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AN ELEMENTARY ANALYSIS OF THE EFFECT OF SWEEP, MACH NUMBER, AND LIFT COEFFICIENT ON WING-STRUCTURE WEIGHT

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SUMMARY

Results are presented from an elementary analysis of the effect of sweep angle on the idealized structural weight of swept wings, with cruise Mach number M and lift coefficient C_L as parameters. The analysis indicates that sweep is unnecessary for cruise Mach numbers below about 0.80, whereas for the higher subsonic speeds, a well defined minimum-weight condition exists at a sweep angle in the neighborhood of 35^o or 40^o, depending on M and C_L. The results further indicate that wing-structure weight increases sharply with Mach number in the high subsonic range, with Mach 0.85 wings weighing half again as much as Mach 0.75 wings. Weight is also shown to increase with cruise lift coefficient, but the effect is not strong for the usual range of design lift coefficients. Minimum wing-structure weight is found to occur at a ratio of thickness to normal chord of about 18 percent, but it is concluded that the thickness ratio for optimum wing design would probably lie in the range of 12 to 15 percent.

INTRODUCTION

The structural weight of wings on swept-wing aircraft is a subject that continues to be widely discussed and little understood except in terms of the results of highly complex design processes. The present memorandum is intended to contribute to the discussion by presenting a first-order analysis which combines the various aerodynamic and structural effects in an elementary way. The result of the analysis is a simple expression that agrees reasonably well with the known trends in the dependence of wing weight on Mach number and sweep angle.

It should be emphasized that this analysis is based on a number of severe assumptions and that the results are, therefore, only of limited value. It is believed, however, that the major interrelationships have been taken into account, and that the trends which are indicated are valid.

SYMBOLS

A	curve-fit coefficient
B	curve-fit coefficient
b	aerodynamic span
c	chord
C_L	wing lift coefficient
C_ℓ	two-dimensional section lift coefficient
h	effective structural depth of wing box
K	combined constant factor for structural weight
k_c	constant factor for wing area
k_h	constant factor for wing-box depth
k_p	constant factor for load on wing
k_w	constant factor for beam weight
ℓ	beam length; structural semispan of wing
M	drag-rise Mach number
P	total load on beam
q	dynamic pressure
S	wing area
t	wing profile thickness
W	weight of wing-box structure to carry bending stresses
W^*	nondimensional weight of wing-box structure
W_G	gross weight of airplane, for determining the lift coefficient of the wing
W_o	"supported weight" of airplane: weight which is supported at center of wing and which thereby contributes to the bending moment at the wing root

Λ_A	sweep angle of the wing for aerodynamics
Λ_S	sweep angle of structural axis of wing

Subscripts

o	reference value
n	referring to flow component normal to axis of swept wing
∞	referring to flow component parallel to free stream
0	curve-fit coefficient
1	curve-fit coefficient

ANALYSIS

Wing structural weight on an aircraft such as a large subsonic transport is made up of a large number of components, all of which must be designed to meet the worst of a variety of design conditions. It is a matter of experience, however, that about two-thirds of the actual weight of the structural wing box on transport aircraft can be accounted for by a simple calculation of the "theoretical" weight of "bending material" in the wing box. This calculated "ideal bending-material" weight is the weight of the volume of structural material which would be required for an idealized box beam of the given dimensions, subjected to the given spanwise loading, and limited to a nominal skin stress which is consistent with the ultimate-strength and fatigue-strength properties of the skin material (usually about 15,000 psi for steady flight with maximum payload).

The remaining one-third of the wing-box weight which is not directly required to carry the bending stresses can largely be accounted for in the weight of the shear-carrying spar-web material and in the ribs or frames necessary to stabilize the cross section of the box. A relatively small portion of the total wing-box weight can also be attributed to design features associated with manufacturing convenience, such as the use of rib-to cover fastener clips and the use of uniform-thickness, standard-gauge material instead of continuously tapered sections as assumed for the idealized weight. Most of this "nonideal" one-third of the wing-box weight varies roughly in proportion to the bending load which also determines the "ideal" two-thirds of the weight. Thus the idealized bending-material weight calculation is a reasonably accurate indicator of the total wing-box weight.

Another characteristic of wing-box design which simplifies the problem of obtaining a first-order weight estimate for conventional swept-wing transport designs is that the bending-material weight estimate is valid independently of planform or other considerations which might be expected to complicate the calculation. Thus, the same "non-optimum" multiplying factor (as described above, roughly 3/2) applies, independently of sweep, taper ratio, engine location, wing loading, aspect ratio, etc. It is worth noting that, for conventional designs, the weight penalties associated with "secondary" structural requirements such as torsional stiffness and flutter are sufficiently small to be neglected in an analysis of this kind. Furthermore, there is no large penalty (as is often suggested) associated with the sweep break at the wing root, compared to a straight wing of the same structural span and depth.

For transport aircraft of generally conventional design, then, the wing-box weight can be considered to be approximately proportional to the idealized weight of bending material, independently of other complicating effects. For the purposes of the present analysis, it can also be assumed that the spanwise distribution of wing-box depth is invariant among the aircraft in a given class, and that the spanwise distribution of aerodynamic loading in the critical flight condition is similarly invariant. These assumptions can be justified on the grounds that wing-planform taper ratio is about 0.3 for most transports, and that the wing profile thickness ratio is usually very nearly constant for most of the span. Furthermore, in the critical loading condition, the spanwise distribution of loading can be considered to be approximately proportional to chord, for purposes of weight analysis.

With the assumptions of similar distributions of wing-box depth and loading, and of uniform bending stress in the idealized wing box, the bending-material weight is amenable to simple structural scaling. The beam scaling relation used here is derived in Appendix A. For beams of similar construction and loading, the weight W is given by

$$W = k_W \cdot P \cdot \ell \cdot (\ell/h) \tag{1}$$

where P is the total load (for which the nondimensional distribution has been predetermined); ℓ is the length of the beam; h is the depth of the beam at some reference station (say, the root); and K_W is a constant of proportionality, which depends on the material density and stress, and on the distributions of load and depth. For the calculation of the weight of a swept wing, the length ℓ is the structural semispan

$$\ell = (b/2)/\cos \Lambda_S \tag{2}$$

where b is the aerodynamic span, and Λ_S is the sweep angle of the structural axis of the wing. This relation is diagrammed in Figure 1.

The load P in equation (1) is the total load on the wing, which is equal to the load supported at the root (i.e. half the weight of fuselage, payload, tail, landing gear, etc., multiplied by a design acceleration-load factor, plus the aerodynamic loads on the tail, for the critical flight condition). The load P should not include the weight of the wing itself or of the fuel in the wing, since these loads contribute almost nothing to the wing bending moment in flight. Similarly, wing-mounted engines add very little to the bending moment and can, therefore, be neglected in this approximate calculation of wing weight. Thus it is assumed that

$$P = k_p \; W_0 \tag{3}$$

where W_0 is the "supported weight" of the airplane and k_p is a constant which is equal to the maximum design load factor times one-half (i.e., each wing supports half of the total maximum load required to support the fuselage etc.).

The effective depth h of the wing box is related to the maximum thickness t of the aerodynamic profile of the wing at each spanwise station by a factor that depends on the geometry of the internal stiffeners (stringers) and on the width of the wing box. For the purpose of preliminary-design structural analysis, the relationship is usually taken to be

$$h = k_h \cdot t, \tag{4}$$

where k_h is normally about 0.85. A wing section with an equivalent box is diagramed in Figure 2. The profile thickness varies along the span, but for most wings, the thickness distribution is such that the profile thickness-to-chord ratio t/c is approximately constant, with the tip somewhat thinner and the root somewhat thicker. The present purposes are served by assuming that the non-dimensional spanwise distribution of wing-box depth is invariant among the various designs within a class of aircraft. A reference wing-box depth h can then be defined to be a fixed proportion of the profile thickness ratio times the chord c_0 taken at some reference station. Thus, from equation (4),

$$h_0 = k_h \cdot (t/c)_0 \cdot c_0 \tag{5}$$

The choice of reference station on the wing is, of course, arbitrary, provided that the same reference station is used when comparing different designs.

For this analysis the reference chord is taken to be the "structural", or "normal", chord at the reference station (i.e., the width of the wing normal to the structural axis). The thickness ratio also corresponds to the normal chord. As shown in Figure 2, the normal chord c_n is related to the streamwise chord c_ω by

$$c_n = c_\omega \cdot \cos \Lambda_S \tag{6}$$

With this notation, equation (5) for the reference beam depth becomes

$$h_0 = k_h \cdot (t/c_n)_0 \cdot (c_n)_0 \tag{7}$$

It is convenient to eliminate the reference chord in favor of the wing area S and the span b. For wings of given taper ratio and reference station, the wing area is proportional to the product of the structural span and the reference chord. Thus

$$S = k_c \cdot \ell \cdot (c_n)_0 \; , \tag{8}$$

where k_c is a constant factor. Making use of equations (8) and (2), then, it becomes possible to express the wing-box depth from equation (7) in terms of span and area:

$$h_o = (k_n/k_c) \cdot (2S/b) \cdot (t/c_n)_o \cdot \cos \Lambda_S \qquad (9)$$

The weight of the wing box can now be expressed in the desired terms. Substituting into equation (1) to eliminate P, ℓ, and h in favor of W, b, and S gives

$$W = \frac{K\ W_o\ b^3}{S} \cdot \frac{1}{\cos^3 \Lambda_S \cdot (t/c_n)_o} \qquad (10)$$

where K stands for the group of multiplicative constants from the contributing relations. Since this weight equation will only be used here in a relative, nondimensional form, it is not necessary to evaluate K. The nondimensional expression for weight which will be plotted is

$$W^* = \frac{W}{(KW_o b^3/S)} = \frac{1}{\cos^3 \Lambda_S \cdot (t/c_n)_o} \qquad (11)$$

Equation (11) gives the relative weights of wing boxes as a function of the sweep angle, provided the profile thickness can be determined. In order to establish a functional relationship which is representative of the profile thickness ratios normally used in design practice, it is necessary to consider briefly the wing design process.

