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A MODEL FOR SIMULATING RANDOM ATMOSPHERES AS A FUNCTION OF LATITUDE, SEASON, AND TIME

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16. Abstract

An empirical stochastic computer model has been developed with the capability of generating random thermodynamic profiles of the atmosphere below an altitude of 99 km which are characteristic of any given season, latitude, and time of day. Samples of temperature, density, and pressure profiles generated by the model are statistically similar to measured profiles in a data base of over 6000 rocket and high-altitude atmospheric soundings; that is, means and standard deviations of modeled profiles and their vertical gradients are in close agreement with data. Model-generated samples can be used for Monte Carlo simulations of aircraft or spacecraft trajectories to predict or account for the effects on a vehicle's performance of atmospheric variability. Other potential uses for the model are in simulating pollutant dispersion patterns, variations in sound propagation, and other phenomena which are dependent on atmospheric properties, and in developing data-reduction software for satellite monitoring systems.

Model-data agreement is particularly good below an altitude of 60 km where data sample sizes are adequate. Accuracy of the model above 60 km is uncertain largely because of the diminished quantity and quality of the data at those heights. As more and better high-altitude data become available, the parameter input tape of the model can readily be updated to improve the accuracy.

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A MODEL FOR SIMULATING RANDOM ATMOSPHERES AS A FUNCTION OF LATITUDE, SEASON, AND TIME

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SUMMARY

An empirical stochastic computer model has been developed with the capability of generating random thermodynamic profiles of the atmosphere below an altitude of 99 km which are characteristic of any given season, latitude, and time of day. Temperature profiles are generated from multivariate normal distributions, and then density and pressure profiles are calculated from the hydrostatic equation and the equation of state.

Model parameters were estimated by using a set of over 6000 meteorological rocket and high-altitude soundings of the atmosphere. These soundings were divided into 17 latitude-season categories and model parameters were estimated for each category.

Means and standard deviations of model temperatures and vertical temperature gradients are controlled by model input parameters, and hence can be forced to match these properties in the data. Means and standard deviations of model pressures, densities, and their vertical gradients were not controlled directly but do agree well with data. Model-data agreement is particularly good below an altitude of 60 km where the data sample sizes are adequate. Accuracy of the model above 60 km, in most latitude-season categories, is uncertain largely because of the diminished quantity and quality of the data at these heights.

Samples of random atmospheres generated by the model can be used in Monte Carlo studies of the effect of atmospheric variability on spacecraft or aircraft. Other potential uses for the model are in simulating pollutant dispersion patterns, variations in sound propagation, and other phenomena which are dependent on atmospheric properties, and in developing data-reduction software for satellite monitoring systems.

INTRODUCTION

The Need for a Stochastic Atmosphere Model

In evaluating the performance of aerospace vehicle designs prior to their initial flight, a primary tool has been the wide variety of available trajectory simulation computer programs. These range from simple point-mass programs whose governing equations have closed-form solutions to complex six-degree-of-freedom models requiring numerical solutions of differential or integral equations. Virtually all simulation programs rely on some atmospheric model to provide values of atmospheric temperatures, densities, and pressures as functions of altitude. These atmospheric models, often the U.S. Standard Atmosphere (refs. 1 and 2), provide average estimates of atmospheric properties but do not account for the atmosphere's natural variability, which is appreciable at some altitudes. In the mesosphere (50 to 90 km), for example, atmospheric densities are sometimes twice their predicted U.S. Standard (1962) value. One can observe in available data sets that atmospheric density at the same altitude varies by at least 10 percent within the same season.

In designing an aircraft or spacecraft, an important task is the determination of performance envelopes within which with high probability all flight parameters are expected to lie. In general, a large number of variables contribute to the width of these performance envelopes - atmospheric variability is only one. A common approach to error analyses (that is, to defining the performance envelopes) is to assume that all sources of variation are additive and uncorrelated, and then proceed to estimate their variances separately. Under this assumption, the total variance is simply the sum of the individual variances. An alternate error analysis approach is to perform Monte Carlo simulations in which the various error sources are treated as random inputs. This latter approach, although usually more costly, is often considered to be preferable to the former because it allows natural nonlinearities and interactions to appear in the system. Regardless of which approach is taken, a method is needed to provide an accurate representation of each error source so that its impact on the various performance parameters of interest can be measured. A stochastic atmospheric model is a tool for assessing the impact of atmospheric variability on a performance envelope of the vehicle.

In past design studies on atmospheric entry vehicles, a common method of accounting for atmospheric density variations (refs. 3 and 4) was to calculate trajectories by using both maximum and minimum density profiles as shown in figure 1. Generally, these are profiles in which atmospheric density is two or three standard deviations above or below its mean at all altitudes simultaneously. This method has two major disadvantages. One is that since density profiles such as these never occur in any real atmosphere, a design parameter based on this method might be overly conservative and, thereby, require unnecessary expense. A second disadvantage, more critical than the first, is that these extreme density profiles do not produce extremes in all entry parameters. For example, it has been shown (ref. 5) that for some vehicles a more severe total heat load is produced when atmospheric density is extremely low as entry begins and suddenly becomes extremely high at lower altitudes. The initial low density causes less deceleration than is normal, and thus the spacecraft encounters an extremely dense atmosphere while traveling at an abnormally high velocity.

To account for the fact that extremes in the various entry parameters are produced by different atmospheric situations, an alternate deterministic approach (ref. 6) has been to determine analytically for each performance parameter the atmospheric profiles which maximize and minimize that parameter, and then design the vehicle to withstand those extremes. As with any deterministic approach, however, this method has the disadvantage that any specific atmospheric profile has a zero probability of occurrence, and thus the design may be overly conservative. Furthermore, the degree of conservatism cannot be ascertained since no knowledge is provided as to the probability of encountering atmospheres similar to the design atmospheres.

The disadvantages associated with deterministic methods act as justification for the use of statistical methods. A Monte Carlo simulation based on realistic random atmospheres can be used to estimate the statistical distribution of any performance parameter, and design values can be selected for any desired risk or exceedance probability.

The advent of reusable spacecraft with the space shuttle will make it even more important to have good estimates of exceedance probabilities. For example, when a spacecraft was used only once, it was not so critical to distinguish between a failure probability of 0.005 and one of 0.001. With the shuttle having an expected lifetime of 100 missions, however, the difference between single-flight failure probabilities of 0.005 and 0.001 makes the difference between a 40-percent and a 10-percent chance of at least one failure during the lifetime of the shuttle. This condition, therefore, is a further justification for using the best available statistical techniques in design studies to acquire as much confidence as possible in the reliability estimates.

The stochastic atmospheric model described in this report was developed for evaluating the effects of atmospheric variability on the performance parameters of a vehicle (spacecraft or aircraft) along any portion of its trajectory below an altitude of 99 km. The terms "stochastic" and "statistical" will be used interchangeably to describe any model capable of generating random atmospheres. From a sample of random atmospheres, one can construct samples of associated performance parameters such as that illustrated in figure 2. Figure 2 shows histograms (frequency distribution plots) of maximum dynamic pressures which occurred in simulated entries of a space shuttle configuration into random seasonal atmospheres (ref. 7).

Other Available Stochastic Models

The term "statistical model" sometimes refers to summaries of statistics (usually means and standard deviations) computed from a particular data set (ref. 8). Usually in statistical summaries of this type, no effort is made to model the data. Those who do model the data (ref. 9) generally concentrate on modeling means and standard deviations of the various atmospheric properties as a function of geographic location, but no attempt is made to simulate atmospheres. One of the more thorough efforts of this type is the four-dimensional worldwide model of Spiegler and Fowler (ref. 10) which provides estimates of the mean and standard deviation of atmospheric properties at any location below an altitude of 25 km as a function of longitude and latitude.

Very few stochastic atmosphere models (by the stricter definition) are available, and the few that do exist typically adhere to the belief (sometimes called the "Principle of Parsimony") that the fewer the parameters, the better the model. For the sake of mathematical simplicity, such models compromise their ability to simulate realistic atmospheres. The "Thermodynamic Atmosphere Model" of Engler and Goldschmidt (ref. 11) is an extreme example of this. All atmospheric variability in this model is attributed to a single random variable which determines the coefficients in an empirical polynomial representing the logarithm of pressure as a function of altitude. Although this model has the advantage of simplicity, its atmospheric profiles, especially that of temperature, approximate realistic profiles only roughly.

An earlier model by the author (ref. 7), although more complex, suffers from the same failing. In this model, systematic variations in stratospheric and mesospheric (30 to 90 km) temperatures were attributed to the variability of ozone concentrations. Ozone variations, generated by an auxiliary model based on four random variables, produced changes in the solar heating rate of ozone, and this process, in turn, produced random temperatures by means of a linear regression scheme. A fundamental shortcoming of this approach is that ozone variations are only one of many sources of variation in the atmosphere. Furthermore, even if all sources of variation were incorporated, the problem of relating their influence to atmospheric temperatures is one which atmospheric scientists have yet to solve.

Another model (ref. 12) with the capability of simulating random atmospheres, although this is not its chief purpose, was developed at the Georgia Institute of Technology under a NASA contract. This model combines the four-dimensional worldwide model of reference 10 with other models of higher regions extending to orbital altitudes. Its primary aim is to provide estimates of means and standard deviations for atmospheric properties as functions of longitude and latitude (globally) for each month of the year. Wide bands of uncertainty exist about the estimates because of the limited amount of data used in their construction. In fact, if all available atmospheric data were used as a data base for a model of such fine spacial and temporal resolution, error bands would still be appreciable over most regions of the glove. To generate random atmospheres, this model uses a set of approximations (ref. 13), which relate means and variances of atmospheric properties to the correlations between properties, based on the assumption that percentage departures from the mean are small. Reference 12 does not comment as to its accuracy in simulating random atmospheres.

The ERA Model

The model reported here will be called the ERA (empirical random atmosphere) model. The central goal throughout the modeling process was to imitate a data set consisting of over 6000 meteorological rocket and high-altitude sounding measurements. As will be demonstrated, the ERA model is capable of generating samples of random atmospheres for different seasons and latitude zones which are statistically similar to samples of measured atmospheres in those seasons and latitude zones. The model attempts to match means and standard deviations not only of the thermodynamic properties themselves, but also of their vertical gradients. Because a unified approach was taken in estimating model parameters, and because of the substantial size of the data base of the model, the ERA model is, in the author's judgment, the best available model for simulating realistic random atmospheres.

Other Applications

Although the ERA model was developed as a tool for evaluating space vehicle performances when exposed to random atmospheres, potential applications of the model are certainly not limited to this area. Other potential uses include studies of the effect of atmospheric variability on sound propagation or pollutant dispersion patterns. The ERA model can also be used to simulate random atmospheres for use in developing data-reduction software for satellite monitoring systems. Any phenomenon which is affected by atmospheric temperatures, densities, or pressures would also be affected by the variability of these properties and hence can be studied by this model. In addition, the large number of summary plots detailing statistical properties of the atmosphere at different latitudes and seasons may be of interest solely for their information content, apart from the model.

SYMBOLS

a_0, a_1, a_2 coefficients of linear regression of sea-level pressure on sea-level temperature

B lower triangular matrix satisfying $BB^t = C$

b_{ij} element in ith row and jth column of B

C correlation matrix for temperature vector T

c_{ij} element in ith row and jth column of C matrix; coefficient of linear correlation between T_i and T_j

c_{ij}^* adjusted element of C matrix required to make C positive definite

D column vector of atmospheric densities at altitude intervals of 3 km between sea level and 99 km, kg/m^3

D_i element of D vector at altitude z_i, kg/m^3

$E(\ )$ mean or expected value operator

G universal gas constant, 8.314×10^3 J/K

g acceleration due to gravity, m/sec^2

g_0 acceleration due to gravity at sea level, m/sec^2

$\bar{g}_i$ "effective" acceleration due to gravity between z_{i-1} and z_i, m/sec^2

$\bar{M}$ mean molecular weight of air, 28.964 kg

n total number of soundings in data sample for a latitude-season category

n_i total number of observations of T_i in data sample

n_{ij} total number of soundings in data sample in which both T_i and T_j are measured

P column vector of atmospheric pressures at altitude intervals of 3 km between sea level and 99 km, N/m^2

P_i element of P vector at altitude z_i, N/m^2

P_{ik} value of P_i at time of kth sounding, N/m^2

$\bar{P}_1$ mean sea-level pressure, N/m^2

p_i standardized pressure at altitude z_i

R_0 radius of Earth, m

r_{TP} coefficient of linear correlation between sea-level temperature and sea-level pressure

$r_{T\beta}$ coefficient of linear correlation between sea-level temperature and zenith angle of Sun

$S(\ )$ standard deviation operator

s_i standard deviation of T_i, K

T column vector of atmospheric temperatures at altitude intervals of 3 km between sea level and 99 km, K

T_i element of T vector at altitude z_i, K

T_{ik} value of T_i at time of kth sounding, K

$\bar{T}_i$ sample mean of all observations of T_i, K

t_i standardized temperature at altitude z_i

x column vector of mutually independent, standardized normal (Gaussian) random variables

x_i element of x vector

z altitude, km

z_i altitude level corresponding to $3(i - 1)$ km

$\alpha_0,\alpha_1,\alpha_2$ coefficients of linear regression of standardized sea-level temperature on zenith angle of Sun

β zenith angle of Sun, deg

$\bar{\beta}$ average of zenith angles associated with soundings in data sample, deg

ΔD_i $= D_i - D_{i-1}$, kg/m^3

ΔP_i $= P_i - P_{i-1}$, N/m^2

ΔT_i $= T_i - T_{i-1}$, K

Δz $= z_i - z_{i-1}$, 3 km

δ_{ik} binary function indicating whether T_{ik} is measured ($\delta_{ik} = 1$) or missing ($\delta_{ik} = 0$)

δ_{ik}^* binary function indicating whether P_{ik} is measured ($\delta_{ik}^* = 1$) or missing ($\delta_{ik}^* = 0$)

σ column vector of standard deviations of temperatures at altitude intervals of 3 km between 0 and 99 km, K

σ_i element of σ vector corresponding to standard deviation of temperature at altitude z_i, K

σ_P standard deviation of sea-level pressure, N/m^2

σ_β standard deviation of Sun zenith angles in data sample, deg

τ column vector of temperature means at altitude intervals of 3 km between 0 and 99 km, K

τ_i element of τ vector corresponding to mean temperature at altitude z_i, K

ϕ latitude, deg

Superscript:

t transpose of matrix

Subscripts:

i altitude level $3(i - 1)$ km ($i = 1, \ldots, 34$)

j altitude level $3(j - 1)$ km ($j = 1, \ldots, 34$)

k sounding number in a sample of n soundings ($k = 1, \ldots, n$)

Abbreviations:

ERA empirical random atmosphere

MRN Meteorological Rocket Network

NCC National Climatic Center, NOAA, Asheville, NC

FORMULATION OF MODEL

Objective

The objective of the ERA model is to provide random atmospheres extending to approximately 100 km which are typical of any given season, latitude, and time of day. Each atmosphere consists of three thermodynamic profiles, where the term "profile" refers to a column vector of dimension 34 whose elements correspond to altitudes 0, 3, . . ., 99 km. The three thermodynamic profiles are

$$T = \begin{bmatrix} T_1 \\ T_2 \\ \cdot \\ \cdot \\ \cdot \\ T_{34} \end{bmatrix} \qquad D = \begin{bmatrix} D_1 \\ D_2 \\ \cdot \\ \cdot \\ \cdot \\ D_{34} \end{bmatrix} \qquad P = \begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ \cdot \\ P_{34} \end{bmatrix} \tag{1}$$

where T_1, D_1, and P_1 are sea-level temperature, density, and pressure, respectively; T_2, D_2, and P_2 are these properties at an altitude of 3 km, and so forth.

The criteria by which model-generated atmospheres are judged to be "typical" are based on certain statistical properties of large samples of model-generated atmospheres. Specifically, the following statistical properties of any model-generated sample should match, within sampling errors, the same properties of atmospheric data samples:

(1) Mean and standard deviation of T_i $(i = 1, . . ., 34)$

(2) Mean and standard deviation of D_i $(i = 1, . . ., 34)$

(3) Mean and standard deviation of P_i $(i = 1, . . ., 34)$

(4) Means and standard deviations of vertical gradients dT/dz, dD/dz, and dP/dz

The three vectors T, D, and P representing any one atmosphere generated by the model are mutually consistent in that they are related by the equation of state and the hydrostatic equation. The equation of state (ideal gas law) determines one thermodynamic property from the other two by the relationship

$$D_i = \frac{\bar{M}P_i}{GT_i} \qquad (i = 1, . . ., 34) \tag{2}$$

where $\bar{M}$ is the mean molecular weight of air (28.964 kg) and G is the universal gas constant (8.314×10^3 J/K).

The hydrostatic equation

$$\left(\frac{dP}{dz}\right)_{z=z_i} = -g(z_i)D_i \tag{3}$$

attributes vertical pressure differentials between two altitudes to the weight of the air between those levels. Although it is known that the equation of state and the hydrostatic equation are approximations (for example, $\bar{M}$ varies at high altitudes and atmospheric dynamics disrupt hydrostatic equilibrium), they are widely used in atmospheric models. (See refs. 1 and 2.) Where the data contained two or three simultaneously measured profiles, it was found that these equations were satisfied quite adequately.

The effect of using equations (2) and (3) is a reduction of the degrees of freedom in the model. Given one profile and a boundary condition for another, the two remaining profiles are uniquely determined. The ERA model starts with a random temperature profile and a random sea-level pressure, which are generated by a stochastic model, and uses these values to calculate the remaining atmospheric properties (P_2, . . ., P_{34}) and (D_1, . . ., D_{34}) by equations (2) and (3). Thus, the ERA model is essentially a stochastic temperature model with deterministically derived pressures and densities.

Experience has shown that temperature is the best choice for the "given" profile used in solving equations (2) and (3). Otherwise, derived temperature profiles with the proper shape are difficult to produce. Pressures or densities which deviate slightly from their "true" shapes can produce temperature profiles of a highly unnatural form. For example, suppose one wished to use pressure as the "given" profile by modeling $\log P$ as a polynomial function in z with random coefficients as in reference 11. A plot of $\log P$ against z appears nearly linear. However, it is possible to show that the order of this polynomial must be five or greater in order that $dT/dz = 0$ at three altitudes (tropopause, stratopause, and mesopause). Thus, one must include higher order terms, even though small, since these are crucial for modeling the T profile adequately.

Temperature Model

Atmospheric temperature at altitude z_i ($i = 1$, . . ., 34) is given by the equation

$$T_i = \tau_i + t_i\sigma_i \tag{4}$$

where τ_i and σ_i are the mean and standard deviation, respectively, of temperature at altitude z_i, and t_i is a standardized (with mean of 0 and standard deviation of 1) random variable from a probability distribution which will be specified. The parameter vectors τ and σ

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \cdot \\ \cdot \\ \cdot \\ \tau_{34} \end{bmatrix} \qquad \sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \cdot \\ \cdot \\ \cdot \\ \sigma_{34} \end{bmatrix} \tag{5}$$

are estimated from data for different seasons and latitude zones.

The t_i values are assumed to have a variance-covariance matrix C whose elements c_{ij} are defined so that $c_{ij} \equiv c_{ji} \equiv$ Covariance of t_i and t_j. The t_i values are standardized temperatures since they represent departures from the mean in units of σ_i; that is,

$$t_i = \frac{T_i - \tau_i}{\sigma_i} \tag{6}$$

Thus, the covariance of t_i and t_j,

$$c_{ij} = E(t_i t_j) = \frac{E\left[(T_i - \tau_i)(T_j - \tau_j)\right]}{\sigma_i \sigma_j} \tag{7}$$

is, by definition, the coefficient of linear correlation between T_i and T_j. Thus, C is also the correlation matrix for the T vector.

The method of generating a vector

$$t = \begin{bmatrix} t_1 \\ t_2 \\ \cdot \\ \cdot \\ \cdot \\ t_{34} \end{bmatrix} \tag{8}$$

of standardized random numbers with correlation matrix C is based on the following theorem (ref. 14):

<u>Theorem</u>.- Let x be a vector of independent standardized random numbers and let C be a correlation matrix (i.e., C is real, symmetric, positive semidefinite, and its elements c_{ij} satisfy $c_{ii} = 1$ and $|c_{ij}| \leq 1$). Let B be a real matrix satisfying

$$C = BB^t \tag{9}$$

where B^t is the transpose of B. Then the vector

$$t = Bx \tag{10}$$

is a vector of standardized random numbers whose covariance matrix is C.

Proof of this theorem, omitted here, is straightforward by using the definitions of statistical moments.

The existence of a real matrix B as defined in equation (9) results from the fact that C is positive semidefinite, but B is not unique. If C is positive definite (that is, there are no linear dependencies among the t_i values), then a convenient choice for B is a unique lower triangular matrix obtained by using Cholesky's decomposition method (discussed in ref. 15). Elements of B are obtained recursively as follows:

$$b_{11} = (c_{11})^{1/2} \tag{11}$$

$$b_{i1} = \frac{c_{i1}}{b_{11}} \qquad (i > 1) \tag{12}$$

$$b_{jj} = \left(c_{jj} - \sum_{k=1}^{j-1} b_{jk}^2\right)^{1/2} \qquad (j > 1) \tag{13}$$

$$b_{ij} = \left\{\begin{array}{ll} 0 & (i < j) \\ \dfrac{c_{ij} - \sum_{k=1}^{j-1} b_{ik}b_{jk}}{b_{jj}} & (i > j > 1) \end{array}\right\} \tag{14}$$

The B matrices for different seasons and latitude bands were calculated from estimates of the correlations c_{ij}. Details of the process of estimating model parameters will be given in the next section of this paper.

So far, the development of the temperature model has been general for any choice of a statistical distribution for T. In the ERA model, T is assumed to be a multivariate normal (Gaussian) random variable. This choice was not made on the basis of observed temperature distributions in the data, although the latter do appear qualitatively to have Gaussian shapes, but the choice was one of mathematical expediency. The additive property of normal random variables, which states that any linear combination of independent normal random variables is itself normal, permits the convenience of using a normally distributed x vector in equation (10). For an arbitrary T distribution not possessing this additive property, the distributions of the x variates must be estimated by using characteristic functions (similar to Fourier transforms) of the T distribution functions. (See ref. 14.) The mathematical complexity involved in such an approach did not appear to be warranted for the purposes for which the ERA model is intended; that is, it is believed that parameter distributions resulting from Monte Carlo simulations using the ERA model are insensitive to the choice of the T distribution provided that its first two moments are estimated with sufficient accuracy.

The triangular form of B gives the following system of equations:

$$\left.\begin{aligned}
t_1 &= b_{11}x_1 \\
t_2 &= b_{21}x_1 + b_{22}x_2 \\
t_3 &= b_{31}x_1 + b_{32}x_2 + b_{33}x_3 \\
&\;\;\vdots \qquad\qquad \vdots \\
t_{34} &= b_{34,1}x_1 + b_{34,2}x_2 + \cdots + b_{34,34}x_{34}
\end{aligned}\right\} \tag{15}$$

where the x_i values are a set of mutually independent, standardized normal random numbers provided by a random number generator. In the first equation, $b_{11} = 1$ since $c_{11} = 1$. Thus, the sea-level standardized temperature t_1 is the standardized normal random number x_1 which is selected first and, in a sequential sense, is independent of all other variables at the time it is generated. Subsequently, t_2 is correlated to t_1; t_3 is correlated to t_1 and t_2; and so forth. If no information regarding calendar date and time of day is provided, then the temperature vector t is constructed in this manner.

However, if the date and time are specified, temperatures are adjusted to reflect their diurnal variation. The zenith angle of the Sun β is calculated (from latitude, longitude, date, and time) and t_1 is correlated to β by an adjustment of x_1. The β-dependent x_1 is given by

$$x_1(\beta) = \alpha_0 + \alpha_1\beta + \alpha_2 x_1 \tag{16}$$

where x_1 on the right-hand side is the former independent random number. The regression coefficients α_0, α_1, and α_2 are constants which have been estimated for different seasons and latitude regions. The adjusted $x_1(\beta)$ is then used in the system of equations (15) to calculate the t vector.

Calculating Atmospheric Pressures and Densities

Once the temperature profile is generated, a sea-level pressure is obtained by the model

$$P_1 = a_0 + a_1 T_1 + a_2 p_1 \tag{17}$$

where the regression coefficients a_0, a_1, and a_2 are estimated from data for each season and latitude region, and p_1 is a standardized normal random number. Thus, sea-level pressure is assumed to be normally distributed and correlated linearly to sea-level temperature.

The sea-level pressure and the temperature profile, together with some assumption about how temperature varies between z_i and z_{i+1}, are sufficient to define the remaining atmospheric properties by using the equation of state (eq. (2)), and the hydrostatic equation (eq. (3)). The following solution is that used in U.S. Standard Atmosphere (refs. 1 and 2).

The density D_i in equation (3) is replaced with its equivalent from equation (2) to give the integral equation

$$P_i = P_1 \exp\left[-\frac{\bar{M}}{G}\int_0^{z_i} \frac{g(z)\ dz}{T(z)}\right] \qquad (i = 2, \ldots, 34) \tag{18}$$

The acceleration due to gravity at altitude z is given by

$$g(z) = g_0\left(\frac{R_0}{R_0 + z}\right)^2 \tag{19}$$

where g_0, the value of g at sea level and latitude ϕ, is

$$g_0 = 9.780\ 356(1 + 0.005\ 288\ 5 \sin^2 \phi) \tag{20}$$

in m/sec^2 and R_0, the Earth's radius at latitude ϕ, is

$$R_0 = 6\ 356\ 798(0.993\ 307\ 0 + 0.006\ 693\ 0 \sin |\phi|)^{-1/2} \tag{21}$$

in meters. By assuming that temperature varies linearly with altitude between the discrete altitudes z_i $(i = 1, \ldots, 34)$, equation (18) can be integrated to give the elements of the pressure profile as

$$P_i = P_{i-1}\left(\frac{T_i}{T_{i-1}}\right)^{-\bar{g}_i \bar{M} \Delta z / G \Delta T_i} \qquad (\Delta T_i \neq 0) \tag{22a}$$

$$P_i = P_{i-1} \exp\left(\frac{-\bar{g}_i \bar{M} \, \Delta z}{G \, \Delta T_i}\right) \qquad (\Delta T_i = 0) \qquad (22b)$$

where $i = 2, \ldots, 34$, $\Delta z = 3$ km, $\Delta T_i = T_i - T_{i-1}$, and $\bar{g}_i$ is the "effective" gravitational acceleration between z_{i-1} and z_i given by

$$\bar{g}_i = \frac{g_0 R_0^2}{(R_0 + z_i)(R_0 + z_{i-1})} \qquad (23)$$

Once T and P are determined, the equation of state gives the density profile as

$$D_i = \frac{\bar{M} P_i}{G T_i} \qquad (i = 1, \ldots, 34) \qquad (24)$$

A program of the model has been written and is available through the Computer Software Management and Information Center (COSMIC) at the University of Georgia under the name ERA (number LAR-12228). The program user can specify latitude, longitude, calendar date, time of day, and the number of random atmospheres required. The program will then simulate that number of atmospheres and store them on an output tape for later use. If the user specifies a requirement of 0 atmospheres, the program sets $T = \tau$ and P_1 equal to its mean and solves the equation of state and the hydrostatic equation with these mean conditions and then stores the resultant profiles on the output tape.

ESTIMATION OF PARAMETERS

The model parameters, as defined in the preceding section, are

(1) τ and σ, the mean and standard deviation (vectors) of the temperature profile T

(2) B, the lower triangular matrix defined so that $BB^t = C$ where C is the correlation matrix of the T vector

(3) α_0, α_1, and α_2, coefficients of the linear regression of standardized sea-level temperature t_1 on the zenith angle of the Sun β

(4) a_0, a_1, and a_2, coefficients of the linear regression of sea-level pressure P_1 on sea-level temperature T_1.

In order to estimate these parameters for different locations and seasons, a set of approximately 6000 rocket and high-altitude soundings of the atmosphere was utilized. The data were divided into 17 latitude-season categories and a complete set of parameters, as listed above, was estimated for each category. These parameter sets are stored on an auxiliary parameter tape which can be linked to any computer program of the model.

Data Base

Measured profiles of the atmosphere comprising the model's data base were acquired from two sources. Approximately 5600 of these soundings were on 12 magnetic tapes furnished by the National Climatic Center (NCC), Environmental Data Service, of the National Oceanic and Atmospheric Administration. The NCC tapes were compiled from Meteorological Rocket Network (MRN) soundings made at numerous sites around the world between 1969 and 1971 (inclusive). Details of the MRN program, including sounding techniques and accuracy, may be found in reference 16. In most cases, these soundings do not extend above 60 km. To extend the data base to higher altitudes, a supplementary tape of 442 high-altitude soundings was acquired from Dale L. Johnson of NASA Marshall Space Flight Center.

The soundings were divided into categories according to the latitude zone and season in which each sounding was made. The five latitude zones

Zone	Latitudes, °N or °S
15° band	0 to 22.5
30° band	22.51 to 37.5
45° band	37.51 to 52.5
60° band	52.51 to 67.5
75° band	67.51 to 90.0

are those used in the 1966 U.S. Standard Atmosphere Supplements (ref. 2). Soundings which fell into one of the nonequatorial latitude bands were also classified by their season. The four season categories are:

Season	Northern hemisphere	Southern hemisphere
Spring	March to May	September to November
Summer	June to August	December to February
Autumn	September to November	March to May
Winter	December to February	June to August

Soundings in the 15° latitude band were not classified by season since seasonal differences are considered to be negligible at these latitudes. Table I lists the resultant 17 latitude-season categories and the number of soundings which belong to each. Since the soundings seldom cover the entire altitude range (0 to 99 km), the number of observations at each altitude is less than the number of profiles shown in table I. Tables II, III, and IV show the actual number of temperature, pressure, and density observations, respectively, at each altitude level after a filter was applied to eliminate unreasonable points.

The drastic reduction in available data above 60 km is readily apparent. Where data are sparse, parameter estimates are very uncertain. Bands of uncertainty in the form of 90-percent confidence intervals will be shown in later figures. Where data are completely missing, "fictitious" parameter estimates are made for the sake of completeness. A later section of this paper discusses that procedure.

Notation and Terminology

It is customary for statisticians to differentiate notationally between true population parameters, which generally remain unknown, and estimates of these parameters which are based on data. For example, if the true mean temperature at altitude z_i is denoted τ_i, an estimate of τ_i based on a data sample might be denoted $\hat{\tau}_i$. The parameter τ_i is a fixed "State of Nature," whereas the estimate $\hat{\tau}_i$ is itself a random variable which changes as the data changes. In the present paper, however, no such distinction is made explicitly. Estimates of the model parameters listed share the same notation as the parameters themselves. It is believed that readers can distinguish, when necessary, between the two meanings by the context in which the symbol is used.

The fact that the data are incomplete in that very few soundings cover the entire altitude range (0 to 99 km) complicates the parameter estimation task and the notation involved in defining the estimators which were used. The system of notation for treating incomplete data which will be used is described as follows.

Let n be the number of soundings in the data base for the latitude-season category in question (from table I), and let T_{ik} $(i = 1, \ldots, 34;$ $k = 1, \ldots, n)$ be the value of T_i at the time of the kth sounding. Define the variable δ_{ik} as

$$\delta_{ik} = \begin{Bmatrix} 1 & (\text{if } T_{ik} \text{ is present}) \\ 0 & (\text{if } T_{ik} \text{ is missing}) \end{Bmatrix} \tag{25}$$

Thus, n_i, the number of temperature observations at altitude z_i (from table II), is given by

$$n_i = \sum_{k=1}^{n} \delta_{ik} \tag{26}$$

and n_{ij}, the number of soundings in which both T_i and T_j are measured, is

$$n_{ij} = \sum_{k=1}^{n} \delta_{ik}\delta_{jk} \tag{27}$$

Estimating τ and σ

The most common estimators for the mean and standard deviation of T_i are the sample mean

$$\bar{T}_i = \frac{\sum_{k=1}^{n} \delta_{ik}T_{ik}}{n_i} \tag{28}$$

and the sample standard deviation

$$s_i = \left[\frac{\sum_{k=1}^{n} \delta_{ik}(T_{ik} - \bar{T}_i)^2}{n_i - 1}\right]^{1/2} \tag{29}$$

If the model parameters τ_i and σ_i are set equal to $\bar{T}_i$ and s_i, respectively, means and standard deviations of model temperatures can be forced to match those in the data exactly. The ERA model, however, does not use $\bar{T}_i$ and s_i as estimators for the mean and standard deviation of T_i. Instead, estimators were selected to improve the fit of Gaussian distributions to the observed temperature distributions.

The parameters τ_i and σ_i were obtained as follows: Temperatures in the data corresponding to cumulative frequencies of 0.26, 0.5, and 0.84 were set equal to $\tau_i - \sigma_i$, τ_i, and $\tau_i + \sigma_i$, respectively, and these three equations were solved for the best (least-squares) values of τ_i and σ_i. The similarity between model (Gaussian) and data temperature distributions is reflected in the fact that τ_i and σ_i agreed closely with $\bar{T}_i$ and s_i at most altitudes.

An advantage of the method used for estimating τ_i and σ_i over the usual estimators $\bar{T}_i$ and s_i is that the former method is much less sensitive to erratic behavior in the tails of the data distribution. For example, if the data samples contain outliers of questionable validity, these tend to inflate s_i whereas they have little effect, in general, on the estimate σ_i based on cumulative frequencies. This is an important consideration in estimation problems such as this where individual point-by-point decisions on data validity are precluded by the large volume of data involved.

Estimating B

The coefficient of correlation between T_i and T_j was estimated as

$$c_{ij} = \frac{\sum_{k=1}^{n} \delta_{ik}\delta_{jk}(T_{ik} - \tau_i)(T_{jk} - \tau_j)}{\left[\sum_{k=1}^{n} \delta_{ik}\delta_{jk}(T_{ik} - \tau_i)^2 \sum_{k=1}^{n} \delta_{ik}\delta_{jk}(T_{jk} - \tau_j)^2\right]^{1/2}} \tag{30}$$

Note that summations in the numerator and denominator include only data where both T_i and T_j are measured; that is, the $\delta_{ik}\delta_{jk}$ product appears in all three sums. This estimator is used because it insures that $|c_{ij}| \leq 1$. To see this, note that c_{ij} takes the form of the cosine of the angle between two n-dimensional vectors whose kth components are $\delta_{ik}\delta_{jk}(T_{ik} - \tau_i)$ and $\delta_{ik}\delta_{jk}(T_{jk} - \tau_j)$.

