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# OUTPUT FEEDBACK REGULATOR DESIGN FOR JET ENGINE CONTROL SYSTEMS

by Walter Merrill
Lewis Research Center
Cleveland, Ohio 44135

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Walter Merrill

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

A multivariable control design procedure based on the output feedback regulator formulation is described and applied to an F100 turbofan engine model. Full order model dynamics, are incorporated in the example design. The effect of actuator dynamics on closed loop performance is investigated. Also, the importance of turbine inlet temperature as an element of the dynamic feedback is studied. Step responses are given to indicate the improvement in system performance with this control. Calculation times for all experiments are given in CPU seconds for comparison purposes.

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Walter Merrill

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

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A multivariable control design procedure based on the output feedback regulator formulation is described and applied to an F100 turbofan engine model. Full order model dynamics, are incorporated in the example design. The effect of actuator dynamics on closed loop performance is investigated. Also, the importance of turbine inlet temperature as an element of the dynamic feedback is studied. Step responses are given to indicate the improvement in system performance with this control. Calculation times for all experiments are given in CPU seconds for comparison purposes.

## 1. INTRODUCTION

Increased performance requirements, more complex engine configurations, and the feasibility of onboard digital engine controllers have generated considerable interest in advanced, multivariable, engine control systems. One program, the F100 Multivariable Control Synthesis (MVCS) program, was initiated jointly by NASA and the Air Force to study the applicability of linear, quadratic regulator (LQR) theory to the design of multivariable F100 turbofan engine controls (Ref. 1). The designed control structure (See Fig. 1) incorporates a scheduled matrix of feedback gains for regulation of engine steady-state conditions. In the MVCS program these gains were synthesized using quadratic regulator theory at each operating point by the following process. First, a linear model, valid for small perturbations about the operating point, was obtained using performance data generated by a nonlinear digital simulation of the engine. Next, since the number of available measurements was much smaller than the number of states the order of the linear engine model was reduced using modal techniques to constrain the reduced state vector to be a subset of the vector of available engine measurements. This step facilitated the use of LQR since LQR requires full state feedback. Finally, a performance index for the reduced state vector was selected that incorporated the control requirements and the gains synthesized using LQR theory and the reduced models. The result was a family of constant feedback gain matrices for regulation at various engine operating conditions. This design procedure gave a control design implementable on a digital computer. Moreover, a real-time hybrid computer simulation evaluation of the engine and digital control (Ref. 2) has demonstrated the ability of the control to perform the prescribed control functions and the flexibility of the LQR design process to incorporate changing control function definitions. Prompted by these simulation results full scale engine tests will be conducted at the Lewis Research Center altitude test facility to further evaluate this control design.

One aspect of the LQR design, the model reduction process, or more specifically the selection of the reduced order state space, was not straightforward and complicated the overall design procedure. In particular the selection of the reduced state space defines a trade-off between control performance and the control implementation complexity. A definitive answer to the adequacy of this trade-off would require many iterations through the design procedure and, therefore, many model reductions. Additionally, there is no a priori assurance that any approximation technique se-

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lected will yield good or even acceptable results when the resultant control design is applied to the original system. Therefore, a design process that incorporates the utility of the LQR theory but eliminates the model reduction step is desirable.

It has been shown (Ref. 3) that output feedback regulator (OFR) theory (Ref. 4) can be applied to the synthesis of feedback control matrices for jet engine controls. In fact the results of Ref. 3 showed that given the same definition of performance and feedback structure of the MVCS study, equivalent feedback matrices could be determined directly from the full state model using OFR theory. Design using the full state space insures that all information in the model is incorporated in the design. Also, any state can be directly weighted in the OFR performance index, while only those states in the reduced order state vector can be weighted in the LQR case. Additionally the OFR formulation retains the performance index formulation and the constant gain feedback structure of the LQR design that have been shown to be of value in engine control design while eliminating the model reduction step. Finally, the OFR formulation allows an efficient study of the performance-complexity trade-off important in any realistic design. These advantages are obtained at the cost of increased computational effort. For example the LQR design at one operating point requires the solution of a reduced order Riccati equation while the OFR approach requires the solution of two full order Riccati equations. In the studies of Refs. 1 and 3 the reduced and full state orders were 5 and 17, respectively.

