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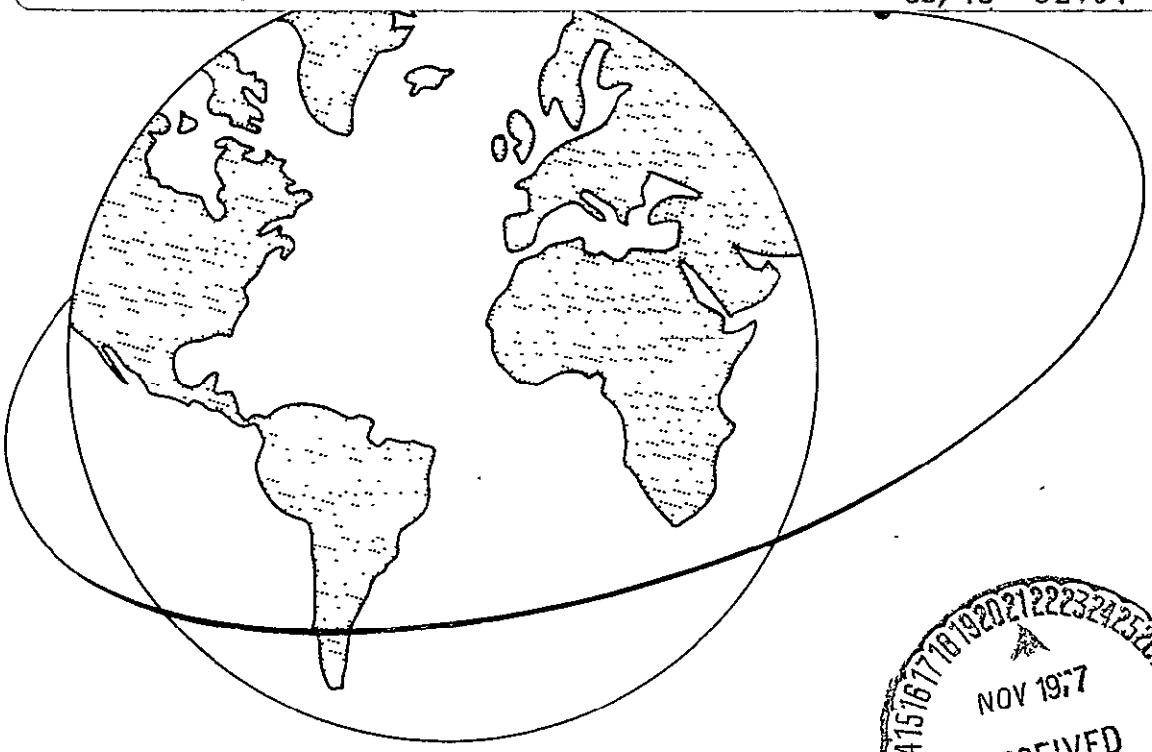
THIRD-ORDER SOLUTION OF AN ARTIFICIAL-SATELLITE THEORY

HIROSHI KINOSHITA



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ABSTRACT

A third-order solution is developed for the motions of artificial satellites moving in the gravitational field of the earth, whose potential includes the second-, third-, and fourth-order zonal harmonics. Third-order periodic perturbations with fourth-order secular perturbations are derived by Hori's perturbation method. All quantities are expanded into power series of the eccentricity, but the solution is obtained so as to be closed with respect to the inclination. A comparison with the results of numerical integration of the equations of motion indicates that the solution can predict the position of a close-earth, small-eccentricity satellite with an accuracy of better than 1 cm over a period of 1 month.

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1. INTRODUCTION

Second-order theories of artificial satellites have been established by many authors during the past 15 years; an excellent review is given by Hori and Kozai (1975). A third-order solution was derived by Deprit and Rom (1970), but their solution does not include J_3 and J_4 terms. According to Aksnes's numerical experiments comparing his second-order solution (Aksnes, 1970) with numerical integration, residuals of a few decimeters remain in position, most of which come from the third-order interaction among J_2, J_3, and J_4. On the other hand, the accuracy of current geodetic satellites equipped with retroreflectors for laser ranging will reach 3 to 5 cm. Therefore, we can expect to obtain more accurate information on satellite motions by taking into account the third-order periodic perturbations.

In this paper, we consider only the zonal-harmonics perturbations of second, third, and fourth order. We chose a Keplerian motion as an intermediate orbit and adopted Hori's (1966) perturbation method. We assumed that the eccentricity of a geodetic satellite requiring highly

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accurate solutions is usually low; therefore, all quantities in the present solution are expanded into power series of the eccentricity, but the solution is obtained in closed form with respect to the inclination. Delaunay variables were selected as the canonical elements used to construct the new Hamiltonian and the determining functions eliminating periodic terms, which do not depend on the chosen canonical variables so long as Keplerian motion is adopted as an intermediate orbit. However, the final expressions of the periodic perturbations are given in $\ell + g$, h, $e \cos g$, $e \sin g$, and L, which are not singular at zero eccentricity.

All literal calculations were carried out by means of the computer algebra program Smithsonian Package for Algebra and Symbolic Mathematics (SPASM) (Hall and Cherniack, 1969). Final results were checked analytically in various ways and were compared with the results of numerical integration.

2. OUTLINE OF THE METHOD OF SOLUTION

Let us consider an artificial satellite orbiting in an axially symmetric gravitational field of the earth, whose force function U is of the form

$$U = \frac{\mu}{r}\left[1 - \sum_{n=2}^{4}\left(\frac{a_e}{r}\right)^n J_n P_n(\sin\beta)\right] \quad , \tag{1}$$

where a_e is the equatorial radius of the earth, r the radius vector of the satellite, and β the declination. In this paper, the coefficient J_2 is assumed to be a small quantity of the first order and J_3 and J_4 are of the second order.

To solve for the motion of the satellite under the force function (1), we adopted Hori's (1966) perturbation method, which utilizes Lie transformation; all formulas are canonically invariant, and the perturbations of any quantity are given by simple formulas and in explicit form. Because of the generality of Hori's method, we can choose any set of canonical variables. In the present paper, we use Delaunay variables as a canonical set for their simplicity, where ℓ is the mean anomaly, g is the argument of perigee, h is the longitude of the ascending node, and

$$L = \sqrt{\mu a} \quad ; \quad G = L\sqrt{1 - e^2} \quad , \quad H = G\cos i \quad .$$

The equations of motion are

$$\frac{d}{dt}(L,G,H) = \frac{\partial F}{\partial(\ell,g,h)} \quad , \qquad \frac{d}{dt}(\ell,g,h) = -\frac{\partial F}{\partial(L,G,H)} \quad , \tag{2}$$

where

$$\begin{aligned} F &= F_0 + F_1 + F_2 \quad , \\ F_0 &= \frac{\mu^2}{2L^2} \quad , \\ F_1 &= -\frac{\mu a_e^2}{r^3} J_2 P_2(\sin\beta) \quad , \\ F_2 &= -\frac{\mu a_e^3}{r^4} J_3 P_3(\sin\beta) - \frac{\mu a_e^4}{r^5} J_4 P_4(\sin\beta) \quad . \end{aligned} \tag{3}$$

Under the assumption that the eccentricity is small, we expanded the disturbing functions F_1 and F_2 into a power series of the eccentricity:

$$\begin{aligned} F_1 &= \frac{\mu^4 J_2}{L^6} \sum_{p=0}^{1} \sum_{k=-\infty}^{\infty} X_k^{2,2p}(e)\, B_{2,2p}(i) \cos(k\ell + 2pg) \quad , \\ F_2 &= \frac{\mu^5 J_3}{L^8} \sum_{p=0}^{1} \sum_{k=-\infty}^{\infty} X_k^{3,2p+1}(e)\, B_{3,2p+1}(i) \sin[k\ell + (2p+1)g] \quad , \\ &\quad + \frac{\mu^6 J_4}{L^{10}} \sum_{p=0}^{2} \sum_{k=-\infty}^{\infty} X_k^{4,2p}(e)\, B_{4,2p}(i) \cos(k\ell + 2pg) \quad , \end{aligned} \tag{4}$$

where $X_k^{n,m}(e)$ is a Hansen coefficient and

$$B_{20} = -\frac{1}{4}(-2 + 3\sin^2 i) \quad , \qquad B_{22} = \frac{3}{4}\sin^2 i \quad ,$$

$$B_{31} = -\frac{3}{8}\sin i\,(-4 + 5\sin^2 i) \quad , \qquad B_{33} = \frac{5}{8}\sin^3 i \quad ,$$

$$B_{40} = -\frac{3}{8}(8 - 40\sin^2 i + 35\sin^4 i) \quad ,$$

$$B_{42} = \frac{5}{16}\sin^2 i\,(-6 + 7\sin^2 i) \quad , \qquad B_{44} = -\frac{35}{64}\sin^4 i \quad .$$

When the lowest powers of e and sin i of the coefficient of $\begin{matrix}\sin\\ \cos\end{matrix}(k\ell + qg)$ in the trigonometric series in equations (4) are α and δ, we have the following relations:

$$\alpha = |k - q| \pmod 2 \quad ,$$
$$\delta = |q| \pmod 2 \quad ,$$

which are called the d'Alembert characteristics.

The algorithm for deriving the new Hamiltonian F^* (Hori, 1966) and the determining function S eliminating short-period terms is

zeroth order:

$$F_0^* = F_0 \quad , \tag{5a}$$

first order:

$$F_1^* = F_{1s} \quad ,$$
$$S_1 = \frac{L'^3}{\mu^2} \int F_{1p}\, d\ell' \quad , \tag{5b}$$

second order:

$$F_2^* = F_{2s} + \frac{1}{2}\left\{F_1 + F_1^*,\, S_1\right\}_s \quad ,$$
$$S_2 = \frac{L'^3}{\mu^3} \int \left(F_{2p} + \frac{1}{2}\left\{F_1 + F_1^*,\, S_1\right\}_p\right) d\ell' \quad , \tag{5c}$$

third order:

$$F_3^* = \frac{1}{12}\left\{\left\{F_{1p},\, S_1\right\},\, S_1\right\}_s + \frac{1}{2}\left\{F_2 + F_2^*,\, S_1\right\}_s + \frac{1}{2}\left\{F_1 + F_1^*,\, S_2\right\}_s \quad ,$$

$$S_3 = \frac{L'^3}{\mu^2} \int \left(\frac{1}{12}\left\{\left\{F_{1p},\, S_1\right\},\, S_1\right\}_p + \frac{1}{2}\left\{F_2 + F_2^*,\, S_1\right\}_p \right. \tag{5d}$$
$$\left. + \frac{1}{2}\left\{F_1 + F_1^*,\, S_2\right\}_p\right) d\ell' \quad ,$$

fourth order:

$$F_4^* = \frac{1}{2}\left\{F_1 + F_1^*,\, S_3\right\}_s + \frac{1}{2}\left\{F_2 + F_2^*,\, S_2\right\}_s + \frac{1}{2}\left\{F_3 + F_3^*,\, S_1\right\}_s$$
$$+ \frac{1}{12}\left\{\left\{F_{1p},\, S_1\right\},\, S_2\right\}_s + \frac{1}{12}\left\{\left\{F_{1p},\, S_2\right\},\, S_1\right\}_s$$
$$+ \frac{1}{12}\left\{\left\{F_{2p},\, S_1\right\},\, S_1\right\}_s \quad , \tag{5e}$$

where the braces represent Poisson brackets and the subscripts s and p indicate the constant and periodic parts in ℓ'. It should be noted that any function of L', G', H', and g' can be added to S.

The algorithm for calculating the new Hamiltonian F^{**} and the determining function S^{*} eliminating long-period terms is given by the following equations:

first order:

$$F_1^{**} = F_1^{*} \quad , \tag{6a}$$

second order:

$$\begin{aligned} F_2^{**} &= F_{2s}^{*} \quad , \\ S_1^{*} &= -\left(\frac{\partial F_1^{*}}{\partial G''}\right)^{-1} \int F_{2p}^{*}\, dg'' \quad , \end{aligned} \tag{6b}$$

third order:

$$\begin{aligned} F_3^{**} &= \frac{1}{2}\left\{F_2^{*} + F_2^{**},\ S_1^{*}\right\}_s \quad , \\ S_2^{*} &= -\frac{1}{2}\left(\frac{\partial F_1^{*}}{\partial G''}\right)^{-1} \int \left\{F_2^{*} + F_2^{**},\ S_1^{*}\right\}_p dg'' \quad , \end{aligned} \tag{6c}$$

fourth order:

$$\begin{aligned} F_4^{**} = \frac{1}{12}\left\{\left\{F_{2p}^{*},\ S_1^{*}\right\},\ S_1^{*}\right\}_s + \frac{1}{2}\left\{F_3^{*} + F_3^{**},\ S_1^{*}\right\}_s \\ + \frac{1}{2}\left\{F_2^{*} + F_2^{**},\ S_2^{*}\right\}_s \quad , \end{aligned} \tag{6d}$$

(eq. (6d) cont. on next page)

$$S_3^* = -\left(\frac{\partial F_1}{\partial G''}\right)^{-} \int \left(\frac{1}{12}\left\{\left\{F_{2p}^*, S_1^*\right\}, S_1^*\right\}_p + \frac{1}{2}\left\{F_3^* + F_3^{**}, S_1^*\right\}_p \right.$$

$$\left. + \frac{1}{2}\left\{F_2^* + F_2^{**}, S_2^*\right\}_p\right) dg'' \quad , \tag{6d}$$

where the subscripts s and p indicate the constant and periodic parts in g″. These algorithms are actually very simple, but calculating them by hand is laborious. Therefore, all computations were carried out by the computer program SPASM (Hall and Cherniack, 1969). SPASM handles the operations in equations (5) and (6) easily, keeping rational fractions for coefficients.

A key operation in equations (5) and (6) is evaluating the Poisson bracket {A,B}:

$$\{A,B\} = \frac{\partial A}{\partial L}\frac{\partial B}{\partial \ell} - \frac{\partial A}{\partial \ell}\frac{\partial B}{\partial L} + \frac{\partial A}{\partial G}\frac{\partial B}{\partial g} - \frac{\partial A}{\partial g}\frac{\partial B}{\partial G} + \frac{\partial A}{\partial H}\frac{\partial B}{\partial h} - \frac{\partial A}{\partial h}\frac{\partial B}{\partial H} \quad . \tag{7}$$

In the present theory, as we seek to obtain a solution that is closed with respect to the inclination, the atomic variables in the computer algebra are L, e, s = sin i, c = cos i, and $\gamma = 1 - 5\cos^2 i$ (γ appears in the denominator of S^*). The derivatives with respect to L, G, and H are

$$\frac{\partial}{\partial L} = \left(\frac{\partial}{\partial L}\right) + \frac{1 - e^2}{eL}\frac{\partial}{\partial e} \quad ,$$

$$\frac{\partial}{\partial G} = \frac{1}{L\sqrt{1-e^2}}\left(-\frac{1 - e^2}{e}\frac{\partial}{\partial e} + \frac{1 - \sin^2 i}{\sin i}\frac{\partial}{\partial s} - \cos i \frac{\partial}{\partial c}\right. \tag{8}$$

$$\left. + 10(1 - \sin^2 i)\frac{\partial}{\partial \gamma}\right) \quad ,$$

(eq. (8) cont. on next page)

$$\frac{\partial}{\partial H} = -\frac{1}{L\sqrt{1-e^2}}\left(\frac{\cos i}{\sin i}\frac{\partial}{\partial s} + \frac{\partial}{\partial c} + 10\cos i\,\frac{\partial}{\partial \gamma}\right) \quad , \tag{8}$$

where the factor $\sqrt{1-e^2}$ will be replaced by a Taylor expansion in powers of e. If both A and B satisfy the d'Alembert characteristics, then even if these derivatives include the terms 1/e and 1/sin i, the Poisson bracket {A,B} keeps the d'Alembert characteristics and does not have 1/e and 1/sin i in the expression. This serves as a good check on literal manipulation by a computer. In Deprit and Rom's (1970) theory, $\cos i = H/L\sqrt{1-e^2}$ is developed in power series of e. Therefore, their determining function W apparently loses the d'Alembert characteristics with respect to the inclination. If A and B are expanded into power series of e and truncated at e^n, the derivatives with respect to L and G are correct up to e^{n-2}; however, {A,B} is correct up to e^{n-1}. In other words, with one operation of the Poisson bracket, only one degree in e (not two) is lost. Our program, which takes this fact into consideration, saves a lot of computer time.

Complete to J_2^3, the analytical solution must take into account all the following terms:

first order:

$$J_2 \quad , \qquad J_3/J_2 \quad , \qquad J_4/J_2 \quad ,$$

second order:

$$J_3 \quad , \quad J_3J_4/J_2^2 \quad , \qquad J_2^2 \quad , \qquad J_4 \quad ,$$

$$(J_3/J_2)^2 \quad , \qquad (J_4/J_2)^2 \quad ,$$

third order:

$$J_2J_3 \quad , \quad J_3J_4^2/J_2^3 \quad , \quad (J_3/J_2)^3 \quad , \quad J_3J_4/J_2 \quad ,$$

$$J_2^3 \quad , \quad J_2J_4 \quad , \quad (J_4/J_2)^3 \quad , \quad J_3^2J_4/J_2^3 \quad ,$$

$$J_3^2/J_2 \quad , \quad J_4^2/J_2 \quad .$$

Most of these terms arise from the interaction among J_2, J_3, and J_4. Tables 1 and 2 list the numbers of terms involved in S_3 and S_3^*, respectively. The total number of terms with the factor J_2^3 in S_3 is 106. On the other hand, Deprit and Rom's W_3 contains 192 terms up to e^4, partly because they chose $\eta = H/L$ as one of the arguments. Explicit expressions of S, F^*, S^*, and F^{**} are given in Appendix A.

Table 1. Numbers of terms in the determining function S_3.

Order of eccentricity	J_2^3	J_2J_3	J_2J_4	Total
e^0	6	6	6	18
e^1	16	8	16	40
e^2	19	18	18	55
e^3	32	19	32	83
e^4	33	30	31	94
Total	106	81	103	290

Table 2. Numbers of terms in the determining function S_3^*.

Order of eccentricity	$J_3J_4^2/J_2^3$	$(J_3/J_2)^3$	J_3J_4/J_2	J_2J_3	$(J_4/J_2)^3$	$J_3^2J_4/J_2^3$	J_4^2/J_2	J_3^2/J_2	J_2J_4	J_2^3	Tot
e^1	5	4	8	9	--	--	--	--	--	--	2
e^2	--	--	--	--	6	7	15	7	15	15	6
e^3	19	14	26	26	--	--	--	--	--	--	8
Total	24	18	34	35	6	7	15	7	15	15	17

When both short- and long-period terms have been eliminated, the new Hamiltonian F^{**} contains no angular variables. The action variables L'', G'', and H'' are constant, and the angular variables ℓ'', g'', and h'' are expressed as follows:

$$\ell'' = -\frac{\partial}{\partial L''} F^{**} + \ell_0'' \quad ,$$

$$g'' = -\frac{\partial}{\partial G''} F^{**} + g_0'' \quad , \tag{9}$$

$$h'' = -\frac{\partial}{\partial H''} F^{**} + h_0'' \quad .$$

The generating functions S and S^* determine a completely canonical transformation from the osculating elements (L, G, H, ℓ, g, and h) to the mean elements (L'', G'', H'', ℓ'', g'', and h''). The periodic perturbations of ℓ and g have negative powers of the eccentricity as a factor, which reflects the fact that the perigee is not well determined when the eccentricity is small. If the eccentricity is smaller than $(-J_3/2J_2a)$ sin i, the argument of perigee librates and does not circulate (Kozai, 1966; Kinoshita and Hori, 1972; Hori, 1973). Because ℓ, g, and e do not satisfy the d'Alembert characteristics with respect to the eccentricity, we selected ℓ + g, h, e cos g, e sin g, L, and H as the set of elements for calculating the ephemeris of a satellite, as was also done by Deprit and Rom (1970). If ε stands for one of these elements, the osculating element ε is obtained from the following formulas through the third order of J_2^3:

$$\varepsilon = \varepsilon' + \{\varepsilon', S\} + \frac{1}{2}\{\{\varepsilon', S\}, S\} + \frac{1}{6}\{\{\{\varepsilon', S\}, S\}, S\} + O(J_2^4) \quad ,$$

$$\varepsilon' = \varepsilon'' + \{\varepsilon'', S^*\} + \frac{1}{2}\{\{\varepsilon'', S^*\}, S^*\} + \frac{1}{6}\{\{\{\varepsilon'', S^*\}, S^*\}, S^*\} \qquad (1$$

$$+ O(J_2^4) \quad .$$

These expressions do not contain negative powers of the eccentricity. Even though neither $\ell + g$ nor h satisfies the d'Alembert characteristics with respect to the inclination, the sum of $\ell + g + h$ does, thus providing another good check on the lengthy calculations. Explicit expressions of $\partial F^{**}/\partial(L,G,H)$, short-period perturbations $\varepsilon - \varepsilon'$, and long period perturbations $\varepsilon' - \varepsilon''$ are given in Appendix B.

3. COMPUTATION OF POSITION AND VELOCITY

Given the mean elements a_0'', e_0'', i_0'', ℓ_0'', g_0'', and h_0'' at the initial epoch, which are determined from observations, the position and velocity of the satellite at an arbitrary epoch t measured from the initial epoch can be calculated from the following procedures:

A. First we compute the secular perturbations from equations (9) plus $L'' = \sqrt{a_0''}$, $G'' = L''\sqrt{1 - e_0''^2}$, and $H'' = G'' \cos i_0''$.

B. From equations (10), we compute the long- and short-period perturbations.

C. Now we have the osculating elements $\xi = e \cos g$, $\eta = e \sin g$, $\lambda = \ell + g$, h, L, and H. The method of calculating the position and velocity from these osculating elements is given in Deprit and Rom (1970).

Kepler's equation in the form

$$\psi = \lambda + \xi \sin \psi - \eta \cos \psi \tag{11}$$

is solved by iteration to obtain $\psi = E + g$, where E is the eccentric anomaly. The eccentricity, the semimajor axis, and the inclination are computed from the relations

$$e^2 = \xi^2 + \eta^2 \quad , \quad a = L^2 \quad , \quad G = L\sqrt{1 - e^2} \quad , \quad \cos i = H/G \quad . \tag{12}$$

The position and the velocity in the nodal frame of reference are given by

$$
\begin{aligned}
X &= a\left[\cos\psi - \xi + \frac{\eta}{1+\sqrt{1-e^2}}\left(\xi\sin\psi - \eta\cos\psi\right)\right] \quad , \\
Y &= a\left[\sin\psi - \eta - \frac{\xi}{1+\sqrt{1-e^2}}\left(\xi\sin\psi - \eta\cos\psi\right)\right] \quad , \\
\dot{X} &= -\frac{1}{G}\left(\xi + \frac{Y}{r}\right) \quad , \\
\dot{Y} &= \frac{1}{G}\left(\eta + \frac{X}{r}\right) \quad ,
\end{aligned}
\tag{13}
$$

where $r = \sqrt{X^2 + Y^2}$. Finally, we have the position and velocity in the inertial frame:

$$
\begin{aligned}
x &= X\cos h - Y\cos i\sin h \quad , \\
y &= X\sin h + Y\cos i\cos h \quad , \\
z &= Y\sin i \quad , \\
\dot{x} &= \dot{X}\cos h - \dot{Y}\cos i\sin h \quad , \\
\dot{y} &= \dot{X}\sin h + \dot{Y}\cos i\cos h \quad , \\
\dot{z} &= \dot{Y}\sin i \quad .
\end{aligned}
\tag{14}
$$

The semimajor axis is usually obtained from the mean motion of the mean anomaly. When the eccentricity is small, the mean anomaly is not well determined, because it is strongly correlated with the argument of perigee. On the other hand, even if the eccentricity is very small, the mean argument of latitude $\ell + g$ is well determined and does not suffer so much from the inaccuracy of the long-period perturbations. The long-period perturbation of $\ell + g$ is expressed as

$$\Delta_{\text{long}} (\ell + g) = eA_1'(e,i) \cos g + e^2 A_2(e,i) \sin 2g + O(e^3) \quad ,$$

which satisfies the d'Alembert characteristics; this expression is also small because the amplitudes of cos g and sin 2g have factors of e and e^2, respectively. On the other hand, since the eccentricity appears in the denominator in the long-period perturbation of the mean anomaly, the semimajor axis is determined with better accuracy from the mean motion of $\ell + g$ than from the mean motion of ℓ.

4. ANALYTICAL AND NUMERICAL CHECK

Hori (1970a) and Yuasa (1971) showed that both Hori's and von Zeipel's perturbation theories give the same canonical transformation, together with the same new Hamiltonian, up to the third order. This allows us to compare the present solution, based on Hori's theory, with Kozai's (1962) solution, based on von Zeipel's theory. Our F_3^{**} completely agrees with Kozai's F_3^{**}. Although Hori (1966) gave a relation between the determining functions of his and von Zeipel's theories, we have not compared these functions, because of the complexity of Kozai's expression of S_2 and the tediousness of the calculations. Instead, we compared our S_2 with that derived later by Hori (1970b). In determining S, we have an ambiguity and can add an arbitrary function of L, G, H, and g to S, giving us $\overline{S}$. In the present theory, the disturbing function is expanded into a Fourier series with arguments of ℓ and g; therefore, it is natural to determine S in such a manner that its mean value with respect to the mean anomaly is zero. On the other hand, the mean value of Hori's S is not zero. The relation between S and $\overline{S}$, both of which are determining functions eliminating short-period terms in the Hamiltonian, is

$$\begin{aligned} \overline{S}_1 &= S_1 + f_1 \quad , \\ \overline{S}_2 &= S_2 + \frac{1}{2}\{S_1, f_1\} + f_2 \quad , \end{aligned} \tag{15}$$

where f_1 and f_2 are arbitrary functions of L, G, H, and g. Then the relation between S^* and $\bar{S}^*$, which are determining functions eliminating long-period terms, is

$$\begin{aligned} \bar{S}_1^* &= S_1^* - f_1 \quad , \\ \bar{S}_2^* &= S_2^* + \frac{1}{2}\{S_1^*, f_1\} - f_2 \quad . \end{aligned} \tag{16}$$

Even if the functional forms of these determining functions are different, the composite canonical transformation of (S, S^*) is identical to that of $(\bar{S}, \bar{S}^*)$. The second-order determining function S_2 of the present theory and that derived by Hori are found to satisfy relation (15).

It is of interest to compare the present solution with that due to Deprit and Rom (1970), in which J_3 and J_4 are zeros. Their solution was obtained by their own perturbation method, which, like Hori's, is based on Lie transformation. Campbell and Jefferys (1970) showed that the perturbation theories of Hori and Deprit are equivalent and derived explicit relations between the determining functions for the two:

$$\begin{aligned} W_1 &= -S_1 \quad , \\ W_2 &= -2S_2 \quad , \\ W_3 &= -6S_3 - \{S_1, S_2\} \quad , \end{aligned} \tag{17}$$

where W_n are the determining functions in Deprit's theory. Using these relations, we compared our solution with Deprit and Rom's, which also serves as an independent check. The few disagreements we found with their terms all seemed to be typographical errors. Kutuzov (1975) also discovered some discrepancies between his solution, obtained by computer algebra, and theirs.

As an internal consistency check, we wanted to make sure that a small divisor $(1 - 5 \cos^2 i)$ disappeared for the even-order harmonics when the geopotential was equal to that in Vinti (1959) [$J_{2n} = (-1)^{n+1} J_2^n$, $n \geq 2$]. In checking the third order, we had to add long-period perturbations arising from J_6 and J_8.

Finally, the present solution was compared with the results obtained from numerical integration. A Taylor-type integrator was adopted, in which the positions and velocities are expanded into a power series of time and the coefficients of the series are determined by recurrent formulas (Rabe, 1961; Deprit and Zahar, 1966). The order of the power series and the step size of time were chosen so as to maintain about 12 significant figures in the integral of energy; the integration step is roughly one-fifth of the convergence radius τ of the two-body problem, $\tau = (1/n) \left\{ \ell n \left[(1/e) + \sqrt{(1/e^2) - 1} \right] - \sqrt{1 - e^2} \right\}$, when the degree of the Taylor series is 16. The numerical calculations were carried out by a CDC 6400 computer in double precision in order to avoid round-off errors. It takes about 0.7 sec to evaluate the series of the present theory and about 0.3 sec to integrate the equations of motion for one step.

The initial conditions were computed from the present theory from a set of mean elements a'', e'', i'', ℓ'', g'', and h''. It should be noted that the mean motion of the mean anomaly of the integrated orbit is expected to be different from the computed mean motion by an order of J_2^4 because the accuracy of the periodic terms is of the order of J_2^3. This discrepancy can be avoided by adjusting the semimajor axis so as to remove the secular term in the residuals of the mean anomaly.

Such comparisons were done for the artificial satellites Geos 3, Starlette, Lageos, and Anna 1B. The results for Starlette are plotted in Figures 1 and 2. Figure 1 shows the in-track, along-track, and across-track errors over two revolutions. The deviations, less than 2×10^{-4} m, are totally negligible. Figure 2 plots these same errors over about 600 revolutions. The deviations along and across the track are totally insignificant, but the in-track error has a secular trend that seems to be proportional to the square of time. The error first increases, then decreases, and finally vanishes at about $t = 22$ days, at which time the semimajor axis was adjusted. This secular trend can be explained by the accumulation of truncation errors, which are caused by replacing infinitesimal operations with finite operations. The round-off errors do not accumulate significantly, because our calculations were carried out in double precision, which amounts to an accuracy of about 30 significant figures in decimal notation. The accumulation error due to discretization (Kinoshita, 1968) is

$$\Delta r_{\text{in-track}} \propto a\ell^2 h^p \quad , \tag{18}$$

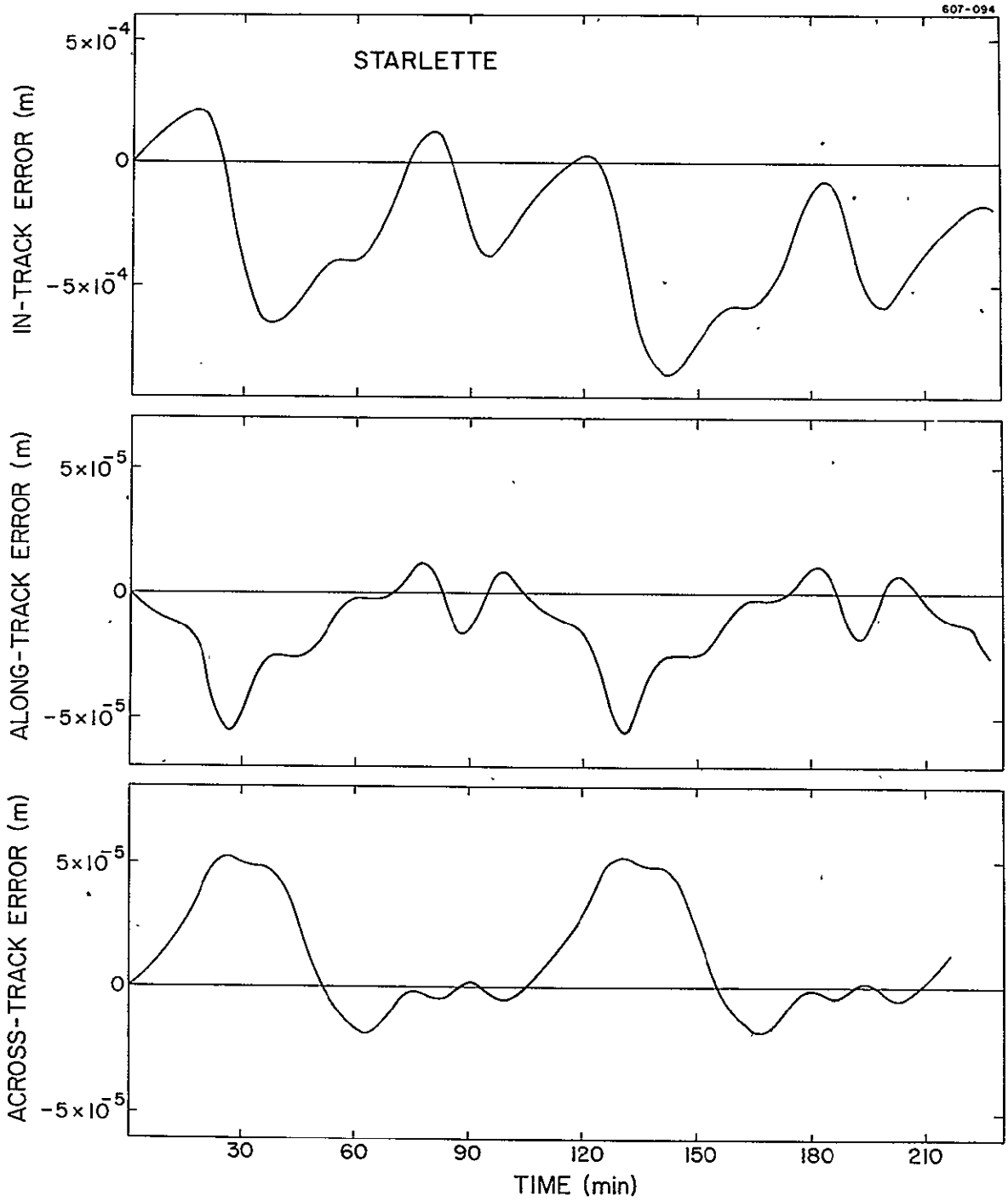

Figure 1. In-track, along-track, and across-track prediction errors of the third-order solution for Starlette plotted over two revolutions: $J_2 = 1.082 \times 10^{-3}$, $J_3 = -2.54 \times 10^{-6}$, $J_4 = -1.619 \times 10^{-6}$, $a'' = 7.335 \times 10^6$ m, $e'' = 0.020636$, $i'' = 49°.8223$, $\ell_0'' = 350°.23968$, $g_0'' = 82°.7702$, $h_0'' = 125°.0266$, P = 104.2 min.

