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A QUALITATIVE ASSESSMENT OF A RANDOM PROCESS PROPOSED AS AN ATMOSPHERIC TURBULENCE MODEL

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INTRODUCTION

Several random processes have been proposed as mathematical models for atmospheric turbulence in aeronautical applications. The amplitude modulated random process or the Press model (references 1 and 2) has been widely applied to the analysis and measurement of aircraft response to atmospheric turbulence (references 3 to 7, for example). The amplitude modulated process accounts for the local variations of the turbulence in combination with a random modulation of the amplitude. In reference 8 this process was extended by the addition of an independent mean value process, based upon a reinterpretation of the random process described in references 9 to 12. The resulting amplitude-modulated-plus-mean (AMPM) random process allows more versatility in modeling the measured properties of atmospheric turbulence. The basic mathematical properties of the AMPM process, including the first and second order properties and the characteristic function of general order, have been developed in reference 8.

The correspondence between the properties of the AMPM process and the associated properties of both atmospheric turbulence and aircraft response are examined on a qualitative basis in the present study, which consists of three parts. The first part is an interpretation of the AMPM random process in terms of the physical structure of atmospheric motions. The second part is an examination of the considerations involved in application of the AMPM process as a model for atmospheric motions. The third part is an evaluation of the effect of the random mean value variation upon aircraft response. A procedure for the development of aircraft strength design criteria is outlined. The mathematical properties of the AMPM process are reviewed in Appendix A.

SYMBOLS

A	standard deviation of either the subscripted process or (without subscript) the R process
a	filter constant of aircraft plunging motion
b	standard deviation of S process
b_i	amplitude intensity parameter
c	standard deviation of M process
c_i	mean value intensity parameter
$\bar{c}$	mean aerodynamic chord length of wing
$E[\ ]$	ensemble average
erf ( )	error function (reference 13)
erfc ( )	complementary error function = 1 - erf ( )
L	scale of turbulence of subscripted process
$M_4(\ )$	fourth order flatness factor or kurtosis (fourth moment ÷ square of second moment)
m, M	mean value random process
$N(\ )$	expected frequency of positive slope crossings of indicated level
N_{or}	expected frequency of positive slope zero crossings of R process
P_i	probability parameter
r, R	local random process
s, S	amplitude random process
T	integral scale of subscripted process
t	time
U_σ	gust intensity paramcter
V	aircraft speed

w, W	amplitude-modulated-plus-mean (AMPM) random process
w	aircraft plunging velocity
w_g	vertical velocity component of atmospheric motion
w_o	deterministic mean value of w
x	intensity random variable
z, Z	amplitude modulated random process
α	ratio of standard deviations of Z and M processes
ε	ratio of N_o of subscripted process to N_{or}
μ_g	aircraft mass parameter
ρ	autocorrelation function of subscripted process, correlation coefficient of subscripted variables
σ	standard deviation of subscripted process
τ	time difference variable
$\Phi(\omega)$	power spectral density function of subscripted process
$\phi(\omega)$	normalized power spectral density function of subscripted process, $0 \leq \omega \leq \infty$
$\Psi(\tau)$	autocovariance function of subscripted process
ω	frequency, Fourier transformation variable of τ
ω_n	natural frequency of dynamic system

Subscripts:

b	breakpoint
in	input random process
out	output random process
q	quasi-steady

PHYSICAL INTERPRETATION OF THE AMPM PROCESS

The AMPM process is formed by the sum of an amplitude modulated and a mean value random process. The amplitude modulated process is formed by the product of a local R component process and a slowly varying amplitude S component process which modulates the local process.

$$z(t) = r(t)\ s(t) \qquad (1;\ A1)^*$$

The amplitude modulated process accounts for some of the Gaussian and non-Gaussian aspects of atmospheric turbulence measurements. For a short sample of the amplitude modulated process, the slowly varying amplitude component is approximately constant; measurements of the process have approximately Gaussian properties. For a long sample, the amplitude component varies as a random function of time; measurements of the process have strongly non-Gaussian properties. The defining relations for the AMPM (amplitude-modulated-plus-mean) or total process are

$$w(t) = z(t) + m(t) \qquad (2;\ A2)$$

$$w(t) = r(t)\ s(t) + m(t) \qquad (3;\ A3)$$

*The dual equation numbers identify equations which are discussed in Appendix A. Equation (1; A1) is both equation (1) of the text and equation (A1) of Appendix A.

The three component processes are identified as the local R, the amplitude S, and the mean value M.* Following the development of references 9 to 12 the three component processes are specified to be independent, stationary, and Gaussian with zero mean values.

A physical interpretation of the AMPM random process, which considers both recorded atmospheric turbulence data and general turbulence properties, is presented in this section. Two aspects of the random process are of primary interest. The first is the addition of the independent mean value component to the amplitude modulated process. The second is the interpretation of the mean value component as slowly varying relative to the amplitude modulated or, more specifically, relative to the local R component process. This aspect is a reinterpretation of the original development of references 9 to 12, which assigned the same integral scale value to the three component processes.

The qualitative effect of slowly varying changes in both the amplitude and the mean value can be evident in measured atmospheric turbulence data. Figure 1 shows an example of measurements of the three velocity components of atmospheric motion. The data show two periods of moderate turbulence superimposed on a background of light turbulence. These two periods show significant increases in the amplitude of the local variations. Slow, random variations of the mean value are also apparent. The vertical velocity component shows a dominant amplitude modulation with relatively little

*The term mean value process is used for $m(t)$, since it appears as a random variation in the mean value of the local R process. Care must be used to distinguish between this mean value process and the mean value of any measurement of the AMPM process.

variation of the mean value. The two horizontal components, particularly the lateral, show large, slow variations in the mean value.

The set of data in figure 1 is a case of clear air turbulence related to high-altitude wind shear. The data were recorded above the Death Valley area of California at an altitude of 13 km with an airspeed of 183 m/s. The average wind direction was almost perpendicular to the flight path. The data were obtained as part of the MAT (Measurement of Atmospheric Turbulence) program at NASA-Langley Research Center (reference 14). Special care was taken in the measurement of the long wavelength part of the atmospheric motion, particularly in removing the low frequency aspects of the aircraft motion (reference 15). Thus the slow variation of the mean value shown in figure 1 is atmospheric motion; the residual aircraft motion in the data is negligible.

The properties of the AMPM process can be interpreted by examining the associated power spectral density function, which is the sum of the functions of the independent amplitude modulated and mean value processes.

$$\Phi_w(\omega) = \Phi_z(\omega) + \Phi_m(\omega) \qquad (4;\ A23)$$

The power spectral density function consists of two terms with significantly different frequency dependence. The composition of the resulting power spectral density function is shown in schematic form in figure 2. The term corresponding to the mean value process has a predominantly low frequency content, since this component is slowly varying with respect to the local R component process. The term corresponding to the amplitude modulated process has a predominantly high frequency content, which is associated with the local R component process. (The effect of the amplitude process upon the spectral properties of this term is relatively small as will be shown subsequently). The higher frequency term, corresponding to the amplitude modulated process, is currently accounted for in aeronautical applications. The lower frequency term is an additional effect which is included in the AMPM process.

The structure of the AMPM process suggests a possible correspondence with the basic composition of atmospheric motion, which consists of several different types of motion (references 16 and 17, for example). The primary classification divides atmospheric flow into two basic kinds of motion: internal gravity waves (winds or drafts) and turbulence. Based upon the associated ranges of frequency values, the internal gravity waves would correspond to the lower frequency mean value process and the turbulence would correspond to the higher frequency amplitude modulated process. The resulting separation of the associated power spectral functions into two terms in the manner of figure 2 appears in studies of measured atmospheric motions (references 18, 19 and 20, for example). A specific example of the separation of the spectral function into two distinct terms is given in reference 21, which includes an examination of the transfer of energy between the terms.

The possible correspondence between the AMPM process and the composition of atmospheric motions must be thoroughly examined before it is established on a firm basis. Such an examination must consider the basic mathematical properties of the random process and the corresponding physical properties of atmospheric motion. In particular, the random process specifies two important properties. First, the AMPM process specifies that the two kinds of atmospheric motion have significantly different probabilistic structures, the lower frequency term being Gaussian and the higher frequency term being formed by the product of two independent Gaussian processes. This difference is reflected in all associated properties, for example, the probability density function and the associated flatness factors. Studies such as references 18 to 21, which show two terms in the spectral functions, do not examine the probabilistic structure associated with these terms. Second, the AMPM process specifies that the two terms are statistically independent. Since the two terms consequently can not interact, the AMPM process can not account for the interaction due to nonlinear coupling between the two kinds of motion, which is an essential physical property (reference 16, for example). One possible means of removing the independence property is the introduction of correlation between the amplitude and mean value component processes. This modification and the properties of the resulting AMPM process are discussed in appendix B, where it is shown that the effects of the correlation upon the probabilistic structure are small if the amplitude and mean value processes are weakly correlated.

An associated question is the determination of the values of the physical parameters associated with the random mean value variation in atmospheric motions. Although the answer to this question requires an extensive examination of atmospheric data, a preliminary estimate is made to determine representative values for use in the subsequent discussion. The first quantity of interest is the α parameter, which is the ratio of the standard deviations of the amplitude modulated and the mean value processes.

$$\alpha = \frac{\sigma_z}{\sigma_m} = \frac{Ab}{c} \qquad (5;\ A14)$$

In reference 9 a value of slightly less than one is suggested for the α parameter of the vertical velocity of atmospheric motion, based upon examination of measured data. The value of one is used as a representative value of the α parameter in the subsequent discussion.* The second quantity of interest is the ratio of the integral scale values of the mean value and the amplitude modulated processes. Based upon published spectral data which show the existence of both processes (references 18 to 21), the value of ten is used in the subsequent discussion as a representative value of this ratio for the vertical velocity component of atmospheric motion.

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*If the α parameter is equal to one, then $b = c$ since A is given a unit value for turbulence processes. The condition of equal standard deviations ($b = c$) does not imply that the values of the amplitude and the mean value processes are equal within any short segment of turbulence, since those values are only member functions of their total ensembles.

APPLICATION OF THE AMPM PROCESS AS AN ATMOSPHERIC TURBULENCE MODEL

The considerations involved in modelling the properties of atmospheric motions with the AMPM process are examined in this section. The primary topic is the effect of the slowly varying mean value component, since the presence of this component is the difference between the amplitude modulated and the AMPM process. The discussion begins with an examination of the correlation and spectral properties of the AMPM process. The relationships between these dynamic properties and the probabilistic structures of both the AMPM process and atmospheric motions are then examined. The effect of the mean value variation upon the exceedance expression of the AMPM process is also examined.

Correlation and Spectral Properties

The mean value variation has a strong effect upon the autocorrelation function of the AMPM process, particularly for large values of the time difference. There are corresponding effects upon the power spectral density function, particularly for small values of the frequency. In the present understanding of atmospheric turbulence properties there are open questions on the spectral properties at low frequency values and consequently on the value of the integral scale (reference 14).

The (normalized) autocorrelation function of the AMPM process is determined from the corresponding functions of the three component processes.

$$\rho_w(\tau) = \frac{\alpha^2}{\alpha^2 + 1} \rho_z(\tau) + \frac{1}{\alpha^2 + 1} \rho_m(\tau) \qquad (6;\ A31a)$$

where $\rho_z(\tau) = \rho_r(\tau)\ \rho_s(\tau)$.

The corresponding relation for the normalized power spectral density function is the Fourier transformation of equation (6).

$$\phi_w(\omega) = \frac{\alpha^2}{\alpha^2 + 1}\,\phi_z(\omega) + \frac{1}{\alpha^2 + 1}\,\phi_m(\omega) \qquad (7;\ A31b)$$

The preceding relations show the relative contributions of the amplitude modulated and the mean value processes to the functions of the AMPM process. The relative contributions are determined by the α parameter. For small values of the α parameter, the functions of the mean value component are dominant; for large values of the α parameter, the functions of the amplitude modulated process are dominant. The autocorrelation and spectral functions of the AMPM process must be distinguished from those of the three component processes. Most discussions in the aeronautical literature and most applications to aircraft design (references 3 to 6, for example) consider only the functions of the local R component process.

One measure of the dynamic properties of a random process is the integral scale, which is the integral of the autocorrelation function over the range of positive time values.

$$T_w = \int_o^\infty \rho_w(\tau)\ d\tau \qquad (8)$$

The integral scale is related to the zero frequency value of the normalized power spectral density function, the relation following from equation (8) and from the definition of the Fourier transformation.

$$T_W = \frac{\pi}{2} \phi_W(0) \qquad (9)$$

These relations for the integral scale follow the notation used in the atmospheric turbulence and the aeronautical literatures (reference 5, for example). However, the autocorrelation is usually defined as a function of the spatial dimension; the corresponding integral scale is referred to as the scale of turbulence. Also, the present notation uses W to denote any of the vector components of the atmospheric velocity, not specifically the vertical component.

The integral scale value of the AMPM process is determined by those of the amplitude modulated and mean value processes. The relationship is obtained by combining the definition of the integral scale, equation (8), and the relation between the autocorrelation functions, equation (6).

$$T_W = \frac{\alpha^2}{\alpha^2 + 1} T_z + \frac{1}{\alpha^2 + 1} T_m \qquad (10)$$

In the concept of the AMPM process, the mean value process is slowly varying relative to the local R component and also relative to the amplitude modulated process. The value of the integral scale of the mean value process is accordingly much greater than that of the amplitude modulated process. Thus by equation (10) the presence of the mean value variation strongly influences the value of the integral scale of the AMPM process. This can be true even if the contribution of the mean value component to the standard deviation of the total process is fairly small, that is, if the value of the α parameter is large.