The purpose of this paper is to demonstrate the application and utility of OFR theory for the design of jet engine controls using the F100 engine model defined in the "Theme Problem Description." Also, the effect on engine performance of input actuators for main burner fuel flow (WFMB), nozzle jet area (AJ) and inlet guide vane position (CIVV) will be demonstrated for this control structure. It will also be shown that one measurement, fan turbine inlet temperature, FTIT, is of limited value as a dynamic feedback element. Although well known, a brief description of the theory and philosophy of the OFR is now given. This is followed by a discussion of the application of the theory to the design example and a discussion of the results of this application.

## 2. THEORY AND PHILOSOPHY

Given a time invariant linear system

$$\dot{x} = Ax + Bu, \qquad x(0) = x_0 \qquad (1)$$
$$y = Cx$$

where the initial state, $x_0$, is a zero-mean random variable with covariance, $X_0$. The OFR problem is to find the time invariant feedback law

$$u = -Fy \qquad (2)$$

which minimizes

$$J = E\left\{\frac{1}{2}\int_0^\infty (x^TQx + u^TRu)dt\right\} \qquad (3)$$

with

$$Q \geq 0$$
$$R > 0 \tag{4}$$

Necessary conditions for optimality and computational solutions are derived for this problem statement in References 4 and 5 and for its discrete time counterpart in Reference 6. The necessary conditions are

$$KA_0 + A_0^T K + Q + C^T F^T RFC = 0 \tag{5a}$$

$$LA_0^T + A_0 L + X_0 = 0 \tag{5b}$$

$$F = R^{-1} B^T KLC^T (CLC^T)^{-1} \tag{5c}$$

where

$$A_0 = A - BFC \tag{6}$$

The solution of (5) gives $F$, and the suboptimal value of $J$ for this $F$ is given by

$$J = \frac{1}{2} \operatorname{Tr}(KX_0) \tag{7}$$

The design philosophy requires the selection of $Q$ and $R$ in the quadratic performance index to establish a suboptimal trade-off between system performance and control energy required to achieve that performance. Control performance is typically thought of as regulation or rejection of unwanted disturbances in the system.

## 3. ENGINE APPLICATION

The OFR formulation was applied directly to the "Theme Problem" which represents an F100 engine at a sea level, static, maximum non-afterburning power condition. Both the three and five control input feedback structures were studied. The feedback variables include fan speed, N1, compressor speed, N2, compressor discharge pressure, PT3, augmentor pressure, PT7M, fan turbine inlet temperature, FTIT, and in one case a measured fuel flow, WFMB. The $Q$ and $R$ matrices were selected as

$$Q = \operatorname{Diag}\left[q_i\right] \tag{8}$$

$$R = \operatorname{Diag}\left[r_i\right] \tag{9}$$

where the $q_i$ and $r_i$ are defined in Table I, for all the cases studied in this paper. These matrix-elements correspond in value to those used in Ref. 1 and 3. The same $Q$ and $R$ were used in each case studied.

The engine model is supplied as

$$\dot{x} = Ax + Bu \tag{10a}$$

$$y = Cx + Du \tag{10b}$$

For purposes of this paper define a vector

$$z = Hx \tag{11}$$

where $z$ represents the measurements used in the feedback law

$$u = -Fz \tag{12}$$

The $F$ matrix for each case was calculated by approximating the continuous system of (10) by its descrete counterpart and applying the numerical algorithms of Ref. 3. A sampling time of $T = 0.0001$ secs/cycle was selected to give a close approximation to the continuous system since the open loop eigenvalues are greater than -600 rad/sec. Also, the initial condition covariance matrix was arbitrarily set as $X_o = I$ for all cases.