In all 17 latitude-season categories the C matrices, as defined by equation (30), failed to be positive definite, a requirement needed to apply the Cholesky decomposition, equations (11) to (14), to obtain B. It is possible to show that had the data set been complete ($\delta_{ik} = 1$ for all i and k) C would have been positive definite. In this case C could be expressed as the product UU^t where U is the $34 \times n$ matrix of rank 34 whose elements are

$$u_{ik} = \frac{T_{ik} - \tau_i}{\left[n \sum_{l=1}^{n} (T_{il} - \tau_i)^2\right]^{1/2}} \tag{31}$$

for $i = 1, \ldots, 34$; $k = 1, \ldots, n$. A matrix of full rank is positive definite if, and only if, it can be expressed as UU^t for some matrix U. (See ref. 17.) This same matrix U does not work for the case of incomplete data because the sums of squares in the denominator of equation (30) are dependent on both i and j indices.

In general, C matrices defined by equation (30) may still be positive definite. In fact, the first 20 rows and columns of the C matrices involved in the ERA model were positive definite. These values correspond to altitudes below 60 km where samples were much more complete. Above 60 km, missing data and possibly other anomalies in the data began to impact the estimates of correlations to such an extent that C no longer was positive definite.

In each of the 17 data sets, the nonpositive nature of C was encountered in calculating the B matrix, recursively, when the right-hand side of equation (13) became the square root of a negative number. In each case j was greater than 20 so that at least the first 20 rows and columns of B had been successfully defined.

A means of overcoming this problem was sought which would keep the successfully defined portion of B intact and modify only the row and column of C where the difficulty was encountered. In addition, it was decided to allow all elements of that row and column to be modified except the diagonal and first off-diagonal elements. That is, if this is the mth row (column), T_{m-1} and T_{m+1} would retain their estimated correlations with T_m but the correlation of the other T_i values with T_m could be changed. Since the variance of the vertical temperature gradient at z_i is dependent on $c_{m,m-1}$ and $c_{m,m+1}$, this condition was imposed to maintain an accurate modeling of the temperature gradient.

The simplest regression model of t_m on t_{m-1} was selected; namely,

$$t_m = \lambda_m t_{m-1} + \varepsilon_m \sqrt{1 - \lambda_m^2} \tag{32}$$

where λ_m is a constant and ε_m is a standardized random variable which is independent of t_i $(i < m)$. It is straightforward to show that $\lambda_m = c_{m,m-1}$ retains the desired correlation between T_m and T_{m-1}. The remaining correlations are

$$E(t_m t_i) = \lambda_m \, E(t_{m-1} t_i) = c_{m,m-1} c_{m-1,i} \tag{33}$$

for $i = 1, \ldots, m - 2$. Thus, when a row m was encountered so that b_{mm} was imaginary, the correlations c_{mi} and c_{im} $(i < m - 1)$, were replaced by

$$c_{mi}^* = c_{im}^* = c_{m,m-1} c_{m-1,i} \tag{34}$$

It is possible to show, by using equations (11) to (14), that the problem of a nonpositive-definite C matrix is thereby solved. The adjusted mth row of the B matrix becomes

$$b_{mi}^* = c_{m,m-1} b_{m-1,i} \qquad (1 \leqq i < m) \tag{35}$$

and

$$b_{mm}^* = \left[c_{m,m} - \sum_{i=1}^{m-1} (b_{mi}^*)^2\right]^{1/2} = (1 - c_{m,m-1}^2)^{1/2} \tag{36}$$

is always real.

The physical significance of the model given by equation (32) is that the conditional distribution of T_m when given T_{m-1} is independent of the temperatures at levels below z_{m-1}. In other words, if one wishes to estimate T_m when given other temperatures lower in the atmospheric profile, knowledge of T_{m-1} is sufficient and the additional knowledge of lower temperatures is of no value. No attempt was made to justify the use of equation (32) on the basis of this physical significance. The model was chosen because it preserves the modeling of $\Delta T_m/\Delta z$ and because it guarantees a real B matrix. It is interesting to note that this adjusted C matrix is optimal in that it corresponds, for a particular choice of weights, to an optimal solution derived by Carswell (ref. 14) for correcting a nonpositive-definite correlation matrix.

In summary, the B matrix elements were estimated by applying equations (11) to (14) to the correlation matrix C whose elements are defined by equation (30). If the correlations in some row M of the C matrix were such that the element b_{mm} defined by equation (13) was not real, then the mth row and column of the C matrix were modified by using equation (34). This procedure corrected the problem in a physically meaningful manner and preserved the modeling of temperature gradients found in the data.

Estimating α_0, α_1, and α_2

A linear regression of the standardized sea-level temperature on β, the zenith angle of the Sun, was estimated as follows: Let $r_{T\beta}$ be the coefficient of linear correlation between T_1 and β defined by

$$r_{T\beta} = \frac{\sum_{k=1}^{n} \delta_{1k}(T_{1k} - \tau_1)(\beta_k - \bar{\beta})}{\left[\sum_{k=1}^{n} \delta_{1k}(T_{1k} - \tau_1)^2 \sum_{k=1}^{n} \delta_{1k}(\beta_k - \bar{\beta})^2\right]^{1/2}} \tag{37}$$

where β_k $(k = 1, \ldots, n)$ is the zenith angle of the Sun at the time of the kth sounding (calculated from dates, times, longitudes, and latitudes) and $\bar{\beta}$ is the average of the β_k values. There were no missing β observations since sufficient information was provided with the data to calculate β for each profile.

A linear regression of T_1 on β is

$$T_1 = \tau_1 + \frac{r_{T\beta}\sigma_1}{\sigma_\beta}(\beta - \bar{\beta}) + \sigma_1(1 - r_{T\beta}{}^2)^{1/2} x_1 \tag{38}$$

where σ_β is the standard deviation of the β_k values and x_1 is an independent standardized normal random number. This equation can be written as

$$t_1 = \alpha_0 + \alpha_1\beta + \alpha_2 x_1 \tag{39}$$

where t_1 is the standardized sea-level temperature defined by equation (6). The regression coefficients are, therefore, given by

$$\alpha_0 = -\frac{r_{T\beta}\bar{\beta}}{\sigma_\beta} \tag{40}$$

$$\alpha_1 = \frac{r_{T\beta}}{\sigma_\beta} \tag{41}$$

$$\alpha_2 = (1 - r_{T\beta}^2)^{1/2} \tag{42}$$

If each sounding had a sea-level temperature and if $\tau_1 = \bar{T}_1$ and $\sigma_1 = s_1$, this method of defining regression coefficients would be equivalent to the usual least-squares method of fitting a line through the points $\{(t_{1k},\beta_k);\ k = 1, \ldots, n\}$, where t_{1k} is the measurement of t_1 in the kth sounding. Note that the mean and standard deviation of t_1, as defined by equation (39), are still 0 and 1, respectively, when averaged over the distributions of β and x_1, if it is assumed that the mean and variance of β are $\bar{\beta}$ and σ_β^2, respectively.

Estimating a_0, a_1, and a_2

The method of estimating the linear regression of P_1 on T_1 (eq. (17)) is similar to that of t_1 on β, the only difference being that P_1 is not standardized. The regression coefficients are

$$a_0 = \bar{P}_1 - \frac{r_{TP}\sigma_P}{\sigma_1}\tau_1 \tag{43}$$

$$a_1 = \frac{r_{TP}\sigma_P}{\sigma_1} \tag{44}$$

$$a_2 = \sigma_P(1 - r_{TP}^2)^{1/2} \tag{45}$$

where $\bar{P}_1$ and σ_P are the sample mean and standard deviation, respectively, of sea-level pressure, and r_{TP} is the coefficient of linear correlation between T_1 and P_1 defined by

$$r_{TP} = \frac{\sum_{k=1}^{n} \delta_{1k}\delta_{1k}^*(T_{1k} - \tau_1)(P_{1k} - \bar{P}_1)}{\left[\sum_{k=1}^{n} \delta_{1k}\delta_{1k}^*(T_{1k} - \tau_1)^2 \sum_{k=1}^{n} \delta_{1k}\delta_{1k}^*(P_{1k} - \bar{P}_1)^2\right]^{1/2}} \tag{46}$$

Notation for the pressure data corresponds to that for temperature data in that P_{1k} is the measurement of P_1 in the kth sounding and δ_{1k}^* is defined as

$$\delta_{1k}^{*} = \begin{cases} 1 & (\text{if } P_{1k} \text{ is present}) \\ 0 & (\text{if } P_{1k} \text{ is missing}) \end{cases} \tag{47}$$

Estimating Parameters Without Data

In order to estimate a mean, a standard deviation, and a correlation coefficient, at least two data points are required. It is apparent from table II that this minimum requirement was not met at high altitudes in most categories. To complete the estimation of model parameters where data were totally missing, the following substitutions were used:

(1) Means: Temperature means were estimated above available data by extrapolating the mean temperature profile by using the vertical temperature gradients from the 1966 U.S. Standard Atmosphere Supplements (ref. 2).

(2) Standard deviations: Standard deviations of temperatures above available data were assumed to be equal to the nearest standard deviation that was based on data.

(3) Coefficients of correlation: In situations where $n_{ij} \leqq 3$, the coefficient of correlation c_{ij} was estimated as

$$c_{ij} = \exp\left(-\frac{|i - j|}{2}\right) \tag{48}$$

This formula was chosen because it gave a fairly reasonable and simple representation of the observed correlation structure.

COMPARISON OF MODEL WITH DATA

In the present model evaluation, the model is compared with the same data set used in estimating the model parameters. Hence, the model's ability to imitate that data set is being demonstrated without any assurance that the data are adequate representations of the true population. As more and better data are acquired, they can be added to the present set and the model parameters updated. The model would then be expected to simulate the updated data sample with approximately the same accuracy as it does the present sample. Nevertheless, the model will, at best, only be as good as the data used to estimate its parameters.

To evaluate the model in terms of meeting its original objectives, samples consisting of 1000 model-generated atmospheres were created for each of the 17 latitude-season categories. In each model-generated sample, the means and standard deviations of T_i, P_i, D_i, $\Delta T_i/\Delta z$, $\Delta P_i/\Delta z$, and $\Delta D_i/\Delta z$ were calculated and compared with corresponding properties in the data.

Figure 3 compares model and data means and standard deviations of T and $\Delta T/\Delta z$ for the 15° latitude category. Solid lines represent model parameters based on the sample of 1000 modeled atmospheres, and the squares represent corresponding parameters for the data sample. Horizontal error bars are drawn to indicate 90-percent confidence intervals about the data estimates where these intervals extend beyond the data symbol. In places where horizontal error bars do not cross the model parameter curve, statistically significant differences between model and data exist. Note that such a significant difference exists between σ_i and s_i in the $S(T)$ plot in figure 3 at altitudes of 96 and 99 km. These differences were deliberate, however, because of the choice of $\sigma_i \neq s_i$. It was believed that use of the higher s_i values would have significantly overestimated the frequency of large departures from the mean at these altitudes.

In the case of the mean and standard deviation of T, what is really being compared in figure 3 is the model's mean temperature τ_i against the usual sample mean $\bar{T}_i$, given by equation (28), and the model's standard deviation σ_i against the sample standard deviation s_i, defined by equation (29). Had these model parameters been defined as $\tau_i = \bar{T}_i$ and $\sigma_i = s_i$, there would have been no real differences found in the temperature comparisons in figure 3.

The amount of agreement between model and data samples of $\Delta T/\Delta z$ is influenced largely by the agreement in model and data temperature samples. It can be shown that for the model samples, the mean and standard deviation of $\Delta T_i/\Delta z$ are, respectively,

$$E\left(\frac{\Delta T_i}{\Delta z}\right) = \frac{\tau_i - \tau_{i-1}}{\Delta z} \tag{49}$$

$$S\left(\frac{\Delta T_i}{\Delta z}\right) = \frac{\left(\sigma_i^2 + \sigma_{i-1}^2 - 2\sigma_i\sigma_{i-1}C_{i-1,i}\right)^{1/2}}{\Delta z} \tag{50}$$

Thus, any poor agreement between model and data temperature gradients could be a result of poor agreement between temperature means, standard deviations, and adjacent layer correlations. However, there may be other reasons for disagreement. Equations (49) and (50) do not have direct counterparts for the data samples, because the data sample of $\Delta T_i/\Delta z$ consists of $n_{i,i-1}$ observations whereas the samples of T_i and T_{i-1} contain, respectively, n_i and n_{i-1} observations.

The importance of this effect can be seen in figure 3 in the significant difference which exists between model and data samples of $\Delta T/\Delta z$ at 66 km, despite good agreement between τ and $\bar{T}$ and between σ and s at 63 and 66 km. In this data set, the samples of T_{22} (at 63 km) and T_{23} (at 66 km) contained 486 and 136 values, respectively, and the sample of $\Delta T_{23}/\Delta z$ contained 130 values. Thus, of the 486 profiles containing measurements of T_{22}, only 130 also contain measurements of T_{23}. The remaining 356 profiles end somewhere between 63 and 66 km. Apparently, the mean and standard deviation of T_{22} based solely on the 130 observations are significantly different from $\bar{T}_{22}$ and s_{22} based on the entire sample of 486 values.

Figure 4 compares model and data means and standard deviations of P and $\Delta P/\Delta z$ for the same latitude-season category. The values of E(P) and S(P) are nondimensionalized by the 1962 Standard Atmosphere pressures, and similarly, for purposes of nondimensionalizing, pressure gradients are expressed as percentage changes in pressure per kilometer, $|\Delta P_i/\Delta z|/P_i$. Because of the fact that pressures vary by several orders of magnitude between sea level and 99 km, dimensionalized plots of pressures and pressure gradients (on semilog paper) tend to mask or minimize differences between model and data parameters which are of the same order of magnitude.

Figure 5 compares model and data means and standard deviations of D and $\Delta D/\Delta z$ for the 15° latitude category. As in the case of pressures, densities are nondimensionalized by using the 1962 Standard Atmosphere densities and vertical gradients are expressed as percentage changes in density per kilometer, $|\Delta D_i/\Delta z|/D_i$.

Means and standard deviations of P_i, D_i, $\Delta P_i/\Delta z$, and $\Delta D_i/\Delta z$ are not directly controlled by the choice of model parameters (except for sea-level pressure), but result, instead, from the behavior of the hydrostatic equation and the equation of state when applied to model-generated random temperature profiles. There is no inherent guarantee with the empirical modeling procedure used that the distributions of pressures and densities in the model will match those in the data. In fact, since the solution of the hydrostatic equation involves an integration, systematic errors could be cumulative. Although a potential for significant disparity existed, the actual agreement seen in figures 4 and 5 is very satisfactory below 60 km.

Figures 6 to 53 represent the same comparisons as figures 3 to 5 for the remaining 16 latitude-season categories. The darkened symbols represent data samples consisting of only a single measurement, where standard deviations are zero and confidence intervals are undefined. These figures are included here not solely for model-data comparison purposes, but also because they contain valuable information about measured means and standard deviations of thermodynamic properties in the atmosphere.

In general, the agreement between model and data is satisfactory when the quality and quantity of the data base are considered. Below 60 km, agreement is excellent. Therefore, it is reasonable to conclude that below 60 km the model is limited only by the representativeness of the data in describing the true populations of interest. The sample sizes were, in general, adequate, but the time interval over which the data were collected (1969 to 1971) may have been too limited, and the sampling may not have been as random in latitude and time as one would desire. Thus, one may wish to add to the data base below 60 km if more representative data are acquired.

Above an altitude of 60 km, agreement between model and data deteriorates for a number of possible reasons. Obviously, the quantity of data at those heights is inadequate, but if sampling limitations are given, other significant differences exist between model and data. One possible reason for these differences is that the hydrostatic equation and the equation of state (with constant $\bar{M}$) are known to become less applicable. Also, the technique used to obtain the high-altitude data on the tape of 442 high-altitude soundings may have produced biases or other faults in the data. Thus, pressure and density distributions derived by applying the hydrostatic equation and the equation of state to lower altitude distributions might actually be more realistic than those for the high-altitude data. Whatever the reason, the fact remains that the model's accuracy in representing atmospheric profiles above 60 km is uncertain.

The ultimate comparison between model and data is the comparison of histograms (frequency plots) of temperatures, pressures, and densities. As stated previously, no special effort was made to model the statistical distributions found in the data beyond an attempt to approximate their first two moments (means and variances). Temperature profiles were modeled as multivariate normal random vectors because of the desirability of the additive property of the normal distribution. Means and standard deviations of temperatures were selected which improved the normal fit somewhat, but no goodness-of-fit tests were made to determine the suitability of the normal distribution. Distributions of pressures and densities in the model (except for the sea-level pressure) were not normal. Figures 54 to 56 show comparisons of model and data temperature, pressure and density distributions at altitude intervals of 15 km between 0 and 90 km in the 15° latitude band. These comparisons are representative of similar comparisons at other altitudes and in other latitude-season categories.

USING THE MODEL FOR MONTE CARLO SIMULATIONS

A computer program of the model entitled ERA, available through COSMIC (number LAR-12228) was developed for the Control Data CYBER system of computers at the NASA Langley Research Center. Random atmospheres generated by ERA are stored on an output file which can subsequently be linked to a Monte Carlo simulation program and a new independent atmosphere read at the beginning of each trajectory replication. For example, Monte Carlo simulations of space shuttle orbiter landings at the Kennedy Space Center during the summer might be of interest in the design of landing gear and runway length since low-density summer air means faster landings and longer braking distances. The ERA program can be called to provide any size sample of midday atmospheres from the 30° summer band. These atmospheres would be stored on a tape, and at the beginning of each landing replication the landing simulation program could read a new random atmosphere from the tape.

ERA generates atmospheres rapidly in spite of the model's use of large matrices. It takes less than 4 sec of central processor time on a CYBER 175 computer to generate 1000 random atmospheres from the same latitude-season category. This time includes the time required to determine the appropriate latitude-season category, read the parameters from the parameter tape, and then generate 1000 atmospheres from that category.

ERA is suitable for situations where one-dimensional atmospheres are needed. If two- or three-dimensional atmospheric variations are required, one should use a set of four computer tapes called REACT (COSMIC number LAR-12227) which contain three-dimensional global random atmospheres produced by the ERA model (ref. 18). The REACT tapes, one for each season, were generated by assuming a correlation structure relating adjacent atmospheric profiles over a global grid. These tapes contain sufficient atmospheres to allow approximately 1400 independent replications of any three-dimensional spacecraft or aircraft trajectory.

CONCLUDING REMARKS

An empirical stochastic computer model, called ERA, has been developed for simulating random thermodynamic profiles in the Earth's atmosphere below an altitude of 100 km. Such profiles can be used in Monte Carlo studies of the effects of atmospheric variability on, for example, spacecraft or aircraft trajectories or on physical phenomena such as sound propagation or pollutant dispersions. Profiles of temperatures, densities, and pressures generated by the model are characteristic of any specified season, latitude zone, and time of day.

The objective of the ERA model is to provide large samples of random atmospheres which have the same statistical properties (for example, means, variances, and correlations) as a large data base containing data from over 6000 soundings. These data were used both to estimate model parameters and to evaluate the model's accuracy. In their former function, the data were divided into 17 latitude-season categories and a different set of parameters was estimated for each category. Model parameters, stored on a model parameter tape, include the mean and standard deviation of temperatures at altitude intervals of 3 km between sea level and 99 km and interlayer temperature correlations. Thus, statistical properties of modeled temperature profiles were controlled by the model input. Densities and pressures were computed by use of the equation of state and the hydrostatic equation.

The model gave an excellent representation of the data below an altitude of 60 km where data samples were of adequate size. Derived statistical properties at the discrete altitude levels which compared well with data were means and standard deviations of pressures and densities and vertical pressure and density gradients. Above an altitude of 60 km, the model's ability to simulate realistic atmospheric profiles is uncertain. Comparisons with data are inconclusive because of the diminished quantity, and possibly poor quality, of the data at those heights.

An efficient computer program of the ERA model, available through COSMIC (number LAR-12228), can generate 1000 random atmospheres from the same latitude-season category in less than 4 sec on a Control Data CYBER 175 computer. The program reads the appropriate model parameters from an auxiliary tape, which is a part of the program. The program and tape were generated under Control Data Network Operating System at the NASA Langley Research Center.

It is believed that the ERA model is the best available model for simulating realistic atmospheres because of the unified statistical approach used in modeling the data and because of the substantial quantity of data, particularly below an altitude of 60 km, used in constructing the model. As new and better data become available, the model parameter tape can readily be updated and the model improved.

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TABLE I.- NUMBER OF PROFILES IN THE DATA BASE FOR

EACH LATITUDE-SEASON CATEGORY

Latitude band, deg	Season				Total
	Spring	Summer	Autumn	Winter	
15	1928				1928
30	495	468	516	504	1983
45	184	193	154	147	678
60	238	176	217	342	973
75	101	128	91	122	442
					6004

TABLE II.- NUMBER OF TEMPERATURE OBSERVATIONS AT EACH ALTITUDE

IN THE 17 LATITUDE-SEASON CATEGORIES

Altitude, km	Number of temperature observations for seasons and latitude bands, deg																
	Annual	Spring				Summer				Autumn				Winter			
	15	30	45	60	75	30	45	60	75	30	45	60	75	30	45	60	75
0	1747	375	165	234	86	319	172	173	103	319	142	214	69	363	119	338	90
3	1759	457	170	234	88	436	174	173	107	454	141	214	74	475	122	338	94
6	1771	456	170	234	88	433	175	173	107	451	141	214	76	476	122	337	96
9	1781	457	170	234	90	435	173	173	108	452	142	214	77	477	122	337	101
12	1778	457	170	233	90	435	176	173	111	451	142	214	79	477	122	337	104
15	1782	456	170	231	90	436	176	173	110	451	142	211	80	477	122	334	104
18	1781	456	171	233	89	436	171	172	110	451	143	212	81	476	129	327	100
21	1772	452	173	233	90	428	175	173	111	450	142	209	81	474	129	324	100
24	1774	451	173	230	90	429	176	171	107	450	142	207	81	470	129	320	97
27	1768	445	172	227	90	427	174	171	108	451	142	205	78	466	132	315	95
30	1749	436	166	224	98	404	168	168	121	436	142	205	87	452	129	309	108
33	1690	419	158	221	99	395	155	166	118	423	140	200	85	427	118	300	105
36	1616	369	137	214	97	354	147	163	113	380	119	190	82	386	106	290	107
39	1569	353	133	210	87	329	136	152	111	350	114	188	78	364	102	281	102
42	1556	364	133	206	79	325	133	150	102	366	111	186	76	363	102	276	95
45	1560	365	133	203	74	330	133	147	96	371	108	187	69	365	102	273	84
48	1530	368	133	199	69	332	133	144	85	371	103	179	63	362	99	265	73
51	1476	365	126	192	64	324	132	144	79	366	94	177	57	353	95	260	67
54	1369	358	108	182	58	308	120	138	77	351	89	168	52	346	88	250	61
57	1284	321	92	155	50	295	96	120	69	328	76	152	49	326	74	229	53
60	986	286	80	126	44	238	76	108	62	284	53	129	44	273	52	180	45
63	486	183	51	80	40	129	38	81	53	181	21	91	37	154	28	99	39
66	136	90	17	42	33	59	17	46	46	88	12	41	28	58	18	33	36
69	64	47	13	25	28	28	16	13	41	48	8	12	24	25	15	15	33
72	67	20	11	8	23	11	14	0	37	31	7	1	21	24	15	3	28
75	69	14	10	0	22	12	15	0	34	27	7	1	21	24	13	0	26
78	67	14	10	0	19	12	13	0	24	28	6	0	18	23	11	0	21
81	65	14	9	0	14	12	13	0	17	27	6	0	16	23	6	0	17
84	66	13	6	0	0	11	13	0	1	27	6	0	4	20	6	0	2
87	59	13	6	0	0	9	12	0	0	22	5	0	2	13	4	0	1
90	56	11	2	0	0	10	10	0	0	21	5	0	0	11	1	0	0
93	40	1	0	0	0	0	5	0	0	5	3	0	0	2	0	0	0
96	26	1	0	0	0	0	3	0	0	3	3	0	0	1	0	0	0
99	25	0	0	0	0	0	3	0	0	2	3	0	0	2	0	0	0

TABLE III.- NUMBER OF PRESSURE OBSERVATIONS AT EACH ALTITUDE

IN THE 17 LATITUDE-SEASON CATEGORIES

Altitude, km	Number of temperature observations for seasons and latitude bands, deg																
	Annual	Spring				Summer				Autumn				Winter			
	15	30	45	60	75	30	45	60	75	30	45	60	75	30	45	60	75
0	1790	375	165	234	87	311	172	170	102	325	142	214	70	370	121	336	88
3	1765	453	169	235	86	431	175	175	105	452	142	215	74	477	123	341	92
6	1776	453	170	236	87	429	175	175	106	452	141	213	75	476	123	340	92
9	1768	455	170	235	87	430	175	175	105	450	142	214	77	478	123	340	95
12	1785	453	170	236	88	432	176	175	107	453	142	215	78	477	123	338	100
15	1784	453	171	236	88	433	166	175	107	451	141	215	80	476	124	337	102
18	1777	453	169	234	87	432	2	171	107	453	143	213	79	475	130	330	102
21	1766	451	170	232	88	431	174	171	109	452	143	209	80	474	129	326	99
24	1741	449	168	230	87	428	175	169	109	443	143	206	79	468	127	316	93
27	1720	439	169	224	87	418	172	168	107	439	140	203	76	463	129	307	91
30	1694	423	160	219	97	395	168	162	118	424	142	198	84	446	127	290	102
33	1600	401	154	204	96	382	151	159	114	408	140	186	80	420	115	276	94
36	1494	336	135	186	91	328	142	150	110	357	119	167	76	367	103	244	89
39	1432	317	132	171	80	295	134	132	105	316	113	155	71	343	99	233	82
42	1418	326	133	170	72	291	132	128	96	332	109	150	70	340	99	225	73
45	1416	329	132	165	67	298	131	126	89	340	106	148	64	343	98	222	62
48	1388	331	131	158	62	300	131	124	78	338	101	141	57	340	96	215	52
51	1337	331	123	152	56	290	129	121	71	337	93	139	51	331	93	211	49
54	1269	327	110	145	50	275	119	113	68	321	87	129	47	323	86	203	44
57	1155	301	95	124	43	259	97	100	64	296	74	120	45	305	72	185	42
60	883	263	82	96	37	205	77	87	59	257	51	102	42	258	52	144	36
63	432	169	52	60	36	111	38	70	52	160	20	73	34	140	28	84	30
66	123	85	17	37	31	52	17	40	46	80	11	35	26	55	18	30	29
69	60	49	12	22	28	27	16	12	40	46	8	11	23	24	15	15	29
72	64	22	11	8	23	13	16	1	37	31	7	2	21	22	15	3	28
75	66	15	10	1	22	11	15	0	34	27	7	1	21	24	13	0	26
78	62	13	10	0	19	11	14	0	24	27	6	1	18	24	12	0	21
81	60	13	10	0	14	11	14	0	17	26	6	1	16	22	8	0	17
84	61	12	7	0	0	11	13	0	1	27	6	1	4	19	6	0	2
87	55	13	6	0	0	10	10	0	0	22	5	0	2	13	5	0	1
90	54	10	4	0	0	10	9	0	0	22	5	0	0	11	2	0	0
93	40	1	0	0	0	1	5	0	0	5	3	0	0	2	1	0	0
96	27	1	0	0	0	1	3	0	0	3	3	0	0	1	1	0	0
99	25	0	0	0	0	2	3	0	0	2	4	0	0	3	1	0	0

TABLE IV.- NUMBER OF DENSITY OBSERVATIONS AT EACH ALTITUDE

IN THE 17 LATITUDE-SEASON CATEGORIES

Altitude, km	Number of temperature observations for seasons and latitude bands, deg																
	Annual	Spring				Summer				Autumn				Winter			
	15	30	45	60	75	30	45	60	75	30	45	60	75	30	45	60	75
0	1744	372	163	232	86	304	172	168	102	316	142	212	68	360	119	335	88
3	1748	452	169	233	86	431	173	173	105	449	140	214	71	473	122	338	91
6	1753	451	170	233	87	429	174	173	105	449	141	212	72	475	122	337	90
9	1755	449	167	233	87	429	173	173	103	445	140	213	74	472	122	337	95
12	1734	453	170	233	87	424	170	173	106	445	142	214	75	476	122	333	100
15	1766	453	169	231	87	431	166	173	107	447	141	211	75	475	122	332	102
18	1762	453	168	232	87	432	2	169	107	449	143	209	77	474	129	326	98
21	1748	447	170	232	88	424	173	170	107	445	141	203	78	471	128	320	95
24	1727	447	166	227	86	423	174	166	105	435	142	200	79	466	127	313	89
27	1710	438	168	220	87	417	170	165	107	435	140	198	76	459	128	304	86
30	1680	421	157	216	96	390	166	162	118	415	142	196	84	445	127	289	99
33	1580	400	154	204	96	381	151	158	114	404	138	185	80	418	112	272	93
36	1487	335	135	186	91	325	139	149	109	351	117	165	76	367	101	241	89
39	1426	312	131	171	79	292	133	131	103	309	112	153	71	342	98	230	82
42	1411	322	131	169	71	285	131	127	92	327	109	150	70	338	99	222	73
45	1409	326	132	163	66	289	129	124	86	332	106	148	63	340	98	222	62
48	1377	328	131	158	61	290	130	123	77	331	101	141	56	336	96	215	52
51	1328	327	123	152	56	289	129	120	70	329	92	139	51	329	93	211	49
54	1257	324	106	145	50	272	117	112	68	313	87	129	47	322	86	202	43
57	1141	292	91	121	43	259	93	98	63	294	74	118	44	303	72	185	41
60	870	261	80	94	37	202	75	85	59	253	51	101	40	254	52	144	35
63	423	168	51	60	36	110	38	66	52	160	20	72	34	137	28	83	29
66	121	85	17	36	31	51	17	40	46	80	11	34	26	55	18	30	28
69	60	45	12	22	28	25	16	11	40	46	8	10	23	24	15	15	28
72	60	18	10	8	23	11	14	0	37	31	7	1	21	22	15	3	26
75	66	13	10	0	22	11	15	0	34	27	7	1	21	23	13	0	25
78	62	13	10	0	19	10	13	0	24	26	6	0	18	23	11	0	21
81	60	13	9	0	14	11	13	0	17	25	6	0	16	22	6	0	17
84	61	12	6	0	0	11	13	0	1	26	6	0	4	19	6	0	2
87	54	12	6	0	0	9	10	0	0	22	5	0	1	13	4	0	1
90	51	10	2	0	0	10	9	0	0	21	5	0	0	11	1	0	0
93	39	1	0	0	0	0	5	0	0	5	3	0	0	2	0	0	0
96	26	1	0	0	0	0	3	0	0	3	3	0	0	1	0	0	0
99	24	0	0	0	0	0	3	0	0	2	3	0	0	2	0	0	0

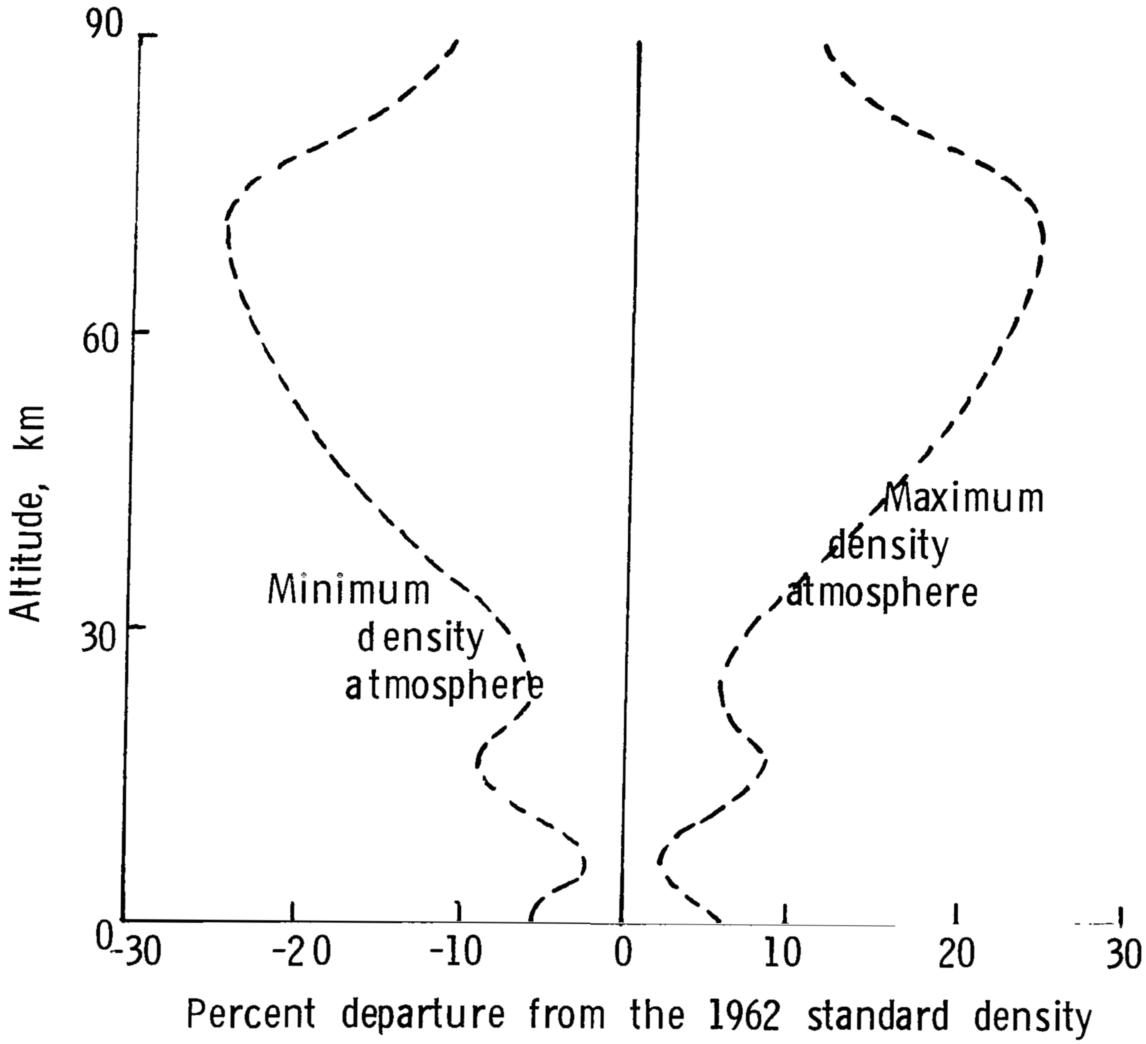

Figure 1.- "Extreme" atmospheres used in entry vehicle design studies.

