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Supplemental Report I

on

A DEPOLARIZATION AND ATTENUATION EXPERIMENT USING THE CTS AND COMSTAR SATELLITES

Mathematical Formulations and Definitions for Dual Polarized Reception of a Wave Passing Through a Depolarizing Medium (A Polarization Primer)

by

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16. Abstract

This report gives the theoretical fundamentals and mathematical definitions for calculations involved with dual polarized radio links. Detailed derivations and results are given for several formulations applied to a general dual polarized radio link.

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Introduction

The use of dual polarized communication links for both operational and research programs has been increasing over the past several years. Frequency reuse can be achieved by using orthogonally polarized channels over the same path and at the same frequency. Dual polarized systems are only effective when a high degree of isolation can be maintained to avoid cross channel interference. When designing hardware systems and/or interpreting experimental data it is important to clearly understand the interaction of the electromagnetic wave and the antenna system. Additionally, for millimeter wave applications above 10 GHz in frequency the propagation medium (rain, snow, etc.) will cause depolarization and thus a reduction in the dual channel isolation. Therefore, mathematical formulations and definitions are required for quantitative evaluations of the medium effects.

The intent of this report is to provide a comprehensive source for the mathematical details required for calculations of antenna and medium effects on the signals of a general dual polarized communication link. The emphasis will be on the obtaining values for the signals at the antenna ports as a function of the antenna (both transmit and receive) and medium parameters. The transmit wave (i.e. transmit antenna) and receive antenna polarization properties are assumed to be known but are not limited to ideal behaviour such as perfect linear or circular. Also this report will not deal with how the propagation medium alters the electromagnetic wave. This requires a propagation model for medium, such as rain, and will be the subject of a subsequent report. We are concerned here with using the properties of communication link components to calculate input-output behaviour.

A review of many elementary polarization concepts is included at the beginning for convenience. Most of the information has been well known for a number of years; however, some of the mathematical representations which are particularly convenient for system calculations are not readily available in the literature. Thus complete derivations of the less familiar forms such as the complex polarization factor and complex vector representations are provided.

1. Characterization of the Polarization State of a Plane Electromagnetic Wave

In general, the tip of the instantaneous electric field vector associated with a plane wave traces out an ellipse in a plane perpendicular to the direction of propagation of the wave. This polarization ellipse together with the sense of rotation of the electric field vector describes the polarization state of the wave. The polarization ellipse shown in Figure 1 uses conventional notation.

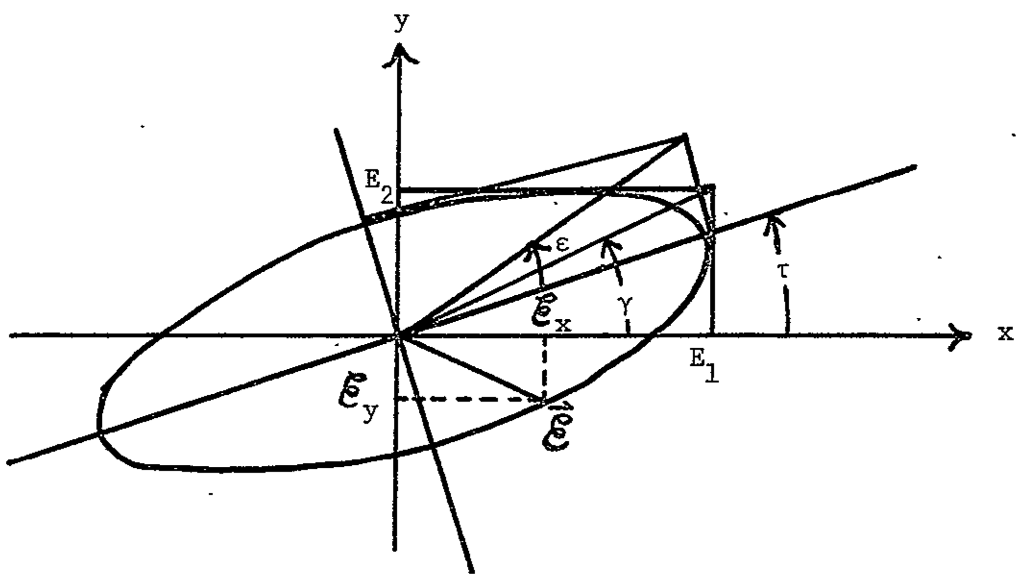


Figure 1. Polarization ellipse: wave approaching.

