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BASIC ANALYSIS OF TERMINAL OPERATION BENEFITS RESULTING FROM REDUCED VORTEX SEPARATION MINIMA

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BASIC ANALYSIS OF TERMINAL OPERATION BENEFITS RESULTING FROM REDUCED VORTEX SEPARATION MINIMA

by

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SUMMARY

An analysis was made to determine the impact on terminal area operation rate of reducing the wake vortex minimum separation required behind heavy jets. The effect on arrival saturation and steady state average delay was determined for various percentages mix of heavy and large jet traffic samples operating under various precision of interarrival spacing. The benefits determined increase with percentage of heavy aircraft and with precision of control. These results demonstrate the payoff possible from research to reduce the severity of the trailing vortex by aerodynamic means.

INTRODUCTION

In general, when an air foil passes through a mass of air and creates lift, energy in the form of vortex turbulence is transmitted to the air mass. Its intensity is related to the required lift, wing span and speed of the airfoil; the heavier and slower the aircraft, the greater the intensity of air circulation in the vortex core. As a result large transport aircraft will cause maximum vortices when heavily loaded and during the takeoff and landing phase of flight where their velocities are lowest.

As a safety measure, current Federal Aviation Administration Instrument Flight Rules (IFR) for wake turbulence separation minima (reference 1) impose additional separation for aircraft (A/C) following a heavy jet. It it were possible to reduce the severity of the vortex by aerodynamic means then the separation standards could be reduced. The goal is to assess the benefits of such changes in the separation regulations; the measure to be employed is the IFR arrival only saturation capacity and its relationship with average delay. This analysis is constrained only by final approach separation, not by any ground limitations such as runway occupancy.

SYMBOLS

A	mean arrival rate to the service queue
f	multiplying factor to change service rate
i	lead aircraft of a pair on final approach
j	trail aircraft of a pair on final approach
L	length of final approach segment
P	mean service or processing rate of a queue
P_E	probability of violation of separation standard
p_i	probability that aircraft type i is in traffic mix
p_j	probability that aircraft type j is in traffic mix
p_{ij}	probability that an aircraft pair will consist of aircraft i followed by aircraft j
S_{ij}	minimum required separation distance between aircraft i and aircraft j according to separation standard
t_B	buffer time added to account for uncertainty of aircraft delivery
t_{ij}	time interval required between aircraft i and aircraft j when aircraft i is at the end of the final approach segment L in order to insure adequate separation everywhere along L
$\bar{t}_{ij}$	mean interarrival time interval
t_{ij_σ}	time interval t_{ij} plus the buffer time t_B
$\bar{t}_q$	mean waiting time in the queue
$\bar{t}_{qf}$	mean waiting time in the queue for factor f
$\bar{t}_{qf}(\frac{1}{P})$	mean waiting time in the queue in units of original service time increments

$\Delta\bar{t}_{qf}(\%)$	percentage change of the delay or waiting time in a queue with a change of f to the service rate
V_i	final approach velocity of aircraft i
V_j	final approach velocity of aircraft j
λ	average flow rate in aircraft per unit time
ρ	utilization factor or ratio of arrival to service rate
σ	standard deviation of the uncertainty of aircraft delivery

ANALYSIS AND ASSUMPTIONS

A. Derivation of Perfect Delivery Capacity

In general when $g(x,y)$ is a function of 2 random variables x and y then the expectation of $g(x,y)$ is

$$E\{g(x,y)\} = \int_x \int_y g(x,y)\ f(x,y)\ dx\ dy \tag{1}$$

with $f(x,y)$ being the joint density function of random variables x and y. For the case where x and y are discrete we can write the expectation of $g(x,y)$ as

$$E\{g(x,y)\} = \sum_k \sum_n g(x_k, y_n)\ p(x_k, y_n) \tag{2}$$

where $p(x_k, y_n)$ is the joint probability function of x and y.

For our case let i and j be the random variables where

$i \triangleq$ lead aircraft of a pair on final approach

$j \triangleq$ trail aircraft of a pair on final approach

Let us define

$g(i,j) \triangleq t_{ij} \triangleq$ time interval between aircraft i and aircraft j when aircraft i is at the end of the final approach segment of length L

For the situation when $V_j \geq V_i$, the minimum required separation S_{ij} for that aircraft pair occurs when aircraft i is at the threshold and is

$$t_{ij}(V_j \geq V_i) = \frac{S_{ij}}{V_j} \tag{3a}$$

For the case when $V_i > V_j$, the minimum required separation S_{ij} for that aircraft occurs when aircraft i is at the beginning of the final approach segment and the separation opens until aircraft i reaches the threshold where

$$t_{ij}(V_i > V_j) = \frac{S_{ij}}{V_j} + L\left(\frac{1}{V_j} - \frac{1}{V_i}\right) \tag{3b}$$

From Eq. 2 we can write the average interarrival spacing $\bar{t}_{ij} = E(t_{ij})$ as

$$\bar{t}_{ij} = \sum_j \sum_i t_{ij}\, p_{ij} \tag{4}$$

where $p_{ij} \triangleq$ probability that an aircraft pair will consist of aircraft i followed by aircraft j.

For independent arrivals and first come first serve control

$$p_{ij} = p_i\, p_j \tag{5}$$

where p_i, p_j are the probabilities of that aircraft in the traffic mix. Thus equation (4) can simply be rewritten as

$$\bar{t}_{ij} = \sum_j \sum_i t_{ij}\, p_i\, p_j \tag{6}$$

and the average flow rate λ is

$$\lambda = \frac{1}{\bar{t}_{ij}} \tag{7}$$

Keep in mind that the separation S_{ij} used in the t_{ij} calculation is the exact separation required, thus the flow rate determined is for perfect delivery precision.

