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SUPERCONDUCTIVITY OF POLAR MANY-VALLEY SEMICONDUCTORS AND SEMIMETALS

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16. Abstract

The superconductivity of a polar, degenerate semiconductor with two equivalent isotropic valleys and of a semimetal with equal effective masses for the holes and electrons is studied within the framework of the modified Gurevich-Larkin-Firsov model (1962) by taking into account the intervalley pairing. Concentration dependences are found for the critical temperature which agree qualititatively with experimental data for $SrTiO_3$ and $BaPb_{1-x}Bi_xO_3$.

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SUPERCONDUCTIVITY OF POLAR MANY-VALLEY SEMICONDUCTORS AND SEMIMETALS

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Studies by Meissner [1] and Buckel and Hilsch [2] laid /696*
the foundation for experimental studies of the superconductivity of chalcogenides of copper [3-5] exhibiting semimetallic properties in the normal state. Over the past decade several superconducting degenerate semiconductors were discovered: strontium titanate [6], type $A^{IV}B^{VI}$ compounds [7-10], chalcogenides of lanthanum [11-15] and indium [16,17], and ternary compounds $In_xSn_{1-x}Te_{1+y}$ [18-20] and $Pb_xSn_{1-x}Te$ [18,21]. Recently the semimetal $BaPb_{1-x}Bi_xO_3$ was synthesized, with the highest critical temperature of the superconducting transition $T_c \approx 13^\circ$ K [22] of compounds not containing transition metal atoms.

According to modern theoretical concepts, superconductivity in compounds that, as a rule, are polar crystals with a high degree of ionic bonding (or that are near-ferroelectrics) is caused mainly by the interaction of free degenerate carriers (electrons and holes) with longitudinal optical phonons [23-35] or with mild transverse mode near the ferroelectric phase transitions [36-38]**. The last point of view was placed in doubt in a study by Allen [42]; however, in recent experiments [10,43,44] a correlation was observed between superconductivity and the structural transition in SnTe with an admixture of PbTe and GeTe, and also in $SrTiO_3$. On the other hand, in the compound $Pb_xSn_{10x}Te$

** In [39-41] the effect of structural instability in type $A^{IV}B^{VI}$ on semiconductivity is also discussed.

* Numbers in the margin indicate pagination in the foreign text.

no structural transition was discovered even in low concentrations of carriers /45/. Still, at low temperatures ($T < 1^\circ$ K), superconductivity was observed in this compound; and on the basis of /21/, it set in at hole concentrations such that the second maximum in the valency zone began to fill in with a large effective mass. The many-valley nature of the zone structure of $Pb_xSn_{1-x}Te$ was confirmed also by experimental measurement of the Hall constant and the mobility of the free carriers /46/.

Cohen /24/ was the first to point to the important role of many-valley effects. He investigated in detail the superconductivity of degenerate semiconductors. However, in an examination /69/ of virtual processes of intervalley scattering of carriers by short-wave acoustic and optical phonons in /24/ and later studies /25-28/, no account was taken of the "crossover" Cooper pairing of electrons (holes) from different valleys with equal Fermi momenta, which can have a substantial effect on T_c. Additionally, as was shown in /34/, in the case of nonequivalent valleys with distinct effective masses and extrema noncoincident in energy, for a certain extent of doping, an additional mechanism of attraction between degenerate "light" carriers comes into play because of their interaction with low-frequency plasma oscillations of "heavy" carriers /47,48/, which in principle may lead to the intensifying of superconductivity in semiconductors. This situation is possibly realized in $Pb_xSn_{1-x}Te$ /21/, whose zone structure can be described by the nonparabolic two-zone model /49,50/ with the ratio of hole masses $m^*_{h2}/m^*_{h1} \approx 10$ when $T = 300^\circ$ K and $m^*_{h2}/m^*_{h1} \approx 25$ when $T = 10^\circ$ K /46/.

In this study, in the framework of a modified Gurevich-Larkin-Firsov (GLF) model /23/, with allowance for intervalley pairing, an examination is made of the superconductivity of a degenerate polar semiconductor with two equivalent isotropic valleys and intrinsic semimetals with equal effective masses of electrons and holes. The resulting concentration dependences of temperature agree qualitatively with experimental data for the compounds $SrTiO_3$ and $BaPb_{1-x}Bi_xO_3$.