The instantaneous electric field vector $\vec{\mathcal{E}}(t)$ can be decomposed into components along the reference axes as

$$\mathcal{E}_x(t) = E_1 \cos \omega t \qquad\qquad \mathcal{E}_y(t) = E_2 \cos(\omega t + \delta) \tag{1}$$

where δ = phase angle by which $\mathcal{E}_y$ leads $\mathcal{E}_x$ $(-180° \leq \delta \leq 180°)$. Then

$$\gamma = \tan^{-1} \frac{E_2}{E_1} \qquad\qquad 0° \leq \gamma \leq 90° \tag{2}$$

The angle ε is seen from Fig. 1 to be given by

$$\varepsilon = \cot^{-1}(AR) \qquad\qquad \begin{matrix} 1 \leq |AR| \leq \infty \\ -45° \leq \varepsilon \leq 45° \end{matrix} \tag{3}$$

where the axial ratio AR is a real number whose magnitude is the ratio of the maximum to the minimum field intensities, or

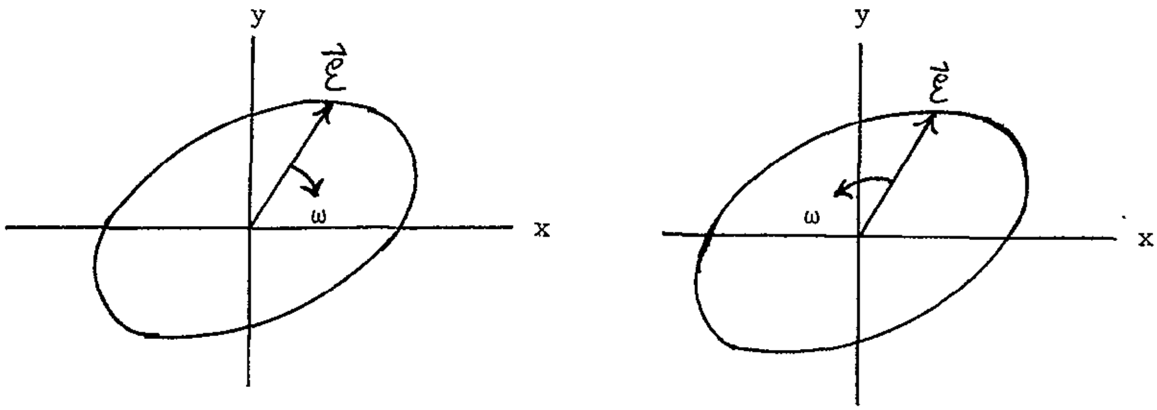
$$|AR| = \frac{\text{polarization ellipse semi-major axis field amplitude}}{\text{polarization ellipse semi-minor axis field amplitude}} \tag{4}$$

and the sign is found from

$$\text{sign}(AR) = \begin{cases} + & \text{for left-hand sense} \\ - & \text{for right-hand sense} \end{cases} \tag{5}$$

When the polarization ellipse is drawn as in Fig. 1 it will always be assumed that the wave is propagating out of the paper, i.e. wave approaching. The sense of rotation of the instantaneous electric field vector $\vec{\mathcal{E}}(t)$ can be either right handed (RH) or left handed (LH). This is determined by placing your thumb in the direction of wave propagation (out of the paper); the sense of rotation of the field vector should then be in the direction of the natural curl of your fingers. If your left (right) hand satisfies these conditions the wave is LH (RH) sensed. See Fig. 2.

Notice that the sign of ε goes as the sign of AR. The sign convention used here is not universal and caution must be exercised when reading the literature.



a) Left hand b) Right hand

Figure 2. Sense of rotation. The electric field vector rotates at radian frequency ω. The wave is propagating in the +z direction.

A thorough discussion of the polarization ellipse (including a proof that it is indeed an ellipse) and many of the mathematical representations of the wave polarization state are found in Kraus [1].

The polarization state of a wave is completely specified relative to a fixed space set of xy-axes by the following three quantities:

1) The shape of the ellipse.
2) The orientation of the ellipse.
3) The sense of rotation of the electric field vector.

An additional quantity is sometimes included but is not usually necessary:

4) The intensity of the wave.

There are many ways to represent the polarization state of a wave. All of them include the first three quantities above. Some of them include the fourth. The various representations are related. In the remainder of this section we will present the essential features of the various polarization state representations and how they are related to each other.

A. Polarization ellipse: ε and τ

The angle ε contains the information $|AR|$ and sign (AR) satisfying 1) and 3). This is seen from (3). The angle τ directly satisfies 2). This is perhaps the most fundamental representation.