The numerical procedure for calculating $F$ is basically successive approximation and requires a stabilizing output feedback matrix as an initial guess. Since the F100 linear models are open loop stable, an initial guess of zero for each feedback element can be conveniently selected to start the iterative process. Once a candidate feedback matrix has been calculated, it may be used in subsequent calculations to speed convergence.

The ten experiments conducted for this paper are summarized in Table II. Several different conditions were studied, i.e., two different input configurations, four different measurement configurations, and several different models of actuator dynamics. The first input configuration includes WFAM, AJ, and CIVV while the second input configuration includes all five inputs in the order given in the "Theme Problem Description." The first measurement configuration includes five measurements N1, N2, PT3, PT6 and FTIT, called the basic set. The second configuration includes four measurements, the basic set minus N2. The third configuration includes four measurements, the basic set minus FTIT. The fourth configuration includes the basic set except for FTIT which is replaced by measured fuel flow. The different actuator models considered include combinations of those given in the "Theme Problem Description" and a first order fuel flow actuator

$$\frac{WFMB(s)}{WFREQ(s)} = \frac{1}{0.1s + 1} \tag{13}$$

Both the original and updated versions of the CIVV actuator dynamics are included. The original model is denoted with an asterisk where appropriate. Actuator dynamics were included by augmenting the original 16th order state vector with additional states representing actuator dynamics. Note that because of the output regulator formulation no changes need be made in the control structure to accommodate this additional information. Sensor dynamics could be included in the same way. However, the sensor dynamics are fast enough so as to have no appreciable effect on the dominant portion of the engine model. This is true except for the FTIT measurement. It will be shown, however, that in this study the inclusion of the FTIT measurement in the feedback structure yields no significant control gain. Thus, the effect of the FTIT sensor dynamics are not included in this study.

## 4. DISCUSSION OF RESULTS

Table II summarizes the results of the ten experiments. The value of the performance index as defined by (3), (7), (8), and (9) is given along with the order of the state vector, the calculation time, and the initial value of the $F$ matrix in the iterative calculation. Calculation time is given in CPU seconds for batch execution on a 1110 UNIVAC computer.

Comparison of the performance indices (A smaller performance index implies better performance) for various cases yields several conclusions. First consider cases 1 and 3. Case 3 represents the feedback structure actually implemented in the F100 MVCS program. Comparison with case 1 shows no difference in performance when using either FTIT or WFMB in the feedback structure. Comparison of cases 1 and 4 shows the degradation in performance when using three control variables rather than five. Comparison of case 4, 5, 6 and 7 shows the relative importance of including actuator dynamics for each input in the model. Only CIVV actuator dynamics and the second WFMB actuator pole seem to be unimportant for the three inputs studied. Comparison of cases 7 and 8 and cases 9 and 10 indicate no significant difference in results when either the original or updated CIVV actuator dynamics are used in the model. Comparison of cases 7 and 10 and cases 8 and 9 indicates the relative importance of FTIT as a feedback variable. The small improvement in performance gained by including FTIT in the feedback structure does not justify the additional cost of making the measurement. A comparison of cases 1 and 2 shows the relative degradation when N2 is not considered in the feedback structure. Such comparisons yield insight into the effect of sensor failures and the relative importance of particular variables in the feedback structure. An evaluation of the trade-off between system performance and control complexity can be readily handled with the OFR formulation.

Based on these results the feedback structure of case 10 was simulated with the full order model and actuator dynamics as shown in Figure 2 to obtain closed loop trajectories. For comparison closed loop and open loop eigenvalues are given in Table III. Note that open loop eigenvalues 3, 4, and 5 which closely correspond to states representing N1 and N2 are decreased giving these states a faster dynamic response. This is accomplished generally by trading-off dynamic performance in the fuel flow actuator eigenvalue (Open Loop-6 to Closed Loop-4) and the area actuator (Open Loop 11-12 to Closed Loop 10-11). Various step responses are given in Figure 3.