The objective of wing design is, of course, to find the geometry that gives the least total drag under the cruise conditions (including the drag associated with structural weight) while satisfying the requirements for aerodynamic lift and structural strength. For the purposes of preliminary design of aircraft in the jet transport category, this complex criterion is often replaced by the use of a highly simplified expedient: the aerodynamic profile is assumed to have the maximum possible thickness consistent with the provision that the "drag-rise Mach number" must remain outside the flight envelope for efficient cruise. (The drag-rise Mach number describes the condition at which the drag imcrement due to compressibility effects begins to become important, and is usually defined in terms of the slope of the curve of drag coefficient vs Mach number). For preliminary design, the compressibility-limited thickness ratio can be assumed to be satisfactorily close to the optimum thickness ratio, thereby eliminating thickness ratio as an independent design variable.

Since the compressiblity effects which produce the drag increase are aggravated by any acceleration of the flow field, the drag-rise Mach number is a function of both the profile thickness ratio and the lift coefficient. Once this functional relationship is established, however, it becomes possible to find immediately the maximum profile thickness which can be used without excessive compressibility drag at the given cruise conditions. Figure 3 shows a semi-empirical relationship between the thickness ratio and the drag-rise Mach number for two-dimensional profiles operating at various values of lift coefficient. This figure, which was taken from Reference 1, shows that the maximum usable thickness ratio decreases as either Mach number or the lift coefficient increases. The limits on thickness indicated by these curves are considered to be slightly optimistic when compared to the current state of the art of supercritical airfoil technology (i.e., (t/c) indicated for a given C_ℓ and drag-rise M is slightly greater than the currently accepted limit), but the basic relationships have been verified by experiment. For the analysis presented here, the thickness ratio was assumed to vary as plotted in the figure. Given the limited objectives of the analysis, the slight shift in the results caused by the use of optimistic aerodynamics is not considered significant.

To represent the two-dimensional thickness ratio for computational purposes, a four-point curve fit was made to the figure in Reference 1. The functional form which was used is a linear variation with Mach number,

$$(t/c) = A\,M + B \tag{12}$$

where the slope A and intercept B are assumed to vary linearly with lift coefficient:

$$\begin{aligned} A &= A_1\,C_\ell + A_0 \\ B &= B_1\,C_\ell + B_0 \end{aligned} \tag{13}$$

The curve fit gives the following values for the coefficients;

$$\begin{aligned} A_0 &= -0.794 \\ A_1 &= -0.296 \\ B_0 &= 0.812 \\ B_1 &= 0.111 \end{aligned} \tag{14}$$

For the wing weight analysis, it was assumed that the two-dimensional values for the profile characteristics could be used directly in the swept-wing problem as the normal-flow characteristics in simple yawed-wing theory. Thus, the two-dimensional relation for constraint on profile thickness, equation (12), is used here directly as the constraint on the normal thickness-ratio for the swept wing. With substitutions indicated from equations (13), the reference thickness ratio for the wing becomes

$$(t/c_n)_{ref} = (A_1 \cdot (C_L)_n + A_0) \cdot M_n + (B_1 \cdot (C_L)_n + B_0) \tag{15}$$

where A_0. . . . . B_1 have the values given by (14), C_L stands for the effective lift coefficient of the wing, and the symbol n indicates the properties of the component of the flow field which is normal to the "aerodynamic axis" of the swept wing. For the purpose of determining the effective aerodynamic sweep angle to be used in applying two-dimensional supercritical aerodynamics to a swept-wing analysis, the best approximation is obtained by using the sweep angle of the high-Mach-number isobars, which lie approximately at the midchord of the wing. Considering the level of approximation involved in the present problem, however, no significant error is introduced by taking the aerodynamic sweep angle to be equal to the structural sweep angle, and this simplification has been made for the numerical computations. The distinction between aerodynamic sweep and structural sweep has been retained in the analysis purely for the purpose of identification of terms.

In the first-order sweep theory, compressibility effects are assumed to be the same as for a two-dimensional (unswept) wing operating at a Mach number determined by the component of the free-stream velocity normal to the wing. Thus the normal Mach number is given by

$$M_n = M_\infty \cos \Lambda_A \tag{16}$$

where the ∞ indicates freestream conditions and the Λ_A is the effective aerodynamic sweep. Associated with this normal component of flow is the normal lift coefficient $(C_L)_n$, which is found by observing that since the component of flow parallel to the wing provides no lift, the lift due to the normal component of flow must equal the total lift:

$$W_G = q_n S(C_L)_n = q_\infty \; S(C_L)_\infty,$$

or

$$(C_L)_n = (C_L)_\infty \, q_\infty/q_n \tag{17}$$

The dynamic pressure q is proportional to M^2, so that

$$(C_L)_n = (C_L)_\infty \, M_\infty^2/M^2, \tag{18}$$

which gives, on substituting for M_∞/M_n from (17),

$$(C_L)_n = (C_L)_\infty/\cos^2 \Lambda_A \tag{19}$$

The relations (16) and (19) allow one to find the normal Mach number and lift coefficient in terms of the free stream values, for a given sweep angle. With the normal values, then, the appropriate value of $(t/c_n)_o$ (i.e., the maximum permissible thickness ratio) for the given freestream conditions can be found from equation (15). Finally, with $(t/c_n)_o$, the value of the nondimensional weight parameter W^* can be found from equation (11). The value of W^* computed in this way represents the nondimensional weight of wing structure which is consistent (within the limitations of the analysis) with the given values of free-stream drag-rise Mach number and free-stream lift coefficient.

RESULTS AND DISCUSSION

The results of the foregoing analysis of wing-weight dependence on sweep angle, drag-rise Mach number, and lift coefficient are shown in Figures 4 and 5. The curves of Figure 4 show the effect of sweep angle on relative wing weight at constant normal thickness ratio, for a range of thickness ratios. This curve set is simply a plot of equation (11), showing the inverse-cosine-cubed relation between weight and sweep angle, with thickness ratio as the parameter.

The identical set of curves is repeated (as dashed lines) in Figure 5, together with the corresponding contours of constant free-stream drag-rise Mach number, for three values of the stream-wise lift coefficient. The constant Mach number curves were obtained from the relationship given in equation (15) (which represents the curve set of Figure 3), making use of equations (16) and (19) to convert the two-dimensional data to swept wings.

Within the constraints and limitations of this elementary analysis, the curves of Figure 5 indicate the manner in which the sweep angle for swept-wing aircraft might be expected to vary with the design cruise conditions. The local minimum-weight values along the contours of constant drag-rise Mach number show the sweep angle which gives the least structural weight for given Mach number and lift coefficient; the "strength" of the local minimum indicates the sensitivity of the wing-box weight to compromises in the sweep angle which might be required to accommodate multiple design requirements.

The results indicate that for cruise Mach numbers below about 0.80 (depending somewhat on lift coefficient), wing sweep is unnecessary from the point of view of wing-box weight, since the weight either increases with sweep angle or is almost independent of sweep out to sweep angles well above the useful range. For the higher Mach numbers, however, a well-defined, minimum-weight design condition exists at a sweep angle somewhat above thirty degrees, depending on Mach number. This local minimum gets progressively stronger as either Mach number or lift coefficient increases. Furthermore, the sweep angle at which the minimum-weight condition occurs can be seen to increase as either the Mach number or the lift coefficient is increased, but the rate of increase is surprisingly mild: the minimum-weight sweep angle varies only about ten degrees for the full range of flight conditions covered by these curves.

The results plotted in Figure 5 give an indication of the cost in structural weight associated with operation at higher Mach numbers. The weight increase is brought about by the increased sweep angle and/or the decreased thickness required for the higher speeds. Figure 5 tends to give a somewhat exaggerated impression of this weight increase, however, since higher cruise Mach numbers are usually accompanied by lower cruise lift coefficients.

A more realistic indication of the relative weight penalty associated with higher Mach numbers can be obtained by varying lift coefficient with Mach number in an appropriate way. An approximate relationship between Mach number and lift coefficient can be derived by considering the total lift on the wing in steady flight at the beginning of cruise:

$$W_G = L = q\, S\, C_L = (\gamma p M^2/2)\, S\, C_L$$

This gives

$$M^2\, C_L = (W_G/S)\,(2/\gamma p) \qquad (20)$$

where γ is a constant ($\gamma = 1.4$) and p is the free-stream static pressure. The design (maximum) value of the product $M\, C_L$ is set by the maximum wing-loading W_G/S and the minimum pressure. The minimum pressure is determined by the maximum altitude for the beginning of cruise, which is in turn determined within fairly narrow limits by engine considerations and air-traffic control requirements. The maximum wing loading is also constrained to lie within a fairly narrow range of values because of landing-field-length requirements. The product M^2C_L can therefore be considered to be approximately constant for a given class of aircraft.

The curves in Figure 6 show the result of replotting weight vs sweep angle, as in Figure 5, holding the product M^2C_L constant (rather than C_L as in Figure 5). The values chosen for these plots (M^2C_L = 0.30 and 0.50) are considered typical; other values of M^2C_L produce similar curve sets, as might be expected from the similarity of Figures 5a, b and c. The rate of increase of weight with Mach number shown by Figure 6 (using the minimum W* for each value of M) amounts to a weight increase of about twenty percent for an increase of Mach number of 0.05. This rate of increase is essentially constant for the full range of interest of M^2C_L .