B. Complex vector representation

This could also be called the rectangular component representation because we deal with the phasor components of wave along the x and y axes. Let

$$\vec{E} = E_x \hat{x} + E_y \hat{y} \tag{6}$$

be the phasor electric field associated with the instantaneous electric field vector $\vec{\mathcal{E}}(t)$ such that

$$\mathcal{E}_x(t) = \mathrm{Re}[E_x e^{j\omega t}] \tag{7}$$

and

$$\mathcal{E}_y(t) = \mathrm{Re}[E_y\ e^{j\omega t}] \tag{7}$$

If further we let

$$E_x = E_1 \qquad \text{and} \qquad E_y = E_2\ e^{j\delta} \tag{8}$$

then

$$|E_x| = E_1 \qquad \text{and} \qquad |E_y| = E_2 \tag{9}$$

and

$$\text{phase }(E_x) = 0 \qquad \text{and} \qquad \text{phase }(E_y) = \delta. \tag{10}$$

It is the relative phase δ between the x and y components that is important and we have arbitrarily set the phase of E_x to zero. Combining the above results we have

$$\mathcal{E}_x(t) = \mathrm{Re}[E_1\ e^{j\omega t}] = E_1 \cos \omega t$$

and

$$\mathcal{E}_y(t) = \mathrm{Re}[E_2\ e^{j\delta}\ e^{j\omega t}] = E_2 \cos(\omega t + \delta)$$

which is (1). The ratio of amplitudes, E_2/E_1, and the relative phase δ will lead to ε and τ which have been shown to completely specify the wave state. From the polarization ellipse we can see that

$$E_2 = E_1 \tan \gamma$$

or

$$\gamma = \tan^{-1} \frac{E_2}{E_1}$$

which was presented earlier as (2). The following relationships exist [1]

$$\sin 2\varepsilon = \sin 2\gamma \sin \delta \tag{11}$$

$$\tan 2\tau = \tan 2\gamma \cos \delta \tag{12}$$

or

$$\varepsilon = \frac{1}{2} \sin^{-1}(\sin 2\gamma \sin \delta) \tag{13}$$

$$\tau = \frac{1}{2} \tan^{-1}(\tan 2\gamma \cos \delta) \tag{14}$$

Thus ε and τ are derivable from γ (which is obtained from E_2/E_1) and δ.

The intensity of the wave is also contained in the complex vector representation. We will now show this. For intensity we shall use the time average Poynting vector (flux density in watts/m^2). In a plane wave the electric and magnetic field phasors are related by

$$\vec{H} = \frac{1}{\eta} \hat{r} \times \vec{E} \tag{15}$$

where $\hat{r}$ is the unit vector in the direction of wave propagation. The complex Poynting vector is

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* \tag{16}$$

Substituting (15) into (16) and using a vector identity for vector triple products gives

$$\vec{S} = \frac{1}{2\eta} \vec{E} \times (\hat{r} \times \vec{E}^*)$$

$$= \frac{1}{2\eta} [(\vec{E} \cdot \vec{E}^*) \hat{r} - (\vec{E} \cdot \hat{r}) \vec{E}^*] \tag{17}$$

But $E \cdot \hat{r} = 0$ since $\vec{E}$ is perpendicular to the direction of propagation

and (17) becomes

$$\vec{S} = \frac{1}{2\eta} \vec{E} \cdot \vec{E}^* \hat{r} \tag{18}$$

The time average Poynting vector in the direction of propagation is the real part of S, or

$$S_{av} = \frac{1}{2\eta} \vec{E} \cdot \vec{E}^* \tag{19}$$

Substituting (6) into (19) gives

$$S_{av} = \frac{1}{2\eta} \vec{E} \cdot \vec{E}^* = \frac{1}{2\eta} (E_x \hat{x} + E_y \hat{y}) \cdot (E_x^* \hat{x} + E_y^* \hat{y})$$

$$= \frac{1}{2\eta} (|E_x|^2 + |E_y|^2)$$

$$= \frac{1}{2\eta} (E_1{}^2 + E_2{}^2) \tag{20}$$

Thus the intensity of the wave is contained in the complex vector representation which includes E_1 and E_2.