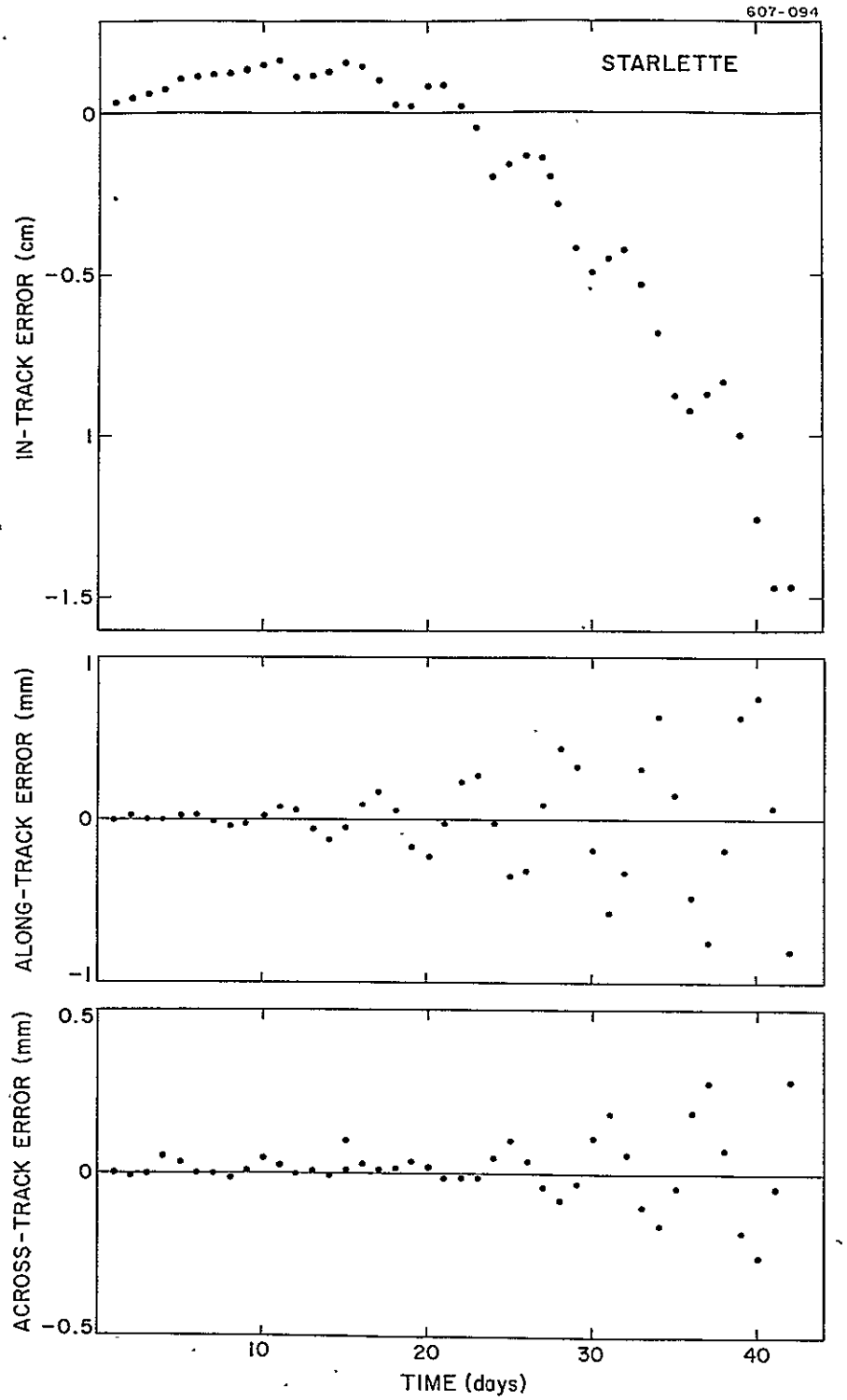

Figure 2. The same as Figure 1 plotted over about 600 revolutions.

where a is the semimajor axis, ℓ the mean anomaly, h the step size, and p the degree of the Taylor expansion. About 1 cm of the in-track error at t = 40 days (1 = 3600 radians) is obtained from using equation (18) with $a = 7.335 \times 10^6$ m and h = 600 sec = 1/10 radians, the order of the error agreeing with that of the numerical experiment. We can avoid discretization errors by employing a much more accurate integrator, but it seems to be an unnecessary use of computer time. The comparisons for Geos 3 and Lageos gave roughly the same results that we got for Starlette.

The errors for Anna 1B are plotted in Figure 3, while Deprit and Rom's (1970) results for Anna 1B are reproduced in Figure 4. Even though the time span of our test is shorter than Deprit and Rom's, a comparison of Figures 3 and 4 indicates that our solution gives better predictions.

We are now confident that the solution presented herein can predict the position of a small-eccentricity geodetic satellite with an accuracy of better than 1 cm over 1 month.

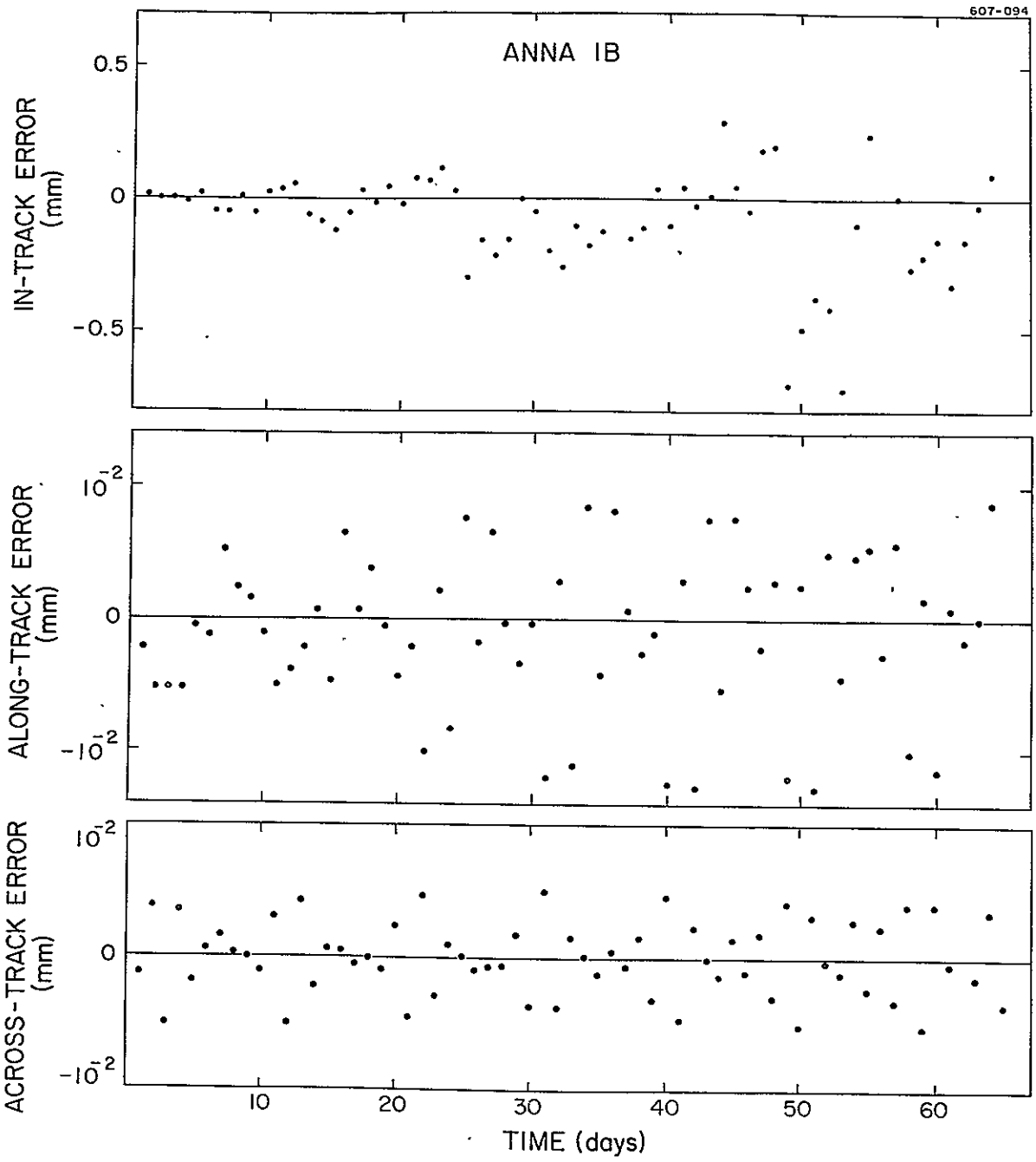

Figure 3. In-track, along-track, and across-track prediction errors of the third-order solution for Anna 1B plotted over 860 revolutions: $J_2 = 1.082 \times 10^{-3}$, $J_3 = 0$, $J_4 = 0$, $a'' = 7.4738 \times 10^6$ m, $e'' = 0.006711$, $i'' = 50.^{\circ}14768$, $\ell_0'' = 307.^{\circ}164$, $g_0'' = 198.^{\circ}303$, $h_0'' = 54.^{\circ}41$, P = 107.15 min.

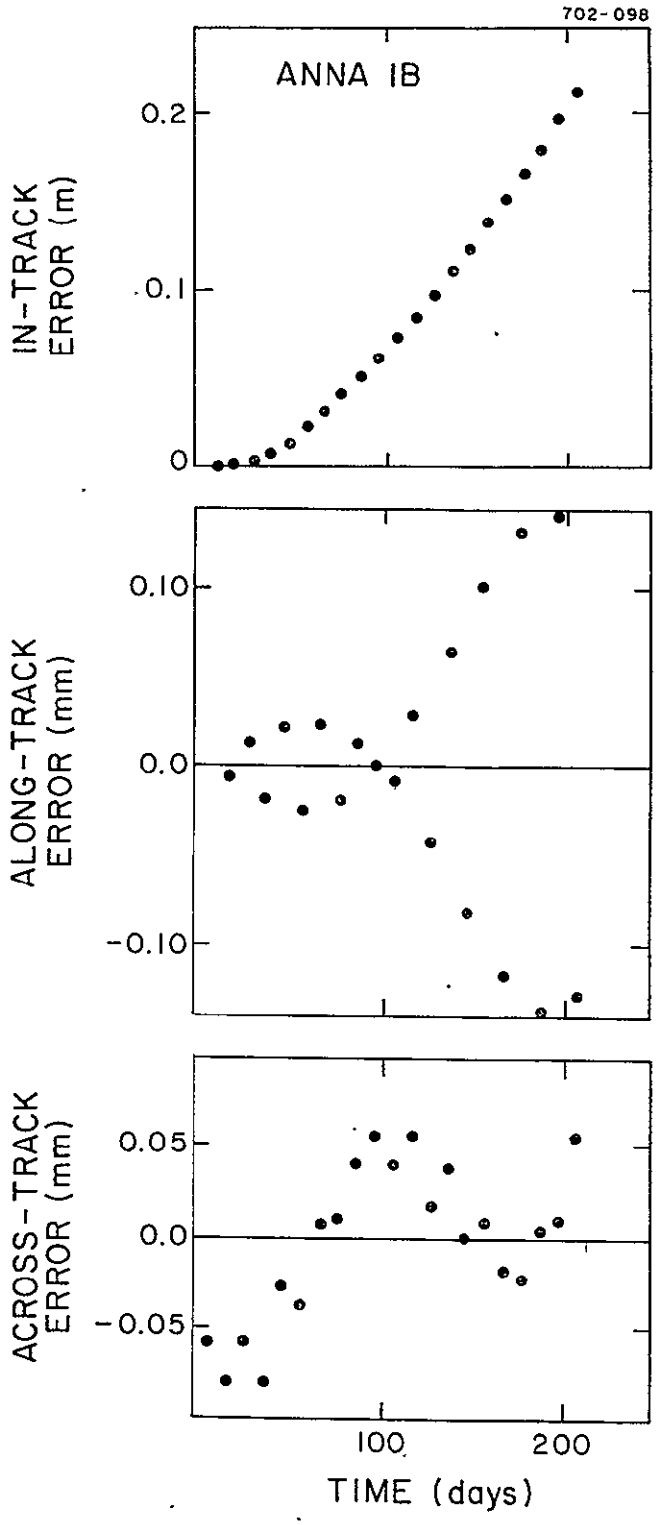

Figure 4. The same as Figure 3 plotted over about 2700 revolutions (from Deprit and Rom, 1970).

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REFERENCES

AKSNES, K.

1970. A second-order artificial satellite theory based on an intermediate orbit. Astron. Journ., vol. 75, pp. 1066-1076.

CAMPBELL, A., and JEFFERYS, W. H.

1970. Equivalence of the perturbation theories of Hori and Deprit. Celest. Mech., vol. 2, pp. 467-473.

DEPRIT, A., and ROM, A.

1970. The main problem of artificial satellite theory for small and moderate eccentricities. Celest. Mech., vol. 2, pp. 166-206.

DEPRIT, A., and ZAHAR, R.

1966. Numerical integration of an orbit and its concomitant variations by recurrent power series. Zeit. Angew. Math. Phys., vol. 17, pp. 426-430.

HALL, N. M., and CHERNIACK, J. R.

1969. Smithsonian Package for Algebra and Symbolic Mathematics. Smithsonian Astrophys. Obs. Spec. Rep. No. 291, 49 pp.

HORI, G.

1966. Theory of general perturbations with unspecified canonical variables. Publ. Astron. Soc. Japan, vol. 18, pp. 287-296.

1970a. Comparison of two perturbation theories based on canonical transformations. Publ. Astron. Soc. Japan, vol. 22, pp. 191-198.

HORI, G.

1970b. Unpublished (paper read by B. Garfinkel at the International Astronomical Union General Assembly, Commission No. 7, Brighton, England, August).

1973. Theory of general perturbations. In Recent Advances in Dynamical Astronomy, ed. by B. D. Tapley and V. Szebehely, D. Reidel Publ. Co., Dordrecht-Holland, pp. 231-249.

HORI, G., and KOZAI, Y.

1975. Analytical theories of the motion of artificial satellites. In Satellite Dynamics, ed. by G. E. O. Giacaglia, Springer-Verlag, Berlin, pp. 1-15.

KINOSHITA, H.

1968. Propagation of the discretization error in the two-body problem. Publ. Astron. Soc. Japan, vol. 20, pp. 1-23.

KINOSHITA, H., and HORI, G.

1972. On the stationary solutions for problems of artificial satellites. Tokyo Astron. Obs. Rep., vol. 16, pp. 230-240.

KOZAI, Y.

1962. Second-order solution of artificial satellite theory without air drag. Astron. Journ., vol. 67, pp. 446-461.

1966. The earth gravitational potential derived from satellite motion. Space Sci. Rev., vol. 5, pp. 818-879.

KUTUZOV, A. L.

1975. Application of symbolic computer operations for solving the main problem in the theory of satellite motion. Soviet Astron. Lett., vol. 1, pp. 42-44.

1961. Determination and survey of periodic Trojan orbits in the restricted problem of three bodies. Astron. Journ., vol. 66, pp. 500-513.

VINTI, J. P.

1959. New method of solution for unretarded satellite orbits. Journ. Res. Nat. Bur. Standards, vol. 63B, pp. 105-116.

YUASA, M.

1971. The comparison of Hori's perturbation theory and von Zeipel's theory. Publ. Astron. Soc. Japan, vol. 23, pp. 399-403.

APPENDIX A

EXPRESSIONS OF S, F^*, S^*, AND F^{**}

Some of the expressions herein are reproduced directly from computer printout, whose format differs from the type written equations. In the printout, the following notation is used:

$A(2) = J_2$, SI = sin i , E = e ,

$A(3) = J_3$, CI = cos i , SG = g ,

$A(4) = J_4$, $ING = 1 - 5\cos^2 i$, M = ℓ .

A.1 Expressions of S

$$S = S_1(J_2) + S_2(J_3) + S_2(J_4) + S_2(J_2^2) + S_3(J_2^3) + S_3(J_2J_3) + S_3(J_2J_4)$$

$$S_1(J_2) = \frac{J_2}{6G^3}\left\{-\frac{3}{2}(1 - 3c^2)(f - \ell + e\sin f) + \frac{3}{4}[3e\sin(f + 2g) + 3\sin 2(f + g) + e\sin(3f + 2g)]\right\}$$
$$+ \frac{J_2}{8G^3}\frac{s^2e^2(1 + 2\eta)}{(1 + \eta)^2}\sin 2g$$

$$S_2(J_3) = -\frac{J_3}{96G^5}s\left\{3(1 - 5c^2)[12e(f - \ell)\sin g + 3e^2\cos(f - g) - 6(2 + e^2)\cos(f + g) - 6e\cos(2f + g)\right.$$
$$\left. - e^2\cos(3f + g)] + s^2[15e^2\cos(f + 3g) + 30e\cos(2f + 3g) + 10(2 + e^2)\cos 3(f + g) + 15e\cos(4f + 3g)\right.$$
$$\left. + 3e^2\cos(5f + 3g)]\right\} + \frac{J_3}{96G^5}\left[6(1 - 5c^2)\frac{se(5 + 5\eta + 2\eta^2)}{1 + \eta}\cos g - \frac{s^3e^3(3 + 9\eta + 8\eta^2)}{(1 + \eta)^3}\cos 3g\right]$$

$$S_2(J_4) = \frac{J_4}{512G^7}\left\{-2(3 - 30c^2 + 35c^4)[6(2 + 3e^2)(f - \ell) + 9e(4 + e^2)\sin f + 9e^2\sin 2f + e^3\sin 3f]\right.$$
$$+ 2s^2(1 - 7c^2)[60e^2(f - \ell)\cos 2g + 10e^3\sin(f - 2g) + 30e(4 + e^2)\sin(f + 2g) + 20(2 + 3e^2)\sin 2(f + g)$$
$$+ 10e(4 + e^2)\sin(3f + 2g) + 15e^2\sin(4f + 2g) + 2e^3\sin(5f + 2g)] - s^4[35e^3\sin(f + 4g)$$
$$+ 105e^2\sin(2f + 4g) + 35e(4 + e^2)\sin(3f + 4g) + 35(2 + 3e^2)\sin 4(f + g) + 21e(4 + e^2)\sin(5f + 4g)$$
$$\left. + 35e^2\sin(6f + 4g) + 5e^3\sin(7f + 4g)]\right\} + \frac{J_4}{512G^7}\left[\frac{2s^2(1 - 7c^2)\,e^2(77 + 154\eta + 101\eta^2 + 28\eta^3)}{(1 + \eta)^2}\sin 2g\right.$$
$$\left. - \frac{s^4e^4(5 + 20\eta + 29\eta^2 + 16\eta^3)}{(1 + \eta)^4}\sin 4g\right]$$

where $c = \cos i$, $s = \sin i$, $\eta = \sqrt{1 - e^2}$.

$$\frac{L^7}{J_2^2} S_2(J_2^2) = -1/163840*E^5*SI^4 \qquad \text{SIN (4*SG-M)}$$

$$+ E^3*SI^2*(427/256-123/64*SI^2+45993/8192*E^2-106341/16384*E^2*SI^2) \qquad \text{SIN (2*SG-M)}$$

$$-1/30720*E^6*SI^4 \qquad \text{SIN (4*SG-2*M)}$$

$$+ E^4*SI^2*(69/64-2529/2048*SI^2+34923/10240*E^2-40221/10240*E^2*SI^2) \qquad \text{SIN (2*SG-2*M)}$$

$$+ E^5*SI^2*(15/16-43731/40960*SI^2) \qquad \text{SIN (2*SG-3*M)}$$

$$+ E^6*SI^2*(3547/3840-1071/1024*SI^2) \qquad \text{SIN (2*SG-4*M)}$$

$$+ E^6*(1317/256-101691/10240*SI^2+170863/40960*SI^4) \qquad \text{SIN (6*M)}$$

$$+ E^5*(41111/10240-40183/5120*SI^2+110003/32768*SI^4) \qquad \text{SIN (5*M)}$$

$$+ E^4*(411/128-1631/256*SI^2+2863/1024*SI^4+1413/256*E^2-29029/2560*E^2 \qquad \text{SIN (4*M)}$$
$$*SI^2+215513/40960*E^2*SI^4)$$

$$+ E^3*(343/128-693/128*SI^2+5037/2048*SI^4+11469/2048*E^2-11829/1024*E^2 \qquad \text{SIN (3*M)}$$
$$*SI^2+178065/32768*E^2*SI^4)$$

$$+ E^2*(39/16-645/128*SI^2+1227/512*SI^4+783/128*E^2-815/64*E^2*SI^2+6265 \qquad \text{SIN (2*M)}$$
$$/1024*E^2*SI^4+3033/256*E^4-25159/1024*E^4*SI^2+47927/4096*E^4*SI^4)$$

$$+ E*(45/16-6*SI^2+777/256*SI^4+1089/128*E^2-2301/128*E^2*SI^2+18237/2048 \qquad \text{SIN (M)}$$
$$*E^2*SI^4+18329/1024*E^4-19189/512*E^4*SI^2+298315/16384*E^4*SI^4)$$

$$-25435/3072*E^6*SI^4 \qquad \text{SIN (4*SG+10*M)}$$

$$-351297/81920*E^5*SI^4 \qquad \text{SIN (4*SG+9*M)}$$

```
+ E**4*SI**4*(-2141/1024+47781/5120*E**2)                                    SIN (4*SG+8*M)

+ E**6*SI**2*(1568921/30720-2915481/40960*SI**2)                             SIN (2*SG+8*M)

+ E**3*SI**4*(-959/1024+138299/32768*E**2)                                   SIN (4*SG+7*M)

+ E**5*SI**2*(244869/8192-3440013/81920*SI**2)                               SIN (2*SG+7*M)

+ E**2*SI**4*(-189/512+1791/1024*E**2-21285/8192*E**4)                       SIN (4*SG+6*M)

+ E**4*SI**2*(8443/512-48047/2048*SI**2-550097/10240*E**2+143615/2048*E**2*SI**2   SIN (2*SG+6*M)
  )

+ E*SI**4*(-15/128+1305/2048*E**2-3785/4096*E**4)                            SIN (4*SG+5*M)

+ E**3*SI**2*(4219/512-12279/1024*SI**2-2061/64*E**2+340401/8192*E**2*SI**2)-  SIN (2*SG+5*M)

+ SI**4*(-3/128+3/16*E**2-291/1024*E**4+199/1536*E**6)                       SIN (4*SG+4*M)

+ E**2*SI**2*(27/8-1323/256*SI**2-2505/128*E**2+6381/256*E**2*SI**2-189/16*E**4   SIN (2*SG+4*M)
  +29151/2048*E**4*SI**2)

+ E*SI**4*(9/256-9/128*E**2+369/16384*E**4)                                  SIN (4*SG+3*M)

+ E*SI**2*(47/64-179/128*SI**2-3089/256*E**2+1943/128*E**2*SI**2-47067/4096*E**4   SIN (2*SG+3*M)
  +115011/8192*E**4*SI**2)

+ E**2*SI**4*(-3/256+1/512*E**2-3/4096*E**4)                                 SIN (4*SG+2*M)

+ SI**2*(-3/8+9/32*SI**2-927/128*E**2+2313/256*E**2*SI**2-4785/512*E**4+5859   SIN (2*SG+2*M)
  /512*E**4*SI**2-82583/6144*E**6+66765/4096*E**6*SI**2)
```

$$+ E^{3}*SI^{4}*(1/2048+5/16384*E^{2}) \qquad \text{SIN} \quad (4*SG+M)$$

$$+ E*SI^{2}*(-45/16+225/64*SI^{2}-2691/512*E^{2}+6507/1024*E^{2}*SI^{2}-309/32*E^{4} \qquad \text{SIN} \quad (2*SG+M)$$
$$+46875/4096*E^{4}*SI^{2})$$

$$\frac{L^{11}}{J_2^3}S_3(J_2^3) = E^{3}*SI^{2}*(-43231/4096+162979/6144*SI^{2}-12785/768*SI^{4}) \qquad \text{SIN} \quad (2*SG-M)$$

$$+ E^{4}*SI^{2}*(-82639/12288+105475/6144*SI^{2}-715997/65536*SI^{4}) \qquad \text{SIN} \quad (2*SG-2*M)$$

$$+ E^{4}*(5171/512-304559/6144*SI^{2}+841529/8192*SI^{4}-33411/512*SI^{6}) \qquad \text{SIN} \quad (4*M)$$

$$+ E^{3}*(8553/1024-240419/6144*SI^{2}+1256523/16384*SI^{4}-4626661/98304*SI^{6} \qquad \text{SIN} \quad (3*M)$$

$$+ E^{2}*(1917/256-16745/512*SI^{2}+239659/4096*SI^{4}-276537/8192*SI^{6}+17741 \qquad \text{SIN} \quad (2*M)$$
$$/512*E^{2}-110983/768*E^{2}*SI^{2}+999653/4096*E^{2}*SI^{4}-2203923/16384*E^{2}*SI^{6}$$
$$)$$

$$+ E*(1077/128-8413/256*SI^{2}+100647/2048*SI^{4}-99747/4096*SI^{6}+44689 \qquad \text{SIN} \quad (M)$$
$$/1024*E^{2}-343335/2048*E^{2}*SI^{2}+4025431/16384*E^{2}*SI^{4}-3896571/32768*E^{2}$$
$$*SI^{6})$$

$$+ 264995/196608*E^{4}*SI^{6} \qquad \text{SIN} \quad (6*SG+10*M)$$

$$+ 29979/65536*E^{3}*SI^{6} \qquad \text{SIN} \quad (6*SG+9*M)$$

$$+ E^{2}*SI^{6}*(2149/16384-108401/98304*E^{2}) \qquad \text{SIN} \quad (6*SG+8*M)$$

$$+ E^{4}*SI^{4}*(733357/12288-2310441/32768*SI^{2}) \qquad \text{SIN} \quad (4*SG+8*M)$$

$$+ E*SI^{6}*(235/8192-19531/65536*E^{2}) \qquad \text{SIN} \quad (6*SG+7*M)$$

```
+ E**3*SI**4*(2870799/114688-6818593/229376*SI**2)                    SIN  (4*SG+7*M)

+ SI**6*(15/4096-63/1024*E**2+14463/65536*E**4)                       SIN  (6*SG+6*M)

+ E**2*SI**4*(18517/2048-44301/4096*SI**2-282515/6144*E**2+105793/2048*E**2   SIN  (4*SG+6*M)
  *SI**2)

+ E**4*SI**2*(397793/4096-5639543/24576*SI**2+26702183/196608*SI**4)  SIN  (2*SG+6*M)

+ E*SI**6*(-61/8192+2731/65536*E**2)                                  SIN  (6*SG+5*M)

+ E*SI**4*(25907/10240-12537/4096*SI**2-1572423/81920*E**2+705243/32768*E**2   SIN  (4*SG+5*M)
  *SI**2)

+ E**3*SI**2*(1015027/20480-7160317/61440*SI**2+3365033/49152*SI**4)  SIN  (2*SG+5*M)

+ E**2*SI**6*(77/16384-901/98304*E**2)                                SIN  (6*SG+4*M)

+ SI**4*(105/256-519/1024*SI**2-14409/2048*E**2+4041/512*E**2*SI**2-52611   SIN  (4*SG+4*M)
  /2048*E**4+122895/4096*E**4*SI**2)

+ E**2*SI**2*(22871/1024-107261/2048*SI**2+501057/16384*SI**4-165433/6144*E**2   SIN  (2*SG+4*M)
  +383039/6144*E**2*SI**2-1238799/32768*E**2*SI**4)

  -63/65536*E**3*SI**6                                                SIN  (6*SG+3*M)

+ E*SI**4*(-11201/6144+8411/4096*SI**2-457999/49152*E**2+359975/32768*E**2   SIN  (4*SG+3*M)
  *SI**2)

+ E*SI**2*(4069/512-28841/1536*SI**2+67621/6144*SI**4-70799/4096*E**2   SIN  (2*SG+3*M)
  +449159/12288*E**2*SI**2-1937753/98304*E**2*SI**4)
```

$$+ 13/196608*E^{4}*SI^{6} \qquad \text{SIN} \quad (6*SG+2*M)$$

$$+ E^{2}*SI^{4}*(-3747/4096+9717/8192*SI^{2}+10157/768*E^{2}-239709/16384*E^{2}*SI^{2} \qquad \text{SIN} \quad (4*SG+2*M)$$

$$)$$

$$+ SI^{2}*(417/256-533/128*SI^{2}+10747/4096*SI^{4}-2051/256*E^{2}+28523/2048*E^{2} \qquad \text{SIN} \quad (2*SG+2*M)$$

$$*SI^{2}-21903/4096*E^{2}*SI^{4}-14995/4096*E^{4}-101629/8192*E^{4}*SI^{2}+1348099$$

$$/65536*E^{4}*SI^{4})$$

$$+ E^{3}*SI^{4}*(85735/16384-193979/32768*SI^{2}) \qquad \text{SIN} \quad (4*SG+M)$$

$$+ E*SI^{2}*(1577/512-1183/128*SI^{2}+27207/4096*SI^{4}+147083/4096*E^{2}-96419 \qquad \text{SIN} \quad (2*SG+M)$$

$$/1024*E^{2}*SI^{2}+2015547/32768*E^{2}*SI^{4})$$

$$\frac{9}{J_3} S_3(J_2J_3) = E^{4}*SI^{3}*(-124105/49152+92385/32768*SI^{2}) \qquad \text{COS} \quad (3*SG-M)$$

$$+ E^{2}*SI*(93/64+801/256*SI^{2}-21565/4096*SI^{4}+1107/256*E^{2}+19481/1024*E^{2} \qquad \text{COS} \quad (SG-M)$$

$$*SI^{2}-651893/24576*E^{2}*SI^{4})$$

$$+ E^{3}*SI*(111/64+325/256*SI^{2}-5515/1536*SI^{4}) \qquad \text{COS} \quad (SG-2*M)$$

$$+ E^{4}*SI*(2241/1024+2043/4096*SI^{2}-217935/65536*SI^{4}) \qquad \text{COS} \quad (SG-3*M)$$

$$+ 369081/65536*E^{4}*SI^{5} \qquad \text{COS} \quad (5*SG+0*M)$$

$$+ 557/256*E^{3}*SI^{5} \qquad \text{COS} \quad (5*SG+0*M)$$

$$+ E^{2}*SI^{5}*(2961/4096-10591/2048*E^{2}) \qquad \text{COS} \quad (5*SG+7*M)$$