Figure 5 also indicates that the weight savings available from the use of thick-profile sections at higher Mach numbers is perhaps more limited than is generally believed. The family of "minimum-weight" designs requires a profile thickness ratio of about 18-to 20-percent for Mach numbers above about 0.80, for the complete range of lift coefficients. This result is shown more clearly in the curves of Figure 7, in which the weight is cross plotted against thickness ratio for constant Mach number. It can be seen from Figure 7 that, in the moderate-to-high Mach number range, minimum structural weights indicated for thicknesses of about 18-percent are not much less than the weights available through the use of relatively thin conventional sections at lower sweep: at a thickness of 15-percent the weight is only about 3 percent greater than the minimum value, while the 12-percent thickness gives weight values which are only about 10-percent greater than the minimum, for given design cruise conditions. Furthermore, the 12-percent thickness allows about 10-degrees less sweep and about 6-percent less streamwise thickness than the minimum at 18-percent. Considering that the thinner streamwise profiles give lower profile drag, and that the use of greater sweep involves a variety of disadvantages which are not directly connected with structural weight (i.e., more complex high-lift devices, undesirable stability and control characteristics, etc.), it appears that the higher-thickness supercritical wing technology may be (on the basis of this analysis) only of very limited value for conventional transports at conventional cruise speeds.

It should perhaps be emphasized that these arguments concerning thick airfoils apply to the structural weight of wings for conventional swept-wing aircraft designed for cruise at Mach numbers greater than about 0.80. For unconventional aircraft, such as cargo-wing transports, for which the wing thickness is determined by constraints other than minimum bending-material weight, these arguments do not apply. Furthermore, for conventional aircraft designed for cruise Mach numbers below about 0.80, for which sweep is not required, the wing structure weight is related to thickness by the simple inverse proportion indicated in equation (1). For these aircraft, the use of a 12-percent thickness wing instead of 18-percent would involve a structural wing box weight penalty of fifty-percent.

One final caution should be added against applying these results out of context: the weight-scaling relation used here has been found to be useful for a certain class of aircraft (the conventional jet transport), and must be viewed with suspicion for aircraft outside this class. Structural weight required to resist bending loads is readily calculated, but for unconventional configurations (e.g. aircraft with very high aspect ratio), considerations other than primary bending can easily become important in the wing-structure weight. Thus the weight "penalty" associated with secondary requirements such as flutter stability or torsional stiffness might easily overshadow the trends in bending-material weight which were explored in this study.

In this connection it should also be remembered that while the wing box is an important item in the weight breakdown of transport aircraft, it is still a relatively minor part - about eight percent of the maximum takeoff gross weight for the C-5 and B-747 and even less for smaller aircraft. By contrast, for example, the fuel weight can approach fifty percent of the gross weight. Since the fuel consumption is strongly influenced by the wing geometry as well as the wing weight, the selection of the "optimum" wing design must be made on the basis of overall system performance rather than simply on the basis of wing weight.

A complete performance analysis, then, is required to find the best combination of values of sweep, Mach number, and lift coefficient (as well as span, wing area, gross weight, etc.). Given the stated limitations of the foregoing analysis, however, it is believed that the results which have been presented give a valid indication of the manner in which the structural weight of transport-aircraft wings varies with the parameters which have been considered.

CONCLUSIONS

By the use of a set of severe and limiting assumptions, an elementary analysis has been made of the effects of sweep angle and design cruise conditions (drag-rise Mach number and lift coefficient) on the structural weight of the wings of swept-wing transport aircraft. While the analysis can only be of limited value because of the level of approximation used, the results appear to substantiate the following general conclusions:

1. The analysis provides a simple formulation for wing structure weight which agrees well with known trends gleaned from sophisticated design studies.

2. For design cruise Mach numbers below about 0.80, wing weight is almost independent of sweep angle in the range from zero to about thirty degrees. Thus, from the standpoint of wing weight, sweep is unnecessary in this speed range.

3. For design cruise Mach numbers above about 0.80, a well-defined minimum weight condition exists at a sweep angle of about 35°. This minimum-weight sweep angle increases as either Mach number or lift coefficient is increased.

4. Wing structure weight increases rapidly as Mach number is increased: holding the product M^2C_L constant (as required for constant wing loading and cruise altitude), it was found that the weight increases about twenty percent for each increment of 0.05 in Mach number.

5. Wing weight increases as the design lift coefficient is increased, but the rate of increase is small: the weight increase is about twenty-percent for an increase in lift coefficient from 0.3 to 0.7.

6. For design Mach numbers above about 0.80, minimum-weight designs involve profile thickness ratios (thickness divided by normal chord) of about 18- to 20-percent, with 15-percent and 12-percent designs having respectively about 3 percent and 10 percent greater weight.

7. The use of profiles which are somewhat thinner than 18 percent permits the use of substantially lower sweep angles at a relatively small cost in weight (a 12-percent thickness allows a reduction in sweep angle of about ten degrees from that of the minimum-weight design at 18 percent). Since reduced sweep is desirable for many reasons other than wing-box weight, the results suggest that the overall "optimum" supercritical wing design will involve profile thicknesses in the neighborhood of 12 percent, which is about the same as the range of profile thickness on current modern transport aircraft.

REFERENCES

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APPENDIX A

DERIVATION OF SIMPLE SCALING LAW FOR WEIGHT OF BOX BEAMS

In determining the relative structural weights of wing boxes of various dimensions and loadings for the swept-wing analysis, the following scaling law was used:

$$W = K_w P \ell^2/h_o \tag{A-1}$$

where P, ℓ, and h_o are the total load, the length, and the characteristic depth of the beam. This relationship is intended as a measure of the weight of the bending-strength material; weights associated with other requirements such as shear strength, torsional stiffness, etc., follow the different scaling laws. As discussed in the main text of this report, however, the bending-material weight determined by this calculation has been found to be a reliable indicator of the relative structural weight of wings among transport aircraft of given class.

The derivation of this expression for weight scaling of wing structure involves the following steps:

a. Approximate the wing cross section by an equivalent ideal box beam.

b. Write the bending moment in terms of the loading function and the beam length.

c. Find the cross-sectional area of the box "covers" (the upper and lower surfaces) which is required to carry the bending moment with the given depth of the box cross section and with the given allowable stress.

d. Find the weight of the covers from the required distribution of cross sectional area.

The notation used in this derivation is as follows:

SYMBOLS

A	cross sectional area of each box cover
h	effective structural depth of box beam
I	area moment of inertia of box cross section
k,K	constant factors

ℓ	beam length
M	bending moment
p(y)	lengthwise load distribution
P	total load on beam
$\bar{t}$	effective thickness of box cover
W	bending material weight
w(y)	bending material weight per unit length along beam
y	lengthwise dimension
y'	dummy variable for lengthwise integration
η	nondimensional lengthwise variable $\eta = y/\ell$
η'	nondimensional dummy variable
σ	bending stress
ρ	material density

Subscripts

o	reference value
p	pertaining to load
w	pertaining to weight

For the purpose of this derivation, an idealized box beam consists of a pair of cover plates which form the upper and lower surfaces of the box and which are connected by weightless, shear-resistant webs forming the sides of the box. The covers are capable of carrying longitudinal loads, either in tension or compression, at the given allowable stress σ . Under a given bending moment M , the covers of the box experience bending stresses according to the well-known formula

$$\sigma = \pm M\ (H/2)/I \qquad \text{(A-2)}$$

where I is the area moment of inertia of the cross section. Here

$$I = b \cdot \bar{t} \cdot h^2/2 = A\ h^2/2 \qquad \text{(A-3)}$$

where b, t, and A are the width, skin thickness, and cross-section area of each cover. These equations can be solved for A to get

$$A = M/\sigma h \qquad \text{(A-4)}$$

which, when multiplied by the density ρ , gives the weight of the box cover per unit length of the beam.

In general, both M and h are functions of lengthwise position y along the beam. Thus, the weight per unit length of the box beam (two covers, with weightless shear webs) is:

$$w(y) = 2\rho \cdot A(y) = (2\rho/\sigma) \cdot M(y)/h(y) \qquad \text{(A-5)}$$

The bending moment along the beam can be expressed in terms of the given load distribution p(y) as:

$$M(y) = \int_y^\ell (y' - y) \cdot p(y') \cdot dy' \qquad \text{(A-6)}$$

With this expression for $M(y)$ and the given depth distribution $H(y)$, the weight of the full length of the beam can be found by integrating (A-5):

$$W = \int_0^\ell w(y)\, dy = 2\rho/\sigma \int_0^\ell 1/h(y) \left(\int_y^\ell (y' - y)\, p(y')\, dy'\right) dy \qquad \text{(A-7)}$$

Equation (A-7) can be simplified to yield the desired form of (A-1) by making the integrand nondimensional

$$W = 2\rho/\sigma \cdot \rho_0 \ell^3/h_0 \left(\int_0^1 h_0/h(\eta) \left(\int_\eta^1 (\eta' - \eta) \cdot \rho(\eta')/\rho_0 \cdot d\eta'\right) d\eta\right) \qquad \text{(A-8)}$$

In this form, the integral within the brackets has the value of a factor which is dependent on the functional forms of h(y) and p(y), that is, on the form of the distribution of the beam depth and the loading. Under the assumption that these distributions are invariant within a given class of aircraft, the integral has a fixed numerical value for all cases within the class. Equation (A-8) can then be written

$$W = 2\rho/\sigma \cdot \rho_0 \ell^3/h_0 \cdot K \qquad \text{(A-9)}$$

To put (A-9) into the form of (A-1), note that for a given distribution $\rho(y)$, the total load P is related to ρ_0 by

$$P = \int_0^\ell \rho(y)\, dy = \rho_0 \ell \int_0^1 \rho(\eta)/\rho_0\, d\eta = \rho_0 \ell\, k_\rho \qquad \text{(A-10)}$$

since this integral also has a fixed numerical value. With the substitution of P for ρ (A-9) becomes

$$W \quad W = \frac{2\rho K}{\sigma k_p} \cdot \frac{P \ell^2}{h_0} = k_w P \ell^2/h_0 \qquad \text{(A-11)}$$

which is the desired form.