The amplitude component process also influences the integral scale of the AMPM process, although to a much smaller extent. This effect can be seen by a specific example. The functions of both the local R and the amplitude component processes are assumed to have the form for the longitudinal velocity component in the Dryden model for turbulence (reference 5). Equivalently, the Gaussian R and S components are assumed to be Markov processes. Their autocorrelation functions are

$$\rho_r(\tau) = e^{-|\tau|/T_r} \tag{11a}$$

$$\rho_s(\tau) = e^{-|\tau|/T_s} \tag{11b}$$

The autocorrelation function of the amplitude modulated process is the product of the autocorrelation functions of the local and amplitude component processes by equation (6).

$$\rho_z(\tau) = e^{-|\tau|/T_z} \tag{12}$$

where $\frac{1}{T_z} = \frac{1}{T_r} + \frac{1}{T_s}$

Under the modulation concept the integral scale of the amplitude process is much larger than that of the local R process. In this case the amplitude modulation has the effect of slightly reducing the integral scale value of the original R process. For example, if the integral scale of the amplitude process is ten times that of the local R process, then the amplitude modulation reduces the integral scale of the original process by about nine percent. The amplitude component accordingly has a minor effect on the correlation and spectral functions of the amplitude modulated process. Conversely, it will be difficult to separate the autocorrelation and spectral functions of the local R and the amplitude components from the measured functions of the amplitude modulated process. The preceding example is a special case where the separation is impossible.

Effect of Dynamic Properties Upon Probabilistic Structure

The mean value variation has significant effects upon the probabilistic structure of the AMPM process. These effects are related to the dynamic properties of the process, since the AMPM process is the sum of two independent processes which have significant differences in both their dynamic properties and probabilistic structures.

As an example of the effects of the mean value variation, the differences between the AMPM random process and its first derivative are considered. The defining relation for the AMPM process is

$$w(t) = r(t)\ s(t) + m(t) \tag{3}$$

The quasi-steady derivative of the process is considered.*

$$\dot{w}_q(t) = \dot{r}(t)\ s(t) \qquad (13;\ A50)$$

*The quasi-steady form of the AMPM process is used in the discussion, that is, the dynamic properties, such as the derivatives, of the slowly varying amplitude and mean value components are omitted. The dynamic properties of the local R component are retained.

For the quasi-steady derivative the value of the α parameter is infinite. Consequently the AMPM process and its first derivative generally have different relative contributions from the amplitude modulated and the mean value processes, that is, they have different values of the α parameter. The difference is shown by the values of the associated fourth order flatness factors.

$$M_4(w) = \frac{3(3\alpha^4 + 2\alpha^2 + 1)}{(\alpha^2 + 1)^2} \qquad (14;\ A16)$$

$$M_4(\dot{w}_q) = 9 \qquad (15)$$

The flatness factor of the derivative follows from the general form of equation (14) with an infinite value of the α parameter. The value of nine also follows from the form of the derivative, equation (13), which is the product of two independent Gaussian processes.

The amplitude modulated process and its quasi-steady first derivative have similar properties.

$$z(t) = r(t)\ s(t) \qquad (1)$$

$$\dot{z}_q(t) = \dot{r}(t)\ s(t) \qquad (16)$$

The amplitude modulated process and its quasi-steady derivative have the same basic form: the product of a Gaussian local process and the same amplitude process. Consequently their probability density functions have the same functional form and their fourth order flatness factors have the same value.

$$M_4(z) = M_4(\dot{z}_q) = 3M_4(s) \tag{17}$$

The flatness factor has a value of nine if the amplitude process is Gaussian. Other distributions have been suggested for the amplitude process, enabling the resulting amplitude modulated process to account for a range of values for the flatness factor. However, in all cases the resulting amplitude modulated process and its quasi-steady derivative must have the same value for the flatness factor.

Consequently one effect of the slow, random mean value variation is that it allows the AMPM random process and its derivative to have different probabilistic structures. This property cannot be accounted for by amplitude modulation alone. There are several studies which show that this property appears in measurements of atmospheric motions. These studies present a strong argument that the inclusion of the mean value variation gives a better representation of the properties of atmospheric motions. One study is reference 22, which examined turbulence related to storms. The probability distribution functions of the atmospheric velocity components and their gradients were examined and presented in graphical form. Comparison of the measured probability distributions shows that the functions of the velocity components and their gradients have significantly different functional forms. The gradients show a much greater deviation from the Gaussian form, a property which is consistent with the form of the flatness factors of the AMPM process, equations (14) and (15).

A more definitive study is that of Chen, reference 23. Atmospheric turbulence data were obtained from several sources: low altitude, severe storm, and high altitude clear air turbulence. The properties of the three velocity components and their gradients were examined, including computation of the fourth order flatness factors. The results show a consistent trend of higher values of the flatness factor for the gradients than for the velocity itself. The flatness factors for the velocity generally had values somewhat greater than three, with a few values being slightly less than or equal to three. The flatness factors for the gradients had values between three and seven. These values are less than the value of nine for the derivative of the quasi-steady process. However values below nine are possible if transition effects* are present or if the quasi-steady approximation is not completely satisfied, particularly since the results are based upon the gradients and not the derivatives of the velocity components. Thus the results of reference 23 are generally consistent with the properties of the AMPM process.

The study of references 24 and 25 presents data for atmospheric motions which were measured by tower-based instrumentation. The probability density functions and the associated flatness factors were determined. The data show higher values of the flatness factor for the velocity gradients than for the velocity itself. Another set of results was obtained in an examination of the effects of high-pass filtering upon the measured data. The filtering consisted of removing a running mean value from the data. The filtering calculation considered several time periods, the shorter time periods removing more of the lower

*Transition effects are related to the development of the slowly varying amplitude and mean value components from their initial values in any measurements of the AMPM process (reference 8).

frequency content of the velocity components. Using the concept of the AMPM process, this operation represents an approximate but direct removal of the random mean value variation. The resulting flatness factors show a definite trend toward higher values as the filtering operation removes more of the low frequency content. These results indicate that the inclusion of a slow mean value variation gives a better representation of the measured atmospheric motions. The highest values of the flatness factor were generally less than the value of nine for the quasi-steady derivative, which represents the complete removal of the mean value variation. The data also show some values of the flatness factor less than three and some relatively large values for the skewness (normalized third moment). These results can not be modelled by the AMPM process in its fully developed form, but could represent the transitional form of the process (reference 8).

The difference between the probabilistic structures of the velocity and the velocity gradient of atmospheric motions has been examined in other studies where possible explanations of this property have been suggested. For example, reference 26 uses this property to recommend that the gradient rather than the turbulence velocity is the more important experimental quantity. The turbulence velocity is considered to be a Gaussian process, thus omitting the amplitude modulation effect. The analysis of reference 26 shows that the Gaussian property is lost in taking the gradient of the process. However the gradient, which is the difference between the values of the process at two time values, is a linear combination of Gaussian random variables and therefore must itself be Gaussian (reference 27). Consequently the development of reference 26 can not explain the difference between the probabilistic structures of the velocity and the velocity gradient of atmospheric motion.

In summary, the probabilistic structure of the AMPM random process possesses some mathematical properties that are not present with amplitude modulation alone. In particular, the probabilistic structure of the AMPM process can vary as different ranges of frequency values are emphasized. Measurements of atmospheric motions show corresponding properties, for example, the velocity gradients show more deviation from Gaussian properties than the velocity itself. Atmospheric motions thus show significant properties which can be accounted for by the AMPM process. It is noted that this conclusion is in disagreement with that of references 11 and 12, which are the original development of the basic random process. However the development of those references did not include the differences between the dynamic properties of the three component processes, which are essential for the existence of the indicated dynamic properties of the AMPM random process.

Exceedance Expression

The mean value variation has significant effects on the exceedance expression of the AMPM process. An analytical relation for the exceedance expression has been developed by using the quasi-steady approximation. The relation is expressed as an exceedance ratio: the ratio of the expected frequency of crossings of a given level of the AMPM process to the expected frequency of crossings of the zero level for the R component process.

$$\frac{N(w)}{N_{or}} = \frac{1}{2} e^{1/2\alpha^2} \{e^{-w/\alpha c}[1 + \mathrm{erf}(\frac{w}{\sqrt{2}c} - \frac{1}{\sqrt{2}\alpha})]$$

$$+ e^{w/\alpha c} \mathrm{erfc}(\frac{w}{\sqrt{2}c} + \frac{1}{\sqrt{2}\alpha})\} \qquad (18;\ A54)$$

where $w \geq 0$

$$N(-w) = N(w)$$

The exceedance ratio is plotted in figure 3 as a function of the ratio of the level of the AMPM process to its standard deviation, and for several values of the α parameter. For large values of the α parameter the exceedance ratio approaches the exponential form of the amplitude modulated process, equation (A57). For small values of α the exceedance ratio approaches a Gaussian form related to the mean value component process, equation (A58). The intermediate cases show a combination of those two functional forms. At the zero level the exceedance ratio is not equal to one since the exceedances are ratioed to the expected number of zero crossings of the R component process and not that of the AMPM process. The exceedance ratio can be strongly influenced by the quasi-steady approximation at low levels of the process and for small α values, since the dynamic properties of the mean value component are absent. The exact form of the exceedance expression is examined in appendix C.

The contribution of the mean value variation to the exceedance ratio of the AMPM process is not clearly shown in figure 3, since the mean value process affects the non-dimensional level of the AMPM process through its standard deviation, which depends upon the α parameter.

$$\sigma_w^2 = c^2(\alpha^2 + 1) \qquad (19;\ A15)$$

The contribution of the mean value variation is more clearly shown in figure 4, where the exceedance ratio is plotted as a function of the ratio of the level of the AMPM process to the standard deviation (Ab) of the amplitude modulated process, which is independent of the α parameter. Figure 4 thus shows the effect of the mean value variation relative to the amplitude modulated process, which corresponds to an infinite value of the α parameter. Figure 4 shows that the mean value variation can either increase or decrease the exceedances depending upon the values of both the process level and the α parameter. The mean value variation can significantly increase the exceedances at large values of the process as the value of the α parameter approaches zero. In the limit of large values of the process, the exceedance ratio approaches the exponential form of the amplitude modulated process, but with the exceedances increased by a constant factor whose value follows from equation (A59).

$$\lim_{w \to \infty} \frac{N(w;\ \alpha)}{N(w;\ \alpha=\infty)} = e^{1/2\alpha^2} \qquad (20)$$

This factor indicates the upward shift, with decreasing value of α, of the limiting exponential form of the exceedance ratio curves of figure 4.

The effect of the mean value variation upon the exceedances can be seen in the measurements of the velocity of atmospheric motion in figure 1. The lateral velocity component shows crossings at high levels which are related to the combination of the rapid local variations superimposed on the large, slow variations of the mean value. For example, the crossings of the -20 m/s level are related to the mean value variation. Crossings of this level do not occur in the other two velocity components which show similar local variations but not the large variations in the mean value. Also, the mean value variations cause the crossings at the zero level of the lateral velocity component to be much less than those of the other two velocity components. The measured data thus show the decreased exceedances at low levels and the increased exceedances at high levels, due to the mean value variations, which are indicated in figure 4.

Experimental Aspects

Intensity Process

It was estimated previously that the standard deviations of the amplitude modulated and mean value processes are approximately equal for the vertical velocity component of atmospheric motion. The condition of equal standard deviations for these two processes suggests an explanation for another aspect of measured atmospheric turbulence data. Attempts have been made to determine the distribution of the "intensity" process directly from measured data, in order to verify the assumed Gaussian distribution of the amplitude process. Reference 28, for example, shows that the measured data suggest a Rayleigh rather than a Gaussian distribution for the intensity. The measured data were obtained from two studies, references 29 and 30, which involved direct measurement of atmospheric motions. The resulting intensity random variable is

$$x = \sqrt{s^2 + m^2} \tag{21}$$

where $E[r^2] = 1$

This assumes that the data samples are of appropriate length to allow the full development of the local R component without significant development of the amplitude and mean value component processes. The resulting intensity, x, is a random variable which depends upon the values of both the amplitude and the mean value processes. Following the basic formulation, the random amplitude and mean value variables are independent and Gaussian with zero mean values. If they also have equal standard deviations, the intensity x of equation (21) has a Rayleigh distribution (reference 27). Thus the distribution of turbulence intensity indicated by the results of reference 28 suggests a correspondence with the properties of the AMPM random process.

Experimental Procedures

The determination of the spectral properties of the mean value variations in atmospheric motions presents a difficult experimental problem. There are several problems in the accurate measurement and analysis of atmospheric motions in the range of low frequency values associated with the mean value process, especially when the data are measured with a flight vehicle. References 5 (appendix C), 15 and 31 give discussions of the difficulties associated with the removal of the extraneous effects of aircraft motion from the measurements. Another problem is the accurate computation of the spectral functions. Reference 32, for example, shows that significant spurious effects can be introduced in the low frequency range by the numerical procedures used to compute the spectral functions. Thus caution must be used in both the measurement and the analysis of atmospheric motions in order to determine the spectral properties of the mean value variation.

EFFECT OF THE MEAN VALUE VARIATION UPON AIRCRAFT RESPONSE

The effect of the mean value variation in atmospheric motions upon aircraft response is examined in the present section, using estimates of the dynamic properties of both the mean value process and significant aircraft response quantities. Also, a procedure for developing design criteria for aircraft structural strength is outlined.

AMPM Process and Aircraft Response

The effect of the AMPM process upon aircraft responses can be examined by considering a simple mathematical model, in which the aircraft is a rigid body in level flight at constant airspeed and is allowed to move in plunging motion only. The corresponding equation of motion is

$$\dot{w} + aw = aw_g \tag{22}$$

where w is the plunging velocity of the aircraft, $\dot{w}$ the plunging acceleration and w_g is the vertical velocity of the atmospheric motion. The plunging velocity has the characteristics of a low-pass filter; the frequency response function is approximately constant for frequency values below the filter constant a and decreases as a minus-one power of frequency above a. The acceleration has the characteristics of a high-pass filter; the frequency response function increases proportionally to frequency below a and is approximately constant for frequency values above a. The filter constant of the first-order system is related to aircraft response parameters.

$$a = \frac{V}{\bar{c}\mu_g} \tag{23}$$

where V is the aircraft speed, $\bar{c}$ the mean aerodynamic chord of the wing, and μ_g the dimensionless aircraft mass parameter (reference 33)*.