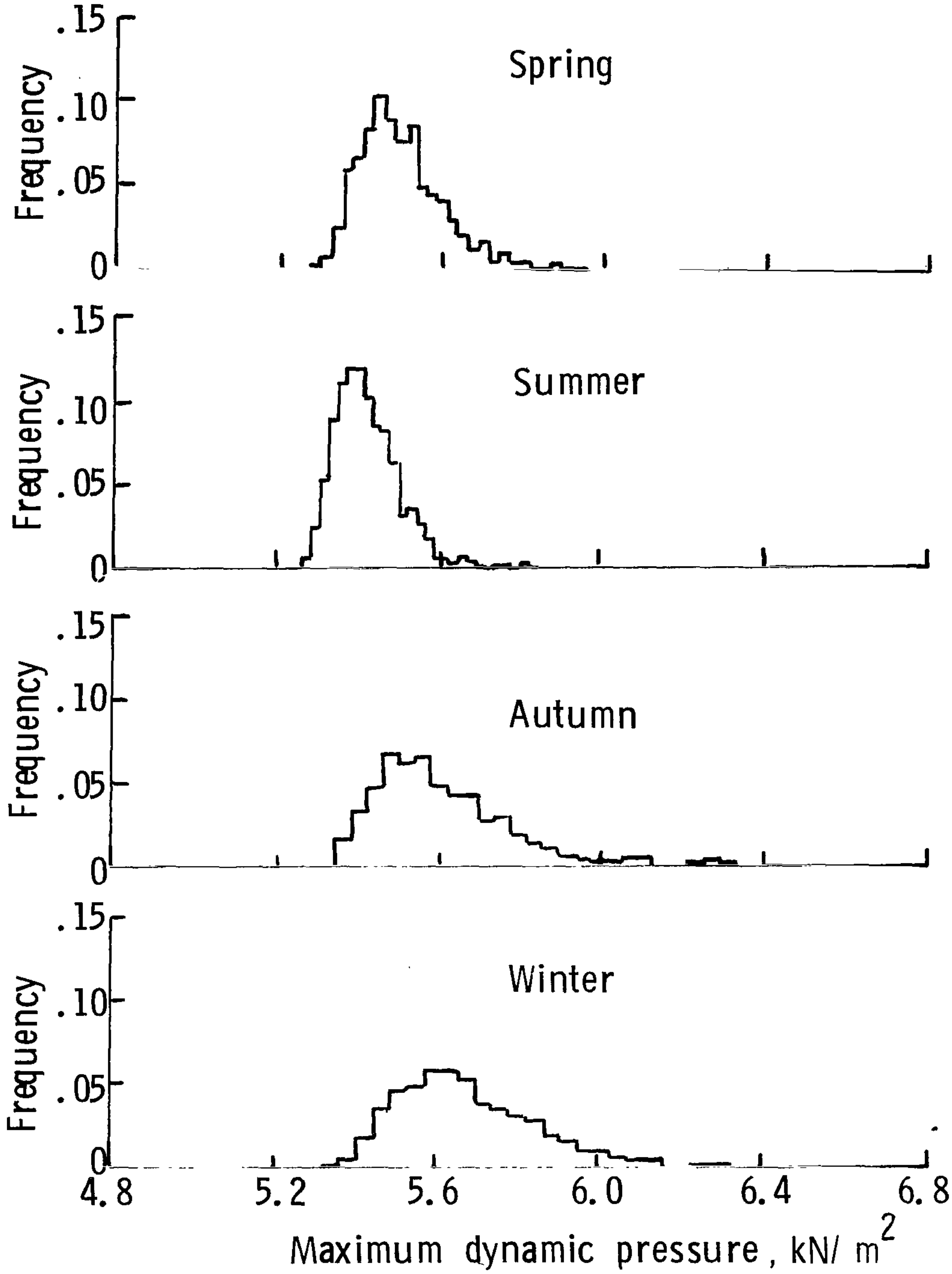

Figure 2.- Seasonal distributions of maximum dynamic pressure based on simulated shuttle entries.

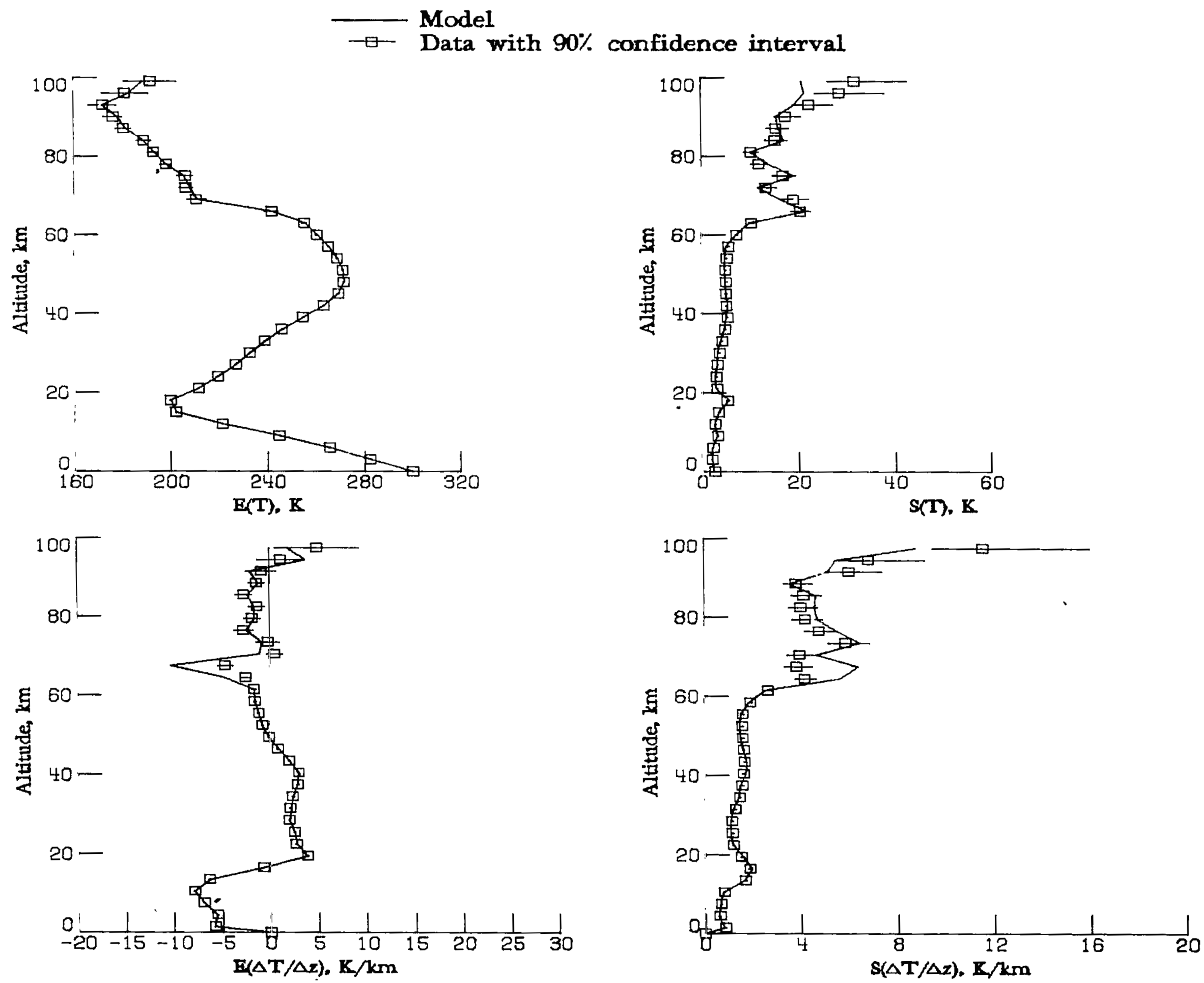

Figure 3.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the 15° latitude category.

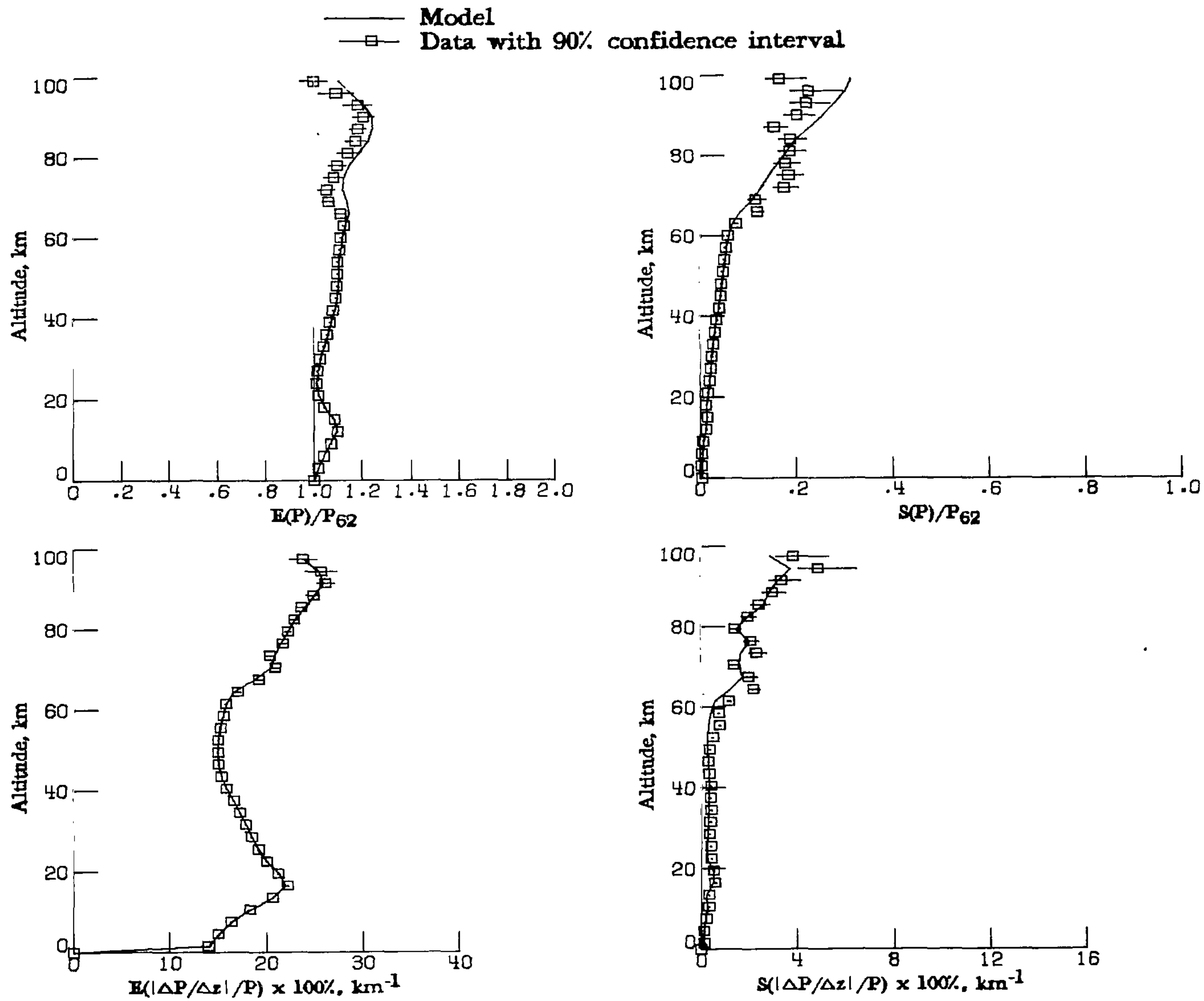

Figure 4.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the 15° latitude category.

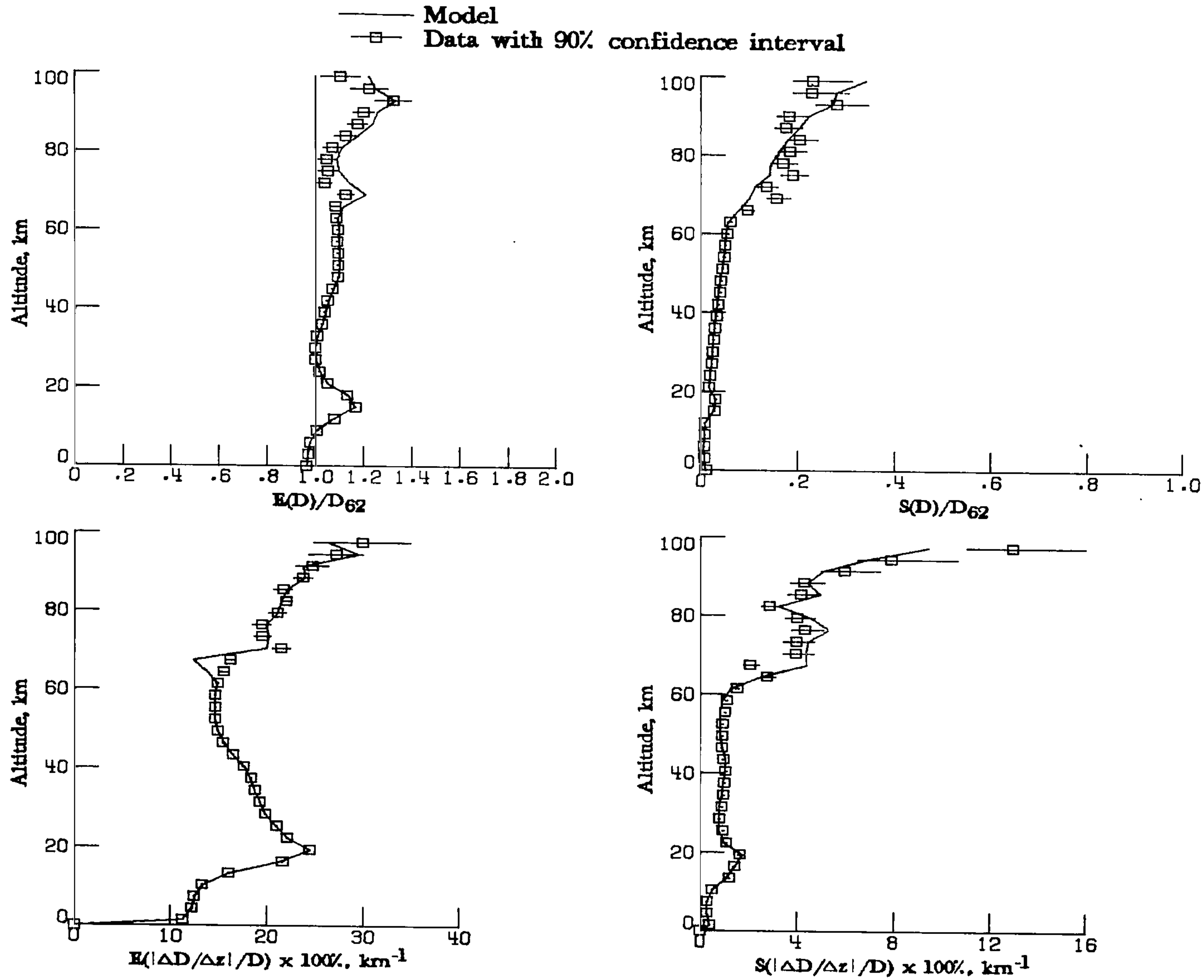

Figure 5.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the 15° latitude category.

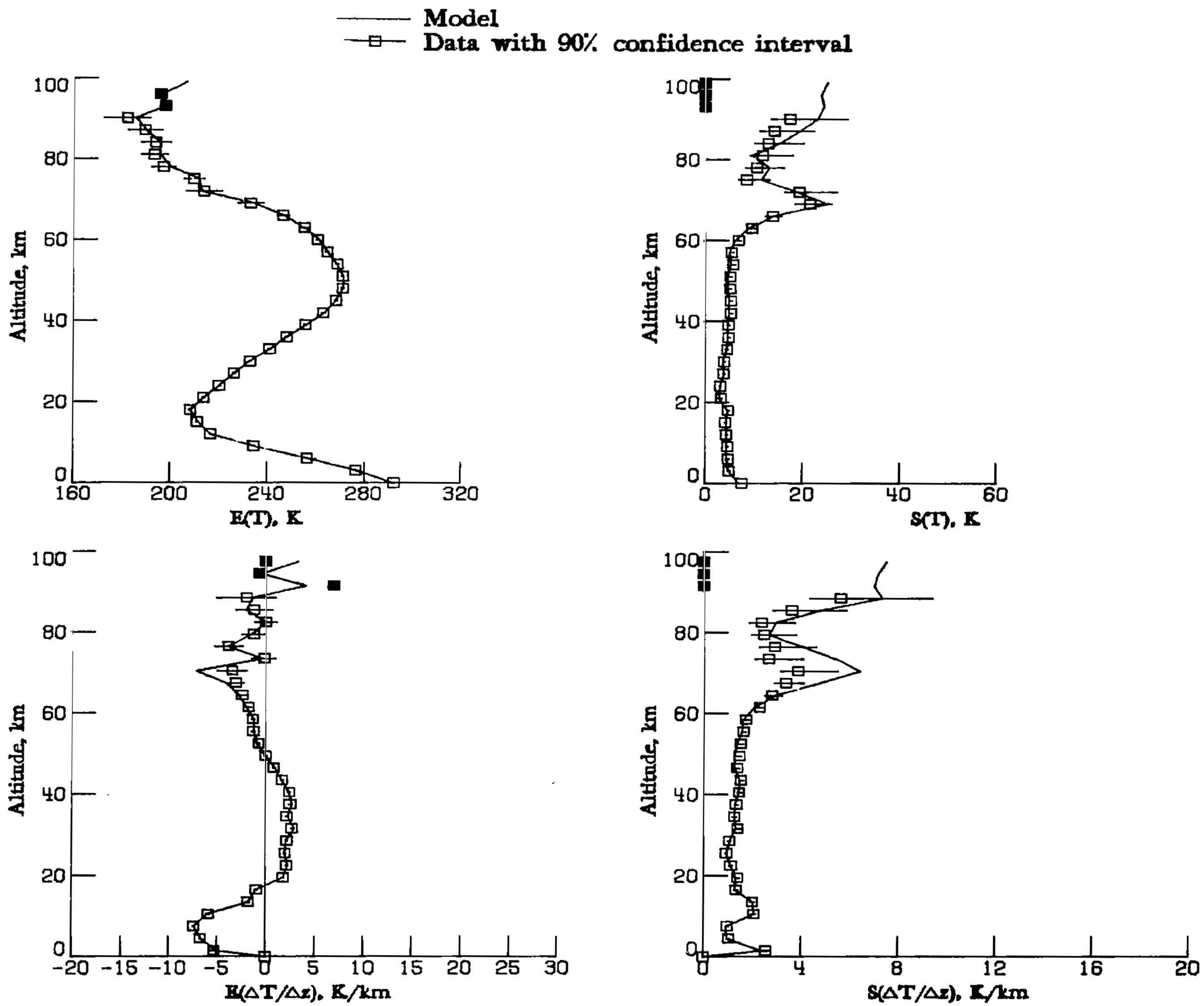

Figure 6.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the spring, 30° latitude category.

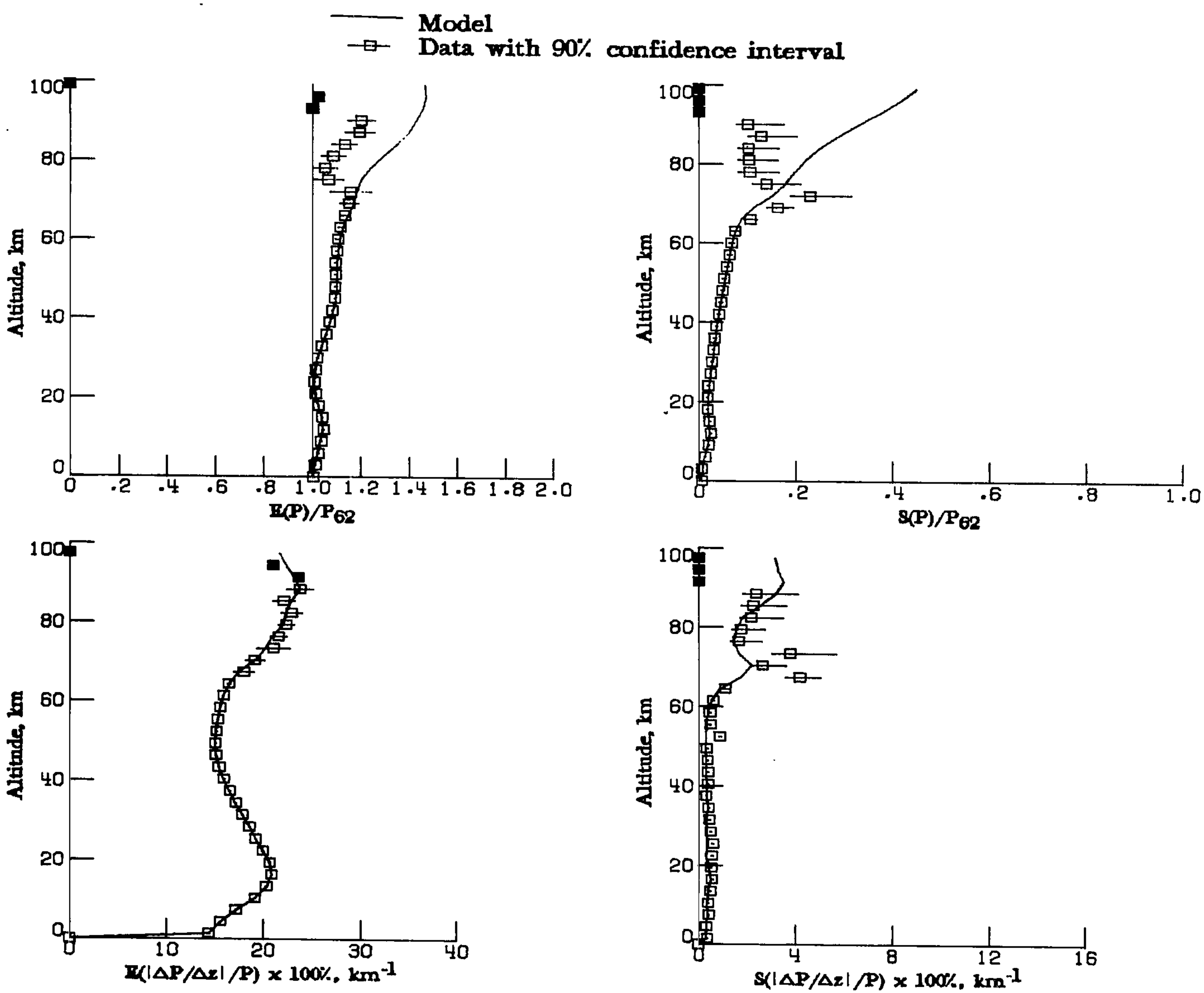

Figure 7.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the spring, 30° latitude category.

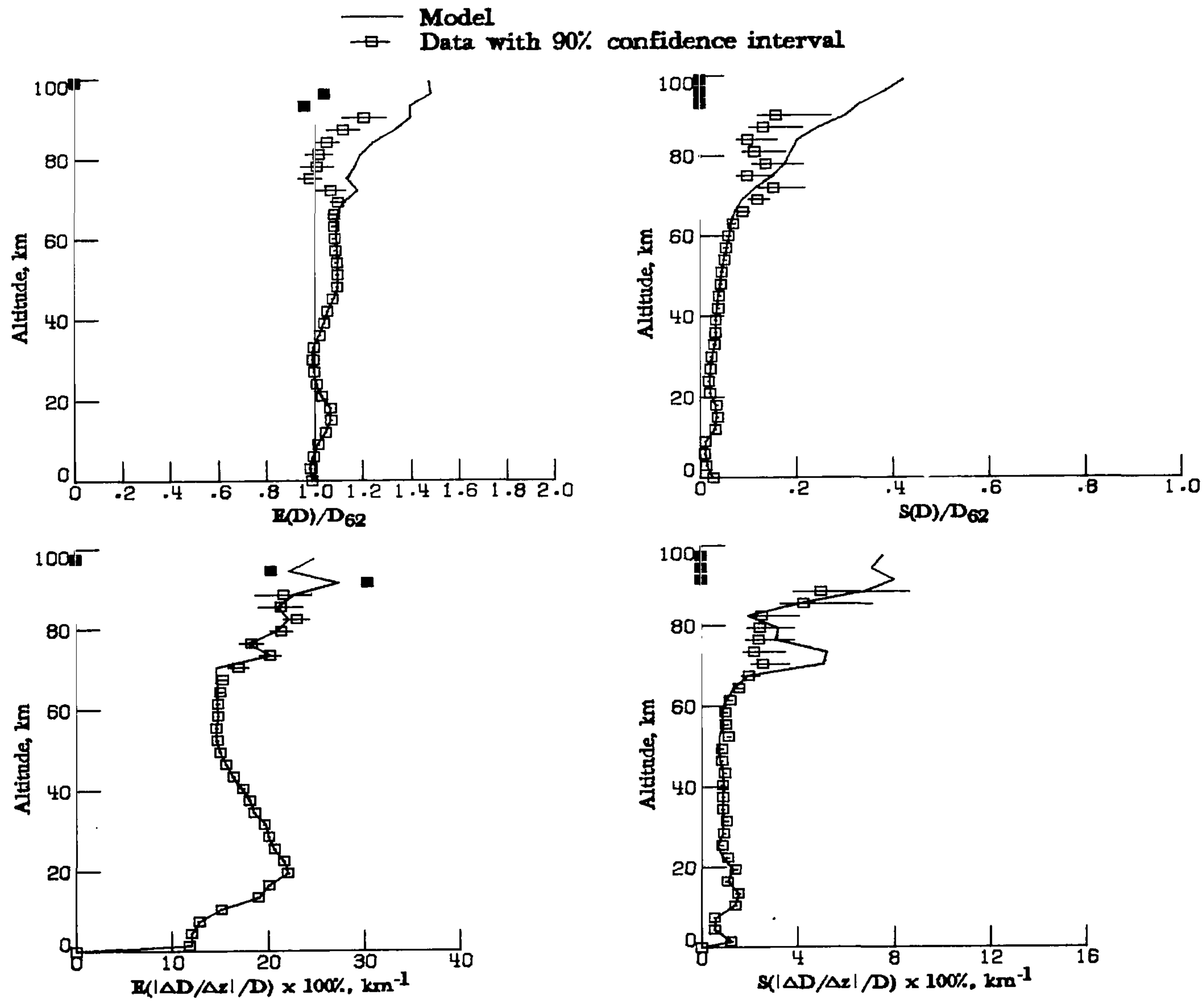

Figure 8.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the spring, 30° latitude category.

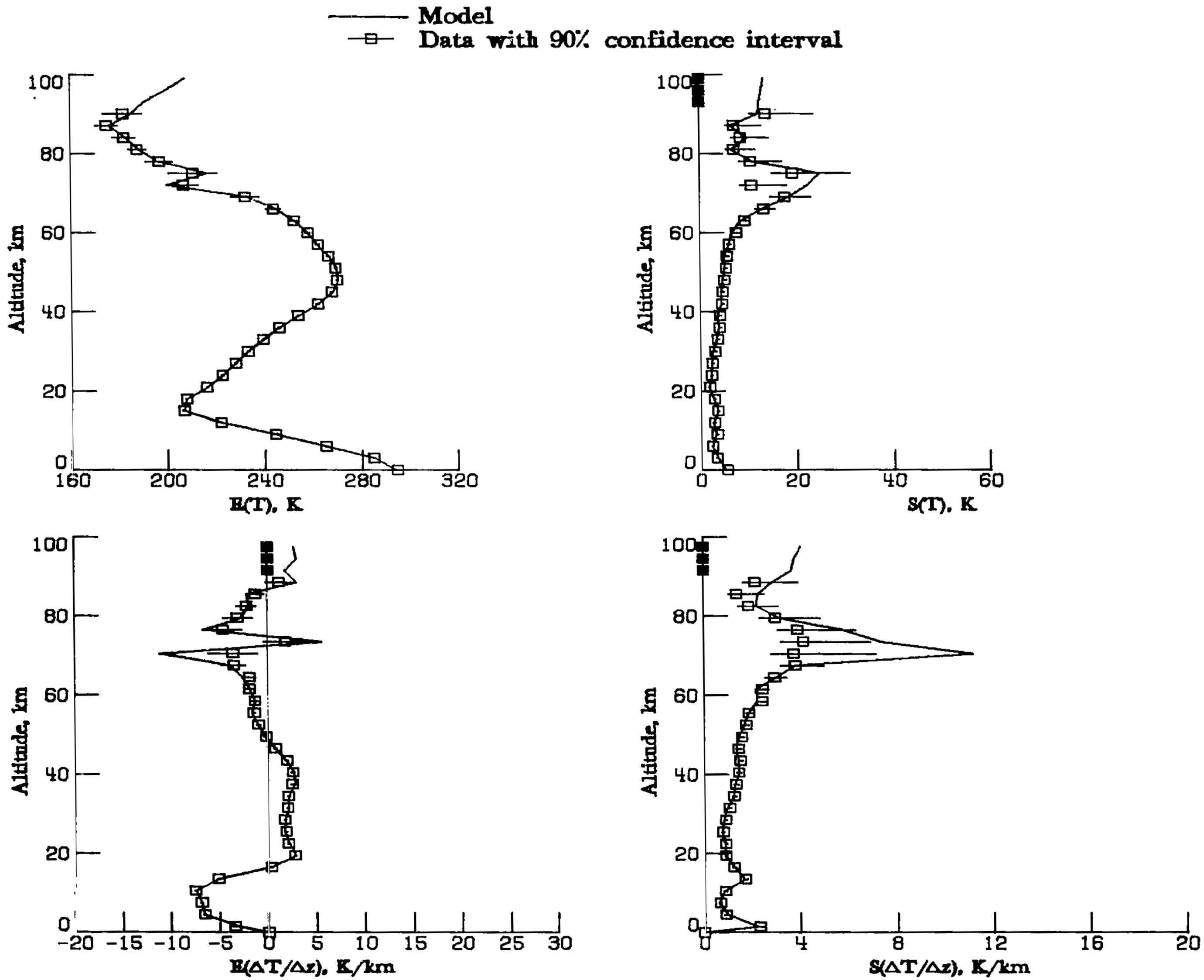

Figure 9.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the summer, 30° latitude category.

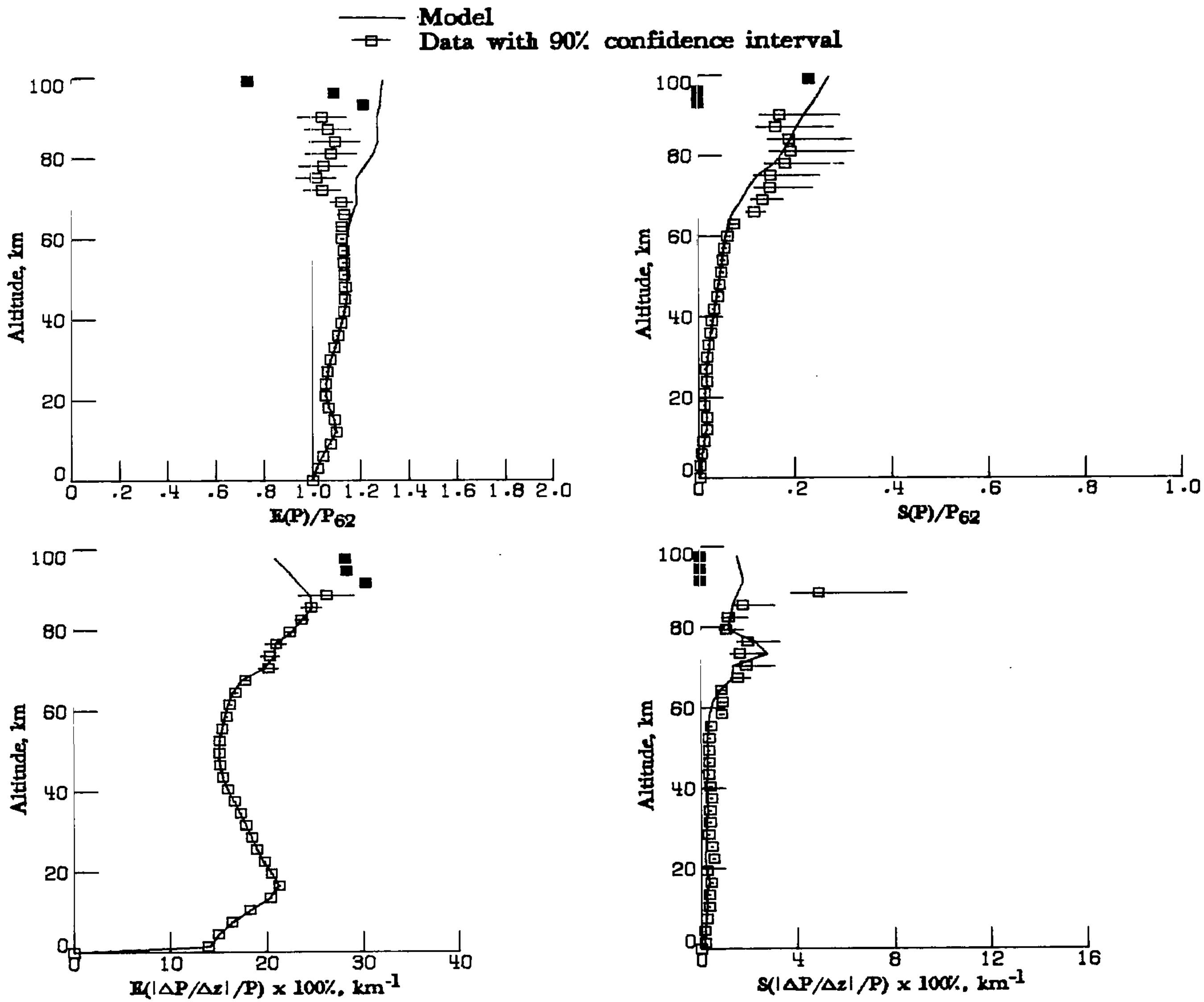

Figure 10.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the summer, 30° latitude category.