In approximating the weight of the box-beam structure for a wing, some allowance must be made for the fact that the wing box is nonideal in that the cover material does not lie at a uniform distance from the neutral axis, and therefore the bending stress is not constant across the box. Since most wing boxes exhibit this property to about the same degree, the required factor can be absorbed in the constant k_w of equation (A-11). Equation (A-11) is, therefore, a satisfactory relation for estimating the relative weights of wing box bending-material among comparable transport aircraft designs.

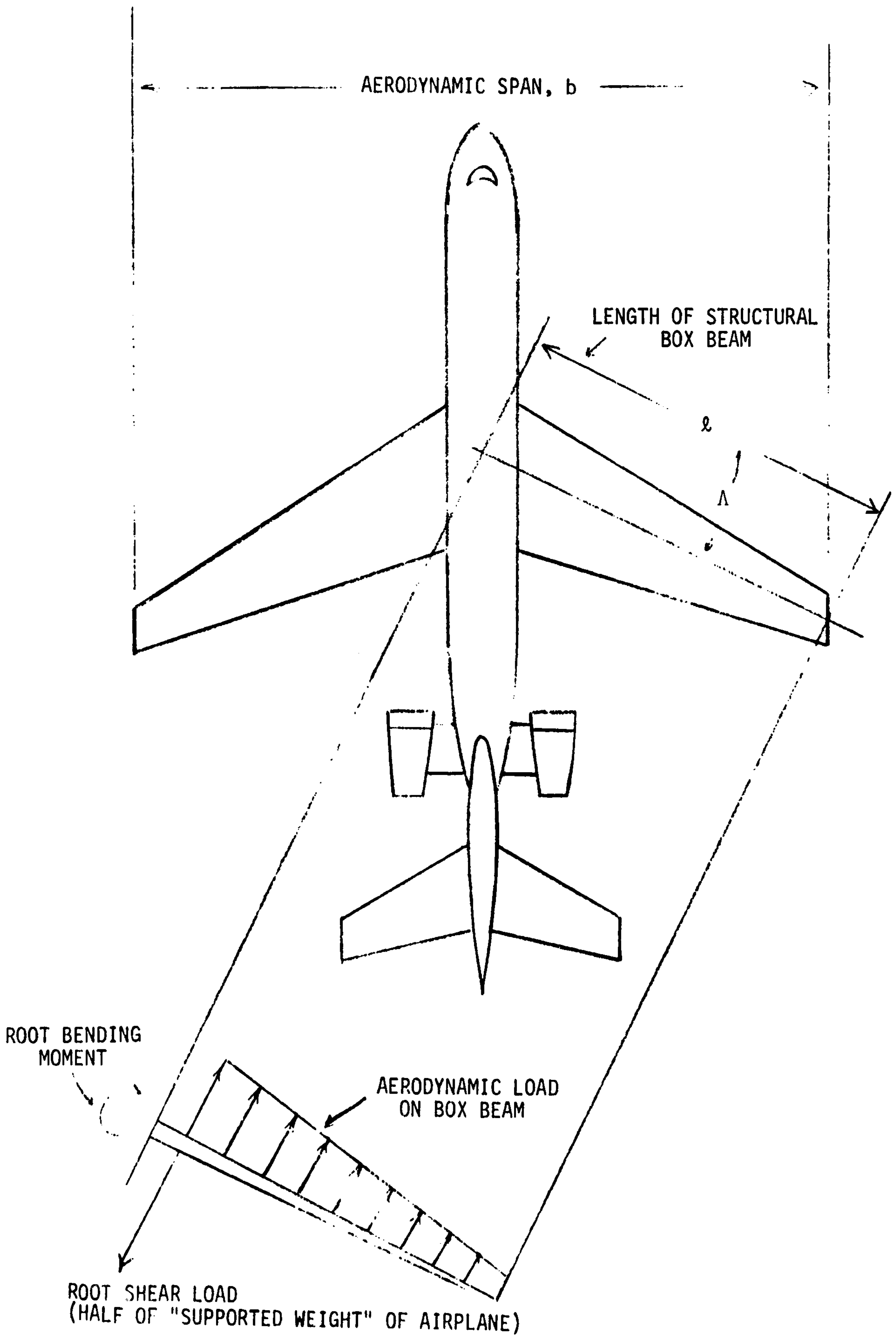
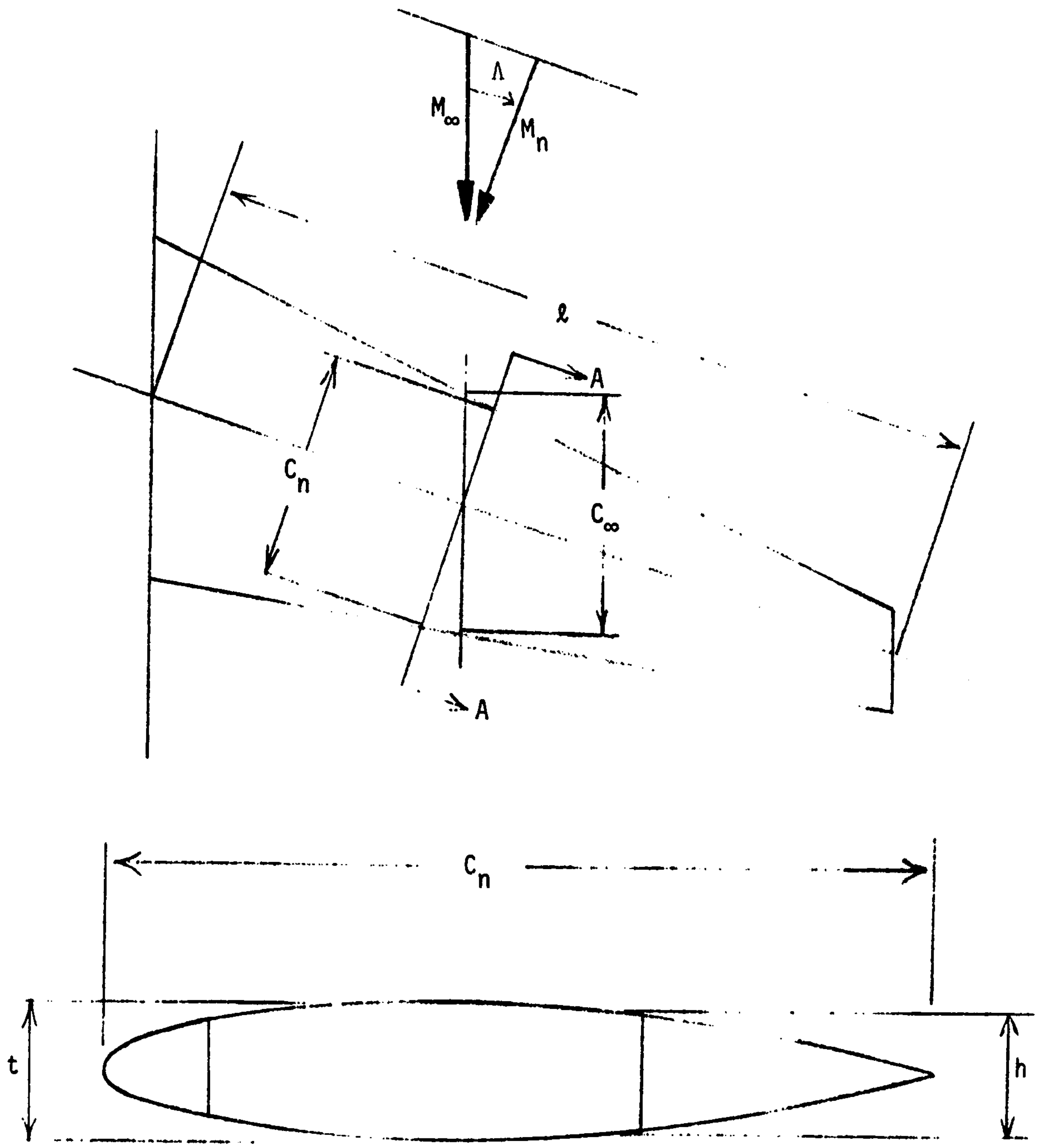


Figure 1. - Wing structure and loads on swept-wing transport.



SECTION A-A

t - THICKNESS OF AERODYNAMIC PROFILE
h - DEPTH OF EQUIVALENT BOX BEAM

Figure 2. - Dimensions for structural model of swept wing.