$$+ E^{4}*SI^{3}*(-1778797/49152+1656401/32768*SI^{2}) \qquad \text{COS} \quad (3*SG+7*M)$$

```
+ E*SI**5*(3/16-207/128*E**2)                                                    COS (5*SG+6*M)

+ E**3*SI**3*(-2163/128+3027/128*SI**2)                                          COS (3*SG+6*M)

+ SI**5*(15/512-405/1024*E**2+36285/32768*E**4)                                  COS (5*SG+5*M)

+ E**2*SI**3*(-7201/1024+20205/2048*SI**2+85847/6144*E**2-170049/8192*E**2*SI**2 COS (3*SG+5*M)

  )

+ E*SI**5*(-15/256+249/1024*E**2)                                                COS (5*SG+4*M)

+ E**4*SI*(-12711/1024+62575/4096*SI**2-327449/196608*SI**4)                     COS (SG+5*M)

+ E*SI**3*(-77/32+867/256*SI**2+2917/512*E**2-4245/512*E**2*SI**2)               COS (3*SG+4*M)

+ E**3*SI*(-387/64+1135/512*SI**2+15659/3072*SI**4)                              COS (SG+4*M)

+ E**2*SI**5*(135/4096-81/2048*E**2)                                             COS (5*SG+3*M)

+ SI**3*(-69/128+195/256*SI**2+177/64*E**2-3803/1024*E**2*SI**2+138837/8192*E**4 COS (3*SG+3*M)
  -314469/16384*E**4*SI**2)

+ E**2*SI*(-141/64-1161/256*SI**2+32297/4096*SI**4+1605/256*E**2-52881/1024      COS (SG+3*M)
  *E**2*SI**2+413139/8192*E**2*SI**4)

  -5/1024*E**3*SI**5                                                             COS (5*SG+2*M)

+ E*SI**3*(425/256-1035/512*SI**2+6185/512*E**2-7077/512*E**2*SI**2)             COS (3*SG+2*M)

+ E*SI*(-1821/256*SI**2+4099/512*SI**4+93/16*E**2-21015/512*E**2*SI**2+39833     COS (SG+2*M)
  /1024*E**2*SI**4)
```

$$+ 5/65536*E^{4}*SI^{5} \qquad \text{COS}\ (5*SG+M)$$

$$+ E^{2}*SI^{3}*(4309/1024-9855/2048*SI^{2}+75941/6144*E^{2}-115403/8192*E^{2}*SI^{2}) \qquad \text{COS}\ (3*SG+M)$$

$$+ SI*(9/8-195/32*SI^{2}+2755/512*SI^{4}+45/8*E^{2}-3891/128*E^{2}*SI^{2}+1717/64*E^{2} \qquad \text{COS}\ (SG+M)$$
$$*SI^{4}+6339/512*E^{4}-158637/2048*E^{4}*SI^{2}+2317405/32768*E^{4}*SI^{4})$$

$$\frac{11}{j_4}S_3(J_2J_4)= E^{3}*SI^{2}*(-40135/2048+225085/4096*SI^{2}-7202783/196608*SI^{4}) \qquad \text{SIN}\ (2*SG-M)$$

$$+ E^{4}*SI^{2}*(-52367/4096+450415/12288*SI^{2}-4874653/196608*SI^{4}) \qquad \text{SIN}\ (2*SG-2*M)$$

$$+ E^{4}*(-8721/512+17453/256*SI^{2}-245719/4096*SI^{4}+32053/4096*SI^{6}) \qquad \text{SIN}\ (4*M)$$

$$+ E^{3}*(-6915/512+58161/1024*SI^{2}-458529/8192*SI^{4}+196861/16384*SI^{6}) \qquad \text{SIN}\ (3*M)$$

$$+ E^{2}*(-735/64+26415/512*SI^{2}-15003/256*SI^{4}+74781/4096*SI^{6}-29595/512 \qquad \text{SIN}\ (2*M)$$
$$*E^{2}+8527/32*E^{2}*SI^{2}-1295737/4096*E^{2}*SI^{4}+437899/4096*E^{2}*SI^{6})$$

$$+ E*(-765/64+7545/128*SI^{2}-81795/1024*SI^{4}+68355/2048*SI^{6}-35475/512*E^{2} \qquad \text{SIN}\ (M)$$
$$+347445/1024*E^{2}*SI^{2}-3718989/8192*E^{2}*SI^{4}+3053841/16384*E^{2}*SI^{6})$$

$$+ 412205/49152*E^{4}*SI^{6} \qquad \text{SIN}\ (6*SG+10*M)$$

$$+ 185415/65536*E^{3}*SI^{6} \qquad \text{SIN}\ (6*SG+9*M)$$

$$+ E^{2}*SI^{6}*(1651/2048-100559/12288*E^{2}) \qquad \text{SIN}\ (6*SG+8*M)$$

$$+ E^{4}*SI^{4}*(-447401/12288+6427969/98304*SI^{2}) \qquad \text{SIN}\ (4*SG+8*M)$$

$$+ E*SI^{6}*(1435/8192-145369/65536*E^{2}) \qquad \text{SIN}\ (6*SG+7*M)$$

```
+ E**3*SI**4*(-1068423/114688+13719047/688128*SI**2)                    SIN  (4*SG+7*M)

+ SI**6*(91/4096-3759/8192*E**2+135369/65536*E**4)                      SIN  (6*SG+6*M)

+ E**2*SI**4*(-3625/4096+32371/8192*SI**2+755605/8192*E**2-2105291/16384*E**2   SIN  (4*SG+6*M)
  *SI**2)

+ E**4*SI**2*(-595769/4096+1738701/4096*SI**2-19034635/65536*SI**4)     SIN  (2*SG+6*M)

+ E*SI**6*(-455/8192+25865/65536*E**2)                                  SIN  (6*SG+5*M)

+ E*SI**4*(7997/10240-371/4096*SI**2+2358127/81920*E**2-1337549/32768*E**2   SIN  (4*SG+5*M)
  *SI**2)

+ E**3*SI**2*(-702799/10240+4231921/20480*SI**2-28358101/196608*SI**4)  SIN  (2*SG+5*M)

+ E**2*SI**6*(91/2048-1505/12288*E**2)                                  SIN  (6*SG+4*M)

+ SI**4*(99/256-329/1024*SI**2+6017/1024*E**2-18207/2048*E**2*SI**2-115609   SIN  (4*SG+4*M)
  /2048*E**4+283537/4096*E**4*SI**2)

+ E**2*SI**2*(-3459/128+43535/512*SI**2-31213/512*SI**4+24863/256*E**2-761599   SIN  (2*SG+4*M)
  /3072*E**2*SI**2+15145223/98304*E**2*SI**4)

  -819/65536*E**3*SI**6                                                 SIN  (6*SG+3*M)

+ E*SI**4*(43/2048-1589/4096*SI**2-306991/16384*E**2+751821/32768*E**2*SI**2)   SIN  (4*SG+3*M)

+ E*SI**2*(-1685/256+11651/512*SI**2-140791/8192*SI**4+144659/2048*E**2   SIN  (2*SG+3*M)
  -802529/4096*E**2*SI**2+8489153/65536*E**2*SI**4)
```

$$+ 91/98304*E^{4}*SI^{0} \qquad \text{SIN} \quad (6*SG+2*M)$$

$$+ E^{2}*SI^{4}*(-9197/4096+24101/8192*SI^{2}+96331/6144*E^{2}-427189/24576*E^{2} \qquad \text{SIN} \quad (4*SG+2*M)$$
$$*SI^{2})$$

$$+ SI^{2}*(75/64-1335/512*SI^{2}+5719/4096*SI^{4}+25275/512*E^{2}-74621/512*E^{2} \qquad \text{SIN} \quad (2*SG+2*M)$$
$$*SI^{2}+822549/8192*E^{2}*SI^{4}+178161/1024*E^{4}-4321633/8192*E^{4}*SI^{2}$$
$$+24202069/65536*E^{4}*SI^{4})$$

$$+ E^{3}*SI^{4}*(101313/16384-691397/98304*SI^{2}) \qquad \text{SIN} \quad (4*SG+M)$$

$$+ E*SI^{2}*(5415/256-31385/512*SI^{2}+342587/8192*SI^{4}+207255/2048*E^{2} \qquad \text{SIN} \quad (2*SG+M)$$
$$-1187761/4096*E^{2}*SI^{2}+12862703/65536*E^{2}*SI^{4})$$

A.2 Expressions of F^*

$$F^* = F_1^* + F_2^* + F_3^* + F_4^*$$

$$F_1^* = -\frac{J_2}{4L^3G^3}(1 - 3c^2)$$

$$F_2^* = F_{2s}^* + F_{2p}^*$$

$$F_{2s}^* = \frac{3J_2^2}{128L^3G^7}\left[-5(1 - 2c^2 - 7c^4) + 4\eta(1 - 3c^2)^2 + \eta^2(5 - 18c^2 + 5c^4)\right] - \frac{3J_4}{128L^3G^7}(3 - 30c^2 + 35c^4)(2 + 3e^2)$$

$$F_{2p}^* = \frac{3J_2^2}{64L^3G^7}e^2s^2\left[1 - 15c^2 + \frac{4(1 + 2\eta)}{(1 + \eta)^2}(1 - 5c^2)\right]\cos 2g - \frac{3J_3}{8L^3G^5}es(1 - 5c^2)\sin g + \frac{15J_4}{64L^3G^7}e^2s^2(1 - 7c^2)\cos 2g$$

COEFFICIENT OF COS(4*SG)

$$A2*A4*E^4/L^{14}*SI^4*(7685/2048-735/512*SI^2)$$

$$+ A2^3*E^4/L^{14}*SI^4*(1155/2048-315/512*SI^2)$$

COEFFICIENT OF SIN(3*SG)

$$A2*A3*E^3/L^{12}*SI^3*(-235/128+525/256*SI^2)$$

COEFFICIENT OF COS(2*SG)

$$A2*A4*E^2/L^{14}*SI^2*(455/64-9285/256*SI^2+12075/512*SI^4+9825/128*E^2-107835/512*E^2*SI^2+70665/512*E^2*SI^4)$$

$$+ A2^3*E^2/L^{14}*SI^2*(573/64-1389/64*SI^2+3375/256*SI^4+11811/256*E^2-115221/1024*E^2*SI^2+140865/2048*E^2*SI^4)$$

COEFFICIENT OF SIN(SG)

$$A2*A3*E/L^{12}*SI*(9/4-39/16*SI^2+15/128*SI^4+9*E^2-693/128*E^2*SI^2-285/64*E^2*SI^4)$$

COEFFICIENT OF COS(0)

$$A2*A4/L^{14}*(-45/16+495/32*SI^2-3105/128*SI^4+3045/256*SI^6-675/32*E^2+7155/64*E^2*SI^2-5355/32*E^2*SI^4+20055/256*E^2*SI^6-5175/64*E^4$$

$$+107325/256*E^4*SI^2-311805/512*E^4*SI^4+565215/2048*E^4*SI^6)$$

$$+ A2^3/L^{14}*(27/8-405/32*SI^2+2091/128*SI^4-111/16*SI^6+621/32*E^2-4647/64*E^2*SI^2+1527/16*E^2*SI^4-42243/1024*E^2*SI^6+8271/128*E^4$$

$$-61959/256*E^4*SI^2+662529/2048*E^4*SI^4-145707/1024*E^4*SI^6)$$

F_4^*

COEFFICIENT OF SIN(3*SG)

$$A3*A4*E^3/L^{16}*SI^3*(-5295/512+8955/512*SI^2-29925/4096*SI^4)$$

$$+ A2^2*A4*E^3/L^{16}*SI^3*(231/8-72645/1024*SI^2+176325/4096*SI^4)$$

COEFFICIENT OF COS(2*SG)

$$A4^2*E^2/L^{18}*SI^2*(-14175/256+239175/1024*SI^2-1320165/4096*SI^4+74235/512*SI^6)$$

$$+ A3^2*E^2/L^{14}*SI^2*(-249/32+1965/128*SI^2-3975/512*SI^4)$$

$$+ A2^2*A4*E^2/L^{18}*SI^2*(-3855/128+191775/1024*SI^2-2602509/8192*SI^4+2626995/16384*SI^6)$$

$$+ A2^4*E^2/L^{18}*SI^2*(11445/1024-23853/2048*SI^2-157131/8192*SI^4+632877/32768*SI^6)$$

COEFFICIENT OF SIN(SG)

$$A3*A4*E/L^{16}*SI*(135/8-2835/32*SI^2+36735/256*SI^4-9345/128*SI^6+1035/8*E^2-345555/512*E^2*SI^2+559095/512*E^2*SI^4-1138515/2048*E^2*SI^6)$$

$$+ A2^2*A3*E/L^{16}*SI*(27/2-4731/64*SI^2+61977/512*SI^4-123735/2048*SI^6+6837/64*E^2-302487/512*E^2*SI^2+500109/512*E^2*SI^4-4027665/8192*E^2*SI^6)$$

COEFFICIENT OF COS(0)

$$A4^2/L^{18}*(225/128-225/16*SI^2+20475/512*SI^4-94675/2048*SI^6+152145/8192*SI^8+675/32*E^2-19575/128*E^2*SI^2+795825/2048*E^2*SI^4-1668765/4096*E^2*SI^6+153615/1024*E^2*SI^8)$$

$$+ A3^2/L^{14}*(9/8-9*SI^2+1095/64*SI^4-4875/512*SI^6+81/8*E^2-291/4*E^2*SI^2+68205/512*E^2*SI^4-74415/1024*E^2*SI^6)$$

$$+ A2^2*A4/L^{18}*(-135/8+7155/64*SI^2-66525/256*SI^4+65325/256*SI^6-23205/256*SI^8-10935/64*E^2+143565/128*E^2*SI^2-675705/256*E^2*SI^4+10808025/4096*E^2*SI^6-7816305/8192*E^2*SI^8)$$

$$+ A2^4/L^{18}*(1755/128-16389/256*SI^2+24189/256*SI^4-94863/2048*SI^6+9201/8192*SI^8+13959/128*E^2-16161/32*E^2*SI^2+1506957/2048*E^2*SI^4-2871795/8192*E^2*SI^6+1761/1024*E^2*SI^8)$$

A.3 Expressions of S^*

$$S^* = S_1^* + S_2^* + S_3^*$$

$$S_1^* = -\frac{J_2}{32G^3}e^2s^2\left[\frac{1-15c^2}{1-5c^2}+\frac{4(1+2n)}{(1+n)^2}\right]\sin 2g - \frac{J_3}{2J_2G}es\cos g - \frac{5J_4}{32J_2G^3}\frac{e^2s^2(1-7c^2)}{1-5c^2}\sin 2g$$

S_2^*

COEFFICIENT OF SIN(4*SG)

$$1/A2^2*A4^2*E^4/L^7*SI^4*(1125/512/ING^3-1875/256/ING^3*SI^2+16625/2048/ING^3*SI^4-6125/2048/ING^3*SI^6-225/256/ING^2+525/256/ING^2*SI^2-1225/1024/ING^2*SI^4)$$

$$+ A4*E^4/L^7*SI^4*(975/256/ING^3-6475/512/ING^3*SI^2+3575/256/ING^3*SI^4-2625/512/ING^3*SI^6-195/128/ING^2+905/256/ING^2*SI^2-525/256/ING^2$$
$$*SI^4-895/2048/ING+245/512/ING*SI^2)$$
$$+ A2^2*E^4/L^7*SI^4*(845/512/ING^3-2795/512/ING^3*SI^2+3075/512/ING^3*SI^4-1125/512/ING^3*SI^6-169/256/ING^2+195/128/ING^2*SI^2-225/256/ING^2$$
$$*SI^4-385/2048/ING+105/512/ING*SI^2)$$

COEFFICIENT OF COS(3*SG)

$$1/A2^2*A3*A4*E^3/L^5*SI^3*(-75/64/ING^2+325/128/ING^2*SI^2-175/128/ING^2*SI^4+145/192/ING-665/768/ING*SI^2)$$
$$+ A3*E^3/L^5*SI^3*(-65/64/ING^2+35/16/ING^2*SI^2-75/64/ING^2*SI^4-181/1152/ING+125/768/ING*SI^2)$$

COEFFICIENT OF SIN(2*SG)

$$1/A2^2*A4^2*E^2/L^7*SI^2*(-75/8/ING^2+3025/64/ING^2*SI^2-2275/32/ING^2*SI^4+8575/256/ING^2*SI^6-2775/64*E^2/ING^2+27475/128*E^2/ING^2*SI^2$$
$$-163625/512*E^2/ING^2*SI^4+153125/1024*E^2/ING^2*SI^6)$$
$$+ 1/A2^2*A3^2*E^2/L^3*SI^2*(-5/16/ING+5/16/ING*SI^2+1/8-15/32*E^2/ING+15/32*E^2/ING*SI^2+3/16*E^2)$$
$$+ A4*E^2/L^7*SI^2*(35/4/ING^2-295/16/ING^2*SI^2+2085/256/ING^2*SI^4+875/512/ING^2*SI^6-285/32/ING+3095/128/ING*SI^2-4025/256/ING*SI^4+315$$
$$/16*E^2/ING^2-1375/128*E^2/ING^2*SI^2-26485/512*E^2/ING^2*SI^4+175/4*E^2/ING^2*SI^6-2135/64*E^2/ING+23565/256*E^2/ING*SI^2-15505/256*E^2$$
$$/ING*SI^4)$$
$$+ A2^2*E^2/L^7*SI^2*(117/8/ING^2-3277/64/ING^2*SI^2+15275/256/ING^2*SI^4-5925/256/ING^2*SI^6-191/32/ING+463/32/ING*SI^2-1125/128/ING*SI^4$$
$$+3283/64*E^2/ING^2-11283/64*E^2/ING^2*SI^2+25765/128*E^2/ING^2*SI^4-78175/1024*E^2/ING^2*SI^6-2409/128*E^2/ING+23591/512*E^2/ING*SI^2$$
$$-28955/1024*E^2/ING*SI^4)$$

COEFFICIENT OF COS(SG)

$$1/A2^2*A3*A4*E/L^5*SI*(5/ING-665/32/ING*SI^2+1085/64/ING*SI^4+75/64*E^2/ING^2*SI^2-325/128*E^2/ING^2*SI^4+175/128*E^2/ING^2*SI^6+625/32$$
$$*E^2/ING-4975/64*E^2/ING*SI^2+15925/256*E^2/ING*SI^4)$$
$$+ A3*E/L^5*SI*(-6/ING+533/32/ING*SI^2-345/32/ING*SI^4+65/64*E^2/ING^2*SI^2-35/16*E^2/ING^2*SI^4+75/64*E^2/ING^2*SI^6-469/32*E^2/ING+5131$$
$$/128*E^2/ING*SI^2-6545/256*E^2/ING*SI^4)$$

S_3^+

COEFFICIENT OF COS(3*SG)

$$1/A2^3*A3*A4^2*E^3/L^9*SI^3*(1375/32/ING^3-203875/768/ING^3*SI^2+143875/256/ING^3*SI^4-1539125/3072/ING^3*SI^6+165375/1024/ING^3*SI^8$$
$$+10525/768/ING^2-48225/1024/ING^2*SI^2+54775/1024/ING^2*SI^4-82075/4096/ING^2*SI^6)$$

$$+ 1/A2^3*A3^3*E^3/L^5*SI^3*(25/64/ING^2-25/32/ING^2*SI^2+25/64/ING^2*SI^4-305/1152/ING+305/1152/ING*SI^2+31,576)$$

$$+ 1/A2*A3*A4*E^3/L^9*SI^3*(-2725/96/ING^3+9775/128/ING^3*SI^2-35575/768/ING^3*SI^4+12225/512/ING^3*SI^6+11475/512/ING^3*SI^8+15755/576$$
$$/ING^2-1061875/9216/ING^2*SI^2+2834725/18432/ING^2*SI^4-812875/12288/ING^2*SI^6-22405/1536/ING+214505/6144/ING*SI^2-21385/1024/ING$$
$$*SI^4)$$

$$+ A2*A3*E^3/L^9*SI^3*(-455/8/ING^3+67695/256/ING^3*SI^2-14645/32/ING^3*SI^4+89525/256/ING^3*SI^6-6375/64/ING^3*SI^8-7933/768/ING^2+142279$$
$$/4608/ING^2*SI^2-557285/18432/ING^2*SI^4+118525/12288/ING^2*SI^6+10103/1536/ING-24985/1536/ING*SI^2+40315/4096/ING*SI^4)$$

COEFFICIENT OF SIN(2*SG)

$$1/A2^3*A4^3*E^2/L^{11}*SI^2*(375/4/ING^3-13375/16/ING^3*SI^2+181375/64/ING^3*SI^4-291375/64/ING^3*SI^6+447125/128/ING^3*SI^8-1586375/1536$$
$$/ING^3*SI^{10})$$

$$+ 1/A2^3*A3^2*A4*E^2/L^7*SI^2*(75/8/ING^2-6375/128/ING^2*SI^2+18925/256/ING^2*SI^4-8575/256/ING^2*SI^6+115/64/ING-1865/768/ING*SI^2+385$$
$$/1536/ING*SI^4)$$

$$+ 1/A2*A4^2*E^2/L^{11}*SI^2*(-1025/4/ING^3+28325/16/ING^3*SI^2-291875/64/ING^3*SI^4+2865575/512/ING^3*SI^6-421325/128/ING^3*SI^8+1549625$$
$$/2048/ING^3*SI^{10}-75/32/ING^2-5225/128/ING^2*SI^2+14425/64/ING^2*SI^4-338975/1024/ING^2*SI^6+305025/2048/ING^2*SI^8+4725/128/ING-79725$$
$$/512/ING*SI^2+440055/2048/ING*SI^4-24745/256/ING*SI^6)$$

$$+ 1/A2*A3^2*E^2/L^7*SI^2*(-105/8/ING^2+5775/128/ING^2*SI^2-13015/256/ING^2*SI^4+4825/256/ING^2*SI^6+635/192/ING-275/32/ING*SI^2+355/64$$
$$/ING*SI^4)$$

$$+ A2*A4*E^2/L^{11}*SI^2*(45/4/ING^3+6035/16/ING^3*SI^2-30035/16/ING^3*SI^4+13235/4/ING^3*SI^6-10467225/4096/ING^3*SI^8+6026125/8192/ING^3*SI$$
$$^{10}-1595/16/ING^2+33245/64/ING^2*SI^2-492185/512/ING^2*SI^4+195565/256/ING^2*SI^6-228375/1024/ING^2*SI^8+1285/64/ING-63925/512/ING*SI^2$$
$$+867503/4096/ING*SI^4-875665/8192/ING*SI^6)$$

$$+ A2^3*E^2/L^{11}*SI^2*(1053/4/ING^3-24633/16/ING^3*SI^2+172861/48/ING^3*SI^4-2156315/512/ING^3*SI^6+10085725/4096/ING^3*SI^8-2358375/4096$$
$$/ING^3*SI^{10}-1215/32/ING^2+22647/128/ING^2*SI^2-153269/512/ING^2*SI^4+454155/2048/ING^2*SI^6-15825/256/ING^2*SI^8-3815/512/ING+7951$$
$$/1024/ING*SI^2+52377/4096/ING*SI^4-210959/16384/ING*SI^6)$$

COEFFICIENT OF COS(SG)

$$1/A2^3*A3*A4^2*E/L^9*SI*(-50/ING^2+3325/8/ING^2*SI^2-616575/512/ING^2*SI^4+723275/512/ING^2*SI^6-3543925/6144/ING^2*SI^8+1125/32*E^2/ING^3$$
$$*SI^2-120375/512*E^2/ING^3*SI^4+16625/32*E^2/ING^3*SI^6-2925125/6144*E^2/ING^3*SI^8+961625/6144*E^2/ING^3*SI^{10}-2925/8*E^2/ING^2+392475$$
$$/128*E^2/ING^2*SI^2-71025/8*E^2/ING^2*SI^4+7975975/768*E^2/ING^2*SI^6-12986225/3072*E^2/ING^2*SI^8)$$

$$+ 1/A2^3*A3^3*E/L^5*SI*(35/32/ING*SI^2-35/32/ING*SI^4+1/6-35/48*SI^2-25/64*E^2/ING^2*SI^2+25/32*E^2/ING^2*SI^4-25/64*E^2/ING^2*SI^6-35/32*E^2$$
$$/ING+1055/128*E^2/ING*SI^2-915/128*E^2/ING*SI^4+47/48*E^2-575/192*E^2*SI^2)$$

$$+ 1/A2*A3*A4*E/L^{9}*SI*(150/ING^{2}-15825/16/ING^{2}*SI^{2}+574175/256/ING^{2}*SI^{4}-1640825/768/ING^{2}*SI^{6}+750575/1024/ING^{2}*SI^{8}+285/4/ING$$

$$-21705/64/ING*SI^{2}+125555/256/ING*SI^{4}-115605/512/ING*SI^{6}+75/32*E^{2}/ING^{3}*SI^{2}-13225/256*E^{2}/ING^{3}*SI^{4}+240175/153A*E^{2}/ING^{3}*SI^{6}$$

$$-258925/1536*E^{2}/ING^{3}*SI^{8}+7875/128*E^{2}/ING^{3}*SI^{10}+13825/16*E^{2}/ING^{2}-707445/128*E^{2}/ING^{2}*SI^{2}+12423585/1024*E^{2}/ING^{2}*SI^{4}-68666125$$

$$/6144*E^{2}/ING^{2}*SI^{6}+15171975/4096*E^{2}/ING^{2}*SI^{8}+25725/64*E^{2}/ING-934995/512*E^{2}/ING*SI^{2}+2583775/1024*E^{2}/ING*SI^{4}-569695/512*E^{2}/ING$$

$$*SI^{6})$$

$$+ A2*A3*E/L^{9}*SI*(-105/ING^{2}+8619/16/ING^{2}*SI^{2}-1519933/1536/ING^{2}*SI^{4}+407615/512/ING^{2}*SI^{6}-30375/128/ING^{2}*SI^{8}-99/4/ING+4329/64/ING$$

$$*SI^{2}-7535/128/ING*SI^{4}+7875/512/ING*SI^{6}-195/8*E^{2}/ING^{3}*SI^{2}+201475/1536*E^{2}/ING^{3}*SI^{4}-386335/1536*E^{2}/ING^{3}*SI^{6}+105975/512*E^{2}/ING^{3}$$

$$*SI^{8}-31875/512*E^{2}/ING^{3}*SI^{10}-7323/16*E^{2}/ING^{2}+51769/24*E^{2}/ING^{2}*SI^{2}-11443601/3072*E^{2}/ING^{2}*SI^{4}+573A015/2048*E^{2}/ING^{2}*SI^{6}-3178525$$

$$/4096*E^{2}/ING^{2}*SI^{8}-6593/64*E^{2}/ING+129189/512*E^{2}/ING*SI^{2}-366959/2048*E^{2}/ING*SI^{4}+108245/4096*E^{2}/ING*SI^{6})$$

A 4 Expressions of F^{**}

$$F^{**} = F_1^{**} + F_2^{**} + F_3^{**} + F_4^{**},$$

$$F_1^{**} = F_1^{*}$$

$$F_2^{**} = F_{2s}^{*}$$

$$F_3^{**}$$

$$1/A2*A4^{2}/L^{14}*(675/256*E^{2}/ING*SI^{4}-1575/256*E^{2}/ING*SI^{6}+3675/1024*E^{2}/ING*SI^{8}+3375/512*E^{4}/ING^{2}*SI^{4}-5625/256*E^{4}/ING^{2}*SI^{6}+49875$$

$$/2048*E^{4}/ING^{2}*SI^{8}-18375/2048*E^{4}/ING^{2}*SI^{10}-675/256*E^{4}/ING*SI^{2}+15525/512*E^{4}/ING*SI^{4}-15225/256*E^{4}/ING*SI^{6}+33075/1024*E^{4}/ING*SI^{8}$$

$$)$$

$$+ 1/A2*A3^{2}/L^{10}*(3/32*ING*SI^{2}-15/32*E^{2}*SI^{2}+15/32*E^{2}*SI^{4}-3/32*E^{2}*ING+39/64*E^{2}*ING*SI^{2}-105/64*E^{4}*SI^{2}+105/64*E^{4}*SI^{4}-21/64*E^{4}*ING$$

$$+441/256*E^{4}*ING*SI^{2})$$

$$+ A2*A4/L^{14}*(-45/16+495/32*SI^{2}-3105/128*SI^{4}+3045/256*SI^{6}+585/128*E^{2}/ING*SI^{4}-2715/256*E^{2}/ING*SI^{6}+1575/256*E^{2}/ING*SI^{8}-675/32*E^{2}$$

$$+7155/64*E^{2}*SI^{2}-5395/32*E^{2}*SI^{4}+20055/256*E^{2}*SI^{6}+2925/256*E^{4}/ING^{2}*SI^{4}-19425/512*E^{4}/ING^{2}*SI^{6}+10725/256*E^{4}/ING^{2}*SI^{8}-7875/512*E^{4}$$

$$/ING^{2}*SI^{10}-585/128*E^{4}/ING*SI^{2}+27135/512*E^{4}/ING*SI^{4}-106215/1024*E^{4}/ING*SI^{6}+114975/2048*E^{4}/ING*SI^{8}-5175/64*E^{4}+107325/256*E^{4}$$

$$*SI^{2}-311805/512*E^{4}*SI^{4}+565215/2048*E^{4}*SI^{6})$$

$$+ A2^{3}/L^{14}*(27/8-405/32*SI^{2}+2091/128*SI^{4}-111/16*SI^{6}+507/256*E^{2}/ING*SI^{4}-585/128*E^{2}/ING*SI^{6}+675/256*E^{2}/ING*SI^{8}+621/32*E^{2}-4647/64$$

$$*E^{2}*SI^{2}+1527/16*E^{2}*SI^{4}-42243/1024*E^{2}*SI^{6}+2535/512*E^{4}/ING^{2}*SI^{4}-8385/512*E^{4}/ING^{2}*SI^{6}+9225/512*E^{4}/ING^{2}*SI^{8}-3375/512*E^{4}/ING^{2}*SI$$

$$^{10}-507/256*E^{4}/ING*SI^{2}+741/32*E^{4}/ING*SI^{4}-46305/1024*E^{4}/ING*SI^{6}+24975/1024*E^{4}/ING*SI^{8}+8271/128*E^{4}-61959/256*E^{4}*SI^{2}+662529$$

$$/2048*E^{4}*SI^{4}-145707/1024*E^{4}*SI^{6})$$

$$F_4$$

$$1/A2^2*A4^3/L^{18}*(-3375/128*E^2/ING^2*SI^4+167625/1024*E^2/ING^2*SI^6-727125/2048*E^2/ING^2*SI^8+1341375/4096*E^2/ING^2*SI^{10}-900375/8192*E^2/ING^2*SI^{12})$$

$$+1/A2^2*A3^2*A4/L^{14}*(-15/16*SI^2+255/64*SI^4-105/32*SI^6-225/128*E^2/ING*SI^4+975/256*E^2/ING*SI^6-525/256*E^2/ING*SI^8+15/16*E^2-165/8*E^2*SI^2+1065/16*E^2*SI^4-12705/256*E^2*SI^6)$$

$$+A4^2/L^{18}*(225/128-225/16*SI^2+20475/512*SI^4-94675/2048*SI^6+152145/8192*SI^8+225/128*E^2/ING^2*SI^4+62475/1024*E^2/ING^2*SI^6-912675/4096*E^2/ING^2*SI^8+531825/2048*E^2/ING^2*SI^{10}-1635375/16384*E^2/ING^2*SI^{12}-12825/256*E^2/ING*SI^4+199125/1024*E^2/ING*SI^6-126525/512*E^2/ING*SI^8+422625/4096*E^2/ING*SI^{10}+675/32*E^2-19575/128*E^2*SI^2+795825/2048*E^2*SI^4-1668765/4096*E^2*SI^6+153615/1024*E^2*SI^8)$$

$$+A3^2/L^{14}*(9/8-135/16*SI^2+939/64*SI^4-975/128*SI^6-195/128*E^2/ING*SI^4+105/32*E^2/ING*SI^6-225/128*E^2/ING*SI^8+153/16*E^2-987/16*E^2*SI^2+25179/256*E^2*SI^4-6135/128*E^2*SI^6)$$

$$+A2^2*A4/L^{18}*(-135/8+7155/64*SI^2-66525/256*SI^4+65325/256*SI^6-23205/256*SI^8+7995/128*E^2/ING^2*SI^4-767825/1024*E^2/ING^2*SI^6+417315/1024*E^2/ING^2*SI^8-1145325/4096*E^2/ING^2*SI^{10}+291375/4096*E^2/ING^2*SI^{12}-9855/128*E^2/ING*SI^4+295455/1024*E^2/ING*SI^6-731235/2048*E^2/ING*SI^8+149625/1024*E^2/ING*SI^{10}-10935/64*E^2+143565/128*E^2*SI^2-675705/256*E^2*SI^4+10808025/4096*E^2*SI^6-7816305/8192*E^2*SI^8)$$

$$+A2^4/L^{18}*(1755/128-16389/256*SI^2+24189/256*SI^4-94863/2048*SI^6+9201/8192*SI^8+4563/128*E^2/ING^2*SI^4-169923/1024*E^2/ING^2*SI^6+1185585/4096*E^2/ING^2*SI^8-459225/2048*E^2/ING^2*SI^{10}+266625/4096*E^2/ING^2*SI^{12}-7449/256*E^2/ING*SI^4+6663/64*E^2/ING*SI^6-127215/1024*E^2/ING*SI^8+50625/1024*E^2/ING*SI^{10}+13959/128*E^2-16161/32*E^2*SI^2+1506957/2048*E^2*SI^4-2871795/8192*E^2*SI^6+1761/1024*E^2*SI^8)$$

APPENDIX B

EXPRESSIONS OF SHORT-PERIOD, LONG-PERIOD, AND SECULAR PERTURBATIONS

The notation used in the following printout is the same as that in Appendix A; refer to page A-1 for definitions.