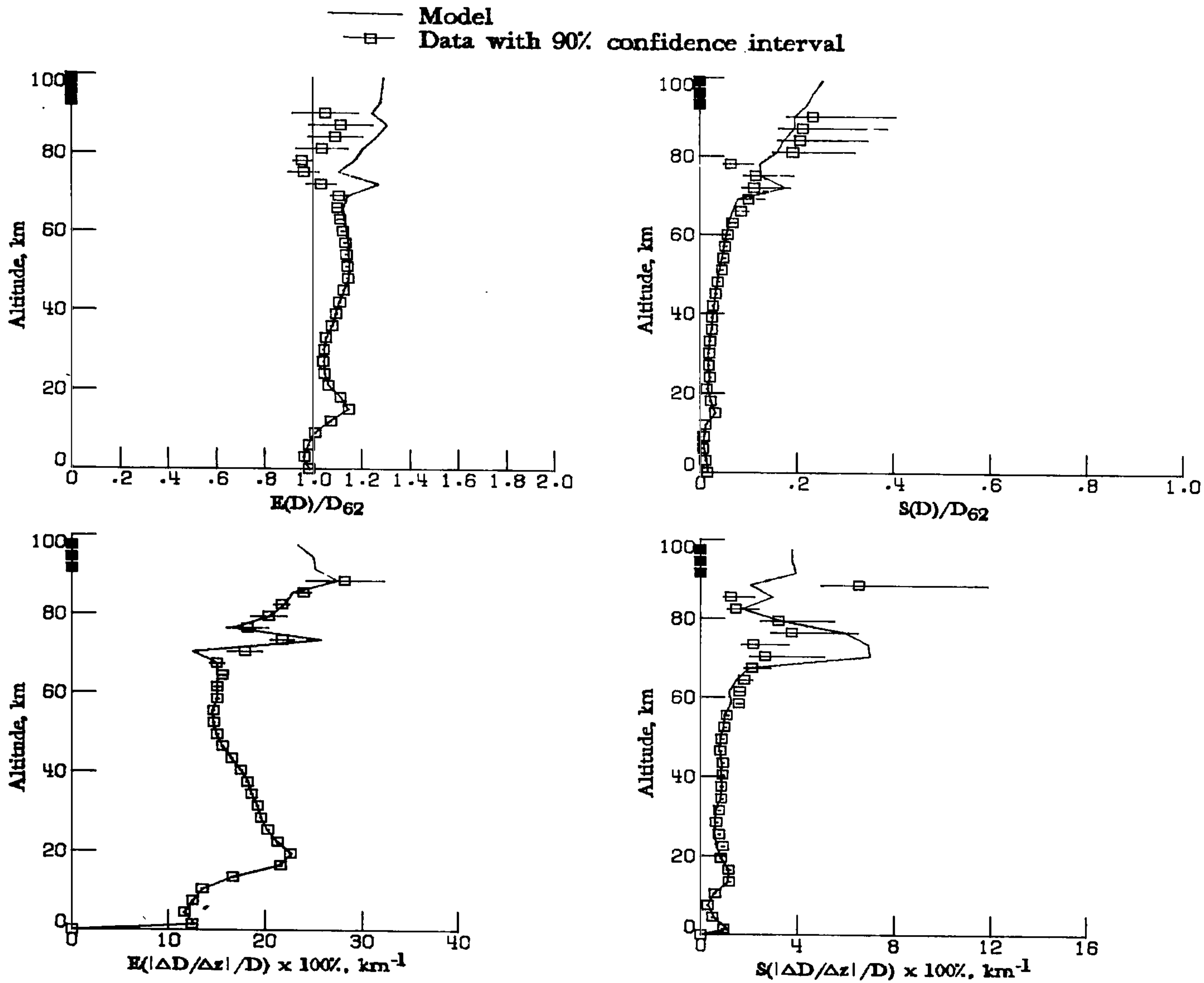

Figure 11.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the summer, 30° latitude category.

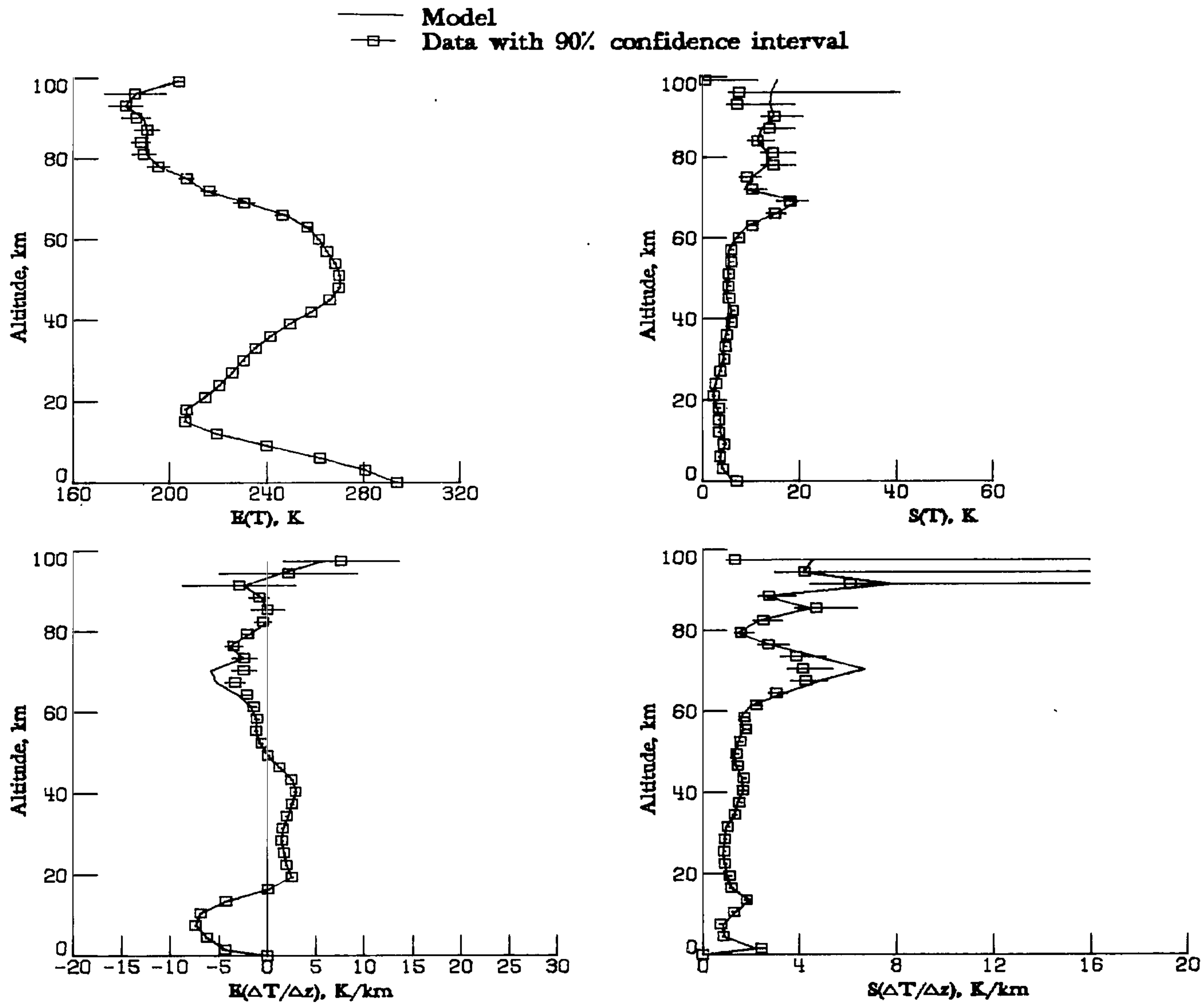

Figure 12.- Comparison of model and data means and standard deviations of T and ΔT/Δz in the autumn, 30° latitude category.

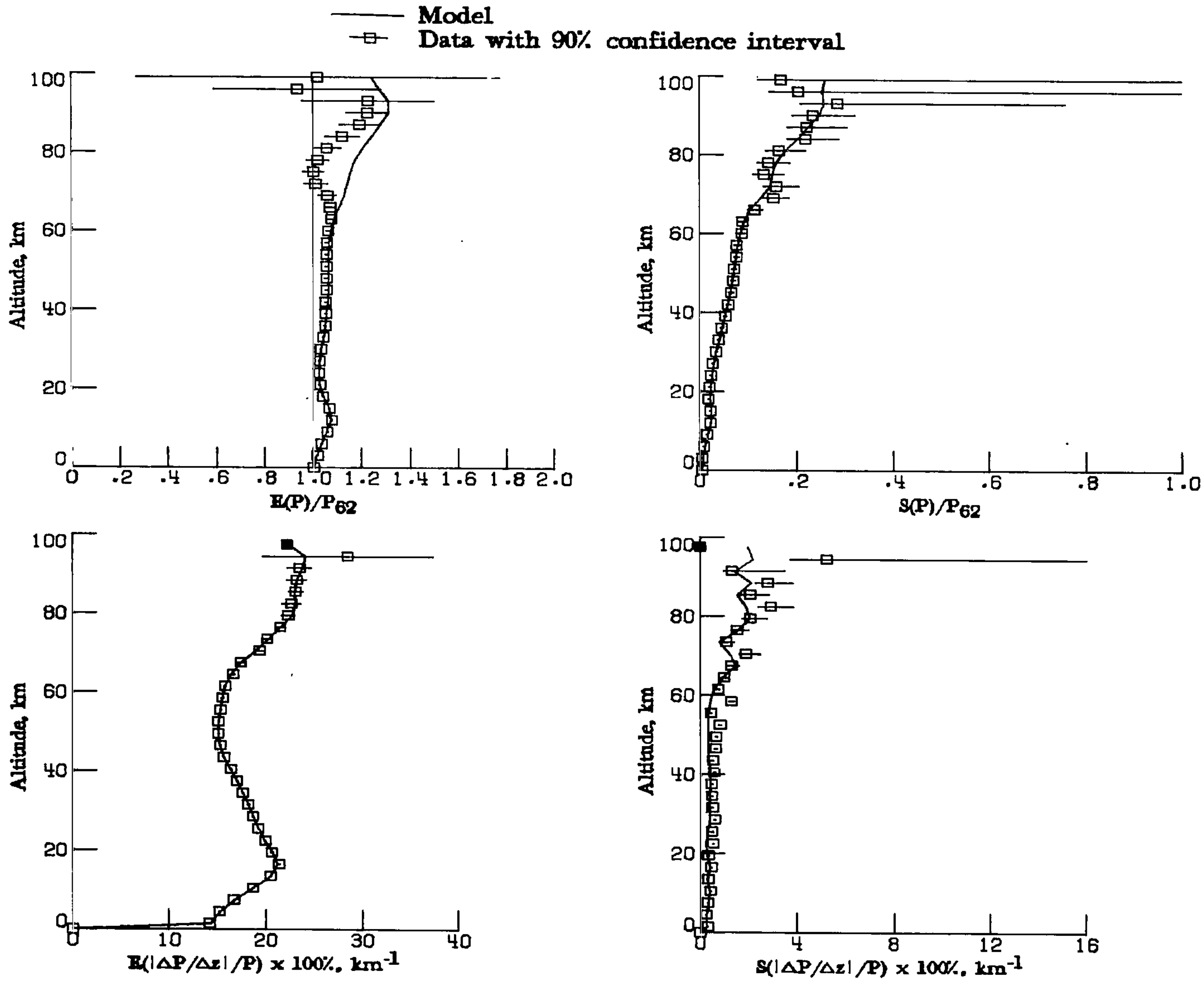

Figure 13.- Comparison of model and data means and standard deviations of P and ΔP/Δz in the autumn, 30° latitude category.

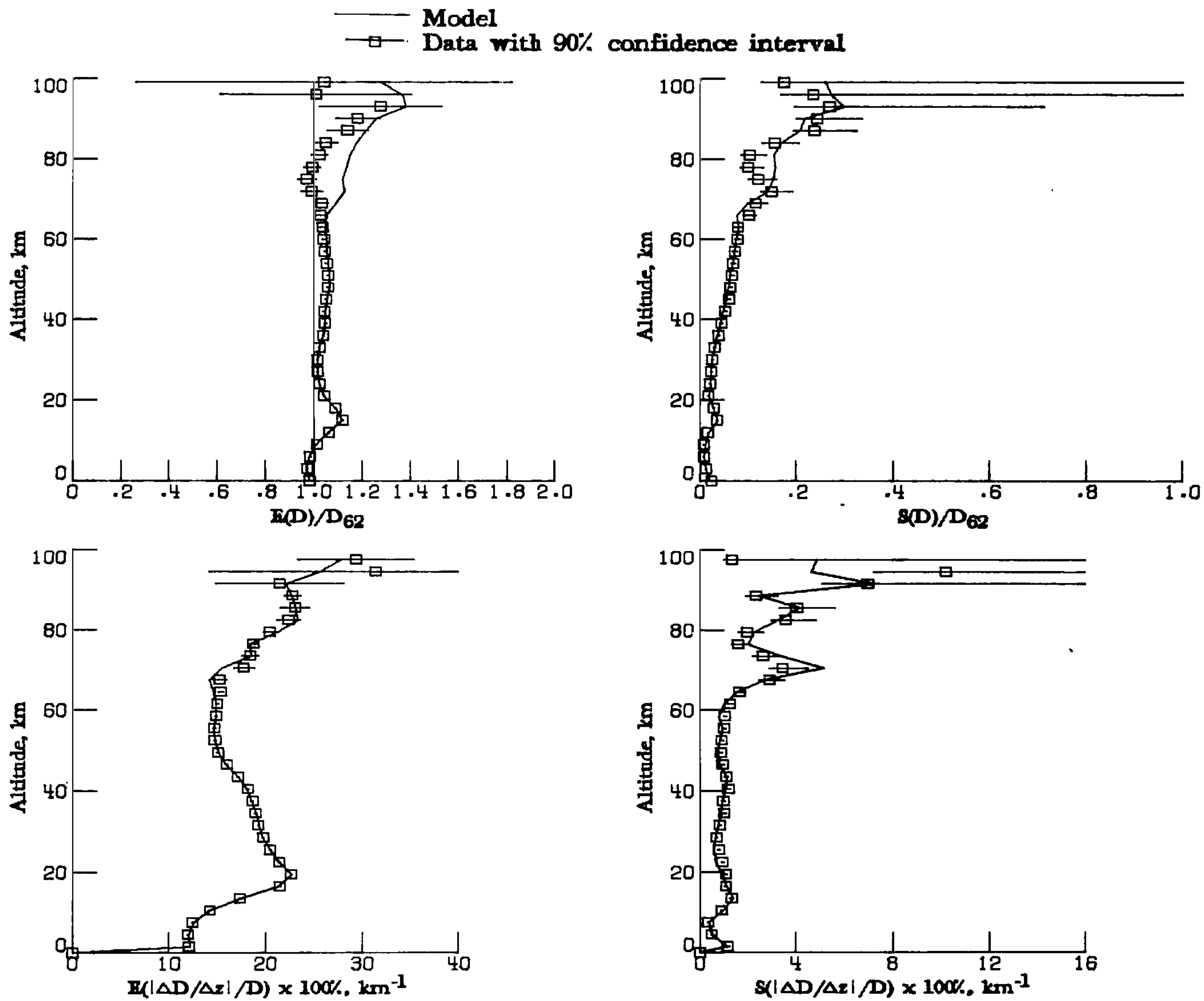

Figure 14.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the autumn, 30° latitude category.

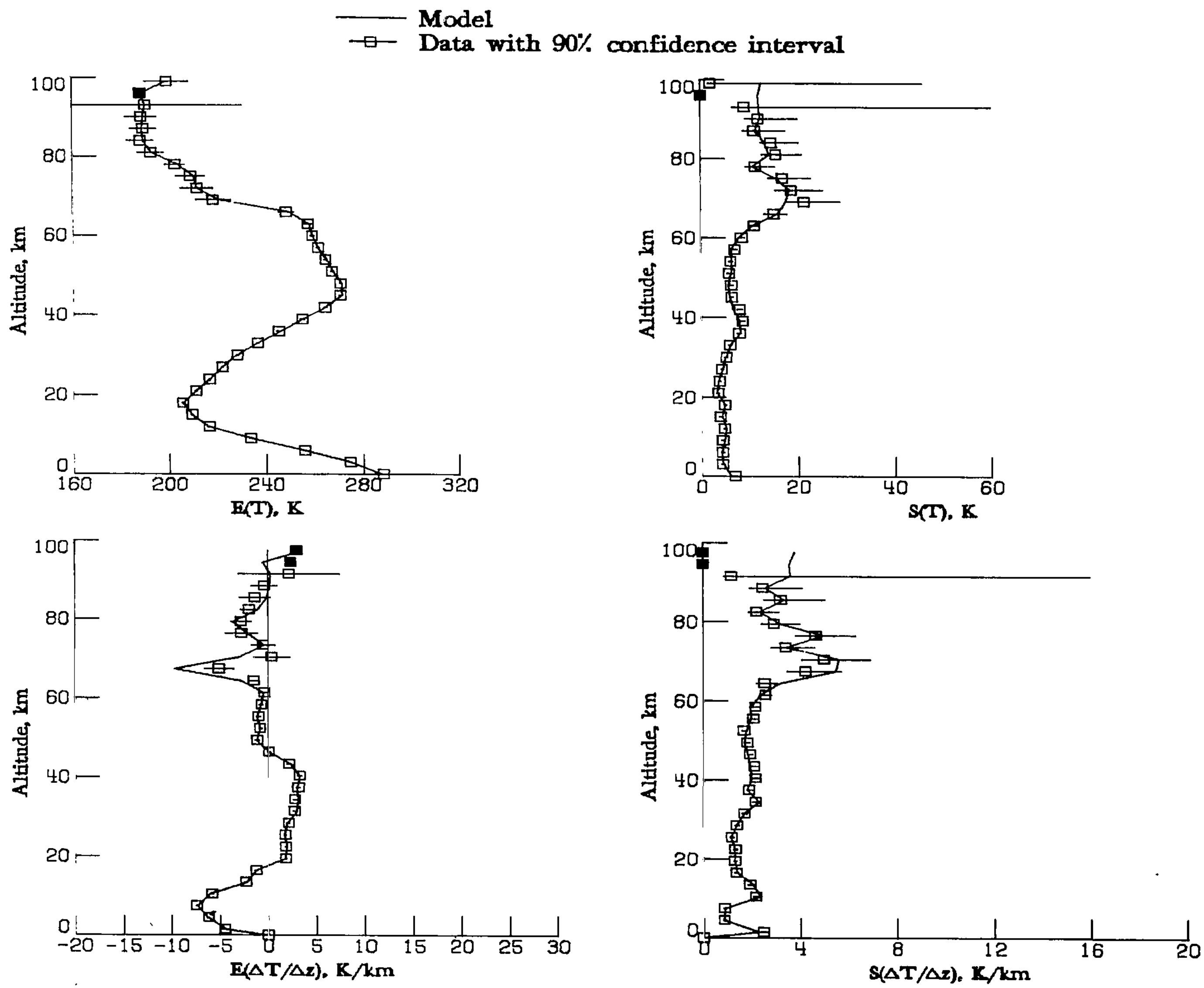

Figure 15.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the winter, 30° latitude category.

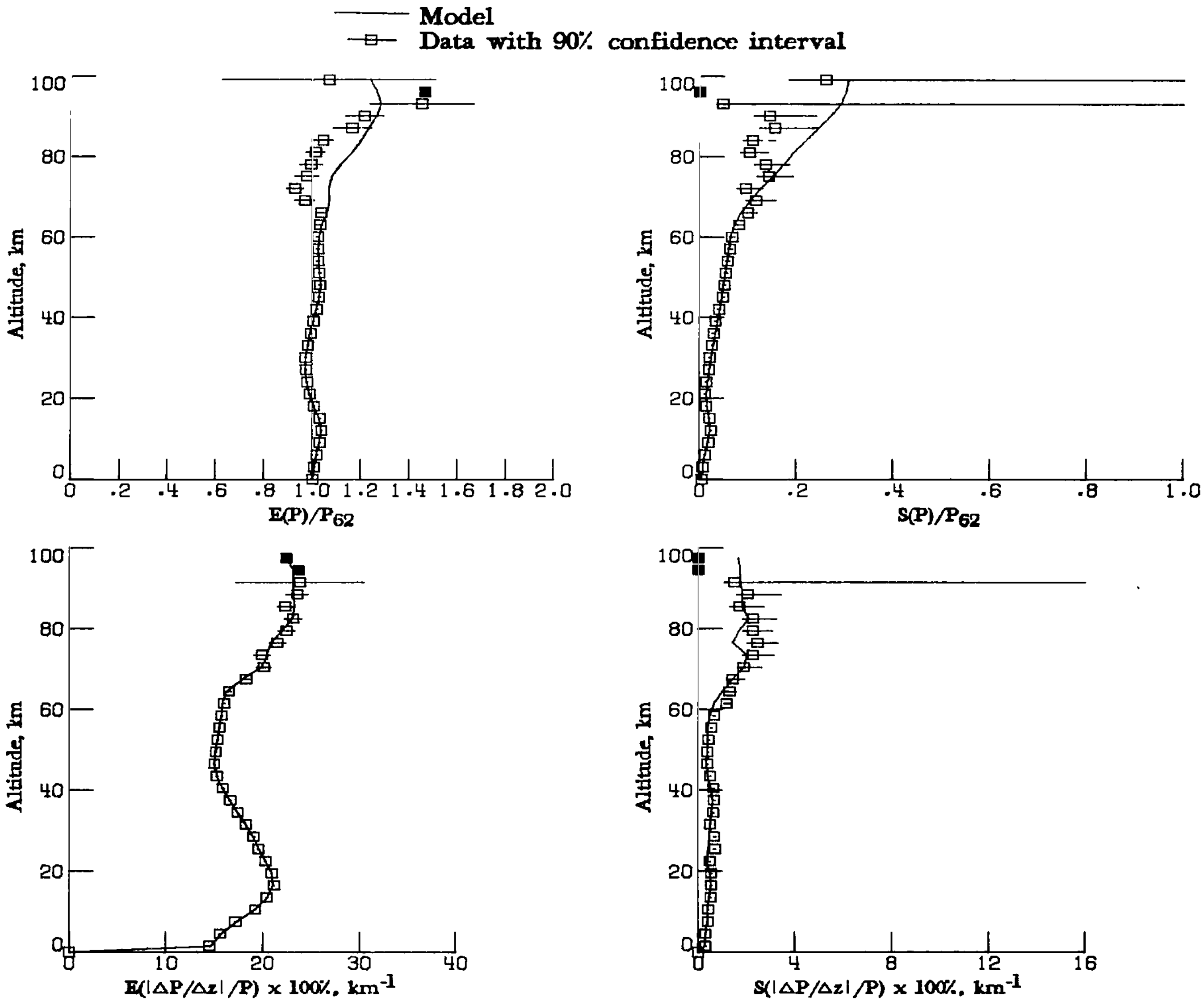

Figure 16.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the winter, 30° latitude category.

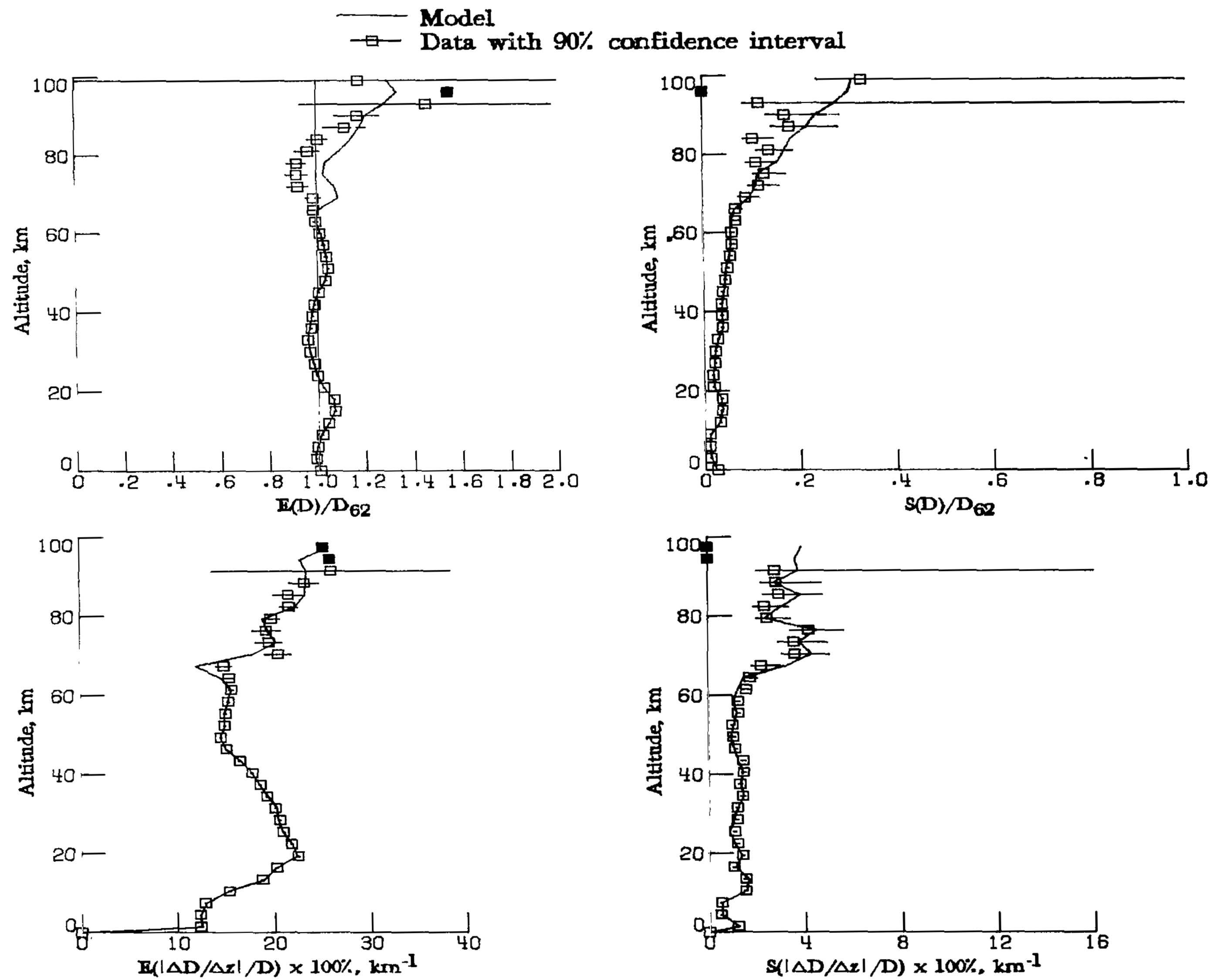

Figure 17.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the winter, 30° latitude category.

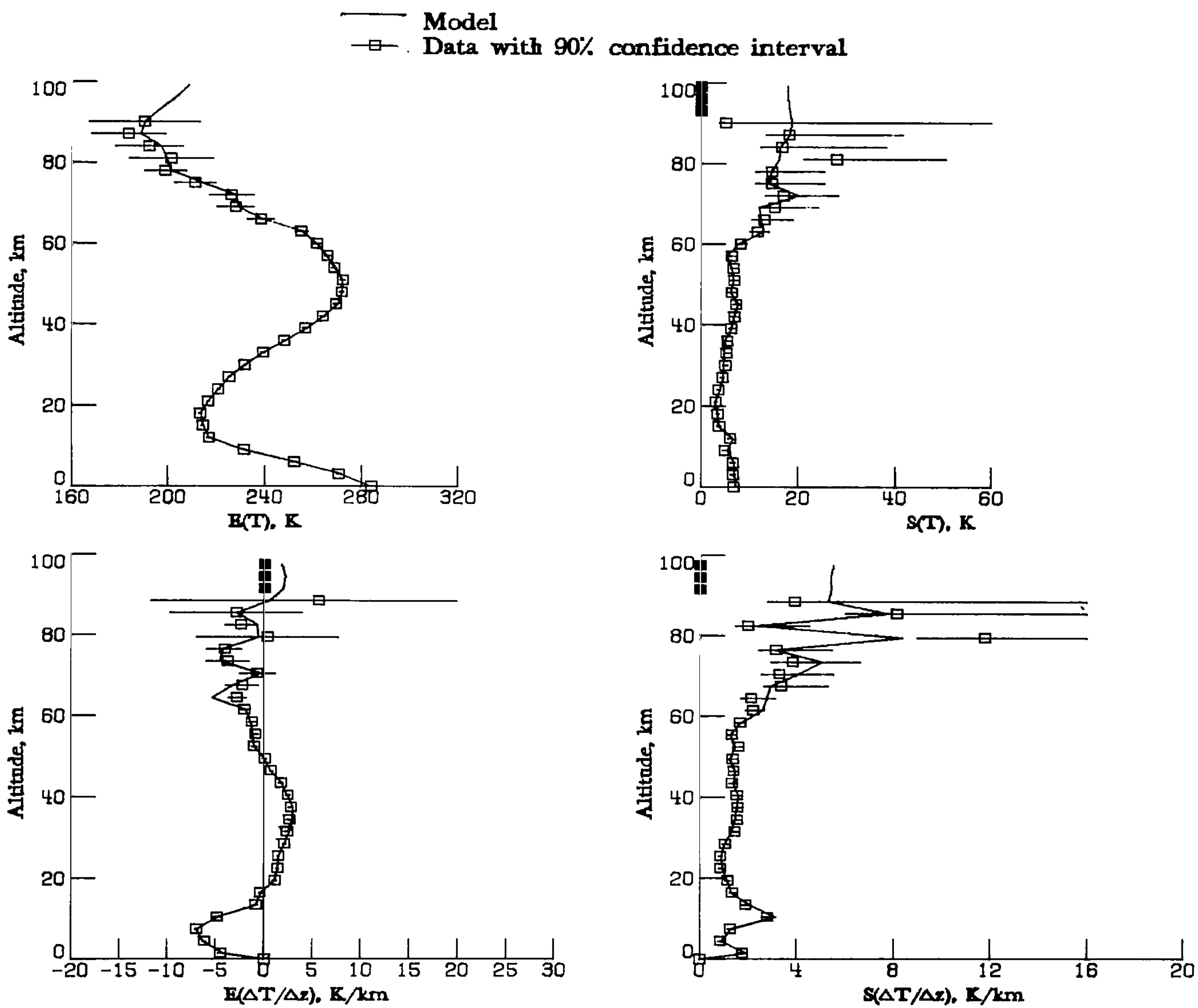

Figure 18.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the spring, 45° latitude category.

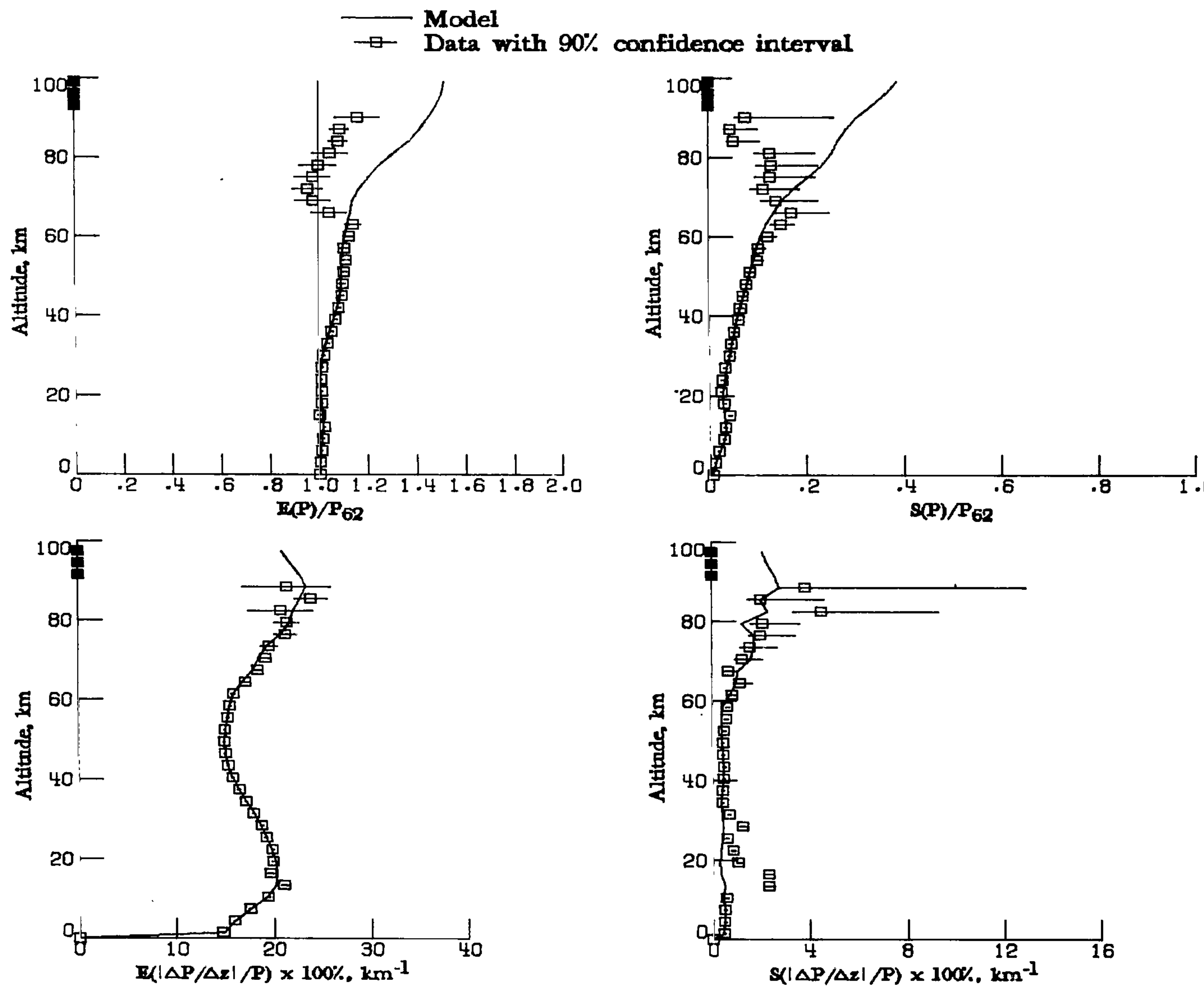

Figure 19.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the spring, 45° latitude category.