The aircraft plunging motion is developed for the response to atmospheric motion which is modeled by the AMPM process. The spectral properties of both the local R and the mean value components are taken to be those of the transverse velocity component in the Dryden model of turbulence (reference 5).

$$\phi_r(\omega) = \frac{\omega_{rb}}{\pi} \cdot \frac{\omega_{rb}^2 + 3\omega^2}{(\omega_{rb}^2 + \omega^2)^2} \tag{24}$$

where $\omega_{rb} = 1/T_r = V/L_r$ and L_r is the scale of turbulence of the R component process. For the first-order differential equation, equation (22), and the normalized spectral density function for the transverse velocity component, equation (24), the variances of the response are (reference 34)

$$A_r^2 = \frac{a + \frac{1}{2}\omega_{rb}}{a(a + \omega_{rb})^2} \tag{25a}$$

$$A_{\dot{r}}^2 = \frac{\omega_{rb}(\frac{3}{2}a + \omega_{rb})}{(a + \omega_{rb})^2} \tag{25b}$$

The aircraft plunging response to the AMPM process is developed from the previous relations by using the quasi-steady approximation for the

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*$\mu_g = 2(W/S)/(g\bar{c}C_{L_\alpha}\rho)$, where (W/S) = wing loading, g = gravity constant, C_{L_α} = lift curve slope, and ρ = air density.

amplitude modulated process. The variance of the response to the AMPM process is the sum of the variances for the responses to the independent amplitude modulated and mean value processes. The α parameters of the response are determined by the standard deviations of the response to the local R and mean value components and by the α parameter of the AMPM process, for example,

$$\alpha_w = \frac{A_r}{A_m} \alpha_{w_g} \tag{26; A48}$$

The α parameters of the plunging response are determined by combining the previous relations and introducing the appropriate notations for the two component processes.*

$$\alpha_w^2 = \frac{a + \frac{1}{2}\omega_{rb}}{a + \frac{1}{2}\omega_{mb}} \left(\frac{a + \omega_{mb}}{a + \omega_{rb}}\right)^2 \alpha_{w_g}^2 \tag{27a}$$

$$\alpha_{\dot{w}}^2 = \frac{\omega_{rb}}{\omega_{mb}} \frac{\frac{3}{2}a + \omega_{rb}}{\frac{3}{2}a + \omega_{mb}} \left(\frac{a + \omega_{mb}}{a + \omega_{rb}}\right)^2 \alpha_{w_g}^2 \tag{27b}$$

The α parameters of the response quantities depend upon the three frequency constants: that of the plunging response, a, and those of the two component processes, ω_{rb} and ω_{mb}.

The preceding analysis is used to estimate the effects of the mean value variations in atmospheric motions upon the aircraft plunging motion

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*The dynamic properties of the mean value process have been included in the variances of the response, but have been omitted in the exceedance expression, equation (18). The quasi-steady values of the variances are obtained by setting ω_{mb} to zero.

for the present class of subsonic transports. The ratio of the frequency constants of the plunging response and the local R component process is

$$\frac{a}{\omega_{rb}} = \frac{L_r}{\bar{c}\ \mu_g} \tag{28}$$

Using typical parameters for subsonic transport aircraft, the mass parameter value is usually between 50 and 100. Using typical values of the wing chord length and the scale of turbulence, the ratio of the frequency constants has a minimum value of about one, that is, $a = \omega_{rb}$. (Larger values of the ratio of equation (28) result in larger values of α and thus less effect of the mean value variation upon the plunging acceleration.) Using the previous estimates of the physical properties of atmospheric motions, the α parameter of the vertical velocity is equal to one, and the ratio of the integral scales of the mean value and the local R components is equal to ten. With these values of the parameters, the resulting values of the α parameters and the fourth order flatness factors of the plunging response are

$$M_4(w;\ \alpha = .66) = 3.55$$

$$M_4(w_g;\ \alpha = 1.00) = 4.50 \qquad (29)^*$$

$$M_4(\dot{w};\ \alpha = 2.18) = 7.09$$

*If the quasi-steady approximation is used in determining the variances of the response to the mean value process, that is, ω_{mb} is set equal to zero, the results are $M_4(w;\ \alpha = .61) = 3.44$
$M_4(\dot{w};\ \alpha = \infty) = 9$

The results show the effects of the dynamic system response upon the relative contributions of the lower frequency mean value process and the higher frequency amplitude modulated process. For the plunging velocity, which has the characteristics of a low-pass filter, the relative contribution of the mean value process is increased, resulting in a lower value of both the α parameter and the fourth order flatness factor. The opposite effects are shown by the plunging acceleration, which has the characteristics of a high-pass filter.

The associated effects upon the quasi-steady exceedance ratios for the three quantities of equation (29) are shown in figure 5, with the process level ratioed to the standard deviation of the response to the amplitude modulated process in the same manner as figure 4. For the original AMPM process (α parameter equal to 1.00) the effect of the mean value variation is small. For the plunging acceleration (α equal to 2.18) the effect is almost negligible. For the plunging velocity (α equal to 0.66) the mean value variation significantly increases the exceedance ratio, except at the lowest response levels. The results of figure 5 show two points. First, with the presence of both the amplitude modulated and the mean value processes, the exceedance expression depends upon the dynamic properties of the system response quantity and consequently can be different for different response quantities. Second, the presence of the mean value variation can significantly increase the exceedances, particularly for a response quantity which acts as a low-pass filter. Since the critical aircraft response quantities are usually either accelerations or incremental loads which are closely related to accelerations, the effect of the mean value variations in atmospheric motions upon aircraft response are generally not significant. The effect of the mean value variation however can be important for response quantities which are related to the velocity or displacement.

The effects of the mean value variation in atmospheric motions upon aircraft response are also examined by considering quantities which have the characteristics of a narrow band-pass filter, such as low damped oscillatory modes, either whole body or elastic. For simplicity both the amplitude modulated (or equivalently the local R) and the mean value processes are assumed to have the Dryden spectral function for the longitudinal component of atmospheric turbulence (reference 5), for example,

$$\phi_r(\omega) = \frac{2\omega_{rb}}{\pi} \cdot \frac{1}{\omega_{rb}^2 + \omega^2} \tag{30}$$

where $\omega_{rb} = 1/T_r$

The power spectral density function has a simple, approximate form: for frequency values below ω_b the function is constant, and for values above ω_b the function decreases as the minus-two power of the frequency. This approximate form is plotted in figure 6 for both the mean value and the amplitude modulated (or the local R) processes. Since the mean value component is considered to be slowly varying, the integral scale value is significantly greater than that of the local R process, giving the opposite relation for the corresponding values of the frequency breakpoint.

The power spectral density function shows the basic effects of the two random processes upon the response of dynamic systems which have the characteristics of narrow band-pass filters. The associated frequency response function is dominated by a single frequency value, which is the natural frequency ω_n of the dynamic system. The relative effect of the two random processes upon the variance of the response is directly related to the values of the two power spectral density functions at the value of the system natural frequency. The effect can be visualized from figure 6. The functional properties of the composite power spectral density are divided into three regions of frequency values. First, for values of the system natural frequency which are below the frequency breakpoint of the mean value process, both spectral functions are constant. The response is dominated by the mean value process; the α parameter of the system response is smaller than that of the excitation process.

$$\omega_n < \omega_{mb} \qquad \alpha^2_{out} \approx \frac{\omega_{mb}}{\omega_{rb}} \alpha^2_{in} < \alpha^2_{in} \tag{31}$$

Second, for values of the system natural frequency which are between the two frequency breakpoints, the spectral function of the amplitude modulated process is constant while the function of the mean value process decreases with increasing frequency. Either process can be dominant in the system response, depending upon the value of the natural frequency.

$$\omega_{mb} < \omega_n < \omega_{rb}, \qquad \alpha^2_{out} \approx \frac{\omega_n^2}{\omega_{rb}\,\omega_{mb}}\,\alpha^2_{in} \tag{32}$$

Third, for values of the system natural frequency which are above the frequency breakpoint of the amplitude modulated process, both spectral functions decrease as a minus-two power of frequency. The response is dominated by the amplitude modulated process; the α parameter of the system response is larger than that of the excitation process.

$$\omega_{rb} < \omega_n \qquad \alpha^2_{out} \approx \frac{\omega_{rb}}{\omega_{mb}}\,\alpha^2_{in} > \alpha^2_{in} \tag{33}$$

Consequently the relative contributions of the two processes to the response of a narrow band-pass system to the AMPM process depend on the value of the natural frequency of the dynamic system relative to the values of the frequency breakpoints of the amplitude modulated and the mean value processes.

The preceding development can be used to estimate the effects of the mean value variation in atmospheric motion upon aircraft dynamic systems. The frequency breakpoint of the amplitude modulated process, which is approximately equal to that of the local R component process, can be estimated from the values of the scale of turbulence currently suggested in the aeronautical literature. There is an open question on the most appropriate value, but estimates vary from about 250 m (800 ft, reference 35) up to the value of 762 m (2500 ft) used in aircraft design criteria, references 3 and 4. For the present class of subsonic transports this gives values of ω_{rb} between about 0.2 and 1.0 radians per second.* The dominant whole body

*The frequency breakpoint is given by the relation $\omega_{rb} = \frac{\gamma V}{L_r}$, where the constant γ locates the "knee" of the spectral curve and has a value between 1.0 and 1.5 for the atmospheric turbulence models used in aeronautical applications.

modes of aircraft, the Dutch roll (lateral) and the short period (longitudinal) modes, have representative values of about one and two radians per second, respectively, for subsonic transport aircraft. The values of the primary response frequency thus fall mostly in the third region of the spectral function, which is dominated by the amplitude modulated process. Consequently, the value of the α parameter for most aircraft response quantities is greater than that of the original atmospheric velocity components.

The effects of the mean value variation in atmospheric motions are thus fairly small for most aspects of the response of current subsonic transport aircraft, for example, response quantities such as accelerations and incremental loads which are related to acceleration quantities. The mean value variation will be more important for response quantities related to the velocity or displacement. The mean value variation may significantly effect the long period or phugoid mode associated with speed and altitude perturbations of aircraft (reference 36). This mode is usually ignored in the calculation of aircraft loads. However, there are cases where the effects of the phugoid mode may be significant. The effects of the mean value variation may become important for aircraft which are significantly larger than the current class of subsonic transports, due to the generally lower values of the frequencies of the whole body modes. The mean value variation is more important for higher flight speeds due to the corresponding higher values of the frequency breakpoint of the amplitude modulated process. A similar conclusion applies to flight at low altitudes due to the associated lower values of the scale of turbulence (reference 35).

The previous conclusions depend upon the condition that the standard deviations of the amplitude modulated and mean value process are approximately equal in atmospheric motions, that is, the value of the α parameter is approximately one. This condition is used as a preliminary estimate of the standard deviation of the mean value process. If measured atmospheric data show cases where the standard deviation of the mean value component is significantly different, then the previous conclusions must be revised accordingly. An associated question is the isotropy of the atmospheric motions. Examination of the measured data of references 18 and 19, which show the separation of the spectral functions into two fairly distinct parts, suggests that the mean value component process of the atmospheric velocity can be strongly anisotropic, giving significantly larger contributions to the horizontal velocity components. These data suggest that the α parameter can have significantly lower values for the lateral than for the vertical velocity component of atmospheric motion. The lateral response of the aircraft may accordingly be more significantly influenced by the mean value variation.

The presence of the mean value variation in atmospheric motions has an indirect effect upon the calculation of aircraft response through the specification of the appropriate integral scale value for the local turbulence variations. The development of the integral scale values in measured atmospheric data requires the determination of two primary quantities: the integral scale of the mean value process and that of the local turbulence variations. Previous determinations of the integral scale value have not directly accounted for the mean value variations. This effect was included in the determination of the integral scale of the local turbulence variations, that is, the local R component of the total process. This suggests that these determinations of the scale of turbulence, and consequently the criteria value of 762 m (2500 ft), are too high. It is noted that suggestions for reducing the present scale of turbulence values have been made elsewhere (reference 35, for example).

Development of Aircraft Design Criteria

The application of the AMPM process to long-term measurements of atmospheric motions and the associated aircraft response requires the introduction of the concept of several types of turbulence. This concept was used in the original development of the amplitude modulated model (reference 1) and has extensive experimental justification. The original random process is modified by introducing a conditional process, which is conditional on the type of turbulence. The probability density functions are weighted by the probability of the occurrence of each type. The quasi-steady exceedance expression of the modified AMPM process is obtained from the original expression, equation (A53).

$$N(w) = \frac{1}{2} N_{or} \sum_i P_i \exp\left(\frac{A_m^2 c_i^2}{2A_r^2 b_i^2}\right)$$

$$\left\{\exp\left(\frac{-|w - w_o|}{A_r b_i}\right)\left[1 + \operatorname{erf}\left(\frac{|w - w_o|}{\sqrt{2}\, A_m c_i} - \frac{A_m c_i}{\sqrt{2}\, A_r b_i}\right)\right]\right. \qquad (34)$$

$$\left. + \exp\left(\frac{|w - w_o|}{A_r b_i}\right)\operatorname{erfc}\left(\frac{|w - w_o|}{\sqrt{2} A_m c_i} + \frac{A_m c_i}{\sqrt{2} A_r b_i}\right)\right\}$$

Historically two types of turbulence are considered. These are often referred to as nonstorm (i=1) and storm (i=2) turbulence. There is an additional term, representing the probability of no turbulence, which is usually omitted in the literature.

The exceedance expression of the modified process, equation (34), can be applied to both the velocity components of atmospheric motion and the resulting dynamic system response under the quasi-steady assumption. For the case of an atmospheric velocity component, the deterministic mean value (w_o) is zero and the two standard deviation factors (A_r and A_m) have unit values by definition.

