NASA TECHNICAL NOTE

NASA TN D-8525

c. 1

NASA TN D-8525

AN OPTIMALITY CRITERION FOR SIZING MEMBERS OF HEATED STRUCTURES WITH TEMPERATURE CONSTRAINTS

G. Venkateswara Rao, Charles P. Shore, and R. Narayanaswami

Langley Research Center
Hampton, Va. 23665

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1977

1. Report No. NASA TN D-8525	2. Government Accession No.
4. Title and Subtitle AN OPTIMALITY CRITERION FOR SIZING MEMBERS OF HEATED STRUCTURES WITH TEMPERATURE CONSTRAINTS	5. Report Date October 1977
	6. Performing Organization Code
7. Author(s) G. Venkateswara Rao, Charles P. Shore, and R. Narayanaswami	8. Performing Organization Report No. L-11601
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665	10. Work Unit No. 505-02-12-01
	11. Contract or Grant No.
	13. Type of Report and Period Covered Technical Note
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546	14. Sponsoring Agency Code

15. Supplementary Notes

G. Venkateswara Rao: NRC-NASA Resident Research Associate.
Charles P. Shore: Langley Research Center.
R. Narayanaswami: Old Dominion University, Norfolk, Virginia.

16. Abstract

A thermal optimality criterion is presented for sizing members of heated structures with multiple temperature constraints. The optimality criterion is similar to an existing optimality criterion for design of mechanically loaded structures with displacement constraints. Effectiveness of the thermal optimality criterion is assessed by applying it to one- and two-dimensional thermal problems where temperatures can be controlled by varying the material distribution in the structure. Results obtained from the optimality criterion agree within 2 percent with results from a closed-form solution and with results from a mathematical programing technique. The thermal optimality criterion augments existing optimality criteria for strength and stiffness related constraints and offers the possibility of extension of optimality techniques to sizing structures with combined thermal and mechanical loading.

17. Key Words (Suggested by Author(s)) Optimization Thermal structures Optimality criterion	18. Distribution Statement Unclassified – Unlimited Subject Category 39

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	42	$4.00

* For sale by the National Technical Information Service, Springfield, Virginia 22161

AN OPTIMALITY CRITERION FOR SIZING MEMBERS OF HEATED STRUCTURES WITH TEMPERATURE CONSTRAINTS

G. Venkateswara Rao,* Charles P. Shore,
and R. Narayanaswami**
Langley Research Center

SUMMARY

A thermal optimality criterion is presented for sizing members of heated structures with multiple temperature constraints. The optimality criterion is similar to an existing optimality criterion for design of mechanically loaded structures with displacement constraints. Effectiveness of the thermal optimality criterion is assessed by applying it to one- and two-dimensional thermal problems where temperatures can be controlled by varying the material distribution in the structure. Results obtained from the optimality criterion agree within 2 percent with results from a closed-form solution and with results from a mathematical programing technique. The thermal optimality criterion augments existing optimality criteria for strength and stiffness related constraints and offers the possibility of extension of optimality techniques to sizing structures with combined thermal and mechanical loading.

INTRODUCTION

Aerospace structures may be subjected to severe mechanical and thermal loading. Efficient designs for such structures increasingly require use of automated design methods to determine minimum-mass structural arrangements that satisfy several mechanical and thermal design requirements. Both mathematical programing methods (ref. 1) and optimality criteria (refs. 2 to 4) have been proposed for automated design of mechanically loaded structures with strength, stiffness, and flutter constraints.

Recent studies have focused on resizing procedures for structures under combined mechanical and thermal loading with strength constraints. In references 5 and 6, steady-state thermal effects were included in an implicit fashion through strength requirements; however, temperatures in the structure were not updated as a result of structural resizing and significant unknown differences can exist between final and initial temperature distributions in the structure. Explicit thermal design requirements and updating of temperatures during the resizing process were included in a finite-element-based mathematical programing method described in reference 7 where membrane element thicknesses and bar element areas were used as design variables to adjust average element temperatures.

*NRC-NASA Resident Research Associate.
**Old Dominion University, Norfolk, Virginia.

The present paper presents an optimality criterion which satisfies temperature requirements and permits updating of structural nodal temperatures during resizing. The optimality criterion resizing formulas are similar to those of reference 3 for design of mechanically loaded structures with displacement constraints. Only temperatures (not stresses) are considered in the present paper, and use of the criterion is illustrated by applying it to several examples with both one- and two-dimensional heat-transfer characteristics. An existing closed-form solution (ref. 8) is used to assess the effectiveness of the optimality criterion for one-dimensional problems, and a mathematical programing technique described in references 9 and 10 is used for a similar assessment for two-dimensional problems.

SYMBOLS

Values are given in both SI and U.S. Customary Units. The measurements and calculations were made in U.S. Customary Units.

A — area of bar element

$\bar{A}$ — $= A/A_o$

a — general design variable

B_i — Biot number, $h l_b^2/kA_o$ for bar and $h l_p^2/kt_o$ for plate

b — convergence parameter

c,c_1,c_2,c_3 — points on bar and plate at which constraints are applied

$[C]$ — conductance matrix

F — constraint function

$[H]$ — convective matrix

h — convective heat-transfer coefficient

k — thermal conductivity

l_b — length of bar

l_p — characteristic length of plate

ℓ — length of bar element

m — mass of structure

N_b — number of bar elements

N_G	heat-generation number, $q l_b^2/kA_oT_o$ for bar and $q l_p/kt_oT_o$ for plate
N_p	number of plate elements
N_T	number of points at which temperatures are constrained
$\{Q\}$	unit thermal load vector
q	rate of heat input
$\{q\}_1$	thermal load vector
$\{q\}_2$	convection load vector
S	surface area of plate element
T	temperature
$\bar{T}$	$= T/T_o$
$\{T\}$	temperature vector
$\{T_Q\}$	temperature vector corresponding to $\{Q\}$
T_a	ambient temperature
$\bar{T}_a$	nondimensional ambient temperature
T_c	specified temperature at point c
$\bar{T}_c, \bar{T}_{c1}, \bar{T}_{c2}, \bar{T}_{c3}$	specified nondimensionalized temperatures at constraint points c, c_1, c_2, and c_3
t	thickness of plate element
V	volume of structure
$\bar{V}$	$\begin{cases} = V/A_o l_b & \text{for bar} \\ = V/t_o l_p^2 & \text{for plate} \end{cases}$
x,y	Cartesian coordinates
$\bar{x},\bar{y}$	nondimensional Cartesian coordinates, $x/l_{b,p}$ and $y/l_{b,p}$, respectively
γ	step-size parameter
λ	Lagrangian multiplier
ρ	density

Subscripts:

b	bar
i	ith bar element
j	jth bar element
k	number of constraint point
o	reference value
p	plate

OPTIMALITY CRITERION

An optimality criterion for thermally loaded structures is developed from the general treatment presented in reference 3 for problems which can be characterized by an objective function m that is dependent on a group of design variables a_i

$$m = m(a_i) \tag{1}$$

and an equality constraint function that may be implicitly dependent on the design variables a_i

$$F(a_i) - F_c = 0 \tag{2}$$

The quantity F_c is the desired value of $F(a_i)$ at point c.

To obtain the necessary condition for a minimum of m subject to equation (2), first append the constraint condition to the objective function with a Lagrangian multiplier

$$m^* = m(a_i) + \lambda\left[F(a_i) - F_c\right] \tag{3}$$

and then equate to zero the first partial derivatives of m^* with respect to the design variables a_i

$$\frac{\partial m^*}{\partial a_i} = \frac{\partial m}{\partial a_i} + \lambda \frac{\partial F}{\partial a_i} = 0 \tag{4}$$

Equation (4) is also called the optimality criterion.

For two-dimensional thermally loaded structures the response quantity $F(a_i)$ is the structural temperature and is given by the following partial differential equation and the appropriate thermal boundary conditions:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + Q = 0 \tag{5}$$

where k_x and k_y are the material thermal conductivities in the x- and y-directions, respectively, and Q is the applied thermal loading.

Equation (4) is specialized in reference 3 for finite-element representations of mechanically loaded structures with deflection constraints. The finite-element representation allows both $m(a_i)$ and $\partial F(a_i)/\partial a_i$ to be expressed as sums of individual member contributions. As a result, the simultaneous equations represented by equation (4) uncouple and can be solved by simple recursive formulas.

For structures with both convective and conductive modes of heat transfer present, equation (5) can be written in finite-element matrix notation as

$$[D]\{T\} = \{q\}_1 + \{q\}_2 \tag{6}$$

where $[D]$ is a heat-transfer matrix and includes terms for both convective and conductive heat transfer, $\{T\}$ is a vector of unknown temperatures, and $\{q\}_1$ and $\{q\}_2$ are applied thermal and convective heat load vectors, respectively. Since equation (6) is analogous to the corresponding equation for displacements in mechanically loaded structures and since use of thermal finite elements also permits uncoupling of the simultaneous equations represented by equation (4), the optimality criterion for deflection constraints developed in reference 3 can be adapted to problems with temperature constraints. Appropriate expressions for the optimality criterion and resizing formulas for bar and plate elements are now restated in terms of thermal quantities. Equation numbers from reference 3 are also given. For clarity and completeness details of the adaptation are presented in the appendix.

Optimality Criterion and Resizing Formulas for a Single Temperature Constraint

For a single temperature constraint the optimality criterion given by equation (4) can be expressed in a form similar to equation (44) of reference 3 as

$$\frac{e_i}{\rho_b} = 1 \tag{7}$$

for bar elements and

$$\frac{e_j}{\rho_p} = 1 \tag{8}$$

for plate elements where

$$e_i = \frac{\lambda\{T\}_i^T[C]_i\{T_Q\}_i}{\ell_i A_i} \tag{9}$$

and

$$e_j = \frac{\lambda\{T\}_j^T[C]_j\{T_Q\}_j}{S_j t_j} \tag{10}$$

In the expressions for e_i and e_j, λ is the unknown Lagrangian multiplier, $\{T\}$ is the vector of nodal temperatures obtained by solution of equation (6) for the applied thermal loading, $[C]$ is the elemental conductivity matrix, $\{T_Q\}$ is the temperature vector obtained by solution of equation (6) for a unit thermal load applied at the constraint point c, ℓ_i and A_i are the length and area of the ith bar element, respectively, and S_j and t_j are the surface area and thickness of the jth plate element, respectively. The subscripts i and j on the quantities $\{T\}$, $[C]$, and $\{T_Q\}$ indicate that these quantities are associated with the ith bar element or the jth plate element.

The resizing formula given by equation (48) of reference 3 is rewritten as

$$A_{i,n+1} = \left(\frac{e_i}{\rho_b}\right)^{\gamma} A_{i,n} \tag{11}$$

for bar elements and

$$t_{j,n+1} = \left(\frac{e_j}{\rho_p}\right)^{\gamma} t_{j,n} \tag{12}$$

for plate elements, where n is the iteration number and γ is a paramter that controls the step size.

Optimality Criterion and Resizing Formulas for Multiple Temperature Constraints

If the structure has N_T temperature constraints at points c_k ($k = 1, 2, \ldots, N_T$), N_T Lagrangian multipliers are required, and equation (22) of reference 3 can be rewritten for bar elements as

$$\frac{E_i}{\rho_b} = 1 \tag{13}$$

and for plate elements as

$$\frac{E_j}{\rho_p} = 1 \tag{14}$$

where

$$E_i = \sum_{k=1}^{N_T} \frac{\lambda_k \{T\}_i^T [C]_i \{T_Q\}_{i,k}}{\ell_i A_i} \tag{15}$$

and

$$E_j = \sum_{k=1}^{N_T} \frac{\lambda_k \{T\}_j^T [C]_j \{T_Q\}_{j,k}}{S_j t_j} \tag{16}$$

The resizing formulas become

$$A_{i,n+1} = \left(\frac{E_i}{\rho_b}\right)^{\gamma} A_{i,n} \tag{17}$$

for bar elements and

$$t_{j,n+1} = \left(\frac{E_j}{\rho_p}\right)^{\gamma} t_{j,n} \tag{18}$$

for plate elements.

In the resizing formulas the Lagrangian multipliers are determined by equation (7) of reference 4 rewritten as

$$\lambda_{k,n+1} = \left(\frac{T_k}{T_{c,k}}\right)^b \lambda_{k,n} \qquad (19)$$

where T_k and $T_{c,k}$ are the temperature and the desired temperature at the kth constraint point, respectively, and b is a convergence parameter. Equation (19) also permits consideration of inequality constraints of the form

$$T_k \leqq T_{c,k} \qquad (20)$$

since for passive constraints successive values of λ_k become small and eventually approach zero and their impact in the resizing formulas becomes negligible.