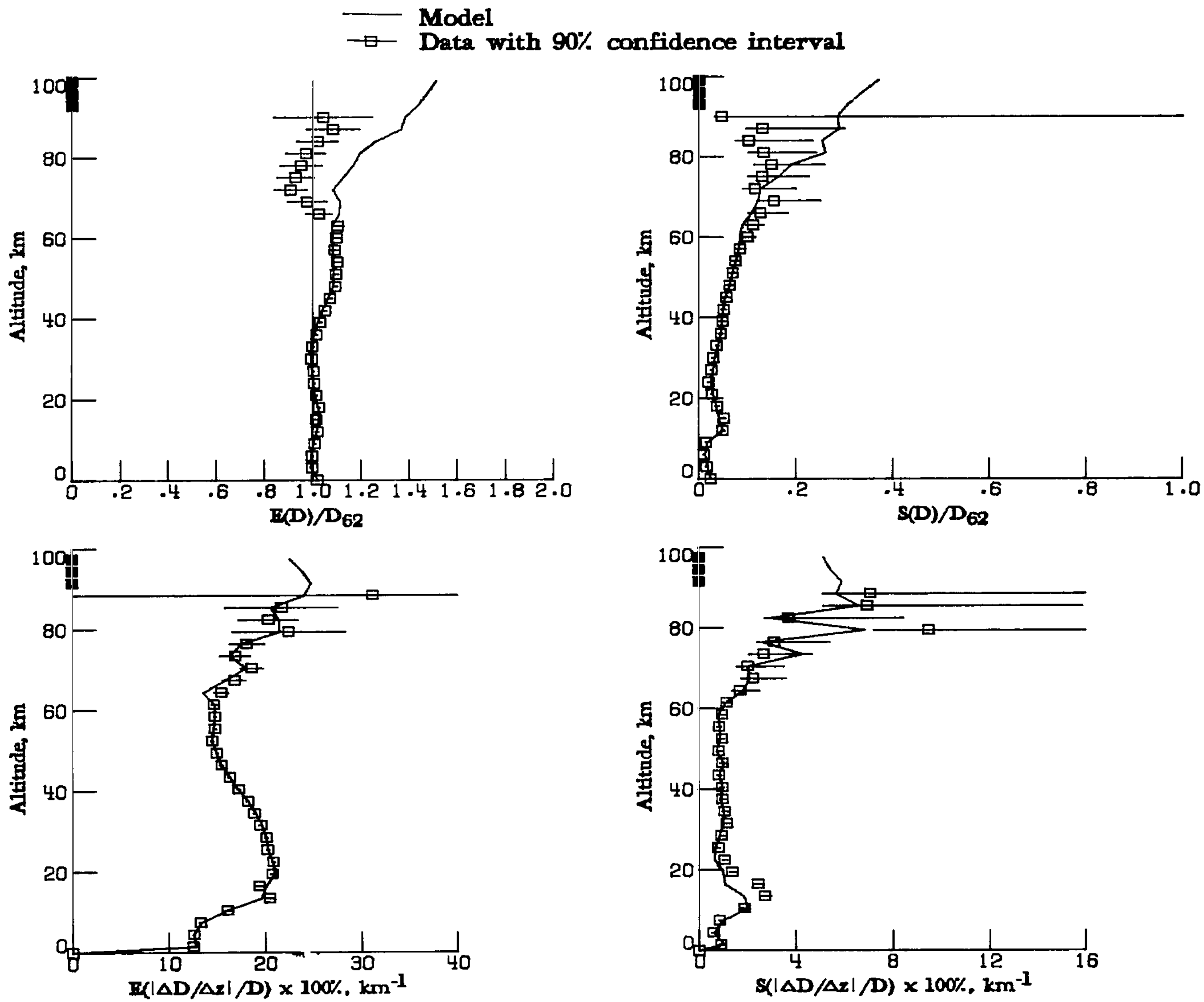

Figure 20.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the spring, 45° latitude category.

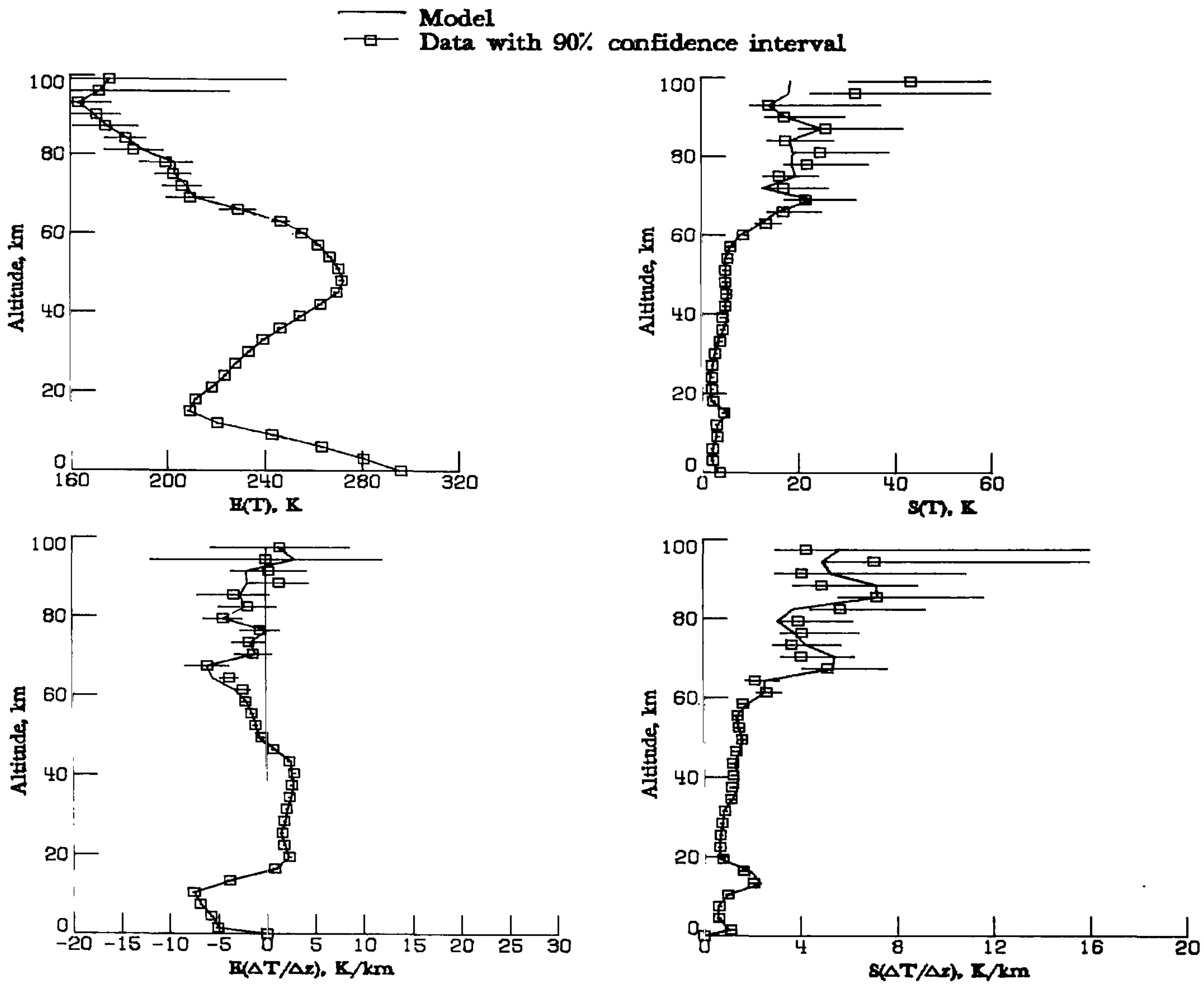

Figure 21.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the summer, 45° latitude category.

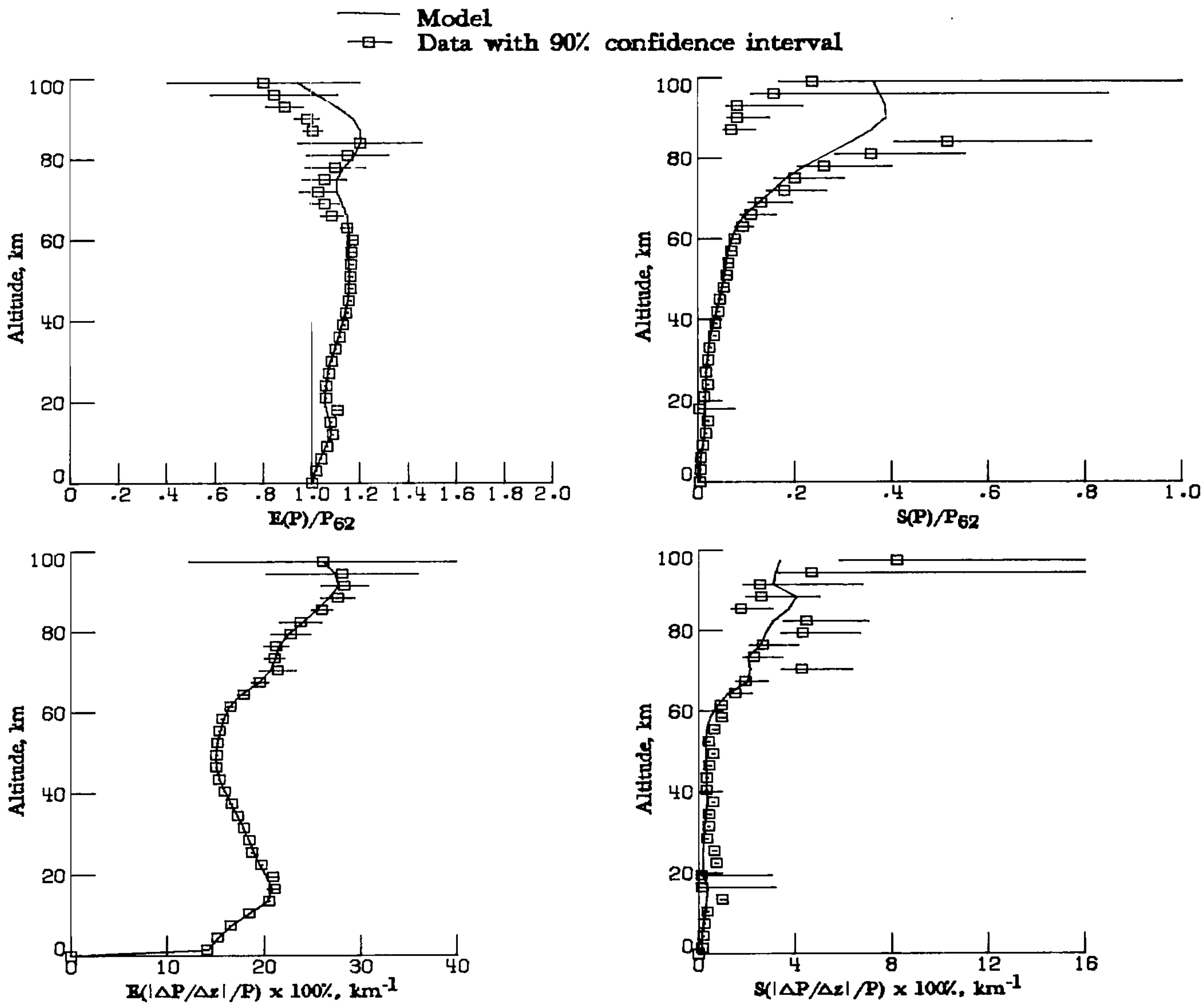

Figure 22.- Comparison of model and data means and standard deviations of P and ΔP/Δz in the summer, 45° latitude category.

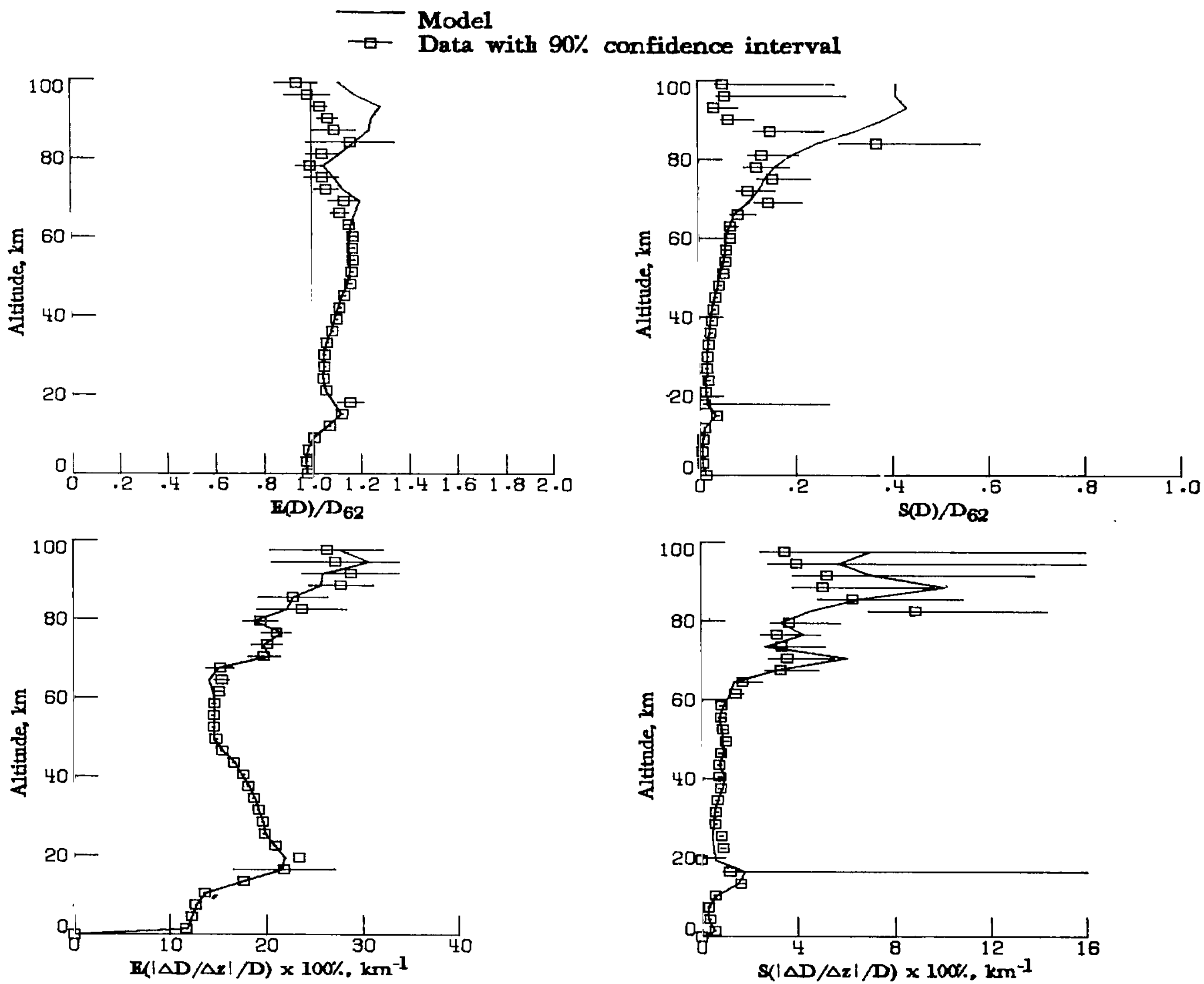

Figure 23.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the summer, 45° latitude category.

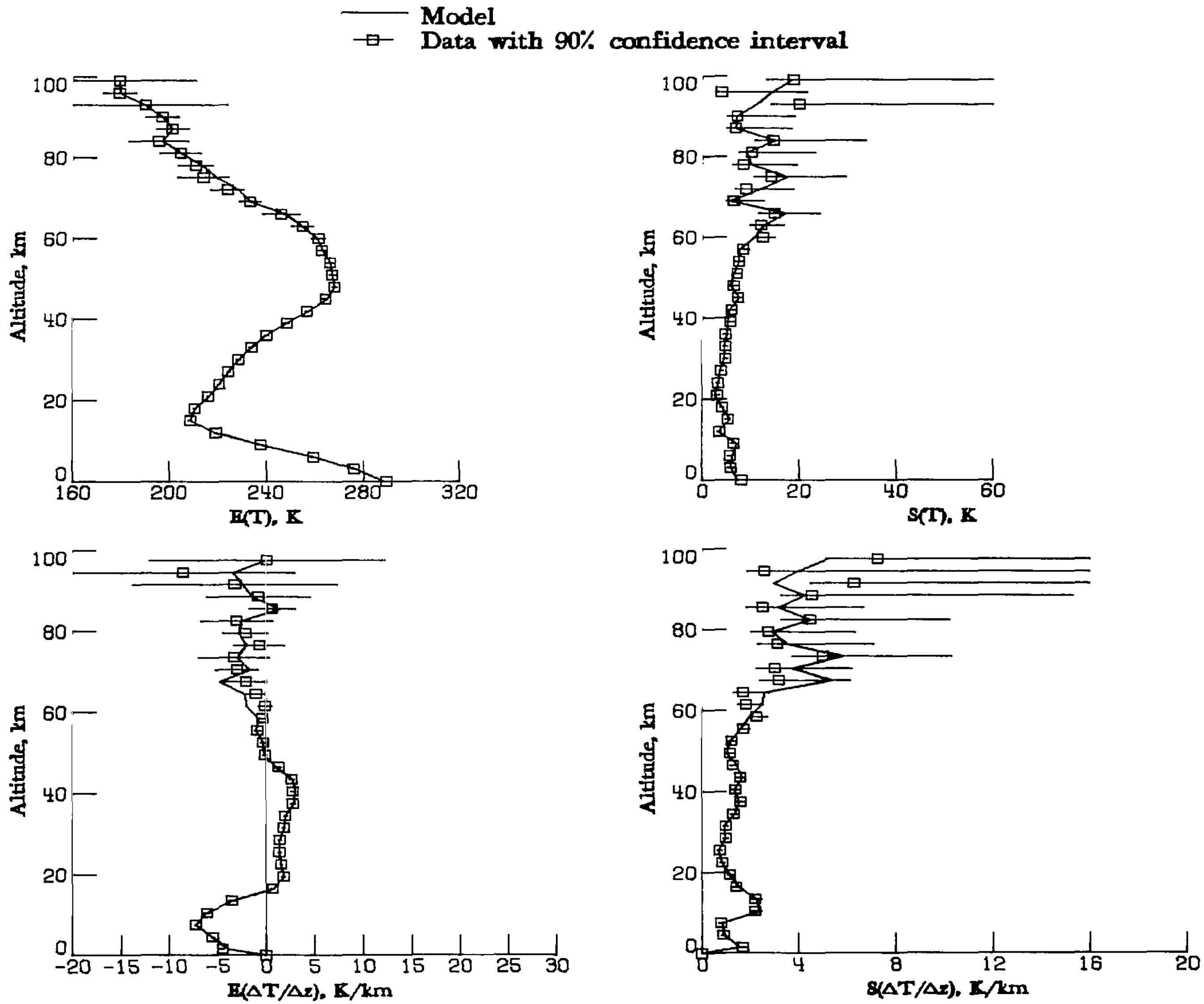

Figure 24.- Comparison of model and data means and standard deviations of T and ΔT/Δz in the autumn, 45° latitude category.

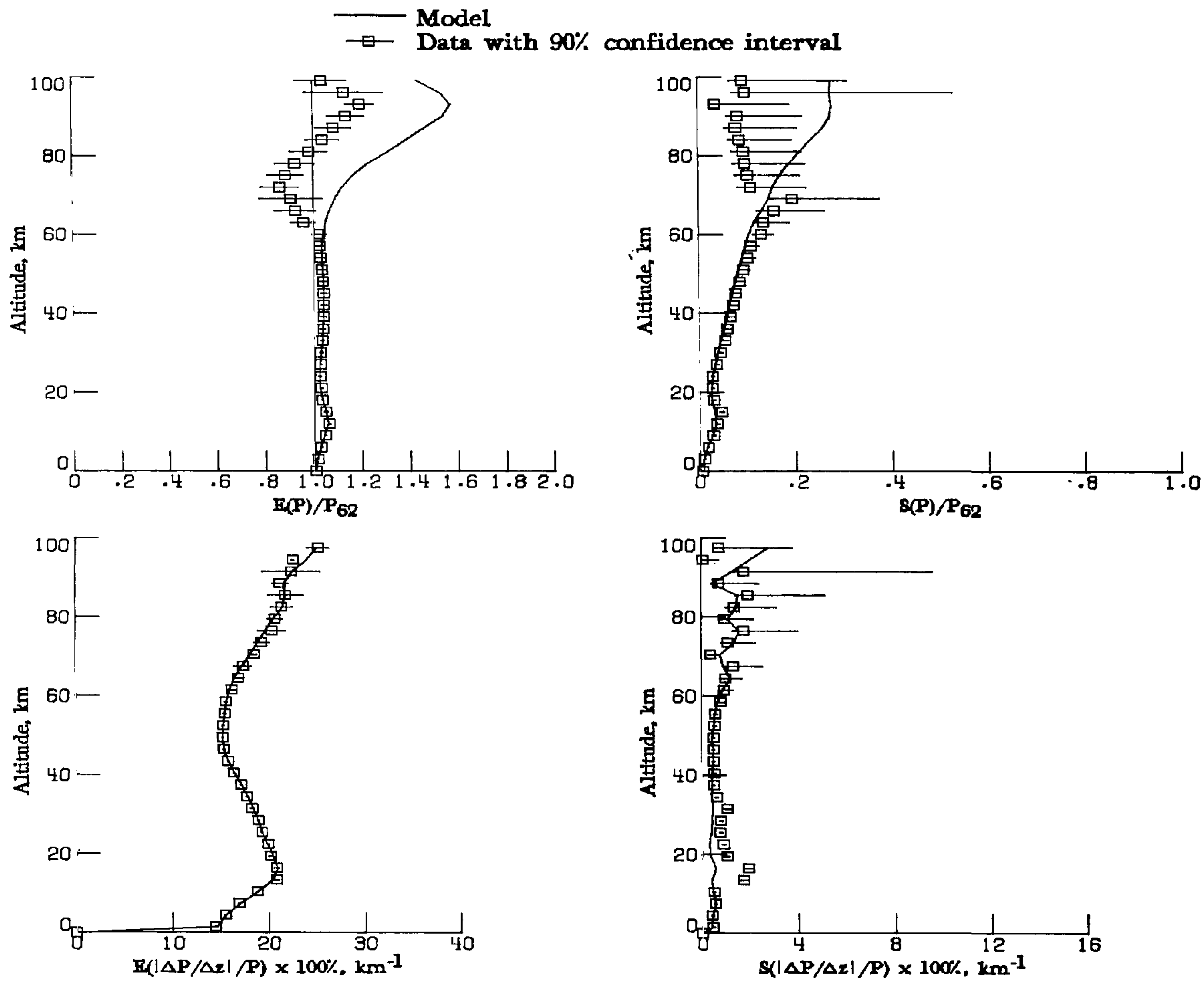

Figure 25.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the autumn, 45^{o} latitude category.

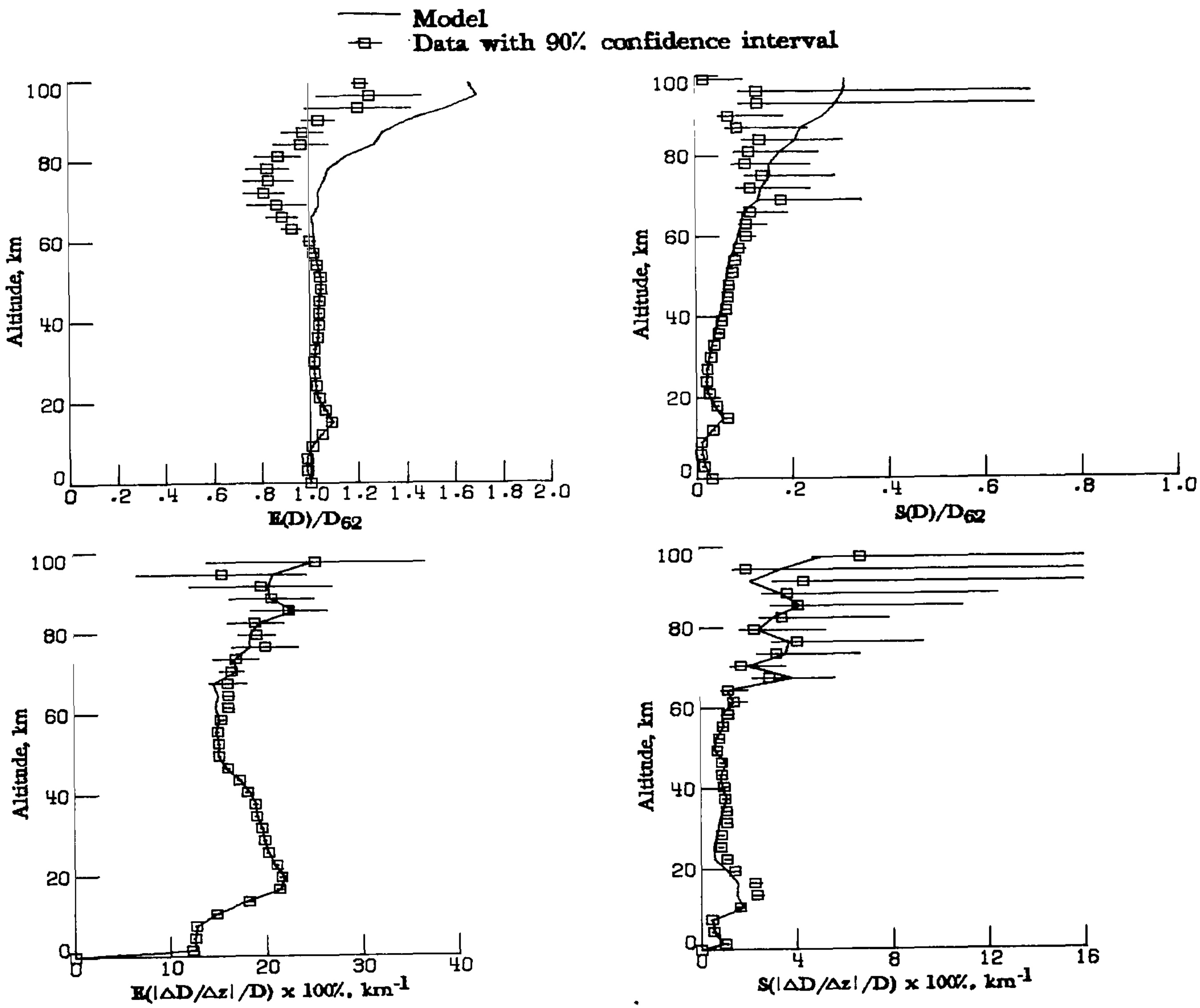

Figure 26.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the autumn, 45° latitude category.

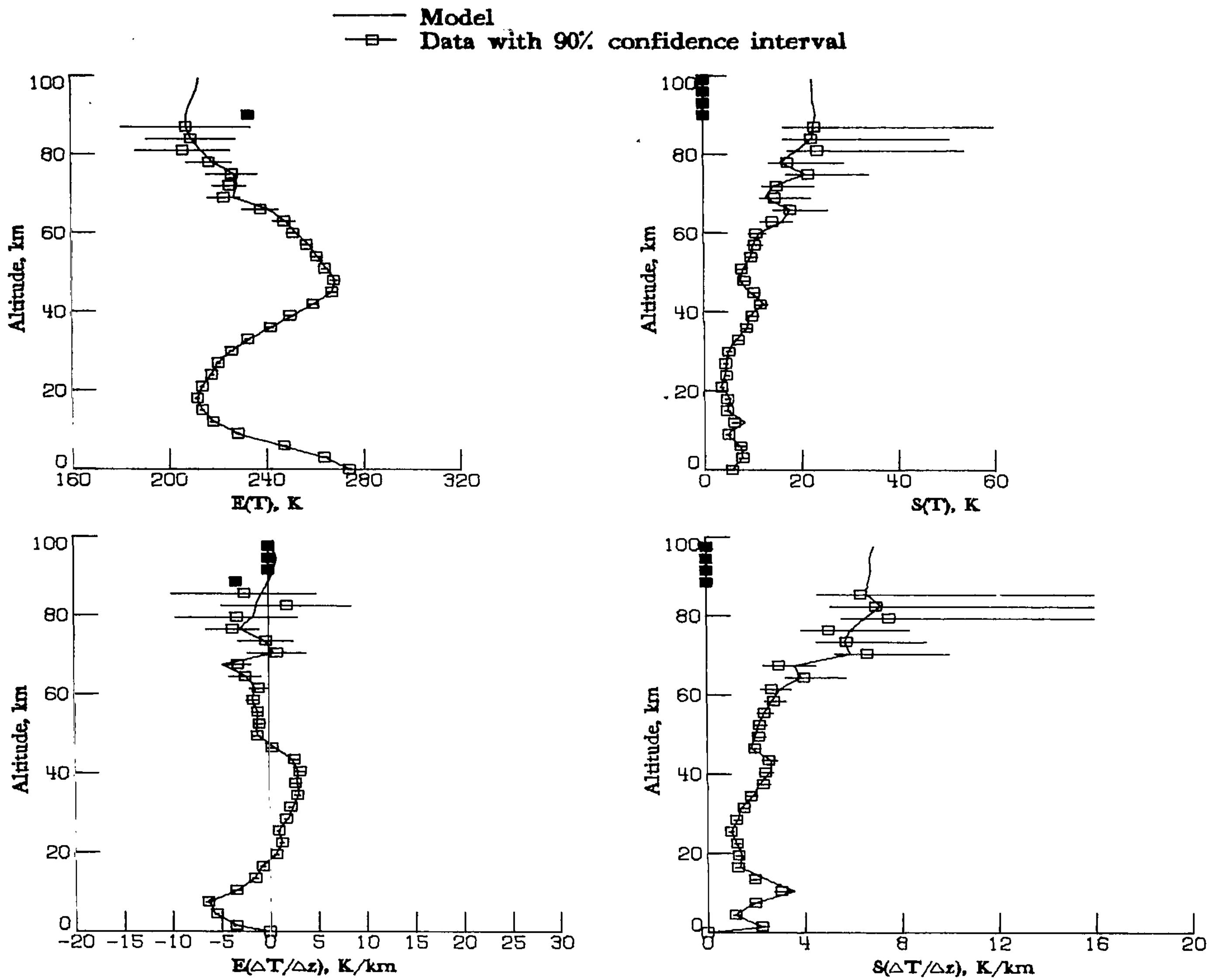

Figure 27.- Comparison of model and data means and standard deviations of T and ΔT/Δz in the winter, 45° latitude category.

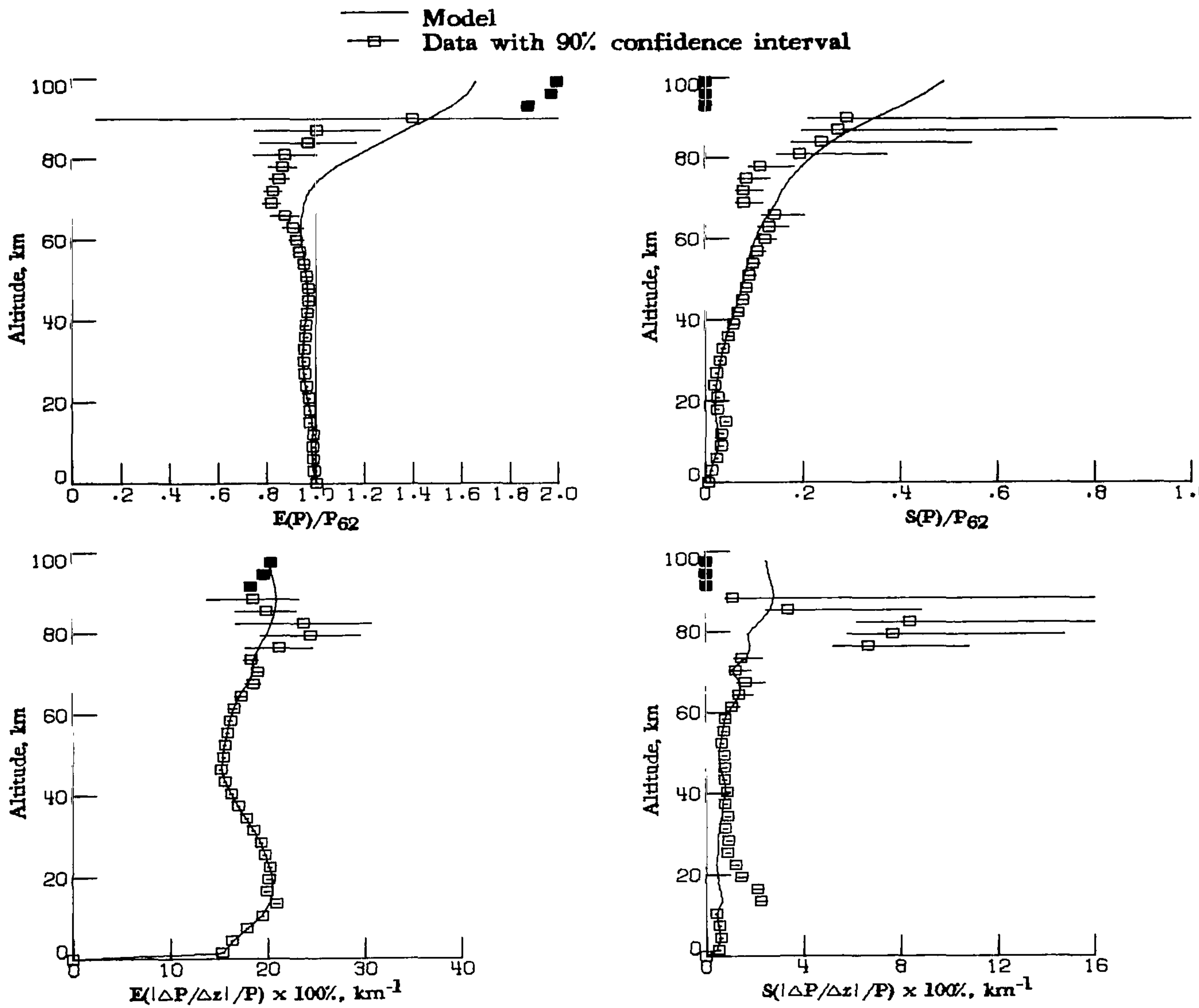

Figure 28.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the winter, 45° latitude category.