Since the intensity is an unnecessary quantity we can use a normalized complex vector representation which does not include it. To do this we define $\vec{e}$ where

$$\vec{E} = |E| \vec{e} \tag{21}$$

such that

$$\vec{e} \cdot \vec{e}^* = 1 \tag{22}$$

Then

$$\vec{E} \cdot \vec{E}^* = |E|^2 \vec{e} \cdot \vec{e}^* = |E|^2 = |E_x|^2 + |E_y|^2 \tag{23}$$

which is the desired result as seen in (20). Now, let

$$\vec{e} = e_x \hat{x} + e_y \hat{y} \tag{24}$$

Then

$$\begin{aligned}\vec{e} \cdot \vec{e}^* &= (e_x \hat{x} + e_y \hat{y}) \cdot (e_x^* \hat{x} + e_y^* \hat{y}) \\ &= |e_x|^2 + |e_y|^2 \\ &= |e_x|^2 \left(1 + \frac{|e_y|^2}{|e_x|^2}\right) \end{aligned} \tag{25}$$

But

$$\frac{|e_y|}{|e_x|} = \frac{E_2}{E_1} = \tan \gamma \tag{26}$$

So

$$\begin{aligned}\vec{e} \cdot \vec{e}^* &= |e_x|^2 (1 + \tan^2 \gamma) \\ &= |e_x|^2 \frac{1}{\cos^2 \gamma}\end{aligned} \tag{27}$$

However, $\vec{e} \cdot \vec{e}^* = 1$ so

$$|e_x|^2 = \cos^2 \gamma \tag{28}$$

If further we let e_x be real as we did E_x, then

$$e_x = \cos \gamma \tag{29}$$

And then to satisfy (22)

$$|e_y| = \sin \gamma \tag{30}$$

The phase of e_y will be the same as E_y, so

$$e_y = \sin\gamma\, e^{j\delta} \tag{31}$$

Thus

$$\vec{e} = \cos\gamma\,\hat{x} + \sin\gamma\, e^{j\delta}\,\hat{y} \tag{32}$$

We could also define $e_1 = e_x$ and $e_2 = |e_y|$.

C. Polarization ellipse: γ and δ

As discussed in the previous section, a knowledge of γ and δ is sufficient for describing the polarization ellipse. The angles ε and τ are derivable from γ and δ using (13) and (14). Also if we know ε and τ we can find γ and δ from

$$\tan\delta = \frac{\tan 2\varepsilon}{\sin 2\tau} \tag{33}$$

$$\cos 2\gamma = \cos 2\varepsilon\,\cos 2\tau \tag{34}$$

So

$$\delta = \tan^{-1}\left(\frac{\tan 2\varepsilon}{\sin 2\tau}\right) \tag{35}$$

$$\gamma = \frac{1}{2}\cos^{-1}\left(\cos 2\varepsilon\,\cos 2\tau\right) \tag{36}$$

D. Stokes Parameters

The Stokes parameters representation is a matrix formulation which is convenient for antenna-wave power transfer calculations. It also can be used to include a description of partially polarized waves, but we will confine our discussions to completely polarized waves. The matrix is

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} \tag{37}$$

where

$$\begin{aligned}
I &= S_{av} = \frac{1}{2\eta}(E_1^{\,2} + E_2^{\,2}) = S_x + S_y \\
Q &= S_x - S_y = S_{av} \cos 2\varepsilon \cos 2\tau \\
U &= (S_x - S_y) \tan 2\tau = S_{av} \cos 2\varepsilon \sin 2\tau \\
V &= (S_x - S_y) \tan 2\varepsilon \sec 2\tau = S_{av} \sin 2\varepsilon
\end{aligned} \tag{38}$$

This representation obviously includes the wave intensity in I. Also

$$I^2 = Q^2 + U^2 + V^2 \tag{39}$$

Normalized Stokes parameters are used frequently and they are obtained by dividing all matrix entries by the wave intensity S_{av}. Thus, the normalized Stokes matrix is

$$\begin{bmatrix} 1 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} \tag{40}$$

where

$$s_1 = \cos 2\varepsilon \cos 2\tau$$

$$s_2 = \cos 2\varepsilon \sin 2\tau \tag{41}$$

$$s_3 = \sin 2\varepsilon$$

and

$$1 = s_1^{\,2} + s_2^{\,2} + s_3^{\,2} \tag{42}$$

The two ellipse angles ε and τ are all that is used in this formulation.

E. Coherency matrix representation

The coherency matrix [1]

$$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \tag{43}$$

is derivable from the normalized Stokes parameters as

$$\begin{aligned} s_{11} &= \frac{1}{2}(1 + s_1), \qquad & s_{12} &= \frac{1}{2}(s_2 + j\ s_3) \\ s_{21} &= \frac{1}{2}(s_2 - j\ s_3), & s_{22} &= \frac{1}{2}(1 - s_1) \end{aligned} \tag{44}$$

The intensity may be included by multiplying (43) by S_{av}.