These trajectories were obtained by commanding a 480 lb step change in thrust. The $K_f$ matrix of Figure 2 represents a reference schedule. It is selected such that the steady-state gain from commanded input to desired output is unity. The trajectories show good transient response with no violation of stall margin limits. Thrust response indicates only a 2% overshoot while the turbine temperature and rotor speeds undergo a relatively small and short transient. Additionally, this increased thrust is obtained by "closing down" nozzle area and by increasing fuel flow. Figure 4 shows a comparison of open and closed loop rotor speeds for a step change in fuel flow to illustrate the improvement in response time and the lack of overshoot in the closed loop trajectories.

## 5. SUMMARY

This paper has presented a multivariable control design procedure based on OFR theory that can be used to design operating point controls for jet engines. The procedure utilizes the benefits of a linear, quadratic approach but eliminates the need for full state feedback. The effect of actuator dynamics on the control design is studied along with the relative importance of various feedback variables. It was demonstrated that sensor failure and complexity-performance trade-off studies, in the form of differing feedback structures, can be handled quite readily with this formulation. Closed loop step responses for a particular design are given to demonstrate the satisfactory performance of the closed loop system.

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TABLE I DEFINITION OF Q AND R

| $i$ | $q_i$ | $r_i$ |
|---|---|---|
| 1 | 0.001 | 0.0001 |
| 2 | .002 | 2000.0 |
| 3 | 5.00 | 2.000 |
| 4 | .000 | |
| 5 | 2.00 | |

$q_i = 0,\ i > 5.$

TABLE II SUMMARY OF EXPERIMENTS AND RESULTS

| Case | Number of inputs | Measurement configuration | Actuator model order | | | Performance index ($\times 10^{-4}$) | State order | Calculation time, sec | $F_{IC} = 0$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | WFAM | AJ | CIVV | | | | |
| 1 | 5 | 1 | 1 | - | -- | 5.516 | 17 | 164 | Yes |
| 2 | 5 | 2 | 1 | - | -- | 5.806 | 17 | --- | --- |
| 3 | 5 | 4 | 1 | - | -- | 5.513 | 17 | 135 | Yes |
| 4 | 3 | 1 | 1 | - | -- | 9.710 | 17 | 56 | Yes |
| 5 | 3 | 1 | 2 | - | -- | 10.13 | 18 | 54 | No |
| 6 | 3 | 1 | 2 | 3 | -- | 14.71 | 21 | 320 | No |
| 7 | 3 | 1 | 2 | 3 | 3 | 14.83 | 24 | 104 | No |
| 8 | 3 | 1 | 2 | 3 | 3* | 14.87 | 24 | 320 | No |
| 9 | 3 | 3 | 2 | 3 | 3* | 15.18 | 24 | 197 | No |
| 10 | 3 | 3 | 2 | 3 | 3 | 15.13 | 24 | 301 | No |

TABLE III OPEN AND CLOSED LOOP EIGENVALUES

| | Open loop | Closed loop |
|---|---|---|
| 1 | -.6477 | -.703 |
| 2 | -1.906 | -1.977 |
| 3 | -2.618 | -5.638 |
| 4 | -6.715 + 1.312j | -8.336 |
| 5 | -6.715 - 1.312j | -10.18 + 3.980j |
| 6 | -10.00 | -10.18 - 3.980j |
| 7 | -13.94 | -12.57 |
| 8 | -17.80 + 4.781j | -15.06 + 5.183j |
| 9 | -17.80 - 4.781j | -15.06 - 5.183j |
| 10 | -18.59 | -18.54 + 30.07j |
| 11 | -21.11 + 31.23j | -18.54 - 30.07j |
| 12 | -21.11 - 31.23j | -18.63 |
| 13 | -21.33 + .8218j | -21.45 + .8720j |
| 14 | -21.33 - .8218j | -21.45 - .8720j |
| 15 | -38.68 | -42.31 + 9.207j |
| 16 | -47.13 | -42.31 - 9.207j |
| 17 | -50.00 | -49.75 |
| 18 | -50.00 | -50.84 + 1.317j |
| 19 | -50.66 | -50.84 - 1.317j |
| 20 | -59.16 | -62.13 |
| 21 | -86.06 | -85.76 |
| 22 | -100.0 | -101.7 |
| 23 | -175.7 | -175.0 |
| 24 | -577.0 | -577.0 |