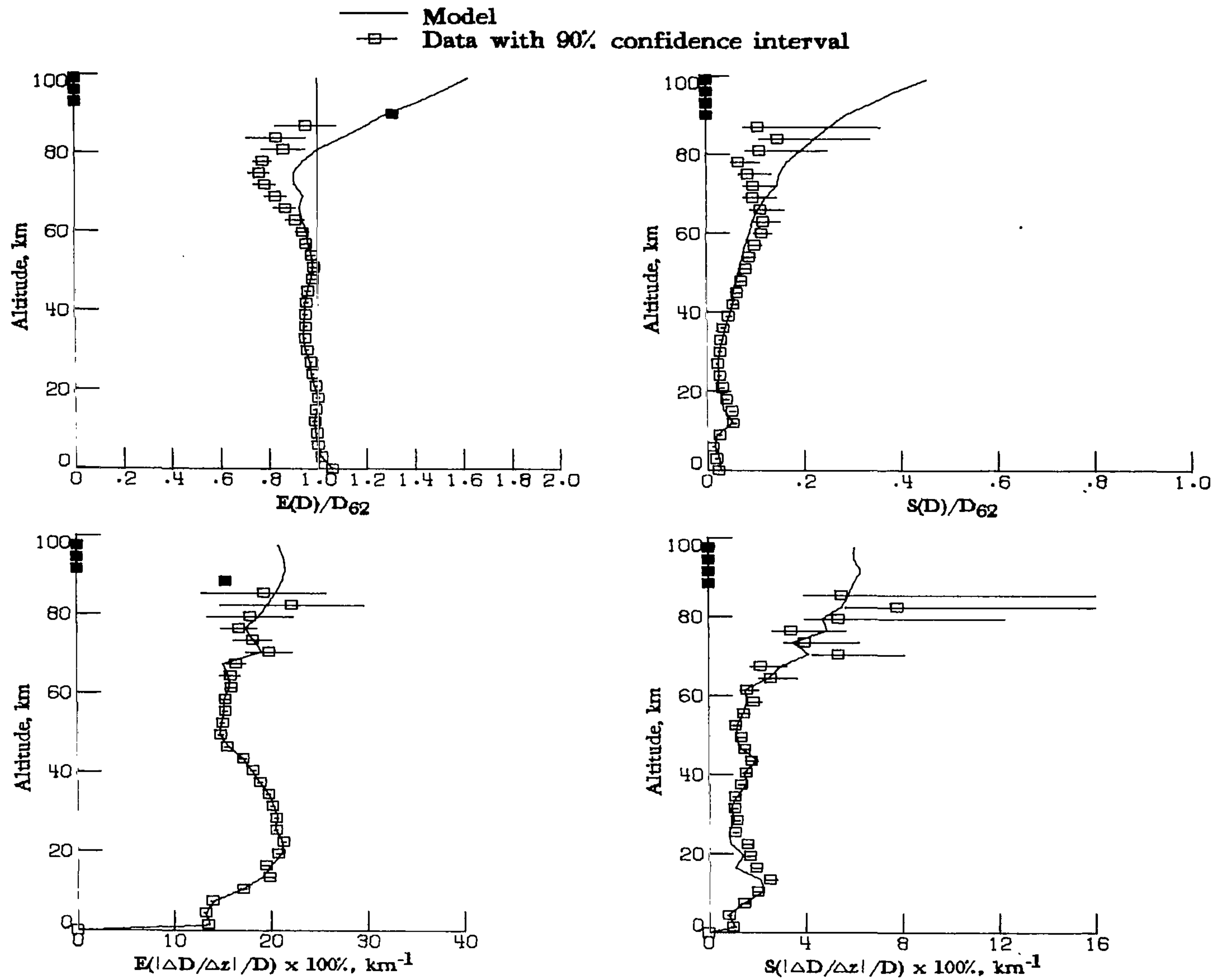

Figure 29.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the winter, 45° latitude category.

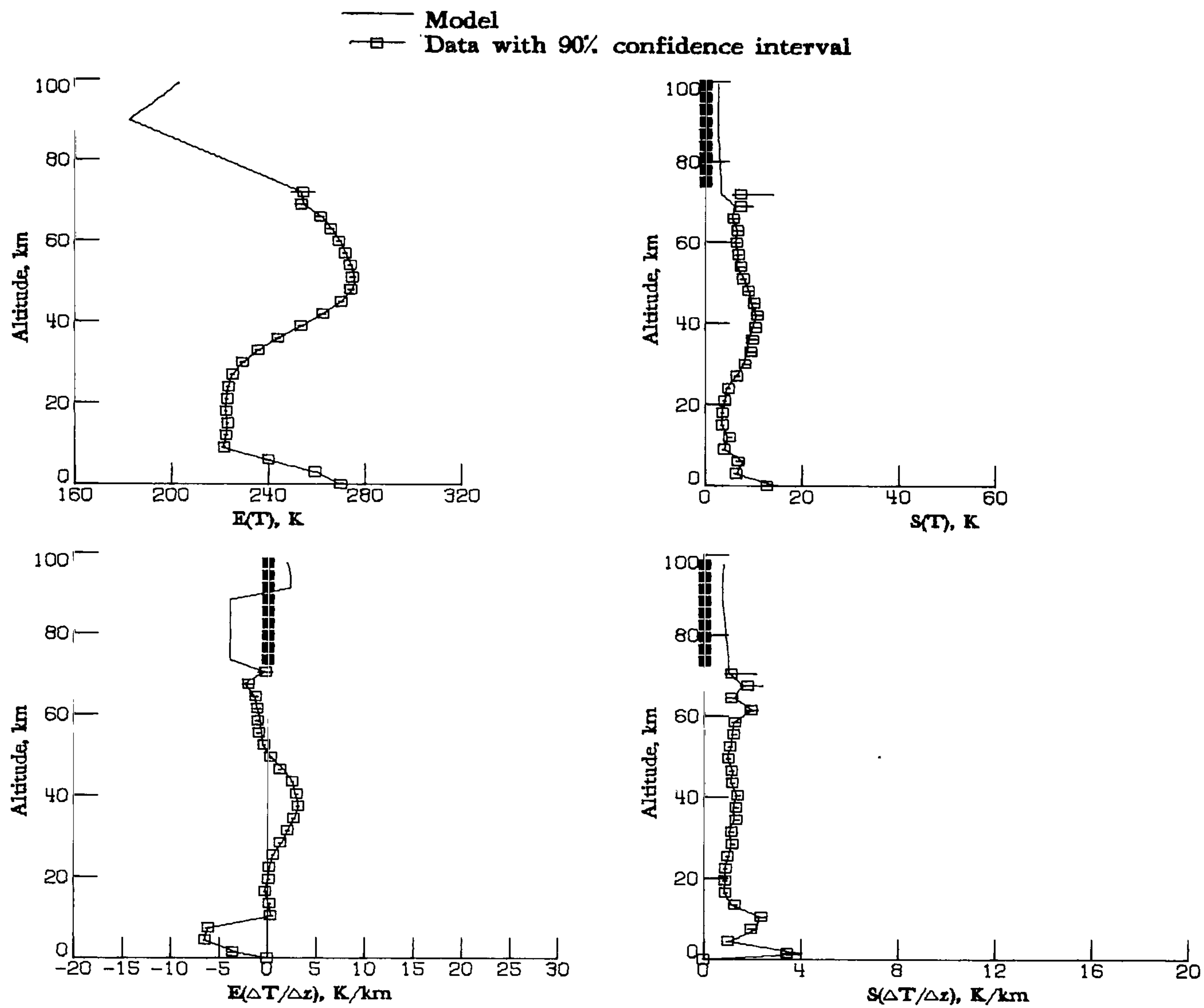

Figure 30.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the spring, 60° latitude category.

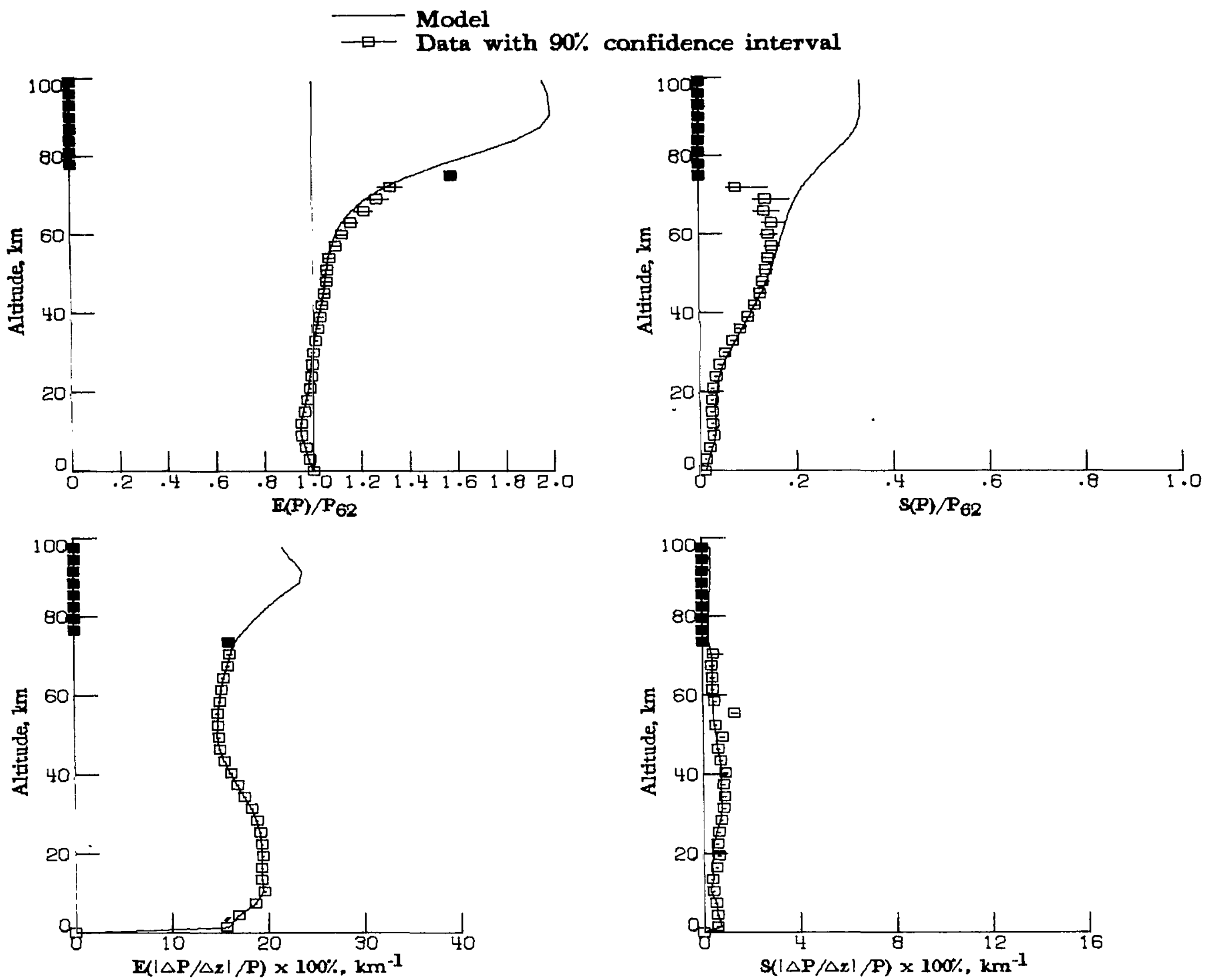

Figure 31.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the spring, 60° latitude category.

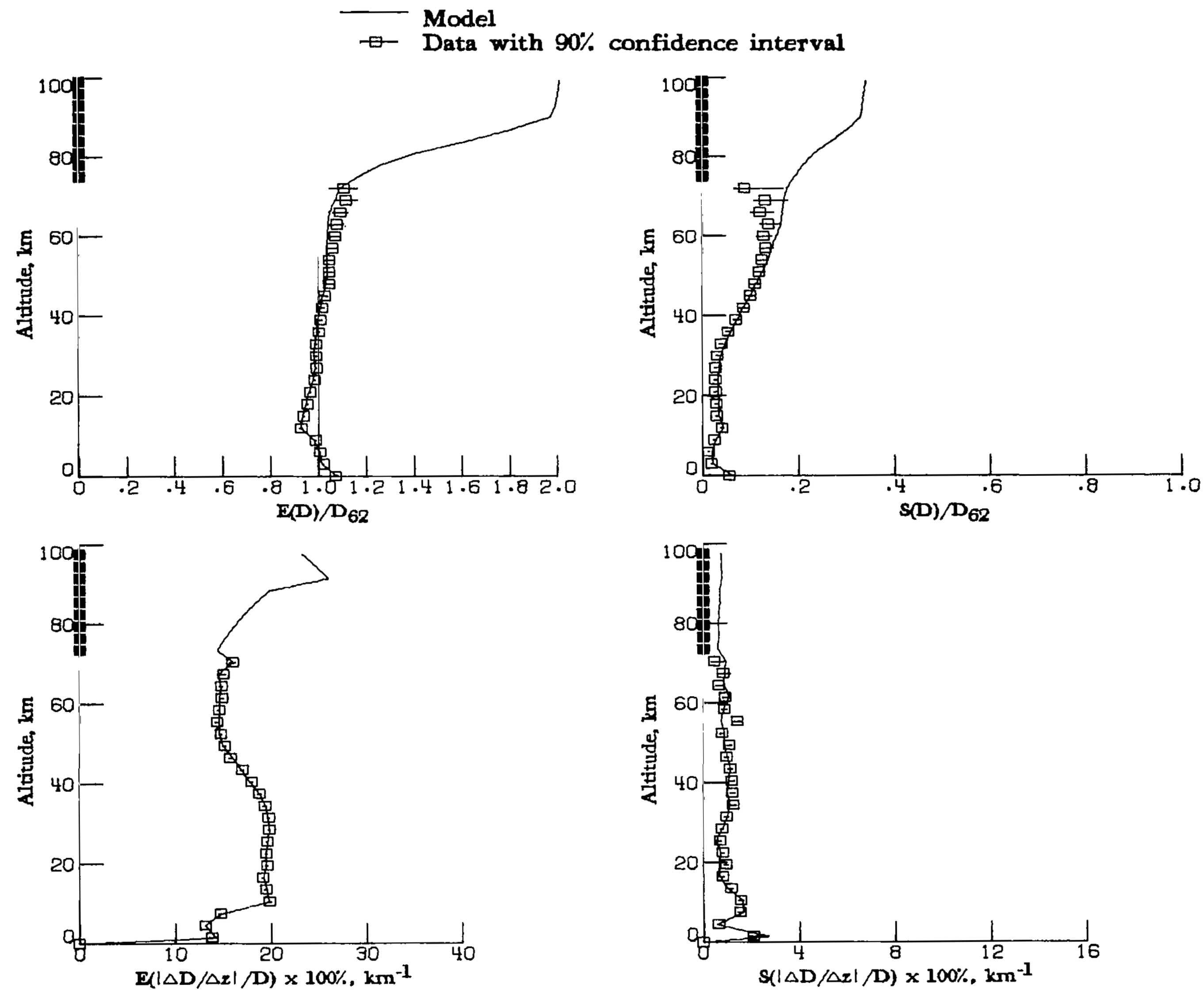

Figure 32.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the spring, 60° latitude category.

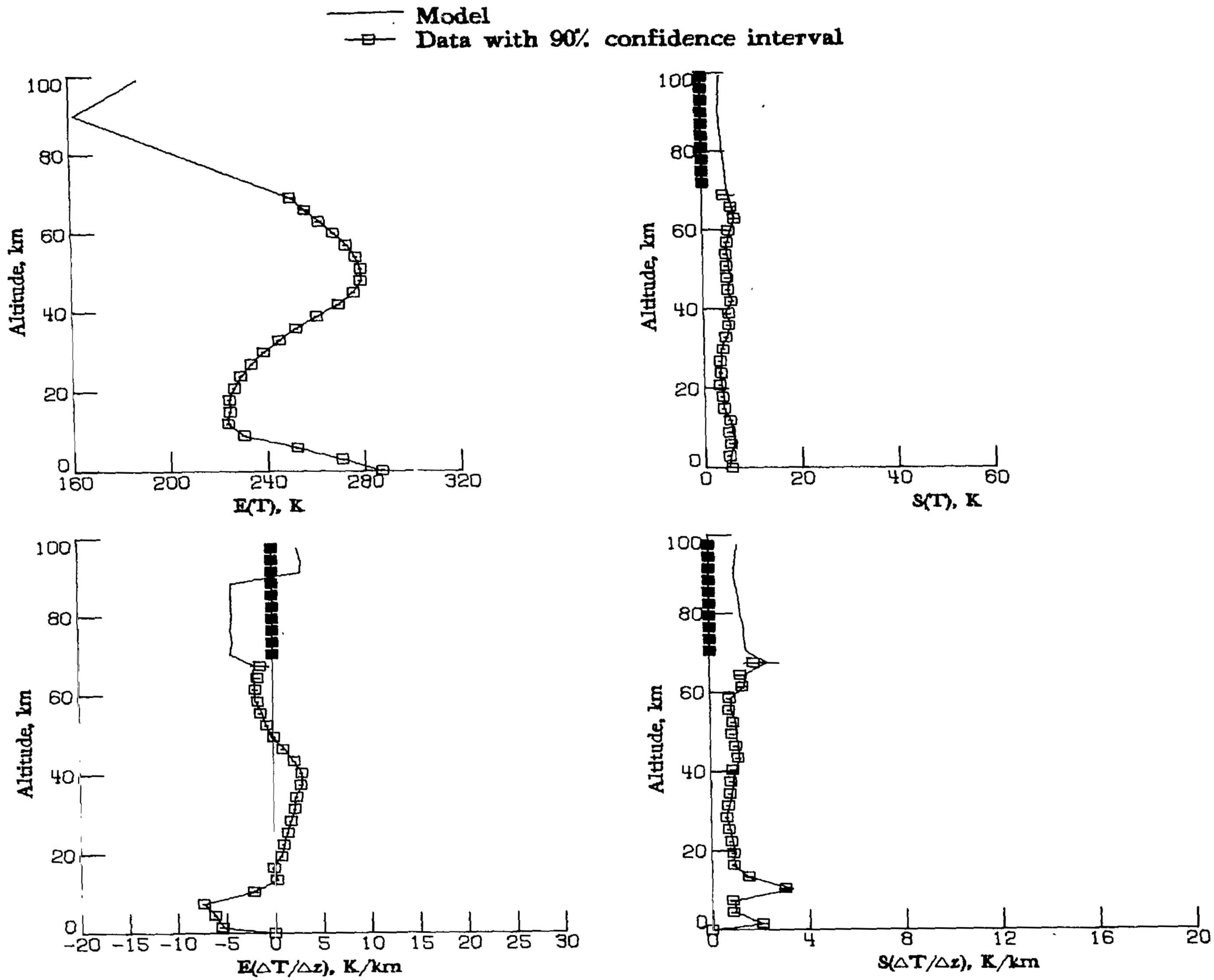

Figure 33.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the summer, 60° latitude category.

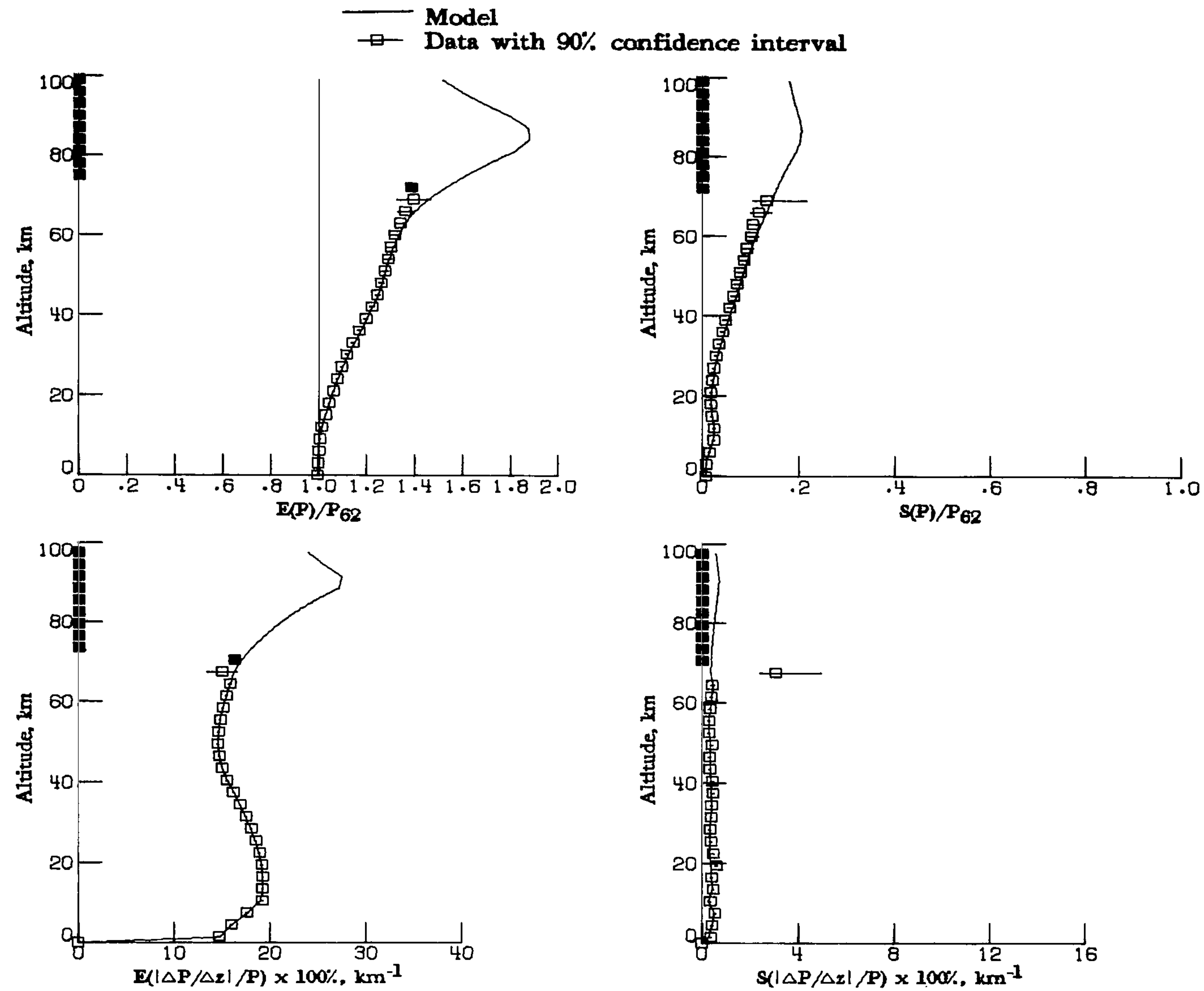

Figure 34.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the summer, 60° latitude category.

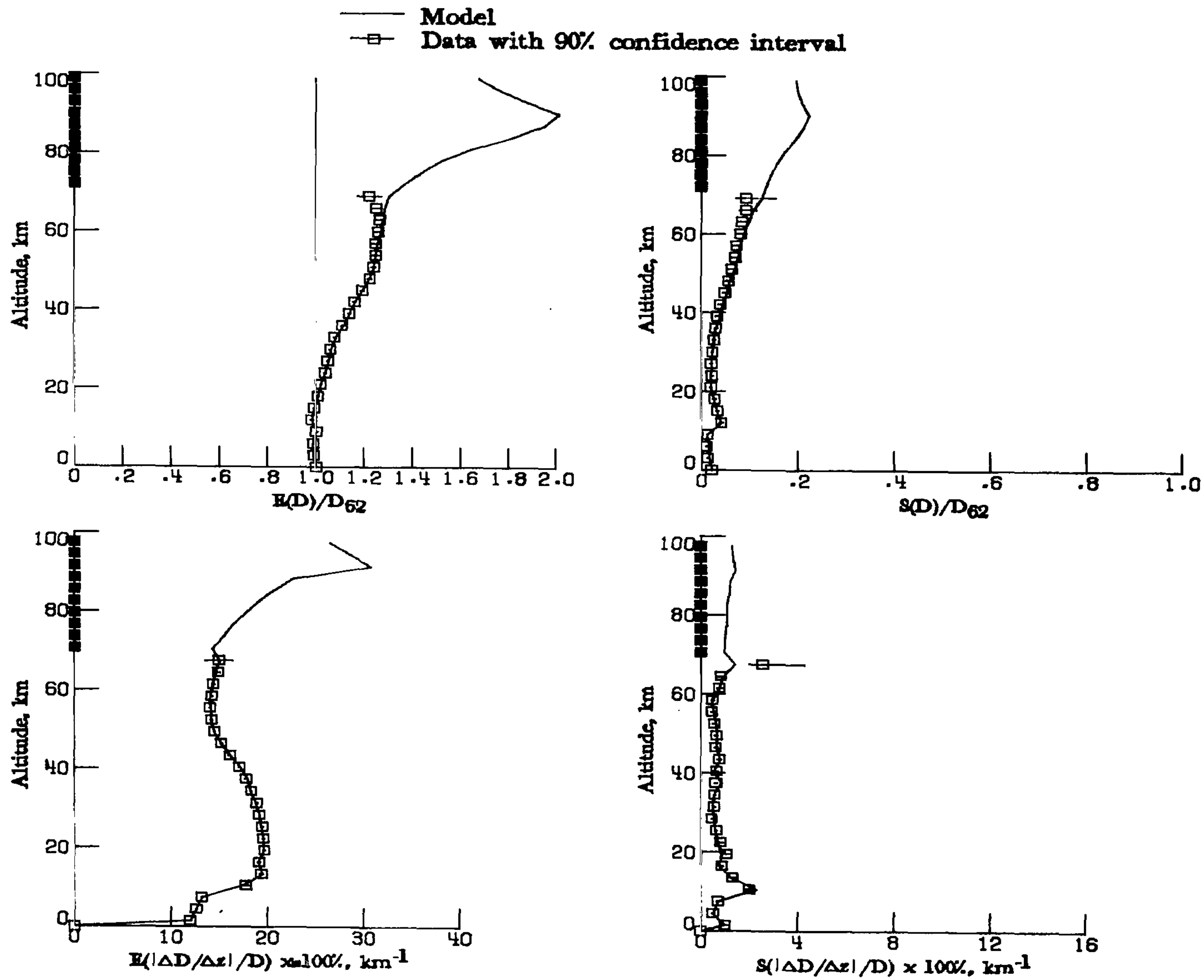

Figure 35.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the summer, 60° latitude category.

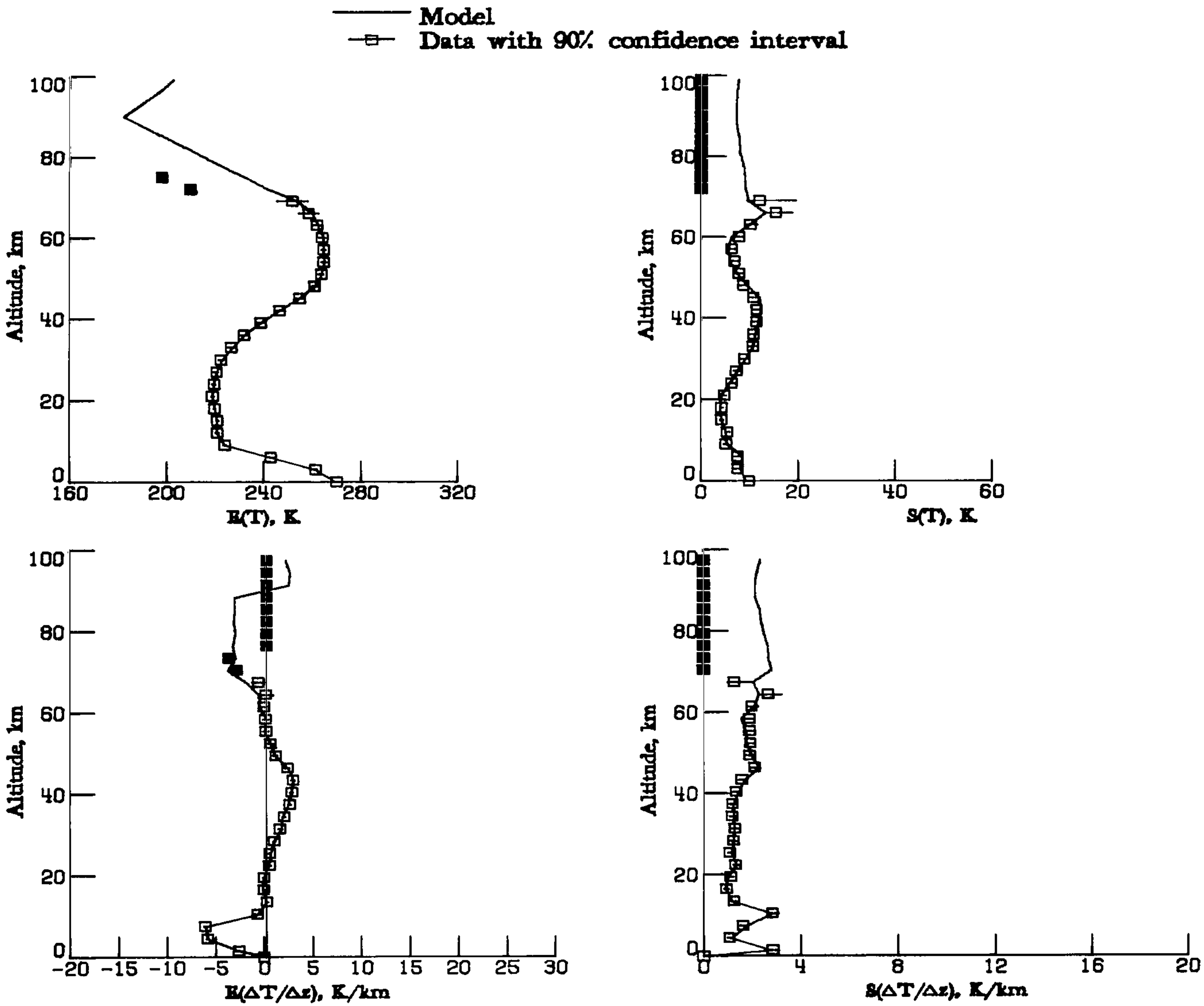

Figure 36.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the autumn, 60° latitude category.

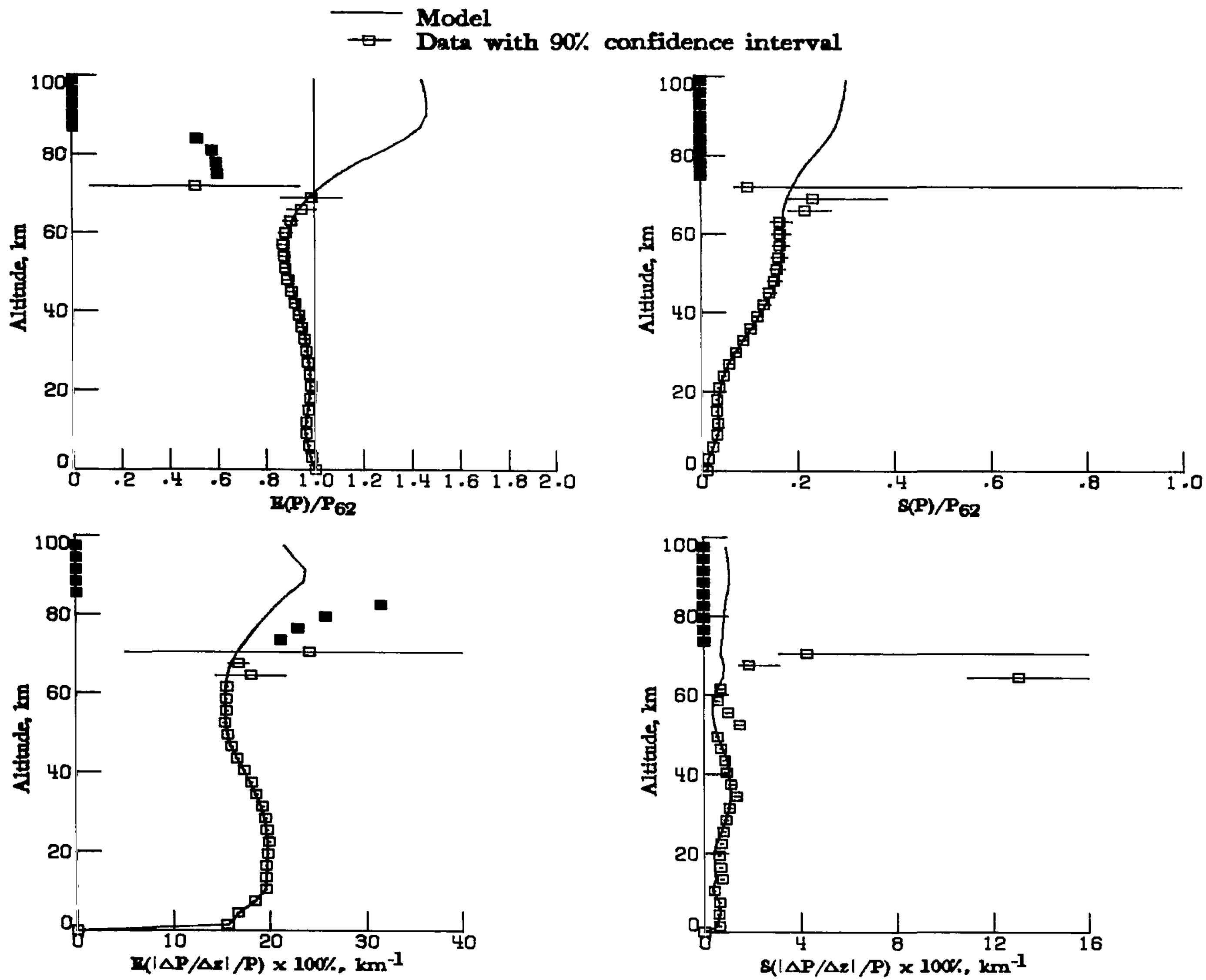

Figure 37.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the autumn, 60° latitude category.

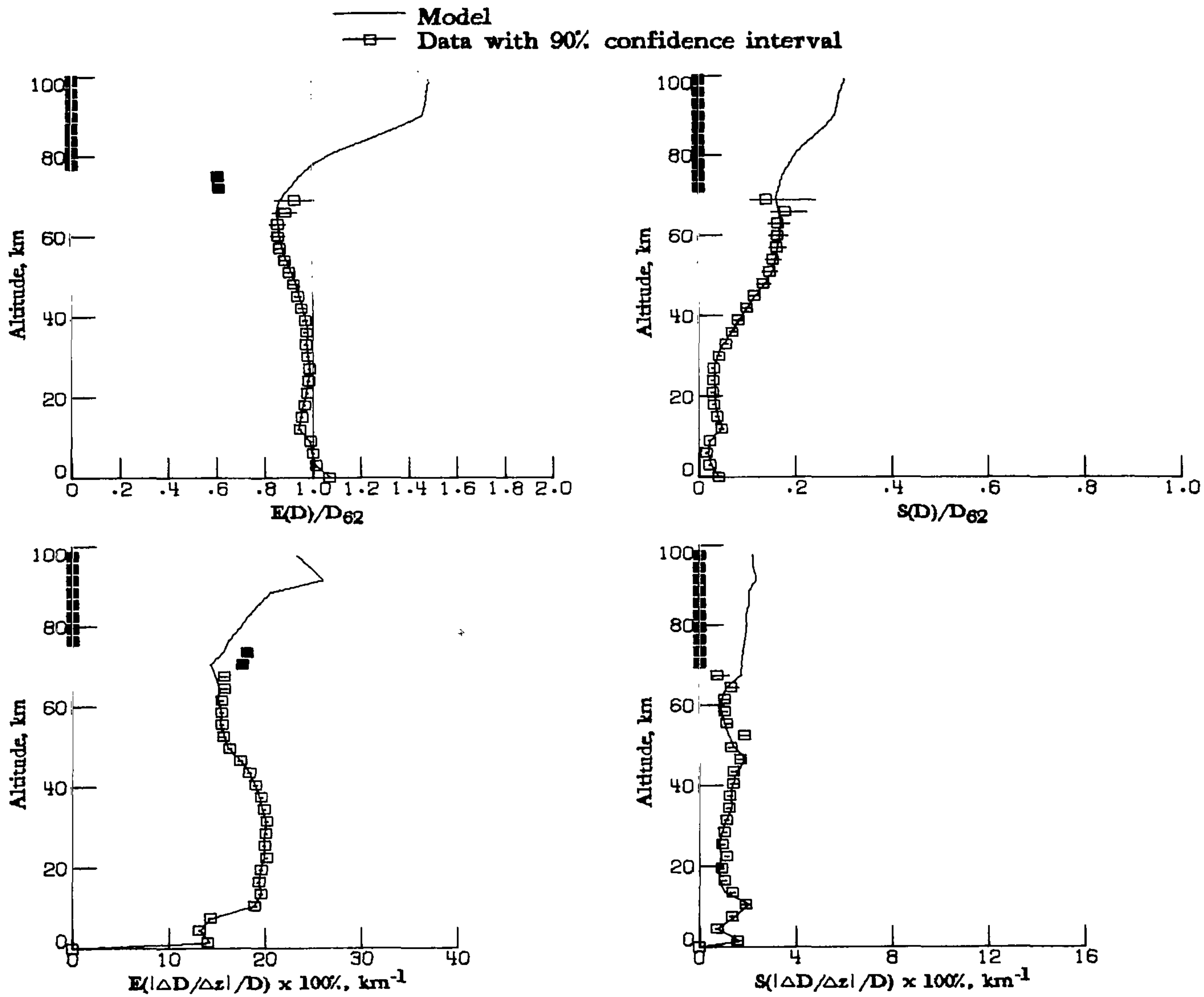

Figure 38.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the autumn, 60° latitude category.