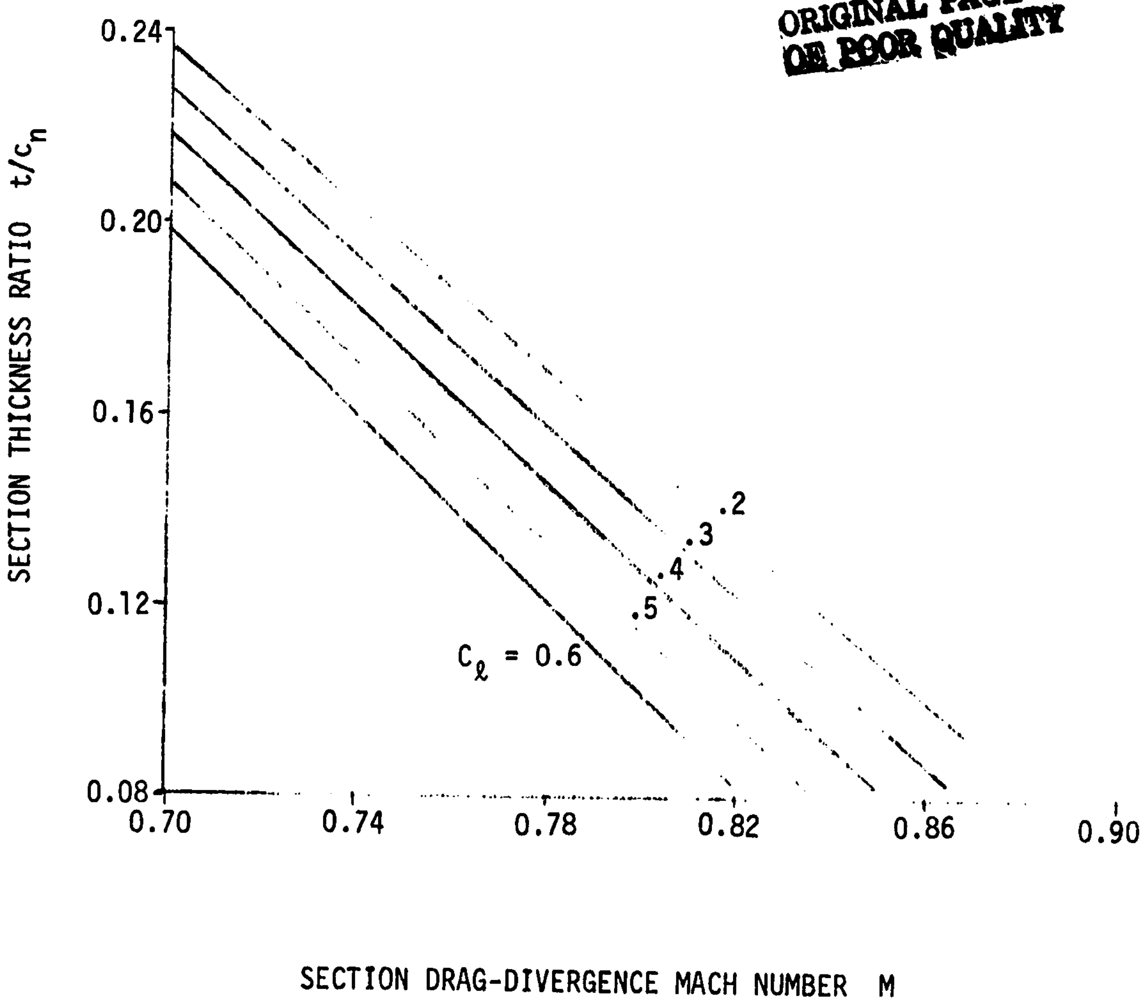


Figure 3. - Drag-rise Mach number for two dimensional supercritical airfoils.

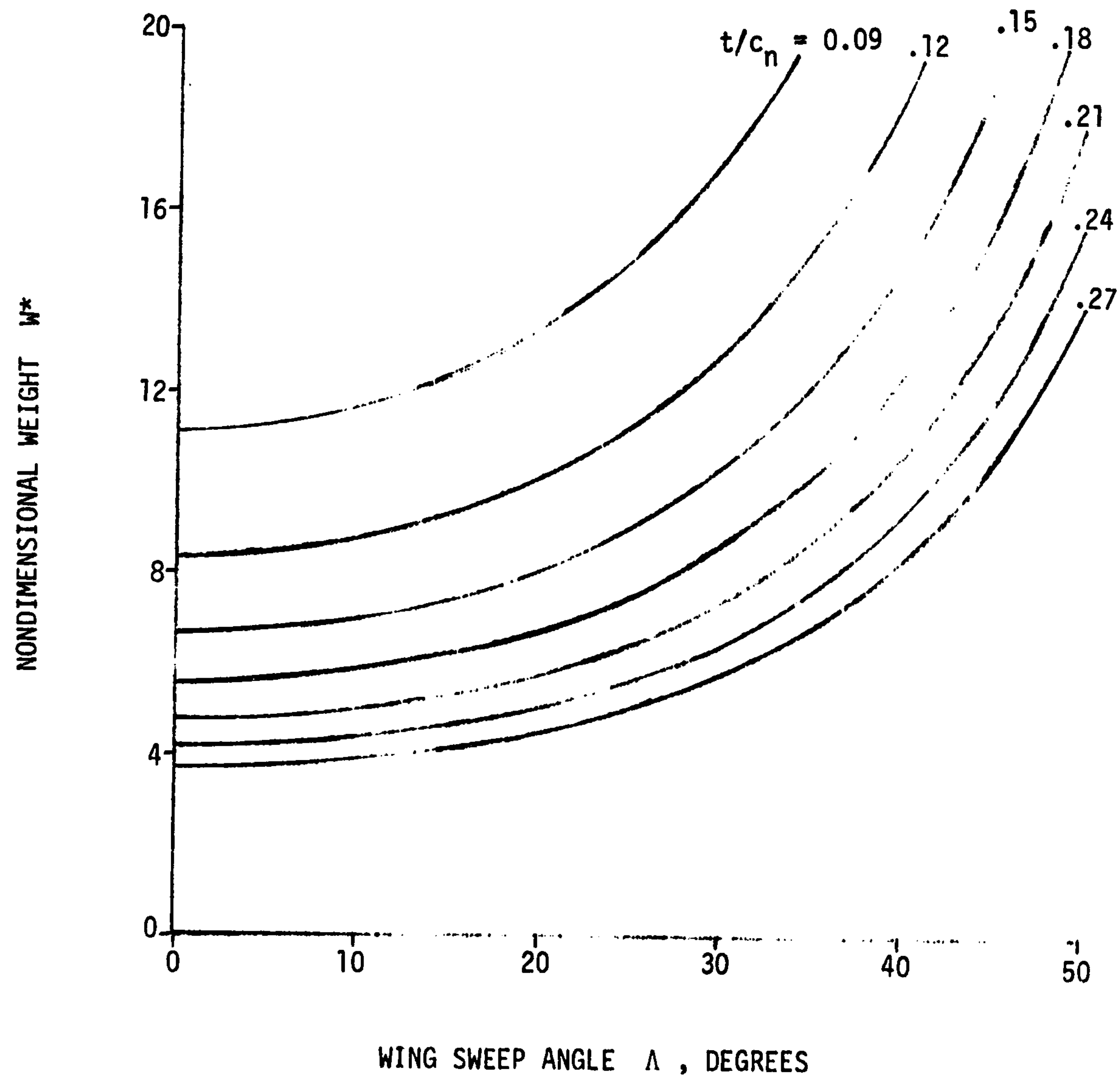


Figure 4. - Nondimensional weight of wing structure for constant normal-thickness ratio.