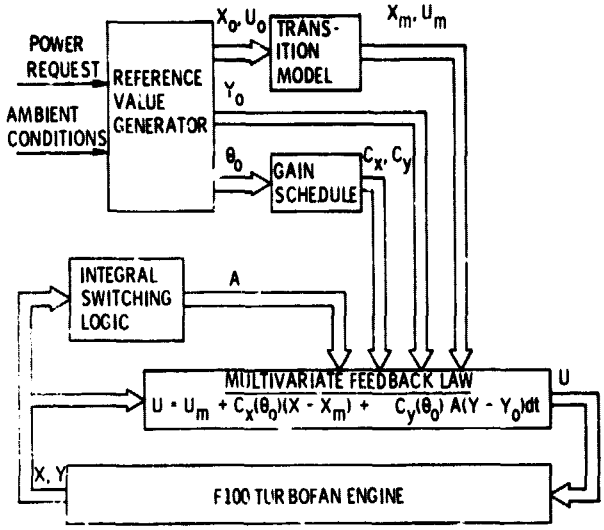


Figure 1. - F100 turbofan multivariable control structure.

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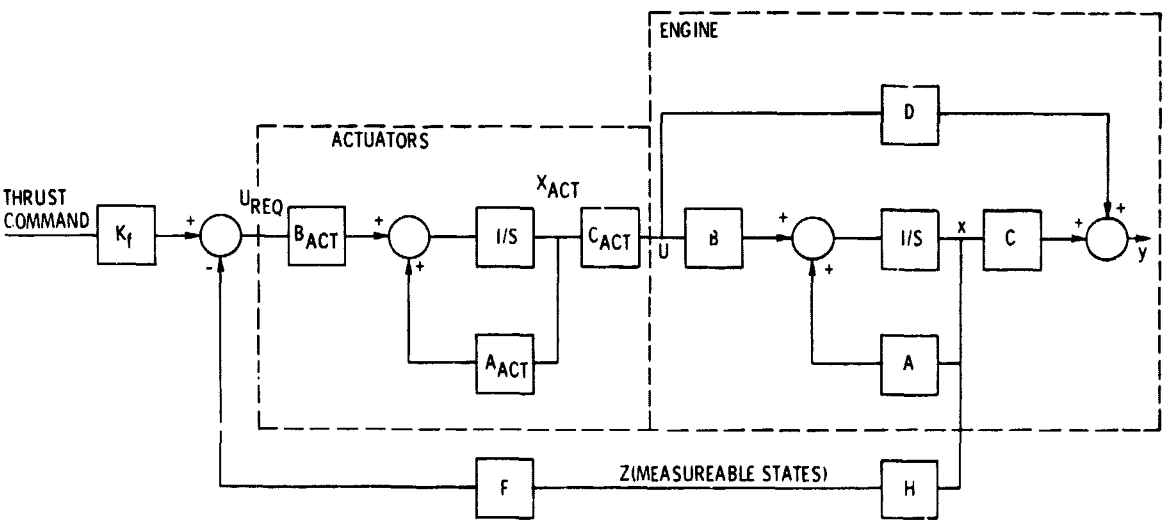


Figure 2. - Engine, actuators, and control block diagram.