B.1 Second-Order Short-Period Perturbations due to J_2^2

$$\frac{r^8}{J_2^2}\Delta_2(e\cos g) = E^4*(-3531/2048+96383/8192*SI^2-180455/16384*SI^4)\qquad COS\ (3*SG-M)$$

$$+E^2*(-7505/128+10041/256*SI^2-9837/512*SI^4-10141/128*E^2+161085/1024\qquad COS\ (SG-M)$$
$$*E^2*SI^2-154871/2048*E^2*SI^4)$$

$$+E^5*(-2607/2560+15381/2048*SI^2-36433/5120*SI^4)\qquad COS\ (3*SG-2*M)$$

$$+E^3*(-1833/256+319/32*SI^2-1075/512*SI^4-31*E^2+49863/1024*E^2*SI^2\qquad COS\ (SG-2*M)$$
$$-15151/1024*E^2*SI^4)$$

$$+E^4*(-7881/2048+2629/2048*SI^2+27701/8192*SI^4)\qquad COS\ (SG-3*M)$$

$$+E^5*(-10837/5120-4411/1024*SI^2+150897/20480*SI^4)\qquad COS\ (SG-4*M)$$

$$+E^3*(-109/128+545/512*SI^2-137/1024*SI^4-1365/512*E^2+1661/512*E^2*SI^2\qquad COS\ (3*SG)$$
$$-631/2048*E^2*SI^4)$$

$$+E*(-405/64+1147/64*SI^2-6479/512*SI^4-4725/128*E^2+3171/32*E^2*SI^2\qquad COS\ (SG)$$
$$-16999/256*E^2*SI^4-110113/1024*E^4+144851/512*E^4*SI^2-95135/512*E^4$$
$$*SI^4)$$

$$+E^5*SI^2*(-59229/512+16897/128*SI^2)\qquad COS\ (5*SG+10*M)$$

$$+E^{4}*SI^{2}*(-419483/8192+966741/16384*SI^{2}) \qquad COS\ (5*SG+9*M)$$

$$+E^{3}*(-10531/512*SI^{2}+24533/1024*SI^{4}+31593/2048*E^{2}+271491/2048*E^{2}*SI^{2} \qquad COS\ (5*SG+8*M)$$
$$-23405/128*E^{2}*SI^{4})$$

$$+E^{5}*(-514627/10240+1955935/2048*SI^{2}-24851673/20480*SI^{4}) \qquad COS\ (3*SG+8*M)$$

$$+E^{2}*(-3663/512*SI^{2}+8637/1024*SI^{4}+3663/512*E^{2}+50585/1024*E^{2}*SI^{2} \qquad COS\ (5*SG+7*M)$$
$$-291431/4096*E^{2}*SI^{4})$$

$$+E^{4}*(-54599/2048+3841207/8192*SI^{2}-9759763/16384*SI^{4}) \qquad COS\ (3*SG+7*M)$$

$$+E*(-251/128*SI^{2}+75/32*SI^{4}+753/256*E^{2}+4047/256*E^{2}*SI^{2}-6165/256*E^{2} \qquad COS\ (5*SG+6*M)$$
$$*SI^{4}-6963/512*E^{4}-76035/2048*E^{4}*SI^{2}+17631/256*E^{4}*SI^{4})$$

$$+E^{3}*(-3387/256+53447/256*SI^{2}-135827/512*SI^{4}-4043/128*E^{2}-1897493 \qquad COS\ (3*SG+6*M)$$
$$/2048*E^{2}*SI^{2}+1302077/1024*E^{2}*SI^{4})$$

$$-21/64*SI^{2}+51/128*SI^{4}+63/64*E^{2}+63/16*E^{2}*SI^{2}-3315/512*E^{2}*SI^{4}-2781 \qquad COS\ (5*SG+5*M)$$
$$/512*E^{4}-40317/4096*E^{4}*SI^{2}+175545/8192*E^{4}*SI^{4}$$

$$+E^{5}*(400941/2560-302529/1024*SI^{2}+156863/1280*SI^{4}) \qquad COS\ (SG+6*M)$$

$$+E^{2}*(-759/128+40605/512*SI^{2}-103293/1024*SI^{4}-9999/512*E^{2}-216449/512 \qquad COS\ (3*SG+5*M)$$
$$*E^{2}*SI^{2}+2399207/4096*E^{2}*SI^{4})$$

$$+E*(27/128+75/128*SI^{2}-69/64*SI^{4}-117/64*E^{2}-465/256*E^{2}*SI^{2}+681/128*E^{2} \qquad COS\ (5*SG+4*M)$$
$$*SI^{4}+2169/1024*E^{4}+1881/1024*E^{4}*SI^{2}-1571/256*E^{4}*SI^{4})$$

$$+E^{4}*(198871/2048-379137/2048*SI^{2}+639881/8192*SI^{4}) \qquad COS\ (SG+5*M)$$

```
+ E*(-285/128+2847/128*SI**2-3633/128*SI**4-1245/128*E**2-45735/256*E**2*SI**2          COS  (3*SG+4*M)
  +31797/128*E**2*SI**4+45573/1024*E**4+2637/1024*E**4*SI**2-41471/512*E**4*SI**4
  )

+E**3*(7413/128-3577/32*SI**2+12323/256*SI**4+26275/1024*E**2-50571/1024*E**2           COS  (5G+4*M)
  *SI**2+45269/2048*E**2*SI**4)

+E**2*(-27/64-63/512*SI**2+855/1024*SI**4+351/512*E**2+45/1024*E**2*SI**2-5049          COS  (5*SG+3*M)
  /4096*E**2*SI**4)

  -9/16+173/64*SI**2-449/128*SI**4-191/64*E**2-8519/128*E**2*SI**2+46525/512            COS  (3*SG+3*M)
  *E**2*SI**4+30277/1024*E**4-158251/4096*E**4*SI**2-5483/8192*E**4*SI**4

+E**2*(4147/128-16235/256*SI**2+14309/512*SI**4+883/32*E**2-52689/1024*E**2            COS  (5G+3*M)
  *SI**2+44167/2048*E**2*SI**4)

+E**3*(27/128-31/256*SI**2-11/64*SI**4+15/128*E**2-15/128*E**2*SI**2+1/1024*E**2      COS  (5*SG+2*M)
  *SI**4)

+E*(9/16-1167/64*SI**2+1443/64*SI**4+2409/128*E**2-8391/256*E**2*SI**2+5361           COS  (3*SG+2*M)
  /512*E**2*SI**4+487/16*E**4-35419/512*E**4*SI**2+2337/64*E**4*SI**4)

+E*(255/16-4041/128*SI**2+453/32*SI**4+5775/256*E**2-159/4*E**2*SI**2+7551            COS  (5G+2*M)
  /512*E**2*SI**4+23929/512*E**4-147703/2048*E**4*SI**2+19167/1024*E**4*SI**4)

+E**4*(-9/512+149/8192*SI**2+41/16384*SI**4)                                          COS  (5*SG+M)

+E**2*(819/128-6075/512*SI**2+4281/1024*SI**4+6005/512*E**2-2177/64*E**2*SI**2        COS  (3*SG+M)
  +89113/4096*E**2*SI**4)
```

$$+ 81/16-75/8*SI^2+231/64*SI^4+87/8*E^2-1269/128*E^2*SI^2-1077/256*E^2*SI^4 \qquad \text{COS} \quad (SG+M)$$

$$+22953/1024*E^4-63/512*E^4*SI^2-133863/4096*E^4*SI^4$$

$$\frac{L^8}{J_2^2}\Delta_2(e \sin g) = E^4*(-3531/2048+96377/8192*SI^2-180445/16384*SI^4) \qquad \text{SIN} \quad (3*SG-M)$$

$$+ E^2*(-2607/128+13485/256*SI^2-17163/512*SI^4-5401/64*E^2+220327/1024 \qquad \text{SIN} \quad (SG-M)$$

$$*E^2*SI^2-275921/2048*E^2*SI^4)$$

$$+ E^5*(-2607/2560+76881/10240*SI^2-7285/1024*SI^4) \qquad \text{SIN} \quad (3*SG-2*M)$$

$$+ E^3*(-2109/256+2971/128*SI^2-8023/512*SI^4-9055/256*E^2+24121/256*E^2 \qquad \text{SIN} \quad (SG-2*M)$$

$$*SI^2-62191/1024*E^2*SI^4)$$

$$+ E^4*(-10887/2048+34583/2048*SI^2-100213/8192*SI^4) \qquad \text{SIN} \quad (SG-3*M)$$

$$+ E^5*(-21113/5120+77861/5120*SI^2-48363/4096*SI^4) \qquad \text{SIN} \quad (SG-4*M)$$

$$+ E^3*(-109/128+545/512*SI^2-137/1024*SI^4-1365/512*E^2+1661/512*E^2*SI^2 \qquad \text{SIN} \quad (3*SG)$$

$$-631/2048*E^2*SI^4)$$

$$+ E*(-441/64+1307/64*SI^2-7535/512*SI^4-4911/128*E^2+26305/256*E^2*SI^2 \qquad \text{SIN} \quad (SG)$$

$$-35075/512*E^2*SI^4-112777/1024*E^4+146279/512*E^4*SI^2-94841/512*E^4$$

$$*SI^4)$$

$$+ E^5*SI^2*(-59229/512+16897/128*SI^2) \qquad \text{SIN} \quad (5*SG+10*M)$$

$$+ E^4*SI^2*(-419483/8192+966741/16384*SI^2) \qquad \text{SIN} \quad (5*SG+9*M)$$

$$+ E^3*(-10531/512*SI^2+24533/1024*SI^4+31593/2048*E^2+271491/2048*E^2*SI^2 \qquad \text{SIN} \quad (5*SG+8*M)$$

$$-23405/128*E^2*SI^4)$$

```
+ E**5*(-409317/10240+9634329/10240*SI**2-4869933/4096*SI**4)                    SIN  (3*SG+8*M)

+ E**2*(-3663/512*SI**2+8637/1024*SI**4+3663/512*E**2+50585/1024*E**2*SI**2      SIN  (5*SG+7*M)
  -291431/4096*E**2*SI**4)

+ E**4*(-44831/2048+3772465/8192*SI**2-9531233/16384*SI**4)                      SIN  (3*SG+7*M)

+ E*(-251/128*SI**2+75/32*SI**4+753/256*E**2+4047/256*E**2*SI**2-6165/256*E**2   SIN  (5*SG+6*M)
  *SI**4-6963/512*E**4-76035/2048*E**4*SI**2+17631/256*E**4*SI**4)

+ E**3*(-2885/256+52257/256*SI**2-132003/512*SI**4-10407/256*E**2-1879953        SIN  (3*SG+6*M)
  /2048*E**2*SI**2+1289037/1024*E**2*SI**4)

  -21/64*SI**2+51/128*SI**4+63/64*E**2+63/16*E**2*SI**2-3315/512*E**2*SI**4-2781 SIN  (5*SG+5*M)
  /512*E**4-40317/4096*E**4*SI**2+175545/8192*E**4*SI**4

+ E**5*(22889/2560-356579/1280*SI**2+108625/512*SI**4)                           SIN  (SG+6*M)

+ E**2*(-675/128+39411/512*SI**2-99567/1024*SI**4-11853/512*E**2-107119/256      SIN  (3*SG+5*M)
  *E**2*SI**2+2374501/4096*E**2*SI**4)

+ E*(27/128+75/128*SI**2-69/64*SI**4-117/64*E**2-465/256*E**2*SI**2+681/128*E**2 SIN  (5*SG+4*M)
  *SI**4+2169/1024*E**4+1881/1024*E**4*SI**2-1571/256*E**4*SI**4)

+ E**4*(122057/2048-374603/2048*SI**2+1126583/8192*SI**4)                        SIN  (SG+5*M)

+ E*(-267/128+2721/128*SI**2-3441/128*SI**4-1401/128*E**2-45219/256*E**2*SI**2  SIN  (3*SG+4*M)
  +31455/128*E**2*SI**4+47019/1024*E**4+2343/1024*E**4*SI**2-41389/512*E**4*SI**4
```

```
+ E**3*(4959/128-14935/128*SI**2+22133/256*SI**4+61879/1024*E**2-141475          SIN  (SG+4*M)
  /1024*E**2*SI**2+156449/2048*E**2*SI**4)

+ E**2*(-27/64-63/512*SI**2+855/1024*SI**4+351/512*E**2+45/1024*E**2*SI**2-5049  SIN  (5*SG+3*M)
  /4096*E**2*SI**4)

  -9/16+155/64*SI**2-395/128*SI**4-209/64*E**2-8411/128*E**2*SI**2+45967/512     SIN  (3*SG+3*M)
  *E**2*SI**4+30745/1024*E**4-158053/4096*E**4*SI**2-7193/8192*E**4*SI**4

+ E**2*(3077/128-18091/256*SI**2+26347/512*SI**4+6203/128*E**2-115779/1024       SIN  (SG+3*M)
  *E**2*SI**2+132369/2048*E**2*SI**4)

+ E**3*(27/128-31/256*SI**2-11/64*SI**4+15/128*E**2-15/128*E**2*SI**2+1/1024*E**2  SIN  (5*SG+2*M)
  *SI**4)

+ E*(9/16-1149/64*SI**2+1419/64*SI**4+2427/128*E**2-8373/256*E**2*SI**2+5277     SIN  (3*SG+2*M)
  /512*E**2*SI**4+1953/64*E**4-2211/32*E**4*SI**2+4653/128*E**4*SI**4)

+ E*(213/16-4845/128*SI**2+1725/64*SI**4+9003/256*E**2-10401/128*E**2*SI**2      SIN  (SG+2*M)
  +23571/512*E**2*SI**4+35693/512*E**4-325343/2048*E**4*SI**2+22365/256*E**4*SI**4
  )

+ E**4*(-9/512+149/8192*SI**2+41/16384*SI**4)                                    SIN  (5*SG+M)

+ E**2*(819/128-6093/512*SI**2+4323/1024*SI**4+5999/512*E**2-17437/512*E**2      SIN  (3*SG+M)
  *SI**2+89339/4096*E**2*SI**4)

+ 63/16-21/2*SI**2+465/64*SI**4+123/8*E**2-3975/128*E**2*SI**2+3657/256*E**2*SI**4  SIN  (SG+M)
  +34863/1024*E**4-34821/512*E**4*SI**2+118983/4096*E**4*SI**4
```

$$\frac{L^8}{J_2^2}\Delta_2(\ell+g) = E^5*(-3/10240+7/40960*SI^2-481/163840*SI^4) \qquad SIN\ (4*SG-M)$$

$$+E^3*(-435/128+6613/256*SI^2-6291/256*SI^4-27111/2048*E^2+795743/8192 \qquad SIN\ (2*SG-M)$$
$$*E^2*SI^2-1507953/16384*E^2*SI^4)$$

$$+E^4*(-135/64+8925/512*SI^2-17079/1024*SI^4) \qquad SIN\ (2*SG-2*M)$$

$$+E^5*(-18137/10240+64573/4096*SI^2-309633/20480*SI^4) \qquad SIN\ (2*SG-3*M)$$

$$-9/4096*E^4*SI^4 \qquad SIN\ (4*SG)$$

$$+E^5*(634449/10240-4943451/40960*SI^2+1894533/32768*SI^4) \qquad SIN\ (5*M)$$

$$+E^4*(5963/128-23497/256*SI^2+89899/2048*SI^4) \qquad SIN\ (4*M)$$

$$+E^2*(-55/64+163/128*SI^2-105/256*SI^4-689/256*E^2+785/256*E^2*SI^2-145 \qquad SIN\ (2*SG)$$
$$/1024*E^2*SI^4)$$

$$+E^3*(4675/128-37401/512*SI^2+72123/2048*SI^4+169959/2048*E^2-1381245 \qquad SIN\ (3*M)$$
$$/8192*E^2*SI^2+2676531/32768*E^2*SI^4)$$

$$+E^2*(999/32-4077/64*SI^2+16047/512*SI^4+11105/128*E^2-45481/256*E^2*SI^2 \qquad SIN\ (2*M)$$
$$+44593/512*E^2*SI^4)$$

$$+E*(543/16-4557/64*SI^2+9303/256*SI^4+14739/128*E^2-122325/512*E^2*SI^2 \qquad SIN\ (M)$$
$$+244461/2048*E^2*SI^4+272121/1024*E^4-2236227/4096*E^4*SI^2+4389747$$
$$/16384*E^4*SI^4)$$

$$+E^5*(419483/10240-34093/10240*SI^2-2619493/20480*SI^4) \qquad SIN\ (4*SG+0*M)$$

$$+E^4*(10531/512-1967/1024*SI^2-31201/512*SI^4) \qquad SIN\ (4*SG+8*M)$$

```
+E**3*(1221/128-131/128*SI**2-27241/1024*SI**4-83903/2048*E**2+44851/8192          SIN  (4*SG+7*M)
  *E**2*SI**2+4014439/32768*E**2*SI**4)

+E**5*(-1595203/10240+5149807/8192*SI**2-49345893/81920*SI**4)                     SIN  (2*SG+7*M)

+E**2*(251/64-31/64*SI**2-10439/1024*SI**4-2321/128*E**2+719/256*E**2*SI**2       SIN  (4*SG+6*M)
  +3201/64*E**2*SI**4)

+E**4*(-177/2+21317/64*SI**2-9993/32*SI**4)                                        SIN  (2*SG+6*M)

+E*(21/16-3/16*SI**2-201/64*SI**4-927/128*E**2+669/512*E**2*SI**2+36897/2048      SIN  (4*SG+5*M)
  *E**2*SI**4+8349/1024*E**4-3029/2048*E**4*SI**2-207343/8192*E**4*SI**4)

+E**3*(-5949/128+82339/512*SI**2-150705/1024*SI**4+221019/2048*E**2                SIN  (2*SG+5*M)
  -2594767/4096*E**2*SI**2+2636283/4096*E**2*SI**4)

+9/32-3/64*SI**2-39/64*SI**4-39/16*E**2+33/64*E**2*SI**2+339/64*E**2*SI**4+723    SIN  (4*SG+4*M)
  /256*E**4-315/512*E**4*SI**2-1989/256*E**4*SI**4

+E**2*(-1353/64+8319/128*SI**2-14823/256*SI**4+8463/128*E**2-185091/512*E**2       SIN  (2*SG+4*M)
  *SI**2+364173/1024*E**2*SI**4)

+E*(-9/16+9/64*SI**2+261/256*SI**4+117/128*E**2-63/256*E**2*SI**2-513/256*E**2    SIN  (4*SG+3*M)
  *SI**4+171/1024*E**4-351/4096*E**4*SI**2+5175/16384*E**4*SI**4)

+E*(-109/16+1025/64*SI**2-1775/128*SI**4+5389/128*E**2-53641/256*E**2*SI**2        SIN  (2*SG+3*M)
  +50597/256*E**2*SI**4+59679/1024*E**4-887819/4096*E**4*SI**2+1431045/8192*E**4
  *SI**4)

+E**2*(9/32-3/32*SI**2-417/1024*SI**4+5/32*E**2-1/16*E**2*SI**2-35/512*E**2*SI**4)  SIN  (4*SG+2*M)
```

$$-15/4*SI^2+99/32*SI^4+423/16*E^2-7593/64*E^2*SI^2+6831/64*E^2*SI^4+2799 \qquad \text{SIN} \quad (2*SG+2*M)$$

$$/64*E^4-86691/512*E^4*SI^2+142785/1024*E^4*SI^4$$

$$+E^3*(-3/128+5/512*SI^2+47/2048*SI^4-73/2048*E^2+1/64*E^2*SI^2+185/8192 \qquad \text{SIN} \quad (4*SG+M)$$

$$*E^2*SI^4)$$

$$+E*(153/16-1377/32*SI^2+1233/32*SI^4+2427/128*E^2-45021/512*E^2*SI^2 \qquad \text{SIN} \quad (2*SG+M)$$

$$+78399/1024*E^2*SI^4+33661/1024*E^4-355327/2048*E^4*SI^2+316359/2048*E^4$$

$$*SI^4)$$

$$\frac{L^8}{J_2^2}\Delta_2 h = CI*E^5*(3/10240-1/40960*SI^2) \qquad \text{SIN} \quad (4*SG-M)$$

$$+CI*E^3*(435/128-123/16*SI^2+27111/2048*E^2-122085/4096*E^2*SI^2) \qquad \text{SIN} \quad (2*SG-M)$$

$$+CI*E^4*(135/64-2529/512*SI^2) \qquad \text{SIN} \quad (2*SG-2*M)$$

$$+CI*E^5*(18137/10240-43731/10240*SI^2) \qquad \text{SIN} \quad (2*SG-3*M)$$

$$+CI*E^5*(-40183/2560+110003/8192*SI^2) \qquad \text{SIN} \quad (5*M)$$

$$+CI*E^4*(-1631/128+2863/256*SI^2) \qquad \text{SIN} \quad (4*M)$$

$$+CI*E^2*(55/64+689/256*E^2) \qquad \text{SIN} \quad (2*SG)$$

$$+CI*E^3*(-693/64+5037/512*SI^2-14601/512*E^2+218361/8192*E^2*SI^2) \qquad \text{SIN} \quad (3*M)$$

$$+CI*E^2*(-645/64+1227/128*SI^2-3905/128*E^2+1873/64*E^2*SI^2) \qquad \text{SIN} \quad (2*M)$$

$$+CI*E*(-12+777/64*SI^2-2685/64*E^2+21345/512*E^2*SI^2-24943/256*E^4 \qquad \text{SIN} \quad (M)$$

$$+389911/4096*E^4*SI^2)$$

```
+ CI*E**5*(-419483/10240-351297/20480*SI**2)                          SIN  (4*SG+9*M)

+ CI*E**4*(-10531/512-2141/256*SI**2)                                 SIN  (4*SG+8*M)

+ CI*E**3*(-1221/128-959/256*SI**2+83903/2048*E**2+122955/8192*E**2*SI**2)   SIN  (4*SG+7*M)

+ CI*E**5*(1595203/10240-3440013/20480*SI**2)                         SIN  (2*SG+7*M)

+ CI*E**2*(-251/64-189/128*SI**2+2321/128*E**2+801/128*E**2*SI**2)    SIN  (4*SG+6*M)

+ CI*E**4*(177/2-48047/512*SI**2)                                     SIN  (2*SG+6*M)

+ CI*E*(-21/16-15/32*SI**2+927/128*E**2+1185/512*E**2*SI**2-8349/1024*E**4   SIN  (4*SG+5*M)
  -665/256*E**4*SI**2)

+ CI*E**3*(5949/128-12279/256*SI**2-221019/2048*E**2+291285/2048*E**2*SI**2)  SIN  (2*SG+5*M)

+ CI*(-9/32-3/32*SI**2+39/16*E**2+45/64*E**2*SI**2-723/256*E**4-51/64*E**4*SI**2)  SIN  (4*SG+4*M)

+ CI*E**2*(1353/64-1323/64*SI**2-8463/128*E**2+11439/128*E**2*SI**2)  SIN  (2*SG+4*M)

+ CI*E*(9/16+9/64*SI**2-117/128*E**2-27/128*E**2*SI**2-171/1024*E**4+9/4096*E**4   SIN  (4*SG+3*M)
  *SI**2)

+ CI*E*(109/16-179/32*SI**2-5389/128*E**2+3707/64*E**2*SI**2-59679/1024*E**4   SIN  (2*SG+3*M)
  +172891/2048*E**4*SI**2)

+ CI*E**2*(-9/32-3/64*SI**2-5/32*E**2-1/64*E**2*SI**2)                SIN  (4*SG+2*M)

+ CI*(9/8*SI**2-423/16*E**2+2349/64*E**2*SI**2-2799/64*E**4+4113/64*E**4*SI**2)   SIN  (2*SG+2*M)

+ CI*E**3*(3/128+1/512*SI**2+73/2048*E**2+9/4096*E**2*SI**2)           SIN  (4*SG+M)
```

$$+ CI*E*(-153/16+225/16*SI^2-2427/128*E^2+8307/256*E^2*SI^2-33661/1024*E^4 \quad SIN \quad (2*SG+M)$$
$$+65289/1024*E^4*SI^2)$$

$$\frac{L^7}{J_2^2}\Delta_2 l = 587/16384*E^5*SI^4 \quad COS \quad (4*SG-M)$$

$$+E^3*SI^2*(-121/128+15/64*SI^2-3381/4096*E^2-22011/8192*E^2*SI^2) \quad COS \quad (2*SG-M)$$

$$+E^4*SI^2*(-99/128-201/512*SI^2) \quad COS \quad (2*SG-2*M)$$

$$+E^5*SI^2*(-2259/5120-28539/20480*SI^2) \quad COS \quad (2*SG-3*M)$$

$$+27/1024*E^4*SI^4 \quad COS \quad (4*SG)$$

$$+E^5*(74285/1024-11243/64*SI^2+2341411/16384*SI^4) \quad COS \quad (5*M)$$

$$+E^4*(1413/32-851/8*SI^2+21683/256*SI^4) \quad COS \quad (4*M)$$

$$+E^2*SI^2*(15/32-9/8*SI^2+225/64*E^2-867/128*E^2*SI^2) \quad COS \quad (2*SG)$$

$$+E^3*(1671/64-4005/64*SI^2+49779/1024*SI^4+65349/1024*E^2-20475/128*E^2 \quad COS \quad (3*M)$$
$$*SI^2+2205063/16384*E^2*SI^4)$$

$$+E^2*(237/16-1131/32*SI^2+1707/64*SI^4+1389/32*E^2-431/4*E^2*SI^2+22627 \quad COS \quad (2*M)$$
$$/256*E^2*SI^4)$$

$$+E*(63/8-75/4*SI^2+1749/128*SI^4+1791/64*E^2-4407/64*E^2*SI^2+56145 \quad COS \quad (M)$$
$$/1024*E^2*SI^4+34655/512*E^4-21839/128*E^4*SI^2+1179919/8192*E^4*SI^4)$$

$$+809433/4096*E^5*SI^4 \quad COS \quad (4*SG+Q*M)$$

```
+ 78973/1024*E**4*SI**4                                                    COS  (4*SG+8*M)

+ E**3*SI**4*(859/32-2286685/16384*E**2)                                   COS  (4*SG+7*M)

+ E**5*SI**2*(13983651/20480-39928347/40960*SI**2)                         COS  (2*SG+7*M)

+ E**2*SI**4*(2025/256-5535/128*E**2)                                      COS  (4*SG+6*M)

+ E**4*SI**2*(42243/128-242427/512*SI**2)                                  COS  (2*SG+6*M)

+ E*SI**4*(57/32-11349/1024*E**2+86557/4096*E**4)                          COS  (4*SG+5*M)

+ E**3*SI**2*(37115/256-107427/512*SI**2-337155/1024*E**2+1739793/4096*E**2
  *SI**2)                                                                  COS  (2*SG+5*M)

+ SI**4*(15/64-267/128*E**2+573/128*E**4)                                  COS  (4*SG+4*M)

+ E**2*SI**2*(873/16-5139/64*SI**2-2385/16*E**2+12003/64*E**2*SI**2)       COS  (2*SG+4*M)

+ E*SI**4*(-27/128+351/512*E**2-1647/8192*E**4)                            COS  (4*SG+3*M)

+ E*SI**2*(489/32-1497/64*SI**2-8373/128*E**2+2571/32*E**2*SI**2-92925/2048*E**4
  +198585/4096*E**4*SI**2)                                                 COS  (2*SG+3*M)

+ E**2*SI**4*(15/256+11/256*E**2)                                          COS  (4*SG+2*M)

+ SI**2*(15/8-27/8*SI**2-837/32*E**2+2025/64*E**2*SI**2-861/32*E**4+1881/64*E**4
  *SI**2)                                                                  COS  (2*SG+2*M)

+ E**3*SI**4*(19/1024+47/512*E**2)                                         COS  (4*SG+M)
```

$$+ E*SI^2*(-27/4+261/32*SI^2-2295/256*E^2+4779/512*E^2*SI^2-4323/512*E^4 \quad \cos(2*SG+M)$$
$$+10287/2048*E^4*SI^2)$$

$$+ 9/8-45/16*SI^2+141/64*SI^4+45/8*E^2-465/32*E^2*SI^2+405/32*E^2*SI^4+525 \quad \cos(0)$$
$$/32*E^4-5565/128*E^4*SI^2+5145/128*E^4*SI^4$$