The number of iterations required to obtain a minimum-mass design depends on the value of the step-size parameter γ in equations (11), (12), (17), and (18) and on the value of the convergence parameter b in equation (19). Generally, a trial-and-error technique is used to find values of γ and b which lead to minimum-mass designs with the fewest iterations. Since these parameters may be problem dependent, new values of γ and b should be determined for each new type of problem. To start the resizing process, an initial vector of design variables and initial values of λ_k are required in the resizing formulas. As a general procedure, for the problems considered herein, a unit initial design vector and unit initial values of λ_k are used to start the process.

RESULTS AND DISCUSSION

The effectiveness of the optimality criterion for problems with temperature constraints is assessed by applying the resizing formulas presented in the previous section to the following problems involving uniform heating:

(1) Cooling fin: A one-dimensional heat conduction problem with a single temperature constraint. Results for no convective heat loss are presented to demonstrate the effect of both the step-size parameter γ and the convergence parameter b on the resizing process and to assess effects of discretization on convergence to the closed-form solution for the minimum-mass design of cooling fins presented in reference 8. Effects of convective heat loss on the minimum fin mass for a given heating condition are also presented.

(2) Square plate: A two-dimensional heat conduction problem with a single temperature constraint. Effects of convective heat loss on minimum plate mass for given heating conditions are considered, and results from the optimality criterion are compared with similar results obtained from the mathematical programing technique of usable feasible directions described in references 9 and 10.

(3) Triangular plate: Similar to the square-plate problem except that convective heat loss is not included and effects of multiple temperature constraints are considered.

(4) Combined plate and bars: A problem with both one- and two-dimensional heat conduction paths and with multiple temperature constraints. Convective heat losses are not considered; however, two materials with different mass densities and thermal conductivities are used for the plates and bars to determine a minimum-mass solution for a problem with mixed materials.

Cooling Fin

The geometry of a cooling fin with the associated boundary conditions is shown in figure 1. Heat is generated in the fin at a constant rate of q per unit length, and convective heat loss may occur over both the top and bottom fin surface. The end $X = 0$ is maintained at a nondimensional temperature of zero, and the insulated end (point c) is constrained to a given value. The objective is to minimize the fin mass (or volume for constant material density) while satisfying the constraint condition.

One-dimensional thermal finite elements with uniform areas and cubic temperature distributions were used to model the cooling fin to determine the temperature distribution. The elements have two nodes with T and dT/dx as nodal degrees of freedom.

Results without convective heat loss.- The cooling fin problem without convective heat loss was used to determine values for the step-size parameter γ and the convergence parameter b in the resizing formulas (eqs. (11) and (19)). Based on results from references 3 and 4, a value of 0.5 was initially used for γ. The fin was idealized by five elements and minimum-volume solutions were obtained for a heat-generation number $N_G = 1.0$, a constraint temperature $\bar{T}_c = 0.3$, and values of b from 0.1 to 0.9. Results for the fin volume $\bar{V}$, temperature $\bar{T}_c$ at the constraint point, and the number of iterations required to reach convergence are given in table I as functions of b. The number of iterations for convergence are also plotted as a function of b in figure 2. The data indicate that each solution converges to the same minimum fin volume; however, values of b from 0.5 to 0.7 require the fewest iterations to reach converged values of the fin volume and simultaneously satisfy the temperature constraint to four significant digits. The convergence parameter $b = 0.5$ was selected as a best value for this problem. Additional solutions were then obtained for $b = 0.5$ and values of the step-size parameter γ of 1 and 1/3. Results from these solutions revealed that $\gamma = 1$ caused the resizing process to diverge and that $\gamma = 1/3$ increased the number of iterations required for convergence. Since the other problems considered in this investigation were similar to the fin problem, values of $\gamma = 0.5$ and $b = 0.5$ were used for the remaining calculations. Values of these parameters, which produce most rapid convergence, however, are problem dependent and should be redetermined for problems which differ significantly from those considered herein.

To study the effects of discretization on the finite-element procedure used for this problem, the fin was modeled with 5, 10, and 20 elements and minimum fin volumes were determined for $N_G = 1.0$ and $\overline{T}_c = 0.3$. The results are given in table II and figure 3 and are compared with similar results from the closed-form solution of reference 8. Converged values of the minimum fin volume as a function of the number of elements are shown in figure 3(a). The finite-element results approach the closed-form solution slowly as the number of elements is increased; the difference between the two solutions decreases from 2.4 to 1.4 percent as the number of elements increases from 5 to 20. Fin volume $\overline{V}$ and constraint temperature $\overline{T}_c$ obtained from the 20-element solution are shown in figures 3(b) and 3(c), respectively, as a function of iteration number. The data indicate that good convergence for $\overline{V}$ and $\overline{T}_c$ is obtained in 5 to 10 iterations. More detailed results are given in table II.

Fin cross-sectional area distributions for the 5-, 10-, and 20-element solutions are compared with the area distribution from the closed-form solution in figure 4. The area distributions from the optimality criterion method slowly approach the closed-form solution as the number of elements increase. The 20-element solution for the fin volume is within 2 percent of the closed-form solution. However, near the fin root, the areas given by the optimality criterion method are less than the closed-form solution. Near the tip, the areas given by the optimality criterion method are greater than the closed-form solution. A finer mesh size near the fin tip or use of tapered elements could improve the agreement between the finite-element solution and the closed-form solution; however, since the objective of this investigation is to demonstrate the use of the optimality criterion, such refinements are considered unwarranted.

The fin temperature distribution from the 20-element solution is compared with the closed-form solution in figure 5. At the center of the fin, the temperatures given by the finite-element solution are approximately 6 percent greater than the temperatures given by the closed-form solution. Again, this difference is attributable to use of constant-area elements and could be removed by use of a finer mesh size or tapered elements.

Results with convective heat loss.- To determine the influence of terms in the heat-transfer matrix [D], which are not functions of the design variables (see discussion in the appendix), solutions for minimum fin volume were obtained by using 5, 10, and 20 elements for values of B_i from 0 to 0.5 and $N_G = 1.0$, $\overline{T}_c = 0.3$, and $\overline{T}_a = 0.05$. Converged values of minimum fin volume were obtained within 20 iterations for each idealization and value of B_i. (See table III.) Since converged solutions were obtained within 20 iterations, effects on the resizing process of including constant terms in the heat-transfer matrix [D] appear to be negligible for this problem. Area and temperature distributions from the 20-element solution for $B_i = 0$ and 0.5 are shown in figures 6(a) and 6(b), respectively. The variation of minimum fin volume $\overline{V}$ with B_i is shown in figure 7. These results indicate that the minimum fin volume decreases and temperatures in the fin increase as convective heat losses increase.

Square Plate

The effectiveness of the optimality criterion resizing formulas for two-dimensional problems with and without convective heat losses was assessed by obtaining minimum-mass (minimum volume for constant-mass density) solutions for a uniformly heated square plate with a single temperature constraint. Two-dimensional triangular thermal finite elements of constant thickness with a quadratic temperature distribution were used to determine the temperature response of the plate. The elements had six nodes, three at the vertices and three at the midpoints of the sides; temperature was the nodal degree of freedom. The geometry, finite-element subdivision, and thermal boundary conditions for the plate are shown in figure 8. Temperature at point c was constrained to $\bar{T}_c = 10.0$ and convective heat loss to the ambient atmosphere could occur from both top and bottom plate surfaces. Minimum-volume solutions were obtained for $N_G = 1.0$, $\bar{T}_a = 5.0$, and values of $B_i = 0$ (no convective heat loss), 0.025, and 0.075. Iteration histories for the plate volume and temperature at the constraint point are given in table IV. Results from the mathematical programing technique of references 9 and 10 are also shown.

Results without convective heat loss.- Results in table IV indicate that in the absence of convective heat loss converged values of the plate volume and temperature at the constraint point are obtained with 20 iterations and agree with the mathematical programing results within approximately 1 percent. The thickness distribution corresponding to the minimum-volume solution for the finite-element mesh shown in figure 8 is presented in figure 9. Since the plate is symmetrical about the diagonal, only half of the plate is shown. The greatest material thickness is required along the insulated edges of the plate where the thickness increases from the constraint point to the heat sink. The temperature distribution in the plate for the minimum-volume solution is shown in figure 10 and indicates that, although the temperature constraint at point c is satisfied, the temperature at one point on the diagonal is approximately $2\frac{1}{2}$ times the constraint temperature. Thus, if it is important that temperatures over the entire plate do not exceed the value at point c, then temperatures at additional points must be constrained. Thickness and temperature distributions from the mathematical programing technique were virtually the same as those from the optimality criterion.

Results with convective heat loss.- Iteration histories of $\bar{V}$ and $\bar{T}$ at the constraint point for the square plate for $B_i = 0$ and $B_i = 0.075$ are shown in figures 11(a) and 11(b), respectively. In contrast to the cooling fin problem, where convergence occurred monotonically, convergence for the two-dimensional plate problem is oscillatory. Furthermore, the presence of the design-variable independent convective terms in the heat-transfer matrix causes the temperature response in the plate to become less sensitive to changes in the design variables and increases the number of iterations required for convergence. Results in table IV indicate that the optimality criterion yields values of $\bar{V}$ and $\bar{T}$ at the constraint point for $B_i = 0.025$ and 0.075 that agree within approximately 1 percent with similar values from the mathematical programing technique.

Triangular Plate

The geometry, boundary conditions, and finite-element idealization for a uniformly heated triangular plate are shown in figure 12. The triangular elements described previously were used to determine temperature response for the plate. Convective heat losses were not included in this problem. Minimum-volume solutions were obtained for the plate for $N_G = 1.0$ with temperature constraints at points c_1, c_2, and c_3. Two constraint combinations were considered; namely, (1) point c_3 was constrained to $\overline{T}_{c3} = 10.0$ with points c_1 and c_2 unconstrained and (2) $\overline{T}_{c3} = 10.0$ with points c_1 and c_2 constrained to $\overline{T}_{c1} = \overline{T}_{c2} = 5.0$. Iteration histories for minimum plate volume $\overline{V}$ and temperatures at the constraint points from the optimality criterion for the two constraint combinations are shown in tables V(a) and V(b), respectively. Final results from the mathematical programing technique of reference 9 are also shown. A comparison of results from the optimality criterion and the mathematical programing technique indicates that the two solutions for the minimum plate volume and constraint temperatures agree within about 1 percent. The iteration histories for plate volumes $\overline{V}$ and temperatures at the constraint points are shown in figures 13(a) and 13(b), respectively. Inclusion of the additional temperature constraints has two significant effects: (1) Since the constraint temperature at points c_1 and c_2 is less than the corresponding temperature from the single constraint solution, additional plate volume is required to satisfy the constraints; and (2) the number of iterations required to reach a converged plate volume and simultaneously satisfy the multiple temperature constraints is almost double that for the single constraint case.

Temperature distributions for the single and multiple temperature constraint conditions are shown in figures 14(a) and 14(b), respectively. Just as in the case of the square plate, when the temperature is constrained only at point c_3, the temperature at other points on the plate greatly exceed the constraint temperature. The additional constraints at points c_1 and c_2 improve but do not eliminate this condition. If it is important that temperatures over the entire plate not exceed the value at point c_3, temperatures at additional points must be constrained.

Combined Plate-Bar Problem

To illustrate further the usefulness of the optimality criterion, it was applied to the combined plate-bar problem shown in figure 15(a). Titanium was used for the plate material and aluminum was used for the bars. This structure was representative of a thermal design problem where one material satisfied strength requirements and the other acted as a highly efficient conductor to transfer the incident heat to a heat sink. The structure was uniformly heated at a rate of $q = 34.1$ kW/m^2 (10 800 Btu/ft^2-hr) and was idealized as shown in figure 15(b) by eight of the previously described triangular thermal finite elements and four bar elements. Three noded constant-area bar elements with quadratic temperature distributions were used to model the bars to insure compatibility between temperatures in the two types of elements. Temperatures in the structure were constrained not to exceed 260° C (500° F). Thus, based on the finite-element idealization, this problem had 20 temperature constraints.

A converged solution for the minimum-mass configuration that satisfied the temperature constraints was obtained with 90 iterations. The large number of iterations required for convergence results from the large number of constraints in the problem. For the idealization used in the problem, a minimum mass of 0.17 kg (0.37 lbm) was obtained. When only the titanium plate was used, a minimum mass of 0.60 kg (1.33 lbm) was required to satisfy the multiple temperature constraints. Thus, addition of the aluminum bars for the purpose of heat conduction reduced the structural mass by about 70 percent.

The plate thickness distribution and bar area distribution for square bars corresponding to the minimum-mass configuration are shown in figure 16. Since the problem is symmetric, only half of the structure is shown. Nodal temperatures for the minimum-mass solution are given in table VI. A complete picture of the temperature field in the structure is shown in figure 17. Since the nodal temperatures presented in table VI are at discrete points, the temperatures shown in figure 17 are calculated over the entire plate at very small intervals. Again, because of the symmetry in the problem, temperatures are shown for only half of the structure. The sharp dips in the temperature distribution near the aluminum bars occur because the bars provide a better conduction path to the heat sink than the titanium plate.