B. Effect of Interarrival Spacing Precision

During IFR conditions on final approach in the real world, the controller usually separates aircraft by the minimum plus some additional buffer separation to account for the uncertainty of aircraft delivery. If one assumes that the uncertainty is Gaussian with a standard deviation of σ, then we can determine the size of the average buffer time t_B needed to keep the probability of separation violation less than some specified probability value P_E. For the probability of violation P_L being less than 5 percent we need a buffer time t_B of 1.65 σ. Figure 1 illustrates this for both the overtaking and opening case. The effect of interarrival spacing precision, parameterized by the standard deviation of the spacing uncertainty, can be determined by rewritting equation (3) as

$$
\begin{aligned}
t_{ij_\sigma}(V_j \geq V_i) &= \frac{S_{ij}}{V_j} + t_B \\
t_{ij_\sigma}(V_i > V_j) &= \frac{S_{ij}}{V_j} + L\left(\frac{1}{V_j} - \frac{1}{V_i}\right) + t_B
\end{aligned}
\qquad (8)
$$

C. Relation of Arrival Rate, Capacity and Delay

The expected delay at a given airport is a function of the runway acceptance rate and the time-varying traffic demand profile, consisting of departures as well as arrivals. The treatment of time dependent demand rate is not developed here but an excellent treatment is presented in references 5 and 6. A development using constant demand as an assumption will illustrate the point that even small percentage gains in the saturation capacity will result in substantial reductions in average delay when operating near capacity.

From queing theory, using Poisson arrivals and first come first serve processing, we get the mean waiting time in the queue (delay time in getting processed) after a system has reached steady state operating condition is

$$\bar{t}_q = \frac{A}{P(P-A)} = \frac{\rho}{P[1-\rho]} \qquad (9)$$

where

$P \triangleq$ mean service or processing rate

$A \triangleq$ mean arrival rate

$\rho \triangleq$ utilization factor or ratio of arrival to service rate (A/P)

Figure 2 (f = 1) is a plot of average delay time in units of service time increments versus utilization rates. The signigicant characteristic is that average delay approaches ∞ as arrival rate approaches service rate. If we change the service rate by some factor f, the mean waiting time in the queue for a constant arrival rate becomes

$$\bar{t}_{qf} = \frac{A}{fP[fP-A]} \tag{10}$$

or in units of the original service time increment

$$\bar{t}_{qf}\left(\frac{1}{P}\right) = \frac{1}{f}\frac{\rho}{[f-\rho]} \tag{11}$$

For illustration, the effect on mean delay when the service rate is increased by 20 percent ($f = 1.2$) is plotted in Figure 2. This considerably reduces the average delay at arrival rates greater than 0.7 of the baseline service rate. To demonstrate the significance of this delay reduction, Figure 3 was plotted with the percentage decrease in delay time versus the arrival rate, as a ratio to baseline service rate, for various percentage increase in the baseline service rate. The percentage decrease in delay time can be written as

$$\begin{aligned} \Delta\bar{t}_{qf}(\%) &= 100\left[1 - \frac{\bar{t}_{qf}}{\bar{t}_q}\right] \\ &= 100\left[1 - \frac{1-\rho}{f(f-\rho)}\right] \end{aligned} \tag{12}$$

For example, an arrival rate which is 0.95 of the saturation rate, even a 1 percent increase in the saturation rate reduces the average delay by 17.5 percent. For the steady state, constant demand case one can use Figure 3 to relate the impact on delay time caused by a percentage change in saturation at various arrival rates.

D. Conditions Used in Benefit Analysis

For this analysis we have chosen to include in the traffic sample only large and heavy aircraft as might be expected at peak traffic hours at a major terminal. The current wake turbulence separation standard (ref. 1) for this sample is given in nautical miles and kilometers by

$$S_{ij} = i \begin{array}{c} \\ \\ 1 \\ 2 \end{array} \begin{array}{c} j \\ \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 3 & 3 \\ 5 & 4 \end{bmatrix} \end{array} \text{ n.mi.} \qquad i \begin{array}{c} \\ \\ 1 \\ 2 \end{array} \begin{array}{c} j \\ \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 5.56 & 5.56 \\ 9.26 & 7.41 \end{bmatrix} \end{array} \text{ km}$$

where $i, j = 1 \triangleq$ Large and $i, j = 2 \triangleq$ Heavy. For example when a heavy aircraft is followed by a large the separation required is 5 nautical miles (9.26 km). If the vortex effect can be reduced then separation standards could also be reduced. This impact can be determined by using for the separations

$$S_{ij} = i \begin{array}{c} \\ \\ 1 \\ 2 \end{array} \begin{array}{c} j \\ \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \end{array} \text{ n.mi.} \qquad i \begin{array}{c} \\ \\ 1 \\ 2 \end{array} \begin{array}{c} j \\ \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 5.56 & 5.56 \\ 5.56 & 5.56 \end{bmatrix} \end{array} \text{ km}$$

and

$$S_{ij} = i \begin{array}{c} \\ \\ 1 \\ 2 \end{array} \begin{array}{c} j \\ \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \end{array} \text{ n.mi.} \qquad i \begin{array}{c} \\ \\ 1 \\ 2 \end{array} \begin{array}{c} j \\ \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 3.70 & 3.70 \\ 3.70 & 3.70 \end{bmatrix} \end{array} \text{ km}$$

The derivation presented is general enough that the performance of any number of types of aircraft in a traffic mix, could be determined each with a specific final approach velocity. However, to determine the impact of percentage of heavy aircraft on the saturation capacity for various separation standards, a traffic mix of only two types of aircraft was used. One was a typical large aircraft with final approach velocity of 127 kts (235.20 km/hr) and the other a representative heavy aircraft with final approach velocity of 137 kts (253.72 km/hr).