B.2 Third-Order Short-Period Perturbations due to J_2^3

$$\frac{L^{12}}{J_2^3}\Delta_3(e\cos g) = E^2*(-101661/1024+3272847/8192*SI^2-2244285/4096*SI^4+8062407/32768 \quad \cos(SG-M)$$
$$*SI^6)$$

$$+ E^3*(-4719/256+60877/512*SI^2-1830619/8192*SI^4+2007615/16384*SI^6) \quad \cos(SG-2*M)$$

$$+ E^3*(-204653/4096+884803/4096*SI^2-5094845/16384*SI^4+14512915/98304 \quad \cos(3*SG)$$
$$*SI^6)$$

$$+ E*(-1431/32+339305/2048*SI^2-408905/2048*SI^4+649501/8192*SI^6 \quad \cos(SG)$$
$$-1405013/4096*E^2+5171899/4096*E^2*SI^2-24808707/16384*E^2*SI^4$$
$$+19533115/32768*E^2*SI^6)$$

$$+ E^3*SI^2*(13797/512-192551/8192*SI^2-77401/8192*SI^4) \quad \cos(7*SG+10*M)$$

$$+ E^2*SI^2*(60477/8192-26809/4096*SI^2-79903/32768*SI^4) \quad \cos(7*SG+9*M)$$

$$+ E*(3141/2048*SI^2-2835/2048*SI^4-483/1024*SI^6-15705/4096*E^2-90549 \quad \cos(7*SG+R*M)$$
$$/4096*E^2*SI^2+12795/512*E^2*SI^4+3507/512*E^2*SI^6)$$

$$+ E^3*(91719/4096-1156757/4096*SI^2+1298749/2048*SI^4-827779/2048*SI^6) \quad \cos(5*SG+R*M)$$

```
+189/1024*SI**2-87/512*SI**4-213/4096*SI**6-945/1024*E**2-8955/2048*E**2*SI**2          COS (7*SG+7*M)
  +669/128*E**2*SI**4+1257/1024*E**2*SI**6

+E**2*(15381/2048-634167/8192*SI**2+9794391/57344*SI**4-25392921/229376                 COS (5*SG+7*M)
  *SI**6)

+E*(-135/1024-513/1024*SI**2+657/1024*SI**4+63/512*SI**6+675/256*E**2                   COS (7*SG+6*M)
  +15039/4096*E**2*SI**2-13959/2048*E**2*SI**4-3807/4096*E**2*SI**6)

+E*(2025/1024-15059/1024*SI**2+31791/1024*SI**4-21249/1024*SI**6+5955                   COS (5*SG+6*M)
  /512*E**2+1251207/4096*E**2*SI**2-3576201/4096*E**2*SI**4+4809669/8192*E**2
  *SI**6)

+E**3*(-40145/256+3934633/2048*SI**2-33813705/8192*SI**4+40690117/16384                 COS (3*SG+6*M)
  *SI**6)

+E**2*(405/1024+2835/8192*SI**2-3405/4096*SI**4-2835/32768*SI**6)                       COS (7*SG+5*M)

+81/256-1209/1024*SI**2+10491/5120*SI**4-6267/4096*SI**6+1035/1024*E**2                 COS (5*SG+5*M)
  +788181/10240*E**2*SI**2-284313/1280*E**2*SI**4+627285/4096*E**2*SI**6

+E**2*(-115743/2048+25554393/40960*SI**2-53795841/40960*SI**4+25709787                  COS (3*SG+5*M)
  /32768*SI**6)

+E**3*(-405/1024+117/2048*SI**2+881/2048*SI**4-1/64*SI**6)                              COS (7*SG+4*M)

+E*(-81/128+6177/512*SI**2-34965/1024*SI**4+6243/256*SI**6-36675/1024*E**2              COS (5*SG+4*M)
  -99/1024*E**2*SI**2+635715/8192*E**2*SI**4-548931/16384*E**2*SI**6)
```

```
+ E*(-1923/128+143967/1024*SI**2-73299/256*SI**4+1390977/8192*SI**6+61653          COS  (3*SG+4*M)
  /2048*E**2-231819/128*E**2*SI**2+72018609/16384*E**2*SI**4-44835489/16384*E**2
  *SI**6)

+ E**3*(815337/2048-2727721/2048*SI**2+26070443/16384*SI**4-5289351/8192           COS  (SG+4*M)
  *SI**6)

+ E**2*(-12231/2048+9679/8192*SI**2+124977/8192*SI**4-309315/32768*SI**6)          COS  (5*SG+3*M)

  -459/256+13397/1024*SI**2-26437/1024*SI**4+64591/4096*SI**6+28453/1024           COS  (3*SG+3*M)
  *E**2-1232385/2048*E**2*SI**2+1398121/1024*E**2*SI**4-3378417/4096*E**2*SI**6

+ E**2*(51137/256-5549135/8192*SI**2+3333259/4096*SI**4-10786333/32768*SI**6       COS  (SG+3*M)
  )

+ E**3*(1791/256-21167/2048*SI**2+304721/8192*SI**4-80417/2048*SI**6)              COS  (5*SG+2*M)

+ E*(10611/1024-102189/1024*SI**2+819621/4096*SI**4-236097/2048*SI**6              COS  (3*SG+2*M)
  +78447/512*E**2-309369/4096*E**2*SI**2-2201055/4096*E**2*SI**4+8029233
  /16384*E**2*SI**6)

+ E*(84963/1024-292167/1024*SI**2+1396197/4096*SI**4-68769/512*SI**6               COS  (SG+2*M)
  +51039/256*E**2-3404661/4096*E**2*SI**2+4901607/4096*E**2*SI**4-9046665
  /16384*E**2*SI**6)

+ E**2*(23961/2048+1275795/8192*SI**2-3550389/8192*SI**4+9027063/32768*SI**6       COS  (3*SG+M)
  )

+ 2349/128-67929/1024*SI**2+21309/256*SI**4-137727/4096*SI**6+40929/1024           COS  (SG+M)
  *E**2-525045/2048*E**2*SI**2+494577/1024*E**2*SI**4-132759/512*E**2*SI**6
```

$$\frac{L^{12}}{J_2^3}\Delta_3(e\sin g) = E^2*(-91389/1024+2164905/8192*SI^2-1035531/4096*SI^4+2303577/32768*SI^6) \quad SIN\ (SG-M)$$

$$+ E^3*(-2235/256+7003/2048*SI^2+239989/8192*SI^4-496561/16384*SI^6) \quad SIN\ (SG-2*M)$$

$$+ E^3*(-204473/4096+880537/4096*SI^2-15090143/49152*SI^4+14198011/98304*SI^6) \quad SIN\ (3*SG)$$

$$+ E*(-1215/32+250687/2048*SI^2-250215/2048*SI^4+308811/8192*SI^6 \quad SIN\ (SG)$$
$$-1299347/4096*E^2+3986665/4096*E^2*SI^2-15143357/16384*E^2*SI^4$$
$$+8638517/32768*E^2*SI^6)$$

$$+ E^3*SI^2*(13797/512-192551/8192*SI^2-77401/8192*SI^4) \quad SIN\ (7*SG+10*M)$$

$$+ E^2*SI^2*(60477/8192-26809/4096*SI^2-79903/32768*SI^4) \quad SIN\ (7*SG+9*M)$$

$$+ E*(3141/2048*SI^2-2835/2048*SI^4-483/1024*SI^6-15705/4096*E^2-90549 \quad SIN\ (7*SG+8*M)$$
$$/4096*E^2*SI^2+12795/512*E^2*SI^4+3507/512*E^2*SI^6)$$

$$+ E^3*(85437/4096-1143413/4096*SI^2+82369/128*SI^4-857985/2048*SI^6) \quad SIN\ (5*SG+8*M)$$

$$+ 189/1024*SI^2-87/512*SI^4-213/4096*SI^6-945/1024*E^2-8955/2048*E^2*SI^2 \quad SIN\ (7*SG+7*M)$$
$$+669/128*E^2*SI^4+1257/1024*E^2*SI^6$$

$$+ E^2*(14625/2048-624765/8192*SI^2+9978099/57344*SI^4-26563587/229376 \quad SIN\ (5*SG+7*M)$$
$$*SI^6)$$

$$+ E*(-135/1024-513/1024*SI^2+657/1024*SI^4+63/512*SI^6+675/256*E^2 \quad SIN\ (7*SG+6*M)$$
$$+15039/4096*E^2*SI^2-13959/2048*E^2*SI^4-3807/4096*E^2*SI^6)$$

```
+ E*(1971/1024-14735/1024*SI**2+32673/1024*SI**4-22671/1024*SI**6+6495          SIN  (5*SG+6*M)
  /512*E**2+1242207/4096*E**2*SI**2-3598017/4096*E**2*SI**4+4876593/8192*E**2
  *SI**6)

+ E**3*(-35595/256+1845369/1024*SI**2-31900973/8192*SI**4+38250885/16384       SIN  (3*SG+6*M)
  *SI**6)

+ E**2*(405/1024+2835/8192*SI**2-3405/4096*SI**4-2235/32768*SI**6)             SIN  (7*SG+5*M)

+ 81/256-1155/1024*SI**2+11211/5120*SI**4-7209/4096*SI**6+1197/1024*E**2       SIN  (5*SG+5*M)
  +781971/10240*E**2*SI**2-286143/1280*E**2*SI**4+636477/4096*E**2*SI**6

+ E**2*(-105729/2048+23810181/40960*SI**2-50257707/40960*SI**4+23929767        SIN  (3*SG+5*M)
  /32768*SI**6)

+ E**3*(-405/1024+117/2048*SI**2+881/2048*SI**4-1/64*SI**6)                    SIN  (7*SG+4*M)

+ E*(-81/128+6123/512*SI**2-35217/1024*SI**4+6345/256*SI**6-36837/1024*E**2    SIN  (5*SG+4*M)
  +99/512*E**2*SI**2+640155/8192*E**2*SI**4-562083/16384*E**2*SI**6)

+ E*(-1797/128+132411/1024*SI**2-134841/512*SI**4+1275681/8192*SI**6           SIN  (3*SG+4*M)
  +25425/2048*E**2-893919/512*E**2*SI**2+70101849/16384*E**2*SI**4-43488213
  /16384*E**2*SI**6)

+ E**3*(573015/2048-2391511/2048*SI**2+28833229/16384*SI**4-7213081/8192        SIN  (SG+4*M)
  *SI**6)

+ E**2*(-12231/2048+10165/8192*SI**2+125949/8192*SI**4-315633/32768*SI**6)     SIN  (5*SG+3*M)

  -405/256+11921/1024*SI**2-24123/1024*SI**4+59635/4096*SI**6+25069/1024       SIN  (3*SG+3*M)
  *E**2-1181561/2048*E**2*SI**2+1344377/1024*E**2*SI**4-3235863/4096*E**2*SI**6
```

$$+ E^2*(77375/512-5052041/8192*SI^2+3702949/4096*SI^4-14438531/32768*SI^6 \quad SIN\ (SG+3*M)$$

$$)$$

$$+ E^3*(1791/256-21185/2048*SI^2+304601/8192*SI^4-80367/2048*SI^6) \quad SIN\ (5*SG+2*M)$$

$$+ E*(10503/1024-96531/1024*SI^2+769389/4096*SI^4-220845/2048*SI^6 \quad SIN\ (3*SG+2*M)$$

$$+80049/512*E^2-202413/4096*E^2*SI^2-1153317/2048*E^2*SI^4+7879293$$

$$/16384*E^2*SI^6)$$

$$+ E*(69189/1024-271701/1024*SI^2+1519335/4096*SI^4-352839/2048*SI^6 \quad SIN\ (5G+2*M)$$

$$+60411/256*E^2-3141609/4096*E^2*SI^2+232623/256*E^2*SI^4-5862405/16384$$

$$*E^2*SI^6)$$

$$+ E^2*(24051/2048+1305351/8192*SI^2-3513519/8192*SI^4+8699187/32768*SI^6 \quad SIN\ (3*SG+M)$$

$$)$$

$$+ 1917/128-57327/1024*SI^2+9789/128*SI^4-141369/4096*SI^6+46383/1024*E^2 \quad SIN\ (SG+M)$$

$$-172491/2048*E^2*SI^2+60603/1024*E^2*SI^4-3045/512*E^2*SI^6$$

$$\frac{L^{12}}{J_2^3}\Delta_3(\ell+g) = E^3*(-6465/1024-524963/4096*SI^2+1375237/4096*SI^4-6813561/32768*SI^6 \quad SIN\ (2*SG-M)$$

$$)$$

$$+ E^2*(-22793/512+190187/1024*SI^2-2183813/8192*SI^4+526185/4096*SI^6) \quad SIN\ (2*SG)$$

$$+ E^3*(2088123/1024-17296009/2048*SI^2+20734303/16384*SI^4-20369749 \quad SIN\ (3*M)$$

$$/32768*SI^6)$$

$$+ E^2*(21255/128-85473/128*SI^2+1959309/2048*SI^4-921519/2048*SI^6) \quad SIN\ (2*M)$$

```
+ E*(21417/128-80691/128*SI**2+1704873/2048*SI**4-1479315/4096*SI**6                SIN  (M)
  +950205/1024*E**2-7185321/2048*E**2*SI**2+76756509/16384*E**2*SI**4
  -67387131/32768*E**2*SI**6)

+ E**3*(-20159/1536+5039/2048*SI**2+27567/16384*SI**4+1388583/65536*SI**6)          SIN  (6*SG+9*M)

+ E**2*(-1047/256+861/1024*SI**2+267/512*SI**4+50907/8192*SI**6)                   SIN  (6*SG+8*M)

+ E*(-63/64+57/256*SI**2+255/2048*SI**4+11445/8192*SI**6+5883/512*E**2-3189        SIN  (6*SG+7*M)
  /1024*E**2*SI**2-23979/16384*E**2*SI**4-1018497/65536*E**2*SI**6)

+ E**3*(47907/512-2231891/14336*SI**2-929099/16384*SI**4+41813809/229376           SIN  (4*SG+7*M)
  *SI**6)

  -9/64+9/256*SI**2+9/512*SI**4+189/1024*SI**6+45/16*E**2-27/32*E**2*SI**2-45     SIN  (6*SG+6*M)
  /128*E**2*SI**4-28035/8192*E**2*SI**6

+ E**2*(7111/256-12615/256*SI**2+3819/2048*SI**4+79555/2048*SI**6)                 SIN  (4*SG+6*M)

+ E*(27/64-9/64*SI**2-105/2048*SI**4-3711/8192*SI**6-1287/512*E**2+1911           SIN  (6*SG+5*M)
  /2048*E**2*SI**2+4875/16384*E**2*SI**4+172281/65536*E**2*SI**6)

+ E*(327/64-13551/1280*SI**2+91581/10240*SI**4+2895/4096*SI**6-68811/512           SIN  (4*SG+5*M)
  *E**2+52737/256*E**2*SI**2+19221903/81920*E**2*SI**4-12201153/32768*E**2*SI**6)

+ E**3*(-1542891/5120+5877149/4096*SI**2-46290221/20480*SI**4+38608515             SIN  (2*SG+5*M)
  /32768*SI**6)

+ E**2*(-27/64+45/256*SI**2+3/64*SI**4+2949/8192*SI**6)                            SIN  (6*SG+4*M)
```

$$-87/128*SI^2+1869/512*SI^4-651/256*SI^6-2667/64*E^2+35607/512*E^2*SI^2 \qquad \text{SIN (4*SG+4*M)}$$

$$+48105/1024*E^2*SI^4-188847/2048*E^2*SI^6$$

$$+E^2*(-14661/128+270387/512*SI^2-208509/256*SI^4+3432525/8192*SI^6) \qquad \text{SIN (2*SG+4*M)}$$

$$+E^3*(81/512-81/1024*SI^2-243/16384*SI^4-6867/65536*SI^6) \qquad \text{SIN (6*SG+3*M)}$$

$$+E*(-459/64+1857/128*SI^2-2325/2048*SI^4-35565/4096*SI^6+10985/512*E^2 \qquad \text{SIN (4*SG+3*M)}$$

$$+18763/1024*E^2*SI^2-3826171/16384*E^2*SI^4+7114431/32768*E^2*SI^6)$$

$$+E*(-3653/128+69185/512*SI^2-108453/512*SI^4+451333/4096*SI^6+333869 \qquad \text{SIN (2*SG+3*M)}$$

$$/1024*E^2-6151813/4096*E^2*SI^2+18519345/8192*E^2*SI^4-9196551/8192*E^2$$

$$*SI^6)$$

$$+E^2*(2001/256-1305/256*SI^2-4635/128*SI^4+9771/256*SI^6) \qquad \text{SIN (4*SG+2*M)}$$

$$-135/64+5601/256*SI^2-22899/512*SI^4+26721/1024*SI^6+4677/32*E^2 \qquad \text{SIN (2*SG+2*M)}$$

$$-265431/512*E^2*SI^2+646419/1024*E^2*SI^4-2184471/8192*E^2*SI^6$$

$$+E^3*(975/512-50743/2048*SI^2+1438511/16384*SI^4-2304997/32768*SI^6) \qquad \text{SIN (4*SG+M)}$$

$$+E*(1881/128+30207/512*SI^2-204207/1024*SI^4+130929/1024*SI^6-98829 \qquad \text{SIN (2*SG+M)}$$

$$/1024*E^2+5297655/4096*E^2*SI^2-22036071/8192*E^2*SI^4+12529017/8192*E^2$$

$$*SI^6)$$

$$\frac{r^{12}}{\mu^3}\Delta_3 h = CI*E^3*(6465/1024+91941/2048*SI^2-131005/2048*SI^4) \qquad \text{SIN (2*SG-M)}$$

$$+CI*E^2*(22793/512-92029/1024*SI^2+384981/8192*SI^4) \qquad \text{SIN (2*SG)}$$

```
+ CI*E**3*(-36641/512+250197/1024*SI**2-1548013/8192*SI**4)                    SIN  (3*M)

+ CI*E**2*(-7899/128+50595/256*SI**2-300093/2048*SI**4)                        SIN  (2*M)

+ CI*E*(-4107/64+46197/256*SI**2-125169/1024*SI**4-186237/512*E**2             SIN  (M)
  +1054935/1024*E**2*SI**2-5798205/8192*E**2*SI**4)

+ CI*E**3*(20159/1536+252201/6144*SI**2+32913/16384*SI**4)                     SIN  (6*SG+9*M)

+ CI*E**2*(1047/256+1233/1024*SI**2+303/512*SI**4)                             SIN  (6*SG+8*M)

+ CI*E*(63/64+69/256*SI**2+273/2048*SI**4-5883/512*E**2-1347/512*E**2*SI**2    SIN  (6*SG+7*M)
  -21105/16384*E**2*SI**4)

+ CI*E**3*(-47907/512+1561193/14336*SI**2+361763/57344*SI**4)                  SIN  (4*SG+7*M)

+ CI*(9/64+9/256*SI**2+9/512*SI**4-45/16*E**2-9/16*E**2*SI**2-9/32*E**2*SI**4) SIN  (6*SG+6*M)

+ CI*E**2*(-7111/256+18119/512*SI**2-4737/2048*SI**4)                          SIN  (4*SG+6*M)

+ CI*E*(-27/64-9/128*SI**2-75/2048*SI**4+1287/512*E**2+663/2048*E**2*SI**2     SIN  (6*SG+5*M)
  +2925/16384*E**2*SI**4)

+ CI*E*(-327/64+10281/1280*SI**2-2577/1024*SI**4+68811/512*E**2-142137         SIN  (4*SG+5*M)
  /1024*E**2*SI**2-366837/8192*E**2*SI**4)

+ CI*E**3*(1542891/5120-3481719/5120*SI**2+774991/2048*SI**4)                  SIN  (2*SG+5*M)

+ CI*E**2*(27/64+9/256*SI**2+3/128*SI**4)                                      SIN  (6*SG+4*M)

+ CI*(87/128*SI**2-15/16*SI**4+2667/64*E**2-24939/512*E**2*SI**2-1005/128*E**2 SIN  (4*SG+4*M)
  *SI**4)
```

$$+ CI*E^2*(14661/128-16647/64*SI^2+73827/512*SI^4) \quad SIN \quad (2*SG+4*M)$$

$$+ CI*E^3*(-81/512-81/16384*SI^4) \quad SIN \quad (6*SG+3*M)$$

$$+ CI*E*(459/64-699/64*SI^2+1371/1024*SI^4-10985/512*E^2-7437/256*E^2*SI^2 \quad SIN \quad (4*SG+3*M)$$
$$+528639/8192*E^2*SI^4)$$

$$+ CI*E*(3653/128-8825/128*SI^2+10189/256*SI^4-333869/1024*E^2+1361781 \quad SIN \quad (2*SG+3*M)$$
$$/2048*E^2*SI^2-2746329/8192*E^2*SI^4)$$

$$+ CI*E^2*(-2001/256+609/512*SI^2+10545/1024*SI^4) \quad SIN \quad (4*SG+2*M)$$

$$+ CI*(135/64-2835/256*SI^2+4893/512*SI^4-4677/32*E^2+32097/128*E^2*SI^2 \quad SIN \quad (2*SG+2*M)$$
$$-94769/1024*E^2*SI^4)$$

$$+ CI*E^3*(-975/512+48793/2048*SI^2-209063/8192*SI^4) \quad SIN \quad (4*SG+M)$$

$$+ CI*E*(-1881/128-4557/256*SI^2+44505/1024*SI^4+98829/1024*E^2-536733 \quad SIN \quad (2*SG+M)$$
$$/1024*E^2*SI^2+4098087/8192*E^2*SI^4)$$

$$\frac{L^{11}}{J_2^3}\Delta_3 L = E^3*SI^2*(48131/2048-116469/2048*SI^2+343403/8192*SI^4) \quad COS \quad (2*SG-M)$$

$$+ F^2*SI^2*(75/4-11463/256*SI^2+62883/2048*SI^4) \quad COS \quad (2*SG)$$

$$+ E^3*(95265/512-745107/1024*SI^2+9181095/8192*SI^4-10647927/16384*SI^6 \quad COS \quad (3*M)$$
$$)$$

$$+ E^2*(3033/32-23577/64*SI^2+569919/1024*SI^4-641805/2048*SI^6) \quad COS \quad (2*M)$$

```
+ E*(2835/64-21609/128*SI**2+249489/1024*SI**4-266001/2048*SI**6+143283          COS  (M)
  /512*E**2-1095771/1024*E**2*SI**2+12932193/8192*E**2*SI**4-14503449/16384*E**2
  *SI**6)

+ 1121445/32768*E**3*SI**6                                                      COS  (6*SG+9*M)

+ 15135/2048*E**2*SI**6                                                         COS  (6*SG+8*M)

+ E*SI**6*(4827/4096-450003/32768*E**2)                                         COS  (6*SG+7*M)

+ E**3*SI**4*(4104127/8192-11810533/16384*SI**2)                                COS  (4*SG+7*M)

+ SI**6*(27/256-3915/2048*E**2)                                                 COS  (6*SG+6*M)

+ E**2*SI**4*(135777/1024-392535/2048*SI**2)                                    COS  (4*SG+6*M)

+ E*SI**6*(-591/4096+36105/32768*E**2)                                          COS  (6*SG+5*M)

+ E*SI**4*(27231/1024-79125/2048*SI**2-1914015/8192*E**2+4894851/16384*E**2     COS  (4*SG+5*M)
  *SI**2)

+ E**3*SI**2*(2628511/2048-3469839/1024*SI**2+73873/32*SI**4)                   COS  (2*SG+5*M)

+ 75/1024*E**2*SI**6                                                            COS  (6*SG+4*M)

+ SI**4*(399/128-291/64*SI**2-13641/256*E**2+33669/512*E**2*SI**2)              COS  (4*SG+4*M)

+ E**2*SI**2*(54651/128-288399/256*SI**2+785169/1024*SI**4)                     COS  (2*SG+4*M)

  -27/32768*E**3*SI**6                                                          COS  (6*SG+3*M)
```

$$+ E*SI^4*(-8853/1024+20889/2048*SI^2-630543/8192*E^2+1409769/16384*E^2 \quad \text{COS} \quad (4*SG+3*M)$$
$$*SI^2)$$

$$+ E*SI^2*(26943/256-17841/64*SI^2+3051/16*SI^4-806829/2048*E^2+61305/64 \quad \text{COS} \quad (2*SG+3*M)$$
$$*E^2*SI^2-8997561/16384*E^2*SI^4)$$

$$+ E^2*SI^4*(-6657/512+15225/1024*SI^2) \quad \text{COS} \quad (4*SG+2*M)$$

$$+ SI^2*(219/16-297/8*SI^2+6681/256*SI^4-18495/128*E^2+177813/512*E^2*SI^2 \quad \text{COS} \quad (2*SG+2*M)$$
$$-407439/2048*E^2*SI^4)$$

$$+ E^3*SI^4*(120431/8192-278423/16384*SI^2) \quad \text{COS} \quad (4*SG+M)$$

$$+ E*SI^2*(-6267/256+15009/256*SI^2-68409/2048*SI^4+38907/2048*E^2 \quad \text{COS} \quad (2*SG+M)$$
$$-122295/2048*E^2*SI^2+1069731/16384*E^2*SI^4)$$

$$+ 27/4-405/16*SI^2+1113/32*SI^4-1131/64*SI^6+1809/32*E^2-13695/64*E^2*SI^2 \quad \text{COS} \quad (0)$$
$$+78483/256*E^2*SI^4-43221/256*E^2*SI^6$$

B.3 Third-Order Short-Period Perturbations due to J_2J_3

$$\frac{L^{10}}{J_2J_3}\Delta_3(e\cos g) = E^3/SI*(1587/256-26159/512*SI^2+164401/3072*SI^4-154145/24576*SI^6) \quad \text{SIN} \quad (2*SG-M)$$

$$+ E^2/SI*(477/64-8507/128*SI^2+275511/2048*SI^4-40177/512*SI^6) \quad \text{SIN} \quad (2*SG)$$

$$+ E^3/SI*(1407/256-3879/128*SI^2-481/4096*SI^4+120627/4096*SI^6) \quad \text{SIN} \quad (3*M)$$

```
+E**2/SI*(75/32-4719/512*SI**2-10547/512*SI**4+7817/256*SI**6)                    SIN  (2*M)

+E/SI*(9/32+15/4*SI**2-18395/512*SI**4+17617/512*SI**6-291/256*E**2+12339        SIN  (M)
  /256*E**2*SI**2-1125105/4096*E**2*SI**4+999603/4096*E**2*SI**6)

+E**3*SI**3*(-251615/4096+608325/8192*SI**2)                                     SIN  (6*SG+9*M)

+E**2*SI**3*(-4687/256+22839/1024*SI**2)                                         SIN  (6*SG+8*M)

+E*SI*(-2147/512*SI**2+5277/1024*SI**4+8109/1024*E**2+101501/2048*E**2*SI**2     SIN  (6*SG+7*M)
  -614649/8192*E**2*SI**4)

+E**3*SI*(-47403/512+9493205/12288*SI**2-6993239/8192*SI**4)                     SIN  (4*SG+7*M)

+SI*(-145/256*SI**2+45/64*SI**4+1095/512*E**2+5399/512*E**2*SI**2-4305/256*E**2  SIN  (6*SG+6*M)
  *SI**4)

+E**2*SI*(-17727/512+141845/512*SI**2-310493/1024*SI**4)                         SIN  (4*SG+6*M)

+E*SI*(45/128+335/256*SI**2-2295/1024*SI**4-2655/512*E**2-30725/4096*E**2        SIN  (6*SG+5*M)
  *SI**2+149475/8192*E**2*SI**4)

+E/SI*(-333/32*SI**2+40417/512*SI**4-87365/1024*SI**6+2151/256*E**2-29511        SIN  (4*SG+5*M)
  /1024*E**2*SI**2-620687/2048*E**2*SI**4+3501737/8192*E**2*SI**6)

+E**3/SI*(-4473/256+88027/256*SI**2-6547549/12288*SI**4+4431275/24576*SI**6      SIN  (2*SG+5*M)
  )

+E**2*SI*(-225/256-175/256*SI**2+1185/512*SI**4)                                 SIN  (6*SG+4*M)

+1/SI*(-63/32*SI**2+445/32*SI**4-945/64*SI**6+207/64*E**2-51/4*E**2*SI**2        SIN  (4*SG+4*M)
  -21055/256*E**2*SI**4+61505/512*E**2*SI**6)
```

$$+E^2/SI*(-549/64+37941/256*SI^2-51289/256*SI^4+23401/512*SI^6) \quad SIN\ (2*SG+4*M)$$

$$+E^3*SI*(675/1024-315/2048*SI^2-6975/8192*SI^4) \quad SIN\ (6*SG+3*M)$$

$$+E/SI*(27/32-405/128*SI^2-4473/256*SI^4+25685/1024*SI^6-27/32*E^2 \quad SIN\ (4*SG+3*M)$$
$$+13311/512*E^2*SI^2-509395/4096*E^2*SI^4+856535/8192*E^2*SI^6)$$

$$+E/SI*(-117/32+1671/32*SI^2-28863/512*SI^4+793/1024*SI^6-5385/256*E^2 \quad SIN\ (2*SG+3*M)$$
$$+11481/256*E^2*SI^2+61003/512*E^2*SI^4-1311789/8192*E^2*SI^6)$$

$$+E^2/SI*(-27/32+1365/128*SI^2-4833/128*SI^4+30555/1024*SI^6) \quad SIN\ (4*SG+2*M)$$

$$+1/SI*(-9/8+93/8*SI^2-2273/256*SI^4-455/128*SI^6-69/8*E^2+5715/512*E^2 \quad SIN\ (2*SG+2*M)$$
$$*SI^2+28935/512*E^2*SI^4-8237/128*E^2*SI^6)$$

$$+E^3/SI*(-297/256+10779/1024*SI^2-86413/6144*SI^4+28375/8192*SI^6) \quad SIN\ (4*SG+M)$$

$$+E/SI*(9/32-759/32*SI^2+4383/64*SI^4-47755/1024*SI^6+3735/256*E^2 \quad SIN\ (2*SG+M)$$
$$-24765/128*E^2*SI^2+1985995/4096*E^2*SI^4-2602449/8192*E^2*SI^6)$$

$$\frac{L^{10}}{J_2J_3}\Delta_3(e\sin g) = E^3/SI*(-1587/256+26553/512*SI^2-239357/3072*SI^4+797765/24576*SI^6) \quad COS\ (2*SG-M)$$

$$+E^2/SI*(-477/64+8475/128*SI^2-275211/2048*SI^4+80399/1024*SI^6) \quad COS\ (2*SG)$$

$$+E^3/SI*(1515/256+9189/256*SI^2-409085/4096*SI^4+120481/2048*SI^6) \quad COS\ (3*M)$$

$$+E^2/SI*(135/32+5865/512*SI^2-18021/512*SI^4+9849/512*SI^6) \quad COS\ (2*M)$$

$$+E/SI*(117/32-687/32*SI^2+23601/512*SI^4-7689/256*SI^6+3567/256*E^2 \quad COS\ (M)$$
$$-39621/256*E^2*SI^2+1480389/4096*E^2*SI^4-475689/2048*E^2*SI^6)$$

```
+ E**3*SI**3*(251615/4096-608325/8192*SI**2)                                    COS  (6*SG+9*M)

+ E**2*SI**3*(4687/256-22839/1024*SI**2)                                        COS  (6*SG+8*M)

+ E*SI*(2147/512*SI**2-5277/1024*SI**4-8109/1024*E**2-101501/2048*E**2*SI**2    COS  (6*SG+7*M)
  +614649/8192*E**2*SI**4)

+ E**3*SI*(11175/128-9269495/12288*SI**2+6759345/8192*SI**4)                    COS  (4*SG+7*M)

+ SI*(145/256*SI**2-45/64*SI**4-1095/512*E**2-5399/512*E**2*SI**2+4305/256*E**2  COS  (6*SG+6*M)
  *SI**4)

+ E**2*SI*(16997/512-138131/512*SI**2+299053/1024*SI**4)                        COS  (4*SG+6*M)

+ E*SI*(-45/128-335/256*SI**2+2295/1024*SI**4+2655/512*E**2+30725/4096*E**2     COS  (6*SG+5*M)
  *SI**2-149475/8192*E**2*SI**4)

+ E/SI*(651/64*SI**2-39227/512*SI**4+83755/1024*SI**6-2151/256*E**2+33051       COS  (4*SG+5*M)
  /1024*E**2*SI**2+598747/2048*E**2*SI**4-3379747/8192*E**2*SI**6)

+ E**3/SI*(3039/256-76365/256*SI**2+7316249/12288*SI**4-7965659/24576*SI**6     COS  (2*SG+5*M)
  )

+ E**2*SI*(225/256+175/256*SI**2-1185/512*SI**4)                                COS  (6*SG+4*M)

+ 1/SI*(63/32*SI**2-215/16*SI**4+225/16*SI**6-207/64*E**2+1707/128*E**2*SI**2   COS  (4*SG+4*M)
  +20195/256*E**2*SI**4-59149/512*E**2*SI**6)

+ E**2/SI*(411/64-33459/256*SI**2+29527/128*SI**4-7007/64*SI**6)                COS  (2*SG+4*M)

+ E**3*SI*(-675/1024+315/2048*SI**2+6975/8192*SI**4)                            COS  (6*SG+3*M)
```

$$+ E/SI*(-27/32+405/128*SI^2+4293/256*SI^4-24695/1024*SI^6+27/32*E^2-423/16*E^2*SI^2+512905/4096*E^2*SI^4-864113/8192*E^2*SI^6) \quad \cos(4*SG+3*M)$$

$$+ E/SI*(99/32-1491/32*SI^2+34515/512*SI^4-24065/1024*SI^6+5529/256*E^2-5241/128*E^2*SI^2-55819/512*E^2*SI^4+1196465/8192*E^2*SI^6) \quad \cos(2*SG+3*M)$$

$$+ E^2/SI*(27/32-1365/128*SI^2+4863/128*SI^4-30855/1024*SI^6) \quad \cos(4*SG+2*M)$$

$$+ 1/SI*(9/8-81/8*SI^2+2641/256*SI^4-65/64*SI^6+147/16*E^2-3231/512*E^2*SI^2-28389/512*E^2*SI^4+14551/256*E^2*SI^6) \quad \cos(2*SG+2*M)$$

$$+ E^3/SI*(297/256-10779/1024*SI^2+86353/6144*SI^4-28285/8192*SI^6) \quad \cos(4*SG+M)$$

$$+ E/SI*(-9/32+429/16*SI^2-4455/64*SI^4+45175/1024*SI^6-3537/256*E^2+48357/256*E^2*SI^2-1824383/4096*E^2*SI^4+2281713/8192*E^2*SI^6) \quad \cos(2*SG+M)$$

$$+ 1/SI*(9/8-405/32*SI^2+461/16*SI^4-18405/1024*SI^6+405/64*E^2-12671/128*E^2*SI^2+463689/2048*E^2*SI^4-70999/512*E^2*SI^6) \quad \cos(0)$$

$$\frac{L^{10}}{J_2 J_3}\Delta_3(\ell+g) = E^2/SI*(-165/32+1593/64*SI^2+43187/1024*SI^4-277925/4096*SI^6) \quad \cos(SG-M)$$

$$+ E^3/SI*(-15/16+763/64*SI^2+146683/3072*SI^4-16405/256*SI^6) \quad \cos(SG-2*M)$$

$$+ E^3/SI*(-27/64+1575/1024*SI^2+315/256*SI^4-10875/4096*SI^6) \quad \cos(3*SG)$$

$$+ E/SI*(-81/16+737/32*SI^2-33877/1024*SI^4+32505/2048*SI^6-453/16*E^2+133835/1024*E^2*SI^2-3123/16*E^2*SI^4+793645/8192*E^2*SI^6) \quad \cos(SG)$$