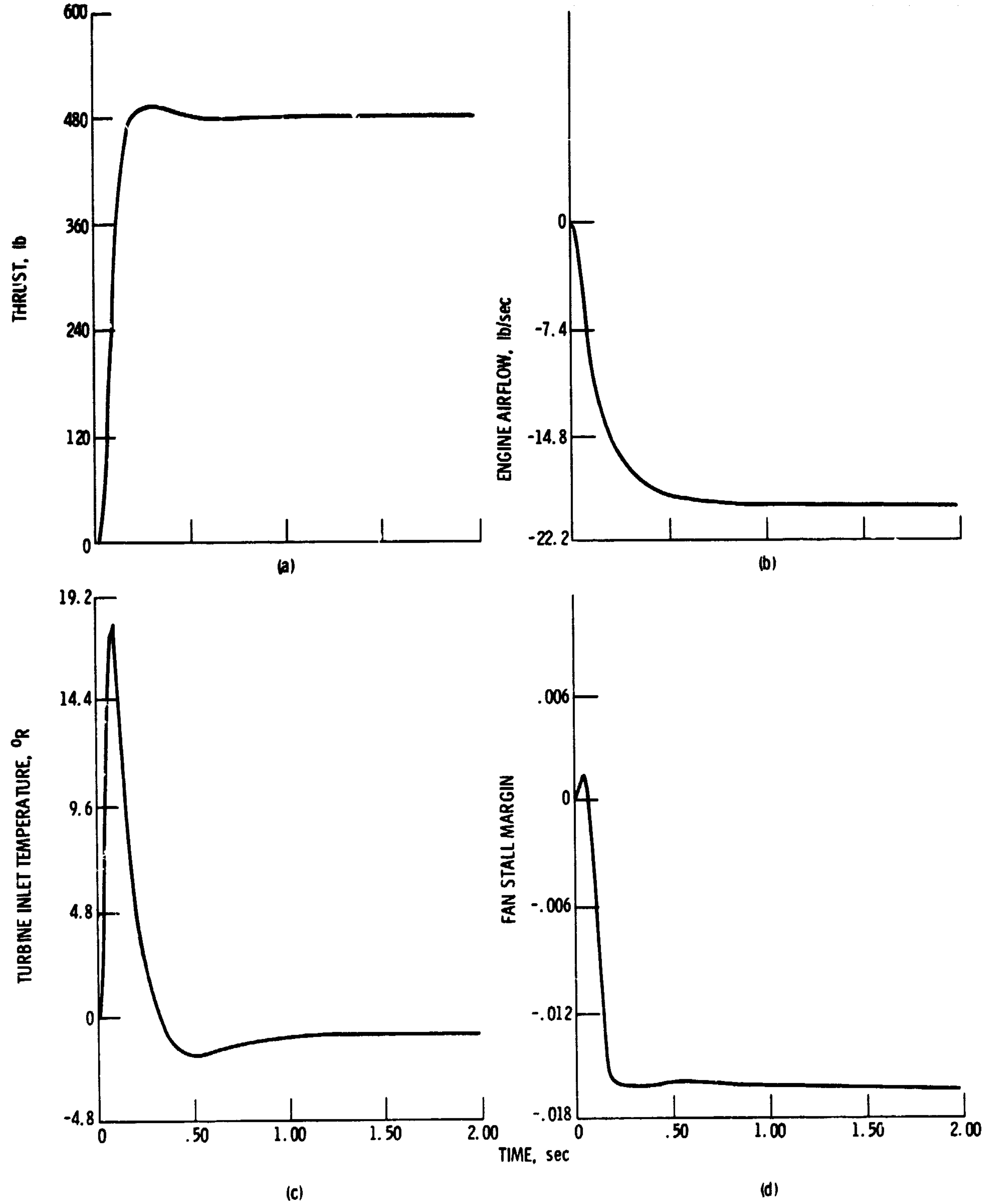


Figure 3. - Commanded thrust response.