The effect of delivery precision is determined by taking three values of standard deviation for the aircraft delivery uncertainty. These are $\sigma = 20$ seconds which is representative of today's manual control, $\sigma = 10$ seconds which is representative of a computer aided metering and spacing system, and $\sigma = 0$ which is the perfect delivery limiting case. The final approach used in all cases was 5 nautical miles (9.26 km).

DISCUSSION AND CONCLUSION

Computer runs with the conditions advanced in the last section were made and the results printed in Table 1. These results are presented in graphical form in Figures 4 through 6. The cross-ruled region indicates the benefit to be gained by eliminating the vortex penalty effect of heavy aircraft. Figure 7 explicitly shows this benefit. When compared to current vortex separation, the gain if all aircraft in our traffic are separated by 3 nautical miles (5.56 km) goes up with increase in percentage of heavy aircraft. This is a significant factor as more heavies go into service.

Another noteworthy feature of Figure 7 is that the capacity gain in going from current separation standards to the all 3 nautical mile (5.56 km) case increases with control precision. Thus the gains to be realized in today's environment by changing separation standards will be even greater with better performing air traffic control systems of the future. The 2 nautical miles (3.70 km) separation was used to illustrate what might be achieved if runway occupancy was reduced and vortex effects further eliminated.

It is difficult to compare gains from current to new separation standards, at various percentages of heavy aircraft and precision of control accuracy, since the saturation capacities at the current separation standards are themselves changing with these factors. One approach is to show the gains in terms of a percentage of the baseline saturation capacity as in Figure 8.

The discussion up to this point has been concerned with the effect on arrival capacity which changing the separation standard has for various percentages of mix of heavy and large jet traffic samples operating under different precision of interarrival spacing. Now using Figure 8 in conjunction with Figure 3, the impact on steady state average delay can be assessed. For instance, with a delivery standard deviation of 20 seconds, a traffic mix consisting of 60 percent heavy jets would experience a 20 percent increase in saturation capacity by going from the current separation standard to an all 3 nautical mile (5.56 km) separation. If the steady state arrival rate at an airport is 70 percent of the saturation capacity, then the average delay at that airport is reduced by 50 percent.

The percentage decrease in average delay are significantly more dramatic for arrival rates nearer the saturation capacity.

A different approach in delay analysis can be taken by using Figure 8 in conjunction with reference 6 to relate changes in saturation capacity to changes in average total daily time for their time-dependent traffic profiles. Results relating capacity changes to delay impact can be translated to the economic field by assigning a dollar cost penalty to delay. Pursuing that line of reasoning, it is possible to do a cost benefit analysis of the aircraft modifications needed to implement wake vortex minimization by aerodynamic techniques.

Another consideration in evaluating the benefits of aerodynamic wake vortex minimization is the FAA's Vortex Advisory System (VAS). This system will improve the airport capacity at those times when wind strength and direction lessen the dangers from lingering trailing wake vortices. However, when those particular wind conditions don't exist then the only recourse from increased separation behind heavy jets is by aerodynamic means. That being the case we have shown that benefits resulting from vortex minimization increase with percentage of heavy aircraft and with precision of interarrival spacing.

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TABLE I.- SATURATION CAPACITY IN AIRCRAFT PER HOUR

Probability of large A/C	Probability of heavy A/C	Standard Deviation of Delivery Uncertainty σ = 0 sec	σ = 10 sec	σ = 20 sec
Current separation standard of 3, 4, 5 n.mi. (5.56, 7.41, 9.26 km)				
0.0	1.0	34.3	29.6	26.1
.1	.9	33.7	29.2	25.8
.2	.8	33.5	29.0	25.6
.3	.7	33.4	29.0	25.6
.4	.6	33.7	29.2	25.7
.5	.5	34.2	29.6	26.0
.6	.4	35.0	30.2	26.5
.7	.3	36.1	31.0	27.1
.8	.2	37.7	32.1	28.0
.9	.1	39.7	33.6	29.1
1.0	0.0	42.3	35.5	30.5
All 3 n.mi. (5.56 km) separation				
0.0	1.0	45.7	37.8	32.2
.1	.9	44.8	37.2	31.8
.2	.8	44.0	36.6	31.4
.3	.7	43.4	36.2	31.1
.4	.6	43.0	35.9	30.8
.5	.5	42.6	35.6	30.6
.6	.4	42.3	35.5	30.5
.7	.3	42.2	35.3	30.4
.8	.2	42.1	35.3	30.4
.9	.1	42.2	35.3	30.4
1.0	0.0	42.3	35.5	30.5
All 2 n.mi. (3.70 km) separation				
0.0	1.0	68.5	52.1	42.1
.1	.9	66.8	51.1	41.4
.2	.8	65.4	50.3	40.9
.3	.7	64.3	49.7	40.5
.4	.6	63.5	49.2	40.1
.5	.5	62.9	48.8	39.9
.6	.4	62.6	48.6	39.8
.7	.3	62.5	48.6	39.7
.8	.2	62.6	48.6	39.8
.9	.1	62.9	48.8	39.9
1.0	0.0	63.5	49.2	40.1

