
100	31.9	32.7	(74)	31.9	(179)	31.8	(355)
95	29.9	30.3	(71)	29.7	(144)	29.7	(355)
90	27.8	27.9	(66)	27.6	(136)	27.6	(355)
85	25.8	25.7	(52)	25.5	(112)	25.6	(355)
80	23.8	23.6	(52)	23.5	(105)	23.5	(355)
75	21.9	21.7	(52)	21.5	(98)	21.6	(355)
70	20.1	19.8	(52)	19.6	(95)	19.8	(355)
65	18.3	17.9	(52)	18.0	(72)	18.0	(355)
60	16.7	16.1	(52)	16.3	(69)	16.3	(355)
55	15.1	14.5	(52)	14.5	(52)	14.8	(355)
50	13.7	13.1	(52)	13.1	(52)	13.3	(355)
45	12.4	11.8	(52)	11.8	(52)	12.0	(355)
40	11.5	10.6	(52)	10.6	(52)	10.8	(293)
35	10.1	9.45	(52)	9.45	(52)	9.73	(220)
30	9.2	8.48	(52)	8.48	(52)	8.84	(206)
25	8.4	7.82	(52)	7.82	(52)	8.07	(171)
20	7.7	7.33	(52)	7.33	(52)	7.42	(139)
15	7.2	6.94	(52)	6.94	(52)	6.90	(136)
10	6.9	6.64	(52)	6.64	(52)	6.51	(134)
5	6.6	6.42	(52)	6.42	(52)	6.26	(114)
0	6.6	6.29	(52)	6.29	(52)	6.15	(112)

2.2.3 CONCLUSIONS

The solar flux calculations for the cone divided into small sections, configuration 1, are exact. When the angle subtended by a conical node was increased, configuration 2, the error also increased. This occurs because the number of surface elements used in the calculation of solar flux is the same regardless of the size of the conical node.

Lowering the accuracy factor from 0.25 to 0.10 improves the albedo and planetary flux calculations without an appreciable increase in the computer time. A continuing decrease in the accuracy factor from 0.10 to 0.01 will use a large amount of computer time while making only small changes in the calculated fluxes.

2.3 DIRECT IRRADIATION TO CIRCULAR PARABOLOID

The solar, albedo, and planetary flux were calculated for two different configurations of a circular paraboloid. In the first configuration, the circular paraboloid was divided into many small nodes. In the second configuration each node was one-quarter of a circular paraboloid.

2.3.1 CONFIGURATION 1

Half a circular paraboloid was divided into 36 equal nodes and was located along the earth-sun line at an altitude of 100 nmi. as shown in figure 5. The focal point (F) is 2.5 ft. The altitude of the circular paraboloid is 2.5 ft.

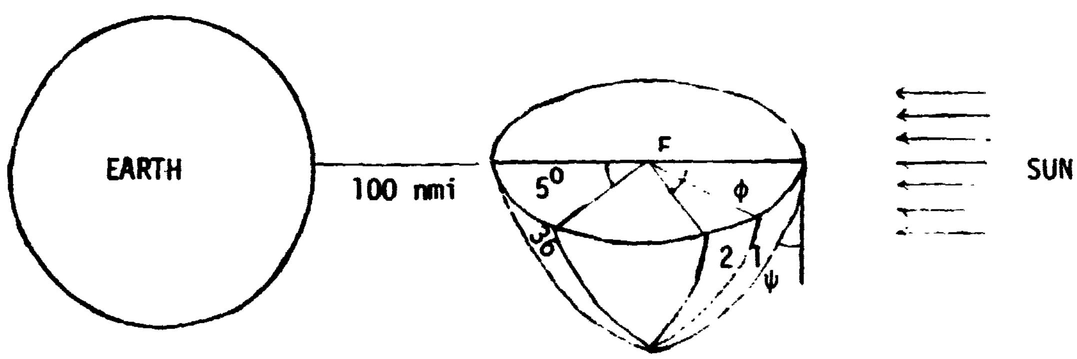

Figure 5. - Circular paraboloid - small nodes

Each node is divided into many small elements for the flux calculations. The flux to each of these small elements is approximately equal to the flux to a rectangle tangent to the element at the midpoint. The flux to each node is calculated by integrating over its entire surface area.