$$+ E^3*SI*(-15739/512+11221/3072*SI^2+1037179/12288*SI^4) \quad \cos(5*SG+3*M)$$

```
+ E**2*SI*(-2703/256+715/512*SI**2+111869/4096*SI**4)                    COS  (5*SG+7*M)

+ E*SI*(-365/128+27/64*SI**2+1763/256*SI**4+3289/128*E**2-2383/512*E**2*SI**2   COS  (5*SG+6*M)
  -128775/2048*E**2*SI**4)

+ E**3/SI*(-49/2+48855/256*SI**2-215471/512*SI**4+21289/64*SI**6)        COS  (3*SG+6*M)

+ SI*(-15/32+5/64*SI**2+535/512*SI**4+885/128*E**2-725/512*E**2*SI**2-7805   COS  (5*SG+5*M)
  /512*E**2*SI**4)

+ E**2/SI*(-717/64+20403/256*SI**2-10127/64*SI**4+121965/1024*SI**6)     COS  (3*SG+5*M)

+ E*SI*(75/64-35/128*SI**2-145/64*SI**4-1215/256*E**2+627/512*E**2*SI**2   COS  (5*SG+4*M)
  +20019/2048*E**2*SI**4)

+ E/SI*(-69/16+1737/64*SI**2-5909/128*SI**4+1029/32*SI**6+87/64*E**2-35307   COS  (3*SG+4*M)
  /512*E**2*SI**2+155723/512*E**2*SI**4-299091/1024*E**2*SI**6)

+ E**3/SI*(2115/64-110099/512*SI**2+178439/768*SI**4-81685/2048*SI**6)    COS  (SG+4*M)

+ E**2*SI*(-225/256+135/512*SI**2+5865/4096*SI**4)                       COS  (5*SG+3*M)

+ 1/SI*(-9/8+189/32*SI**2-15/2*SI**4+555/128*SI**6+9/8*E**2-3735/128*E**2*SI**2   COS  (3*SG+3*M)
  +62009/512*E**2*SI**4-57333/512*E**2*SI**6)

+ E**2/SI*(543/32-5583/64*SI**2+45049/1024*SI**4+143131/4096*SI**6)      COS  (SG+3*M)

+ E**3*SI*(25/128-55/768*SI**2-1565/6144*SI**4)                          COS  (5*SG+2*M)

+ E/SI*(9/8-195/16*SI**2+665/16*SI**4-2265/64*SI**6+219/32*E**2-14745/256*E**2   COS  (3*SG+2*M)
  *SI**2+47773/256*E**2*SI**4-299649/2048*E**2*SI**6)
```

$$+ E/SI*(27/4-1959/128*SI^2-6047/128*SI^4+15827/256*SI^6+69/32*E^2+19965/128*E^2*SI^2-616881/1024*E^2*SI^4+969275/2048*E^2*SI^6) \quad \cos(SG+2*M)$$

$$+ E^2/SI*(99/64-4059/256*SI^2+29911/512*SI^4-1515/32*SI^6) \quad \cos(3*SG+M)$$

$$+ 1/SI*(87/4*SI^2-9785/128*SI^4+29445/512*SI^6-189/16*E^2+11697/64*E^2*SI^2-17159/32*E^2*SI^4+98051/256*E^2*SI^6) \quad \cos(SG+M)$$

$$\frac{L^{10}}{J_2^2 J_3^3}\Delta_3^h = CI*E^2/SI*(165/32+27/64*SI^2-22355/1024*SI^4) \quad \cos(SG-M)$$

$$+ CI*E^3/SI*(15/16+781/128*SI^2-64675/3072*SI^4) \quad \cos(SG-2*M)$$

$$+ CI*E^3/SI*(27/64-1359/1024*SI^2-1695/2048*SI^4) \quad \cos(3*SG)$$

$$+ CI*E/SI*(81/16-193/16*SI^2+7685/1024*SI^4+453/16*E^2-70059/1024*E^2*SI^2+90773/2048*E^2*SI^4) \quad \cos(SG)$$

$$+ CI*E^3*SI*(15739/512+8999/768*SI^2) \quad \cos(5*SG+8*M)$$

$$+ CI*L^2*SI*(2703/256+497/128*SI^2) \quad \cos(5*SG+7*M)$$

$$+ CI*E*SI*(365/128+257/256*SI^2-3289/128*E^2-4195/512*E^2*SI^2) \quad \cos(5*SG+6*M)$$

$$+ CI*E^3/SI*(49/2-45719/256*SI^2+141469/1024*SI^4) \quad \cos(3*SG+6*M)$$

$$+ CI*SI*(15/32+5/32*SI^2-885/128*E^2-1045/512*E^2*SI^2) \quad \cos(5*SG+5*M)$$

$$+ CI*E^2/SI*(717/64-18969/256*SI^2+55395/1024*SI^4) \quad \cos(3*SG+5*M)$$

$$+ CI*E*SI*(-75/64-5/16*SI^2+1215/256*E^2+147/128*E^2*SI^2) \quad \cos(5*SG+4*M)$$

$$+ CI*E/SI*(69/16-1599/64*SI^2+531/32*SI^4-87/64*E^2+34959/512*E^2*SI^2 \quad \cos\ (3*SG+4*M)$$
$$-88809/1024*E^2*SI^4)$$

$$+ CI*E^3/SI*(-2115/64+35951/512*SI^2-75449/3072*SI^4) \quad \cos\ (SG+4*M)$$

$$+ CI*E^2*SI*(225/256+45/256*SI^2) \quad \cos\ (5*SG+3*M)$$

$$+ CI/SI*(9/8-171/32*SI^2+375/128*SI^4-9/8*E^2+3663/128*E^2*SI^2-18491 \quad \cos\ (3*SG+3*M)$$
$$/512*E^2*SI^4)$$

$$+ CI*E^2/SI*(-543/32+657/32*SI^2+7007/1024*SI^4) \quad \cos\ (SG+3*M)$$

$$+ CI*E^3*SI*(-25/128-5/192*SI^2) \quad \cos\ (5*SG+2*M)$$

$$+ CI*E/SI*(-9/8+93/8*SI^2-105/8*SI^4-219/32*E^2+13869/256*E^2*SI^2-59571 \quad \cos\ (3*SG+2*M)$$
$$/1024*E^2*SI^4)$$

$$+ CI*E/SI*(-27/4-801/128*SI^2+5413/256*SI^4-69/32*E^2-7605/64*E^2*SI^2 \quad \cos\ (SG+2*M)$$
$$+170973/1024*E^2*SI^4)$$

$$+ CI*E^2/SI*(-99/64+3861/256*SI^2-18315/1024*SI^4) \quad \cos\ (3*SG+M)$$

$$+ CI/SI*(-129/8*SI^2+2765/128*SI^4+189/16*E^2-7413/64*E^2*SI^2+2287/16*E^2 \quad \cos\ (SG+M)$$
$$*SI^4)$$

$$\frac{L^9}{J_2J_3}\Delta_3 L = E^2*SI*(471/32-4437/128*SI^2+52775/2048*SI^4) \quad \sin\ (SG-M)$$

$$+ E^3*SI*(921/32-1133/16*SI^2+40765/768*SI^4) \quad \sin\ (SG-2*M)$$

$$+ E^3*SI^3*(185/128-75/32*SI^2) \quad \sin\ (3*SG)$$

```
+ E*SI*(45/8-255/16*SI**2+1725/128*SI**4+315/8*E**2-15165/128*E**2*SI**2+3255     SIN  (SG)
  /32*E**2*SI**4)

+ 19243/256*E**3*SI**5                                                           SIN  (5*SG+8*M)

+ 39573/2048*E**2*SI**5                                                          SIN  (5*SG+7*M)

+ E*SI**5*(477/128-18729/512*E**2)                                               SIN  (5*SG+6*M)

+ E**3*SI**3*(27117/64-296181/512*SI**2)                                         SIN  (3*SG+6*M)

+ SI**5*(105/256-3195/512*E**2)                                                  SIN  (5*SG+5*M)

+ E**2*SI**3*(73505/512-200925/1024*SI**2)                                       SIN  (3*SG+5*M)

+ E*SI**5*(-75/128+807/256*E**2)                                                 SIN  (5*SG+4*M)

+ E*SI**3*(617/16-6753/128*SI**2-4631/128*E**2+7185/128*E**2*SI**2)              SIN  (3*SG+4*M)

+ E**3*SI*(537/4-40733/128*SI**2+170687/768*SI**4)                               SIN  (SG+4*M)

+ 495/2048*E**2*SI**5                                                            SIN  (5*SG+3*M)

+ SI**3*(411/64-1125/128*SI**2-99/16*E**2+4989/512*E**2*SI**2)                   SIN  (3*SG+3*M)

+ E**2*SI*(1629/32-13203/128*SI**2+133809/2048*SI**4)                            SIN  (SG+3*M)

  -5/512*E**3*SI**5                                                              SIN  (5*SG+2*M)

+ E*SI**3*(-143/64+45/16*SI**2-4649/128*E**2+20253/512*E**2*SI**2)               SIN  (3*SG+2*M)
```

$$+ E*SI*(117/8-1293/64*SI^2+1121/128*SI^4+1245/32*E^2-5589/128*E^2*SI^2 \quad \text{SIN} \quad (5G+2*M)$$
$$+191/8*E^2*SI^4)$$

$$+ E^2*SI^3*(-5041/512+11115/1024*SI^2) \quad \text{SIN} \quad (3*SG+M)$$

$$+ SI*(9/4-3/16*SI^2-415/256*SI^4+117/8*E^2-1203/64*E^2*SI^2+199/16*E^2*SI^4) \quad \text{SIN} \quad (SG+M)$$

B.4 Third-Order Short-Period Perturbations due to J_2J_4

$$\frac{L^{12}}{J_2J_4}\Delta_3(e\cos g) = E^2*(79125/512-164755/256*SI^2+818471/1024*SI^4-10219251/32768*SI^6) \quad \text{COS} \quad (5G-M)$$

$$+E^3*(81675/1024-544083/2048*SI^2+173101/768*SI^4-464317/12288*SI^6) \quad \text{COS} \quad (5G-2*M)$$

$$+E^3*(-2415/128+529565/4096*SI^2-997039/4096*SI^4+560259/4096*SI^6) \quad \text{COS} \quad (3*SG)$$

$$+E*(1665/64-18225/128*SI^2+489195/2048*SI^4-127939/1024*SI^6+4605/16 \quad \text{COS} \quad (SG)$$
$$*E^2-6031205/4096*E^2*SI^2+2394605/1024*E^2*SI^4-9605253/8192*E^2*SI^6)$$

$$+E^3*SI^4*(2156539/24576-1243043/12288*SI^2) \quad \text{COS} \quad (7*SG+10*M)$$

$$+E^2*SI^4*(46765/2048-871389/32768*SI^2) \quad \text{COS} \quad (7*SG+6*M)$$

$$+E*SI^2*(9177/2048*SI^2-1351/256*SI^4-42063/4096*E^2-75367/1024*E^2*SI^2 \quad \text{COS} \quad (7*SG+8*M)$$
$$+428297/4096*E^2*SI^4)$$

$$+E^3*SI^2*(810175/4096-13709063/12288*SI^2+7505665/6144*SI^4) \quad \text{COS} \quad (5*SG+8*M)$$

$$+SI^2*(259/512*SI^2-2471/4096*SI^4-2373/1024*E^2-55755/4096*E^2*SI^2 \quad \text{COS} \quad (7*SG+7*M)$$
$$+10339/512*E^2*SI^4)$$

```
+E**2*SI**2*(128147/2048-17749069/57344*SI**2+75952967/229376*SI**4)                COS  (5*SG+7*M)

+E*SI**2*(-315/1024-2947/2048*SI**2+1155/512*SI**4+14091/2048*E**2+27167            COS  (7*SG+4*M)
  /2048*E**2*SI**2-112749/4096*E**2*SI**4)

+E*(15789/1024*SI**2-129559/2048*SI**4+133217/2048*SI**6-19305/1024*E**2            COS  (5*SG+4*M)
  -575/1024*E**2*SI**2+3513771/4096*E**2*SI**4-9263269/8192*E**2*SI**6)

+E**3*(75147/1024-3890757/2048*SI**2+39873859/8192*SI**4-25515061/8192              COS  (3*SG+4*M)
  *SI**6)

+E**2*SI**2*(945/1024+2415/2048*SI**2-96635/32768*SI**4)                           COS  (7*SG+5*M)

+585/256*SI**2-35639/5120*SI**4+26929/4096*SI**6-45/8*E**2+14903/5120*E**2         COS  (5*SG+5*M)
  *SI**2+3443977/20480*E**2*SI**4-465633/2048*E**2*SI**6

+E**2*(15075/512-6826353/10240*SI**2+70432409/40960*SI**4-36440695/32768           COS  (3*SG+5*M)
  *SI**6)

+E**3*SI**2*(-945/1024-329/3072*SI**2+9793/6144*SI**4)                             COS  (7*SG+4*M)

+E*(-135/128+165/256*SI**2+8357/512*SI**4-47285/2048*SI**6+315/256*E**2            COS  (5*SG+4*M)
  -39239/1024*E**2*SI**2-411795/2048*E**2*SI**4+647409/2048*E**2*SI**6)

+E*(1245/128-44391/256*SI**2+227063/512*SI**4-590373/2048*SI**6+32145              COS  (3*SG+4*M)
  /256*E**2+632609/1024*E**2*SI**2-2583225/1024*E**2*SI**4+7525537/4096*E**2*SI**6)

+E**3*(-11061/32+1527879/1024*SI**2-11083529/6144*SI**4+32077141/49152             COS  (SG+4*M)
  *SI**6)
```

$$+ E^2*SI^2*(-8555/2048-184105/8192*SI^2+1204455/32768*SI^4) \quad \text{COS} \quad (5*SG+3*M)$$

$$+ 135/64-5375/256*SI^2+49259/1024*SI^4-122969/4096*SI^6+1285/32*E^2 \quad \text{COS} \quad (3*SG+3*M)$$
$$+270153/1024*E^2*SI^2-4306389/4096*E^2*SI^4+1593431/2048*E^2*SI^6$$

$$+ E^2*(-89545/512+790485/1024*SI^2-30613/32*SI^4+11728331/32768*SI^6) \quad \text{COS} \quad (SG+3*M)$$

$$+ E^3*(135/128+3995/1024*SI^2+473153/12288*SI^4-1275715/24576*SI^6) \quad \text{COS} \quad (5*SG+2*M)$$

$$+ E*(45/16+93225/1024*SI^2-593129/2048*SI^4+417669/2048*SI^6-90*E^2 \quad \text{COS} \quad (3*SG+2*M)$$
$$+1443941/2048*E^2*SI^2-5867751/4096*E^2*SI^4+6901699/8192*E^2*SI^6)$$

$$+ E*(-2415/32+351465/1024*SI^2-908501/2048*SI^4+180635/1024*SI^6 \quad \text{COS} \quad (SG+2*M)$$
$$-369945/1024*E^2+93401/64*E^2*SI^2-3431575/2048*E^2*SI^4+1180547/2048$$
$$*E^2*SI^6)$$

$$+ E^2*(-20655/512+553905/2048*SI^2-3801145/8192*SI^4+7742287/32768*SI^6 \quad \text{COS} \quad (3*SG+M)$$

)

$$-1215/64+10875/128*SI^2-54515/512*SI^4+167503/4096*SI^6-4845/32*E^2 \quad \text{COS} \quad (SG+M)$$
$$+521465/1024*E^2*SI^2-1681973/4096*E^2*SI^4+51247/1024*E^2*SI^6$$

$$\frac{L^{12}}{J_2^2 J_4}\Delta_3(e\sin g) = E^2*(85635/512-438195/512*SI^2+1342951/1024*SI^4-20783917/32768*SI^6) \quad \text{SIN} \quad (SG-M)$$

$$+ E^3*(95055/1024-934483/2048*SI^2+1409591/2048*SI^4-4086215/12288*SI^6 \quad \text{SIN} \quad (SG-2*M)$$

)

$$+ E^3*(-2415/128+523785/4096*SI^2-976415/4096*SI^4+544243/4096*SI^6) \quad \text{SIN} \quad (3*SG)$$

```
+ E*(2205/64-26385/128*SI**2+739665/2048*SI**4-49469/256*SI**6+20895/64*E**2          SIN  (SG)
  -7484755/4096*E**2*SI**2+3105387/1024*E**2*SI**4-12757829/8192*E**2*SI**6)

+ E**3*SI**4*(2156539/24576-1243043/12288*SI**2)                                     SIN  (7*SG+10*M)

+ E**2*SI**4*(46765/2048-871389/32768*SI**2)                                         SIN  (7*SG+9*M)

+ E*SI**2*(9177/2048*SI**2-1351/256*SI**4-42063/4096*E**2-75367/1024*E**2*SI**2      SIN  (7*SG+8*M)
  +428297/4096*E**2*SI**4)

+ E**3*SI**2*(782133/4096-13340897/12288*SI**2+2400717/2048*SI**4)                   SIN  (5*SG+8*M)

+ SI**2*(259/512*SI**2-2471/4096*SI**4-2373/1024*E**2-55755/4096*E**2*SI**2          SIN  (7*SG+7*M)
  +10339/512*E**2*SI**4)

+ E*SI**2*(-315/1024-2947/2048*SI**2+1155/512*SI**4+14091/2048*E**2+27167            SIN  (7*SG+6*M)
  /2048*E**2*SI**2-112749/4096*E**2*SI**4)

+ E*(15579/1024*SI**2-124197/2048*SI**4+124789/2048*SI**6-19305/1024*E**2            SIN  (5*SG+6*M)
  +2061/512*E**2*SI**2+3430065/4096*E**2*SI**4-9013373/8192*E**2*SI**6)

+ E**3*(62277/1024-3724467/2048*SI**2+40664479/8192*SI**4-27698925/8192              SIN  (3*SG+6*M)
  *SI**6)

+ E**2*SI**2*(945/1024+2415/2048*SI**2-96635/32768*SI**4)                            SIN  (7*SG+5*M)

+ 585/256*SI**2-33539/5120*SI**4+24339/4096*SI**6-45/8*E**2+18053/5120*E**2          SIN  (5*SG+5*M)
  *SI**2+3337227/20480*E**2*SI**4-450093/2048*E**2*SI**6
```

```
+ E**2*(13155/512-6610091/10240*SI**2+71954243/40960*SI**4-39349291/32768          SIN  (3*SG+5*M)
  *SI**6)

+ E**3*SI**2*(-945/1024-329/3072*SI**2+9793/6144*SI**4)                           SIN  (7*SG+4*M)

+ E*(-135/128+165/256*SI**2+7937/512*SI**4-44877/2048*SI**6+315/256*E**2          SIN  (5*SG+4*M)
  -39869/1024*E**2*SI**2-406111/2048*E**2*SI**4+639975/2048*E**2*SI**6)

+ E*(1155/128-43329/256*SI**2+231277/512*SI**4-632471/2048*SI**6+32355            SIN  (3*SG+4*M)
  /256*E**2+560877/1024*E**2*SI**2-1301925/512*E**2*SI**4+8137157/4096*E**2*SI**6
  )

+ E**3*(-12783/64+1143361/1024*SI**2-3459609/2048*SI**4+38426411/49152*SI**6      SIN  (SG+4*M)
  )

+ E**2*SI**2*(-8555/2048-180325/8192*SI**2+1184421/32768*SI**4)                   SIN  (5*SG+3*M)

+ 135/64-5165/256*SI**2+48585/1024*SI**4-129045/4096*SI**6+1285/32*E**2           SIN  (3*SG+3*M)
  +261805/1024*E**2*SI**2-4411889/4096*E**2*SI**4+1696485/2048*E**2*SI**6

+ E**2*(-59855/512+662637/1024*SI**2-508193/512*SI**4+15387541/32768*SI**6)       SIN  (SG+3*M)

+ E**3*(135/128+3995/1024*SI**2+472313/12288*SI**4-424557/8192*SI**6)             SIN  (5*SG+2*M)

+ E*(45/16+94245/1024*SI**2-607457/2048*SI**4+435211/2048*SI**6-5715/64*E**2      SIN  (3*SG+2*M)
  +1512093/2048*E**2*SI**2-5928441/4096*E**2*SI**4+6670657/8192*E**2*SI**6)

+ E*(-1905/32+337515/1024*SI**2-1061479/2048*SI**4+259525/1024*SI**6              SIN  (SG+2*M)
  -399165/1024*E**2+2033395/1024*E**2*SI**2-3029077/1024*E**2*SI**4+2838713
  /2048*E**2*SI**6)
```

$$+E^{2}*(-20655/512+563255/2048*SI^{2}-3740387/8192*SI^{4}+7258979/32768*SI^{6} \quad \text{SIN} \quad (3*SG+M)$$

$$)$$

$$-945/64+10965/128*SI^{2}-70605/512*SI^{4}+282737/4096*SI^{6}-5355/32*E^{2} \quad \text{SIN} \quad (SG+M)$$

$$+868495/1024*E^{2}*SI^{2}-4987947/4096*E^{2}*SI^{4}+560623/1024*E^{2}*SI^{6}$$

$$\frac{L^{12}}{J_2 J_4}\Delta_3(\ell+g)=E^{3}*(45/2-441425/1024*SI^{2}+50292523/49152*SI^{4}-123596767/196608*SI^{6}) \quad \text{SIN} \quad (2*SG-M)$$

$$+E^{2}*(-75/4+101555/1024*SI^{2}-86663/512*SI^{4}+752115/8192*SI^{6}) \quad \text{SIN} \quad (2*SG)$$

$$+E^{3}*(-176211/512+2756649/2048*SI^{2}-12254569/8192*SI^{4}+8100827/16384 \quad \text{SIN} \quad (3*M)$$

$$*SI^{6})$$

$$+E^{2}*(-69945/256+287357/256*SI^{2}-86233/64*SI^{4}+1026473/2048*SI^{6}) \quad \text{SIN} \quad (2*M)$$

$$+E*(-16995/64+299735/256*SI^{2}-1580455/1024*SI^{4}+1322825/2048*SI^{6} \quad \text{SIN} \quad (M)$$

$$-832605/512*E^{2}+14534509/2048*E^{2}*SI^{2}-75222755/8192*E^{2}*SI^{4}+61449157$$

$$/16384*E^{2}*SI^{6})$$

$$+E^{3}*SI^{2}*(-95309/2048+47981/16384*SI^{2}+8852143/65536*SI^{4}) \quad \text{SIN} \quad (6*SG+\alpha*M)$$

$$+E^{2}*SI^{2}*(-14021/1024+1101/1024*SI^{2}+308243/8192*SI^{4}) \quad \text{SIN} \quad (6*SG+\beta*M)$$

$$+E*SI^{2}*(-791/256+609/2048*SI^{2}+65401/8192*SI^{4}+84007/2048*E^{2}-85729 \quad \text{SIN} \quad (6*SG+7*M)$$

$$/16384*E^{2}*SI^{2}-6975311/65536*E^{2}*SI^{4})$$

$$+E^{3}*(-17313/256+4646225/14336*SI^{2}-25766597/49152*SI^{4}+311665759 \quad \text{SIN} \quad (4*SG+7*M)$$

$$/688128*SI^{6})$$

```
+ SI**2*(-105/256+49/1024*SI**2+2023/2048*SI**4+4697/512*E**2-357/256*E**2*SI**2          SIN  (6*SG+6*M)
  -5579/256*E**2*SI**4)

+ E**2*(-6435/256+12639/128*SI**2-106895/1024*SI**4+348487/4096*SI**6)                    SIN  (4*SG+6*M)

+ E*SI**2*(315/256-455/2048*SI**2-21525/8192*SI**4-18235/2048*E**2+29995                  SIN  (6*SG+5*M)
  /16384*E**2*SI**2+1281035/65536*E**2*SI**4)

+ E*(-15/2+27489/1280*SI**2+1681/10240*SI**4-10325/4096*SI**6+2685/256*E**2               SIN  (4*SG+5*M)
  -156971/512*E**2*SI**2+96626583/81920*E**2*SI**4-36019327/32768*E**2*SI**6)

+ E**3*(126939/320-2631909/1024*SI**2+1144303303/245760*SI**4-499521881                   SIN  (2*SG+5*M)
  /196608*SI**6)

+ E**2*SI**2*(-315/256+77/256*SI**2+18697/8192*SI**4)                                     SIN  (6*SG+4*M)

  -45/32+135/64*SI**2+3659/512*SI**4-6769/1024*SI**6+105/64*E**2-18885/256                SIN  (4*SG+4*M)
  *E**2*SI**2+9119/32*E**2*SI**4-136479/512*E**2*SI**6

+ E**2*(2715/16-530281/512*SI**2+1887375/1024*SI**4-8206471/8192*SI**6)                   SIN  (2*SG+4*M)

+ E**3*SI**2*(945/2048-2331/16384*SI**2-47271/65536*SI**4)                                SIN  (6*SG+3*M)

+ E*(-235/32*SI**2+62207/2048*SI**4-125363/4096*SI**6-135/128*E**2+222505                 SIN  (4*SG+3*M)
  /2048*E**2*SI**2-7359939/16384*E**2*SI**4+13020707/32768*E**2*SI**6)

+ E*(1715/32-9377/32*SI**2+1025337/2048*SI**4-2201661/8192*SI**6-35715                    SIN  (2*SG+3*M)
  /256*E**2+868577/512*E**2*SI**2-61072565/16384*E**2*SI**4+145906003/65536*E**2
  *SI**6)

+ E**2*(45/32+3535/256*SI**2-67245/1024*SI**4+245329/4096*SI**6)                          SIN  (4*SG+2*M)
```

$$+ 135/32-765/256*SI^2-10019/1024*SI^4+16807/2048*SI^6-2115/16*E^2 \qquad \text{SIN (2*SG+2*M)}$$

$$+624267/512*E^2*SI^2-647381/256*E^2*SI^4+378693/256*E^2*SI^6$$

$$+ E^3*(375/128-15795/512*SI^2+5368529/49152*SI^4-8715553/98304*SI^6) \qquad \text{SIN (4*SG+M)}$$

$$+ E*(-1935/32+34605/64*SI^2-2247247/2048*SI^4+5178929/8192*SI^6-81645 \qquad \text{SIN (2*SG+M)}$$

$$/256*E^2+1474669/512*E^2*SI^2-95603025/16384*E^2*SI^4+219493897/65536$$

$$*E^2*SI^6)$$

$$\frac{L^{12}}{J_2^2 J_4}\Delta_3 h = CI*E^3*(-45/2+377125/2048*SI^2-9801883/49152*SI^4) \qquad \text{SIN (2*SG-M)}$$

$$+ CI*E^2*(75/4-47285/1024*SI^2+70847/2048*SI^4) \qquad \text{SIN (2*SG)}$$

$$+ CI*E^3*(37113/256-629511/2048*SI^2+600761/4096*SI^4) \qquad \text{SIN (3*M)}$$

$$+ CI*E^2*(30615/256-143509/512*SI^2+158571/1024*SI^4) \qquad \text{SIN (2*M)}$$

$$+ CI*E*(1965/16-85505/256*SI^2+110355/512*SI^4+191385/256*E^2-4117639 \qquad \text{SIN (M)}$$

$$/2048*E^2*SI^2+5180301/4096*E^2*SI^4)$$

$$+ CI*E^3*SI^2*(95309/2048+333255/16384*SI^2) \qquad \text{SIN (6*SG+2*M)}$$

$$+ CI*E^2*SI^2*(14021/1024+11819/2048*SI^2) \qquad \text{SIN (6*SG+2*M)}$$

$$+ CI*E*SI^2*(791/256+2555/2048*SI^2-84007/2048*E^2-250299/16384*E^2*SI^2 \qquad \text{SIN (6*SG+2*M)}$$

$$)$$

$$+ CI*E^3*(17313/256-4161461/14336*SI^2+37129717/172032*SI^4) \qquad \text{SIN (4*SG+2*M)}$$

$$+ CI*SI^2*(105/256+161/1024*SI^2-4697/512*E^2-3269/1024*E^2*SI^2) \qquad \text{SIN (6*SG+2*M)}$$

```
+ CI*E**2*(6435/256-44121/512*SI**2+109441/2048*SI**4)                    SIN  (4*SG+6*M)

+ CI*E*SI**2*(-315/256-805/2048*SI**2+18235/2048*E**2+42945/16384*E**2*SI**2)   SIN  (6*SG+5*M)

+ CI*E*(15/2-22689/1280*SI**2+6251/1024*SI**4-2685/256*E**2+77143/256*E**2     SIN  (4*SG+5*M)
  *SI**2-2978867/8192*E**2*SI**4)

+ CI*E**3*(-126939/320+12321417/10240*SI**2-41966339/49152*SI**4)             SIN  (2*SG+5*M)

+ CI*E**2*SI**2*(315/256+161/512*SI**2)                                       SIN  (6*SG+4*M)

+ CI*(45/32-45/32*SI**2-623/512*SI**4-105/64*E**2+18675/256*E**2*SI**2-1463   SIN  (4*SG+4*M)
  /16*E**2*SI**4)

+ CI*E**2*(-2715/16+129707/256*SI**2-363979/1024*SI**4)                       SIN  (2*SG+4*M)

+ CI*E**3*SI**2*(-945/2048-1449/16384*SI**2)                                  SIN  (6*SG+3*M)

+ CI*E*(235/32*SI**2-10479/1024*SI**4+135/128*E**2-221425/2048*E**2*SI**2     SIN  (4*SG+3*M)
  +1049895/8192*E**2*SI**4)

+ CI*E*(-1715/32+38611/256*SI**2-207641/2048*SI**4+35715/256*E**2-1413941     SIN  (2*SG+3*M)
  /2048*E**2*SI**2+10366189/16384*E**2*SI**4)

+ CI*E**2*(-45/32-3715/256*SI**2+40831/2048*SI**4)                            SIN  (4*SG+2*M)

+ CI*(-135/32+585/256*SI**2+3269/1024*SI**4+2115/16*E**2-298347/512*E**2*SI**2   SIN  (2*SG+2*M)
  +518959/1024*E**2*SI**4)

+ CI*E**3*(-375/128+15045/512*SI**2-782467/24576*SI**4)                       SIN  (4*SG+M)
```

$$+ CI*E*(1935/32-66585/256*SI^2+463687/2048*SI^4+81645/256*E^2-2844421 \quad SIN \quad (2*SG+M)$$

$$/2048*E^2*SI^2+20128241/16384*E^2*SI^4)$$

$$\frac{L^{11}}{J_2^2 J_4} \Delta_3 L = E^3*SI^2*(15085/1024-66205/2048*SI^2+1231223/98304*SI^4) \quad COS \quad (2*SG-M)$$

$$+ E^2*SI^2*(225/128+525/256*SI^2-15015/2048*SI^4) \quad COS \quad (2*SG)$$

$$+ E^3*(-27135/256+260073/512*SI^2-3106467/4096*SI^4+3079503/8192*SI^6) \quad COS \quad (3*M)$$

$$+ E^2*(-1785/32+34875/128*SI^2-103707/256*SI^4+202041/1024*SI^6) \quad COS \quad (2*M)$$

$$+ E*(-855/32+8715/64*SI^2-106995/512*SI^4+106155/1024*SI^6-43845/256*E^2 \quad COS \quad (M)$$

$$+455175/512*E^2*SI^2-5853069/4096*E^2*SI^4+6105561/8192*E^2*SI^6)$$

$$-3174705/32768*E^3*SI^6 \quad COS \quad (6*SG+9*M)$$

$$-45089/2048*E^2*SI^6 \quad COS \quad (6*SG+8*M)$$

$$+ E*SI^6*(-15155/4096+1719137/32768*E^2) \quad COS \quad (6*SG+7*M)$$

$$+ E^3*SI^4*(-4892583/8192+42593687/49152*SI^2) \quad COS \quad (4*SG+7*M)$$

$$+ SI^6*(-357/1024+16443/2048*E^2) \quad COS \quad (6*SG+6*M)$$

$$+ E^2*SI^4*(-150135/1024+452013/2048*SI^2) \quad COS \quad (4*SG+6*M)$$

$$+ E*SI^6*(2765/4096-188195/32768*E^2) \quad COS \quad (6*SG+5*M)$$

$$+ E*SI^4*(-24739/1024+80465/2048*SI^2+3346951/8192*E^2-9138025/16384*E^2 \quad COS \quad (4*SG+5*M)$$

$$*SI^2)$$

```
+ E**3*SI**2*(-1047037/1024+6321673/2048*SI**2-217028465/98304*SI**4)                COS  (2*SG+5*M)

  -847/2048*E**2*SI**6                                                               COS  (6*SG+4*M)

+ SI**4*(-189/128+203/64*SI**2+19609/256*E**2-54999/512*E**2*SI**2)                  COS  (4*SG+4*M)

+ E**2*SI**2*(-44997/128+17405/16*SI**2-1616321/2048*SI**4)                         COS  (2*SG+4*M)

+ 2583/32768*E**3*SI**6                                                             COS  (6*SG+3*M)

+ E*SI**4*(6009/1024-19047/2048*SI**2-1160493/8192*E**2+2824983/16384*E**2          COS  (4*SG+3*M)
  *SI**2)

+ E*SI**2*(-11325/128+72633/256*SI**2-861693/4096*SI**4+256107/1024*E**2            COS  (2*SG+3*M)
  -1272387/2048*E**2*SI**2+11606619/32768*E**2*SI**4)

+ E**2*SI**4*(-20657/1024+50961/2048*SI**2)                                          COS  (4*SG+2*M)

+ SI**2*(-75/8+4155/128*SI**2-25991/1024*SI**4+17355/128*E**2-49541/128*E**2        COS  (2*SG+2*M)
  *SI**2+518889/2048*E**2*SI**4)

+ E**3*SI**4*(129993/8192-873677/49152*SI**2)                                        COS  (4*SG+M)

+ E*SI**2*(4965/128-29825/256*SI**2+326627/4096*SI**4+163245/1024*E**2              COS  (2*SG+M)
  -954601/2048*E**2*SI**2+10039463/32768*E**2*SI**4)

  -45/16+495/32*SI**2-3375/128*SI**4+3675/256*SI**6-405/16*E**2+4455/32*E**2        COS  (0)
  *SI**2-63225/256*E**2*SI**4+71925/512*E**2*SI**6
```