2. Superconductivity in Many-Valley Polar Semiconductors

Let us examine superconductivity in many-valley polar semiconductors caused by the interaction of degenerate carriers with longitudinal optical phonons, with allowance for virtual intervalley transitions, which lead to greater effective attraction.

Emphasis must be given to the fact that valleys in a semiconductor can be viewed as independent only when the carrier mobility is great enough, when the time of a real intervalley scattering τ_{12} by the lattice defects is much greater than the characteristic time of Cooper pairing $\sim \Delta_i^{-1}$ (Δ_i is the energy gap in the spectrum of quasiparticles in the i-th valley, $\hbar = 1$). As a rule, the time of intravalley relaxation in terms of momenta $\tau_i < \tau_{12}$, so that in addition to the inequality $\Delta_i\tau_{12} >> 1$, the condition $\Delta_i\tau_i << 1$ ("dirty" superconductor) can be satisfied. In addition, the presence of randomly arranged charged impurities or vacancies must not spoil the zone structure of the semiconductors and must not lead to localization of carriers in the random field [51]. These conditions are satisfied, in particular, in $SrTiO_3$ [52] and in chalcogenides of rare-earth elements [14, 53].

Let us examine the case of a doped semiconductor with two isotropic equivalent valleys in the conduction zone or in the valency zone, whose extrema are shifted by the momentum $q_o \approx \pi/a$ (a is the lattice constant), significantly exceeding the Fermi momentum $k_F = (3\pi^2 n)^{1/3}$, where n is the concentration of carriers in the valley. The Gorkov system of equations describing this two-component superconductor has the form (cf. /54,29/)

$$
\begin{aligned}
&(\omega-\xi)\,G - i\Delta F^{+} - i\tilde{\Delta}\tilde{F}^{+} = 1;\\
&(\omega-\xi)\,\tilde{G} - i\tilde{\Delta}F^{+} - i\Delta\tilde{F}^{+} = 0;\\
&(\omega+\xi)\,F^{+} + i\Delta^{+}G + i\tilde{\Delta}^{+}\tilde{G} = 0;\\
&(\omega+\xi)\,\tilde{F}^{+} + i\tilde{\Delta}^{+}G + i\Delta^{+}\tilde{G} = 0,
\end{aligned}
\tag{1}
$$

where G and F+ are the normal and anormal Green functions in each of the valleys, G and F^+ are the corresponding intervalley Green functions; ξ is the energy of electrons (holes) measured from the Fermi level, and the intravalley Δ and "crossover" $\tilde{\Delta}$ anomalous intrinsic-energy components satisfy the nonlinear integral equations* /698

$$\Delta(p) = \frac{1}{(2\pi)^4}\int d^4p'\,\{D_0(p-p')\,F^{+}(p') + 2D_1(p-p')\,\tilde{F}^{+}(p')\}; \tag{2}$$

$$\tilde{\Delta}(p) = \frac{1}{(2\pi)^4}\int d^4p'\,\{D_0(p-p')\,\tilde{F}^{+}(p') + 2D_1(p-p')\,F^{+}(p')\}. \tag{3}$$

Here D_0 is the apical part (quadrupole) of direct polar interaction between carriers, in the plane-wave approximation being equal /23/ to

$$D_0(\mathbf{q},\omega) = \frac{4\pi e^2}{q^2\varepsilon(\mathbf{q},\omega)}, \tag{4}$$

where

$$\varepsilon(\mathbf{q},\omega) = \varepsilon_\infty\frac{\omega^2-\omega_l^2}{\omega^2-\omega_t^2} + \frac{8\pi e^2}{q^2}\Pi(\mathbf{q},\omega); \tag{5}$$