The angle between the normal to each element and the sun's rays is

$$\delta = \cos^{-1} (\cos \phi \cos \psi)$$

The solar flux to each element from eq. (4) is

$$\dot{q}_s = S \cdot \cos \delta = S \cdot \cos \phi \cos \psi$$

The solar flux to each node is

$$\dot{q}_s = S \cdot \int_{\phi_1}^{\phi_2} \int_{\psi_1}^{\psi_2} \cos \phi \cos \psi \, d\psi \, d\phi \tag{11}$$

The albedo flux to each element from eq. (2) is

$$\dot{q}_a = A_E \cdot S \cdot F_p(\delta) \cos \theta \cos \beta$$

The albedo flux to each node is

$$\dot{q}_a = A_E \cdot S \cos \theta \cos \beta \int_{\phi_1}^{\phi_2} \int_{\psi_1}^{\psi_2} F_p(\delta) \, d \psi \, d \phi \tag{12}$$

The planetary flux to each element from eq. (3) is

$$\dot{q}_p = \frac{(1-A_E) \cdot S \cdot F_p(\delta)}{4}$$

The planetary flux to each node is

$$\dot{q}_p = \frac{(1-A_E)\cdot S}{4} \int_{\phi_1}^{\phi_2} \int_{\psi_1}^{\psi_2} F_p(\delta)\, d\psi\; d\,\phi \qquad (13)$$

2.3.1.1 Solar Flux

The solar flux was calculated to the circular paraboloid shown in figure 5. Each node was divided into nine elements.

The solar flux calculated by the program was 1.3 percent greater than the fluxes calculated analytically from eq. (11). The largest error was 3.22 Btu/hr-ft^2 at node 1. The average error was 2.03 Btu/hr-ft^2. The analytical and calculated solar flux are shown in table IX.

TABLE IX. - SOLAR FLUX TO CIRCULAR PARABOLOID (SMALL NODES)

Node	Analytical solar flux, (Btu/hr-ft^2)	Calculated solar flux, (Btu/hr-ft^2)
1	242.44	245.66
2	240.60	243.79
3	236.92	240.07
4	231.44	234.52
5	224.20	227.18
6	215.25	218.11
7	204.67	207.39
8	192.53	195.08
9	178.92	181.30
10	163.95	166.13
11	147.73	149.69
12	130.39	132.12
13	112.05	113.54
14	92.87	94.101
15	72.97	73.943
16	52.52	53.222
17	31.68	32.096
18	10.59	10.726
19	0.0	0.0
⋮	⋮	⋮
36	0.0	0.0

2.3.1.2 Albedo and Planetary Flux

The albedo and planetary flux was calculated for the circular paraboloid shown in figure 5. Three cases were run with the accuracy factor set equal to 0.25, 0.10 and 0.01. The orbit angle (θ) and the orbit inclination (β) are both 0^o for the circular paraboloid.

A summary of the data for albedo and planetary flux for these three accuracy factors are shown in table X. Table XI shows the analytical values, the calculated values, and the number of planetary elements used in the albedo flux calculations. For the planetary flux calculations, the analytical fluxes, the calculated fluxes and the number of planetary elements used are shown in table XII.

TABLE X. - ALBEDO AND PLANETARY FLUX TO CIRCULAR PARABOLOID (SMALL NODES)

Accuracy factor	Elements		Albedo		Planetary		Computer time	
	Planet	Node	Largest error ($Btu/hr\text{-}ft^2$)	Average error ($Btu/hr\text{-}ft^2$)	Largest error ($Btu/hr\text{-}ft^2$)	Average error ($Btu/hr\text{-}ft^2$)	Total(%)	Calc(%)
0.25	52-136	9	3.4	1.43	1.6	0.72	---	---
0.10	52-355	9	1.6	.97	1.0	0.51	+19.8	+31.4
0.01	207-355	9	1.5	.78	0.6	0.39	+46.6	+83.9

2.3.1.2.1 Case 1 DIACC = 0.25

The first case was run with DIACC set equal to the default value of 0.25. The number of elements placed upon the surface nodes was 9. The number of planetary elements ranged from 52 for nodes 1 through 18 to 136 for nodes 34 through 36.