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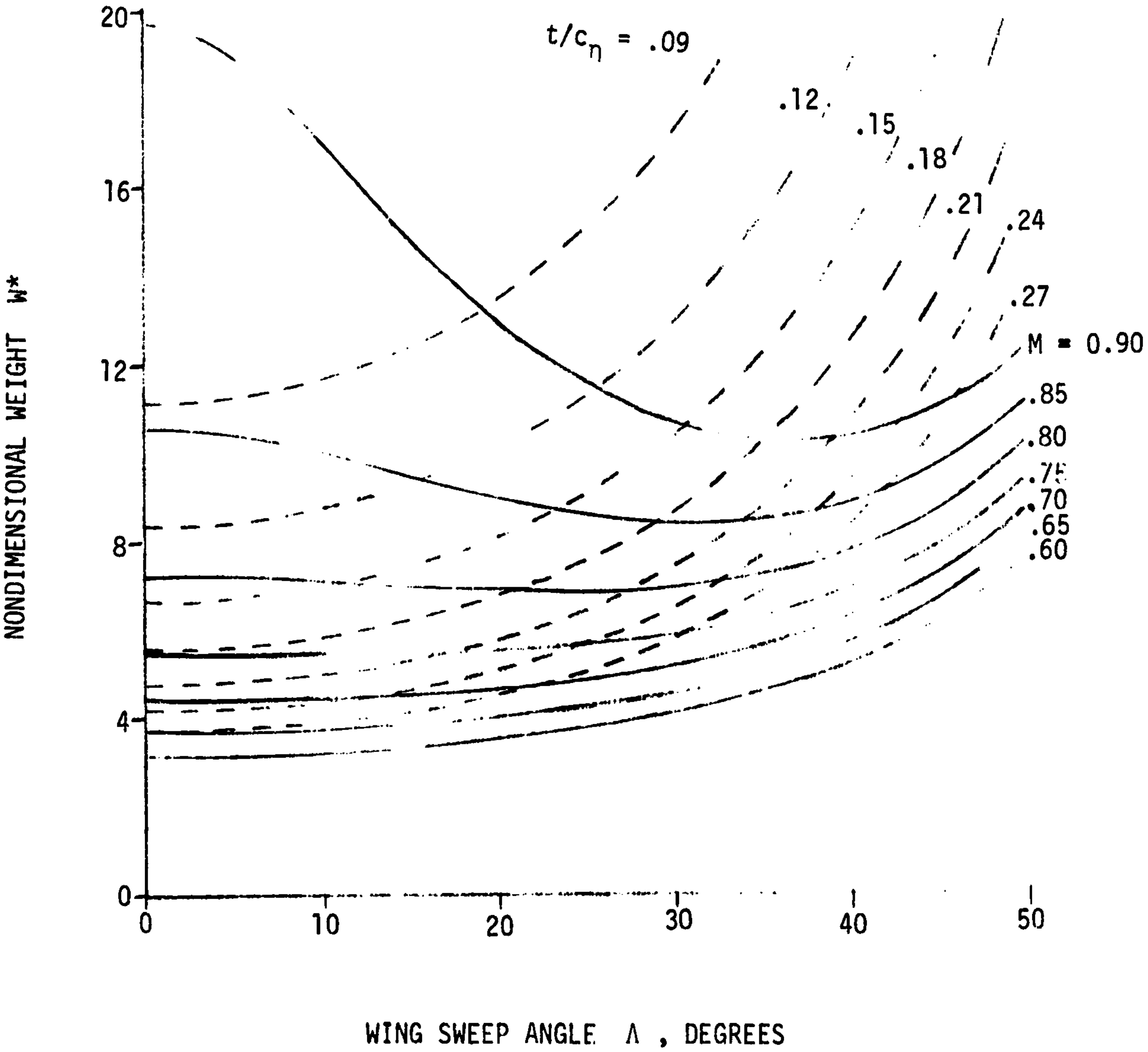
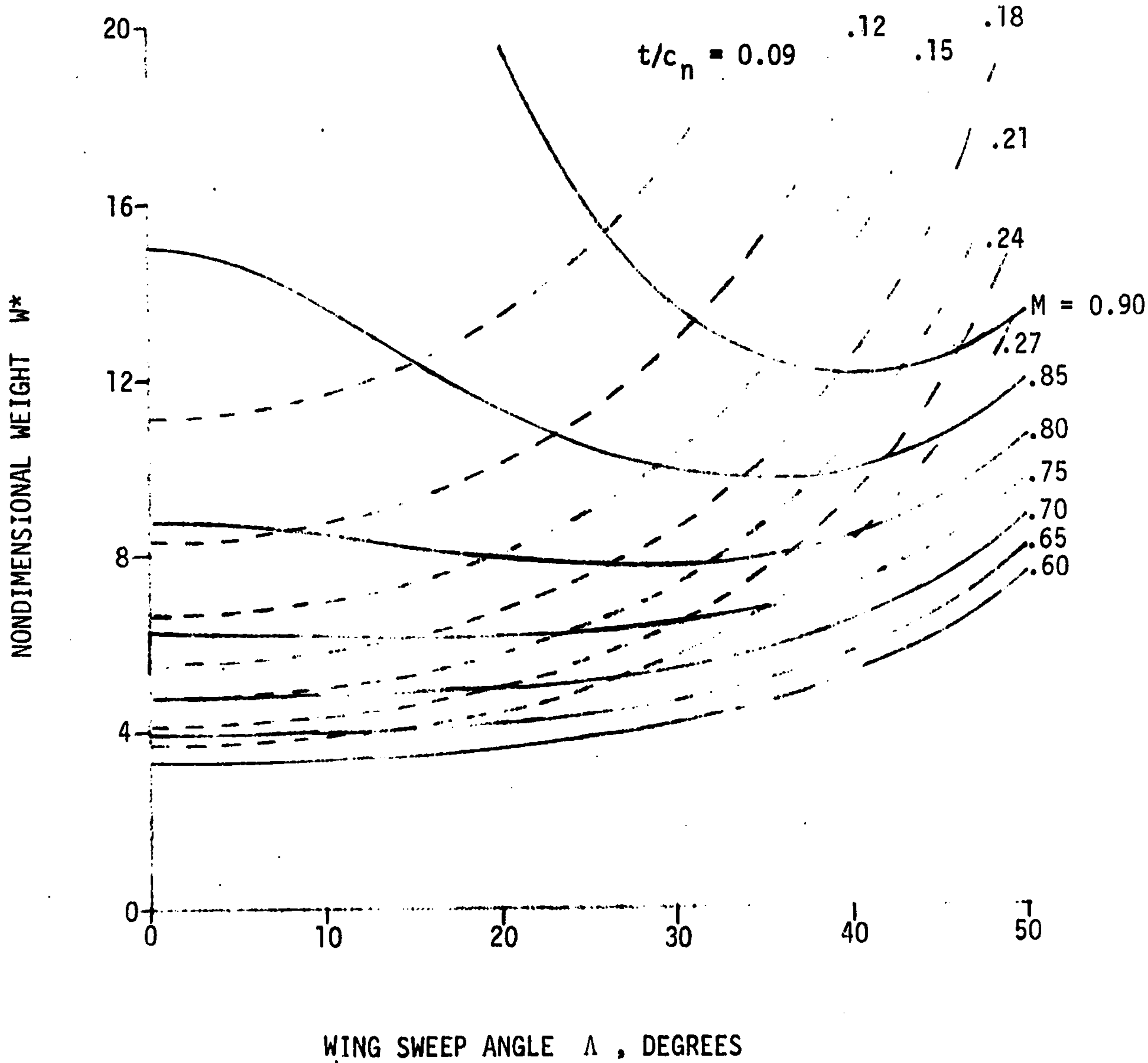


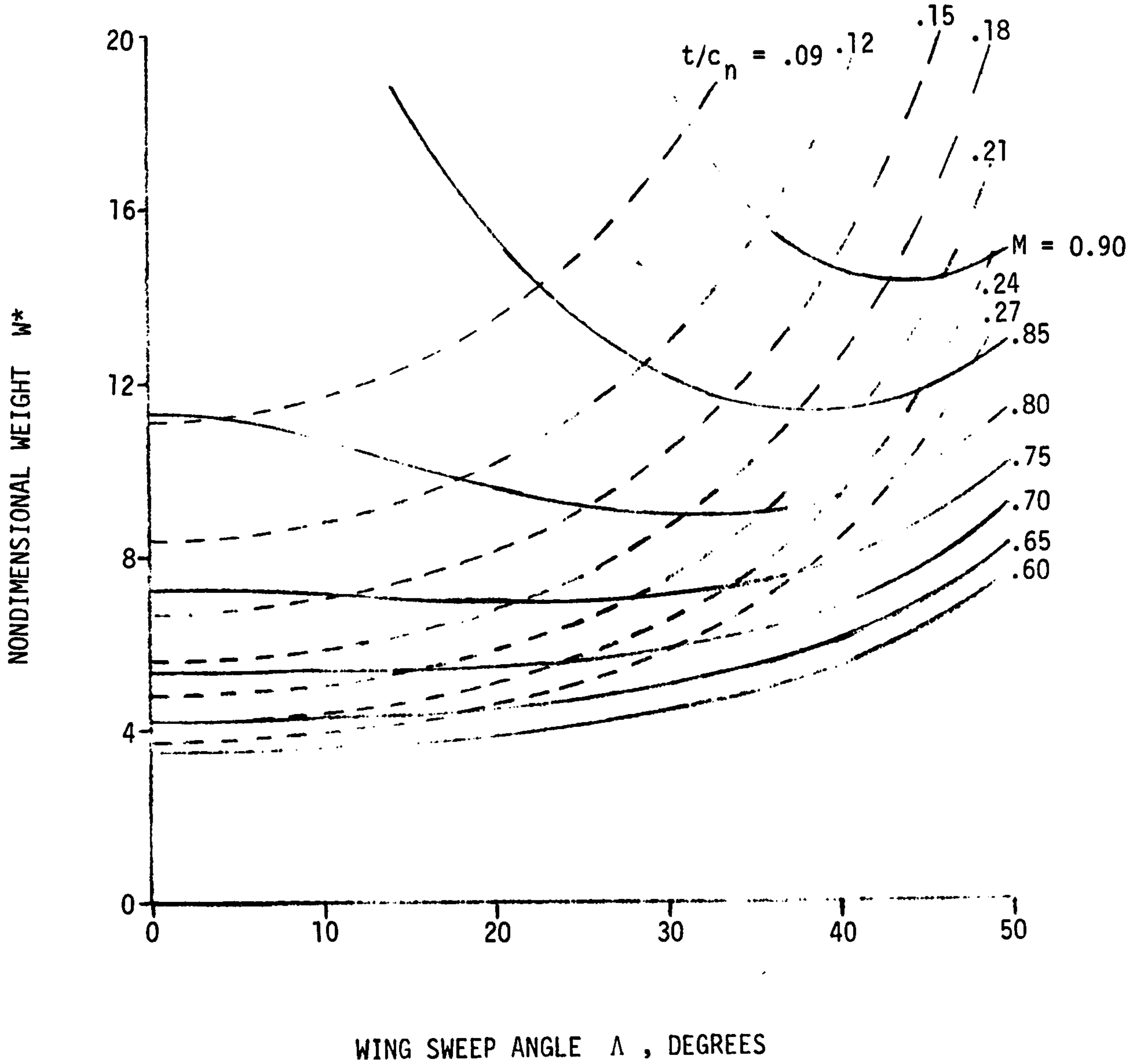
(a) Freestream lift coefficient = 0.3

Figure 5. - Effect of sweep angle and Mach number on nondimensional weight of wing structure.



(b) Freestream lift coefficient = 0.5

Figure 5. - Continued.



(c) Freestream lift coefficient = 0.7.