CONCLUDING REMARKS

An optimality criterion is presented for minimum-mass design of heated structures with constraints on temperature. The thermal optimality criterion is similar to an existing optimality criterion for mechanically loaded structures with displacement constraints. Resizing formulas from the thermal optimality criterion are applied to both one- and two-dimensional problems where temperatures can be controlled by varying material distribution in the structure. A comparison of results obtained from the optimality criterion with results from a closed-form solution and with results from a mathematical programing technique reveal that the optimality criterion results agreed within 2 percent of those from the other methods.

For the one-dimensional problems the optimality criterion converges monotonically. However, for the two-dimensional problems convergence occurs in an oscillatory manner, and the number of iterations required for convergence increases significantly with either the number of temperature constraints or degree of convective heat loss present in the problem.

The thermal optimality criterion is used to size components of a heated structure composed of titanium plates and aluminum bars with temperatures constrained to 260° C (500° F) or less. Based on thermal considerations only, adding aluminum bars reduces the combined mass by about 70 percent of that for titanium plates alone.

The thermal optimality criterion augments optimality criteria for mechanically loaded structures with strength and stiffness related constraints and offers the possibility of extension of optimality techniques to resizing structures with combined thermal and mechanical loading.

Langley Research Center
National Aeronautics and Space Administration
Hampton, VA 23665
August 9, 1977

APPENDIX

THERMAL OPTIMALITY CRITERION

Single Temperature Constraint

For structures with both conductive and convective modes of heat transfer present, the equation governing temperature response in the structure can be written in finite-element matrix notation as

$$[D]\{T\} = \{q\}_1 + \{q\}_2 \tag{A1}$$

where $[D]$ is a heat-transfer matrix composed of terms for both conductive and convective heat transfer, $\{T\}$ is a vector of unknown temperatures, and $\{q\}_1$ and $\{q\}_2$ are applied thermal and convective heat load vectors, respectively. The matrix $[D]$ can be written as

$$[D] = [C] + [H] \tag{A2}$$

where $[C]$ is a conduction matrix and $[H]$ is a convection matrix. For structures which can be represented by N_b bar elements of constant area and N_p plate elements of constant thickness, the total mass can be expressed as

$$m = \sum_{i=1}^{N_b} \rho_b A_i \ell_i + \sum_{j=1}^{N_p} \rho_p t_j S_j \tag{A3}$$

where ρ_b and ρ_p are the mass densities of the bar and plate materials, respectively; A_i and ℓ_i are the cross-sectional area and length of the ith bar element, respectively; t_j and S_j are the thickness and surface area of the jth plate element, respectively. The design variables are A_i and t_j.

The temperature at some point c on the structure is constrained to a specified value T_c. The equation of constraint is

$$T - T_c = 0 \tag{A4}$$

or

$$\{T\}^T\{Q\} - T_c = 0 \tag{A5}$$

where $\{T\}$ is the nodal temperature vector obtained by solution of equation (A1) and $\{Q\}$ is a vector of zeros except for a value of unity at the node corresponding to point c. The optimality criterion is given by equation (4) in the main text and is rewritten in terms of the thermal problem as

$$\rho_b \ell_i + \lambda \frac{\partial \{T\}^T}{\partial A_i} \{Q\} = 0 \tag{A6}$$

for bar elements and

$$\rho_p S_j + \lambda \frac{\partial \{T\}^T}{\partial t_j} \{Q\} = 0 \tag{A7}$$

for plate elements where λ is an unknown Lagrangian multiplier.

The vector $\{Q\}$ can be written as

$$\{Q\} = [D] \{T_Q\} \tag{A8}$$

where $\{T_Q\}$ is a temperature vector obtained by solution of equation (A1) for a unit thermal load applied at point c. Replacing $\{Q\}$ in equations (A6) and (A7) with its equivalent from equation (A8) gives

$$\rho_b \ell_i + \lambda \frac{\partial \{T\}^T}{\partial A_i} [D] \{T_Q\} = 0 \tag{A9}$$

and

$$\rho_p S_j + \lambda \frac{\partial \{T\}^T}{\partial t_j} [D] \{T_Q\} = 0 \tag{A10}$$

as expressions of the optimality criterion for bars and plates, respectively.

Expressions for the partial derivatives of the temperature vector can be obtained by differentiation of equation (A1). Note that, for the type of thermal finite elements used to model the structure, only the conduction matrix $[C]$

is proportional to A_i and t_j. Differentiation of equation (A1) with respect to A_i gives

$$\frac{\partial [C]}{\partial A_i}\{T\} + [D]\frac{\partial \{T\}}{\partial A_i} = 0 \tag{A11}$$

or

$$\frac{\partial \{T\}^T}{\partial A_i}[D] = -\{T\}_i^T \frac{\partial [C]_i}{\partial A_i} \tag{A12}$$

Similarly for t_j

$$\frac{\partial \{T\}^T}{\partial t_j}[D] = -\{T\}_j^T \frac{\partial [C]_j}{\partial t_j} \tag{A13}$$

Since, for the type elements used herein, the conduction matrix $[C]$ is proportional to A_i and t_j, the partials of $[C]_i$ with respect to A_i and t_j can be written as

$$\frac{\partial [C]_i}{\partial A_i} = \frac{[C]_i}{A_i} \tag{A14}$$

and

$$\frac{\partial [C]_j}{\partial t_j} = \frac{[C]_j}{t_j} \tag{A15}$$

Thus, the optimality criterion can finally be written as

$$\rho_b \ell_i - \lambda\{T\}_i^T \frac{[C]_i}{A_i}\{T_Q\}_i = 0 \tag{A16}$$

for bars and

$$\rho_p S_j - \lambda\{T\}_j^T \frac{[C]_j}{t_j}\{T_Q\}_j = 0 \qquad \text{(A17)}$$

for plates. The subscripts i and j on the quantities $\{T\}$, $[C]$, and $\{T_Q\}$ indicate that those quantities are associated with the ith bar element or jth plate element.

Multiple Temperature Constraints

If the structure has N_T temperature constraints at points c_k $(k = 1, 2, \ldots, N_T)$, the constraint equations are written as

$$\{T\}^T\{Q\}_k - T_{c,k} = 0 \qquad (k = 1, 2, \ldots, N_T) \qquad \text{(A18)}$$

The functional given by equation (3) in the main text becomes

$$m^* = \sum_{i=1}^{N_b} \rho_b A_i \ell_i + \sum_{j=1}^{N_p} \rho_p t_j S_j + \sum_{k=1}^{N_T} \lambda_k\left[\{T\}^T\{Q\}_k - T_{c,k}\right] \qquad \text{(A19)}$$

where λ_k are the unknown Lagrangian multipliers for each constraint k. Application of the same procedure used for a single constraint leads to the optimality criterion for multiple constraints

$$\rho_b \ell_i - \sum_{k=1}^{N_p} \lambda_k\{T\}_i^T \frac{[C]_i}{A_i}\{T_Q\}_{i,k} = 0 \qquad \text{(A20)}$$

for bars and

$$\rho_p S_j - \sum_{k=1}^{N_T} \lambda_k\{T\}_j^T \frac{[C]_j}{t_j}\{T_Q\}_{j,k} = 0 \qquad \text{(A21)}$$

for plates.

REFERENCES

1. Schmit, Lucien A., Jr.; and Miura, Hirokazu: Approximation Concepts for Efficient Structural Synthesis. NASA CR-2552, 1976.

2. Venkayya, V. B.; Khot, N. S.; and Berke, L.: Application of Optimality Criteria Approaches to Automated Design of Large Practical Structures. Second Symposium on Structural Optimization, AGARD-CP-123, Nov. 1973, pp. 3-1 - 3-19.

3. Berke, L.; and Khot, N. S.: Use of Optimality Criteria Methods for Large Scale Systems. Structural Optimization, AGARD-LS-70, Sept. 1974, pp. 1-1 - 1-29.

4. Khot, N. S.; Venkayya, V. B.; and Berke, L.: Optimum Design of Composite Structures With Stress and Displacement Constraints. AIAA J., vol. 14, no. 2, 1976, pp. 131-132.

5. Adelman, Howard M.; Walsh, Joanne L.; and Narayanaswami, R.: An Improved Method for Optimum Design of Mechanically and Thermally Loaded Structures. NASA TN D-7965, 1975.

6. Adelman, Howard M.; and Narayanaswami, R.: Resizing Procedure for Structures Under Combined Mechanical and Thermal Loading. AIAA J., vol 14, no. 10, Oct. 1976, pp. 1484-1486.

7. Adelman, Howard M.; and Sawyer, Patricia L.: Inclusion of Explicit Thermal Requirements in the Optimum Structural Design. NASA TM X-74017, 1977.

8. Ahmadi, G.; and Razani, A.: Some Optimization Problems Related to Cooling Fins. Int. J. Heat & Mass Transfer, vol. 16, no. 12, Dec. 1973, pp. 2369-2375.

9. Vanderplaats, Garret N.; and Moses, Fred: Structural Optimization by Methods of Feasible Directions. Comput. & Struct., vol. 3, no. 4, July 1973, pp. 739-755.

10. Vanderplaats, Garret N.: CONMIN - A Fortran Program for Constrained Function Minimization: User's Manual. NASA TM X-62,282, 1973.

TABLE I.- EFFECT OF b ON CONVERGENCE OF RESIZING PROCESS

FOR COOLING FIN PROBLEM IDEALIZED WITH FIVE ELEMENTS

[N_G = 1.0; B_i = 0; $\bar{T}_c$ = 0.3]

b	Converged value of volume $\bar{V}$	Converged value of temperature at constant point $\bar{T}$	No. of iterations to reach convergence
0.1	1.5168	0.3000	139
.3	1.5168	.3000	38
.5	1.5168	.3000	16
.7	1.5168	.3000	16
.9	1.5168	.3000	23

TABLE II.- EFFECT OF FINITE-ELEMENT MESH ON CONVERGENCE OF

RESIZING PROCESS FOR COOLING FIN PROBLEM

[N_G = 1.0; B_i = 0; T_c = 0.3; γ = 0.5; b = 0.5]

Iteration	5 elements		10 elements		20 elements	
	$\bar{V}$	$\bar{T}_c$	$\bar{V}$	$\bar{T}_c$	$\bar{V}$	$\bar{T}_c$
1	0.6719	0.6691	0.6686	0.6674	0.6674	0.6669
5	1.4441	.3148	1.4342	.3146	1.4306	.3146
10	1.5145	.3004	1.5039	.3004	1.5001	.3004
15	1.5167	.3000	1.5061	.3000	1.5022	.3000
20	1.5168	.3000	1.5061	.3000	1.5023	.3000
(a)	1.4815	.3000	1.4815	.3000	1.4815	.3000

[a]Obtained from closed-form solution of reference 8.

TABLE III.- EFFECT OF CONVECTION ON MINIMUM VOLUME FOR COOLING FIN

[N_G = 1.0; $\overline{T}_c$ = 0.3; $\overline{T}_a$ = 0.05; 20 iterations]

No. of elements	Converged fin volume, $\overline{V}$			
	B_i = 0	B_i = 0.05	B_i = 0.2	B_i = 0.5
5	1.5168	1.4878	1.4005	1.2232
10	1.5061	1.4774	1.3906	1.2147
20	1.5023	1.4736	1.3870	1.2115

TABLE IV.- EFFECT OF CONVECTION ON MINIMUM VOLUME FOR HEATED SQUARE PLATE WITH SINGLE TEMPERATURE CONSTRAINT

$\left[N_G = 1.0;\ \bar{T}_c = 10.0;\ \bar{T}_a = 5.0\right]$

Iteration	$B_i = 0$		$B_i = 0.025$		$B_i = 0.075$	
	Plate volume	Temperature at constraint point	$\bar{V}$	$\bar{T}$	$\bar{V}$	$\bar{T}$
0	1.0000	0.59	1.0000	0.66	1.0000	0.78
1	.7521	.74	.7890	.78	.8530	.86
2	.1787	2.99	.1970	2.90	.2330	2.78
3	.0478	10.70	.0523	9.42	.0618	7.77
4	.0259	19.80	.0246	16.70	.0242	12.70
5	.0268	18.80	.0199	19.00	.0135	15.20
6	.0375	13.30	.0239	16.50	.0097	16.10
7	.0512	9.65	.0347	12.30	.0091	16.10
8	.0588	8.31	.0491	9.22	.0115	15.10
9	.0574	8.41	.0581	7.91	.0134	13.00
10	.0521	9.18	.0571	7.93	.0324	10.20
11	.0475	9.95	.0505	8.76	.0510	7.93
12	.0455	10.30	.0442	9.80	.0625	6.95
13	.0453	10.30	.0405	10.50	.0598	7.15
14	.0461	10.20	.0395	10.70	.0492	8.08
15	.0470	10.00	.0403	10.60	.0388	9.28
16	.0476	9.91	.0419	10.20	.0317	10.30
17	.0477	9.91	.0434	9.90	.0278	11.00
18	.0476	9.95	.0443	9.80	.0263	11.30
19	.0474	10.00	.0444	9.89	.0269	11.20
20	.0474		.0444	10.00	.0289	10.30
21	.0473		.0444	10.00	.0318	10.30
22	.0474		.0444	10.00	.0347	9.89
23			.0444	9.98	.0368	9.58
24			.0442	9.95	.0378	9.46
25			.0439	9.95	.0374	9.53
26			.0436	9.97	.0363	9.71
27			.0434	10.00	.0350	9.92
28			.0434		.0339	10.10
29			.0435		.0332	10.20
30			.0437		.0330	10.20
31			.0439		.0333	10.20
32			.0440	9.98	.0337	10.10
33			.0439	9.97	.0342	10.00
34			.0438	9.97	.0346	9.95
35			.0437	9.99	.0348	9.91
36			.0436	10.00	.0349	9.91
37			.0436		.0347	9.93
38			.0436		.0345	9.90
39			.0437		.0343	10.00
40			.0438		.0341	
41			.0438		.0340	
42			.0438	9.99	.0340	
43			.0438	9.99	.0341	
44			.0437	10.00	.0342	
45					.0343	
46					.0344	9.99
47					.0344	9.98
48					.0344	9.98
49					.0344	9.99
50					.0343	10.00
51					.0343	10.00
(a)	.0479	9.98	.0442	9.98	.0342	9.97

[a]Obtained by method of references 9 and 10.