Δ_2^*(e cos g)

COEFFICIENT OF COS(3*SG)

```
1/A2**2*A4**2*E**3/L**8*(-375/4/ING**3*SI**2+18125/32/ING**3*SI**4-37875/32/ING**3*SI**6+133875/128/ING**3*SI**8-42875/128/ING**3*SI**10+75/8/ING**2
-19075/128/ING**2*SI**2+137125/256/ING**2*SI**4-353325/512/ING**2*SI**6+302575/1024/ING**2*SI**8)
+ 1/A2**2*A3**2*E**3/L**4*(-25/16/ING**2*SI**2+25/8/ING**2*SI**4-25/16/ING**2*SI**6+5/16/ING-45/32/ING*SI**2+35/32/ING*SI**4-3/32/SI**2+1/32+1/4*SI**2)
+ A4*E**3/L**8*(175/2/ING**3*SI**2-2175/8/ING**3*SI**4+34025/128/ING**3*SI**6-16475/256/ING**3*SI**8-4375/256/ING**3*SI**10-35/4/ING**2+1135/64/ING**2*SI**2
+14885/128/ING**2*SI**4-35065/128/ING**2*SI**6+153475/1024/ING**2*SI**8+285/32/ING-725/8/ING*SI**2+96935/512/ING*SI**4-14035/128/ING*SI**6)
+ A2**2*E**3/L**8*(585/4/ING**3*SI**2-21065/32/ING**3*SI**4+141915/128/ING**3*SI**6-6625/8/ING**3*SI**8+29625/128/ING**3*SI**10-117/8/ING**2+17557/128
/ING**2*SI**2-88465/256/ING**2*SI**4+173625/512/ING**2*SI**6-118825/1024/ING**2*SI**8+191/32/ING-6877/128/ING*SI**2+13073/128/ING*SI**4-56795
/1024/ING*SI**6)
```

COEFFICIENT OF SIN(2*SG)

```
1/A2**2*A3*A4*E**2/L**6*(-25/ING**2*SI+525/4/ING**2*SI**3-775/4/ING**2*SI**5+175/2/ING**2*SI**7+5/2/ING/SI-435/8/ING*SI+1275/8/ING*SI**3-7175/64
/ING*SI**5)
+ A3*E**2/L**6*(30/ING**2*SI-445/4/ING**2*SI**3+2125/16/ING**2*SI**5-825/16/ING**2*SI**7-3/ING/SI+333/8/ING*SI-8389/96/ING*SI**3+3115/64/ING*SI**5)
```

COEFFICIENT OF COS(SG)

```
1/A2**2*A4**2*E/L**8*(75/4/ING**2*SI**2-11875/128/ING**2*SI**4+17675/128/ING**2*SI**6-33075/512/ING**2*SI**8+375/4*E**2/ING**3*SI**2-71375/128*E**2/ING**3
*SI**4+73875/64*E**2/ING**3*SI**6-518875/512*E**2/ING**3*SI**8+165375/512*E**2/ING**3*SI**10-75/8*E**2/ING**2+28525/128*E**2/ING**2*SI**2-228175/256*E**2
/ING**2*SI**4+620375/512*E**2/ING**2*SI**6-550025/1024*E**2/ING**2*SI**8)
+ 1/A2**2*A3**2*E/L**4*(5/4/ING*SI**2-5/8/ING*SI**4+1/8-1/2*SI**2+25/16*E**2/ING**2*SI**2-25/8*E**2/ING**2*SI**4+25/16*E**2/ING**2*SI**6-5/16*E**2/ING+65/32
*E**2/ING*SI**2-55/32*E**2/ING*SI**4-1/32*E**2/SI**2+19/32*E**2-5/4*E**2*SI**2)
+ A4*E/L**8*(-35/2/ING**7*SI**2+2555/64/ING**2*SI**4-1495/64/ING**2*SI**6+175/256/ING**2*SI**8+285/16/ING*SI**2-3095/64/ING*SI**4+4025/128/ING*SI**6
-175/2*E**2/ING**3*SI**2+18375/64*E**2/ING**3*SI**4-10125/32*E**2/ING**3*SI**6+30775/256*E**2/ING**3*SI**8-875/256*E**2/ING**3*SI**10+35/4*E**2/ING**2-3485
/64*E**2/ING**2*SI**2-1075/16*E**2/ING**2*SI**4+2435/8*E**2/ING**2*SI**6-198625/1024*E**2/ING**2*SI**8-285/32*E**2/ING+2375/16*E**2/ING*SI**2-695/2*E**2
/ING*SI**4+54565/256*E**2/ING*SI**6)
```

$$+ A2^2*E/L^8*(-117/4/ING^2*SI^2+13277/128/ING^2*SI^4-15665/128/ING^2*SI^6+3075/64/ING^2*SI^8+191/16/ING*SI^2-463/16/ING*SI^4+1125/64/ING$$
$$*SI^6-585/4*E^2/ING^3*SI^2+85105/128*E^2/ING^3*SI^4-72355/64*E^2/ING^3*SI^6+109075/128*E^2/ING^3*SI^8-15375/64*E^2/ING^3*SI^{10}+117/8*E^2/ING^2$$
$$-29155/128*E^2/ING^2*SI^2+170769/256*E^2/ING^2*SI^4-367335/512*E^2/ING^2*SI^6+270075/1024*E^2/ING^2*SI^8-141/32*E^2/ING+10931/128*E^2/ING$$
$$*SI^2-91681/512*E^2/ING*SI^4+104865/1024*E^2/ING*SI^6)$$

Δ_2^*(e sin g)

COEFFICIENT OF SIN(3*SG)

$$1/A2^2*A4^2*E^3/L^8*(-375/4/ING^3*SI^2+18125/32/ING^3*SI^4-37875/32/ING^3*SI^6+133875/128/ING^3*SI^8-42875/128/ING^3*SI^{10}+75/8/ING^2$$
$$-18625/128/ING^2*SI^2+130375/256/ING^2*SI^4-329525/512/ING^2*SI^6+278075/1024/ING^2*SI^8)$$
$$+ 1/A2^2*A3^2*E^3/L^4*(-25/16/ING^2*SI^2+25/8/ING^2*SI^4-25/16/ING^2*SI^6+5/16/ING-45/32/ING*SI^2+35/32/ING*SI^4-3/32/SI^2+1/32+1/4*SI^2)$$
$$+ A4*E^3/L^8*(175/2/ING^3*SI^2-2175/8/ING^3*SI^4+34025/128/ING^3*SI^6-16475/256/ING^3*SI^8-4375/256/ING^3*SI^{10}-35/4/ING^2+1525/64/ING^2*SI^2$$
$$+2255/32/ING^2*SI^4-49495/256/ING^2*SI^6+111125/1024/ING^2*SI^8+285/32/ING-725/8/ING*SI^2+99145/512/ING*SI^4-13545/128/ING*SI^6)$$
$$+ A2^2*E^3/L^8*(585/4/ING^3*SI^2-21065/32/ING^3*SI^4+141915/128/ING^3*SI^6-6625/8/ING^3*SI^8+29625/128/ING^3*SI^{10}-117/8/ING^2+17695/128$$
$$/ING^2*SI^2-93561/256/ING^2*SI^4+191515/512/ING^2*SI^6-137125/1024/ING^2*SI^8+191/32/ING-6877/128/ING*SI^2+25761/256/ING*SI^4-55115$$
$$/1024/ING*SI^6)$$

COEFFICIENT OF COS(2*SG)

$$1/A2^2*A3*A4*E^2/L^6*(25/ING^2*SI-525/4/ING^2*SI^3+775/4/ING^2*SI^5-175/2/ING^2*SI^7-5/2/ING/SI+105/2/ING*SI-295/2/ING*SI^3+6475/64$$
$$/ING*SI^5)$$
$$+ A3*E^2/L^6*(-30/ING^2*SI+445/4/ING^2*SI^3-2125/16/ING^2*SI^5+825/16/ING^2*SI^7+3/ING/SI-173/4/ING*SI+8915/96/ING*SI^3-3375/64/ING*SI^5$$
$$)$$

COEFFICIENT OF SIN(SG)

$$1/A2^2*A4^2*E/L^8*(-75/4/ING^2*SI^2+12325/128/ING^2*SI^4-18725/128/ING^2*SI^6+35525/512/ING^2*SI^8-375/4*F^2/ING^3*SI^2+73625/128*E^2/ING^3$$
$$*SI^4-77625/64*E^2/ING^3*SI^6+552125/512*E^2/ING^3*SI^8-117625/512*E^2/ING^3*SI^{10}+75/8*E^2/ING^2-28975/128*E^2/ING^2*SI^2+234925/256*E^2$$
$$/ING^2*SI^4-644175/512*E^2/ING^2*SI^6+574525/1024*E^2/ING^2*SI^8)$$
$$+ 1/A2^2*A3^2*F/L^4*(-5/8/ING*SI^2+5/8/ING*SI^4-1/8+3/8*SI^2-25/16*E^2/ING^2*SI^2+25/8*E^2/ING^2*SI^4-25/16*E^2/ING^2*SI^6+5/16*E^2/ING-65/32$$
$$*E^2/ING*SI^2+55/32*E^2/ING*SI^4-3/32*E^2/SI^2-3/32*E^2+5/8*E^2*SI^2)$$

$$+ A4*E/L^{8}*(35/2/ING^{2}*SI^{2}-2165/64/ING^{2}*SI^{4}+295/32/ING^{2}*SI^{6}+1925/256/ING^{2}*SI^{8}-285/16/ING*SI^{2}+3095/64/ING*SI^{4}-4025/128/ING*SI^{6}$$
$$+175/2*E^{2}/ING^{3}*SI^{2}-16425/64*E^{2}/ING^{3}*SI^{4}+13775/64*E^{2}/ING^{3}*SI^{6}-2175/256*E^{2}/ING^{3}*SI^{8}-9625/256*E^{2}/ING^{3}*SI^{10}-35/4*E^{2}/ING^{2}+3095$$
$$/64*E^{2}/ING^{2}*SI^{2}+14525/128*E^{2}/ING^{2}*SI^{4}-98845/256*E^{2}/ING^{2}*SI^{6}+241675/1024*E^{2}/ING^{2}*SI^{8}+285/32*E^{2}/ING-2375/16*E^{2}/ING*SI^{2}+695/2$$
$$*E^{2}/ING*SI^{4}-54565/256*E^{2}/ING*SI^{6})$$
$$+ A2^{2}*E/L^{8}*(117/4/ING^{2}*SI^{2}-12939/128/ING^{2}*SI^{4}+14885/128/ING^{2}*SI^{6}-1425/32/ING^{2}*SI^{8}-191/16/ING*SI^{2}+463/16/ING*SI^{4}-1125/64/ING$$
$$*SI^{6}+585/4*E^{2}/ING^{3}*SI^{2}-83415/128*E^{2}/ING^{3}*SI^{4}+8695/8*E^{2}/ING^{3}*SI^{6}-102925/128*E^{2}/ING^{3}*SI^{8}+7125/32*E^{2}/ING^{3}*SI^{10}-117/8*E^{2}/ING^{2}$$
$$+28817/128*E^{2}/ING^{2}*SI^{2}-165569/256*E^{2}/ING^{2}*SI^{4}+348945/512*E^{2}/ING^{2}*SI^{6}-251175/1024*E^{2}/ING^{2}*SI^{8}+191/32*E^{2}/ING-10931/128*E^{2}/ING$$
$$*SI^{2}+91681/512*E^{2}/ING*SI^{4}-104865/1024*E^{2}/ING*SI^{6})$$

COEFFICIENT OF COS(0)

$$1/A2^{2}*A3*A4/L^{6}*(5/ING*SI-85/4/ING*SI^{3}+35/2/ING*SI^{5}+25*E^{2}/ING^{2}*SI-525/4*E^{2}/ING^{2}*SI^{3}+775/4*E^{2}/ING^{2}*SI^{5}-175/2*E^{2}/ING^{2}*SI^{7}-5/2$$
$$*E^{2}/ING/SI+1135/16*E^{2}/ING*SI-7125/32*E^{2}/ING*SI^{3}+1295/8*E^{2}/ING*SI^{5})$$
$$+ A3/L^{6}*(-6/ING*SI+65/4/ING*SI^{3}-165/16/ING*SI^{5}-30*E^{2}/ING^{2}*SI+445/4*E^{2}/ING^{2}*SI^{3}-2125/16*E^{2}/ING^{2}*SI^{5}+825/16*E^{2}/ING^{2}*SI^{7}+3*E^{2}$$
$$/ING/SI-859/16*E^{2}/ING*SI+3847/32*E^{2}/ING*SI^{3}-2225/32*E^{2}/ING*SI^{5})$$

$\Delta_2^*(t+g)$

COEFFICIENT OF COS(3*SG)

$$1/A2^{2}*A3*A4*E^{3}/L^{6}*(75/32/ING^{2}*SI+25/48/ING^{2}*SI^{3}-1125/128/ING^{2}*SI^{5}+2275/384/ING^{2}*SI^{7}-15/16/ING/SI-35/32/ING*SI+945/64/ING*SI^{3}$$
$$-5215/384/ING*SI^{5})$$
$$+ A3*E^{3}/L^{6}*(65/32/ING^{2}*SI-4325/576/ING^{2}*SI^{3}+2665/288/ING^{2}*SI^{5}-725/192/ING^{2}*SI^{7}-13/16/ING/SI+281/192/ING*SI+727/1152/ING*SI^{3}$$
$$-1045/768/ING*SI^{5})$$

COEFFICIENT OF SIN(2*SG)

$$1/A2^{2}*A4^{2}*E^{2}/L^{8}*(-375/2/ING^{3}*SI^{2}+18125/16/ING^{3}*SI^{4}-37875/16/ING^{3}*SI^{6}+133875/64/ING^{3}*SI^{8}-42875/64/ING^{3}*SI^{10}+75/4/ING^{2}-4525$$
$$/16/ING^{2}*SI^{2}+3975/4/ING^{2}*SI^{4}-40425/32/ING^{2}*SI^{6}+8575/16/ING^{2}*SI^{8})$$
$$+ 1/A2^{2}*A3^{2}*E^{2}/L^{4}*(-25/8/ING^{2}*SI^{2}+25/4/ING^{2}*SI^{4}-25/8/ING^{2}*SI^{6}+5/8/ING-25/8/ING*SI^{2}+5/2/ING*SI^{4}-1/8/SI^{2}-3/16+55/64*SI^{2})$$
$$+ A4*E^{2}/L^{8}*(175/ING^{3}*SI^{2}-2175/4/ING^{3}*SI^{4}+34025/64/ING^{3}*SI^{6}-16475/128/ING^{3}*SI^{8}-4375/128/ING^{3}*SI^{10}-35/2/ING^{2}+1155/16/ING^{2}*SI^{2}$$
$$+7775/128/ING^{2}*SI^{4}-19115/64/ING^{2}*SI^{6}+23625/128/ING^{2}*SI^{8}+285/16/ING-5945/32/ING*SI^{2}+49215/128/ING*SI^{4}-28175/128/ING*SI^{6})$$
$$+ A2^{2}*E^{2}/L^{8}*(585/2/ING^{3}*SI^{2}-21065/16/ING^{3}*SI^{4}+141915/64/ING^{3}*SI^{6}-6625/4/ING^{3}*SI^{8}+29625/64/ING^{3}*SI^{10}-117/4/ING^{2}+2331/8/ING^{2}*SI^{2}$$
$$-98313/128/ING^{2}*SI^{4}+100855/128/ING^{2}*SI^{6}-18075/64/ING^{2}*SI^{8}+191/16/ING-1881/16/ING*SI^{2}+14487/64/ING*SI^{4}-7875/64/ING*SI^{6})$$

COEFFICIENT OF COS(SG)

$$1/A2^2*A3*A4*E/L^6*(50/ING^2*SI-525/2/ING^2*SI^3+775/2/ING^2*SI^5-175/ING^2*SI^7-5/ING/SI+1525/16/ING*SI-4235/16/ING*SI^3+11585/64$$
$$/ING*SI^5+7125/32*E^2/ING^2*SI-17975/16*E^2/ING^2*SI^3+208225/128*E^2/ING^2*SI^5-92925/128*E^2/ING^2*SI^7-45/2*E^2/ING/SI+13385/32*E^2$$
$$/ING*SI-145845/128*E^2/ING*SI^3+197155/256*E^2/ING*SI^5)$$
$$+A3*E/L^6*(-60/ING^2*SI+445/2/ING^2*SI^3-2125/8/ING^2*SI^5+825/8/ING^2*SI^7+6/ING/SI-1417/16/ING*SI+6165/32/ING*SI^3-885/8/ING*SI^5$$
$$-5585/32*E^2/ING^2*SI+41045/64*E^2/ING^2*SI^3-6075/8*E^2/ING^2*SI^5+18725/64*E^2/ING^2*SI^7+69/4*E^2/ING/SI-16165/64*E^2/ING*SI+8669/16$$
$$*E^2/ING*SI^3-78545/256*E^2/ING*SI^5)$$

$\Delta_2^* h$

COEFFICIENT OF COS(3*SG)

$$1/A2^2*A3*A4*CI*E^3/L^6*(-75/32/ING^2*SI-1225/192/ING^2*SI^3+1925/192/ING^2*SI^5+15/16/ING/SI+25/16/ING*SI-2135/384/ING*SI^3)$$
$$+A3*CI*E^3/L^6*(-65/32/ING^2*SI+175/72/ING^2*SI^3-25/96/ING^2*SI^5+13/16/ING/SI-203/192/ING*SI-115/384/ING*SI^3)$$

COEFFICIENT OF SIN(2*SG)

$$1/A2^2*A4^2*CI*E^2/L^8*(375/2/ING^3*SI^2-15125/16/ING^3*SI^4+11375/8/ING^3*SI^6-42875/64/ING^3*SI^8-75/4/ING^2+3025/16/ING^2*SI^2-6825/16$$
$$/ING^2*SI^4+8575/32/ING^2*SI^6)$$
$$+1/A2^2*A3^2*CI*E^2/L^4*(25/8/ING^2*SI^2-25/8/ING^2*SI^4-5/8/ING+5/4/ING*SI^2+1/8/SI^2+1/4)$$
$$+A4*CI*E^2/L^8*(-175/ING^3*SI^2+1475/4/ING^3*SI^4-10425/64/ING^3*SI^6+4375/128/ING^3*SI^8+35/2/ING^2+245/16/ING^2*SI^2-24695/128/ING^2*SI^4$$
$$+21875/128/ING^2*SI^6-285/16/ING+3095/32/ING*SI^2-12075/128/ING*SI^4)$$
$$+A2^2*CI*E^2/L^8*(-585/2/ING^3*SI^2+16385/16/ING^3*SI^4-76375/64/ING^3*SI^6+29625/64/ING^3*SI^8+117/4/ING^2-1161/8/ING^2*SI^2+27305/128$$
$$/ING^2*SI^4-6225/64/ING^2*SI^6-191/16/ING+463/8/ING*SI^2-3375/64/ING*SI^4)$$

COEFFICIENT OF COS(5G)

$$1/A2^2*A3*A4*CI*E/L^6*(-50/ING^2*SI+425/2/ING^2*SI^3-175/ING^2*SI^5+5/ING/SI-1005/16/ING*SI+2765/32/ING*SI^3-7125/32*E^2/ING^2*SI$$
$$+57875/64*E^2/ING^2*SI^3-46725/64*E^2/ING^2*SI^5+45/2*E^2/ING/SI-8585/32*E^2/ING*SI+46235/128*E^2/ING*SI^3)$$
$$+A3*CI*E/L^6*(60/ING^2*SI-325/2/ING^2*SI^3+825/8/ING^2*SI^5-6/ING/SI+793/16/ING*SI-105/2/ING*SI^3+5585/32*E^2/ING^2*SI-1855/4*E^2/ING^2$$
$$*SI^3+4625/16*E^2/ING^2*SI^5-69/4*E^2/ING/SI+9077/64*E^2/ING*SI-18935/128*E^2/ING*SI^3)$$

Δ_3^*(e cos g)

COEFFICIENT OF SIN(2*SG)

$$1/A2^3*A3*A4^2*E^2/L^{10}*(500/ING^3*SI-4750/ING^3*SI^3+67125/4/ING^3*SI^5-109625/4/ING^3*SI^7+21000/ING^3*SI^9-6125/ING^3*SI^{11}-25/ING^2/SI$$
$$+2075/2/ING^2*SI-110925/16/ING^2*SI^3+281575/16/ING^2*SI^5-1209075/64/ING^2*SI^7+231525/32/ING^2*SI^9)$$
$$+\ 1/A2^3*A3^3*E^2/L^6*(-25/4/ING^2*SI^3+25/2/ING^2*SI^5-25/4/ING^2*SI^7+25/8/ING*SI-365/24/ING*SI^3+145/12/ING*SI^5+1/32/SI-37/16*SI+113$$
$$/24*SI^3)$$
$$+\ 1/A2*A3*A4*F^2/L^{10}*(-1500/ING^3*SI+11450/ING^3*SI^3-259775/8/ING^3*SI^5+1403725/32/ING^3*SI^7-915775/32/ING^3*SI^9+116375/16/ING^3*SI^{11}$$
$$+75/ING^2/SI-11175/4/ING^2*SI+1419955/96/ING^2*SI^3-11357495/384/ING^2*SI^5+9844145/384/ING^2*SI^7-8323175/1024/ING^2*SI^9+285/8/ING$$
$$/SI-29925/32/ING*SI+434435/128/ING*SI^3-534305/128/ING*SI^5+1729595/1024/ING*SI^7)$$
$$+\ A2*A3*E^2/L^{10}*(1080/ING^3*SI-6345/ING^3*SI^3+14715/ING^3*SI^5-539975/32/ING^3*SI^7+1226675/128/ING^3*SI^9-276375/128/ING^3*SI^{11}-54/ING^2$$
$$/SI+1413/ING^2*SI-45407/8/ING^2*SI^3+290327/32/ING^2*SI^5-4980205/768/ING^2*SI^7+1770125/1024/ING^2*SI^9-99/8/ING/SI+3589/16/ING*SI$$
$$-141751/256/ING*SI^3+482271/1024/ING*SI^5-259525/2048/ING*SI^7)$$

COEFFICIENT OF COS(SG)

$$1/A2^3*A4^3*E/L^{12}*(-375/2/ING^3*SI^2+52375/32/ING^3*SI^4-5581625/1024/ING^3*SI^6+17686375/2048/ING^3*SI^8-26845875/4096/ING^3*SI^{10}$$
$$+15735125/8192/ING^3*SI^{12})$$
$$+\ 1/A2^3*A3^2*A4*E/L^8*(-25/4/ING^2*SI^2+2175/64/ING^2*SI^4-6525/128/ING^2*SI^6+2975/128/ING^2*SI^8-5/2/ING+1985/64/ING*SI^2-11235/128$$
$$/ING*SI^4+15995/256/ING*SI^6)$$
$$+\ 1/A2*A4^2*E/L^{12}*(1025/2/ING^3*SI^2-113225/32/ING^3*SI^4+9420375/1024/ING^3*SI^6-11756325/1024/ING^3*SI^8+28463575/4096/ING^3*SI^{10}$$
$$-3362625/2048/ING^3*SI^{12}+75/16/ING^2*SI^2+6175/128/ING^2*SI^4-164425/512/ING^2*SI^6+254625/512/ING^2*SI^8-469175/2048/ING^2*SI^{10}-4725$$
$$/64/ING*SI^2+79725/256/ING*SI^4-440055/1024/ING*SI^6+24745/128/ING*SI^8)$$
$$+\ 1/A2*A3^2*E/L^8*(45/4/ING^2*SI^2-2215/64/ING^2*SI^4+4515/128/ING^2*SI^6-1525/128/ING^2*SI^8+3/ING-2229/64/ING*SI^2+587/8/ING*SI^4-10795$$
$$/256/ING*SI^6)$$
$$+\ A2*A4*E/L^{12}*(-45/2/ING^3*SI^2-21475/32/ING^3*SI^4+3484845/1024/ING^3*SI^6-12422185/2048/ING^3*SI^8+9683275/2048/ING^3*SI^{10}-5621875$$
$$/4096/ING^3*SI^{12}+1595/8/ING^2*SI^2-69775/64/ING^2*SI^4+1082855/512/ING^2*SI^6-1808265/1024/ING^2*SI^8+191125/256/ING^2*SI^{10}-1285/32$$
$$/ING*SI^2+63925/256/ING*SI^4-867503/2048/ING*SI^6+875665/4096/ING*SI^8)$$
$$+\ A2^3*E/L^{12}*(-1053/2/ING^3*SI^2+100053/32/ING^3*SI^4-7602699/1024/ING^3*SI^6+4511495/512/ING^3*SI^8-10703875/2048/ING^3*SI^{10}+2538375$$
$$/2048/ING^3*SI^{12}+1215/16/ING^2*SI^2-47777/128/ING^2*SI^4+171037/256/ING^2*SI^6-538965/1024/ING^2*SI^8+80175/512/ING^2*SI^{10}+3815/256$$
$$/ING*SI^2-7951/512/ING*SI^4-52377/2048/ING*SI^6+210959/8192/ING*SI^8)$$

A_3^*(e sin g)

COEFFICIENT OF COS(2*SG)

$$1/A2^3*A3*A4^2*E^2/L^{10}*(-500/ING^3*SI+4750/ING^3*SI^3-67125/4/ING^3*SI^5+109625/4/ING^3*SI^7-21000/ING^3*SI^9+6125/ING^3*SI^{11}+25/ING^2/SI$$
$$-1000/ING^2*SI+837725/128/ING^2*SI^3-2086325/128/ING^2*SI^5+8824025/512/ING^2*SI^7-1669675/256/ING^2*SI^9)$$
$$+1/A2^3*A3^3*E^2/L^6*(25/4/ING^2*SI^3-25/2/ING^2*SI^5+25/4/ING^2*SI^7-5/2/ING*SI+545/48/ING*SI^3-425/48/ING*SI^5-1/16/SI+2*SI-91/24*SI^3)$$
$$+1/A2*A3*A4*E^2/L^{10}*(1500/ING^3*SI-11450/ING^3*SI^3+259775/8/ING^3*SI^5-1403725/32/ING^3*SI^7+915775/32/ING^3*SI^9-116375/16/ING^3*SI^{11}$$
$$-75/ING^2/SI+2770/ING^2*SI-2799805/192/ING^2*SI^3+11196955/384/ING^2*SI^5-405685/16/ING^2*SI^7+8276275/1024/ING^2*SI^9-285/8/ING/SI$$
$$+30495/32/ING*SI-464585/128/ING*SI^3+598475/128/ING*SI^5-2026325/1024/ING*SI^7)$$
$$+A2*A3*E^2/L^{10}*(-1080/ING^3*SI+6345/ING^3*SI^3-14715/ING^3*SI^5+539975/32/ING^3*SI^7-1226675/128/ING^3*SI^9+276275/128/ING^3*SI^{11}+54$$
$$/ING^2/SI-5847/4/ING^2*SI+770789/128/ING^2*SI^3-3804757/384/ING^2*SI^5+1406045/192/ING^2*SI^7-2077725/1024/ING^2*SI^9+99/8/ING/SI$$
$$-1699/8/ING*SI+129371/256/ING*SI^3-420835/1024/ING*SI^5+210085/2048/ING*SI^7)$$

COEFFICIENT OF SIN(SG)

$$1/A2^3*A4^3*E/L^{12}*(3752/ING^3*SI^2-54625/32/ING^3*SI^4+6028625/1024/ING^3*SI^6-19625375/2048/ING^3*SI^8+30422875/4096/ING^3*SI^{10}$$
$$-18136125/8192/ING^3*SI^{12})$$
$$+1/A2^3*A3^2*A4*E/L^8*(25/4/ING^2*SI^2-2325/64/ING^2*SI^4+7175/128/ING^2*SI^6-3325/128/ING^2*SI^8+525/2/ING-1855/64/ING*SI^2+10155/128/ING$$
$$*SI^4-14245/256/ING*SI^6)$$
$$+1/A2*A4^2*E/L^{12}*(-1025/2/ING^3*SI^2+113375/32/ING^3*SI^4-9253775/1024/ING^3*SI^6+11147875/1024/ING^3*SI^8-25627175/4096/ING^3*SI^{10}$$
$$+704375/512/ING^3*SI^{12}-75/16/ING^2*SI^2-14725/128/ING^2*SI^4+297175/512/ING^2*SI^6-423325/512/ING^2*SI^8+750925/2048/ING^2*SI^{10}+4725$$
$$/64/ING*SI^2-79725/256/ING*SI^4+440055/1024/ING*SI^6-24745/128/ING*SI^8)$$
$$+1/A2^2*A3^2*E/L^8*(-45/4/ING^2*SI^2+2085/64/ING^2*SI^4-3955/128/ING^2*SI^6+1225/128/ING^2*SI^8-3/ING+2011/64/ING*SI^2-4055/64/ING*SI^4$$
$$+9055/256/ING*SI^6)$$
$$+A2*A4*E/L^{12}*(45/2/ING^3*SI^2+26805/32/ING^3*SI^4-4199045/1024/ING^3*SI^6+14647865/2048/ING^3*SI^8-11210375/2048/ING^3*SI^{10}+6398875$$
$$/4096/ING^3*SI^{12}-1595/8/ING^2*SI^2+63205/64/ING^2*SI^4-885885/512/ING^2*SI^6+1320775/1024/ING^2*SI^8-44625/128/ING^2*SI^{10}+1285/32/ING$$
$$*SI^2-63925/256/ING*SI^4+867503/2048/ING*SI^6-875665/4096/ING*SI^8)$$
$$+A2^3*E/L^{12}*(1053/2/ING^3*SI^2-97011/32/ING^3*SI^4+7149571/1024/ING^3*SI^6-10290075/128/ING^3*SI^8+9479275/2048/ING^3*SI^{10}-2182875/2048$$
$$/ING^3*SI^{12}-1215/16/ING^2*SI^2+42811/128/ING^2*SI^4-135501/256/ING^2*SI^6+369345/1024/ING^2*SI^8-46425/512/ING^2*SI^{10}-3815/256/ING*SI^2$$
$$+7951/512/ING*SI^4+52377/2048/ING*SI^6-210959/8192/ING*SI^8)$$