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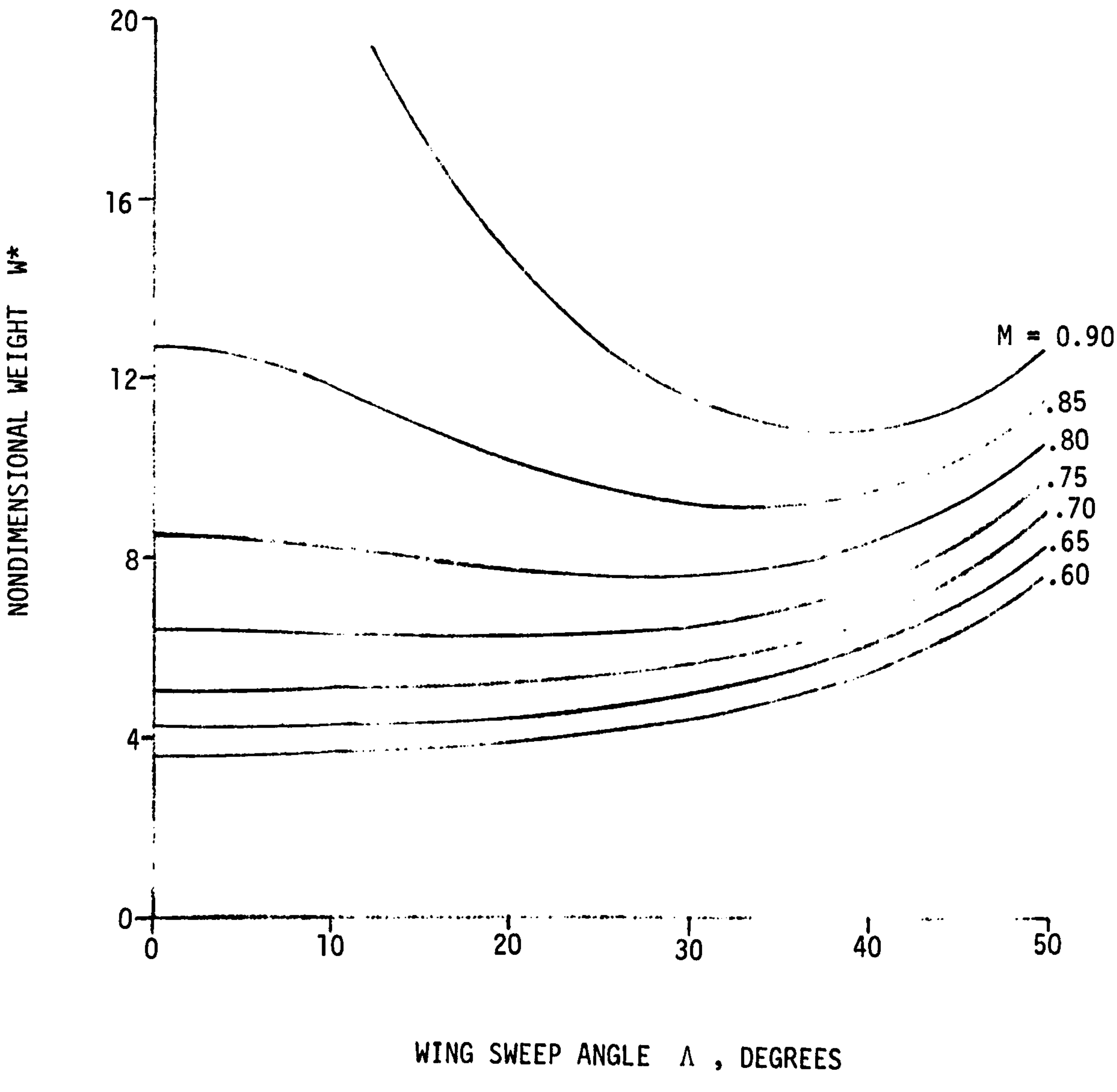
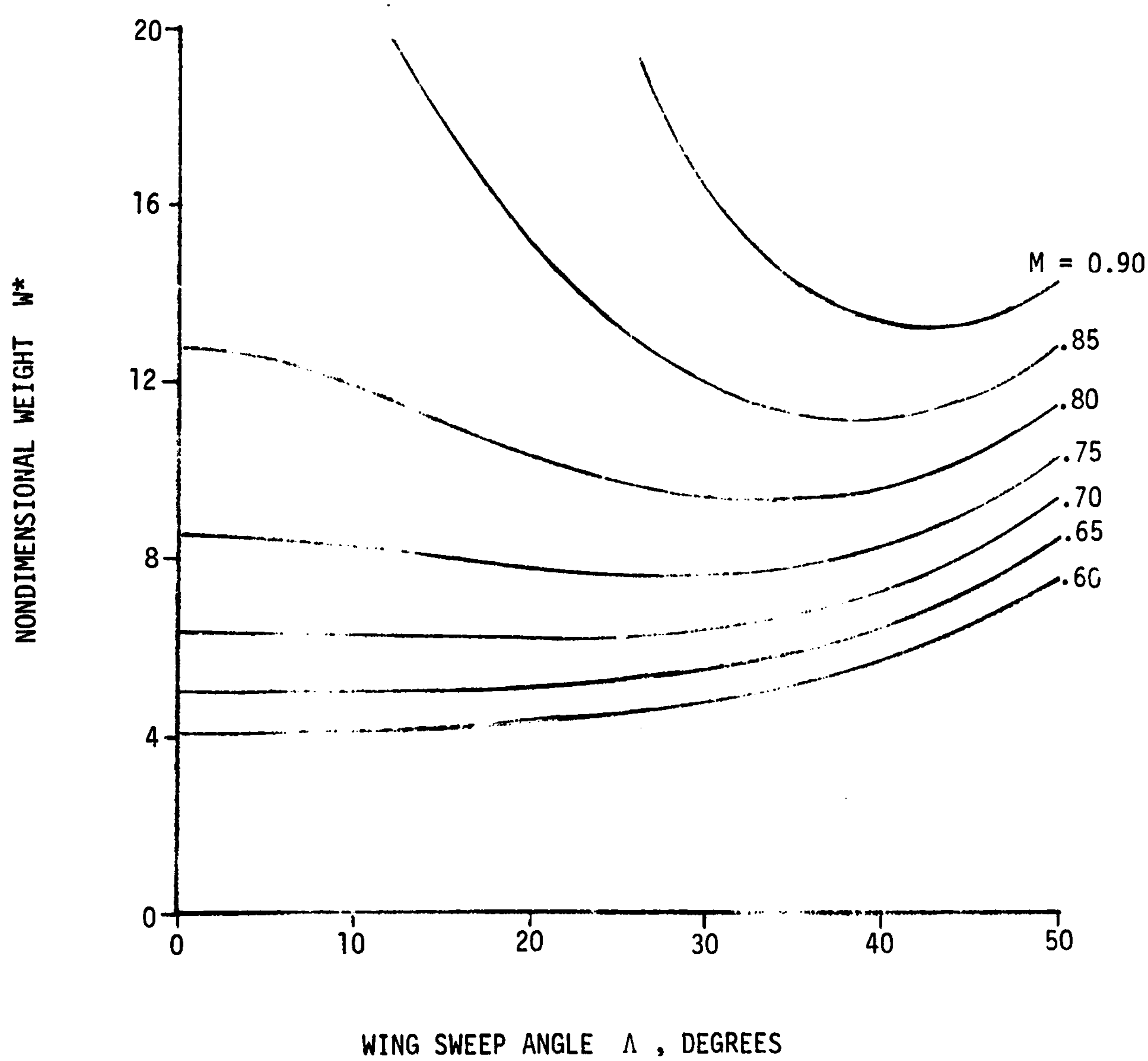
Figure 5. - Continued.



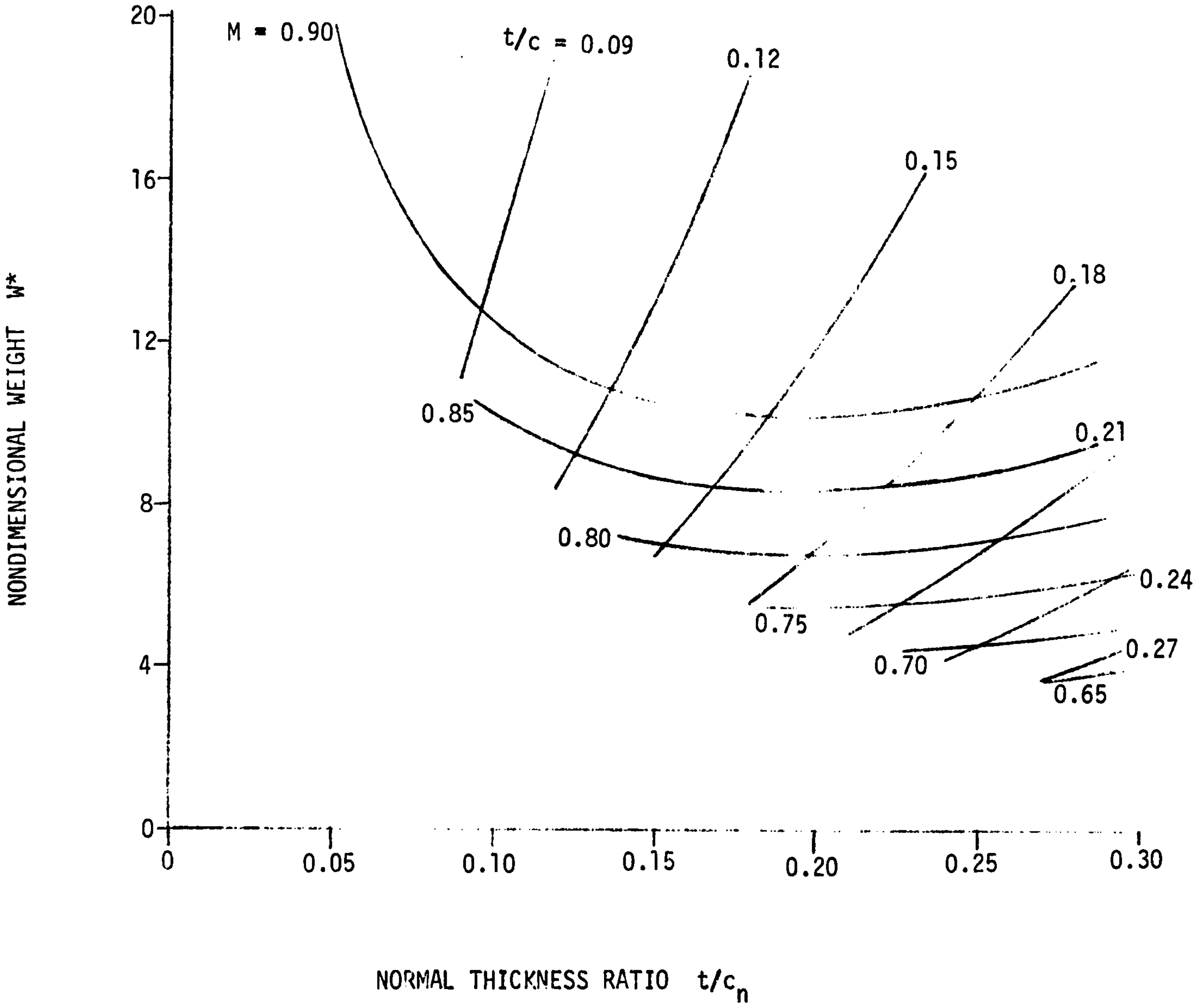
(a) Constant $M^2C_L = 0.30$.