The largest error in the albedo flux calculations was 3.4 Btu/hr-ft^2 at node 31. The flux calculated using eq. (12) was 79.9 Btu/hr-ft^2 and the flux calculated by the program was 83.3 Btu/hr-ft^2. The average error was 1.43 Btu/hr-ft^2

For the planetary flux the greatest error was 1.6 Btu/hr-ft^2 at node 31. The value calculated using eq. (13) was 46.6 Btu/hr-ft^2 and the flux calculated by the program was 48.2 Btu/hr-ft^2. The average error was 0.72 Btu/hr-ft^2.

2.3.1.2.2 Case 2 DIACC = 0.10

For the second case, DIACC was set equal to 0.10. The number of elements placed upon the surface nodes was 9. The minimum number of planetary elements used in the program was 52 for nodes 1 through 12. The maximum number of planetary elements used was 355 for node 36. This was the maximum number of planetary elements allowed by the program for this altitude.

The largest error for albedo flux was reduced to 1.6 Btu/hr-ft^2 and occurred at node 12. The analytical value was 29.2 Btu/hr-ft^2 and the calculated value was 27.6 Btu/hr-ft^2. The average error dropped to 0.97 Btu/hr-ft^2.

The greatest error in the planetary flux calculations was 1.0 Btu/hr-ft^2 at nodes 11 and 12. At node 11, the analytical flux was 15.8 Btu/hr-ft^2 and the calculated value was 14.8 Btu/hr-ft^2. The average error was reduced to 0.51 Btu/hr-ft^2.

The amount of computer time used in this run was 19.8 percent greater than the amount of time used in the previous run. The amount of time used in the albedo and planetary flux calculations was 31.4 percent greater than the time used in the previous run.

2.3.1.2.3 Case 3 DIACC = 0.01

The third case was run with DIACC lowered to 0.01. Each surface node was divided into nine elements. The number of planetary elements ranged from 207 for node 1 to 355 for nodes 10 through 36

The largest error for albedo flux was 1.5 Btu/hr-ft^2 for nodes 34 and 36. For node 34, the flux calculated analytically was 83.6 Btu/hr-ft^2 while the flux calculated by the program was 85.1 Btu/hr-ft^2. The average error was reduced to 0.78 Btu/hr-ft^2.

For planetary flux, the greatest error was lowered to 0.6 Btu/hr-ft^2 at nodes 1, 2, and 36. The analytical flux was 9.3 Btu/hr-ft^2 and the calculated flux was 8.7 Btu/hr-ft^2 at node 1. The average error dropped to 0.39 Btu/hr-ft^2.

The amount of computer time for this run was 46.6 percent greater than the time for the default case. The amount of computer time used in the albedo and planetary flux calculations was 83.9 percent greater than the amount of time used in the default case.

TABLE XI. - ALBEDO FLUX TO CIRCULAR PARABOLOID (SMALL NODES)

Node	Analytical flux (Btu/hr-ft^2)	Calculated flux: Case 1 (Planetary elements) (Btu/hr-ft^2)		Case 2 (Planetary elements) (Btu/hr-ft^2)		Case 3 (Planetary elements) (Btu/hr-ft^2)	
36	84.6	86.9	(136)	86.1	(355)	86.1	(355)
35	84.3	86.6	(136)	85.7	(351)	85.7	(355)
34	83.6	85.8	(136)	85.1	(351)	85.1	(355)
33	82.7	84.7	(134)	84.1	(349)	84.1	(355)
32	81.4	83.4	(134)	82.8	(345)	82.8	(355)
31	79.9	83.3	(114)	81.3	(338)	81.3	(355)
30	78.1	81.3	(112)	79.3	(300)	79.4	(355)
29	76.1	79.0	(108)	77.2	(293)	77.3	(355)
28	73.9	76.5	(105)	74.8	(288)	74.9	(355)
27	71.5	73.7	(105)	72.5	(233)	72.4	(355)
26	68.9	70.7	(98)	69.8	(223)	69.7	(355)
25	66.2	67.5	(98)	66.9	(220)	66.9	(355)
24	63.3	64.2	(95)	63.9	(210)	63.9	(355)
23	60.4	61.0	(93)	60.7	(206)	60.8	(355)
22	57.4	59.0	(77)	57.6	(179)	57.6	(355)
21	54.4	55.6	(72)	54.4	(170)	54.4	(355)
20	51.3	52.1	(69)	51.0	(137)	51.3	(355)
19	48.3	48.9	(66)	47.8	(134)	48.1	(355)
18	45.3	45.5	(52)	44.7	(112)	44.9	(355)
17	42.3	42.0	(52)	41.7	(105)	41.9	(355)
16	39.5	38.8	(52)	38.5	(98)	39.0	(355)
15	36.7	35.8	(52)	35.6	(95)	36.2	(355)
14	34.1	32.9	(52)	32.8	(72)	33.6	(355)
13	31.6	30.0	(52)	30.2	(69)	31.1	(355)
12	29.2	27.6	(52)	27.7	(52)	28.7	(355)
11	27.0	25.5	(52)	25.5	(52)	26.4	(355)
10	25.0	23.7	(52)	23.7	(52)	24.4	(355)
9	23.2	22.2	(52)	22.2	(52)	22.5	(351)
8	21.6	20.7	(52)	20.7	(52)	20.8	(305)
7	20.2	19.3	(52)	19.3	(52)	19.4	(288)
6	18.9	18.0	(52)	18.0	(52)	18.0	(223)
5	17.9	17.0	(52)	17.0	(52)	17.0	(223)
4	17.1	16.2	(52)	16.2	(52)	16.2	(219)
3	16.5	15.6	(52)	15.6	(52)	15.6	(210)
2	16.1	15.1	(52)	15.1	(52)	15.2	(210)
1	15.9	15.0	(52)	15.0	(52)	15.0	(207)