The mean value variation introduces an additional atmospheric parameter. Consequently there are three atmospheric parameters for each type of turbulence: the probability parameter P_i, the amplitude intensity parameter b_i, and the mean value intensity parameter c_i. No data are available on representative values of the mean value intensity parameters since previous examainations of measured data have not considered the mean value variation. Also, these parameters cannot be obtained from long-term measurements of aircraft vertical acceleration, since the aircraft acceleration acts as a high-pass filter which largely removes the mean value variation.

There remains the question of the effects of the mean value variation on the related problem of the specification of structural criteria for turbulence induced loads. The present approach is the specification of a maximum exceedance level for all aircraft loads. For the amplitude modulated process this requires the computation of two quantities: the standard deviation and the expected number of zero crossings of the response to the local R component process. From these quantities, the deterministic mean value (one-g flight response) and a given set of atmospheric parameters, the exceedances of the response can be calculated. The introduction of the mean value variation requires the calculation of one additional quantity: the standard deviation of the response to the mean value process. This quantity can be determined by the same computational procedures used for determining the response to the local R component process. Once the response quantities are known, the associated exceedances can be computed from equation (34) for a given set of atmospheric parameters.

Aircraft design criteria are usually presented in terms of a gust intensity parameter.

$$U_\sigma = \frac{|w - w_o|}{A_r} \tag{35}$$

For the amplitude modulated process the exceedance ratio depends solely upon U_σ for given values of the atmospheric parameters. Consequently the aircraft design criteria can be specified as a required value of U_σ, which is a function, through the atmospheric parameters, of the altitude only. For the AMPM process this procedure is more complicated since the exceedance ratio of the response quantities depends additionally upon the α parameter. This is shown by figures 7 and 8. Figure 7 shows a set of exceedance ratios as functions of the process level, and for several values of the α parameter. The exceedance ratios are computed for an altitude of 6.1 km (20,000 ft) using the atmospheric parameters of references 4 and 37. No assumption on the values of the atmospheric parameters of the mean value process have been made, except that the α

parameters of the two types of turbulence are equal. From figure 7 the combinations of the gust intensity parameter U_σ and the α parameter which correspond to a given value of the exceedance ratio are determined. These combinations are plotted in figure 8, which shows the U_σ and α parameter combinations which correspond to constant values of the exceedance ratio at the given altitude. It is noted that figure 8 also shows that the effect of the mean value variation upon the exceedance expression is very small if the α parameter of the response quantity has a value above two.

The approach outlined above is based upon the exceedance ratio and not directly upon the exceedances, since the secondary effect of the expected number of zero crossings of the local R process for the response quantities is omitted. In order to use the number of exceedances directly, all three parameters of the system response (A_r, A_m, N_{or}) must be accounted for. The resulting exceedances can be computed from equation (34).

CONCLUSIONS

The application of the AMPM (amplitude-modulated-plus-mean) random process as an atmospheric turbulence model is examined and evaluated on a qualitative basis. The effect of the mean value variation is of primary interest since this is the distinction from the amplitude modulated process (the Press model, references 1 and 2) which is currently used to model atmospheric turbulence in aeronautical applications. It is concluded that the combination of amplitude modulation and a slow, random variation of the mean value is a better representation of measured properties of atmospheric motions. In particular, the AMPM process can account for the differences in the statistical properties of atmospheric velocity components and their gradients; these differences cannot be accounted for by amplitude modulation alone.

The correspondence between the properties of the AMPM process and the physical properties of atmospheric motions are examined. The structure of the random process suggests a possible correspondence with the structure of atmospheric motion: the lower frequency mean value process corresponding to the internal gravity waves (winds or drafts) and the higher frequency amplitude modulated process corresponding to the turbulence. The separation of the atmospheric motion into two elements appears in the power spectral density functions of atmospheric motions, which have the same general form as the functions of the AMPM process.

The response of linear dynamic systems to the AMPM process is examined. Due to the structure of the process, the relative contribution of the mean value variation to the total system response is strongly influenced by the dynamic properties of the response quantity. For response quantities which have the characteristics of a high-pass filter, such as aircraft accelerations, the relative contribution of the mean value variation is considerably reduced. For this reason, the gradients of the AMPM process show a larger deviation from Gaussian properties than the process itself. For response quantities which have significant response in the lower frequency range, such as aircraft displacements and velocities, the mean value variation can significantly increase the exceedances of the response at high response levels. Using estimates of aircraft dynamic properties, it is concluded that the effects of the mean value variation upon aircraft loads are small in most cases. However, the effects can be important in the measurement and interpretation of atmospheric motions.

APPENDIX A

SUMMARY OF MATHEMATICAL PROPERTIES OF AMPM PROCESS

The mathematical properties of the amplitude-modulated-plus-mean or the AMPM random process are reviewed in this appendix.

Process Definition and General Properties

The AMPM process is formed from three independent random processes, which are identified as the local R, the amplitude S, and the mean value M components. The modulation of the local process by the amplitude component forms the amplitude modulated process Z.

$$z(t) = r(t)\ s(t) \qquad \text{(A1)*}$$

The sum of the amplitude modulated and the mean value processes forms the AMPM process W.

$$w(t) = z(t) + m(t) \qquad \text{(A2)}$$

$$w(t) = r(t)\ s(t) + m(t) \qquad \text{(A3)}$$

The properties of the AMPM process are determined by the defining relation, equation (A3), and the properties of the three component processes, which are specified to be stationary and Gaussian with zero mean values. The notation for the variances of the component processes is

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*The numbering of the equations corresponds to that in reference 8; equation (A1) is equation (1) of that reference.

$$E[r^2] = A^2$$

$$E[s^2] = b^2 \qquad (A10)$$

$$E[m^2] = c^2$$

The moments of the AMPM process are determined by the defining relation; for example, the variance is

$$E[w^2] = A^2 b^2 + c^2 \qquad (A13)$$

The ratio of the standard deviations of the independent amplitude modulated and mean value processes is a basic parameter of the AMPM process.

$$\alpha = \frac{Ab}{c} \qquad (A14)$$

The moments of the AMPM process can be expressed in terms of the α parameter; for example, the variance is

$$E[w^2] = \sigma_w^2 = c^2(\alpha^2 + 1) \qquad (A15)$$

The fourth order flatness factor (or kurtosis) shows the dependence of the probabilistic structure of the AMPM process upon the relative contributions of the amplitude modulated and the mean value processes.

$$M_4(w) = \frac{E[w^4]}{E^2[w^2]} = \frac{3(3\alpha^4 + 2\alpha^2 + 1)}{(\alpha^2 + 1)^2} \qquad (A16)$$

For a zero value of the α parameter, the flatness factor has the value of three for the Gaussian mean value process. For an infinite value of the α parameter, the flatness factor has the value of nine for the amplitude modulated process, the value of nine resulting from the product of two independent Gaussian processes.

The autocovariance function of the AMPM process is related to the autocovariance functions of the amplitude modulated and the mean value processes. Using the defining relation and the independence property, the relationship is

$$\Psi_w(\tau) = \Psi_z(\tau) + \Psi_m(\tau) \tag{A22}$$

where

$$\Psi_z(\tau) = \Psi_r(\tau)\, \Psi_s(\tau)$$

The corresponding relation for the power spectral density function is

$$\Phi_w(\omega) = \Phi_z(\omega) + \Phi_m(\omega) \tag{A23}$$

The autocovariance functions can be expressed in terms of the (normalized) autocorrelation functions.

$$\Psi_z(\tau) = A^2 b^2 \rho_z(\tau)$$

(A29)

$$\Psi_m(\tau) = c^2 \rho_m(\tau)$$

where

$$\rho_z(\tau) = \rho_r(\tau)\rho_s(\tau)$$

Combining the previous relations and introducing the α parameter, the corresponding relations for the autocorrelation function and the normalized power spectral density function, which is the Fourier transformation of the autocorrelation function, are

$$\rho_w(\tau) = \frac{\alpha^2}{\alpha^2 + 1}\rho_z(\tau) + \frac{1}{\alpha^2 + 1}\rho_m(\tau) \qquad \text{(A31a)}$$

$$\phi_w(\omega) = \frac{\alpha^2}{\alpha^2 + 1}\phi_z(\omega) + \frac{1}{\alpha^2 + 1}\phi_m(\omega) \qquad \text{(A31b)}$$

Quasi-Steady Approximation

In application as an atmospheric turbulence model, the AMPM process is interpreted as the combination of a rapidly varying local component with slowly varying amplitude and mean value components. This concept leads to the quasi-steady approximation in which the dynamic properties of the amplitude process, and possibly the mean value process, are omitted in developing the dynamic properties of the AMPM process. The quasi-steady approximation presents a simple method for the analysis of the response of linear dynamic systems to the AMPM process. Assuming that the amplitude process affects the dynamic response of the system only in a static manner, the dynamic response (or output) of a linear system to an AMPM process is also an AMPM process.

$$w_{in}(t) = r_{in}(t)\ s_{in}(t) + m_{in}(t) \qquad \text{(A3')}$$

$$w_{out}(t) \approx r_{out}(t)\ s_{in}(t) + m_{out}(t) \qquad \text{(A43)}$$

Using the quasi-steady approximation, it is necessary to develop the response of the system to only the local R and the mean value components, which are stationary and Gaussian random processes. The variances of the three components of the input process are those of equation (A10), except that a unit variance is specified for the input R component. The notation for the variances of the components of the output process is

$$E[r^2_{out}] = A^2_r$$
$$\text{(A45)}$$
$$E[m^2_{out}] = A^2_m\ c^2$$

The moments of the input and output processes are developed from equations (A3') and (A43), and from the independence of the component processes.

$$E[w^2_{in}] = c^2(\alpha^2_{in} + 1)$$
$$\text{(A47)}$$
$$E[w^2_{out}] \approx A^2_m\ c^2(\alpha^2_{out} + 1)$$

The relation between the α parameters of the input and output processes is obtained from equations (A14) and (A45).

$$\alpha_{out} = \frac{A_r b}{A_m c} = \frac{A_r}{A_m} \alpha_{in} \quad \text{(A48)}$$

The values of the α parameters and consequently the probabilistic structures of the input and output AMPM processes can be different, depending upon the relative response of the dynamic system to the local R and to the mean value component processes.

Another application of the quasi-steady approximation is the development of the derivative of the AMPM process. The exact derivative, which is developed from equation (A3), depends upon the derivatives of all three component processes. In the quasi-steady form, the derivatives of the amplitude and mean value components are omitted. The resulting quasi-steady derivative is

$$\dot{w}_q(t) = \dot{r}(t)\ s(t) \quad \text{(A50)}$$

Quasi-Steady Exceedance Expression

The exceedance expression (the expected frequency of the crossings of a given level) of a random process is developed from the joint probability density function of the process and its first derivative (references 38 and 39). An analytical form can be developed for the exceedance expression of the AMPM process by omitting the derivatives of the amplitude and mean value components. The resulting quasi-steady exceedance expression is

$$\frac{N(w)}{N_{or}} = \frac{1}{2} e^{c^2/2A^2b^2} \{e^{-\frac{w}{Ab}} [1 + \text{erf}(\frac{w}{\sqrt{2}c} - \frac{c}{\sqrt{2}Ab})] + e^{w/Ab} \text{erfc}(\frac{w}{\sqrt{2}c} + \frac{c}{\sqrt{2}Ab})\} \quad (A53)$$

$$\frac{N(w)}{N_{or}} = \frac{1}{2} e^{1/2\alpha^2} \{e^{-w/\alpha c} [1 + \text{erf}(\frac{w}{\sqrt{2}c} - \frac{1}{\sqrt{2}\alpha})] + e^{w/\alpha c} \text{erfc}(\frac{w}{\sqrt{2}c} + \frac{1}{\sqrt{2}\alpha})\} \quad (A54)$$

where $w \geq 0$

$$N(-w) = N(w)$$

$$N_{or} = \frac{1}{2\pi} \frac{\dot{A}_r}{A_r}$$

The exceedance expression contains both the exponential dependence of the amplitude modulated process and the Gaussian dependence of the mean value process. The exponential dependence is dominant for large values of the α parameter.

$$\lim_{\alpha \to \infty} N(w) = N_{or} e^{-|w|/Ab} \quad (A57)$$

The Gaussian dependence is dominant for small values of both the α parameter and the level of the process.

$$N(|w|<<c;\ \alpha<<1) \approx \sqrt{\frac{2}{\pi}}\, N_{or}\ \alpha\ e^{-w^2/2c^2} \qquad \text{(A58)}$$

The exceedance expression approaches the exponential form of the amplitude modulated process for large values of the AMPM process.

$$N(|w|>>c\alpha) \approx N_{or}\ e^{1/2\alpha^2}\ e^{-|w|/\alpha c} \qquad \text{(A59)}$$

Since the dynamic properties of the mean value component have been eliminated by the quasi-steady approximation, the value of the expected number of crossings of the zero level becomes zero in the limit of small values of the α parameter, as indicated by equation (A58). The exact form of the exceedance expression, which includes the dynamic properties of all three component processes, is examined in appendix C.

APPENDIX B. EFFECTS OF CORRELATION BETWEEN THE AMPLITUDE AND MEAN VALUE COMPONENT PROCESSES

In the formulation of the AMPM or total process, the amplitude and mean value component processes are specified to be independent. The resulting AMPM process is the sum of the amplitude modulated and the mean value processes which are independent. However turbulence theory introduces the concept of interaction between the two processes, specifically the flow of energy from the lower to the higher frequency process. This interaction raises the question of the effect of the assumed independence upon the properties of the resulting total process. In this appendix the formulation of a total process with correlated amplitude and mean value component processes is outlined. The effects of this correlation upon properties of the resulting total process are examined.