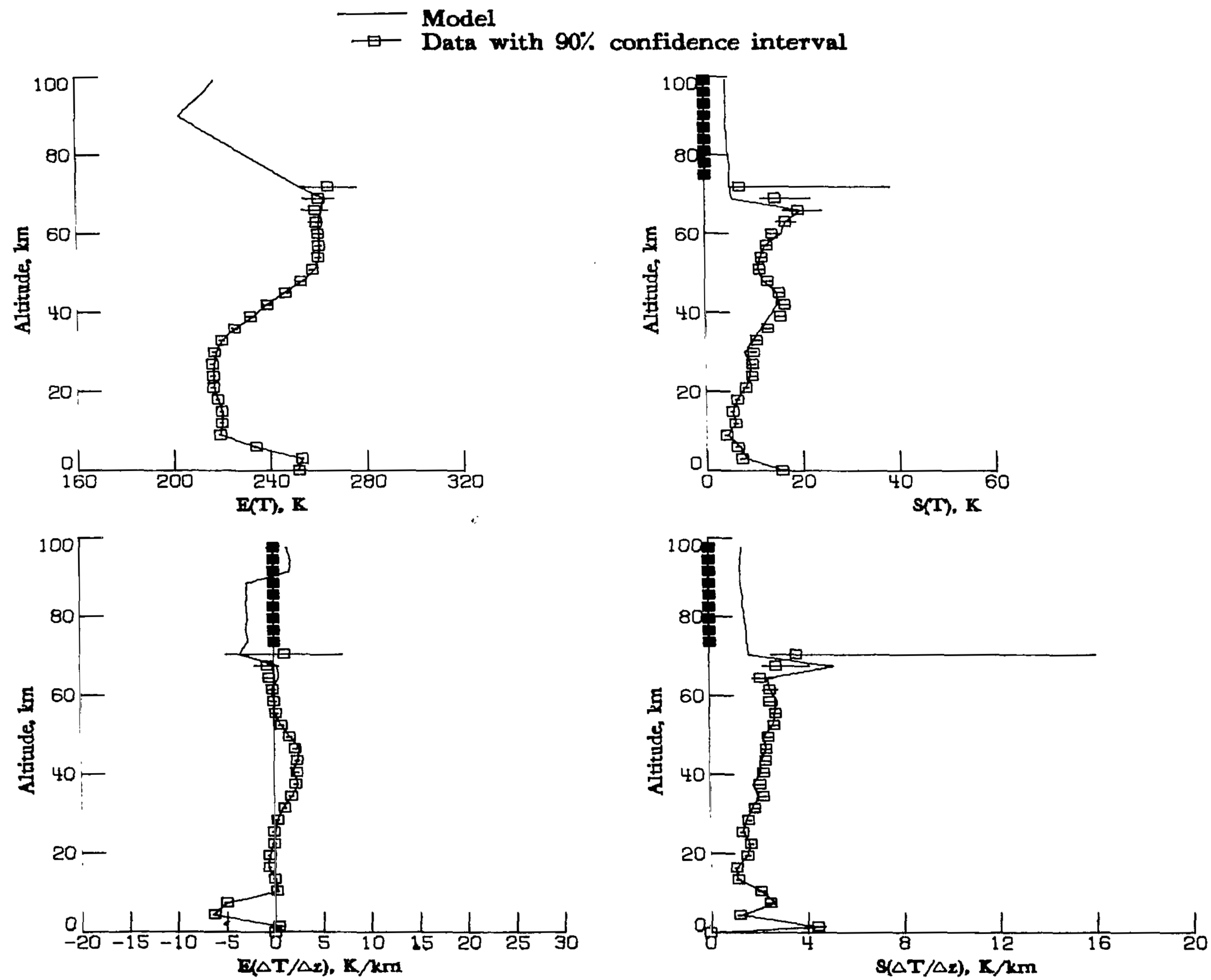

Figure 39.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the winter, 60° latitude category.

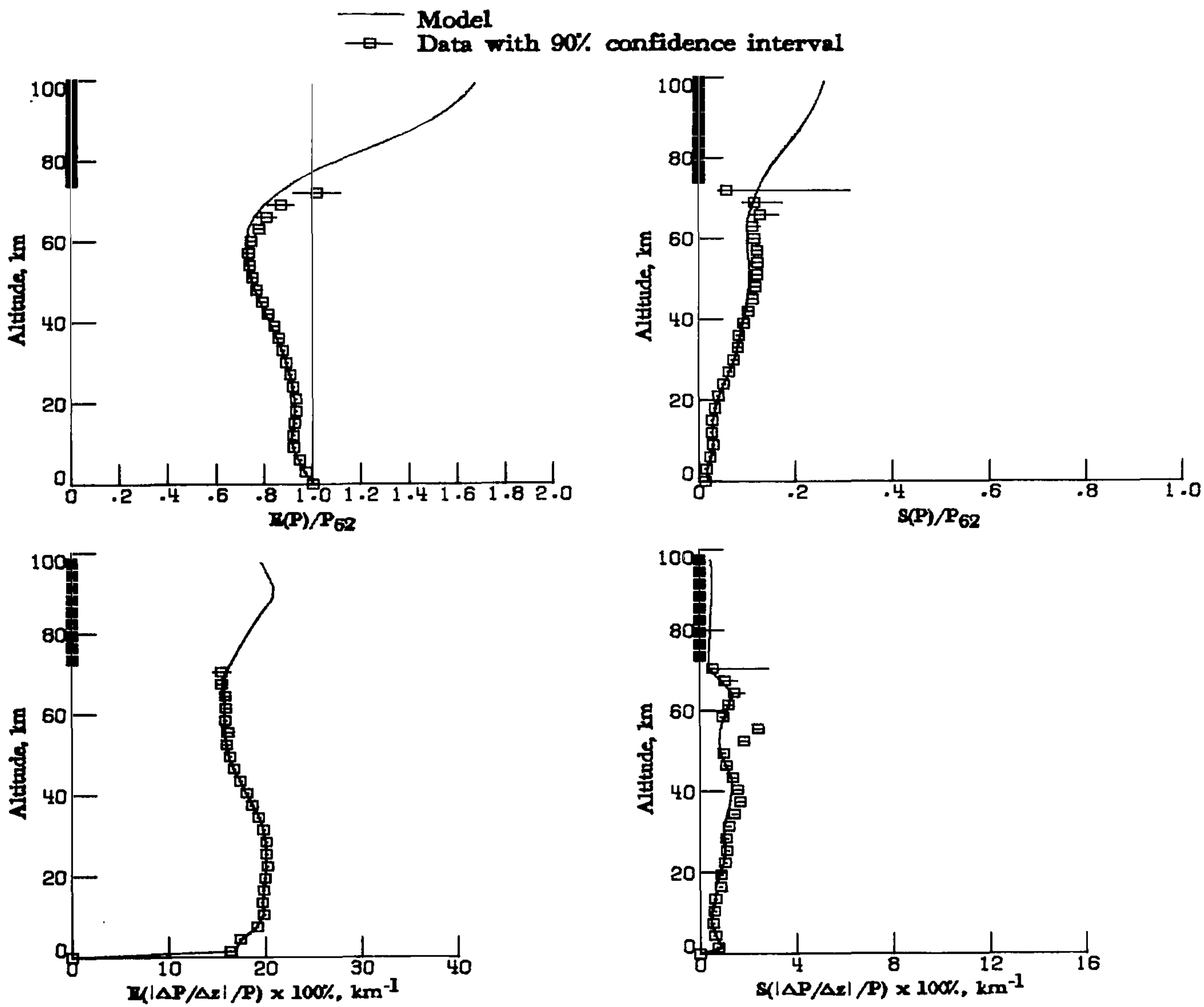

Figure 40.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the winter, 60° latitude category.

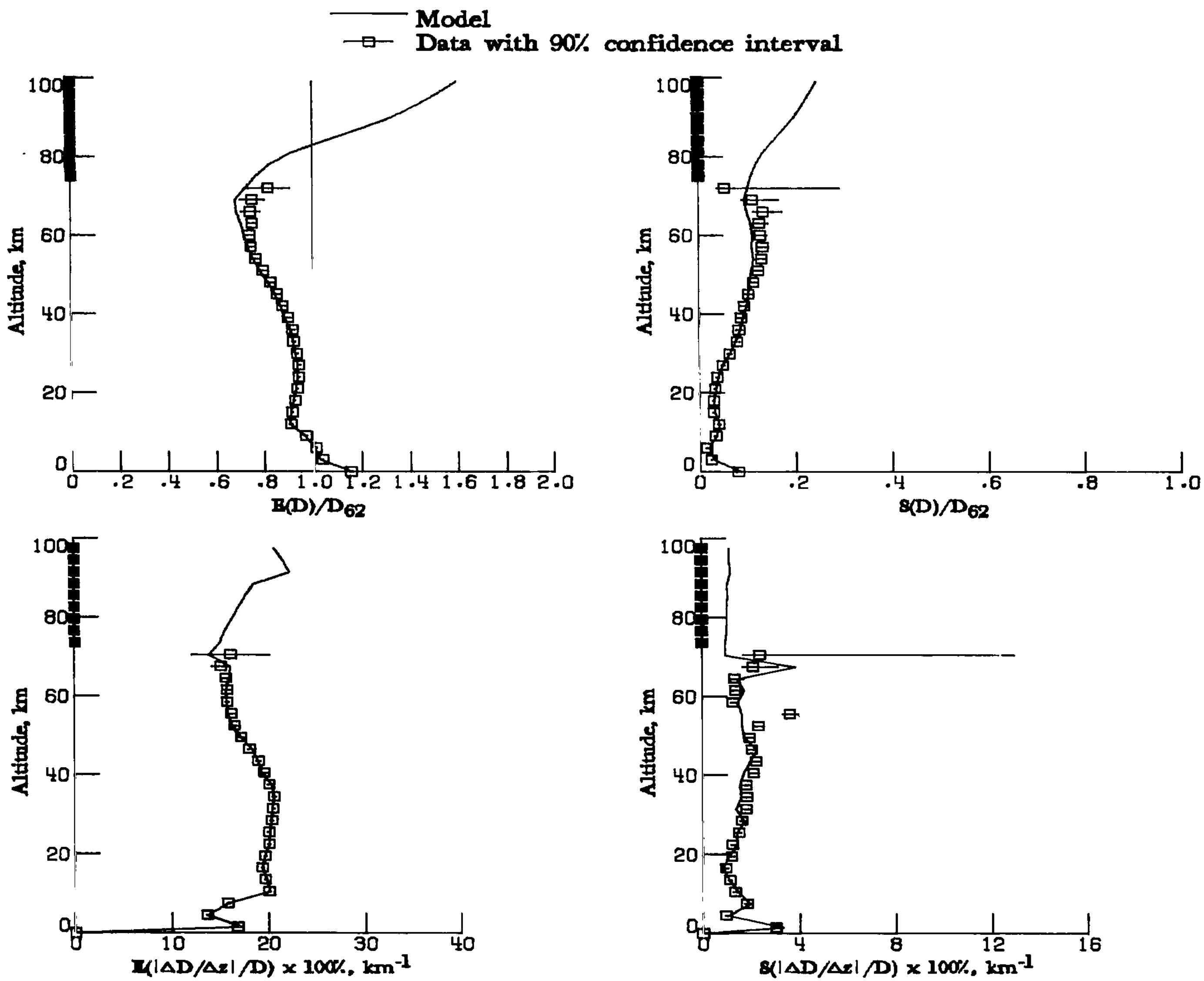

Figure 41.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the winter, 60° latitude category.

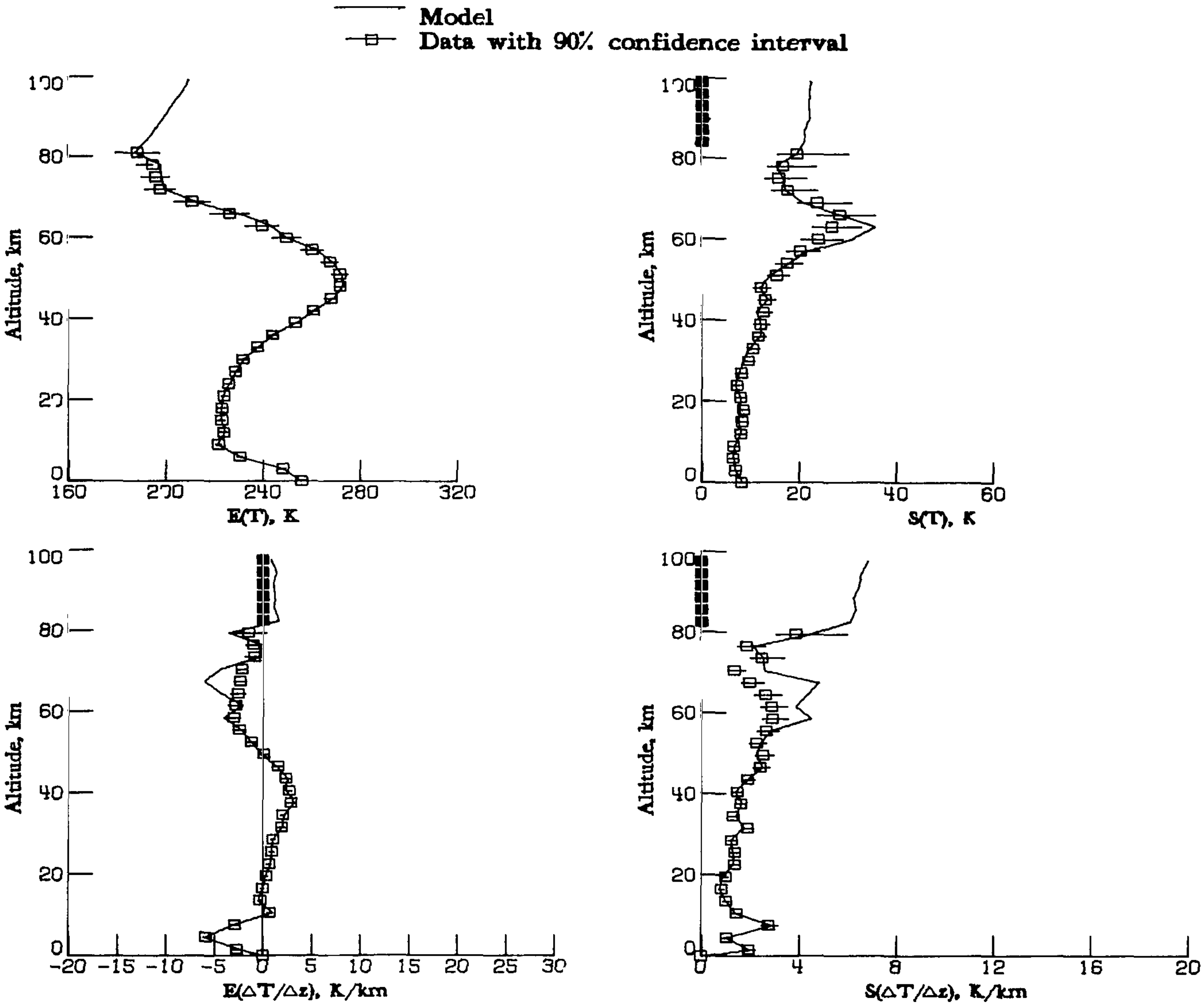

Figure 42.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the spring, 75° latitude category.

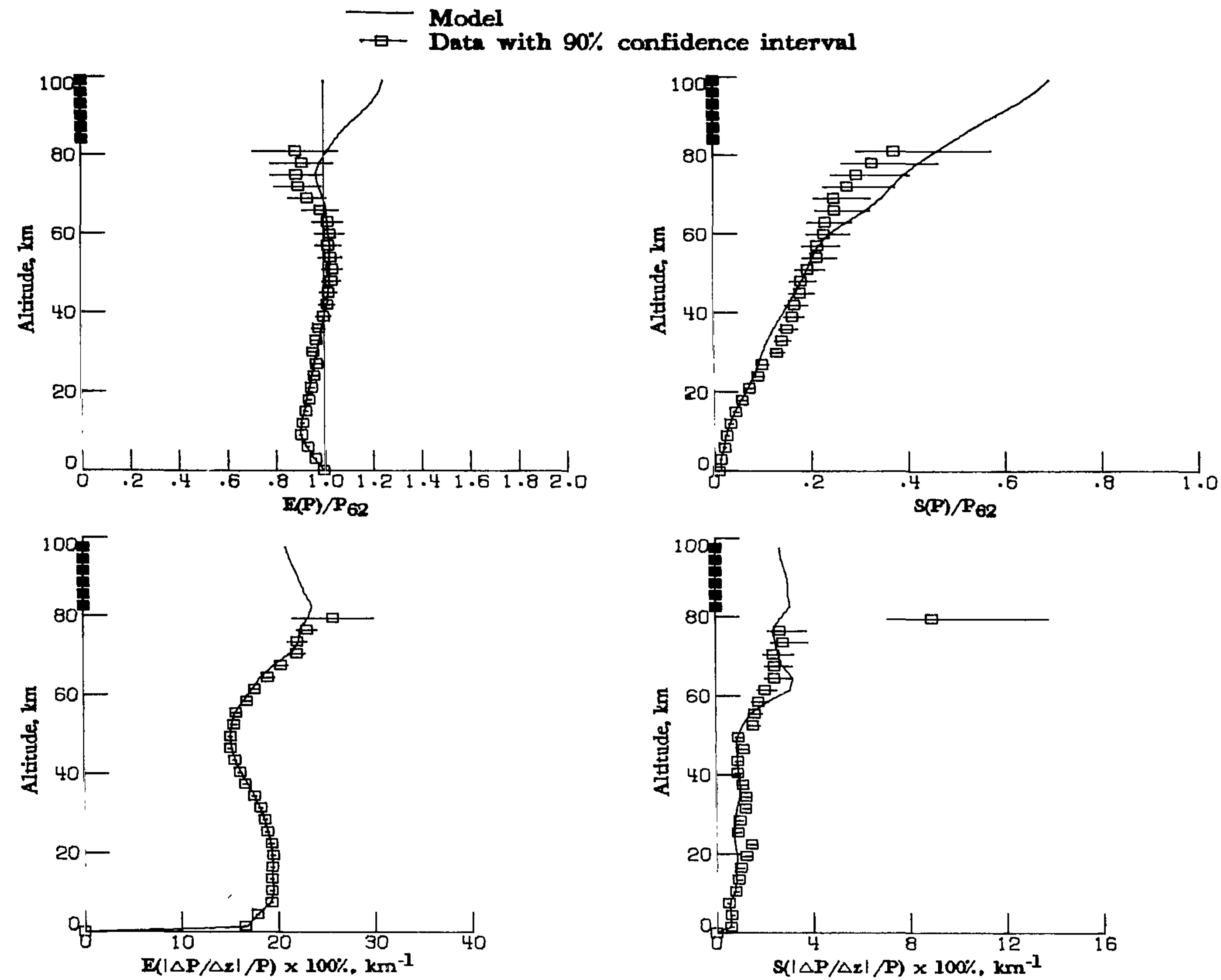

Figure 43.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the spring, 75° latitude category.

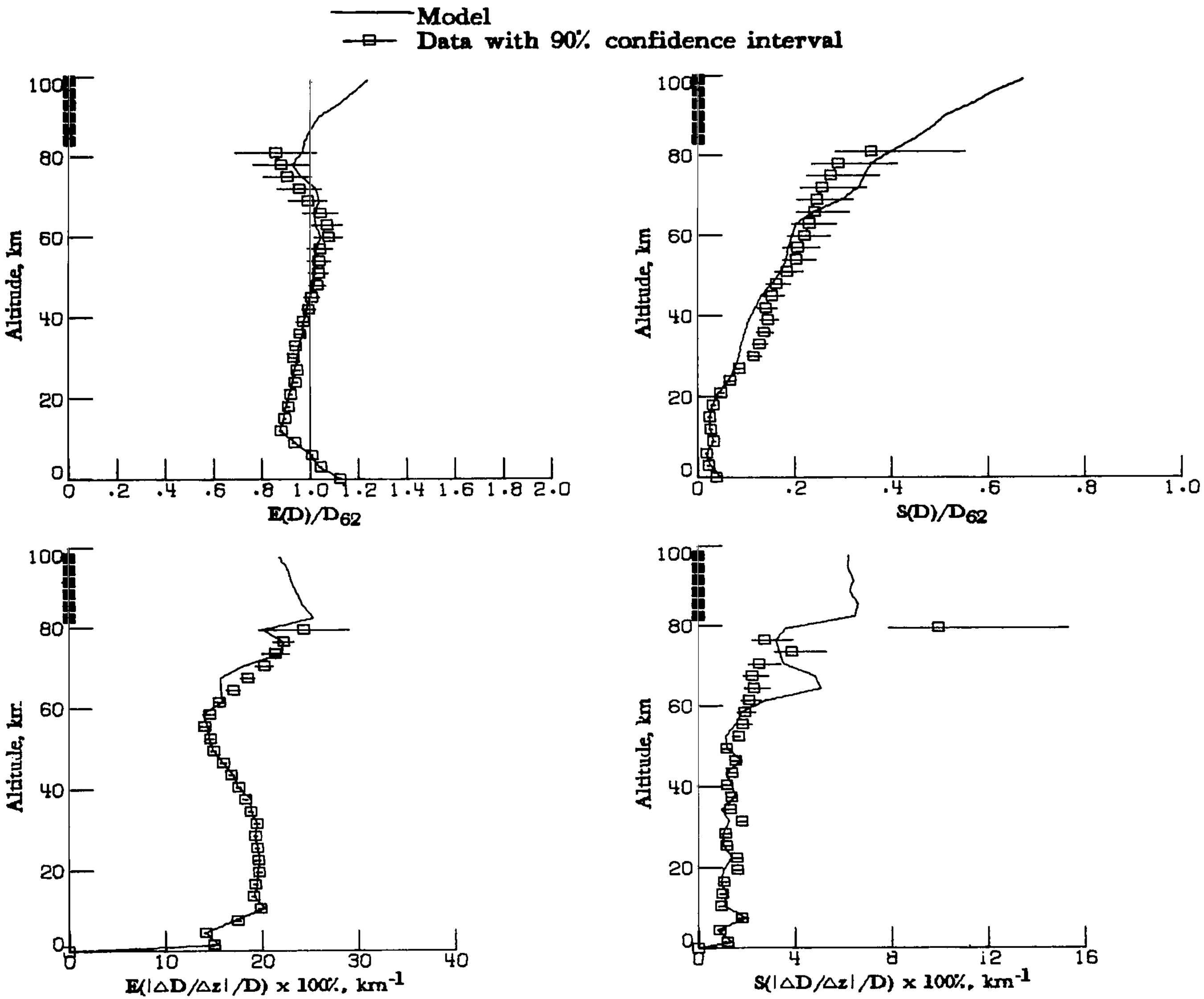

Figure 44.- Comparison of model and data means and standard deviations of D and ΔD/Δz in the spring, 75° latitude category.

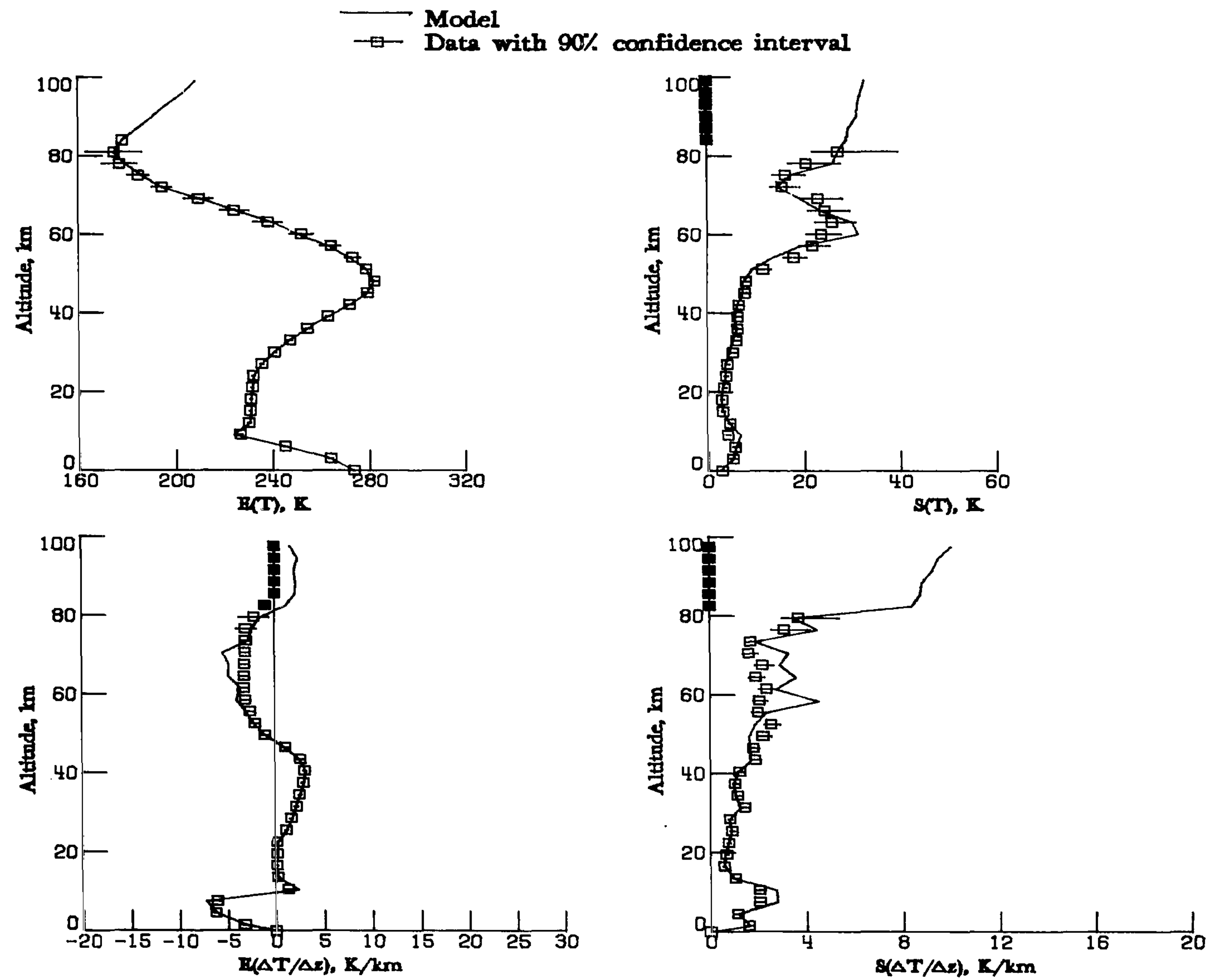

Figure 45.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the summer, 75° latitude category.

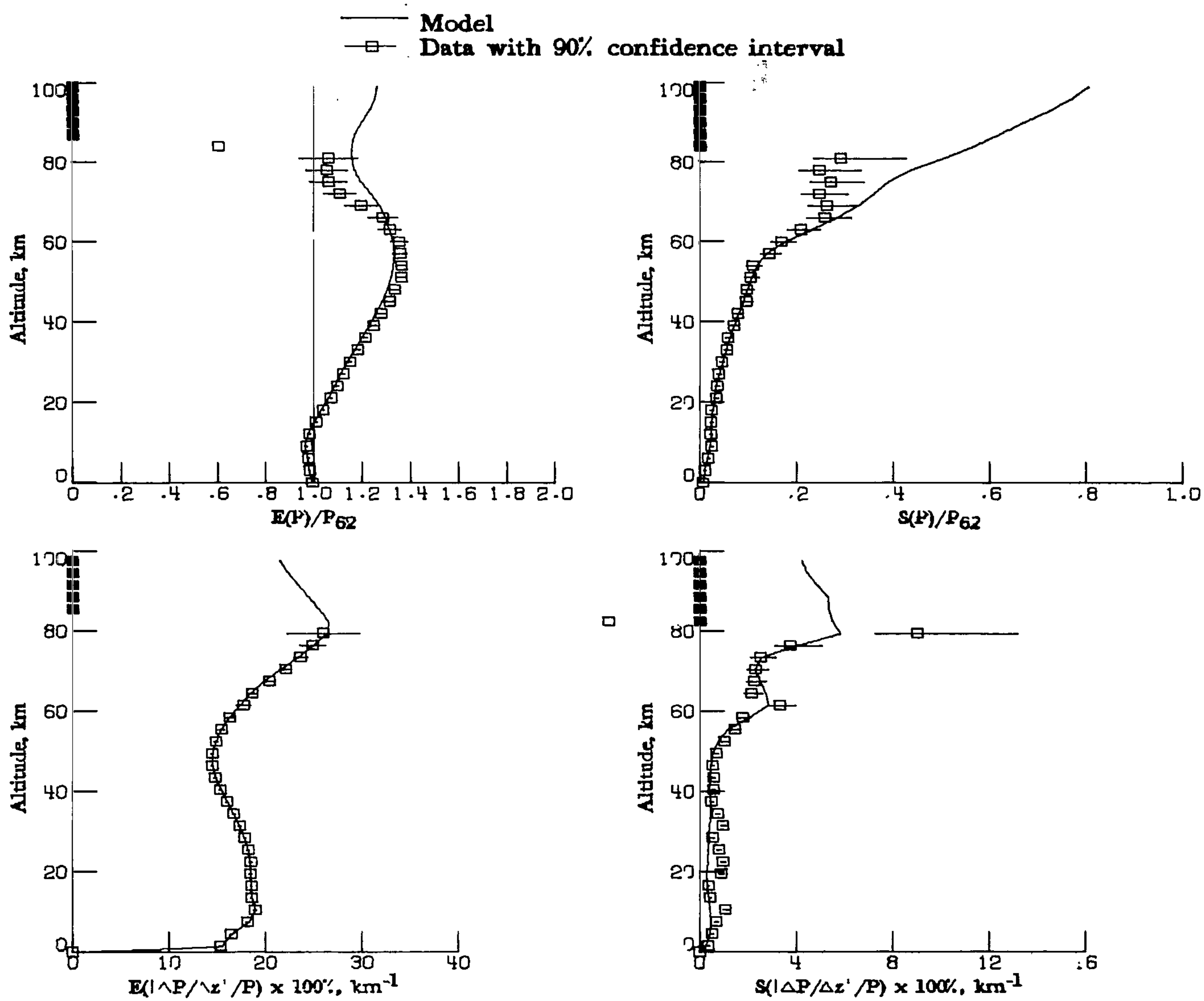

Figure 46.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the summer, 75° latitude category.

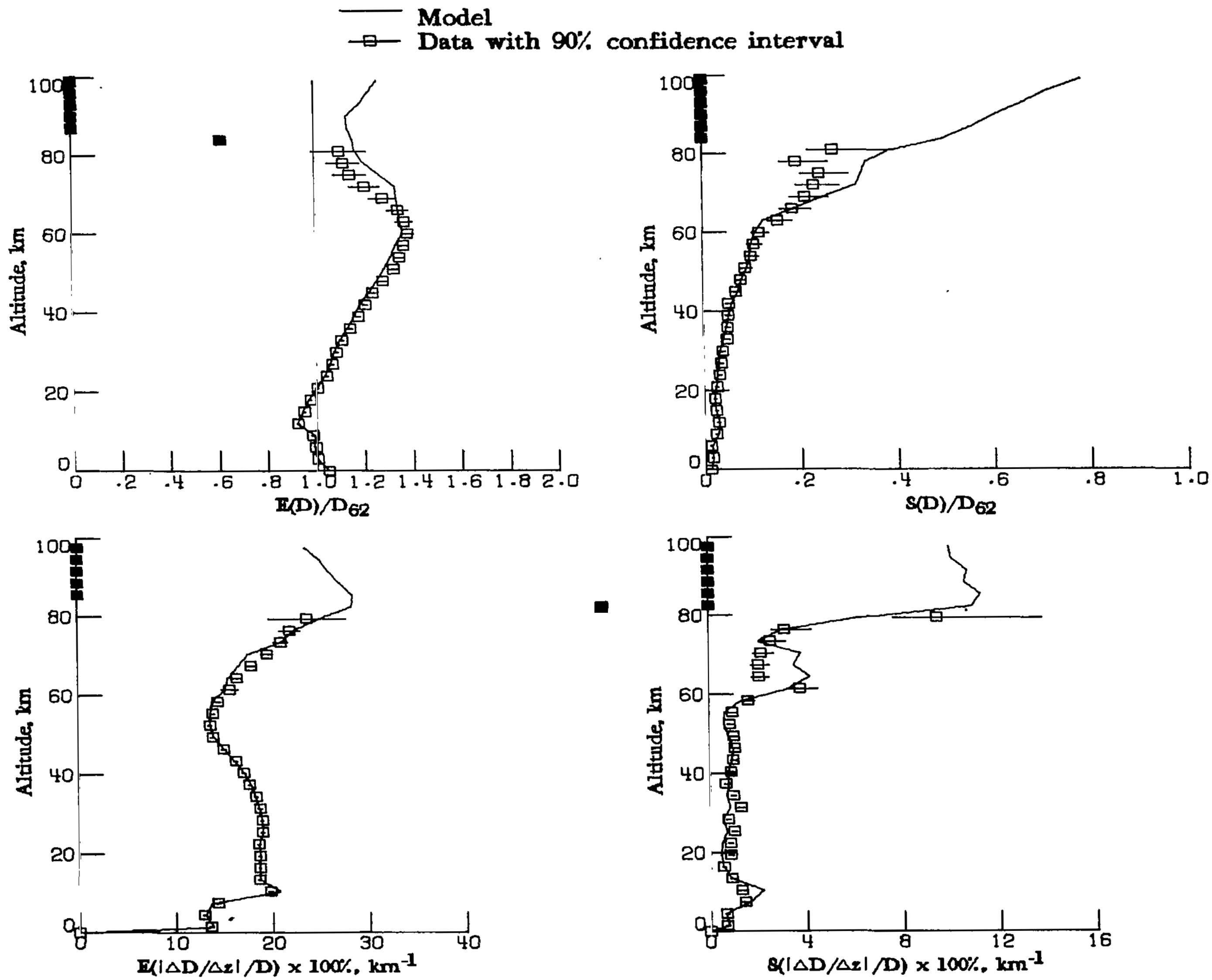

Figure 47.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the summer, 75° latitude category.