TABLE V.- EFFECT OF MULTIPLE TEMPERATURE CONSTRAINTS

ON HEATED TRIANGULAR PLATE

$[N_G = 1.0; \quad B_i = 0]$

(a) Single constraint

$[\bar{T}_{c3} = 10.0]$

Iteration	$\bar{V}$	$\bar{T}_{c1} = \bar{T}_{c2}$	$\bar{T}_{c3}$
0	0.5000	0.38	0.53
1	.5140	.36	.51
2	.1180	1.57	2.21
3	.0266	6.97	9.76
4	.0125	14.80	20.70
5	.0123	15.00	20.90
6	.0176	10.50	14.50
7	.0255	7.25	10.00
8	.0307	6.03	8.31
9	.0307	6.03	8.29
10	.0279	6.63	9.08
11	.0254	7.29	9.97
12	.0242	7.66	10.40
13	.0241	7.69	10.40
14	.0247	7.54	10.20
15	.0252	7.38	9.99
16	.0255	7.32	9.87
17	.0254	7.34	9.87
18	.0252	7.40	9.93
19	.0251	7.47	9.99
20	.0250	7.41	10.00
21	.0249	7.41	10.00
22	.0249	7.45	10.00
23	.0250	7.47	9.99
24	.0249	7.43	10.00
25	.0249	7.43	10.00
(a)	.0252	7.41	9.99

[a]Obtained by method of references 9 and 10.

TABLE V.- Concluded

(b) Multiple constraints

$\left[\bar{T}_{c3} = 10.0;\ \ \bar{T}_{c1} = \bar{T}_{c2} = 5.0\right]$

Iteration	$\bar{V}$	$\bar{T}_{c1} = \bar{T}_{c2}$	$\bar{T}_{c3}$
0	0.5000	0.38	0.53
1	.8120	.21	.33
2	.2070	.81	1.33
3	.0409	4.07	6.80
4	.0160	10.30	17.50
5	.0142	11.60	20.00
6	.0200	8.19	14.30
7	.0301	5.43	9.62
8	.0381	4.27	7.67
9	.0393	4.13	7.48
10	.0360	4.49	8.22
11	.0325	4.96	9.16
12	.0307	5.25	9.77
13	.0304	5.29	9.91
14	.0310	5.17	9.74
15	.0318	5.04	9.54
16	.0323	4.96	9.44
17	.0323	4.95	9.45
18	.0321	4.97	9.52
19	.0319	5.00	9.59
20	.0318	5.02	9.64
21	.0318	5.02	9.66
22	.0318	5.01	9.66
23	.0318	5.00	9.66
24	.0319		9.67
25	.0319		9.69
26	.0318		9.71
27			9.73
28			9.75
29			9.77
30			9.78
31			9.80
32	.0319		9.81
33	.0319		9.83
34	.0318		9.85
35	.0318		9.88
36	.0318		9.91
37	.0318		9.93
38	.0317		9.95
39	.0317		9.97
40	.0317		9.98
41	.0317		9.99
42	.0317		9.99
43	.0316		10.00
44	.0316		10.00
45	.0316		10.00
(a)	.0312	4.96	9.94

[a]Obtained by method of references 9 and 10.

TABLE VI.- NODAL TEMPERATURE DISTRIBUTION FOR COMBINED PLATE-BAR PROBLEM

[Constraint: T ≦ 260° C (500° F) for all nodal points]

Nodal coordinates				Temperature	
x		y			
cm	in.	cm	in.	°C	°F
0.00	0.00	0.00	0.00	260	500
1.91	.75	.00	.00	211	411
3.81	1.50	.00	.00	105	221
5.72	2.25	.00	.00	211	411
7.62	3.00	.00	.00	260	500
.00	.00	1.91	.75	258	497
1.91	.75	1.91	.75	193	380
3.81	1.50	1.91	.75	92	198
5.72	2.25	1.91	.75	193	380
7.62	3.00	1.91	.75	258	497
.00	.00	3.81	1.50	129	264
1.91	.75	3.81	1.50	111	231
3.81	1.50	3.81	1.50	54	130
5.72	2.25	3.81	1.50	111	231
7.62	3.00	3.81	1.50	129	264
.00	.00	5.72	2.25	259	499
1.91	.75	5.72	2.25	244	472
3.81	1.50	5.72	2.25	21	70
5.72	2.25	5.72	2.25	244	472
7.62	3.00	5.72	2.25	259	499
.00	.00	7.62	3.00	-18	0
1.91	.75	7.62	3.00	-18	0
3.81	1.50	7.62	3.00	-18	0
5.72	2.25	7.62	3.00	-18	0
7.62	3.00	7.62	3.00	-18	0

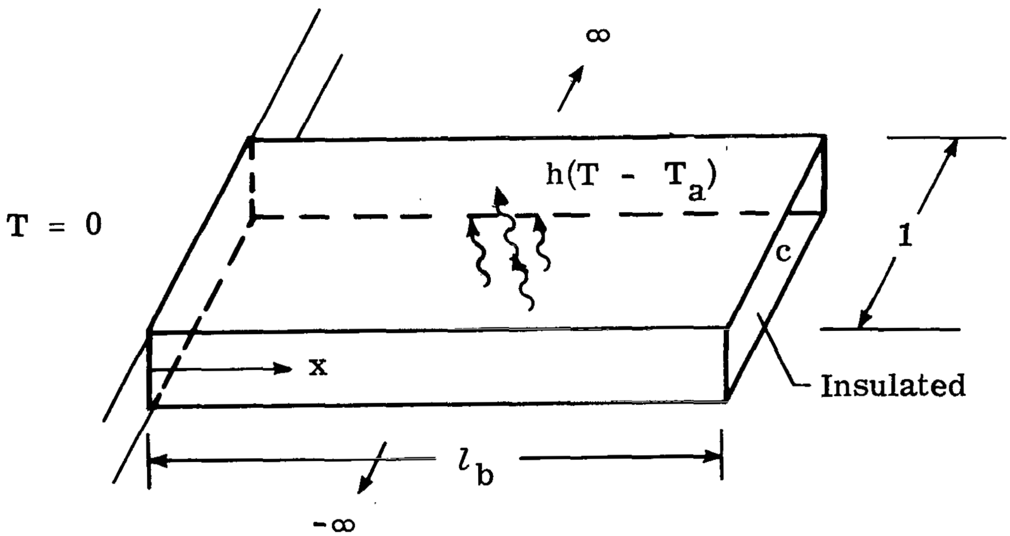


Figure 1.- Fin geometry and boundary conditions.

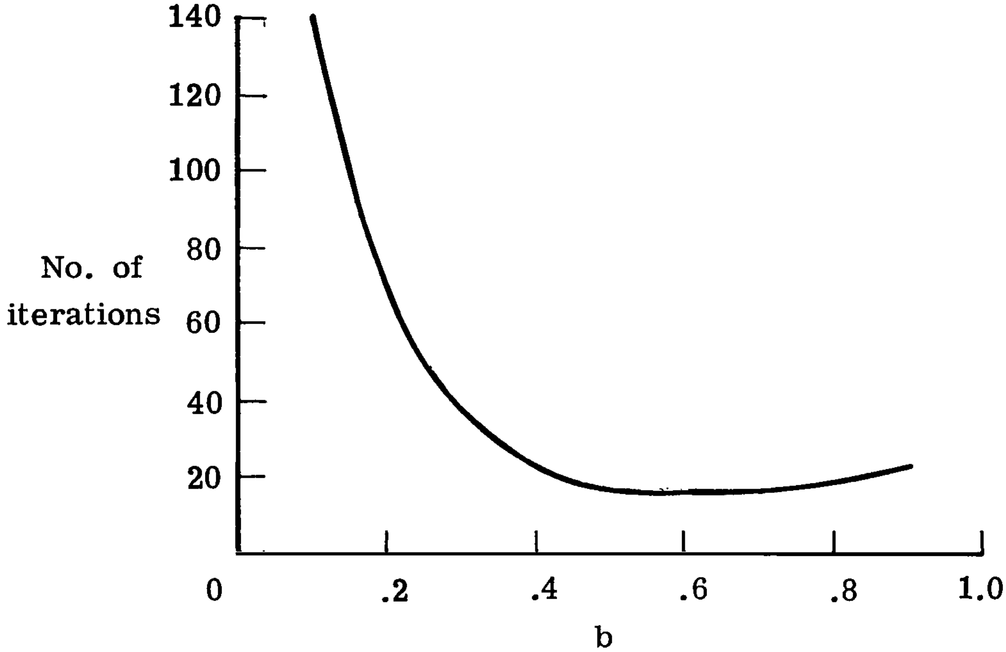
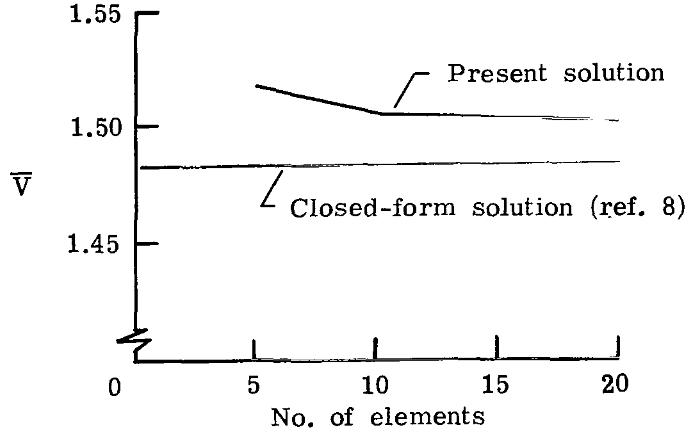
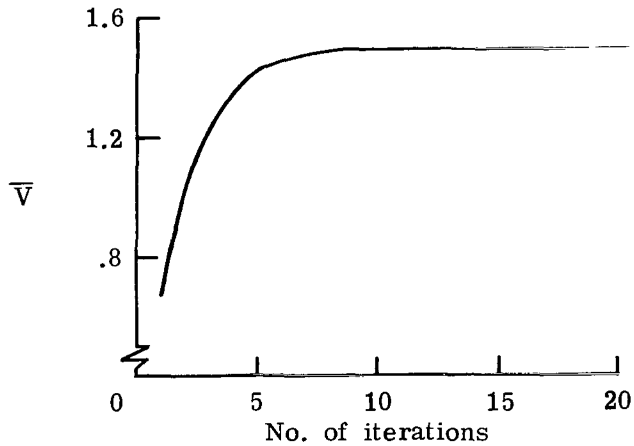


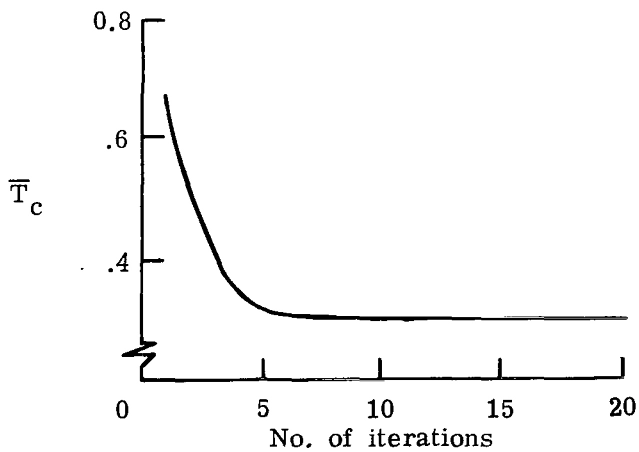
Figure 2.- Effects of parameter b for fin problem, five-element solution.
$N_G = 1.0$; $B_i = 0$; $\overline{T}_c = 0.3$.