The defining relation of the total process is

$$w(t) = z(t) + m(t) \tag{B1a}$$

$$z(t) = r(t)\ s(t) \tag{B1b}$$

The amplitude and the mean value components are specified to be correlated Gaussian processes.

$$E[sm] = \rho_{sm}\ bc \tag{B2}$$

The local R component is specified to be independent of the other two component processes. The resulting total process is the sum of the amplitude modulated and the mean value processes. The relation between these two processes is shown by their joint moments, for example,

$$E[zm] = 0 \qquad \text{(B3a)}$$

$$E[z^2m^2] = A^2b^2c^2(1 + 2\rho_{sm}^2) \qquad \text{(B3b)}$$

Equation (B3a), which follows from the independence and zero-mean properties of the R component, shows that the amplitude modulated and the mean value processes are uncorrelated. Equation (B3b) shows that the amplitude modulated and the mean value processes are not independent (for a non-zero value of the correlation coefficient).

The moments of the corresponding total process are developed from the defining relation, equations (B1a) and (B1b). All odd order moments are zero. The relations for some of the even order moments are:

$$E[w^2] = c^2(\alpha^2 + 1) \qquad \text{(B4a)}$$

$$E[w^4] = 3c^4[3\alpha^4 + 2\alpha^2(1 + 2\rho_{sm}^2) + 1] \qquad \text{(B4b)}$$

$$E[w^6] = 15c^6[15\alpha^6 + (9\alpha^4 + 3\alpha^2)(1 + 4\rho_{sm}^2) + 1] \qquad \text{(B4c)}$$

Examination of the basic form of the moments shows that the correlation coefficient always appears as an even power. The correlation is thus a second order effect for small correlation values.

The effect of the correlation is also shown by the associated fourth order flatness factor.

$$M_4(w) = \frac{3[3\alpha^4 + 2\alpha^2(1 + 2\rho_{sm}^2) + 1]}{(\alpha^2 + 1)^2} \qquad \text{(B5)}$$

The correlation increases the value of the flatness factor, thus showing tendency away from a Gaussian distribution toward that of the amplitude modulated process. However, the minimum value of three and the maximum value of nine for the flatness factor are not changed by the correlation. In order to estimate the numerical effects of the correlation upon the flatness factor, the case of equal values of the standard deviations of the amplitude modulated and mean value process is considered.

$$M_4(w;\ \alpha = 1) = 4.5 + 3\ \rho_{sm}^2 \qquad \text{(B6)}$$

The correlation coefficient must have an appreciable value in order to significantly change the value of the fourth order flatness factor. Conversely, if the amplitude and mean value component processes are weakly correlated, it will be difficult to determine the extent of correlation from measured data by use of the flatness factor.

The relations for the characteristic and probability density functions of the total process follow from the defining relations, equations (B1a) and (B1b). The relation for the characteristic function is

$$C_w(\theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{im\theta} C_r(s\theta) \; p(s,m) \; ds \; dm \qquad (B7)$$

Since the required functions of the three Gaussian component processes are known, the characteristic function of the total process can be determined.

The effect of the correlation upon the exceedances of the total process is an important question for aeronautical applications. The exceedance expression is developed from the joint distribution of the process and its first derivative. Using the quasi-steady approximation (reference 8),

$$N(w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} N_r\left(\frac{w - m}{s}\right) p(s,m) \; ds \; dm \qquad (B8)$$

Equation (B8) is integrated numerically since the required integrations appear to be intractable. Figure 9 shows the resulting exceedance expression as a function of the ratio of the level of the total process to the standard deviation (Ab) of the amplitude modulated process. The exceedance curve is shown for an α parameter of .60, which is approximately the value for the maximum differences; the effects of the correlation upon the exceedance expression vanish in the limits of zero and infinite α values. The correlation has the effect of increasing the exceedances at high values of the process level. For a value of the non-dimensional process level equal to six, which is about the critical level in aeronautical applications, the exceedances are increased by less than 20% for a correlation coefficient of .30.

In summary, the introduction of correlation between the amplitude and mean value component processes eliminates the independence property of the amplitude modulated and mean value processes. Thus the two processes can interact as required by basic turbulence theory. If the amplitude and mean value processes are weakly correlated, then the correlation is a second order effect; the effects of the correlation on the fourth order flatness factor and on the exceedance expression of the total process are fairly small.

APPENDIX C. COMPARISON OF THE EXACT AND QUASI-STEADY EXCEEDANCE EXPRESSIONS

The exact exceedance expression of the AMPM or total process is examined and compared with the quasi-steady form of the exceedance expression, equation (A54), in this appendix. In the quasi-steady approximation the dynamic properties of the local R component process are considered, while those of the amplitude and mean value component processes are omitted. In the exact form of the exceedance expression the dynamic properties of all three component processes are considered. The present development extends that of references 11 and 12. Although the exceedance expression is examined in general form, the modulated form of the total process is of primary interest, that is, the amplitude and mean value component processes are slowly varying relative to the local R process, with the quasi-steady approximation being a limiting case.

The exceedance expression is developed from the joint probability density function of the random process and its first derivative (references 38 and 39).

$$N(w) = \int_{0}^{\infty} \dot{w} \; p(w, \dot{w}) \; d\dot{w} \tag{C1}$$

The indicated integration operation essentially gives one-half of the first absolute moment of the first derivative. (This moment is assumed to exist in the subsequent development.) For the total process it is convenient to develop the exceedance expression from the joint characteristic function of the process and its first derivative.

$$C_{w\dot{w}}(\theta_w,\theta_d) = E[e^{iw\theta_w + i\dot{w}\theta_d}] \qquad (C2)$$

The absolute moments of a random variable are developed from the characteristic function (reference 40).

$$E[|\ x\ |^n] = \int_{-\infty}^{\infty} |x|^n\ p(x)\ dx$$

$$= \frac{1}{2\pi i^{n+1}} \int_0^{\infty} \frac{\partial^n}{\partial \xi^n} [C(\xi) + C(-\xi)] \frac{d\xi}{\xi} \qquad (C3)$$

where n = odd.

The relation for the Fourier transformation of the exceedance expression is obtained by combining the previous relations.

$$F\{N(w)\} = \frac{-1}{\pi} \int_0^{\infty} \frac{\partial}{\partial \theta_d} [C(\theta_w,\theta_d)] \frac{d\theta_d}{\theta_d} \qquad (C4)$$

where $C(\theta_w, -\theta_d) = C(\theta_w, \theta_d)$

Introducing the inverse Fourier transformation, the relation for the exceedance expression is

$$N(w) = \frac{-1}{\pi^2} \int_0^{\infty}\int_0^{\infty} \frac{\partial}{\partial \theta_d} [C(\theta_w, \theta_d)]$$

$$\cos(w\theta_w)d\theta_w \frac{d\theta_d}{\theta_d} \qquad (C5)$$

The joint characteristic function of the total process and its first derivative is obtained from equation (C2). Since the total process is the sum of the independent product and mean value processes, the characteristic function is the product of the functions of these two processes.

$$C_{w\dot{w}}(\theta_w,\theta_d) = C_{z\dot{z}}(\theta_w,\theta_d)C_{m\dot{m}}(\theta_w,\theta_d) \tag{C6}$$

The joint characteristic function of the product process Z and its first derivative is given in reference 7. Since the three components of the total process are stationary and Gaussian, it is convenient to express the standard deviations of their first derivatives in terms of the expected number of positive slope zero crossings. The appropriate notation for the local R component process is

$$N_{or} = \frac{1}{2\pi}\frac{A_{\dot{r}}}{A_r} \tag{C7}$$

where

$$A_r^2 = E[r^2]$$

$$A_{\dot{r}}^2 = E[\dot{r}^2]$$

The notation for the amplitude component process is

$$\varepsilon_s = \frac{N_{os}}{N_{or}} = \sqrt{\frac{E[\dot{s}^2]}{E[s^2]}}\,\frac{A_r}{A_{\dot{r}}} \tag{C8}$$

The notation for the mean value component process is

$$\varepsilon_m = \frac{N_{om}}{N_{or}} = \sqrt{\frac{E[\dot{m}^2]}{E[m^2]}}\,\frac{A_r}{A_{\dot{r}}} \tag{C9}$$

The concepts of amplitude modulation and the slow variation of the mean value require that the ε parameters have values much less than one, with zero values corresponding to the quasi-steady approximation.

The joint characteristic function of the total process and its first derivative is obtained by combining the previous relations.

$$C(\theta_w,\theta_d) = [\alpha^2x^2 + (1 + \alpha^2y^2)(1 + \varepsilon_s^2\alpha^2y^2)]^{-1/2}$$

$$\cdot \exp(-\frac{1}{2}x^2 - \frac{1}{2}\varepsilon_m^2y^2) \qquad \text{(C10)}$$

where

$$x = A_r b\theta_w/\alpha$$

$$y = A_{\dot{r}} b\theta_d/\alpha$$

The relation for the exceedance expression is obtained by combining the joint characteristic function and equation (C5).

$$\frac{N(w)}{N_{or}} = \frac{2}{\pi\alpha}\int_o^\infty\int_o^\infty fD^{-1/2}\exp(-\frac{1}{2}x^2 - \frac{1}{2}\varepsilon_m^2y^2) \qquad \text{(C11)}$$

$$\cdot\cos(\frac{w}{c}x)\,dx\,dy$$

where

$$D = x^2 + (\alpha^{-2} + y^2)(1 + \varepsilon_s^2\alpha^2y^2)$$

$$f = \varepsilon_m^2 + [1 + \varepsilon_s^2(1 + 2\alpha^2)y^2]D^{-1}$$

In the special case of both ε parameters equal to zero the integrals reduce to a standard form, giving the quasi-steady exceedance expression, equation (A54). In the general case the integration appears to be intractable and is evaluated numerically.

The exact exceedance expression is plotted in figures 10 and 11 as a function of the level of the total process (ratioed by the standard deviation, Ab, of the product process), and for several values of the ε_m parameter. The dynamic properties of the amplitude component process have been eliminated by setting the value of the ε_s parameter to zero. Figure 10 shows the exact exceedance expression for the value of the α parameter equal to one, in which case the variances of the product and mean value processes are equal. Figure 11 shows the exact exceedance expression for the value of the α parameter equal to .5, in which case the mean value component is the dominant contributor to the total process. The exceedance expressions for the quasi-steady case correspond to the zero value of the ε_m parameter. As the value of that parameter is increased, the values of the exceedance expression are increased at all levels. Thus the dynamic properties of the mean value process uniformly increase the exceedances of the total process. The relative increase is greater for smaller values of the α parameter, that is, for larger static contributions of the mean value component to the total process. The largest relative increase in the exceedances occurs at the zero level of the total process. The exact and quasi-steady exceedance expressions are equal in the limit of large values of the level of the total process. The results of figures 10 and 11 show the following relation for the special case of equal values of the expected number of zero crossings of the local R and mean value component processes, that is, the ε_m parameter equal to one.

$$N(w = 0; \alpha, \varepsilon_m = 1, \varepsilon_s = 0) = N_{or} = N_{om} \tag{C12}$$

This relation follows from the general expression, equation (C11), which can be integrated analytically in this special case.

The concept of the AMPM process specifies that the mean value process is slowly varying relative to the local R process. Consequently the value of the ε_m parameter is generally much less than one. However there can be exceptions to this which are within the concept of the AMPM process. One example is the response of a narrow band-pass system to the AMPM process. The frequency content of the system response, including both the amplitude modulated and mean value processes, will be dominated by the frequency of the narrow band-pass system. Consequently the expected number of zero crossings of the local R and the mean value processes of the response will be almost equal; the value of ε_m parameter of the system response process will be approximately one.

The effects of the dynamic properties of the amplitude process, which are expressed through the ε_s parameter, equation (C8), were examined by numerical integration of the exact exceedance expression. Numerical results show the following general effects for values of the ε_s parameter less than .25. If the value of the α parameter is one or greater, the effect of the ε_s parameter upon the exceedances is always less than 5%.

The effect is larger for smaller values of the α parameter. The effect of the ε_s parameter can significantly increase the exceedances in the case of very small values of both the α and the ε_m parameters.