ε_∞ is the high-frequency dielectric constant of the crystal lattice; ω_ℓ ω_t are the frequencies of the longitudinal and transverse optical phonons; $\Pi(\mathbf{q},\omega)$ is the polarization operator of free carriers from valleys in the random-phase approximation. The quantity D_1, describing scattering with large transmitted momenta of the order of $q_0 \gg k_F$, which is accompanied by the transfer of a quasiparticle from valley to valley, with an accuracy to terms $\sim (k_F a)^2 \ll 1$, is /28/

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* Note that intervalley pairing is accompanied by the appearance of the nonuniform parameter of the order $\tilde{\Delta}(x,x')$ with the space period $2\pi/q_0 \approx 2a$.

$$D_1(q_0, \omega) \approx \frac{4\pi e^2}{q_0^2 \epsilon(q_0, \omega)}. \tag{6}$$

We note that in Eqs. (2) and (3) we have omitted the small terms that are insignificant in this case, describing intervalley processes of the transfer of Cooper pairs (cf. [35]).

Using expressions for the functions F^+ and $\tilde{F}^+$, we reduce Eqs. (2) and (3) to the form

$$\Delta(\xi) = \frac{1}{2}\int_{-E_F}^{E_F} d\xi' \left\{\frac{Q_0(\xi-\xi')\Delta(\xi')}{\omega_+(\xi')+\omega_-(\xi')}\left[1+\frac{\xi'^2+\Delta^2(\xi')-\tilde{\Delta}^2(\xi')}{\omega_+(\xi')\omega_-(\xi')}\right]+ \right. \\ \left. + \frac{Q_1(\xi-\xi')\tilde{\Delta}(\xi')}{\omega_+(\xi')+\omega_-(\xi')}\left[1+\frac{\xi'^2+\tilde{\Delta}^2(\xi')-\Delta^2(\xi')}{\omega_+(\xi')\omega_-(\xi')}\right]\right\}; \tag{7}$$

$$\tilde{\Delta}(\xi) = \frac{1}{2}\int_{-E_F}^{E_F} d\xi' \left\{\frac{Q_0(\xi-\xi')\tilde{\Delta}(\xi')}{\omega_+(\xi')+\omega_-(\xi')}\left[1+\frac{\xi'^2+\tilde{\Delta}^2(\xi')-\Delta^2(\xi')}{\omega_+(\xi')\omega_-(\xi')}\right]+ \right. \\ \left. + \frac{Q_1(\xi-\xi')\Delta(\xi')}{\omega_+(\xi')+\omega_-(\xi')}\left[1+\frac{\xi'^2+\Delta^2(\xi')-\tilde{\Delta}^2(\xi')}{\omega_+(\xi')\omega_-(\xi')}\right]\right\}. \tag{8}$$

Here

$$\omega_\pm(\xi) = \{\xi^2 + [\Delta(\xi) \pm \tilde{\Delta}(\xi)]^2\}^{1/2}; \tag{9}$$

$$Q_0(\omega) \equiv -\int_0^{2k_F} \frac{q\,dq}{(2\pi)^2 v_F} D_0(q, \omega) = \frac{\beta(\omega)}{4}\ln\left|1-\frac{1}{\beta(\omega)}\right|; \tag{10}$$

$$Q_1(\omega) \equiv -2\int_0^{2k_F} \frac{q\,dq}{(2\pi)^2 v_F} D_1(q_0, \omega) = \alpha_\infty x \frac{\omega_l^2-\omega^2}{(\omega^2-\omega_l^2)+\frac{2}{3}\alpha_\infty x^2(\omega^2-\omega_l^2)}, \tag{11}$$

where

$$\beta(\omega) = \frac{2\alpha_\infty(\omega_l^2-\omega^2)}{\omega^2(1-\alpha_\infty)-(\omega_l^2-\omega_l^2\alpha_\infty)}; \quad x = \left(\frac{2k_F}{q_0}\right)^2; \tag{12}$$