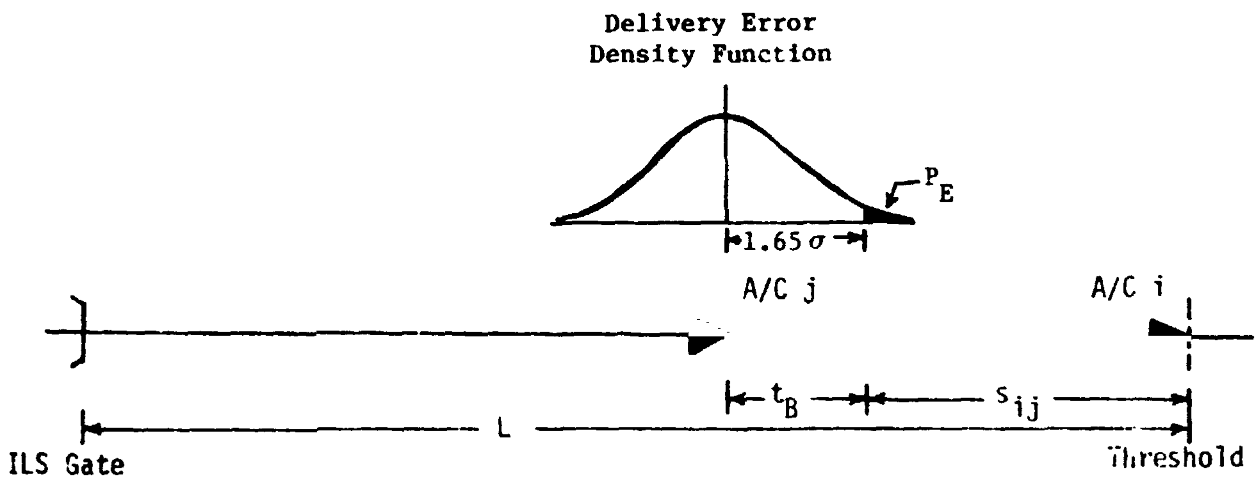

Figure 1a. Overtaking Case When $v_j \geq v_i$

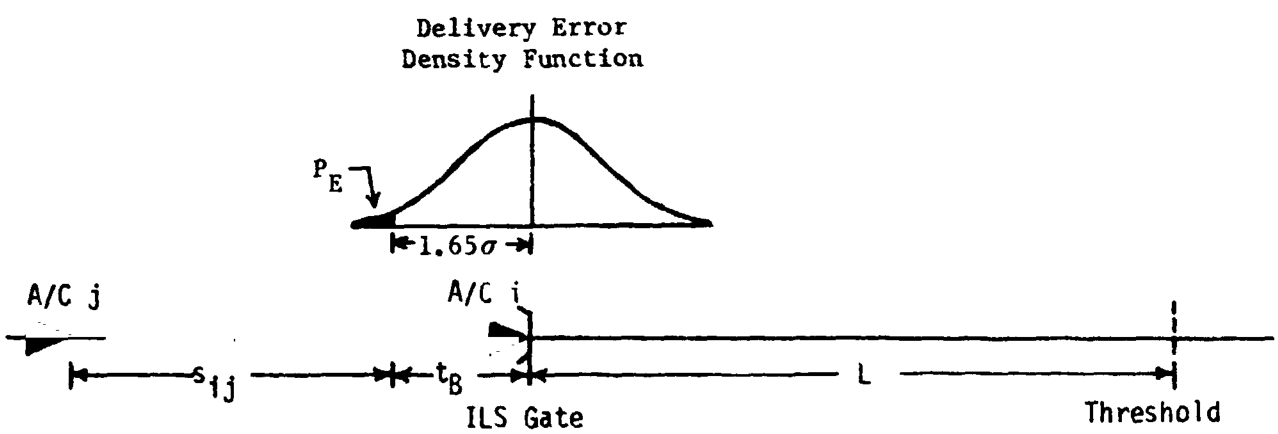

Figure 1b. Opening Case When $v_i > v_j$

Figure 1. Illustration of Separation and Buffering on Final Approach

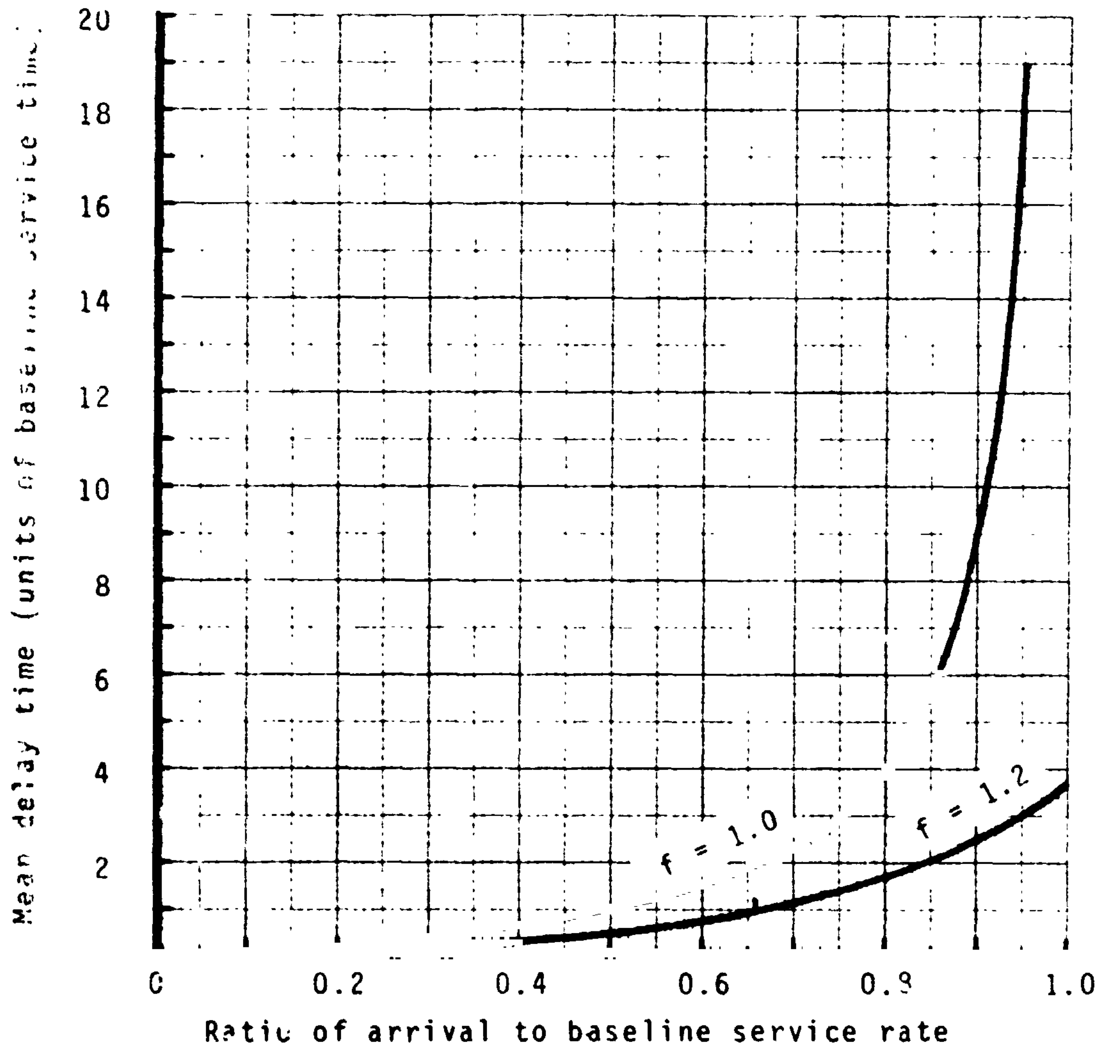