(a) Discretization effects on optimum volume.

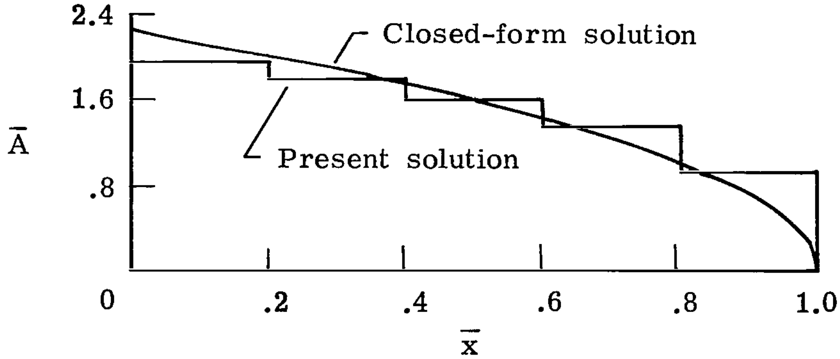


(b) Iterations required to reach optimum volume; 20-element solution.

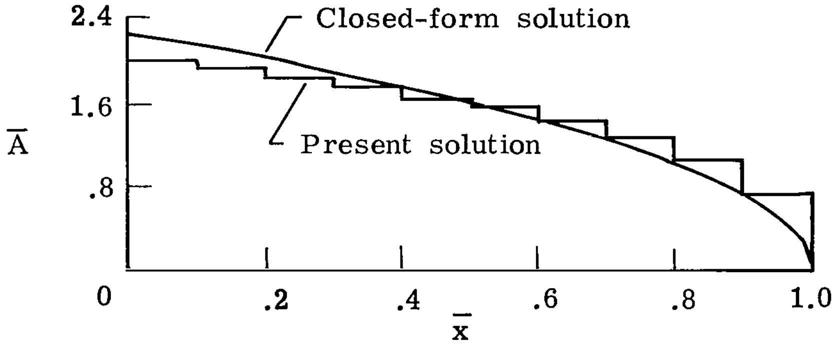


(c) Iterations required to satisfy temperature constraint; 20-element solution.

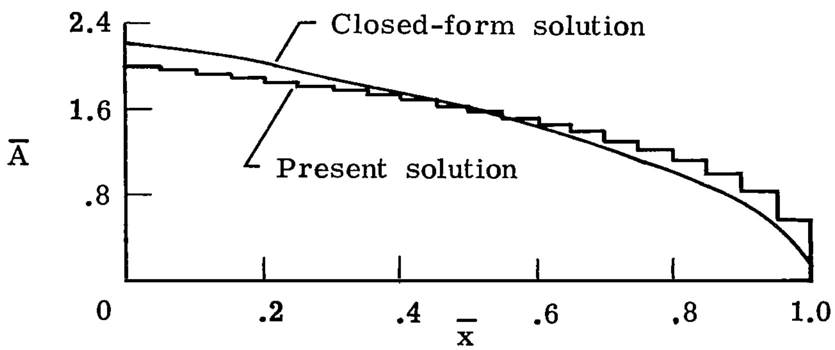
Figure 3.- Finite-element convergence studies for fin problem.
$N_G = 1.0$; $B_i = 0$; $\overline{T}_c = 0.3$.



(a) 5-element solution.



(b) 10-element solution.



(c) 20-element solution.

Figure 4.- Area distribution for fin problem.
$N_G = 1.0$; $B_i = 0$; $\bar{T}_c = 0.3$.

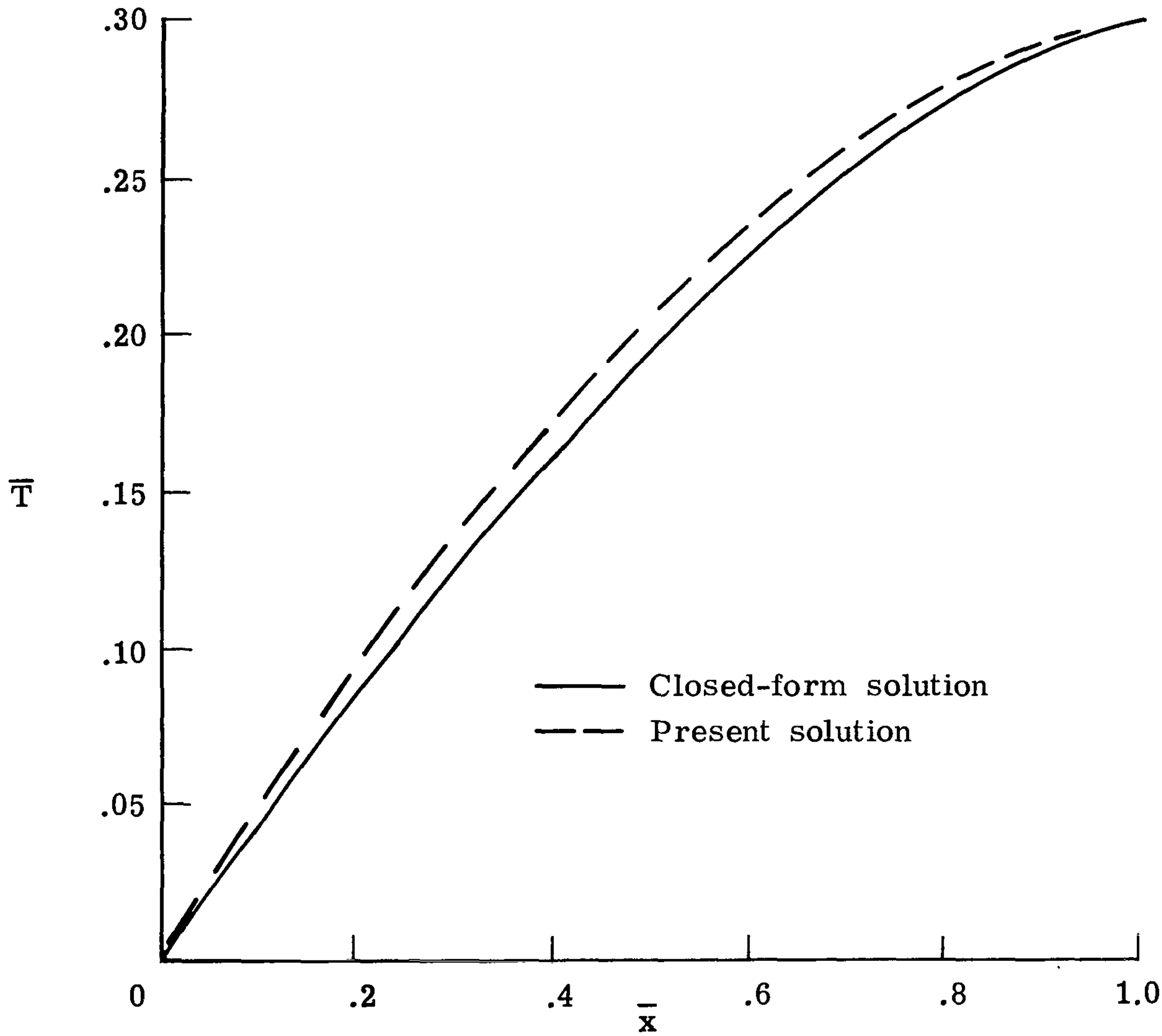
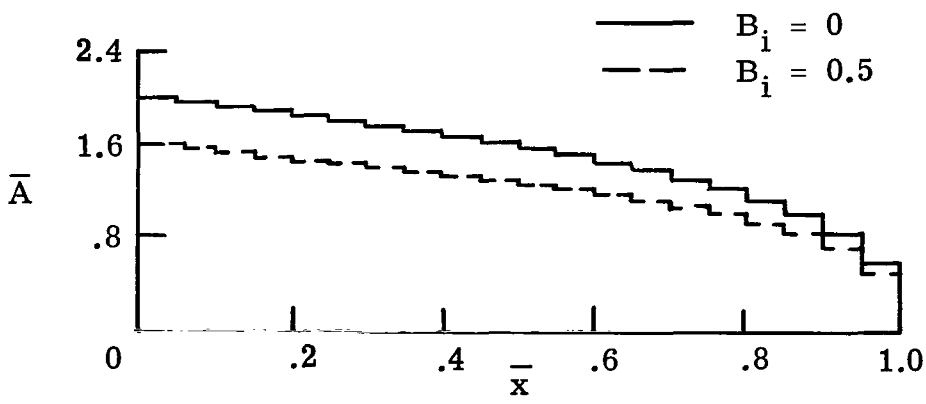
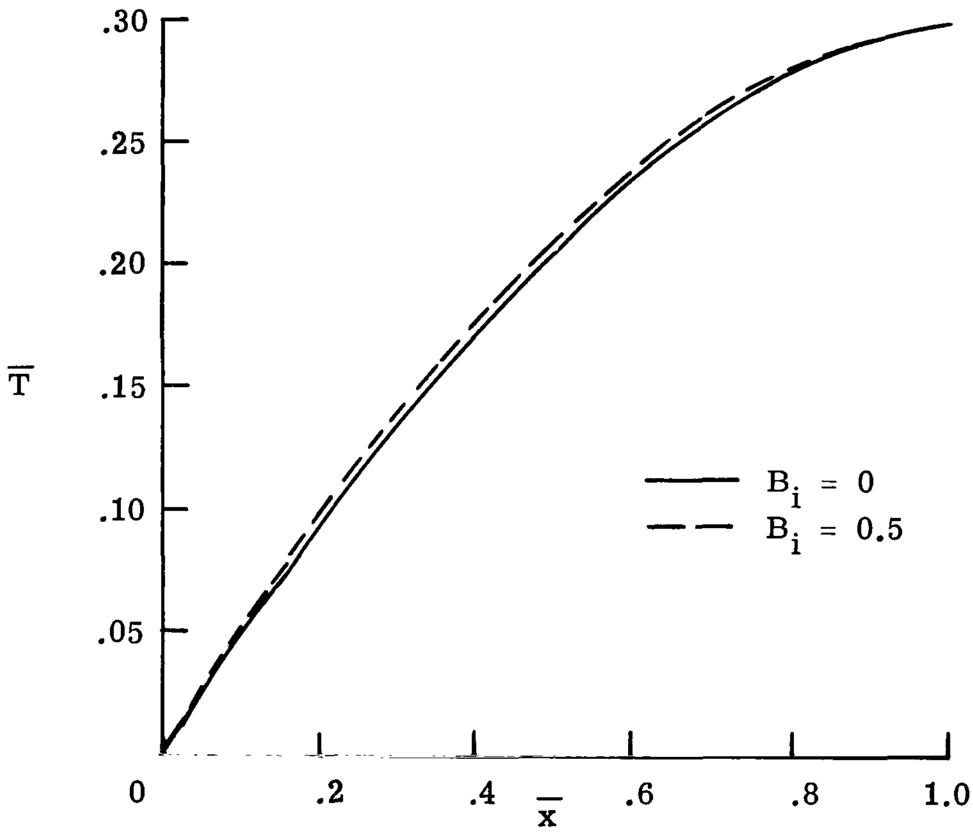


Figure 5.- Temperature distribution for fin problem.
$N_G = 1.0$; $B_i = 0$; $\overline{T}_c = 0.3$.



(a) Area distribution.



(b) Temperature distribution.

Figure 6.- Effects of convection for fin problem.
$N_G = 1.0$; $\bar{T}_c = 0.3$; $\bar{T}_a = 0.05$.