$\alpha_\infty = e^2/\pi v_F \epsilon_\infty$ is the dimensionless density parameter; v_F and E_F are the Fermi velocity and energy of the degenerate carriers in the valleys. In the calculation of the kernel (10), screening of coulombic interaction was taken into account in the Migdal approximation /55/ for the static polarization operator, and in the calculation of Eq. (11), use was made of the known asymptote of the dielectric constant for large momenta /56/. The dependence of $Q_0(\omega)\delta$ and $C_1(\omega)$ on energy ω is shown in Fig. 1 (solid curves). As we see, close to the Fermi surface, when $\omega < \omega_t$, the kernels $Q_0(\omega)$ and $Q_\ell(\omega)$ are negative, which corresponds to repulsion. At the same time, $Q_0(\omega) \geq 0$ (attraction) in the region $\omega_t \leq \omega \leq \omega_0$, where

$$\omega_0 = \omega_l \sqrt{\left(2 + 3\alpha_\infty \frac{\varepsilon_\infty}{\varepsilon_0}\right) / (2 + 3\alpha_\infty)},$$

and $Q_1(\omega) \geq 0$ in the region $\omega_t \leq \omega \leq \omega_1$, where

$$\omega_1 = \omega_l \sqrt{\left(3 + 2\alpha_\infty x^2 \frac{\varepsilon_\infty}{\varepsilon_0}\right) / (3 + 2\alpha_\infty x^2)}$$

(ε_0 is the static dielectric constant of the lattice, where $\omega_\ell^2/\omega_t^2 = \varepsilon_0/\varepsilon_\infty$).

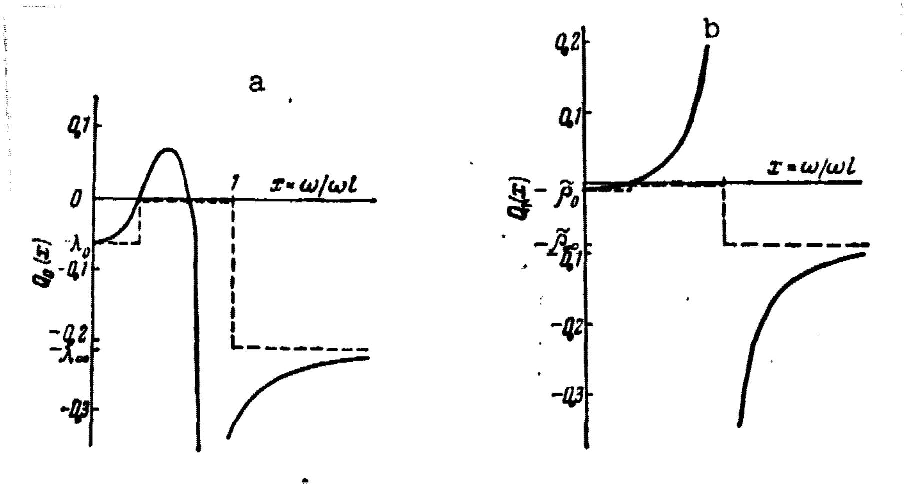

Fig. 1. Kernels of direct polar interaction $Q_0(\omega)$ (a) and interzone (intervalley) interaction with transfer $Q_1(\omega)$ (b) as a function of frequency ω and their approximation by three square wells (– – –)

We can easily see that with the substitution $\Delta \rightleftarrows (\pm\lambda)$, /700
Eqs. (7) and (8) coincide with each other and can be reduced to the single equation

$$\Delta(\xi) = \frac{1}{2}\int_{-E_F}^{E_F} d\xi' \Delta(\xi') \frac{Q_0(\xi-\xi') \pm Q_1(\xi-\xi')}{\sqrt{\xi'^2 + 4\Delta^2(\xi')}}. \tag{13}$$

However, it follows, in particular, that the energy gap in the spectrum of quasiparticles at the Fermi surface is $2\,\Delta(0)$.