TABLE XII. - PLANETARY FLUX TO CIRCULAR PARABOLOID (SMALL NODES)

Node	Analytical flux (Btu/hr-ft^2)	Calculated flux		
		Case 1 (Planetary elements) (Btu/hr-ft^2)	Case 2 (Planetary elements) (Btu/hr-ft^2)	Case 3 (Planetary elements) (Btu/hr-ft^2)
36	49.3	50.4 (136)	49.9 (355)	49.9 (355)
35	49.2	50.2 (136)	49.7 (351)	49.7 (355)
34	48.8	49.7 (136)	49.3 (351)	49.3 (355)
33	48.2	49.1 (134)	48.7 (349)	48.7 (355)
32	47.5	48.3 (134)	48.0 (345)	48.0 (355)
31	46.6	48.2 (114)	47.1 (338)	47.1 (355)
30	45.6	47.1 (112)	46.0 (300)	46.0 (355)
29	44.4	45.8 (108)	44.7 (293)	44.8 (355)
28	43.1	44.3 (105)	43.4 (288)	43.4 (355)
27	41.7	42.7 (105)	42.0 (233)	42.0 (355)
26	40.2	41.0 (98)	40.5 (233)	40.4 (355)
25	38.6	39.1 (98)	38.8 (220)	38.8 (355)
24	36.9	37.2 (95)	37.0 (210)	37.0 (355)
23	35.2	35.4 (93)	35.2 (206)	35.2 (355)
22	33.5	34.2 (77)	33.4 (179)	33.4 (355)
21	31.7	32.2 (72)	31.5 (170)	31.6 (355)
20	29.9	30.2 (69)	29.6 (137)	29.7 (355)
19	28.2	28.4 (66)	27.7 (134)	27.9 (355)
18	26.4	26.4 (52)	25.9 (112)	26.1 (355)
17	24.7	24.4 (52)	24.2 (105)	24.3 (355)
16	23.0	22.5 (52)	22.4 (98)	22.6 (355)
15	21.4	20.8 (52)	20.7 (95)	21.0 (355)
14	19.9	19.1 (52)	19.0 (72)	19.5 (355)
13	18.4	17.4 (52)	17.5 (69)	18.0 (355)
12	17.0	16.0 (52)	16.0 (52)	16.7 (355)
11	15.8	14.8 (52)	14.8 (52)	15.3 (355)
10	14.6	13.8 (52)	13.8 (52)	14.1 (355)
9	13.5	12.9 (52)	12.9 (52)	13.0 (351)
8	12.6	12.0 (52)	12.0 (52)	12.1 (305)
7	11.8	11.2 (52)	11.2 (52)	11.3 (288)
6	11.0	10.5 (52)	10.5 (52)	10.5 (233)
5	10.4	9.89 (52)	9.89 (52)	9.90 (223)
4	10.0	9.40 (52)	9.40 (52)	9.41 (219)
3	9.6	9.04 (52)	9.04 (52)	9.04 (210)
2	9.4	8.79 (52)	8.79 (52)	8.80 (210)
1	9.3	8.69 (52)	8.69 (52)	8.70 (210)

2.3.2 CONFIGURATION 2

The second configuration was for a node which was one-quarter of a circular paraboloid. This node was located along the earth-sun line as shown in figure 6. The orientation is defined as the angle to the midpoint of the node (ϕ). The node was rotated from $\phi = 0^{o}$ (facing the sun) to $\phi = 180^{o}$ (facing the earth) in 5^{o} increments. The circular paraboloid was located at an altitude of 100 nmi. The focal point (F) is 2.5 ft. and the altitude of the circular paraboloid is 2.5 ft.