Figure 2. Mean delay time as a function of arrival to baseline saturation service rate ratio (f is baseline service rate multiplier)

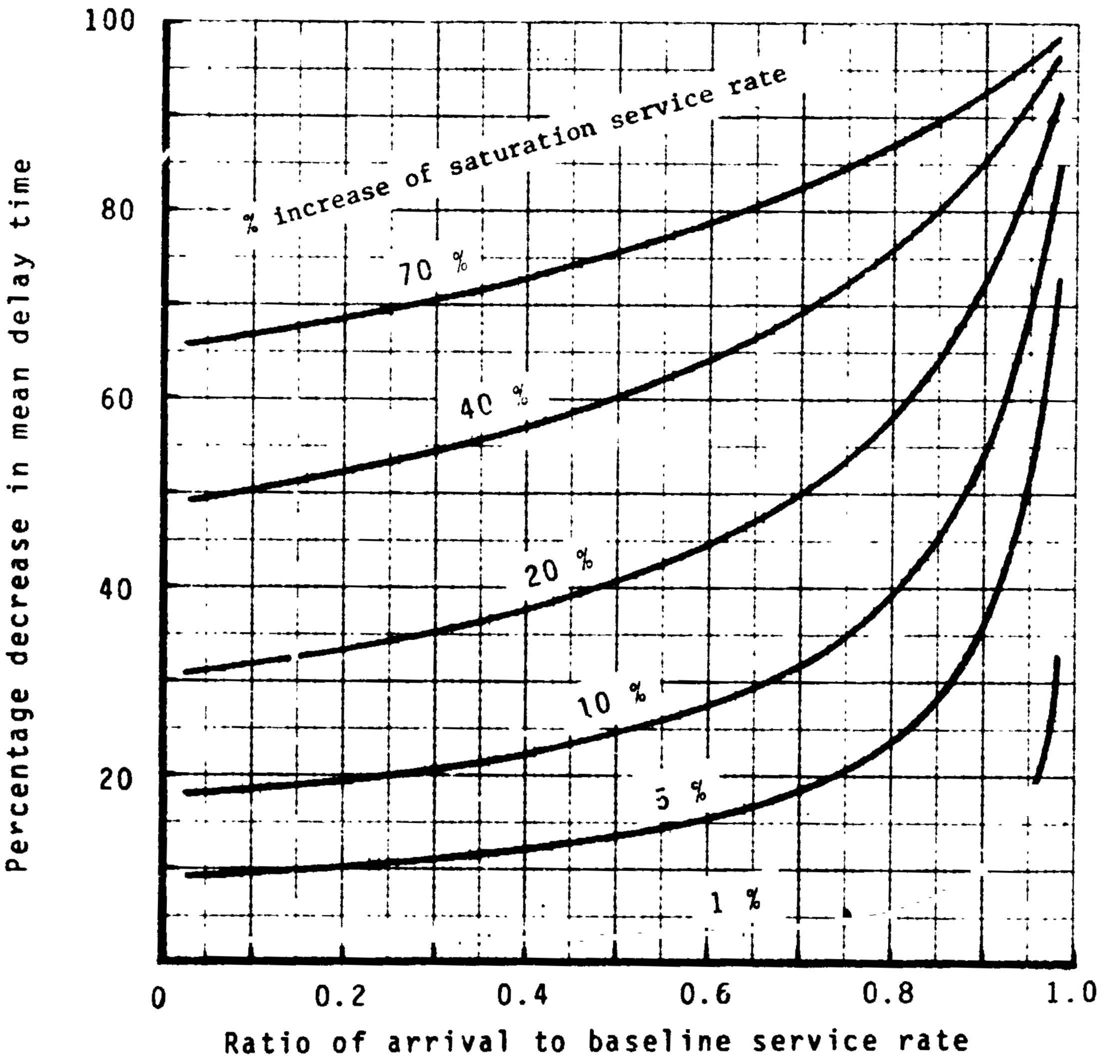

Figure 3. Effect on mean delay time of increasing the baseline saturation service rate by various percentages