To get an approximate solution to Eq. (13), let us approximate the kernels $Q_0(\xi-\xi')$ and $Q_1(\xi-\xi')$ with three square wells (cf. [5]):

$$\tilde{Q}_0(\xi, \xi') = \begin{cases} -\lambda_0, & 0 < |\xi|, |\xi'| < \omega_l; \\ 0, & \omega_l < |\xi|, |\xi'| < \omega_i; \\ -\lambda_\infty, & \omega_i < |\xi|, |\xi'| < E_F; \end{cases} \tag{14}$$

$$\tilde{Q}_1(\xi, \xi') = \begin{cases} -\tilde{\rho}_0, & 0 < |\xi|, |\xi'| < \omega_l; \\ 0, & \omega_l < |\xi|, |\xi'| < \omega_i; \\ -\tilde{\rho}_\infty, & \omega_i < |\xi|, |\xi'| < E_F. \end{cases} \tag{15}$$

Here

$$\lambda_0 \equiv -Q_0(0) = \frac{a_0}{2(1-a_0)} \ln\left(\frac{1+a_0}{2a_0}\right); \tag{16}$$

$$\lambda_\infty \equiv -Q_0(\infty) = \frac{a_\infty}{2(1-a_\infty)} \ln\left(\frac{1+a_\infty}{2a_\infty}\right);$$

$$\tilde{\rho}_0 \equiv -Q_1(0) = \frac{\rho_0}{1+2\rho_0\rho_\infty/3a_\infty}; \quad \tilde{\rho}_\infty \equiv -Q_1(\infty) = \frac{\rho_\infty}{1+2\rho_\infty^2/3a_\infty}; \tag{17}$$

$$\rho_0 = \frac{4\nu\varepsilon_\infty}{\pi^2 a_\infty \varepsilon_0}; \quad \nu = (q_0 a_\infty^*)^{-2}; \quad \rho_\infty = \rho_0\frac{\varepsilon_0}{\varepsilon_\infty}; \quad a_0 = a_\infty\frac{\varepsilon_\infty}{\varepsilon_0}, \tag{18}$$

where $a_\infty^* = \varepsilon_\infty v_F/e^2 k_F$ is the effective Bohr radius. The approximation (14) and (15) is shown in Fig. 1 with dashed lines.

We will seek the solution to Eq. (13) with reference to Eqs. (14) and (15) in the form of the three-stage function

$$\Delta(\xi) = \begin{cases} \Delta_0, & 0 < |\xi| < \omega_l; \\ \Delta_1, & \omega_l < |\xi| < \omega_i; \\ \Delta_\infty, & \omega_i < |\xi| < E_F. \end{cases} \tag{19}$$

By substituting (14) and (15) instead of $Q_0(\xi-\xi')$ and $Q_1(\xi-\xi')$ into (13), with reference to (19), we arrive at a system of three equations in the quantities Δ_0, Δ_1, and Δ_∞:

$$\begin{gathered}\Delta_0 = -(\lambda_0 \pm \tilde{\rho}_0)\Delta_0 J_0 - (\lambda_\infty \pm \tilde{\rho}_\infty)\Delta_\infty J_\infty;\\ \Delta_1 = -(\lambda_\infty \pm \tilde{\rho}_\infty)\Delta_\infty J_\infty;\\ \Delta_\infty = -(\lambda_\infty \pm \tilde{\rho}_\infty)[\Delta_0 J_0 + \Delta_1 J_1 + \Delta_\infty J_\infty],\end{gathered} \tag{20}$$

where

$$J_0 = \int_0^{\omega_t} \frac{d\xi}{\sqrt{\xi^2 + 4\Delta_0^2}}; \quad J_1 = \int_{\omega_t}^{\omega_\ell} \frac{d\xi}{\sqrt{\xi^2 + 4\Delta_1^2}}; \quad J_\infty = \int_{\omega_\ell}^{E_F} \frac{d\xi}{\sqrt{\xi^2 + 4\Delta_\infty^2}}. \tag{21}$$