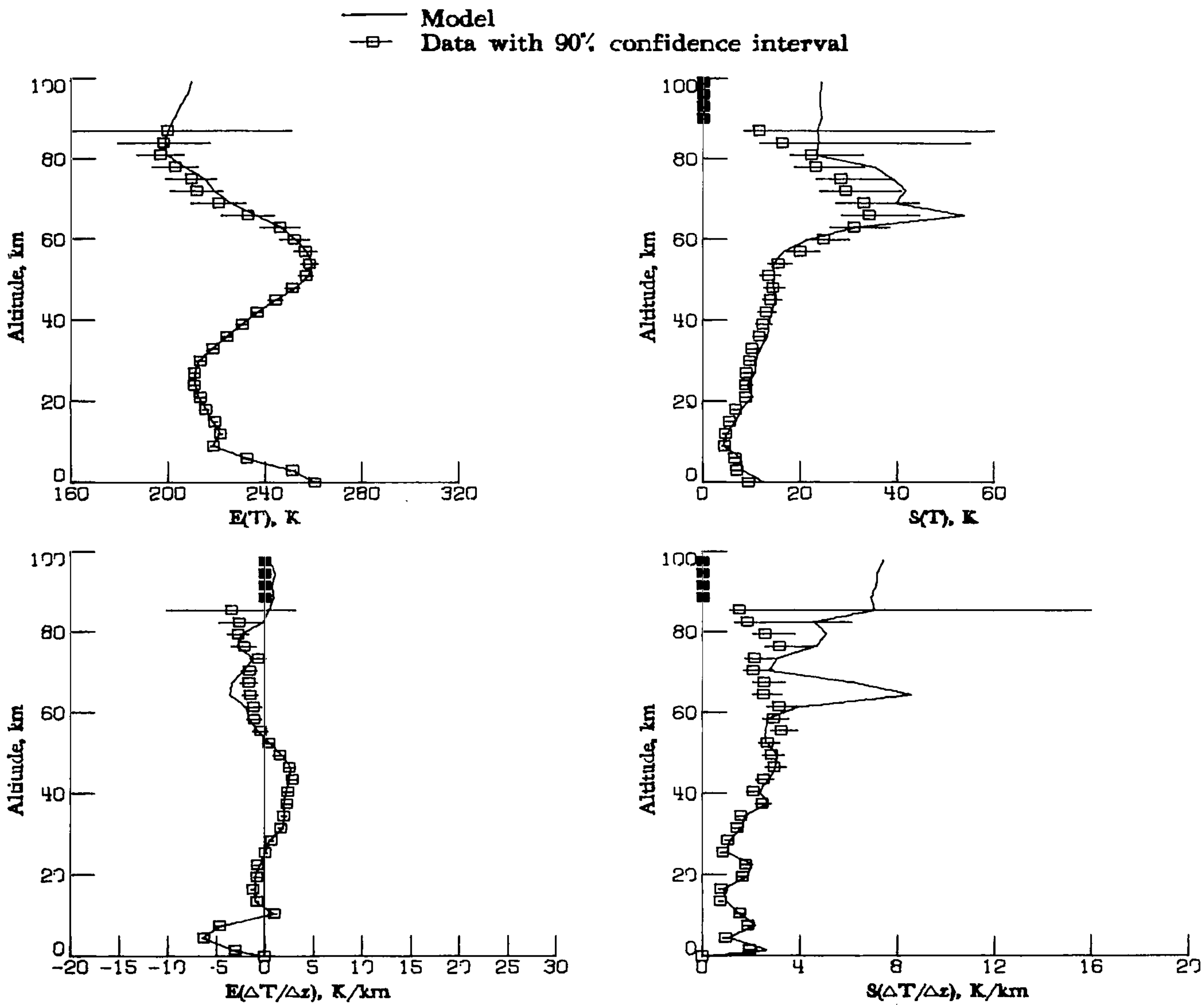

Figure 48.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the autumn, 75° latitude category.

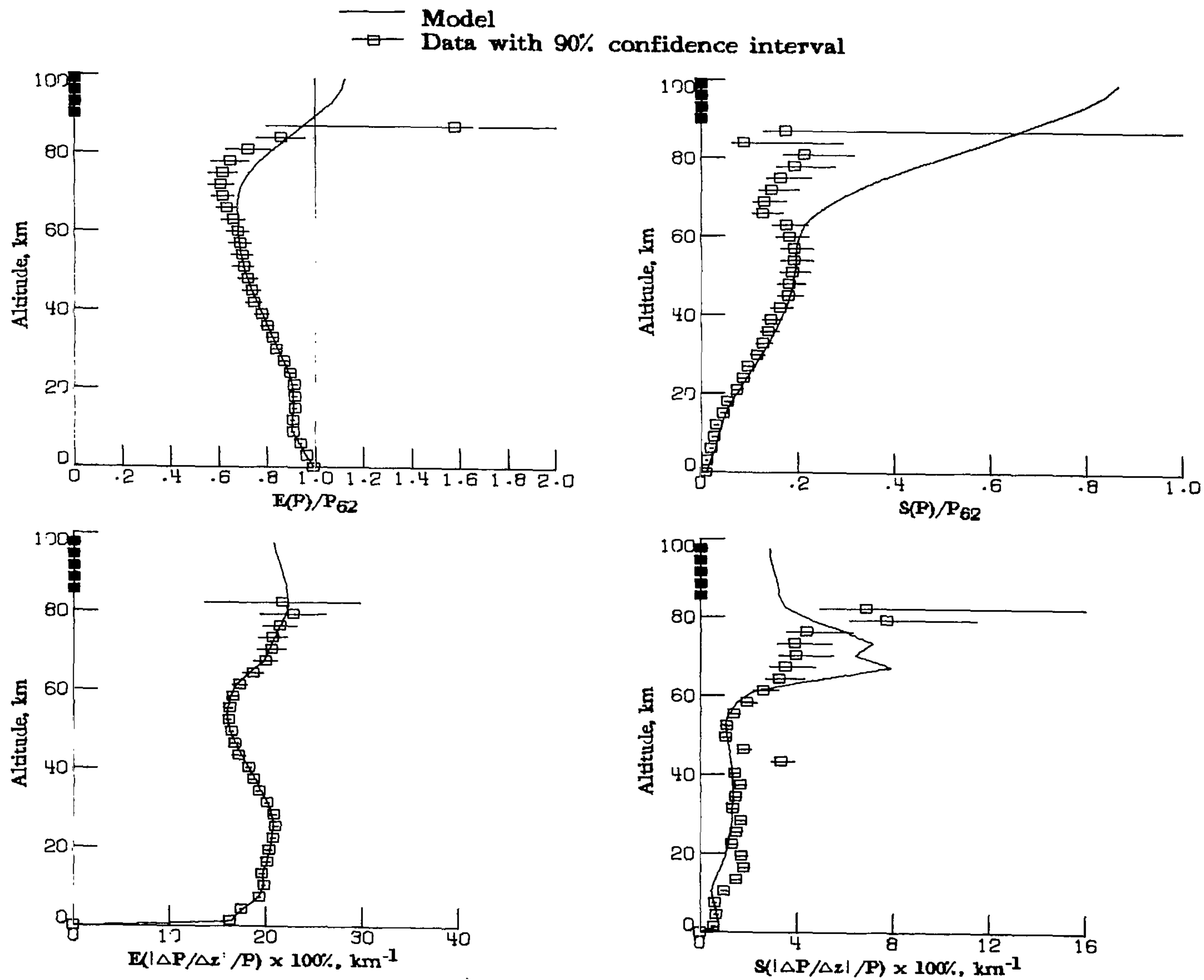

Figure 49.- Comparison of model and data means and standard deviations of P and ΔP/Δz in the autumn, 75° latitude category.

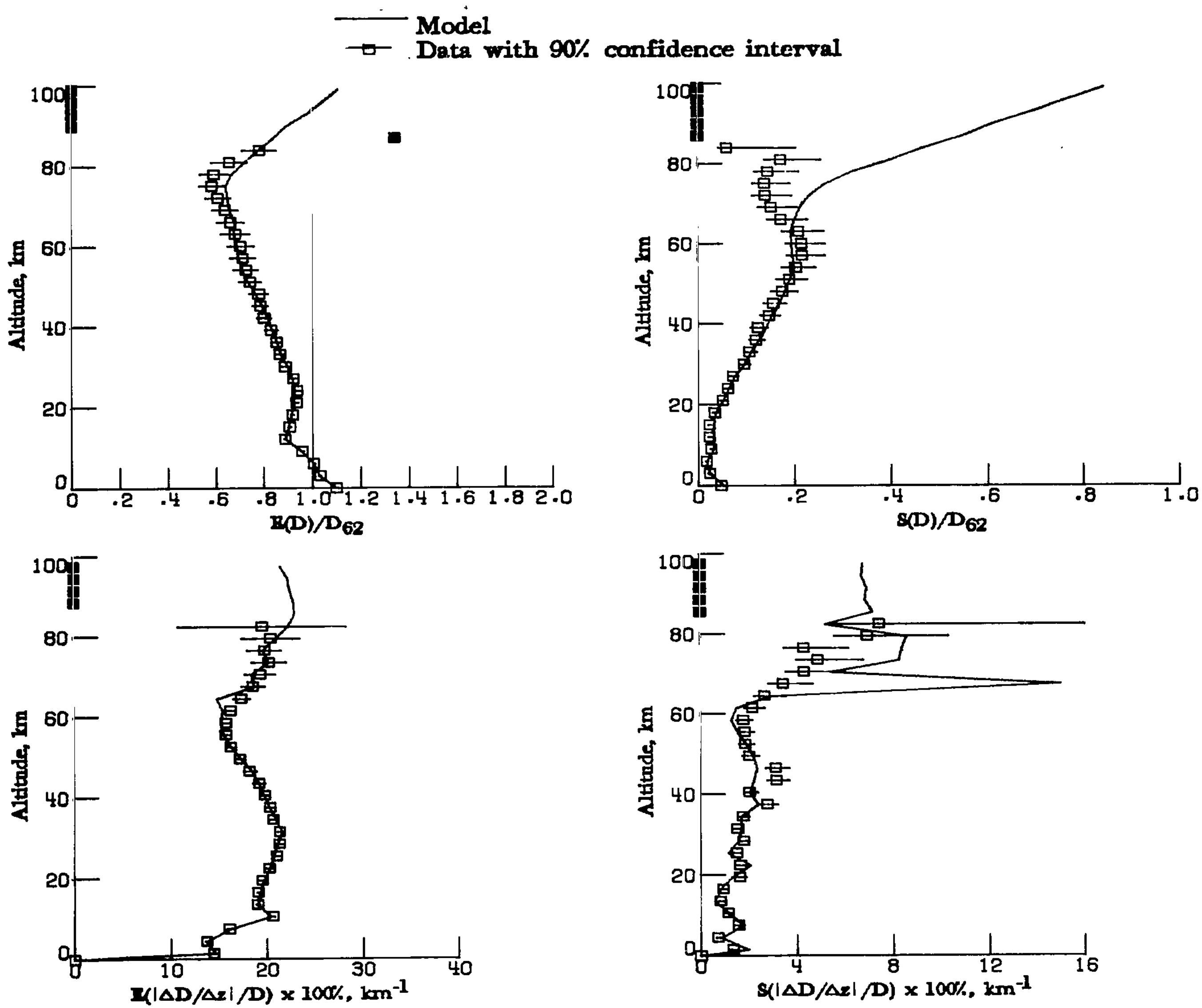

Figure 50.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the autumn, 75° latitude category.

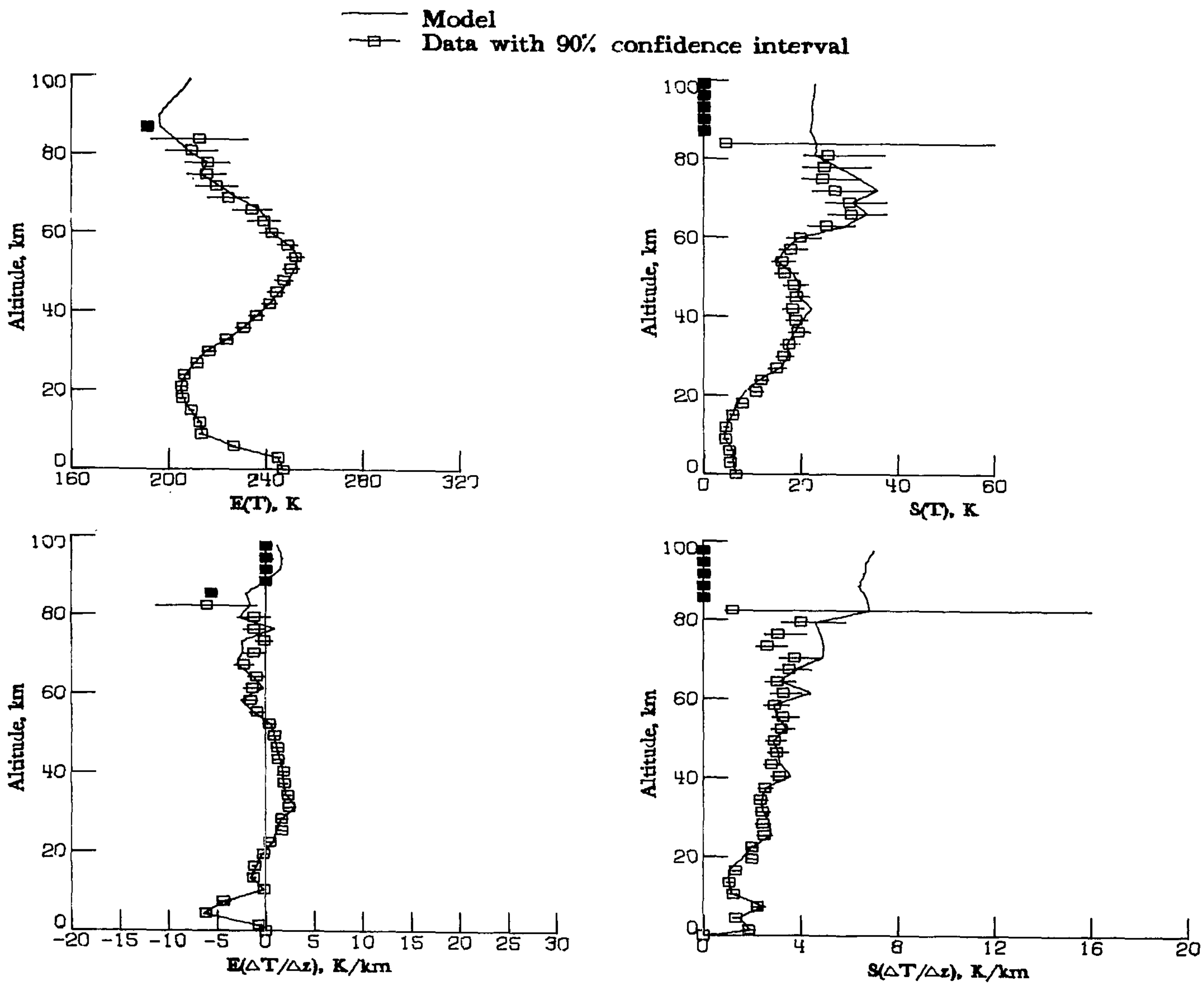

Figure 51.- Comparison of model and data means and standard deviations of T and $\Delta T/\Delta z$ in the winter, 75° latitude category.

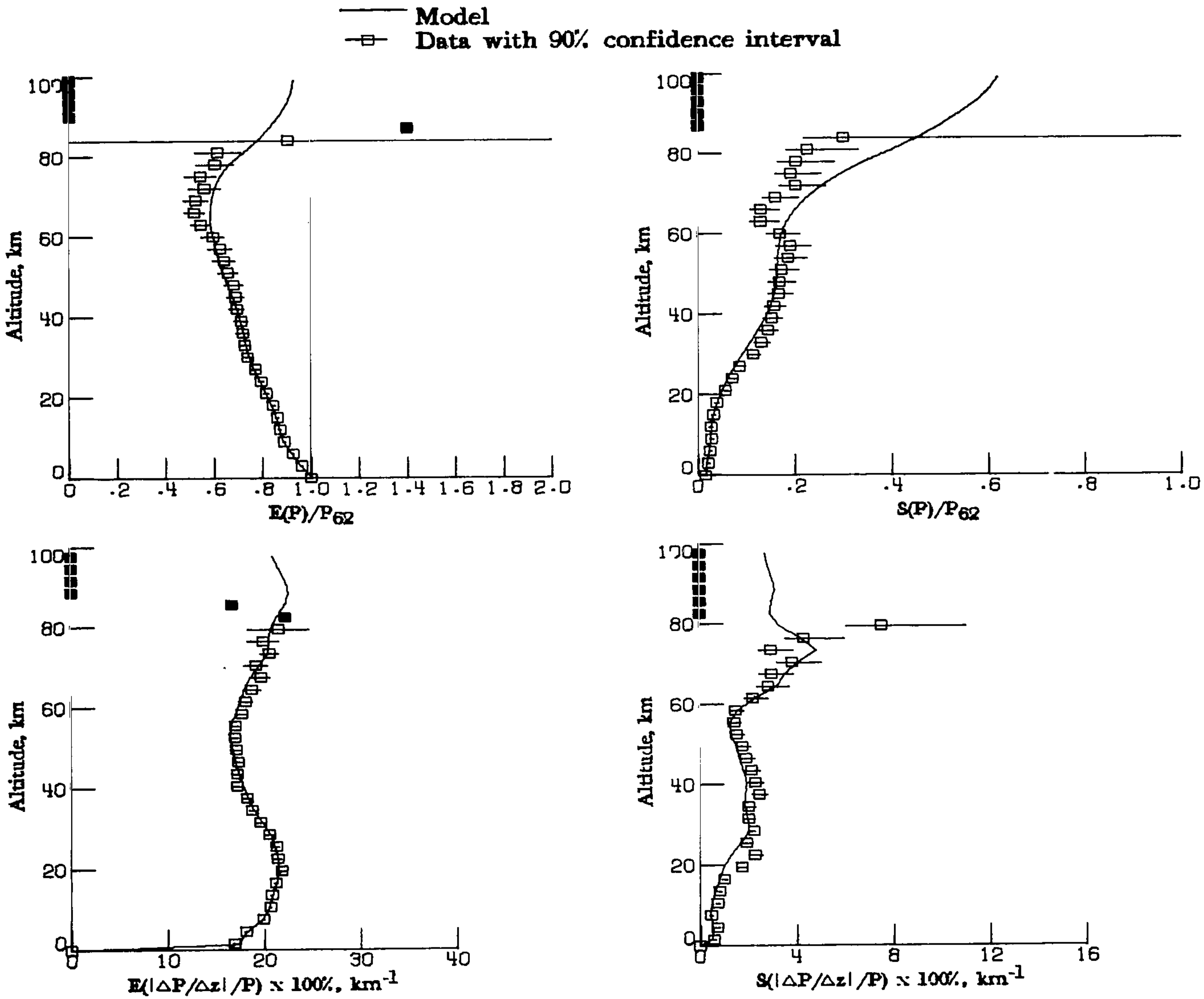

Figure 52.- Comparison of model and data means and standard deviations of P and $\Delta P/\Delta z$ in the winter, 75° latitude category.

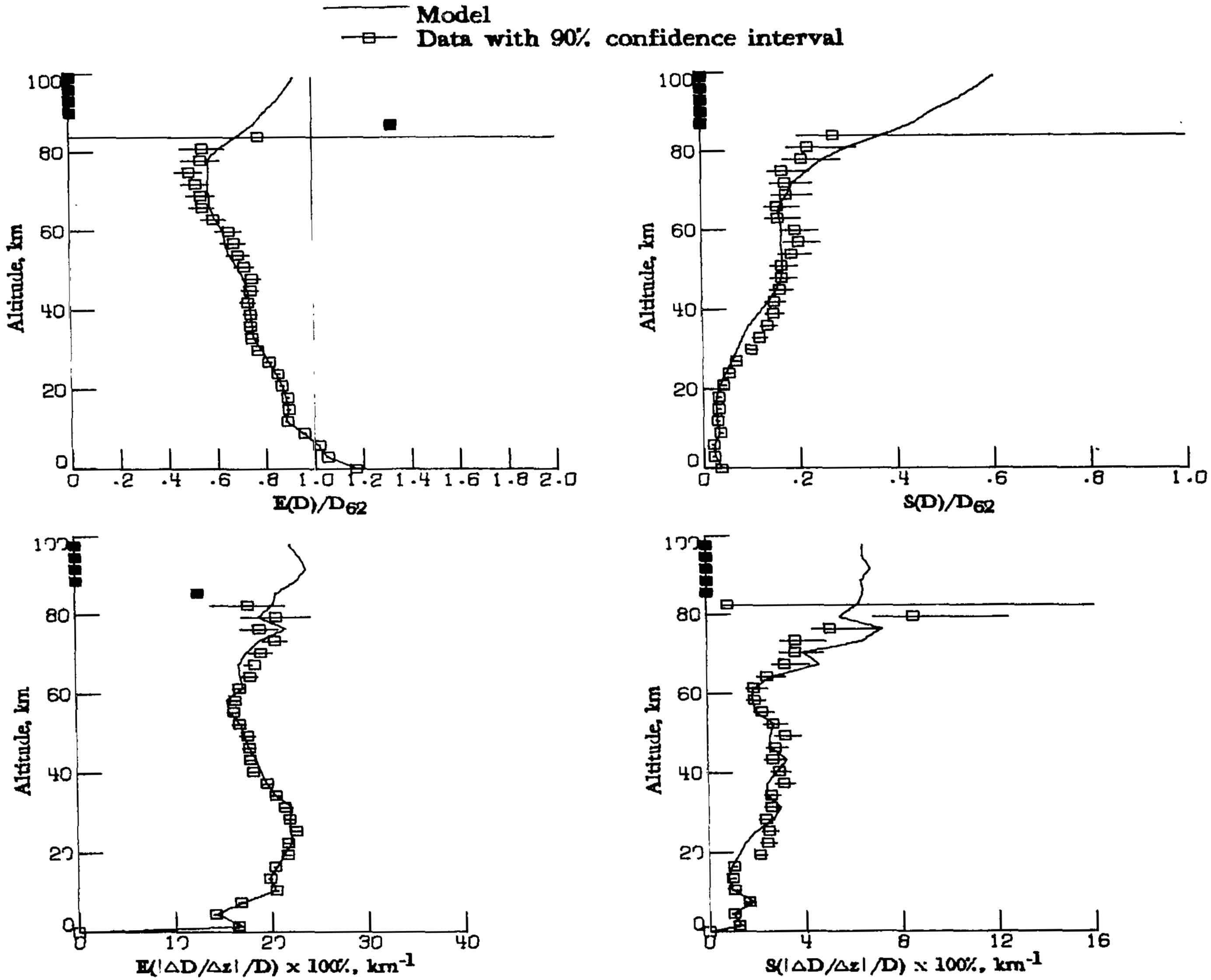

Figure 53.- Comparison of model and data means and standard deviations of D and $\Delta D/\Delta z$ in the winter, 75° latitude category.

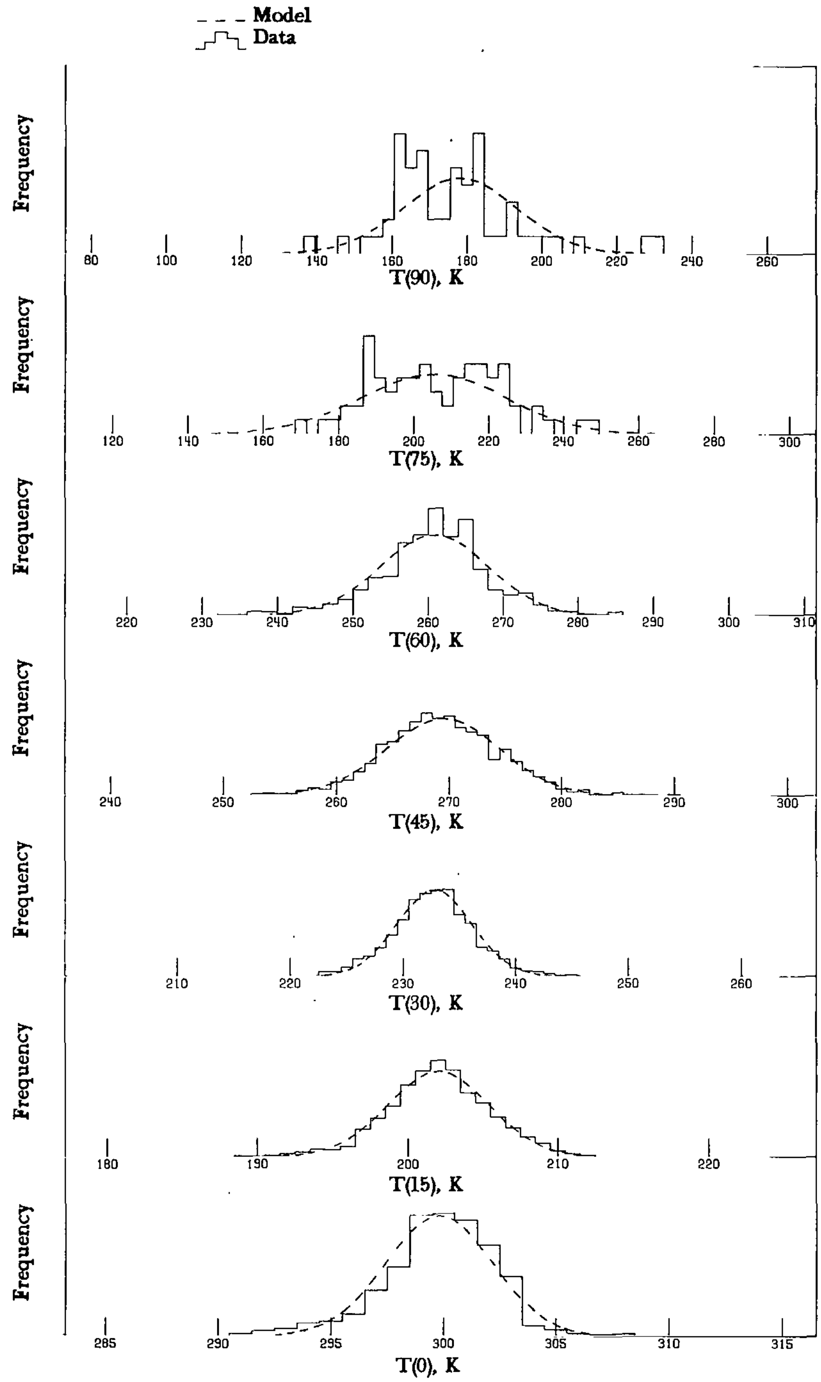

Figure 54.- Comparison of model and data temperature distributions at discrete altitude levels in the 15° latitude category.

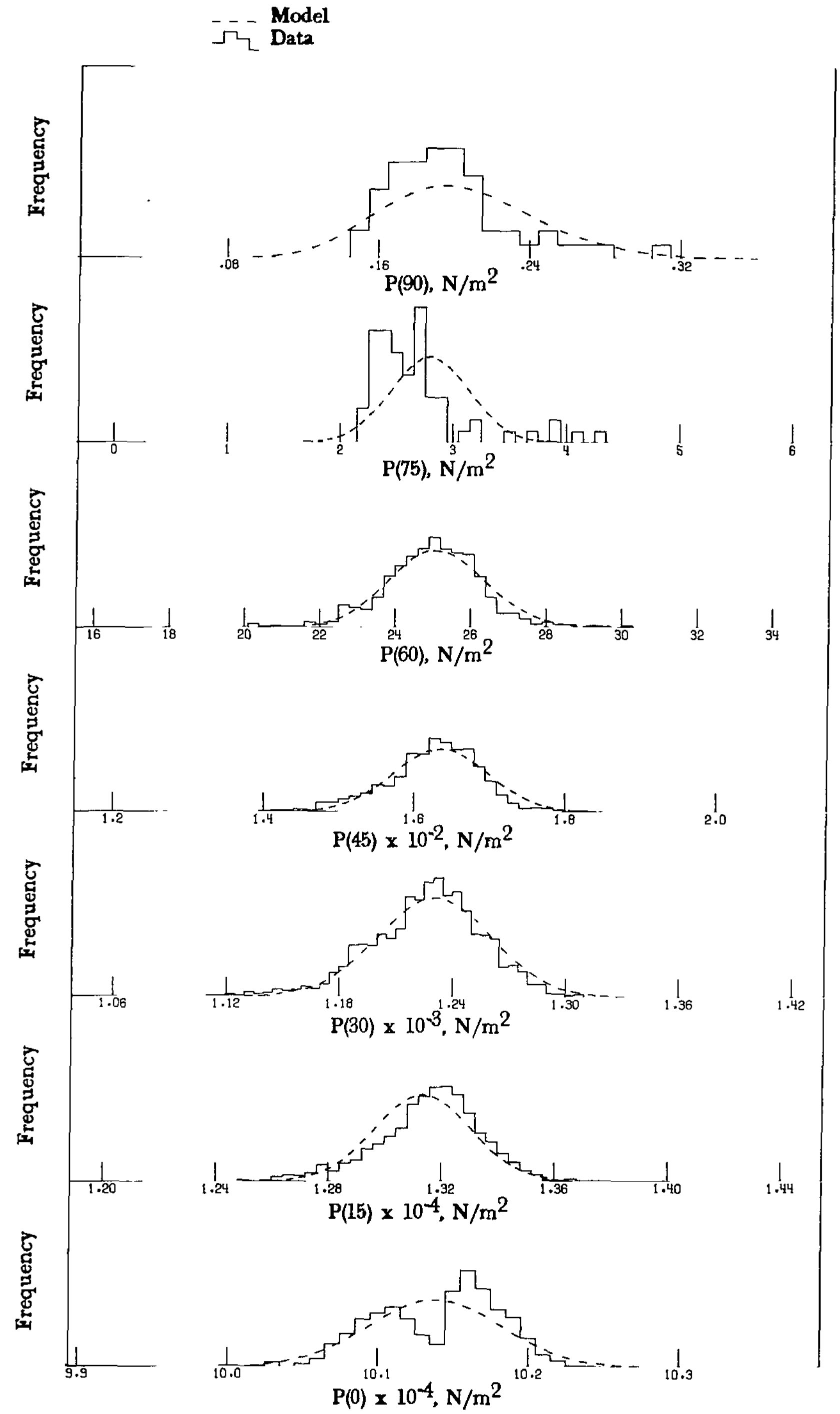

Figure 55.- Comparison of model and data pressure distributions at discrete altitude levels in the 15° latitude category.

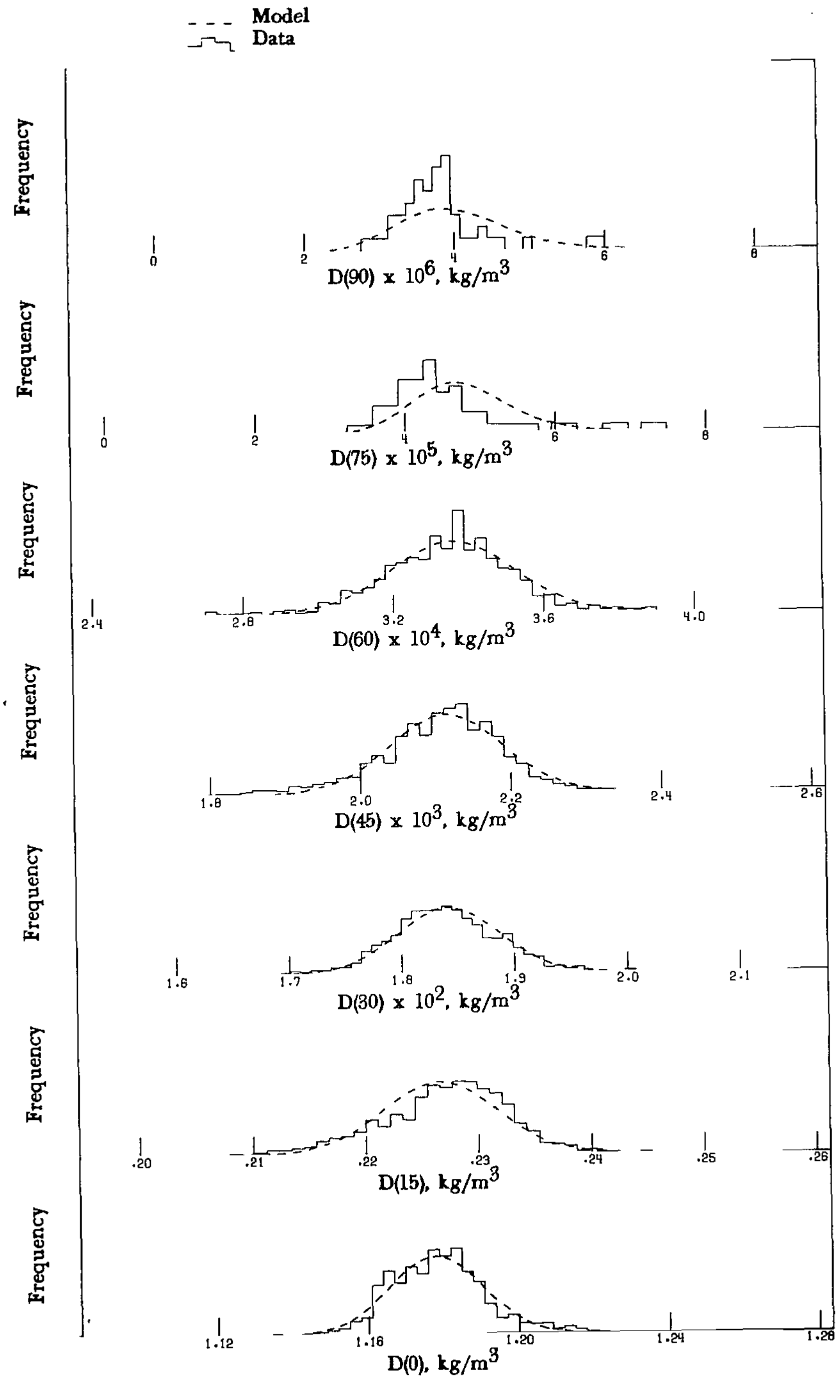

Figure 56.- Comparison of model and data density distributions at discrete altitude levels in the 15° latitude category.

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