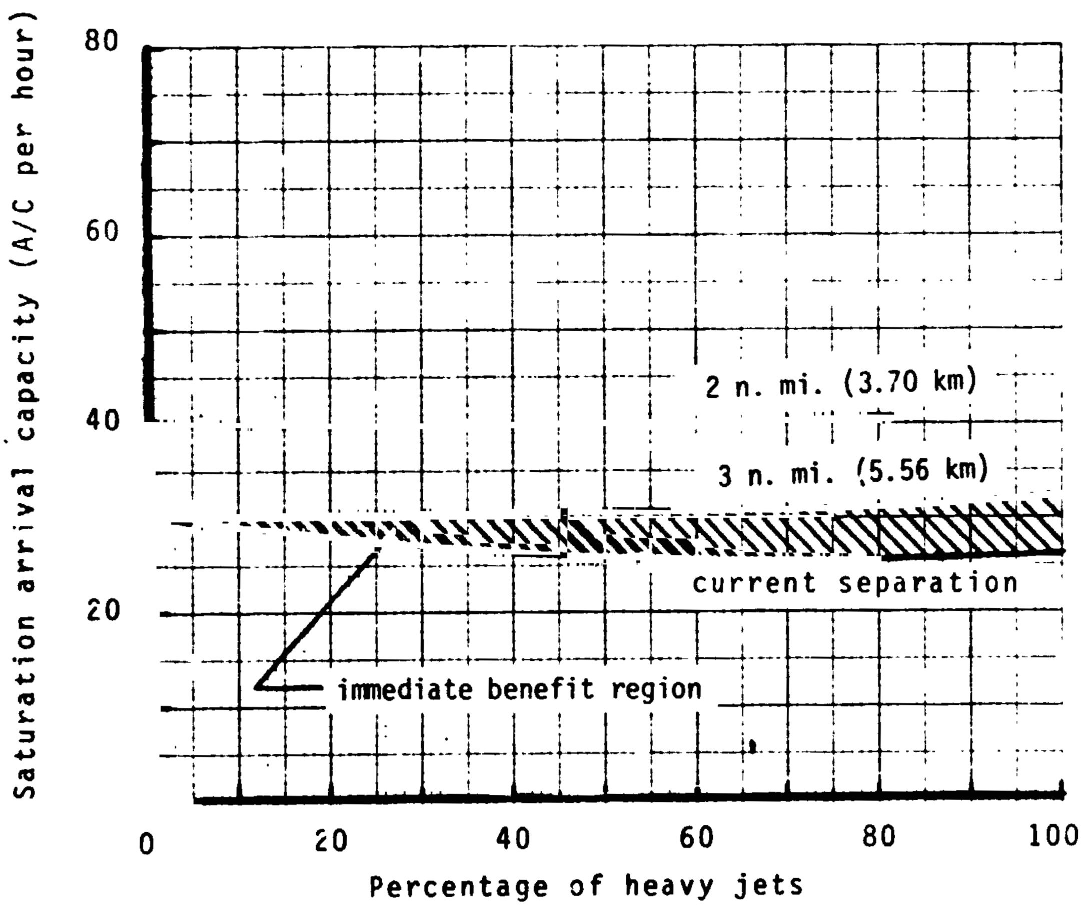

Figure 4. Effect on changing current vortex separation to all 3 n. mi. (5.56 km) and all 2 n. mi. (3.70 km) for delivery standard deviation of 20 seconds

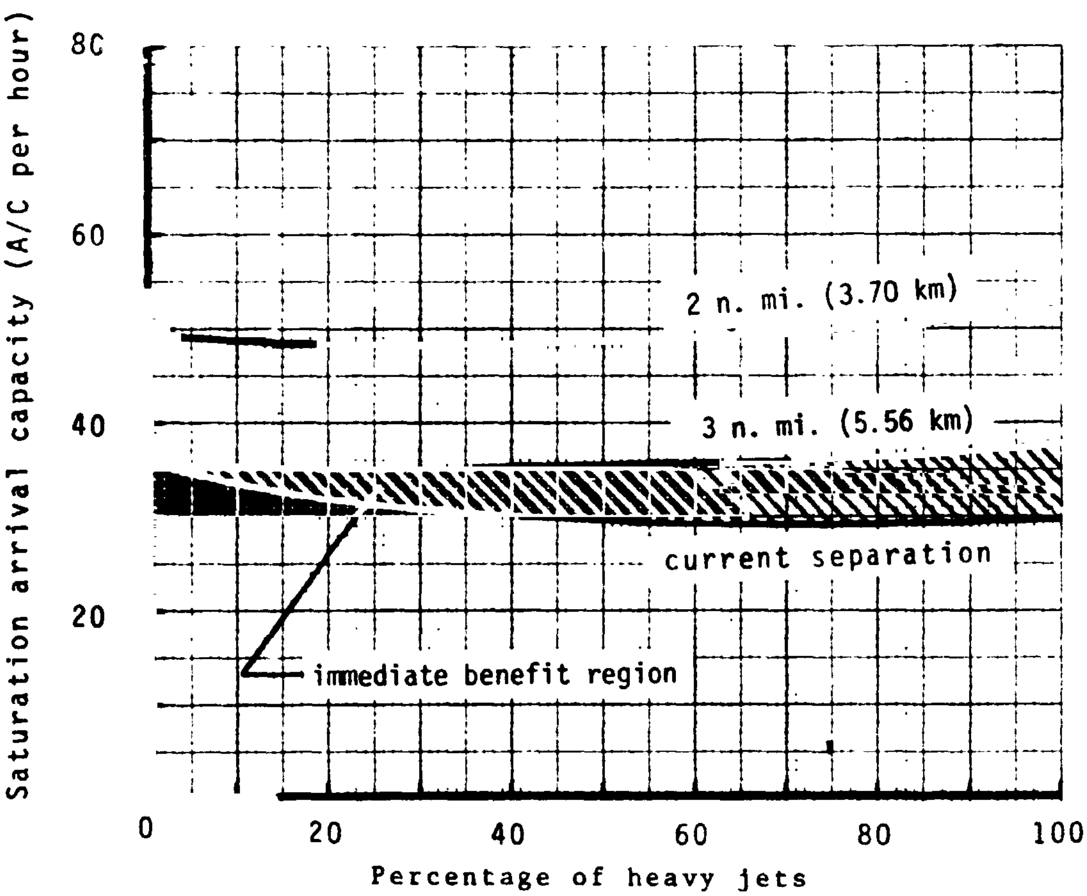

Figure 5. Effect of changing current vortex separation to all 3 n. mi. (5.56 km) and all 2 n. mi. (3.70 km) for delivery standard deviation of 10 seconds