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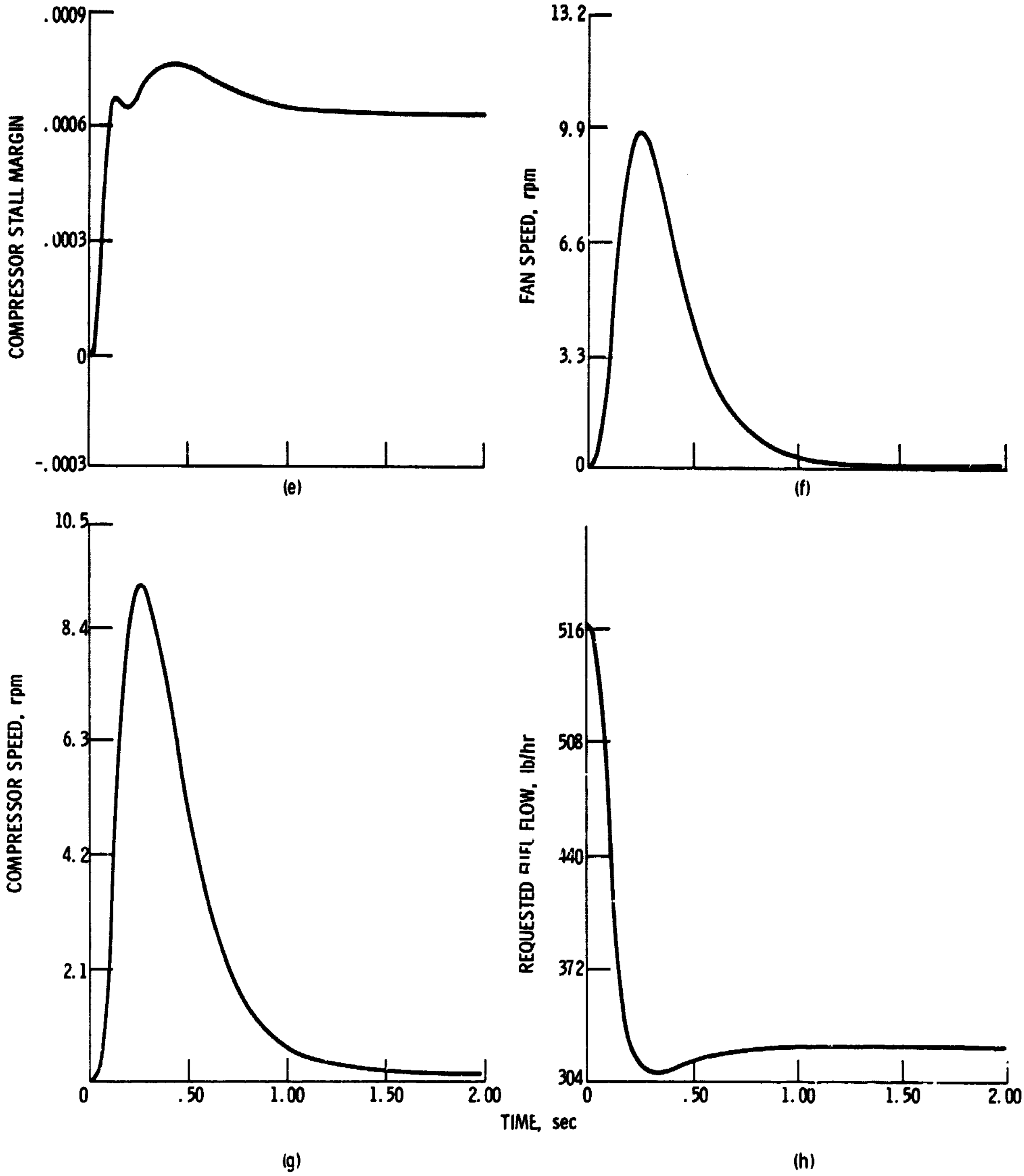


Figure 3. - Continued.