COEFFICIENT OF COS(0)

$$1/A2^{3}*A3*A4^{2}/L^{10}*(-50/ING^{2}*SI+425/ING^{2}*SI^{3}-10025/8/ING^{2}*SI^{5}+2975/2/ING^{2}*SI^{7}-1225/2/ING^{2}*SI^{9}-500*E^{2}/ING^{3}*SI+19375/4*E^{2}/ING^{3}*SI^{3}-278125/16*E^{2}/ING^{3}*SI^{5}+918625/32*E^{2}/ING^{3}*SI^{7}-709625/32*E^{2}/ING^{3}*SI^{9}+104125/16*E^{2}/ING^{3}*SI^{11}+25*E^{2}/ING^{2}/SI-5475/4*E^{2}/ING^{2}*SI+312275/32*E^{2}/ING^{2}*SI^{3}-3307675/128*E^{2}/ING^{2}*SI^{5}+7313775/256*E^{2}/ING^{2}*SI^{7}-2862825/256*E^{2}/ING^{2}*SI^{9})$$

$$+1/A2^{3}*A3^{3}/L^{6}*(5/4/ING*SI^{3}-5/4/ING*SI^{5}+3/16*SI-13/16*SI^{3}+25/4*E^{2}/ING^{2}*SI^{3}-25/2*E^{2}/ING^{2}*SI^{5}+25/4*E^{2}/ING^{2}*SI^{7}-65/16*E^{2}/ING*SI+345/16*E^{2}/ING*SI^{3}-35/2*E^{2}/ING*SI^{5}-1/16*E^{2}/SI+51/16*E^{2}*SI-113/16*E^{2}*SI^{3})$$

$$+1/A2*A3*A4/L^{10}*(150/ING^{2}*SI-995/ING^{2}*SI^{3}+36035/16/ING^{2}*SI^{5}-136605/64/ING^{2}*SI^{7}+23275/32/ING^{2}*SI^{9}+285/4/ING*SI-5355/16/ING*SI^{3}+30615/64/ING*SI^{5}-27895/128/ING*SI^{7}+1500*E^{2}/ING^{3}*SI-45925/4*E^{2}/ING^{3}*SI^{3}+32500*E^{2}/ING^{3}*SI^{5}-279875/64*E^{2}/ING^{3}*SI^{7}+3632825/128*E^{2}/ING^{3}*SI^{9}-917875/128*E^{2}/ING^{3}*SI^{11}-75*E^{2}/ING^{2}/SI+28615/8*E^{2}/ING^{2}*SI-39455/2*E^{2}/ING^{2}*SI^{3}+10303455/256*E^{2}/ING^{2}*SI^{5}-18078865/512*E^{2}/ING^{2}*SI^{7}+5775875/512*E^{2}/ING^{2}*SI^{9}-285/8*E^{2}/ING/SI+20895/16*E^{2}/ING*SI-328015/64*E^{2}/ING*SI^{3}+423175/64*E^{2}/ING*SI^{5}-711795/256*E^{2}/ING*SI^{7})$$

$$+A2*A3/L^{10}*(-108/ING^{2}*SI+1053/2/ING^{2}*SI^{3}-945/ING^{2}*SI^{5}+47515/64/ING^{2}*SI^{7}-55275/256/ING^{2}*SI^{9}-99/4/ING*SI+565/8/ING*SI^{3}-8461/128/ING*SI^{5}+10125/512/ING*SI^{7}-1080*E^{2}/ING^{3}*SI+12495/2*E^{2}/ING^{3}*SI^{3}-227855/16*E^{2}/ING^{3}*SI^{5}+1025625/64*E^{2}/ING^{3}*SI^{7}-1141475/128*E^{2}/ING^{3}*SI^{9}+251625/128*E^{2}/ING^{3}*SI^{11}+54*E^{2}/ING^{2}/SI-14631/8*E^{2}/ING^{2}*SI+7604*E^{2}/ING^{2}*SI^{3}-3162851/256*E^{2}/ING^{2}*SI^{5}+1141815/128*E^{2}/ING^{2}*SI^{7}-613025/256*E^{2}/ING^{2}*SI^{9}+99/8*E^{2}/ING/SI-9983/32*E^{2}/ING*SI+197889/256*E^{2}/ING*SI^{3}-652983/1024*E^{2}/ING*SI^{5}+162805/1024*E^{2}/ING*SI^{7})$$

$\Delta_3^*(z+g)$

COEFFICIENT OF SIN(2*SG)

$$1/A2^{3}*A4^{3}*E^{2}/L^{12}*(5625/2/ING^{4}*SI^{2}-223125/8/ING^{4}*SI^{4}+3523125/32/ING^{4}*SI^{6}-3545625/16/ING^{4}*SI^{8}+15448125/64/ING^{4}*SI^{10}-34759375/256/ING^{4}*SI^{12}+7931875/256/ING^{4}*SI^{14}-375/2/ING^{3}+18625/4/ING^{3}*SI^{2}-972125/32/ING^{3}*SI^{4}+2797875/32/ING^{3}*SI^{6}-8063125/64/ING^{3}*SI^{8}+11423125/128/ING^{3}*SI^{10}-1586375/64/ING^{3}*SI^{12})$$

$$+1/A2^{3}*A3^{2}*A4*E^{2}/L^{8}*(125/2/ING^{3}*SI^{2}-6625/16/ING^{3}*SI^{4}+28375/32/ING^{3}*SI^{6}-3125/4/ING^{3}*SI^{8}+7875/32/ING^{3}*SI^{10}-75/4/ING^{2}+1925/16/ING^{2}*SI^{2}-2075/8/ING^{2}*SI^{4}+59825/256/ING^{2}*SI^{6}-19425/256/ING^{2}*SI^{8}+5/2/ING/SI^{2}-125/8/ING-2275/32/ING*SI^{2}+73735/256/ING*SI^{4}-27125/128/ING*SI^{6})$$

$$+1/A2*A4^{2}*E^{2}/L^{12}*(-15375/2/ING^{4}*SI^{2}+486375/8/ING^{4}*SI^{4}-6077625/32/ING^{4}*SI^{6}+7800825/256/ING^{4}*SI^{8}-6838125/256/ING^{4}*SI^{10}+124842375/1024/ING^{4}*SI^{12}-23244375/1024/ING^{4}*SI^{14}+1025/2/ING^{3}-85725/8/ING^{3}*SI^{2}+219675/4/ING^{3}*SI^{4}-7778575/64/ING^{3}*SI^{6}+34273375/256/ING^{3}*SI^{8}-4623325/64/ING^{3}*SI^{10}+7772625/512/ING^{3}*SI^{12}+75/16/ING^{2}+31975/64/ING^{2}*SI^{2}-1006525/256/ING^{2}*SI^{4}+10660975/1024/ING^{2}*SI^{6}-2873675/256/ING^{2}*SI^{8}+4345075/1024/ING^{2}*SI^{10}-4725/64/ING+145875/128/ING*SI^{2}-3871365/1024/ING*SI^{4}+4752335/1024/ING*SI^{6}-123725/64/ING*SI^{8})$$

$$+ 1/A2*A3^{2}*E^{2}/L^{8}*(-225/2/ING^{3}*SI^{2}+7175/16/ING^{3}*SI^{4}-42675/64/ING^{3}*SI^{6}+14025/32/ING^{3}*SI^{8}-6875/64/ING^{3}*SI^{10}+105/4/ING^{2}-1825/16$$
$$/ING^{2}*SI^{2}+19095/128/ING^{2}*SI^{4}-14635/256/ING^{2}*SI^{6}-1075/256/ING^{2}*SI^{8}-3/ING/SI^{2}+23/8/ING+12517/96/ING*SI^{2}-231293/768/ING*SI^{4}$$
$$+44285/256/ING*SI^{6})$$

$$+ A2*A4*E^{2}/L^{12}*(675/2/ING^{4}*SI^{2}+87825/8/ING^{4}*SI^{4}-270525/4/ING^{4}*SI^{6}+1244625/8/ING^{4}*SI^{8}-360297975/2048/ING^{4}*SI^{10}+404408625/4096$$
$$/ING^{4}*SI^{12}-90391875/4096/ING^{4}*SI^{14}-45/2/ING^{3}-3345/ING^{3}*SI^{2}+474895/16/ING^{3}*SI^{4}-11503925/128/ING^{3}*SI^{6}+258527725/2048/ING^{3}*SI^{6}$$
$$-86821625/1024/ING^{3}*SI^{10}+22645875/1024/ING^{3}*SI^{12}+1595/8/ING^{2}-104725/32/ING^{2}*SI^{2}+1616605/128/ING^{2}*SI^{4}-41058965/2048/ING^{2}*SI^{6}$$
$$+58662445/4096/ING^{2}*SI^{8}-15718675/4096/ING^{2}*SI^{10}-1285/32/ING+99905/128/ING*SI^{2}-6093709/2048/ING*SI^{4}+9558857/2048/ING*SI^{6}$$
$$-4378325/2048/ING*SI^{8})$$

$$+ A2^{3}*E^{2}/L^{12}*(15795/2/ING^{4}*SI^{2}-432675/8/ING^{4}*SI^{4}+154225/ING^{4}*SI^{6}-60002485/256/ING^{4}*SI^{8}+41043675/2048/ING^{4}*SI^{10}-46665375/512$$
$$/ING^{4}*SI^{12}+35375625/2048/ING^{4}*SI^{14}-1053/2/ING^{3}+72675/8/ING^{3}*SI^{2}-1342165/32/ING^{3}*SI^{4}+11390673/128/ING^{3}*SI^{6}-201589205/2048$$
$$/ING^{3}*SI^{8}+113478125/2048/ING^{3}*SI^{10}-6442125/512/ING^{3}*SI^{12}+1215/16/ING^{2}-336331/256/ING^{2}*SI^{2}+2446927/512/ING^{2}*SI^{4}-14565743$$
$$/2048/ING^{2}*SI^{6}+39294065/8192/ING^{2}*SI^{8}-10086005/8192/ING^{2}*SI^{10}+3815/256/ING-1083/8/ING*SI^{2}+97301/2048/ING*SI^{4}+42647/128/ING$$
$$*SI^{6}-1054795/4096/ING*SI^{8})$$

COEFFICIENT OF COS(SG)

$$1/A2^{3}*A3*A4^{2}*E/L^{10}*(-1000/ING^{3}*SI+37625/4/ING^{3}*SI^{3}-527375/16/ING^{3}*SI^{5}+1712375/32/ING^{3}*SI^{7}-1306375/32/ING^{3}*SI^{9}+189875/16/ING^{3}$$
$$*SI^{11}+50/ING^{2}/SI-7125/4/ING^{2}*SI+2897125/256/ING^{2}*SI^{3}-14174475/512/ING^{2}*SI^{5}+29614375/1024/ING^{2}*SI^{7}-22217825/2048/ING^{2}*SI^{9})$$

$$+ 1/A2^{3}*A3^{3}*E/L^{6}*(25/2/ING^{2}*SI^{3}-25/ING^{2}*SI^{5}+25/2/ING^{2}*SI^{7}-55/16/ING*SI+505/32/ING*SI^{3}-395/32/ING*SI^{5}-1/8/SI+53/16*SI-413/64$$
$$*SI^{3})$$

$$+ 1/A2*A3*A4*E/L^{10}*(3000/ING^{3}*SI-91475/4/ING^{3}*SI^{3}+519325/8/ING^{3}*SI^{5}-5623675/64/ING^{3}*SI^{7}+7356475/128/ING^{3}*SI^{9}-1875125/128/ING^{3}$$
$$*SI^{11}-150/ING^{2}/SI+42085/8/ING^{2}*SI-3546125/128/ING^{2}*SI^{3}+14268245/256/ING^{2}*SI^{5}-25027215/512/ING^{2}*SI^{7}+16119775/1024/ING^{2}*SI^{9}$$
$$-285/4/ING/SI+56355/32/ING*SI-847595/128/ING*SI^{3}+546425/64/ING*SI^{5}-1861195/512/ING*SI^{7})$$

$$+ A2*A3*E/L^{10}*(-2160/ING^{3}*SI+25575/2/ING^{3}*SI^{3}-478465/16/ING^{3}*SI^{5}+2214225/64/ING^{3}*SI^{7}-1269275/64/ING^{3}*SI^{9}+144375/32/ING^{3}*SI^{11}$$
$$+108/ING^{2}/SI-23883/8/ING^{2}*SI+3200581/256/ING^{2}*SI^{3}-10716335/512/ING^{2}*SI^{5}+8069435/512/ING^{2}*SI^{7}-2981125/512/ING^{2}*SI^{9}+99/4/ING$$
$$/SI-14905/32/ING*SI+151543/128/ING*SI^{3}-538593/512/ING*SI^{5}+311625/1024/ING*SI^{7})$$

Δ_3^*h

COEFFICIENT OF SIN(2*SG)

$$1/A2^{3}*A4^{3}*CI*E^{2}/L^{12}*(-5625/2/ING^{4}*SI^{2}+200625/8/ING^{4}*SI^{4}-2720625/32/ING^{4}*SI^{6}+4370625/32/ING^{4}*SI^{8}-6706875/64/ING^{4}*SI^{10}$$
$$+7931875/256/ING^{4}*SI^{12}+375/2/ING^{3}-13375/4/ING^{3}*SI^{2}+544125/32/ING^{3}*SI^{4}-291375/8/ING^{3}*SI^{6}+2235625/64/ING^{3}*SI^{8}-1586375/128/ING^{3}$$
$$*SI^{10})$$

$$+ 1/A2^{3}*A3^{2}*A4*CI*E^{2}/L^{8}*(-125/2/ING^{3}*SI^{2}+5625/16/ING^{3}*SI^{4}-17125/32/ING^{3}*SI^{6}+7875/32/ING^{3}*SI^{8}+75/4/ING^{2}-925/16/ING^{2}*SI^{2}-75/2$$
$$/ING^{2}*SI^{4}+22575/256/ING^{2}*SI^{6}-5/2/ING/SI^{2}+115/8/ING+1425/32/ING*SI^{2}-23275/256/ING*SI^{4})$$

$$+ 1/A2*A4^{2}*CI*E^{2}/L^{12}*(15375/2/ING^{4}*SI^{2}-424875/8/ING^{4}*SI^{4}+4378125/32/ING^{4}*SI^{6}-4298625/256/ING^{4}*SI^{8}+6349875/64/ING^{4}*SI^{10}$$
$$-23244375/1024/ING^{4}*SI^{12}-1025/2/ING^{3}+57025/8/ING^{3}*SI^{2}-212375/8/ING^{3}*SI^{4}+2577075/64/ING^{3}*SI^{6}-6771625/256/ING^{3}*SI^{8}+1561875$$
$$/256/ING^{3}*SI^{10}-75/16/ING^{2}-34075/64/ING^{2}*SI^{2}+744825/256/ING^{2}*SI^{4}-4912075/1024/ING^{2}*SI^{6}+2514925/1024/ING^{2}*SI^{8}+4725/64/ING$$
$$-79725/128/ING*SI^{2}+1320165/1024/ING*SI^{4}-24745/32/ING*SI^{6})$$

$$+ 1/A2*A3^{2}*CI*E^{2}/L^{8}*(225/2/ING^{3}*SI^{2}-5375/16/ING^{3}*SI^{4}+21175/64/ING^{3}*SI^{6}-6875/64/ING^{3}*SI^{8}-105/4/ING^{2}+325/16/ING^{2}*SI^{2}+11385/128$$
$$/ING^{2}*SI^{4}-21975/256/ING^{2}*SI^{6}+3/ING/SI^{2}-11/8/ING-6115/96/ING*SI^{2}+18835/256/ING*SI^{4})$$

$$+ A2*A4*CI*E^{2}/L^{12}*(-675/2/ING^{4}*SI^{2}-90525/8/ING^{4}*SI^{4}+450525/8/ING^{4}*SI^{6}-198525/2/ING^{4}*SI^{8}+157008375/2048/ING^{4}*SI^{10}-90391875$$
$$/4096/ING^{4}*SI^{12}+45/2/ING^{3}+7005/2/ING^{3}*SI^{2}-346435/16/ING^{3}*SI^{4}+5849085/128/ING^{3}*SI^{6}-83626525/2048/ING^{3}*SI^{8}+27213375/2048/ING^{3}$$
$$*SI^{10}-1595/8/ING^{2}+60065/32/ING^{2}*SI^{2}-578465/128/ING^{2}*SI^{4}+8178645/2048/ING^{2}*SI^{6}-4756675/4096/ING^{2}*SI^{8}+1285/32/ING-63925/128$$
$$/ING*SI^{2}+2602509/2048/ING*SI^{4}-875665/1024/ING*SI^{6}).$$

$$+ A2^{3}*CI*E^{2}/L^{12}*(-15795/2/ING^{4}*SI^{2}+369495/8/ING^{4}*SI^{4}-864305/8/ING^{4}*SI^{6}+32344725/256/ING^{4}*SI^{8}-151285875/2048/ING^{4}*SI^{10}$$
$$+35375625/2048/ING^{4}*SI^{12}+1053/2/ING^{3}-43191/8/ING^{3}*SI^{2}+578209/32/ING^{3}*SI^{4}-3546285/128/ING^{3}*SI^{6}+41345525/2048/ING^{3}*SI^{8}$$
$$-5809125/1024/ING^{3}*SI^{10}-1215/16/ING^{2}+200251/256/ING^{2}*SI^{2}-959369/512/ING^{2}*SI^{4}+3371355/2048/ING^{2}*SI^{6}-4009205/8192/ING^{2}*SI^{8}$$
$$-3815/256/ING+7951/256/ING*SI^{2}+157131/2048/ING*SI^{4}-210959/2048/ING*SI^{6})$$

COEFFICIENT OF COS(SG)

$$1/A2^{3}*A3*A4^{2}*CI*E/L^{10}*(1000/ING^{3}*SI-33625/4/ING^{3}*SI^{3}+392875/16/ING^{3}*SI^{5}-926625/32/ING^{3}*SI^{7}+189875/16/ING^{3}*SI^{9}-50/ING^{2}/SI$$
$$+5025/4/ING^{2}*SI-1558725/256/ING^{2}*SI^{3}+2570225/256/ING^{2}*SI^{5}-5415725/1024/ING^{2}*SI^{7})$$

$$+ 1/A2^{3}*A3^{3}*CI*E/L^{6}*(-25/2/ING^{2}*SI^{3}+25/2/ING^{2}*SI^{5}+55/16/ING*SI-95/16/ING*SI^{3}+1/8/SI-9/4*SI)$$

$$+ 1/A2*A3*A4*CI*E/L^{10}*(-3000/ING^{3}*SI+79475/4/ING^{3}*SI^{3}-360375/8/ING^{3}*SI^{5}+2740675/64/ING^{3}*SI^{7}-1875125/128/ING^{3}*SI^{9}+150/ING^{2}/SI$$
$$-29485/8/ING^{2}*SI+168325/128/ING^{2}*SI^{3}-1265375/64/ING^{2}*SI^{5}+4493475/512/ING^{2}*SI^{7}+285/4/ING/SI-32415/32/ING*SI+309245/128/ING$$
$$*SI^{3}-394555/256/ING*SI^{5})$$

$$+ A2*A3*CI*E/L^{10}*(2160/ING^{3}*SI-21255/2/ING^{3}*SI^{3}+308425/16/ING^{3}*SI^{5}-980525/64/ING^{3}*SI^{7}+144375/32/ING^{3}*SI^{9}-108/ING^{2}/SI+14811/8$$
$$/ING^{2}*SI-1426837/256/ING^{2}*SI^{3}+1560155/256/ING^{2}*SI^{5}-72375/32/ING^{2}*SI^{7}-99/4/ING/SI+6589/32/ING*SI-40453/128/ING*SI^{3}+66375/512$$
$$/ING*SI^{5})$$

B 7 Secular Perturbations

$$\frac{\partial F^{**}}{\partial(L,G,H)} = \frac{\partial F_1^{**}}{\partial(L,G,H)} + \frac{\partial F_2^{**}}{\partial(L,G,H)} + \frac{\partial F_3^{**}}{\partial(L,G,H)} + \frac{\partial F_4^{**}}{\partial(L,G,H)}$$

$$\frac{\partial F_1^{**}}{\partial L} = \frac{3J_2}{4L^4G^3}(1 - 3c^2)$$

$$\frac{\partial F_1^{**}}{\partial G} = \frac{3J_2}{4L^3G^4}(1 - 5c^2)$$

$$\frac{\partial F_1^{**}}{\partial H} = \frac{3J_2c}{2L^3G^4}$$

$$\frac{\partial F_2^{**}}{\partial L} = \frac{3}{128L^4G^7}\left\{J_2^2\left[15(1 - 2c^2 - 7c^4) - 16\eta(1 - 3c^2)^2 - 5\eta^2(5 - 18c^2 + 5c^4)\right] + 15J_4e^2(3 - 30c^2 + 35c^4)\right\}$$

$$\frac{\partial F_2^{**}}{\partial G} = \frac{3}{128L^3G^8}\left\{J_2^2\left[5(7 - 18c^2 - 77c^4) - 24\eta(1 - 3c^2)(1 - 5c^2) + \eta^2(-25 + 126c^2 - 45c^4)\right] + 5J_4\left[4(3 - 36c^2 + 49c^4) + 3e^2(3 - 42c^2 + 63c^4)\right]\right\}$$

$$\frac{\partial F_2^{**}}{\partial H} = \frac{3c}{32L^3G^8}\left\{J_2^2\left[5(1 + 7c^2) + 12\eta(-1 + 3c^2) + \eta^2(-9 + 5c^2)\right] + 5J_4(3 - 7c^2)(2 + 3e^2)\right\}$$

$$\frac{\partial F_3^{**}}{\partial L}$$

```
1/A2**A4**2/L**15*(675/128/ING*SI**4-1575/128/ING*SI**6+3675/512/ING*SI**8+3375/128*E**2/ING**2*SI**4-5625/64*E**2/ING**2*SI**6+49875/512*E**2/ING**2
*SI**8-18375/512*E**2/ING**2*SI**10-675/64*E**2/ING*SI**2+10125/128*E**2/ING*SI**4-8925/64*E**2/ING*SI**6+18375/256*E**2/ING*SI**8)
+ 1/A2**A3**2/L**11*(-15/16*SI**2+15/16*SI**4-3/16*ING+9/32*ING*SI**2-15/16*E**2*SI**2+15/16*E**2*SI**4-3/16*E**2*ING-27/64*E**2*ING*SI**2)
+ A2*A4/L**15*(585/64/ING*SI**4-2715/128/ING*SI**6+1575/128/ING*SI**8-45/16+225/32*SI**2+315/64*SI**4-315/32*SI**6+2925/64*E**2/ING**2*SI**4
-19425/128*E**2/ING**2*SI**6+10725/64*E**2/ING**2*SI**8-7875/128*E**2/ING**2*SI**10-585/32*E**2/ING*SI**2+17775/128*E**2/ING*SI**4-62775/256*E**2/ING
*SI**6+64575/512*E**2/ING*SI**8+225/16*E**2-7155/64*E**2*SI**2+30915/128*E**2*SI**4-76545/512*E**2*SI**6)
+ A2**3/L**15*(507/128/ING*SI**4-585/64/ING*SI**6+675/128/ING*SI**8-135/16+1023/32*SI**2-2421/64*SI**4+7485/512*SI**6+2535/128*E**2/ING**2*SI**4
-8385/128*E**2/ING**2*SI**6+9225/128*E**2/ING**2*SI**8-3375/128*E**2/ING**2*SI**10-507/64*E**2/ING*SI**2+975/16*E**2/ING*SI**4-27585/256*E**2/ING*SI**6
+14175/256*E**2/ING*SI**8-1665/32*E**2+12393/64*E**2*SI**2-119295/512*E**2*SI**4+23265/256*E**2*SI**6)
```

$$\frac{\partial F_3^{**}}{\partial G}$$

```
1/A2**A4**2/L**15*(-675/128/ING*SI**4+1575/128/ING*SI**6-3675/512/ING*SI**8-3375/64*E**2/ING**2*SI**4+5625/32*E**2/ING**2*SI**6-49875/256*E**2/ING**2
*SI**8+18375/256*E**2/ING**2*SI**10+675/32*E**2/ING*SI**2-42525/256*E**2/ING*SI**4+76125/256*E**2/ING*SI**6-158025/1024*E**2/ING*SI**8)
```

$$+ 1/A2*A3^{2}/L^{11}*(15/8*SI^{2}-15/8*SI^{4}+3/8*ING-45/32*ING*SI^{2}-15/8*E^{2}+525/32*E^{2}*SI^{2}-465/32*E^{2}*SI^{4}+81/32*E^{2}*ING-243/32*E^{2}*ING*SI^{2})$$

$$+ A2*A4/L^{15}*(-585/64/ING*SI^{4}+2715/128/ING*SI^{6}-1575/128/ING*SI^{8}+585/8-5625/16*SI^{2}+64395/128*SI^{4}-14595/64*SI^{6}-2925/32*E^{2}/ING^{2}$$
$$*SI^{4}+19425/64*E^{2}/ING^{2}*SI^{6}-10725/32*E^{2}/ING^{2}*SI^{8}+7875/64*E^{2}/ING^{2}*SI^{10}+585/16*E^{2}/ING*SI^{2}-37035/128*E^{2}/ING*SI^{4}+66195/128*E^{2}/ING$$
$$*SI^{6}-137025/512*E^{2}/ING*SI^{8}+17325/32*E^{2}-161415/64*E^{2}*SI^{2}+894015/256*E^{2}*SI^{4}-784035/512*E^{2}*SI^{6})$$

$$+ A2^{3}/L^{15}*(-507/128/ING*SI^{4}+585/64/ING*SI^{6}-675/128/ING*SI^{8}-513/8+1887/8*SI^{2}-9531/32*SI^{4}+63555/512*SI^{6}-2535/64*E^{2}/ING^{2}*SI^{4}$$
$$+8385/64*E^{2}/ING^{2}*SI^{6}-9225/64*E^{2}/ING^{2}*SI^{8}+3375/64*E^{2}/ING^{2}*SI^{10}+507/32*E^{2}/ING*SI^{2}-32253/256*E^{2}/ING*SI^{4}+57555/256*E^{2}/ING*SI^{6}$$
$$-7425/64*E^{2}/ING*SI^{8}-6351/16*E^{2}+93939/64*E^{2}*SI^{2}-481617/256*E^{2}*SI^{4}+815355/1024*E^{2}*SI^{6})$$

$$\frac{\partial F_3^{**}}{\partial H}$$

$$1/A2*A4^{2}*CI/L^{15}*(3375/128*E^{2}/ING^{2}*SI^{4}-7875/128*E^{2}/ING^{2}*SI^{6}+18375/512*E^{2}/ING^{2}*SI^{8}-675/64*E^{2}/ING*SI^{2}+4725/128*E^{2}/ING*SI^{4}-3675$$
$$/128*E^{2}/ING*SI^{6})$$

$$+ 1/A2*A3^{2}*CI/L^{11}*(-15/16*SI^{2}-3/16*ING+15/8*E^{2}-135/16*E^{2}*SI^{2}-21/16*E^{2}*ING)$$

$$+ A2*A4*CI/L^{15}*(-495/16+3105/32*SI^{2}-9135/128*SI^{4}+2925/64*E^{2}/ING^{2}*SI^{4}-13575/128*E^{2}/ING^{2}*SI^{6}+7875/128*E^{2}/ING^{2}*SI^{8}-585/32*E^{2}/ING$$
$$*SI^{2}+8145/128*E^{2}/ING*SI^{4}-1575/32*E^{2}/ING*SI^{6}-3825/16*E^{2}+45945/64*E^{2}*SI^{2}-129465/256*E^{2}*SI^{4})$$

$$+ A2^{3}*CI/L^{15}*(405/16-2091/32*SI^{2}+333/8*SI^{4}+2535/128*E^{2}/ING^{2}*SI^{4}-2925/64*E^{2}/ING^{2}*SI^{6}+3375/128*E^{2}/ING^{2}*SI^{8}-507/64*E^{2}/ING*SI^{2}$$
$$+1755/64*E^{2}/ING*SI^{4}-675/32*E^{2}/ING*SI^{6}+1263/8*E^{2}-26523/64*E^{2}*SI^{2}+137385/512*E^{2}*SI^{4})$$

$$\frac{\partial F_4^{**}}{\partial L}$$

$$1/A2^{2}*A4^{3}/L^{19}*(-3375/64/ING^{2}*SI^{4}+167625/512/ING^{2}*SI^{6}-727125/1024/ING^{2}*SI^{8}+1341375/2048/ING^{2}*SI^{10}-900375/4096/ING^{2}*SI^{12})$$

$$+ 1/A2^{2}*A3^{2}*A4/L^{15}*(-225/64/ING*SI^{4}+975/128/ING*SI^{6}-525/128/ING*SI^{8}+15/8-225/8*SI^{2}+2475/32*SI^{4}-6825/128*SI^{6})$$

$$+ A4^{2}/L^{19}*(225/64/ING^{2}*SI^{4}+62475/512/ING^{2}*SI^{6}-912675/2048/ING^{2}*SI^{8}+531825/1024/ING^{2}*SI^{10}-1635375/8192/ING^{2}*SI^{12}-12825/128/ING$$
$$*SI^{4}+199125/512/ING*SI^{6}-126525/256/ING*SI^{8}+422625/2048/ING*SI^{10}+675/64-3375/64*SI^{2}+58725/1024*SI^{4}+35385/2048*SI^{6}-140385$$
$$/4096*SI^{8})$$

$$+ A3^{2}/L^{15}*(-195/64/ING*SI^{4}+105/16/ING*SI^{6}-225/64/ING*SI^{8}+27/8-21/4*SI^{2}-1113/128*SI^{4}+345/32*SI^{6})$$

$$+ A2^{2}*A4/L^{19}*(7995/64/ING^{2}*SI^{4}-267825/512/ING^{2}*SI^{6}+417315/512/ING^{2}*SI^{8}-1145325/2048/ING^{2}*SI^{10}+291375/2048/ING^{2}*SI^{12}-9855/64$$
$$/ING*SI^{4}+295455/512/ING*SI^{6}-731235/1024/ING*SI^{8}+149625/512/ING*SI^{10}-1215/32+14775/64*SI^{2}-19245/32*SI^{4}+1401225/2048*SI^{6}$$
$$-1133265/4096*SI^{8})$$

$$+ A2^{4}/L^{19}*(4563/64/ING^{2}*SI^{4}-169923/512/ING^{2}*SI^{6}+1185585/2048/ING^{2}*SI^{8}-459225/1024/ING^{2}*SI^{10}+266625/2048/ING^{2}*SI^{12}-7449/128$$
$$/ING*SI^{4}+6663/32/ING*SI^{6}-127215/512/ING*SI^{8}+50625/512/ING*SI^{10}-459/16+18213/128*SI^{2}-234651/1024*SI^{4}+543273/4096*SI^{6}-68721$$
$$/4096*SI^{8})$$

$$\frac{\partial F_4^{**}}{\partial G}$$

$$1/A2^2*A4^3/L^{19}*(3375/64/ING^2*SI^4-167625/512/ING^2*SI^6+727125/1024/ING^2*SI^8-1341375/2048/ING^2*SI^{10}+900375/4096/ING^2*SI^{12})$$

$$+1/A2^2*A3^2*A4/L^{15}*(225/64/ING*SI^4-975/128/ING*SI^6+525/128/ING*SI^8-15/4+945/16*SI^2-675/4*SI^4+15255/128*SI^6)$$

$$+A4^2/L^{19}*(-225/64/ING^2*SI^4-62475/512/ING^2*SI^6+912675/2048/ING^2*SI^8-531825/1024/ING^2*SI^{10}+163537\text{5}/8192/ING^2*SI^{12}+12825/128$$

$$/ING*SI^4-199125/512/ING*SI^6+126525/256/ING*SI^8-422625/2048/ING*SI^{10}-1125/16+63225/128*SI^2-621875/512*SI^4+2941105/2048*SI^6$$

$$-459375/1024*SI^8)$$

$$+A3^2/L^{15}*(195/64/ING*SI^4-105/16/ING*SI^6+225/64/ING*SI^8-36+3183/16*SI^2-38541/128*SI^4+2265/16*SI^6)$$

$$+A2^2*A4/L^{19}*(-7995/64/ING^2*SI^4+267825/512/ING^2*SI^6-417315/512/ING^2*SI^8+1145325/2048/ING^2*SI^{10}-291375/2048/ING^2*SI^{12}+9855/64$$

$$/ING*SI^4-295455/512/ING*SI^6+731235/1024/ING*SI^8-149625/512/ING*SI^{10}+9045/16-14025/4*SI^2+502365/64*SI^4-15428745/2048*SI^6$$

$$+10786545/4096*SI^8)$$

$$+A2^4/L^{19}*(-4563/64/ING^2*SI^4+169923/512/ING^2*SI^6-1185585/2048/ING^2*SI^8+459225/1024/ING^2*SI^{10}-266425/2048/ING^2*SI^{12}+7449/128$$

$$/ING*SI^4-6663/32/ING*SI^6+127215/512/ING*SI^8-50625/512/ING*SI^{10}-44307/128+194055/128*SI^2-1089285/512*SI^4+4046955/4096*SI^6$$

$$-12723/1024*SI^8)$$

$$\frac{\partial F_4^{**}}{\partial H}$$

$$1/A2^2*A3^2*A4*CI/L^{19}*(15/8-255/16*SI^2+315/16*SI^4)$$

$$+A4^2*CI/L^{19}*(225/8-20475/128*SI^2+284025/1024*SI^4-152145/1024*SI^6)$$

$$+A3^2*CI/L^{15}*(135/8-939/16*SI^2+2925/64*SI^4)$$

$$+A2^2*A4*CI/L^{19}*(-7155/32+66525/64*SI^2-195975/128*SI^4+23205/32*SI^6)$$

$$+A2^4*CI/L^{19}*(16389/128-24189/64*SI^2+284589/1024*SI^4-9201/1024*SI^6)$$

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

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