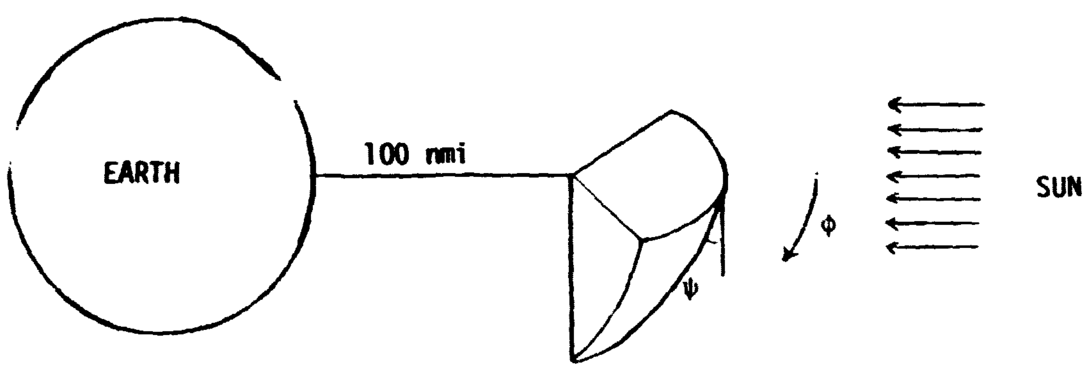

Figure 6. - Circular paraboloid - large node

For this node $\phi_1 = \phi - 45^0$ and $\phi_2 = \phi + 45^0$. The solar flux to the node as calculated from eq. (11) is

$$\dot{q}_s = S \cdot \int_{\phi-45^0}^{\phi+45^0} \int_{\psi_1}^{\psi_2} \cos\phi \cos\psi \, d\psi \, d\phi \tag{14}$$

The albedo flux to the node from eq. (12) is

$$\dot{q}_a = A_E \cdot S \cdot \cos\theta \quad \cos\beta \int_{\phi-45^0}^{\phi+45^0} \int_{\psi_1}^{\psi_2} F_p(\delta) \, d\psi \, d\phi \tag{15}$$

The planetary flux to the node from eq. (13) is

$$\dot{q}_p = \frac{\left(1 - A_E\right) \cdot S}{4} \int_{\phi-45^0}^{\phi+45^0} \int_{\psi_1}^{\psi_2} F_p(\delta) \, d\psi \, d\phi \tag{16}$$

2.3.2.1 Solar Flux

The solar flux was calculated to the circular paraboloid shown in figure 6. Nine elements were placed on the node for each of the orientations.

The largest error in the solar flux occurred at $\phi = 0^0$ and was 5.45 Btu/hr-ft^2. The calculated value was 224.00 Btu/hr-ft^2 and analytically from eq. (14) the value should be 218.55 Btu/hr-ft^2. The average error was 2.92 Btu/hr-ft^2. Table XIII gives the analytical values and the calculated values.

TABLE XIII. - SOLAR FLUX TO CIRCULAR PARABOLOID (LARGE NODE)

Angle (ϕ)	Analytical solar flux ($Btu/hr\text{-}ft^2$)	Calculated solar flux ($Btu/hr\text{-}ft^2$)
0^o	218.55	224.00
5	217.72	223.14
10	215.23	220.60
15	211.10	216.37
20	205.37	210.49
25	198.08	203.01
30	189.27	193.99
35	179.03	183.49
40	167.42	171.59
45	154.54	158.39
50	141.07	143.98
55	127.70	128.48
60	114.54	112.00
65	101.68	101.81
70	89.23	90.849
75	77.27	79.195
80	65.90	66.939
85	55.20	54.173
90	45.26	40.995
95	36.16	34.650
100	27.95	28.042
105	20.70	21.220
110	14.48	14.237
115	9.32	7.1458
120	5.27	0.0
125	2.35	0.0
130	.59	0.0
135	.00	0.0
⋮	⋮	⋮
180	.00	0.0