Figure 6. - Effect of sweep angle and Mach number on nondimensional weight of wing structure.



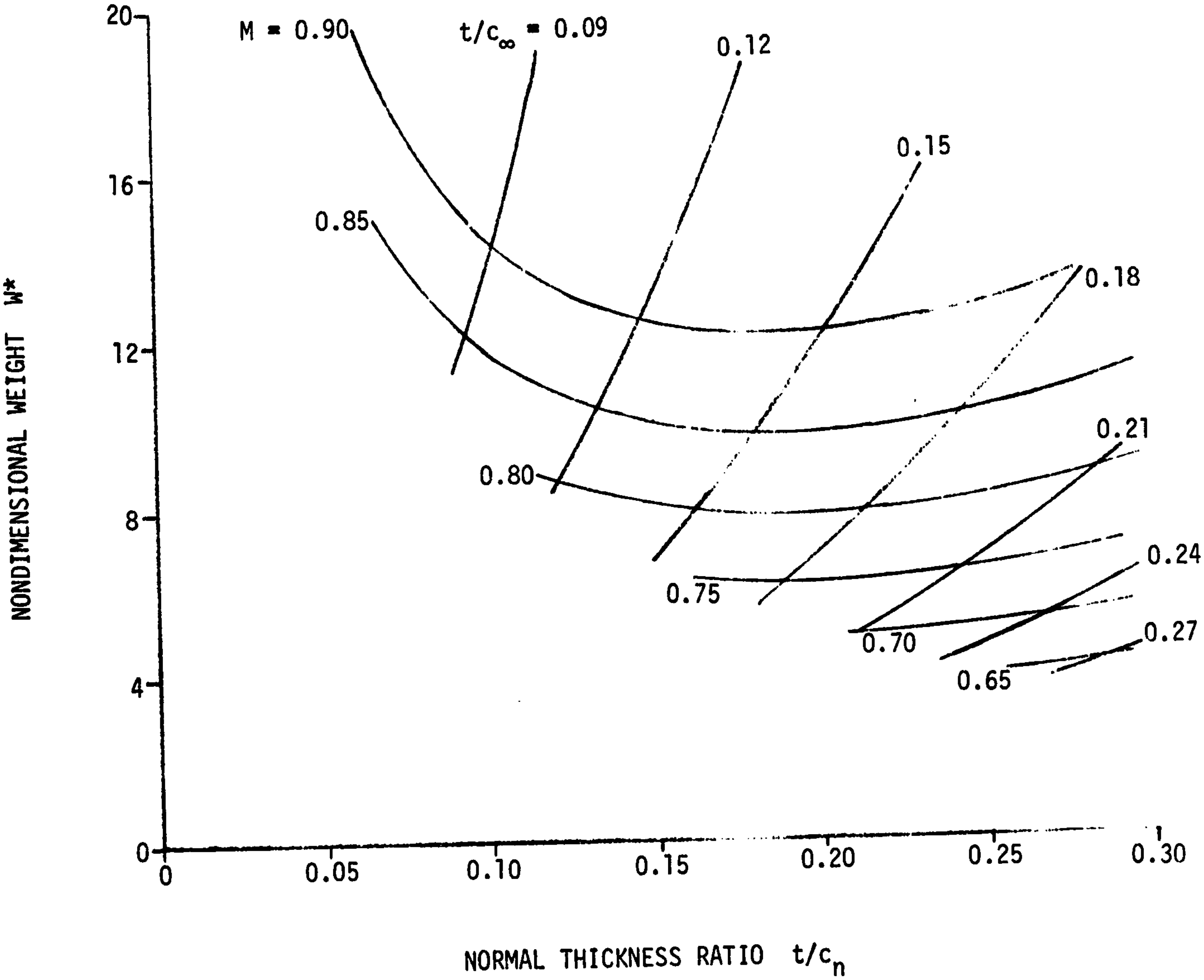
(b) Constant $M^2C_L = 0.50$

Figure 6. - Continued.



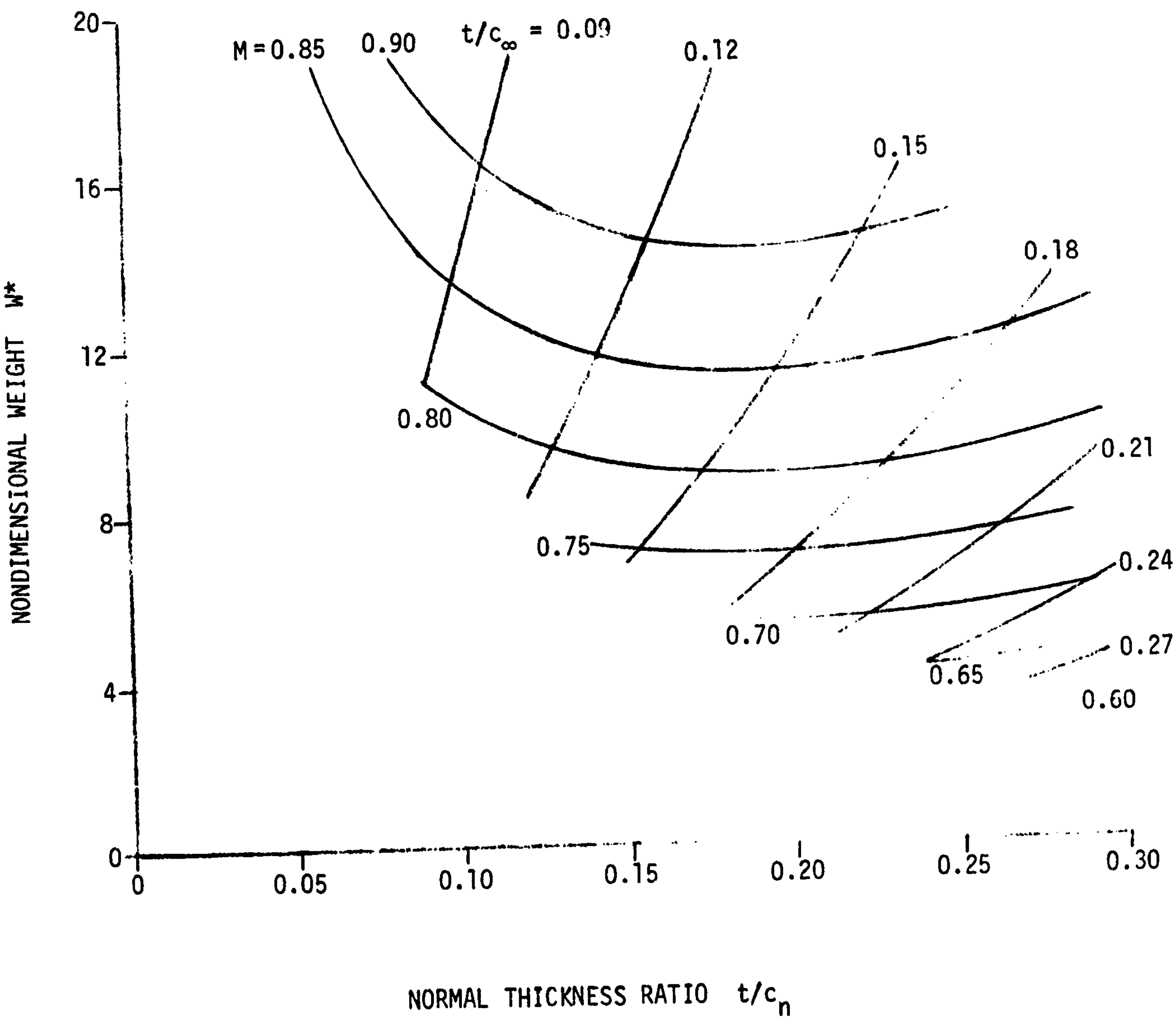
(a) Freestream lift coefficient = 0.3

Figure 7. - Effect of thickness ratio and Mach number on nondimensional weight of wing structure.



(b) Freestream lift coefficient = 0.5.

Figure 7. - Continued.



(c) Freestream lift coefficient = 0.7.

Figure 7. - Continued.

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16. Abstract

Results are presented from an elementary analysis of the effect of sweep angle on the idealized structural weight of swept wings, with cruise Mach number M and lift coefficient C_L as parameters. The analysis indicates that sweep is unnecessary for cruise Mach numbers below about 0.80, whereas for the higher subsonic speeds, a well defined minimum-weight condition exists at a sweep angle in the neighborhood of 35° or 40°, depending on M and C_L. The results further indicate that wing-structure weight increases sharply with Mach number in the high subsonic range, with Mach 0.85 wings weighing half again as much as Mach 0.75 wings. Weight is also shown to increase with cruise lift coefficient, but the effect is not strong for the usual range of design lift coefficients. Minimum wing-structure weight is found to occur at a ratio of thickness to normal chord of about 18 percent, but it is concluded that the thickness ratio for optimum wing design would probably lie in the range of 12 to 15 percent.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
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