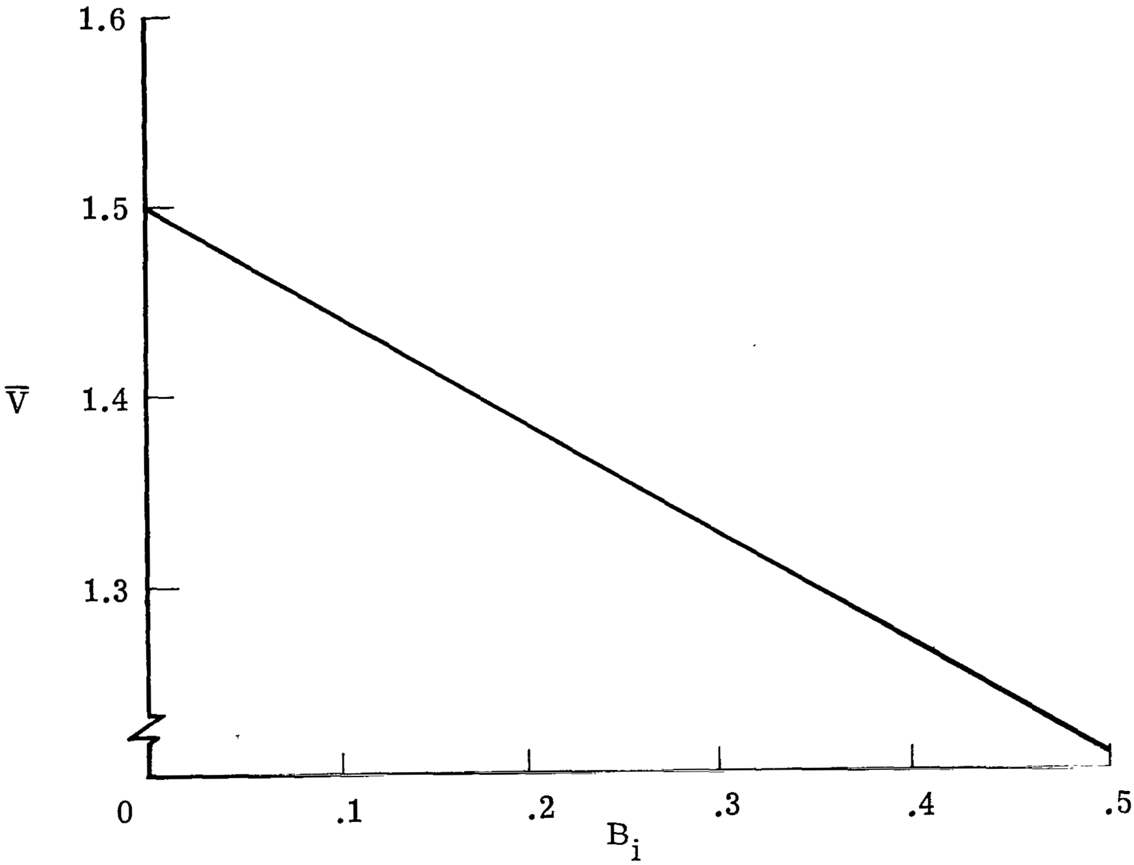


Figure 7.- Effect of convection on minimum fin volume.
$N_G = 1.0$; $\overline{T}_c = 0.3$; $\overline{T}_a = 0.05$.

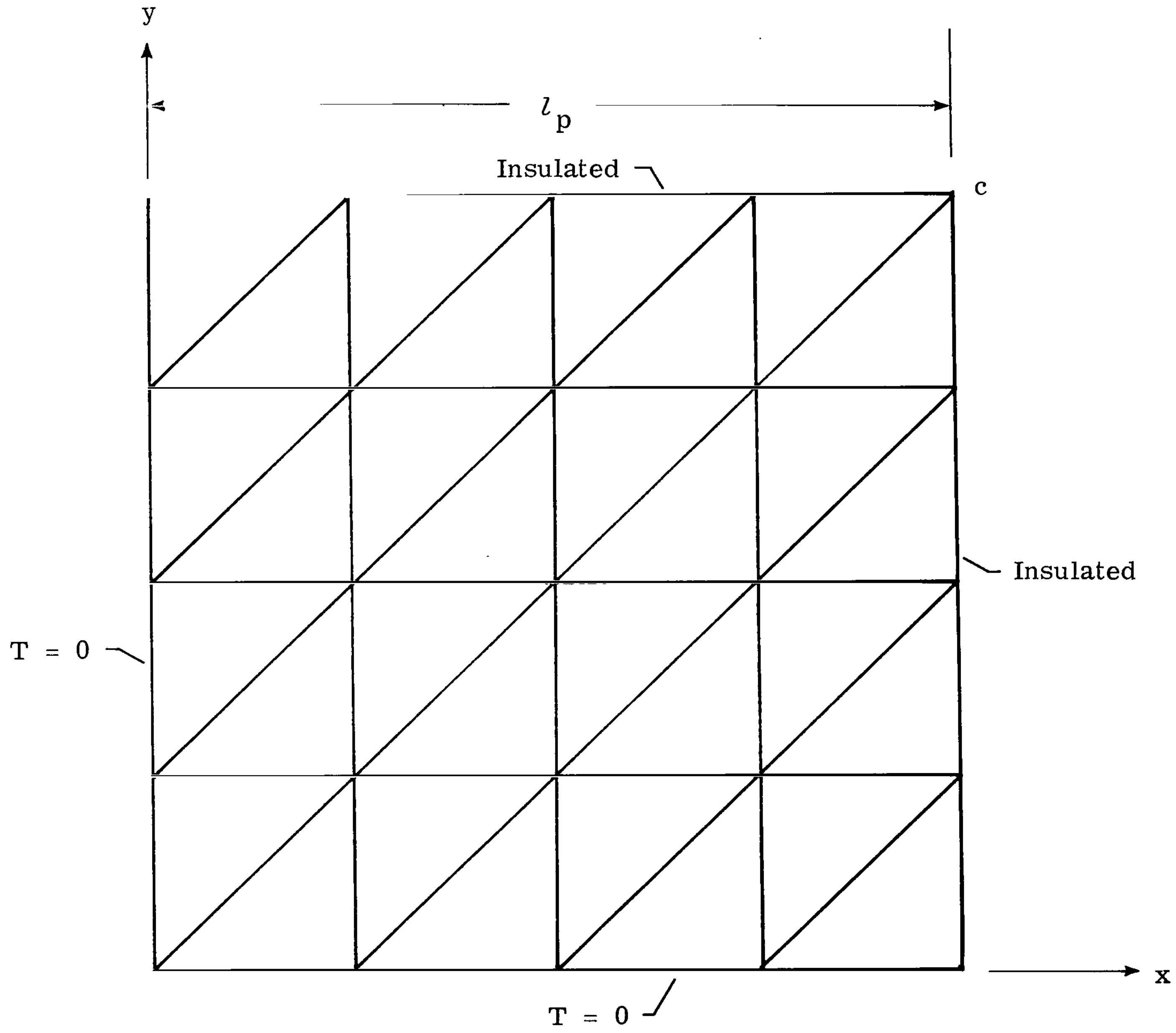


Figure 8.- Square plate with finite-element discretization.

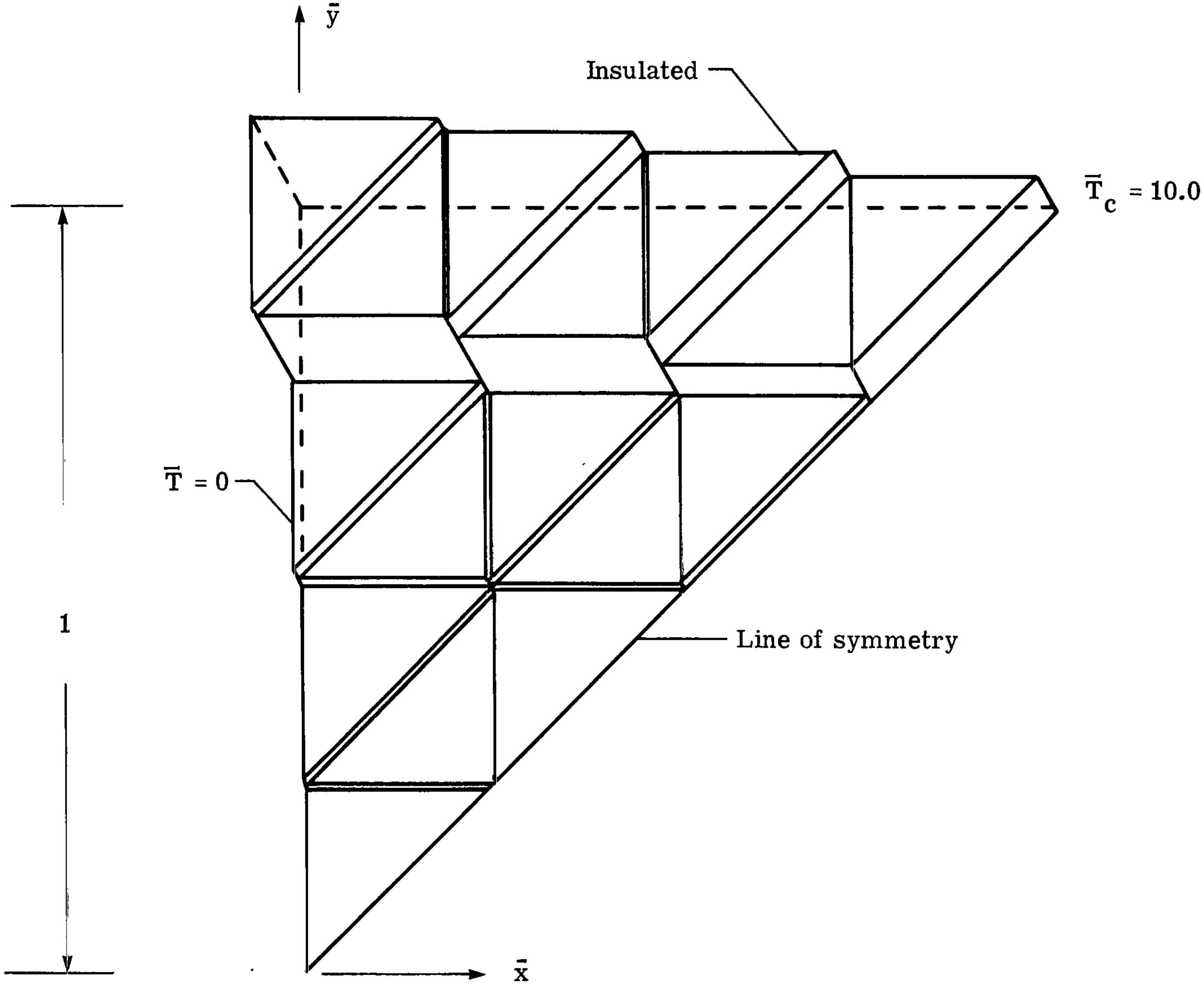


Figure 9. Square plate thickness distribution for minimum volume.