2.3.2.2 Albedo and Planetary Flux

The albedo and planetary flux was calculated for the circular paraboloid shown in figure 6. Three cases were run with the accuracy factor set equal to 0.25, 0.10, and 0.01. The orbit angle (θ) and the orbit inclination (β) are both 0^{o} for this surface. Table XIV gives a summary of the data for albedo and planetary flux for the three accuracy factors. The analytical fluxes, the calculated fluxes, and the number of planetary elements used in the calculations are given in table XV for albedo flux and in table XVI for planetary flux.

TABLE XIV. - ALBEDO AND PLANETARY FLUX TO CIRCULAR PARABOLOID (LARGE NODE)

Accuracy factor	Elements		Albedo		Planetary		Computer time	
	Planet	Node	Largest error (Btu/hr-ft^2)	Average error (Btu/hr-ft^2)	Largest error (Btu/hr-ft^2)	Average error (Btu/hr-ft^2)	Total(%)	Calc(%)
0.25	52-133	6-9	3.7	1.61	1.8	.83	---	---
0.10	52-345	6-9	1.8	1.07	0.9	.56	+13.1	+35.3
0.01	230-355	6	1.8	.95	0.8	.49	+31.7	+83.8

2.3.2.2.1 Case 1 DIACC = 0.25

DIACC was set equal to the default value of 0.25 for the first case. The node was divided into either six or nine elements for the different orientations. The number of planetary elements ranged from 52 for $\phi = 0^{o}$ through $\phi = 85^{o}$ to 133 for $\phi = 170^{o}$ through $\phi = 180^{o}$.

The greatest error for albedo flux was 3.7 Btu/hr-ft^2 at $\phi = 165^{o}$. The flux calculated analytically using eq. (15) was 79.3 Btu/hr-ft^2 and the flux calculated by the program was 83.0 Btu/hr-ft^2. The average error was 1.61 Btu/hr-ft^2.

For planetary flux, the largest error was 1.8 Btu/hr-ft^2 at $\phi = 160^{o}$ and $\phi = 165^{o}$. At $\phi = 160^{o}$, the analytical flux was 45.7 Btu/hr-ft^2 and the calculated flux was 47.5 Btu/hr-ft^2. The average flux error was 0.83 Btu/hr-ft^2.

2.3.2.2 Case 2 DIACC = 0.10

For this case, the accuracy factor was reduced to 0.10. The number of elements placed upon the surface node was either six or nine. The number of planetary elements used ranged from 52 for $\phi = 0^{o}$ through $\phi = 45^{o}$ to 345 for $\phi = 180^{o}$.

The largest error for albedo flux dropped to 1.8 Btu/hr-ft^2 at $\phi = 175^{o}$ and $\phi = 180^{o}$. At $\phi = 175^{o}$, the analytical value was 80.4 Btu/hr-ft^2 and the calculated value was 82.2 Btu/hr-ft^2. The average error was reduced to 1.07 Btu/hr-ft^2.

For planetary flux, the greatest error was reduced to 0.9 Btu/hr-ft^2 and occurred at $\phi = 20^{o}$ and $\phi = 25^{o}$. The flux calculated analytically using eq. (16) was 11.8 Btu/hr-ft^2 and the flux calculated by the program was 10.9 Btu/hr-ft^2 at $\phi = 20^{o}$. The average error dropped to 0.56 Btu/hr-ft^2.

The amount of computer time used in this run was 13.1 percent greater than the amount of time used in the default case. The amount of time used in the albedo and planetary flux calculations was 35.3 percent greater than the time used in the default case.

2.3.2.2.3 Case 3 DIACC = 0.01

For the third case, DIACC was lowered to 0.01. The number of surface elements was six for all orientations. The minimum number of planetary elements was 230 for $\phi = 0^{o}$ and $\phi = 5^{o}$ and the maximum number was 355 for $\phi = 35^{o}$ through $\phi = 180^{o}$.

The greatest error for albedo flux remained 1.8 Btu/hr-ft^2 and occurred at $\phi = 175^{o}$ and $\phi = 180^{o}$. At $\phi = 175^{o}$, the analytical flux was 80.4 Btu/hr-ft^2 and the calculated flux was 82.2 Btu/hr-ft^2. The average error was reduced to 0.95 Btu/hr-ft^2.