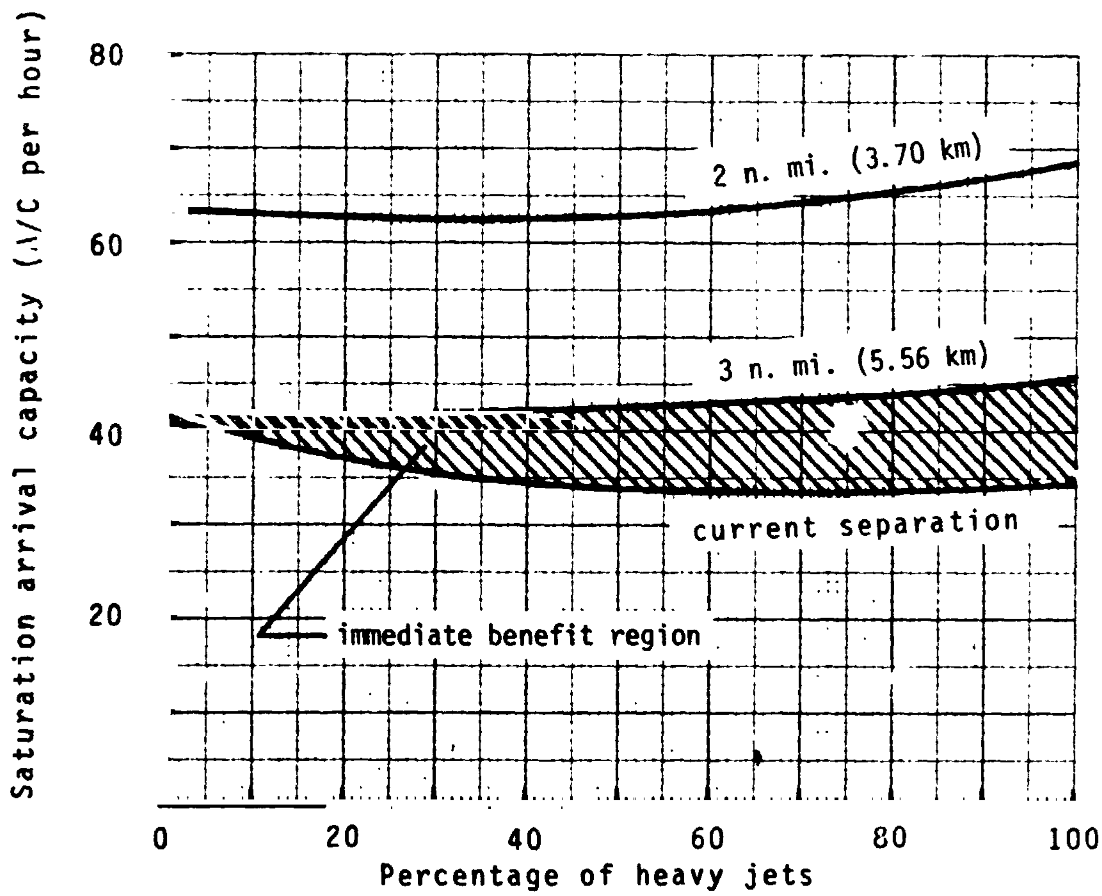

Figure 6. Effect of changing current vortex separation to all 3 n. mi. (5.56 km) and all 2 n. mi. (3.70 km) for delivery standard deviation of 0 seconds