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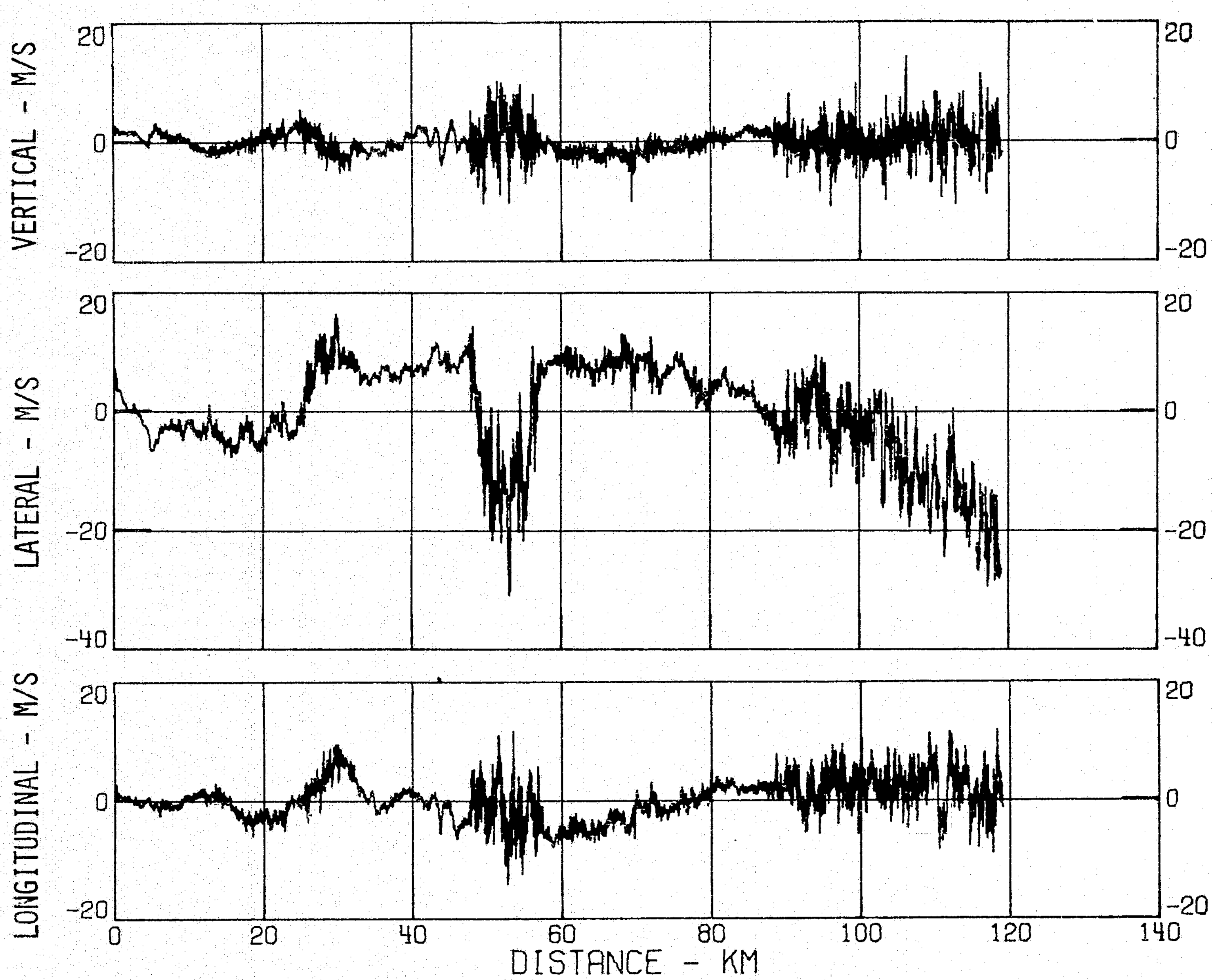

Figure 1. Example of atmospheric velocity components showing random amplitude and mean value variations (flight 32, run 2 from NASA-Langley MAT program, reference 14)

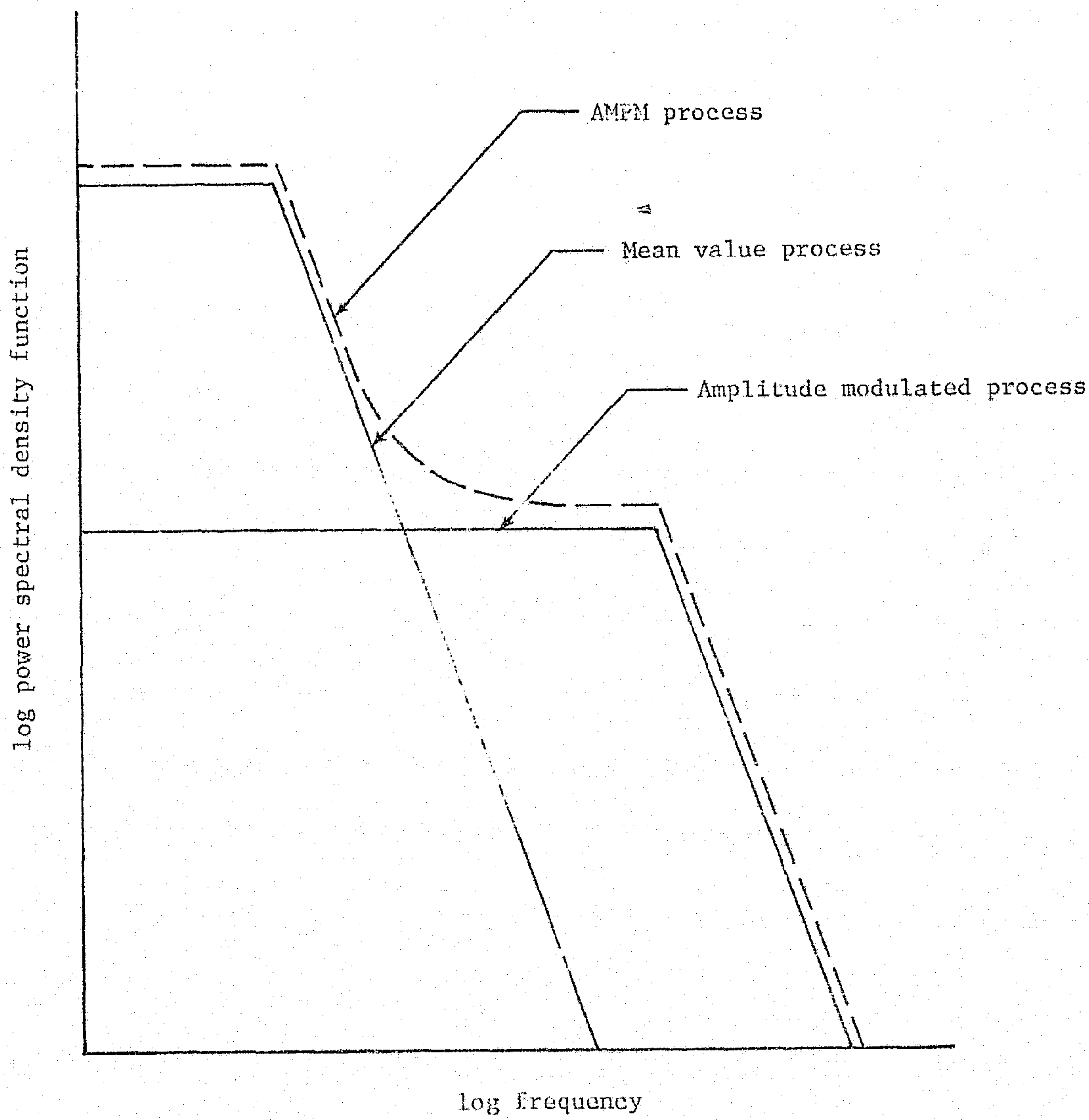

Figure 2. General form of the two terms of the power spectral density function of the AMPM process

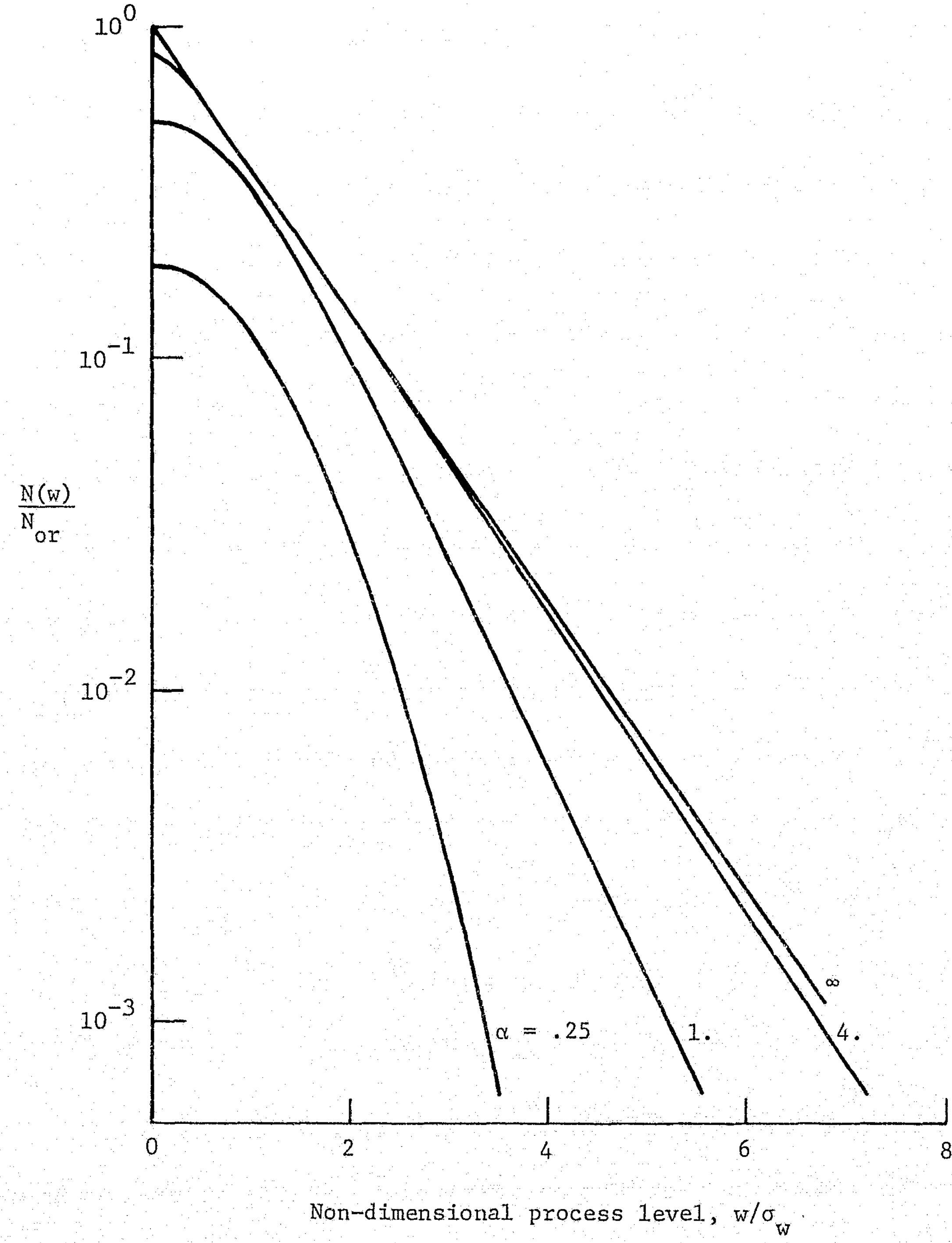

Figure 3. Exceedance ratio as a function of the process level non-dimensionalized for the AMPM process, and for several values of the α parameter

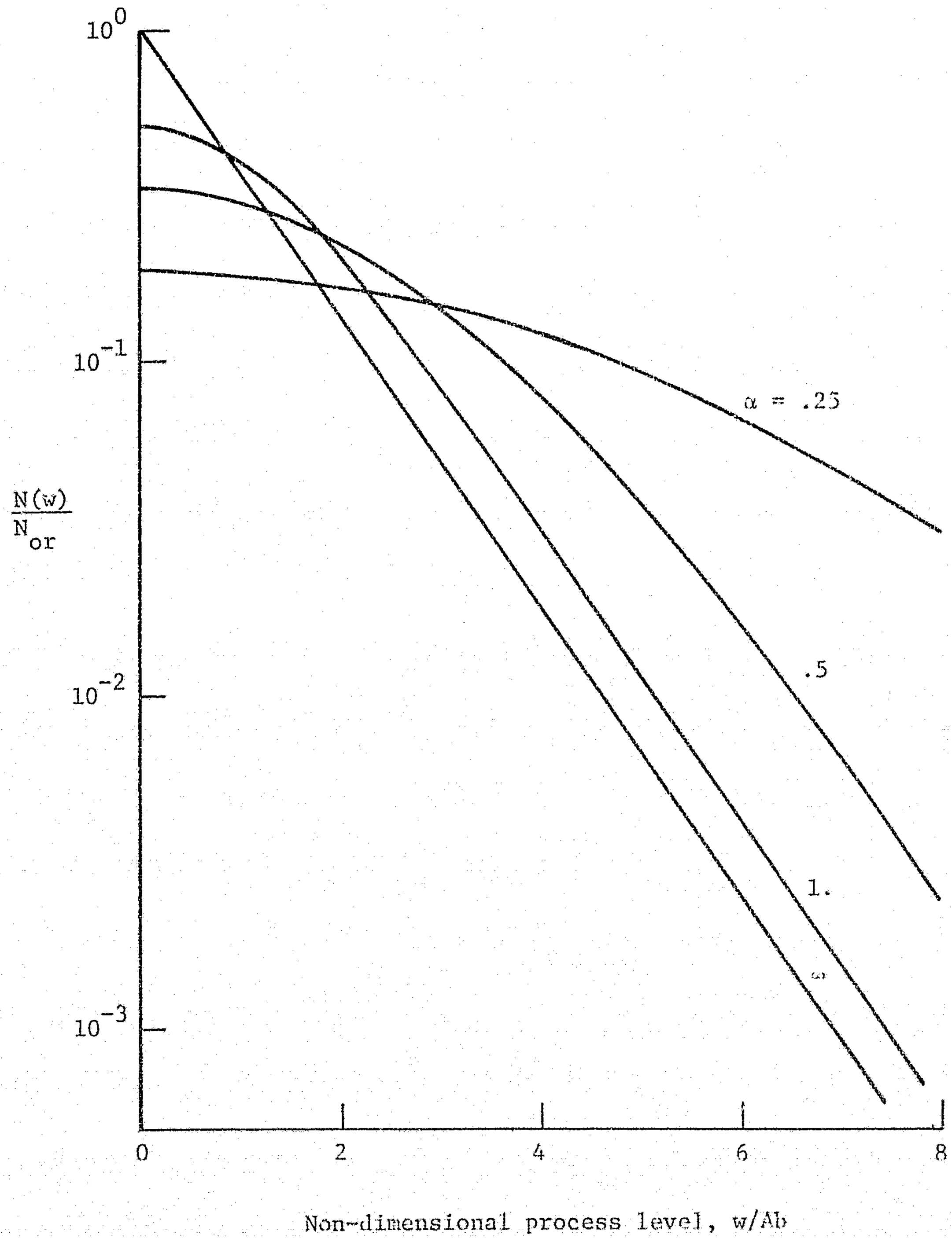

Figure 4. Exceedance ratio as a function of the process level non-dimensionalized for the amplitude modulated process, and for several values of the α parameter