As a result, for the superconducting gap at the Fermi surface $\Delta_S = 2\Delta_0$, under the condition $\Delta_0^2, \Delta_1^2 \ll \omega_t^2$ and $\Delta_\infty^2 \ll \omega_\ell^2$, we get the exponential formula

$$\Delta_S^{\pm} = 2\omega_t \exp\{-1/\lambda_\pm\}, \tag{22}$$

where /701

$$\lambda_\pm = -(\lambda_0 \pm \tilde{\rho}_0) + \frac{(\lambda_\infty \pm \tilde{\rho}_\infty)^2 L}{1 + (\lambda_\infty \pm \tilde{\rho}_\infty) L\,[1 - (\lambda_\infty \pm \tilde{\rho}_\infty) H]}; \tag{23}$$

$$L = \ln\frac{E_F}{\omega_\ell} \equiv \ln\frac{\sigma}{a_\infty^2}; \; H = \ln\frac{\omega_\ell}{\omega_t} \equiv \frac{1}{2}\ln\frac{\varepsilon_0}{\varepsilon_\infty}; \; \sigma = \frac{m^* e^4}{2\pi^2 \varepsilon_\infty^2 \omega_\ell}; \tag{24}$$

m^* is the effective mass of carriers in the parabolic-zone approximation. Of the two possible solutions (22), when $\lambda_\pm > 0$, the solution with the larger coupling constant, corresponding to the minimum free energy. We that that in this case, the usual ratio of the BCS /Bardeen-Cooper-Schrieffer/ theory /58/ between the gap and the critical temperature of the superconducting transition: $T_c = (\gamma/\pi)\max \overset{+}{\underset{S}{-}}$, where $\ln\gamma \equiv C$ is Euler's constant*

* We note that T_c concides for the semiconductor and the semimetal.

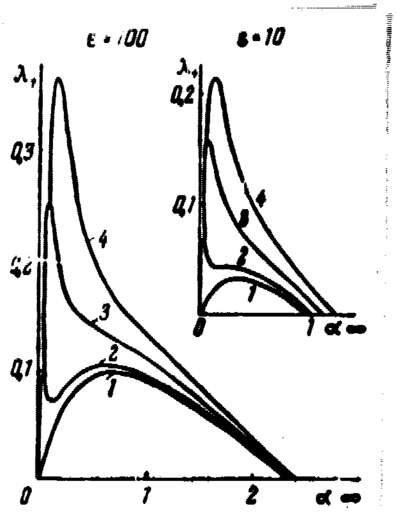

Fig. 2. Coupling constant λ_+ as a function of the density parameter α_∞ when $\varepsilon = 10$ and 100, for $\nu = 0$ (1); 0.01 (2); 0.05 (3); 0.1 (4).

Fig. 2 shows the maximum effective coupling constant λ_+ as a function of α_∞ for different values of parameters ν and $\varepsilon \equiv \varepsilon_0/\varepsilon_\infty$ when $\sigma = 9$. In the region of low concentrations, when α_∞ exceeds some critical value α_∞^*, dependent on ε and ν, the quantity λ_+ is negative; this is associated with the predominant role of repulsion at the Fermi surface. On the other hand, when $\alpha_\infty < \alpha_\infty^*$, the constant λ_+ is positive and increases rapidly both with an increase in the parameter ν, which determines--according to Eq. (18)--the contribution of the intervalley transitions, as well as with an increase in parameter ε characterizing the ionicity of the crystal.

However, near the structural (ferroelectric) transition, when $\varepsilon_0 \to \infty$ and when the frequency of the mild transverse mode $\omega_t \to 0$, repulsion at the Fermi surface becomes insignificant (under the condition that $\omega_t \lesssim \Delta_0$), and attraction caused by the interaction of carriers with longitudinal optical phonons is at a maximum. In this case, instead of Eq. (22), we get an expression for the superconducting gap:

$$\Delta_S^{\mp} = 2\omega_t \exp\left\{-\frac{1 + (\lambda_\infty \pm \tilde{\mu}_\infty) L}{(\lambda_\infty \pm \tilde{\mu}_\infty)^2 L}\right\}, \qquad (25)$$

which corresponds approximately to two square wells:

$$K_0(\xi, \xi') = \begin{cases} 0, & 0 < |\xi|, |\xi'| < \omega_l; \\ -\lambda_\infty, & \omega_l < |\xi|, |\xi'| < E_F; \end{cases} \tag{14a}$$

$$K_1(\xi, \xi') = \begin{cases} 0, & 0 < |\xi|, |\xi'| < \omega_l; \\ -\tilde{\rho}_\infty, & \omega_l < |\xi|, |\xi'| < E_F. \end{cases} \tag{15a}$$

We note that Eq. (25), in the absence of intervalley processes of "transfer" ($\tilde{\rho}_\infty = 0$), coincides with the result of the "gel" model [48,59] (see also [29,33]).