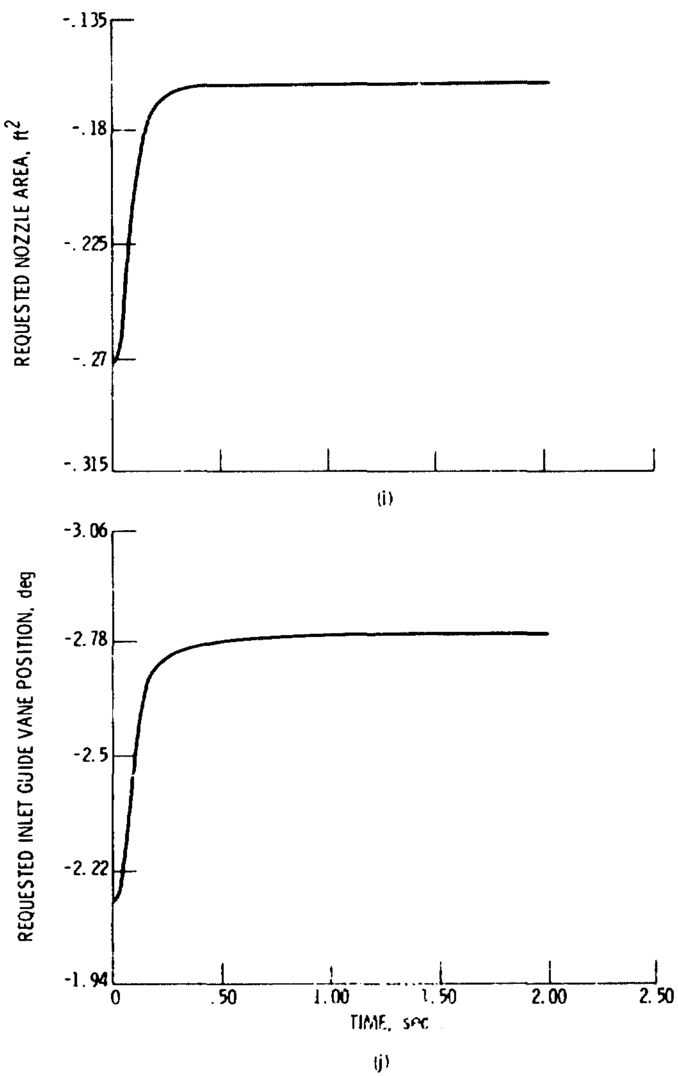


Figure 3. - Concluded.

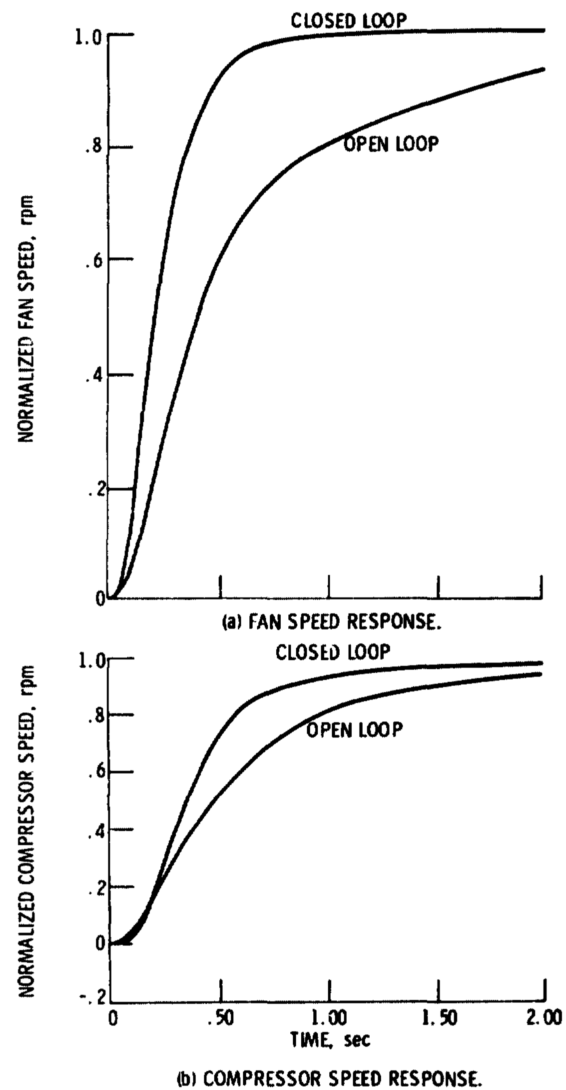


Figure 4. - Fan and compressor speed response to a step change in fuel flow.