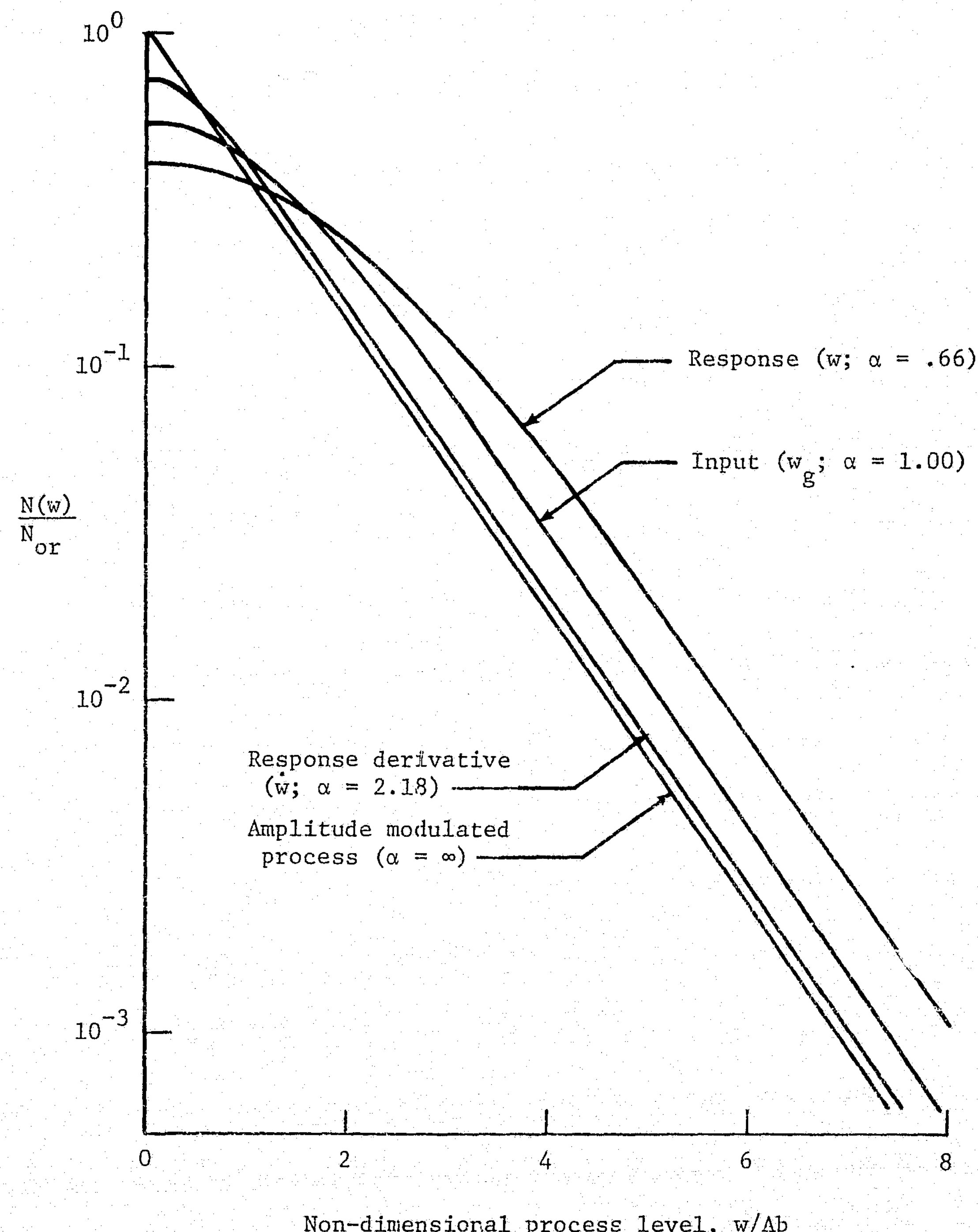

Figure 5. Exceedance ratios for the quantities of a first-order differential equation with the parameters of equation (29)

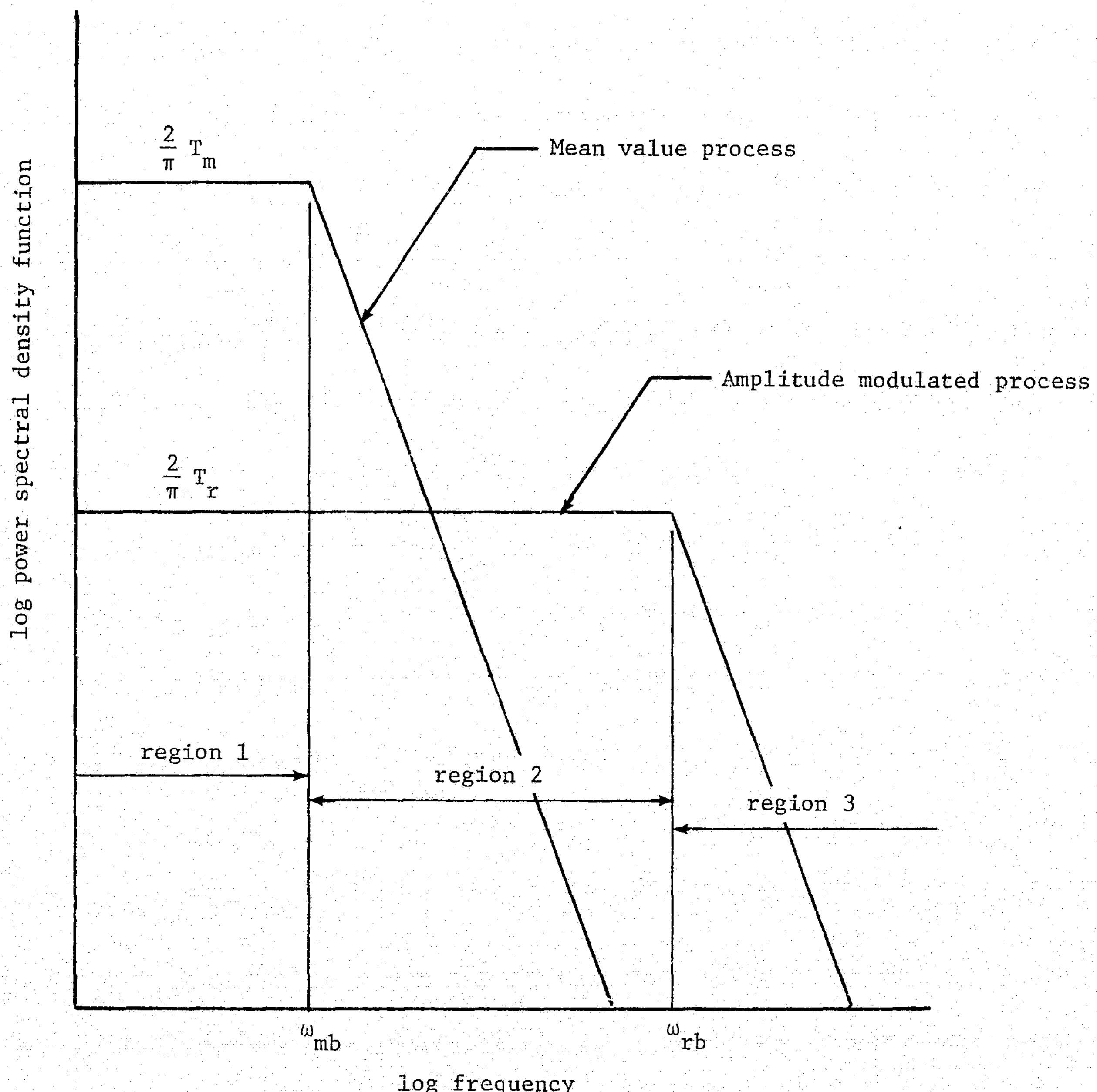

Figure 6. General relation between the power spectral density functions of the two terms of the AMPM process

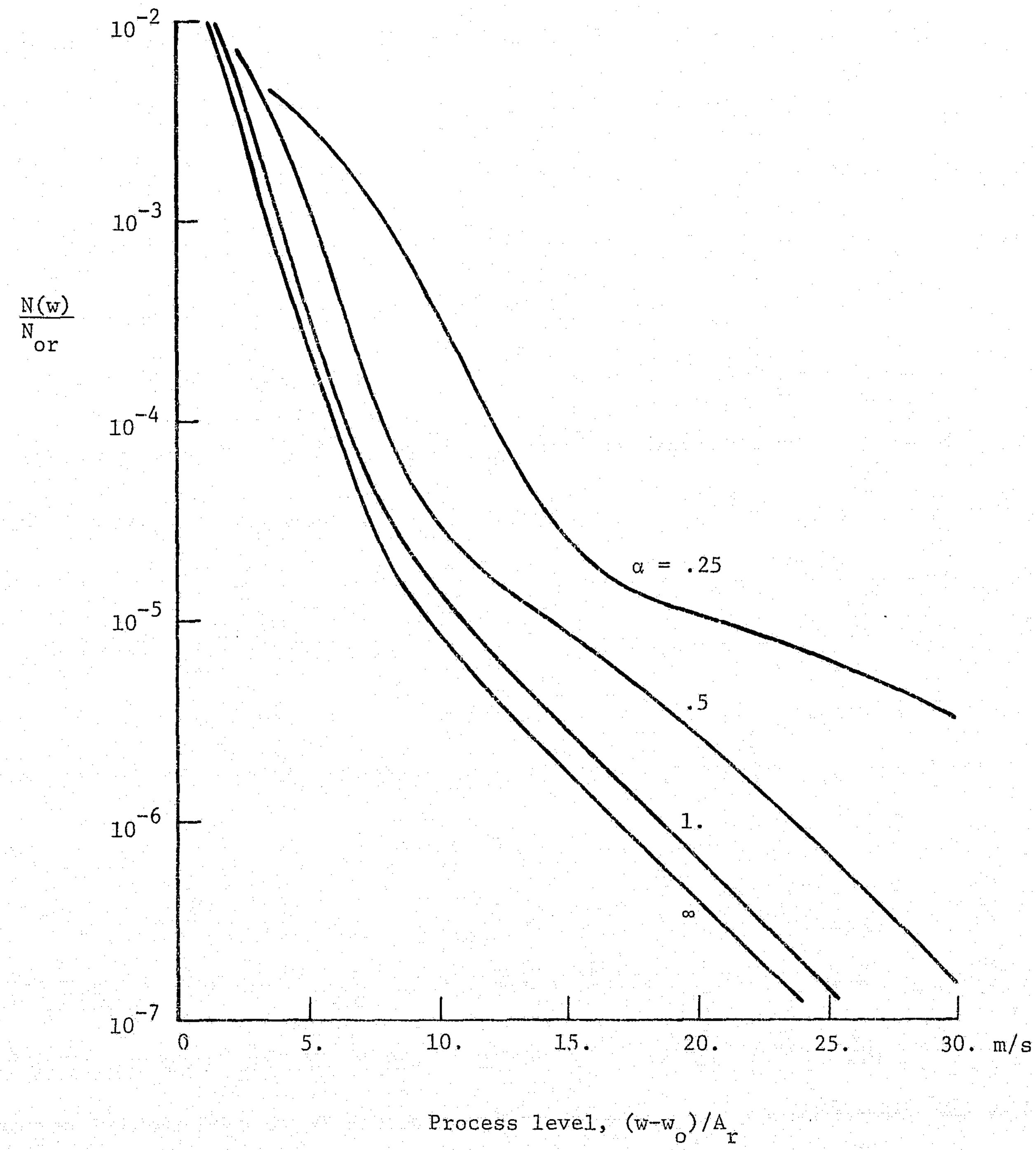

Figure 7. Exceedance ratio as a function of process level, for several values of the α parameter (6.1 km altitude)

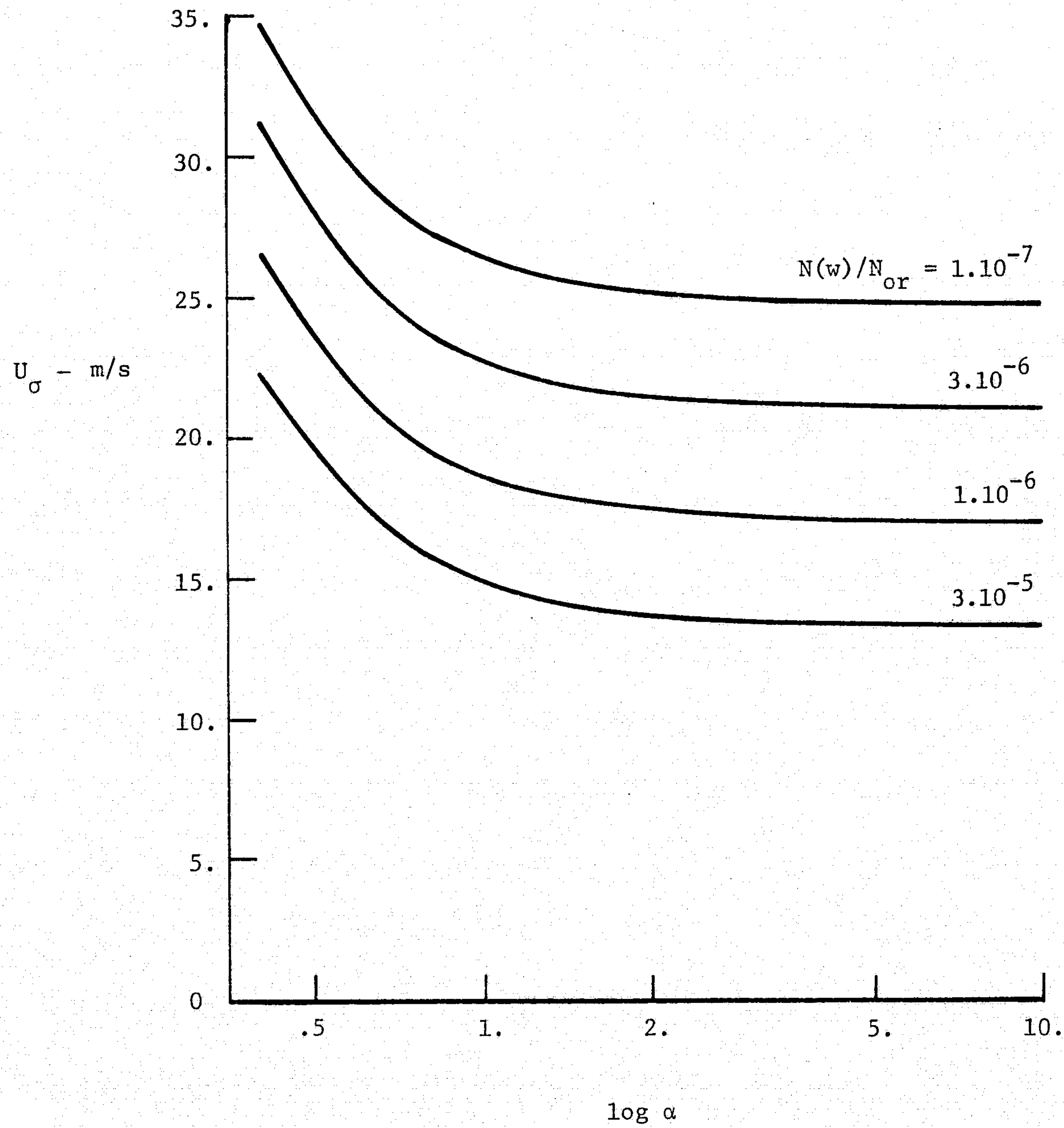

Figure 8. U_σ as a function of the α parameter, for constant values of the exceedance ratio (6.1 km altitude)