The maximum error for planetary flux was 0.8 Btu/hr-ft^2 at $\phi = 10^o$. The analytical flux was 11.1 Btu/hr-ft^2, and the calculated flux was 10.3 Btu/hr-ft^2. The average error dropped to 0.49 Btu/hr-ft^2.

Lowering the accuracy factor to 0.01 increased the amount of computer time used in the run by 31.7 percent. The amount of computer time used in the albedo and planetary flux calculations was 83.8 percent greater than the time used in the first case.

TABLE XV. - ALBEDO FLUX TO CIRCULAR PARABOLOID (LARGE NODE)

Angle	Analytical flux (Btu/hr/ft^2)	Calculated flux					
		Case 1 (Planetary elements) (Btu/hr-ft^2)		Case 2 (Planetary elements) (Btu/hr-ft^2)		Case 3 (Planetary elements) (Btu/hr-ft^2)	
180°	80.5	83.0	(133)	82.3	(345)	82.3	(355)
175	80.4	82.8	(133)	82.2	(338)	82.2	(355)
170	80.0	82.4	(133)	81.7	(338)	81.7	(355)
165	79.3	83.0	(114)	80.9	(305)	81.0	(355)
160	78.4	81.9	(114)	80.0	(303)	80.0	(355)
155	77.2	80.5	(112)	78.7	(300)	78.8	(355)
150	75.8	78.9	(112)	77.2	(293)	77.3	(355)
145	74.2	77.1	(107)	75.5	(288)	75.6	(355)
140	72.4	75.0	(105)	73.5	(285)	73.6	(355)
135	70.3	72.7	(105)	71.5	(233)	71.5	(355)
130	68.2	70.3	(98)	69.2	(230)	69.2	(355)
125	65.9	67.6	(98)	66.7	(220)	66.7	(355)
120	63.4	64.9	(95)	64.1	(216)	64.1	(355)
115	60.8	62.0	(95)	61.3	(207)	61.4	(355)
110	58.2	60.5	(78)	58.7	(180)	58.6	(355)
105	55.5	57.3	(72)	55.9	(176)	55.8	(355)
100	52.8	54.1	(72)	52.8	(144)	52.9	(355)
95	50.1	50.9	(69)	49.9	(136)	50.1	(355)
90	47.3	47.7	(66)	47.0	(134)	47.3	(355)
85	44.7	44.8	(52)	44.2	(112)	44.4	(355)
80	42.0	41.9	(52)	41.5	(105)	41.7	(355)
75	39.5	39.2	(52)	38.8	(98)	39.0	(355)
70	37.0	36.6	(52)	36.2	(95)	36.4	(355)
65	34.7	34.1	(52)	33.7	(93)	34.0	(355)
60	32.4	31.5	(52)	31.3	(72)	31.7	(355)
55	30.3	29.2	(52)	29.1	(69)	29.5	(355)
50	28.4	27.2	(52)	27.0	(66)	27.5	(355)
45	26.6	25.3	(52)	25.3	(52)	25.6	(355)
40	24.9	23.7	(52)	23.7	(52)	23.9	(355)
35	23.5	22.1	(52)	22.1	(52)	22.5	(355)
30	22.2	20.8	(52)	20.8	(52)	21.2	(349)
25	21.1	19.7	(52)	19.7	(52)	20.0	(300)
20	20.2	18.8	(52)	18.8	(52)	19.1	(288)
15	19.5	18.2	(52)	18.2	(52)	18.4	(256)
10	19.0	17.8	(52)	17.8	(52)	17.8	(233)
5	18.7	17.5	(52)	17.5	(52)	17.5	(230)
0	18.6	17.4	(52)	17.4	(52)	17.4	(230)

TABLE XVI. - PLANETARY FLUX TO CIRCULAR PARABOLOID (LARGE NODE)