As follows from Eqs. (24) and (25), given the condition $\varepsilon_\infty \to \infty$, superconductivity is absent in the region of low carrier concentrations, when $\alpha_\infty > \sqrt{\sigma}$, since the condition of adiabaticity is not satisfied for optical phonons $\omega_\ell < E_F$ (cf. [23]).

Fig. 3 shows lg (T_c/ω_ℓ) as functions of α_∞ for different /702 values of ν and ε, and Fig. 4 shows T_c/ω_ℓ as functions of the parameter $\equiv$ lg $(1/\alpha_\infty^3) \sim$ lg n for different values of ε when $\nu = 0.1$ and $\nu = 9$. The increment in T_c simultaneously with the concentration n is caused by the increase in the density of the states of degenerate carriers, while the subsequent rapid decrease in T_c is associated with the screening of the polar interaction at high densities [27]. As we can see, T_c/ω_ℓ increases with increase in the parameter ν; for realistic values $\nu = 0.1$, as $\varepsilon \to \infty$, T_c/ω_ℓ can reach the maximum values $T_c/\omega_\ell \precsim 10^{-2}$, corresponding to the interval $T_c \precsim (1\text{-}10)^\circ$ K.

3. Discussion of Results

Let us present, briefly, the main results from this study.

In the framework of the modified model of polar interaction of conduction electrons (holes) with longitudinal optical phonons, allowing for virtual interzone transitions, we obtain expressions for the gap at the Fermi surface Δ_S and the critical transition temperature in the superconducting state T_c of the degenerate superconductor with two equivalent valleys. The effective bonding constants and T_c are plotted as functions of the carrier concentration, dielectric constants, and other system parameters. It is shown that the intervalley transitions lead to a substantial rise in T_c.

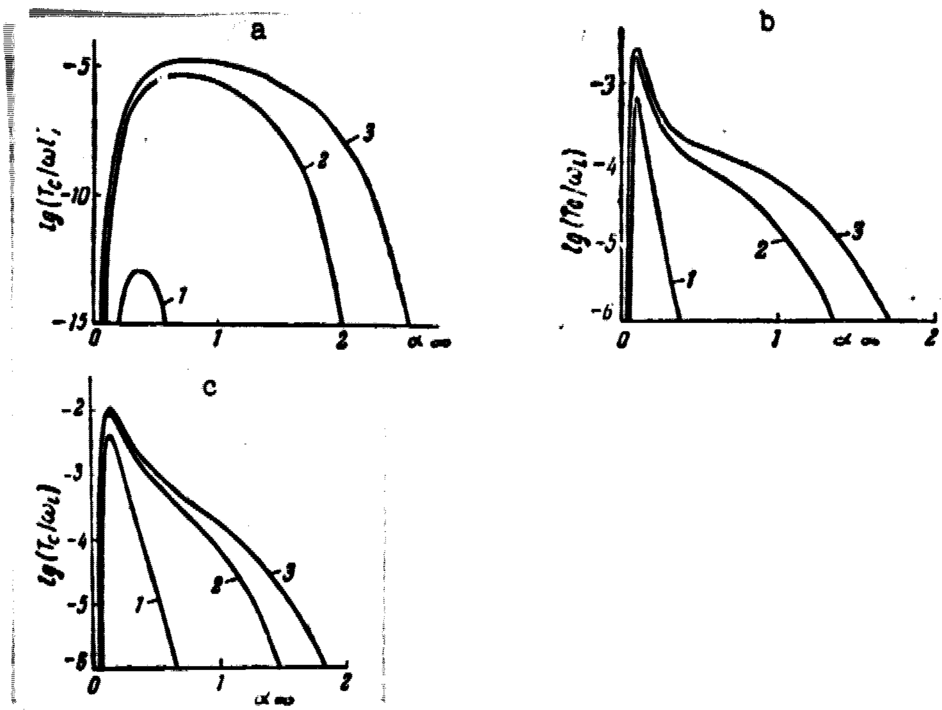

Fig. 3. Lg (T_c/ω_ℓ) as a function of α_∞ when $\sigma = 9$ and $\varepsilon = 10$ (1); 100 (2); ∞ (3) for $\nu = 0$ (a); 0.05 (b); and 0.1 (c).