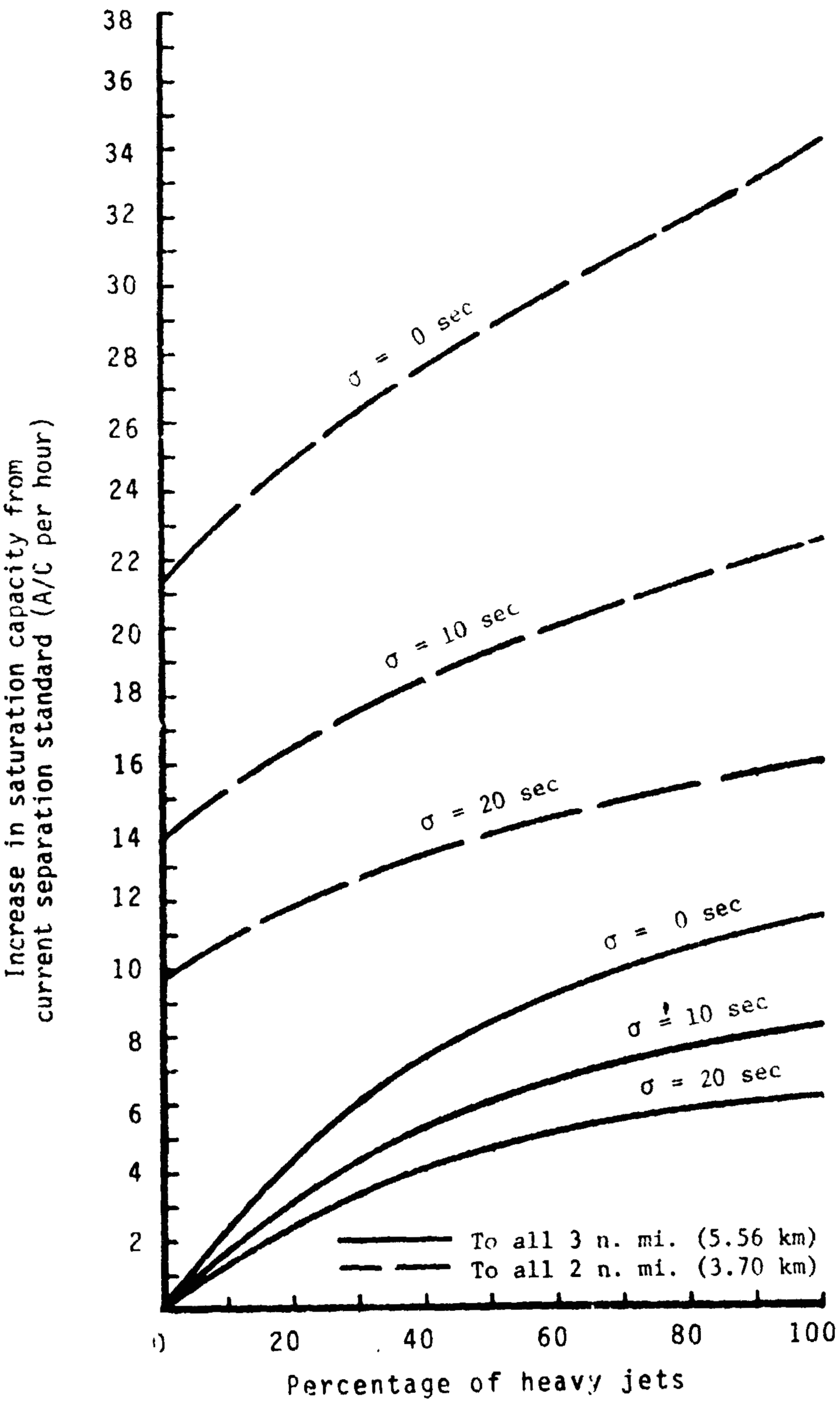

Figure 7. Capacity increase from changing the current vortex separation to all 3 n. mi. (5.56 km) and all 2 n. mi. (3.70 km) for various delivery standard deviations

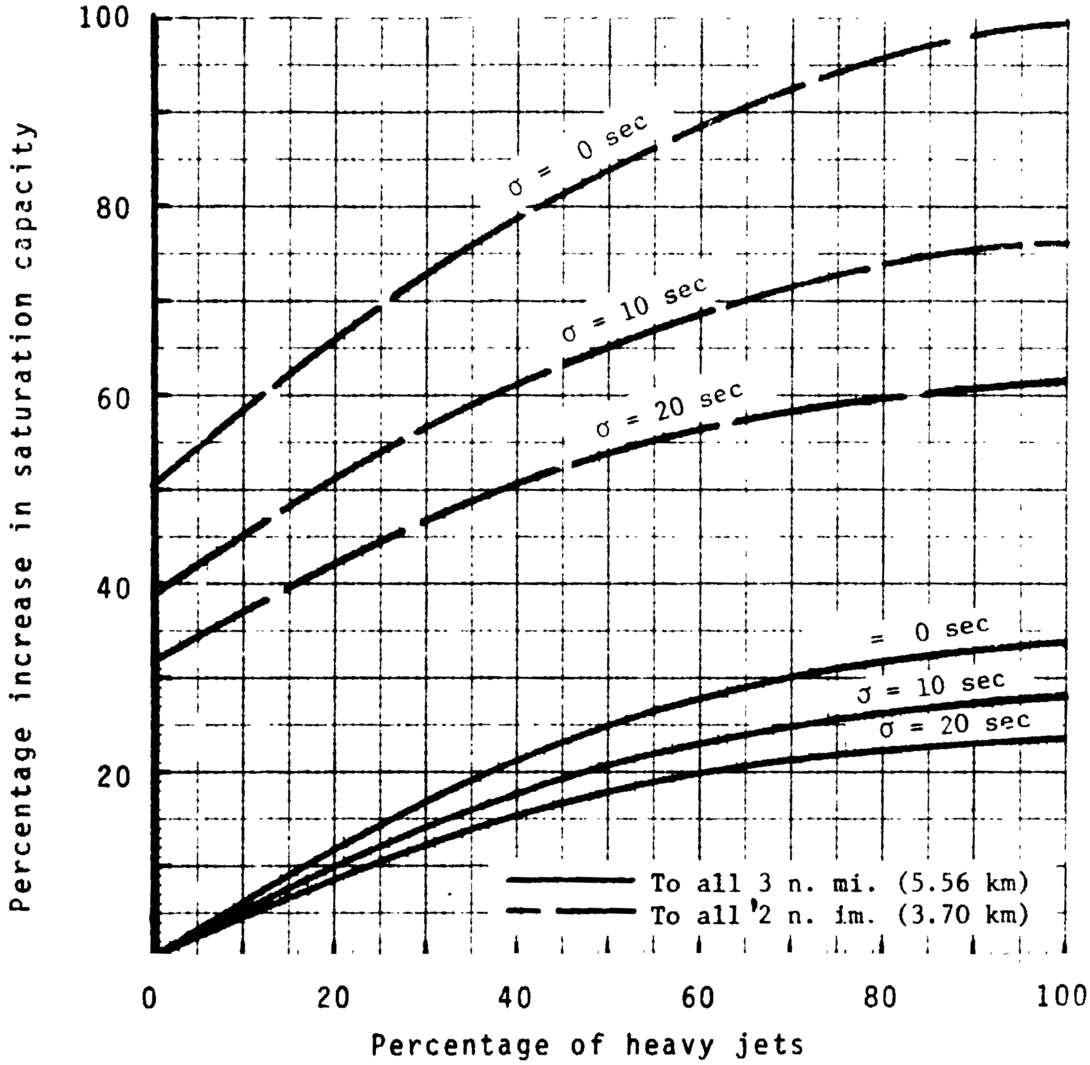

Figure 8. Percentage capacity increase from changing the current vortex separation to all 3 n. mi. (5.56 km) and all 2 n. mi. (3.70 km) for various delivery standard deviations

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15. Supplementary Notes

16. Abstract

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