Angle	Analytical flux (Btu/hr-ft^2)	Calculated flux: Case 1 (Planetary elements) (Btu/hr-ft^2)		Case 2 (Planetary elements) (Btu/hr-ft^2)		Case 3 (Planetary elements) (Btu/hr-ft^2)	
180°	47.0	48.1	(133)	47.7	(345)	47.7	(355)
175	46.9	48.0	(133)	47.6	(338)	47.6	(355)
170	46.7	47.7	(133)	47.4	(338)	47.3	(355)
165	46.3	48.1	(114)	46.9	(305)	46.9	(355)
160	45.7	47.5	(114)	46.3	(303)	46.4	(355)
155	45.0	46.7	(112)	45.6	(300)	45.7	(355)
150	44.2	45.7	(112)	44.8	(293)	44.8	(355)
145	43.3	44.7	(107)	43.7	(288)	43.8	(355)
140	42.2	43.4	(105)	42.6	(285)	42.7	(355)
135	41.0	42.1	(105)	41.5	(233)	41.4	(355)
130	39.8	40.7	(98)	40.1	(230)	40.1	(355)
125	38.4	39.2	(98)	38.7	(220)	38.7	(355)
120	37.0	37.6	(95)	37.2	(216)	37.2	(355)
115	35.5	36.0	(95)	35.6	(207)	35.6	(355)
110	33.9	35.1	(78)	34.1	(180)	34.0	(355)
105	32.4	33.2	(72)	32.4	(176)	32.3	(355)
100	30.8	31.3	(72)	30.6	(144)	30.7	(355)
95	29.2	29.5	(69)	28.9	(136)	29.1	(355)
90	27.6	27.7	(66)	27.3	(134)	27.4	(355)
85	26.1	26.0	(52)	25.6	(112)	25.8	(355)
80	24.5	24.3	(52)	24.0	(105)	24.2	(355)
75	23.0	22.7	(52)	22.5	(98)	22.6	(355)
70	21.6	21.2	(52)	21.0	(95)	21.1	(355)
65	20.2	19.8	(52)	19.6	(93)	19.7	(355)
60	18.9	18.3	(52)	18.2	(72)	18.4	(355)
55	17.7	17.0	(52)	16.9	(69)	17.1	(355)
50	16.5	15.8	(52)	15.7	(66)	15.9	(355)
45	15.5	14.7	(52)	14.7	(52)	14.9	(355)
40	14.5	13.7	(52)	13.7	(52)	13.9	(355)
35	13.7	12.9	(52)	12.9	(52)	13.0	(355)
30	12.9	12.1	(52)	12.1	(52)	12.3	(349)
25	12.3	11.4	(52)	11.4	(52)	11.6	(300)
20	11.8	10.9	(52)	10.9	(52)	11.1	(288)
15	11.4	10.6	(52)	10.6	(52)	10.7	(256)
10	11.1	10.3	(52)	10.3	(52)	10.3	(233)
5	10.9	10.2	(52)	10.2	(52)	10.2	(230)
0	10.8	10.1	(52)	10.1	(52)	10.1	(230)

2.3.3 CONCLUSIONS

A few significant errors occur in the solar flux calculations for a circular paraboloid. For the small node configuration the calculated fluxes increased 1.3 percent higher than the analytical values. When the node size was increased, as in configuration 2, the error varied from 2.5 percent at the maximum heating orientation to showing no heating on some orientations at high θ angles.

Lowering the accuracy factor from 0.25 to 0.10 improves the albedo and planetary fluxes. A decrease in the accuracy factor to 0.01 from 0.10 greatly increases the amount of computer time used but does not change the calculated fluxes appreciably.

3. CONCLUSIONS FROM THE STUDY

From the results of this study and that of reference 1, it is possible to draw some general conclusions.

The calculated unshadowed solar fluxes are exact for the planar surfaces, i.e., rectangles, discs, and trapezoids. For cylinders and cones, the solar fluxes are exact when the surfaces are divided into small nodes. As the angle subtended by the node increases, the error increases. The calculated solar heating rates for spheres and circular paraboloids had some significant errors, especially when the nodal areas are large.

An accuracy factor between 0.10 and 0.25 will give reasonable answers for unshadowed albedo and planetary flux. Accuracy factors greater than 0.25 should be avoided because the errors are large without an appreciable savings in computer time. Lowering the accuracy factor below 0.10 improves the answers by only a small amount but uses a large amount of computer time in doing so.

For surfaces not along the earth-sun line, the albedo flux and the error will be smaller since either the orbit angle (θ) or the orbit inclination (β) is not 0^{o}. The calculated planetary flux will be the same for any orbit angle or orbit inclination.

4. REFERENCES

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4. An Introduction to Spacecraft Thermal Control, LR18901, Lockheed - California.