The model of polar interaction between carriers examined in section 2 can be used in describing the superconducting properties of several degenerate semiconductor compounds. Thus, the semiconductors GeTe, SnTe, and $Pb_xSn_{1-x}Te$ are compounds with a high degree of ionic bonding /60/, and $SrTiO_3$--when $T < 20^\circ$ K--

exhibit the anomalous static dielectric constant $\varepsilon_0 = 2 \cdot 10^4$ [60,61]. At the same time, the compounds $Pb_xSn_{1-x}Te$ [21,46,49, 50], GeTe and SnTe [60], and chalcogenides of rare-earth elements [62] have a many-valley zone structure. From theoretical calculations [63] and experimental data [64,65], $SrTiO_3$ also has a many-valley structure of zones with energy minima when $\mathbf{k} = (\pi/\mathbf{a}, 0, 0)$*.

The critical temperature of the superconducting transition /703
T_c in strontium titanate with n-type conductivity, produced either by heat treatment or with a doping impurity (for example, Nb), with an increase in the concentration of free carriers rises beginning at $n \overset{\sim}{\sim} 7 \cdot 10^{18}$ cm^{-3}, reaches a maximum $T_{cmax} \overset{\sim}{\sim} 0.3^\circ$ K when $n \overset{\sim}{\sim} 2 \cdot 10^{20}$ cm^{-3}, and then drops off sharply down to zero [6,28]. Analogous behavior of T_c was observed in $BaPb_{1-x}Bi_xO_3$ [22] in the interval $0.05 \leq x < 0.3$, where the maximum $T_c \overset{\sim}{\sim} 13^\circ$ K corresponds to x = 0.3, and when x = 0.35, when the carrier concentration rises sharply, $T_c \overset{<}{\sim} 4.2^\circ$ K. The concentration dependence of the critical temperature of the superconducting transition arrived at in this present study for a polar many-valley semiconductor or semimetal (Fig. 4) closely fits these experimental data. We note that tunnel experiments with $SrTiO_3$ [44] indicate that the BKS ratio is satisfied, between T_c and the energy gap Δ .

On the other hand, the monotonic rise in T_c with increase in carrier concentration, observed in the compounds $Pb_xSn_{1-x}Te$ [21], SnTe and GeTe [28], and in chalcogenides of lanthanum [14,67], is evidently associated with the filling of valleys that have a large effective mass [21,46]. Nor is it precluded that the superconductivity of copper sulfide [1-5] is caused by the filling of the second hole maximum in the 3p-zone in the

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* This was placed in doubt by Mattheiss [66], whose calculations lead to a minimum at the point $\mathbf{k} = 0$.

transition from the semiconductor Cu_2S to the semimetal CuS /68/. And in addition to polar electron-phonon interaction, a major role may be played by the interaction of degenerate "light" carriers with acoustic plasma oscillations of "heavy" carriers /34/.

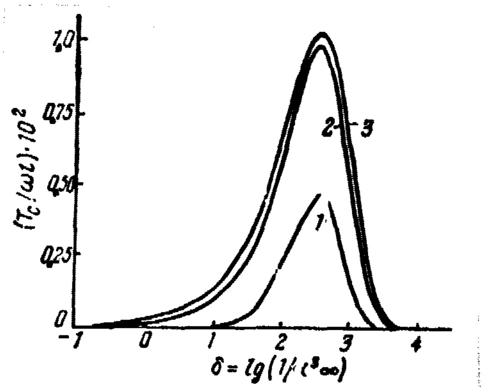

Fig. 4. T_c/ω_ℓ as a function of the parameter $\delta = \lg(1/\alpha_\infty^3)$, when $\nu = 0.1$ and $\sigma = 9$ for $\varepsilon = 10$ (1); 100 (2); ∞ (3).

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