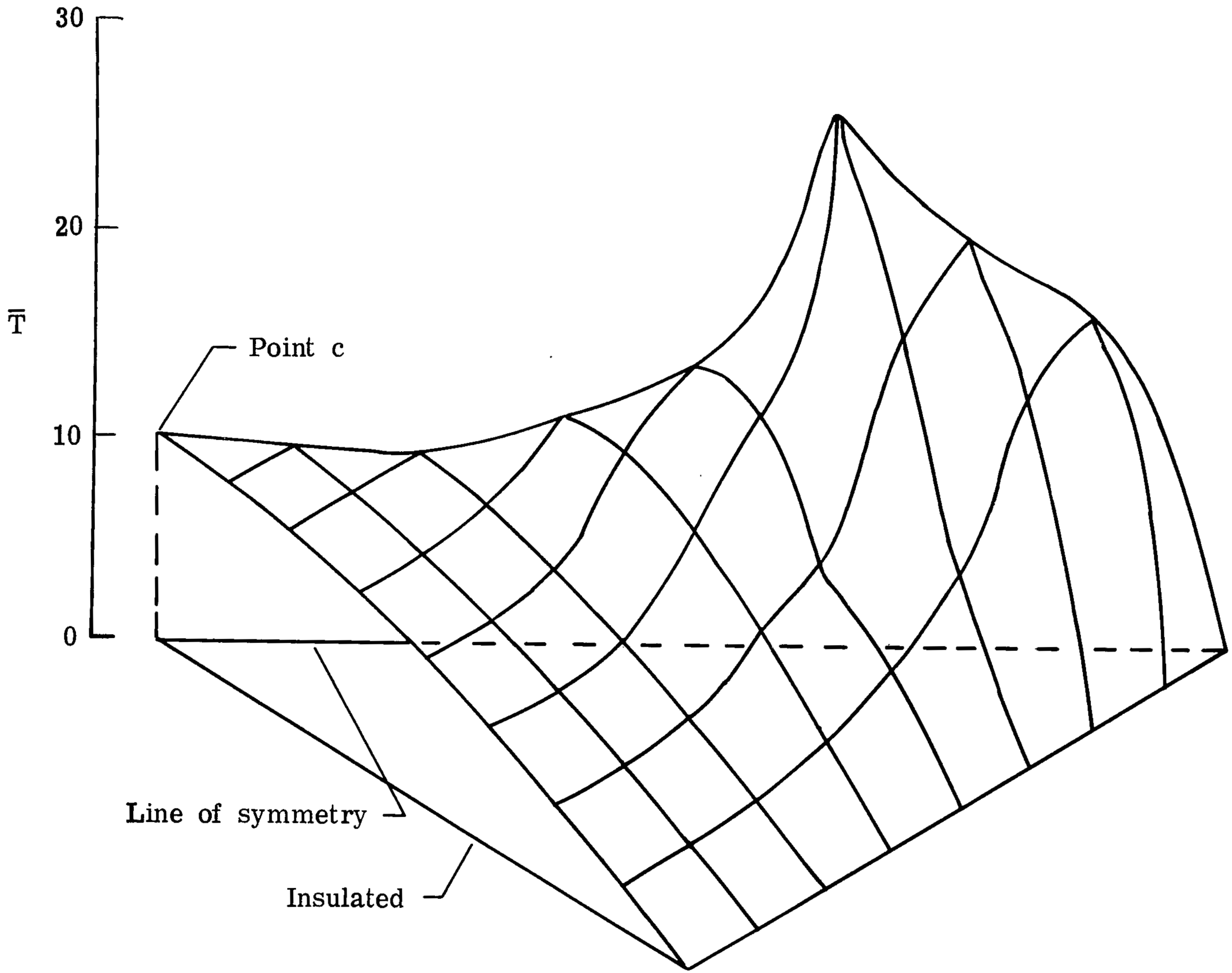
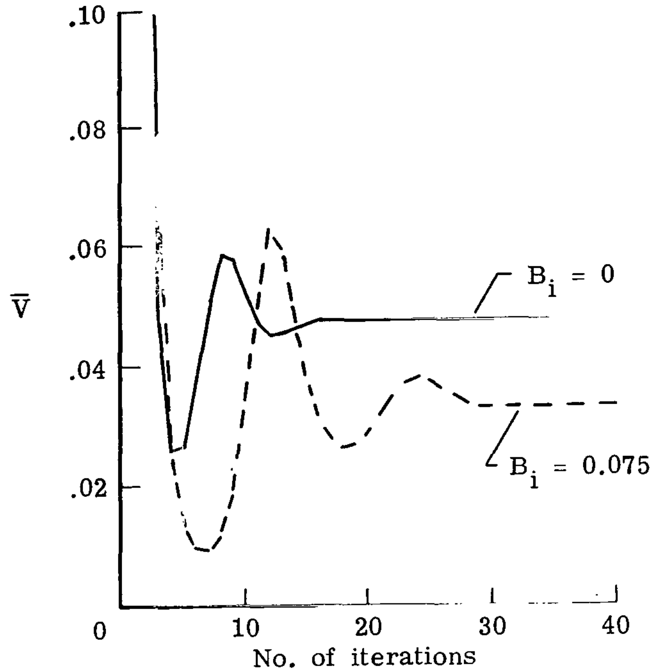
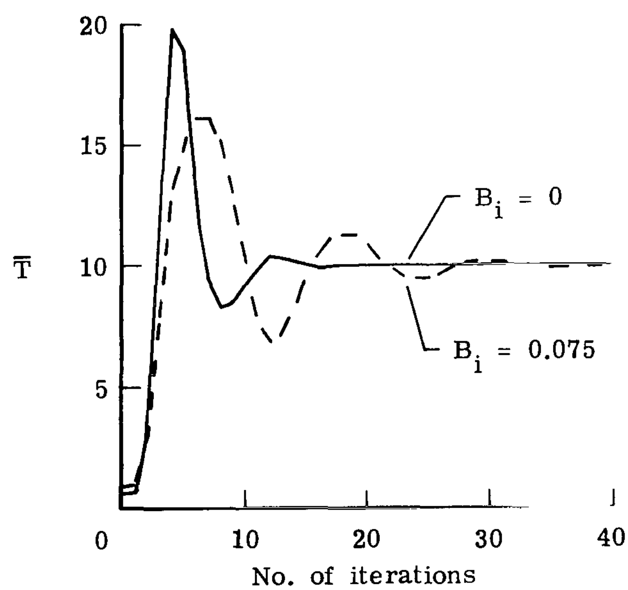


Figure 10.- Temperature distribution for minimum volume square plate.



(a) Convergence of volume.



(b) Convergence of temperature at constraint point.

Figure 11.- Effect of convection on square plate.
$N_G = 1.0$; $\overline{T}_c = 10.0$; $\overline{T}_a = 5.0$.

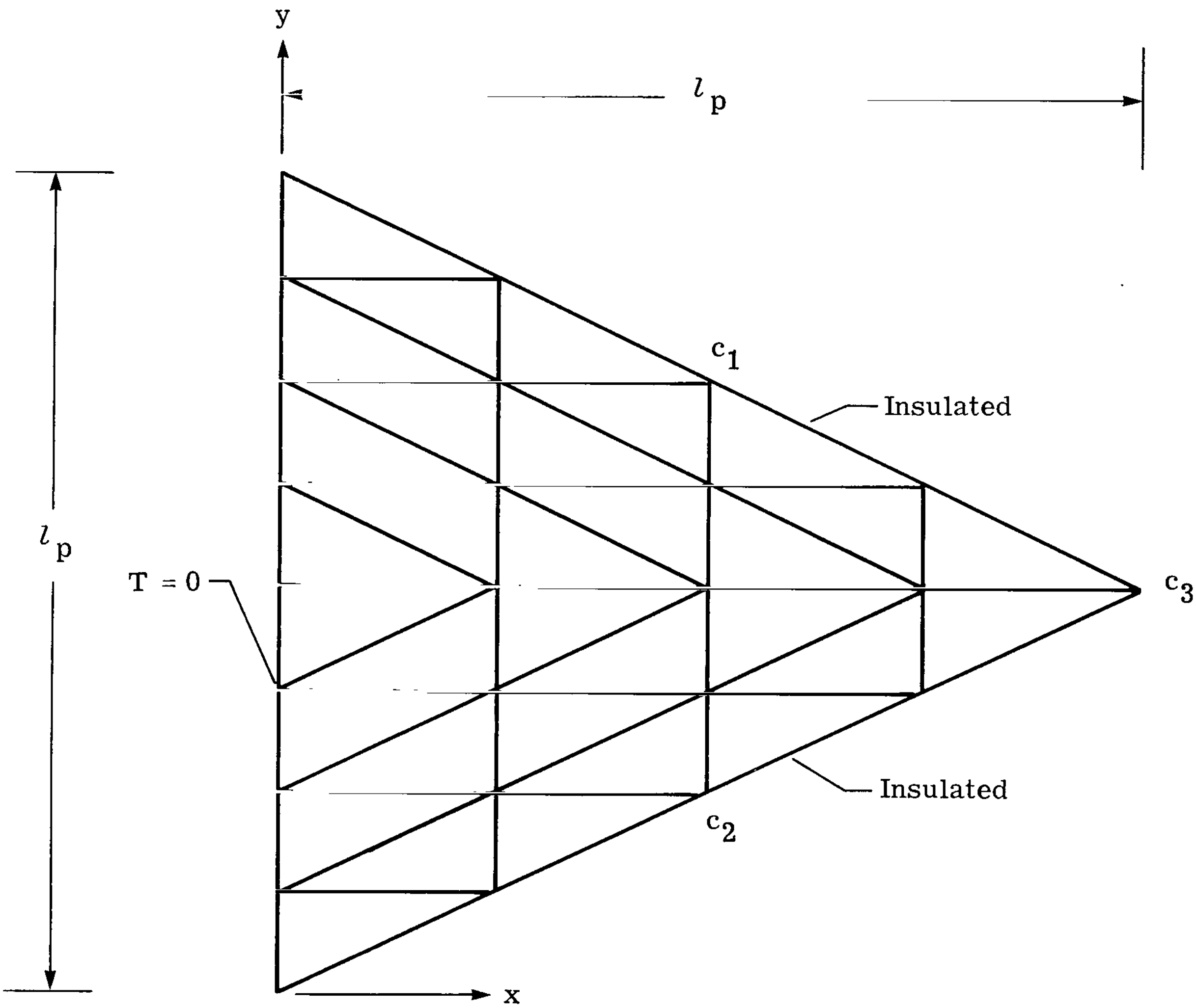
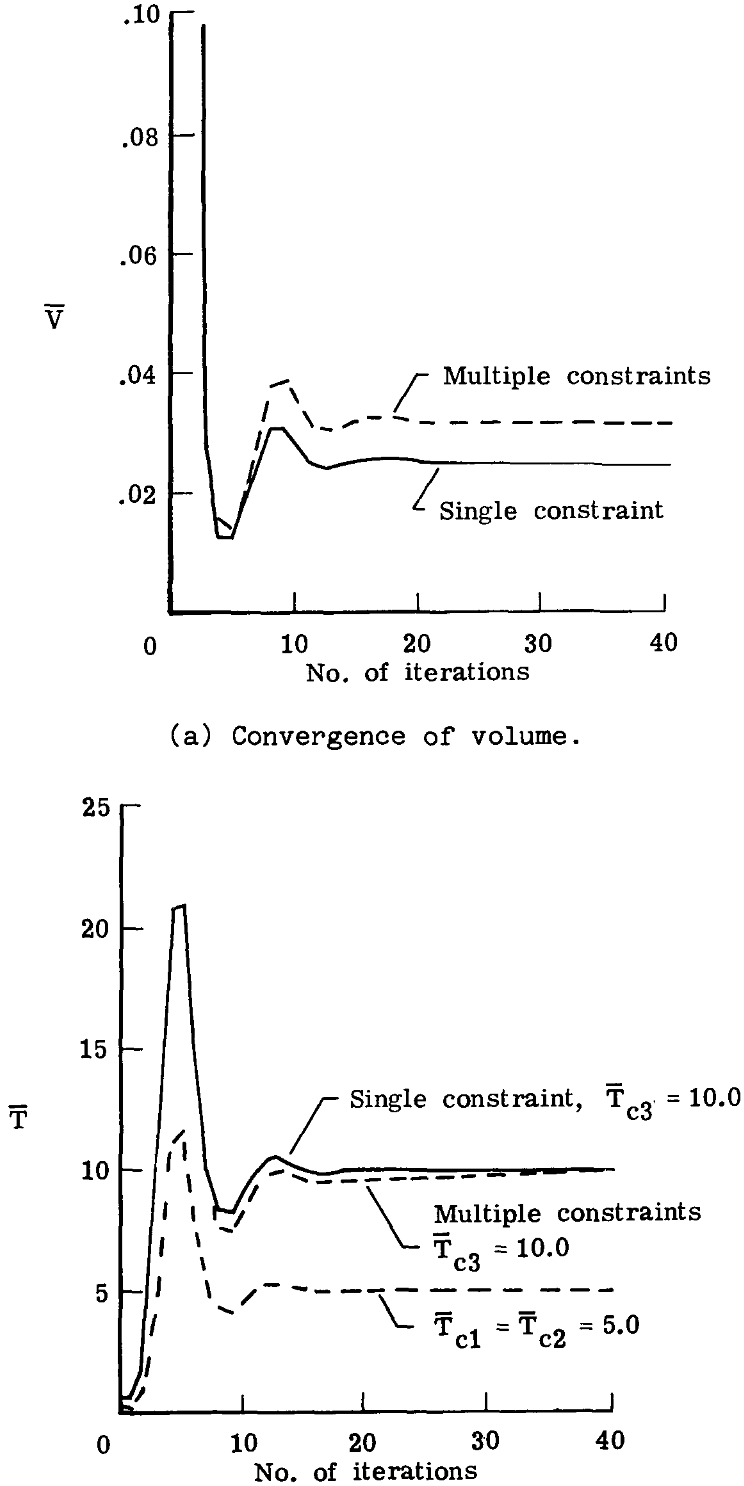


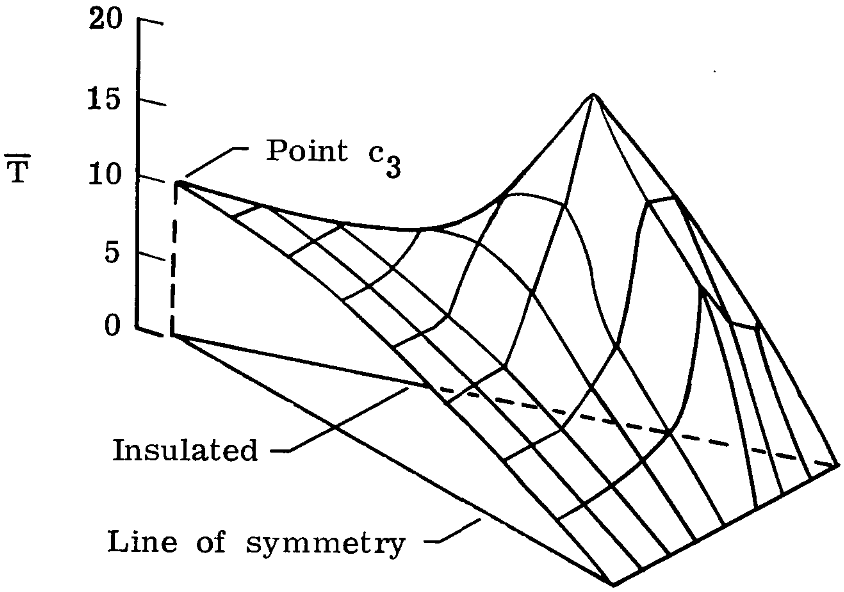
Figure 12.- Triangular plate with finite-element discretization.