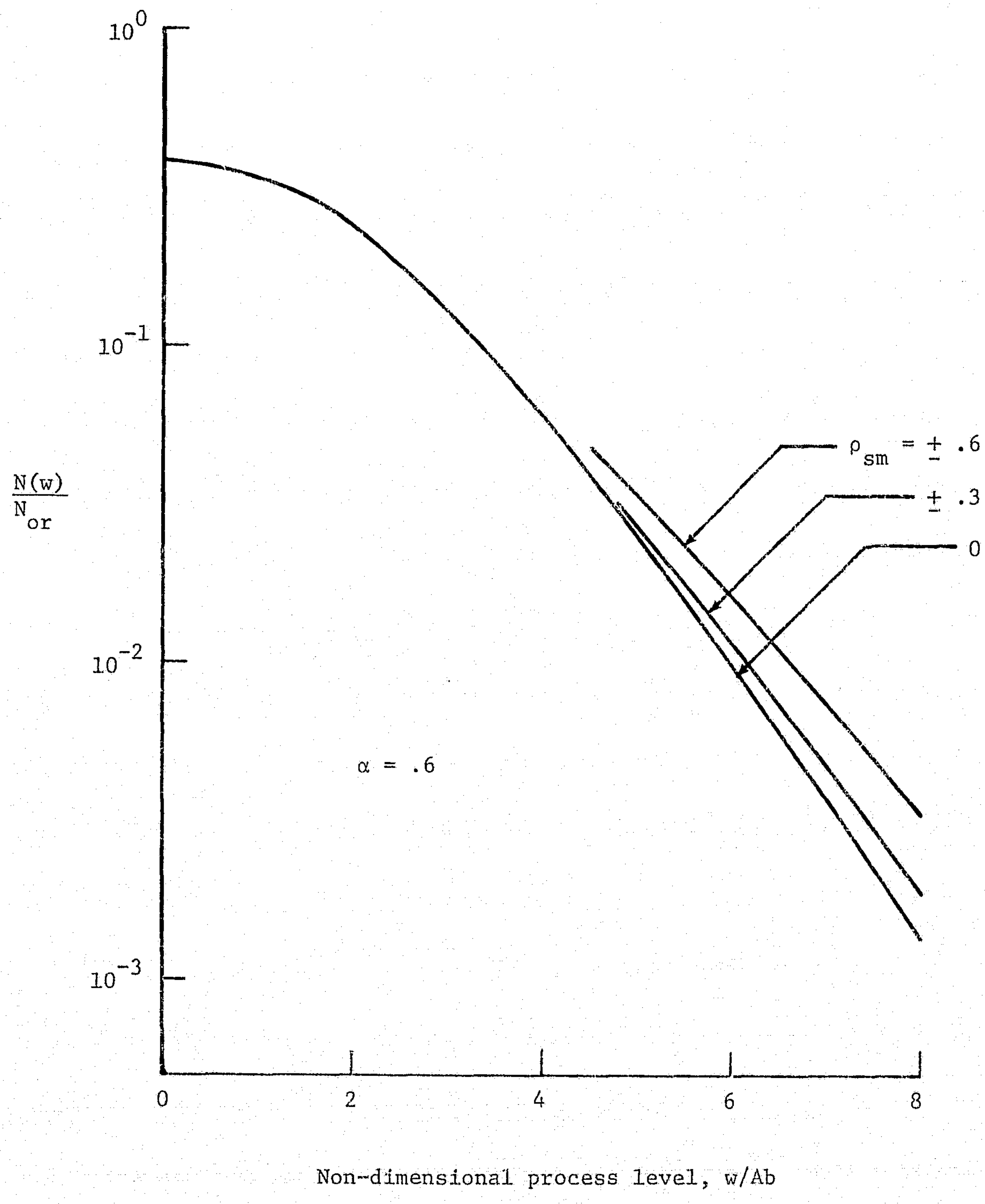

Figure 9. Exceedance ratio - effect of correlation between the amplitude and mean value processes

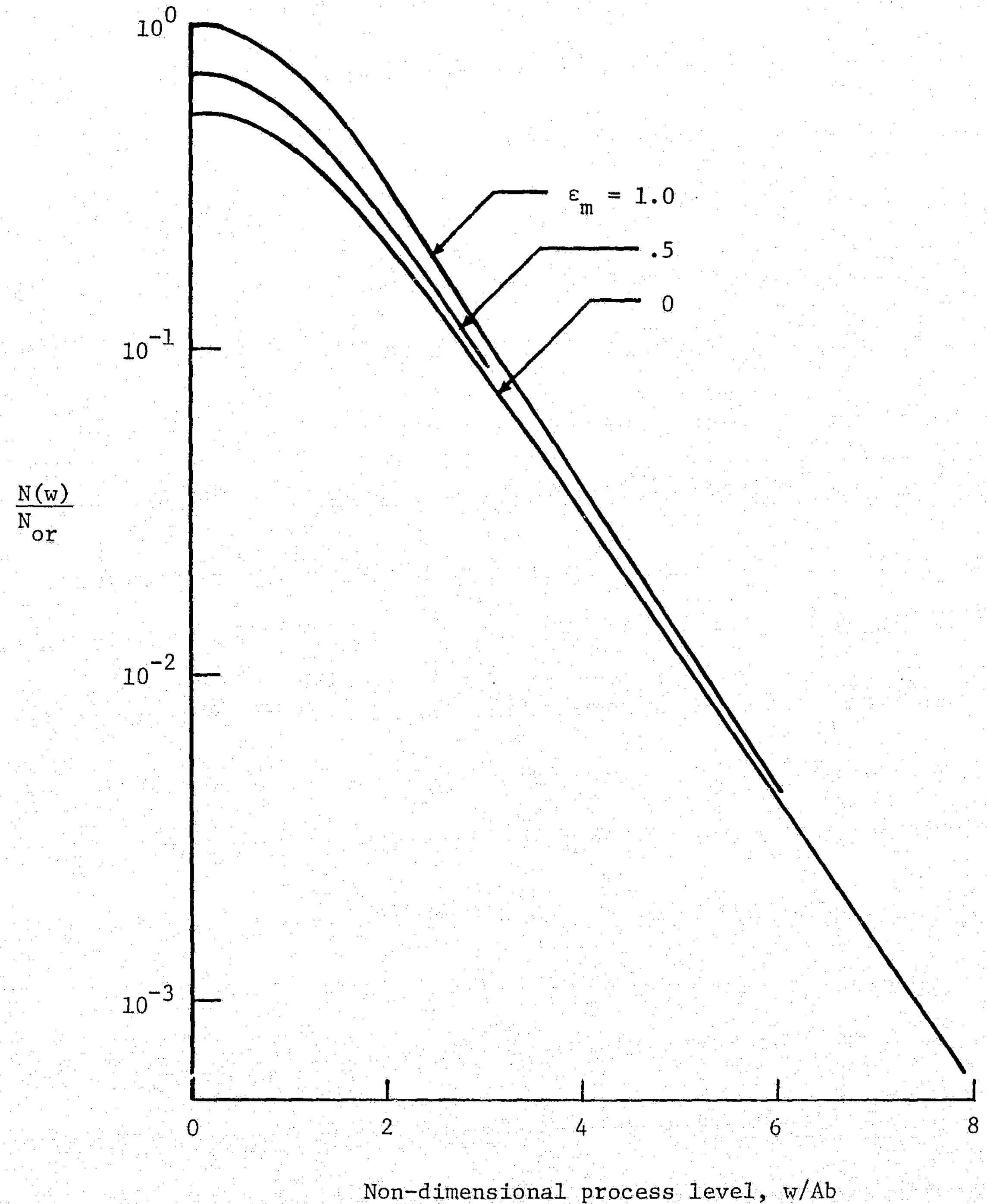

Figure 10. Exact exceedance ratio for several values of the ε_m parameter (α = 1.0 and ε_s = 0)

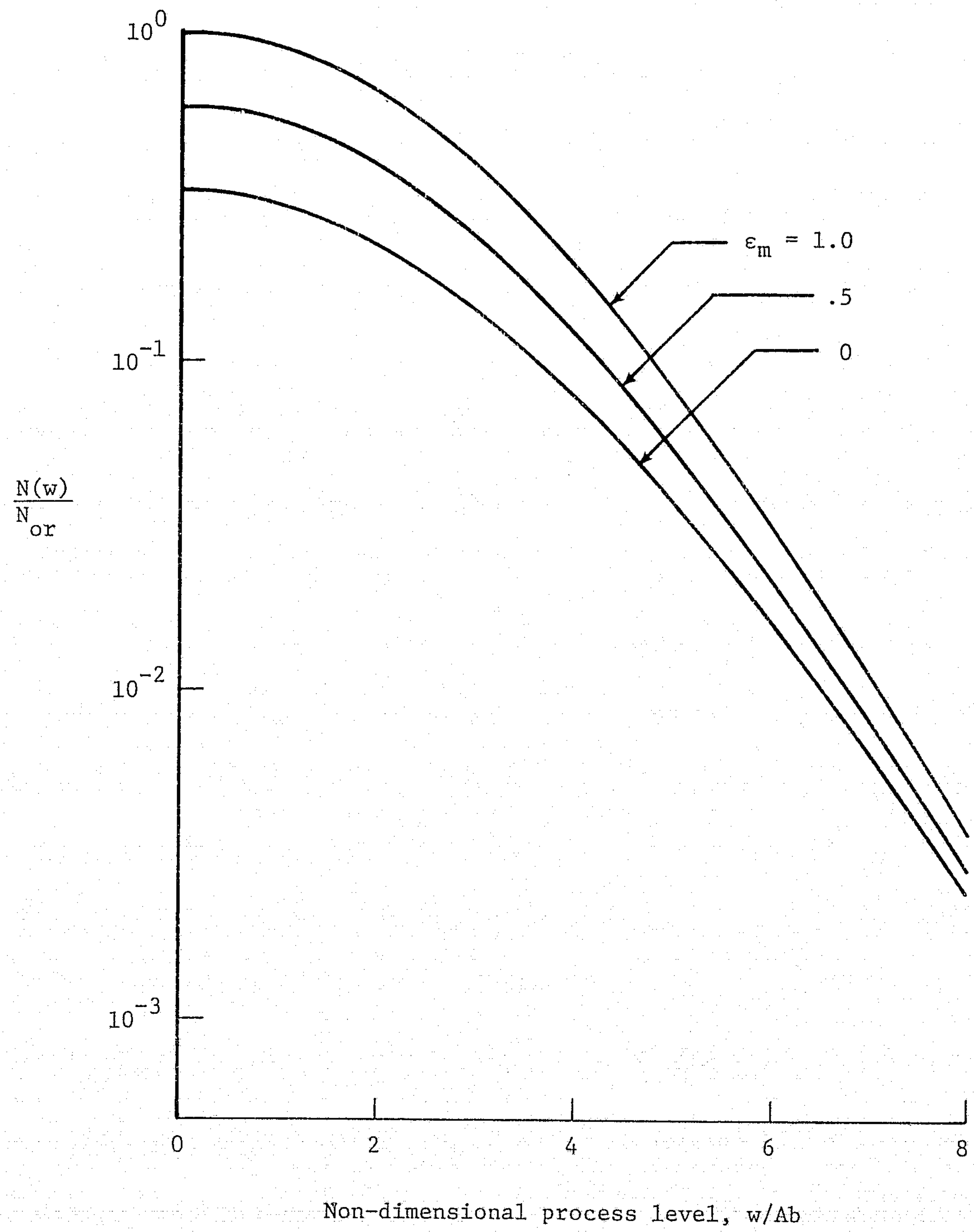

Figure 11. Exact exceedance ratio for several values of the ε_m parameter (α = 0.5 and ε_s = 0)

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15. Supplementary Notes

16. Abstract

A random process is formed by the product of two Gaussian processes and the sum of that product with a third Gaussian process. The resulting total random process is interpreted as the sum of an amplitude modulated process and a slowly varying, random mean value. The properties of the process are examined, including an interpretation of the process in terms of the physical structure of atmospheric motions. The inclusion of the mean value variation gives an improved representation of the properties of atmospheric motions, since the resulting process can account for the differences in the statistical properties of atmospheric velocity components and their gradients. The application of the process to atmospheric turbulence problems, including the response of aircraft dynamic systems, is examined. The effects of the mean value variation upon aircraft loads are small in most cases, but can be important in the measurement and interpretation of atmospheric turbulence data.

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