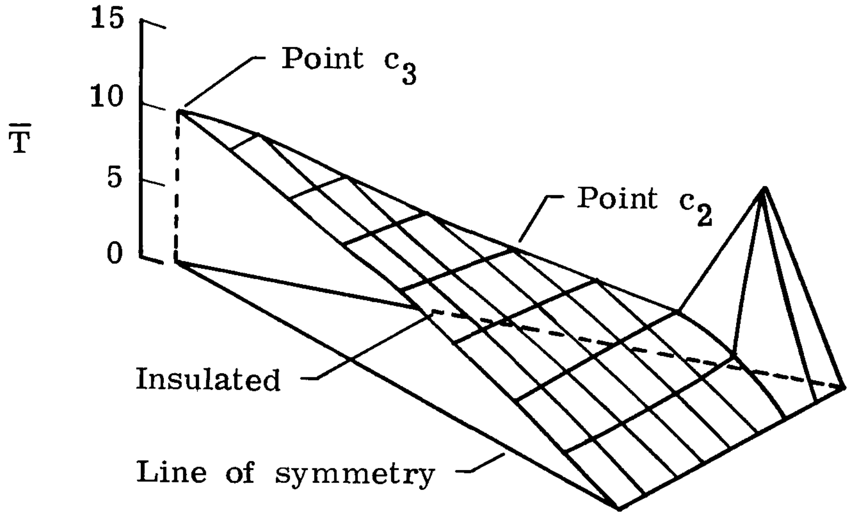
(a) Convergence of volume.

(b) Convergence of temperature at constraint points.

Figure 13.- Effects of multiple constraints on triangular plate. $N_G = 1.0$; $B_i = 0$.

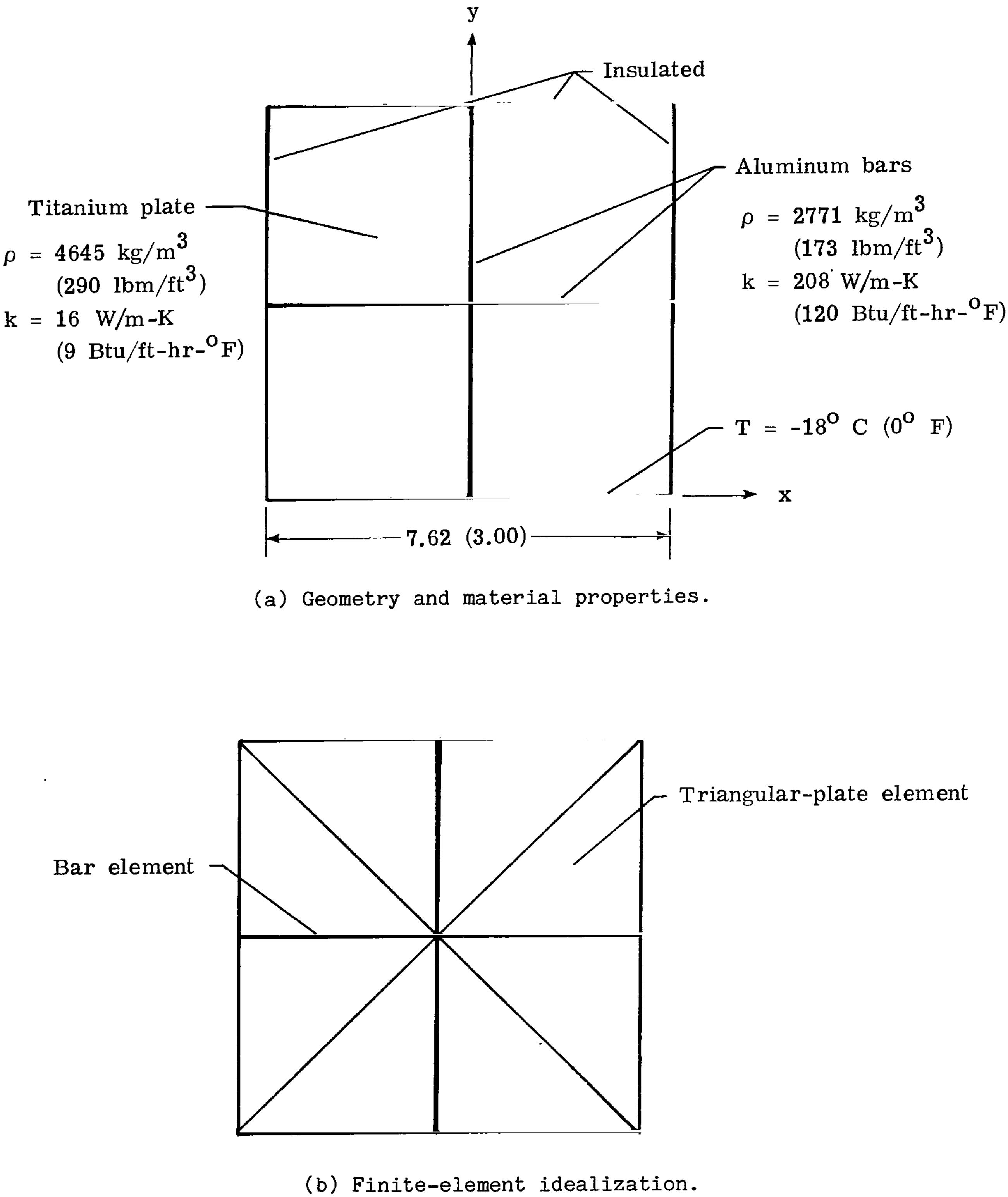


(a) Single constraint.



(b) Multiple constraints.

Figure 14.- Temperature distributions for heated triangular plate.



(a) Geometry and material properties.

(b) Finite-element idealization.

Figure 15.- Combined plate-bar problem. Heat flux, 34.1 kW/m^2 (10 800 Btu/ft^2-hr). (Dimensions in cm (in.).)

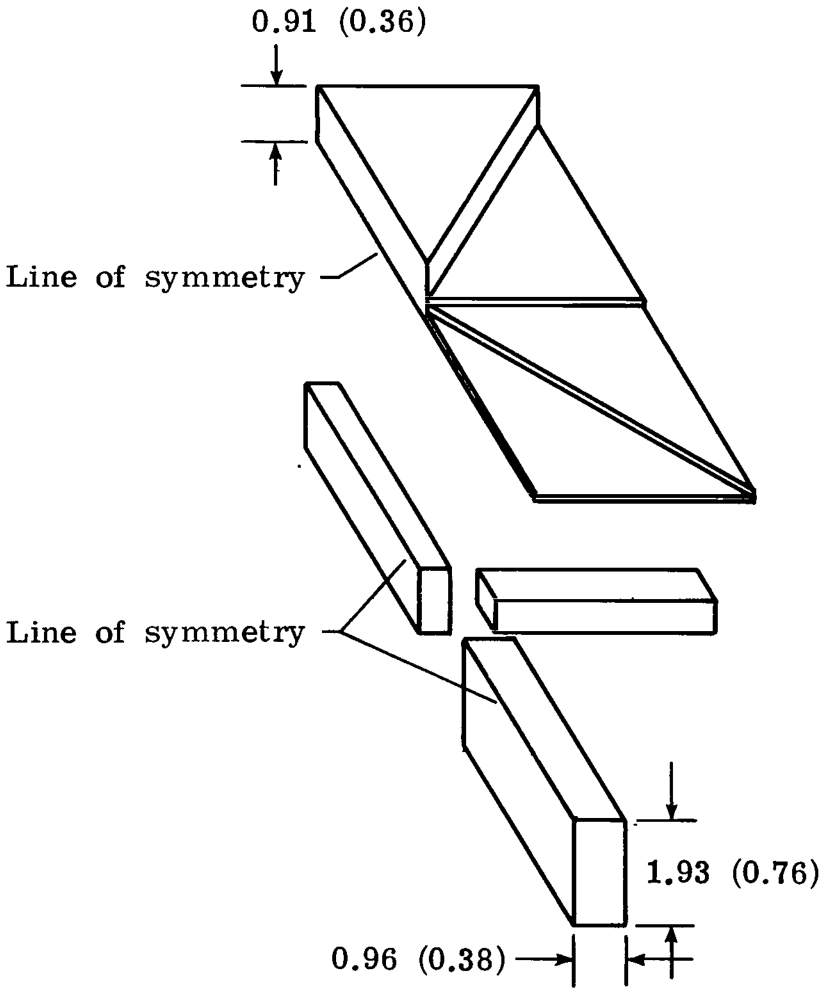


Figure 16.- Material distribution for combined plate-bar problem. (Dimensions in cm (in.).)

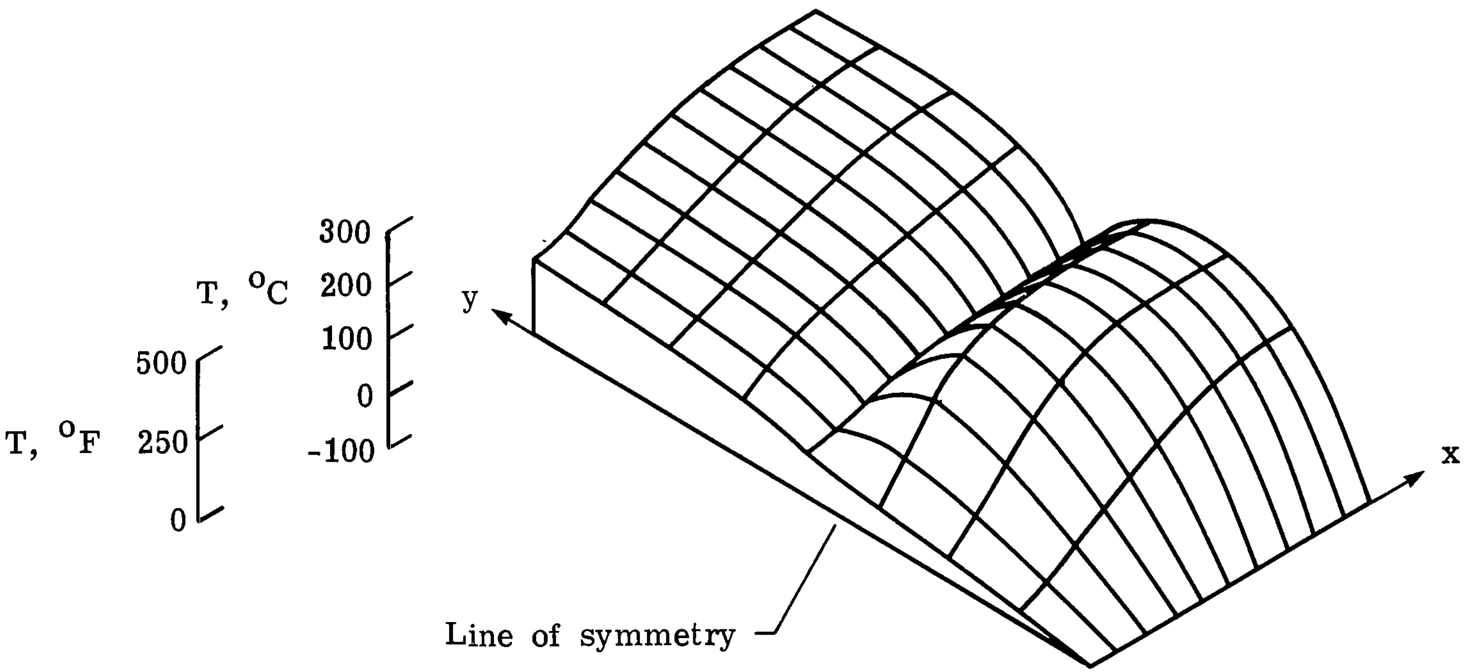


Figure 17.- Temperature distribution for combined plate-bar problem.

NASA-Langley, 1977 L-11601

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546

OFFICIAL BUSINESS
PENALTY FOR PRIVATE USE $300

THIRD-CLASS BULK RATE

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION
451

U.S.MAIL

2 1 1U,D, 100577 S00903DS
DEPT OF THE AIR FORCE
AF WEAPONS LABORATORY
ATTN: TECHNICAL LIBRARY (SUL)
KIRTLAND AFB NM 87117

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

"The aerona. S ? activities of the United States shall be conducted so as . . . to the expansion of human knowledge of phenome.. atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546