F. Poincare Sphere

Every possible polarization state for a completely polarized wave can be assigned a point on the surface of a sphere. Fig. 3 shows this Poincare sphere. [1]

(ε positive, δ positive) and the southern hemisphere corresponds to right hand senses (ε negative, δ negative), while the equator represents linear polarizations.

G. Complex Polarization Factor

The complex polarization factor is a single complex number which represents the polarization state of a wave. In other words, there is a one-to-one correspondence between all possible polarization states and all points on the complex plane. Fig. 4 shows the complex-p plane where p is the complex polarization factor.

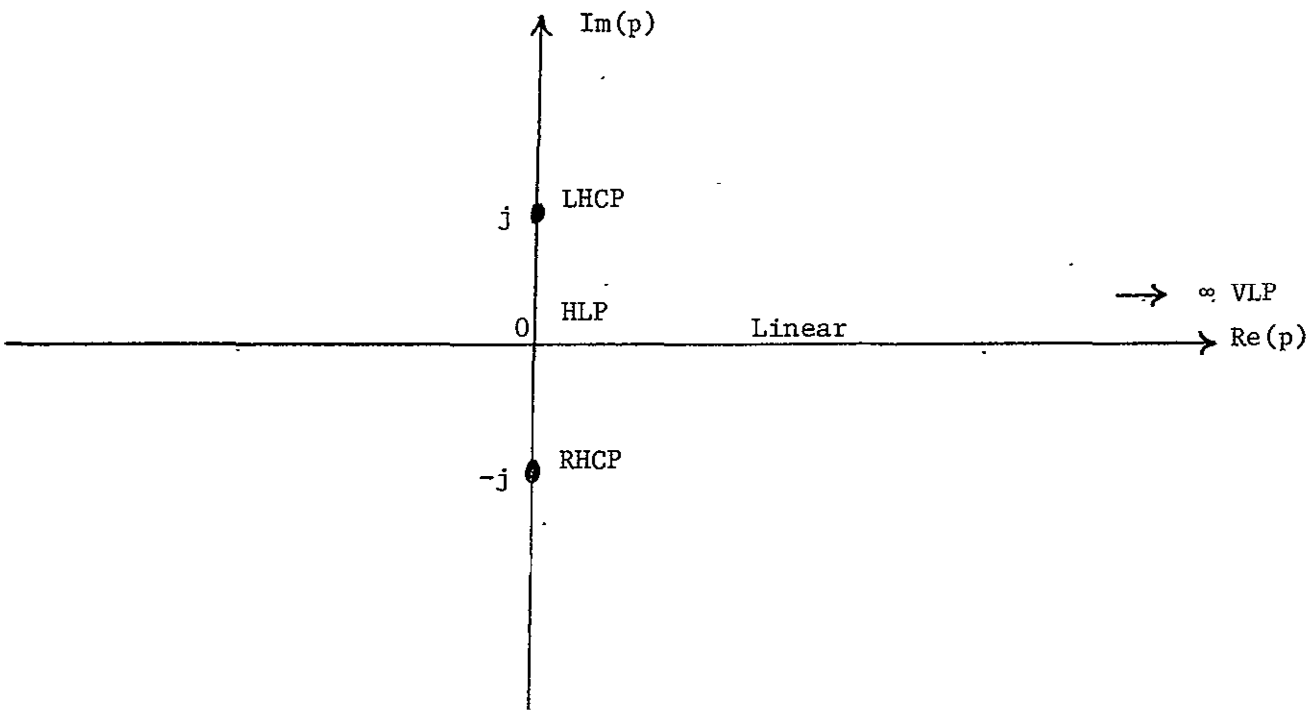


Figure 4. Complex polarization factor plane, or complex p-plane.

Most of the work for this representation was done by Beckmann [2]. In this discussion we have adapted his results to our notation and also corrected his errors.

The definition of p is

$$p = \frac{E_y}{E_x} = \frac{e_y}{e_x} \tag{45}$$

So

$$p = \frac{E_2}{E_1} e^{j\delta} = \frac{e_2}{e_1} e^{j\delta} \tag{46}$$

The following special cases for p help to illustrate its relationship to the polarization states of the polarization ellipse.

<u>p</u>	<u>State</u>	<u>Comment</u>
0	Horizontal Linear (HLP)	$E_2 = 0$
∞	Vertical Linear (VLP)	$E_1 = 0$
j	LHCP	$E_1 = E_2$, $\delta = 90°$
-j	RHCP	$E_1 = E_2$, $\delta = -90°$
Im(p) = 0	Linear	$\delta = 0$
Im(p) > 0	Left Hand Elliptical	$0 < \delta < 180°$
Im(p) < 0	Right Hand Elliptical	$-180 < \delta < 0$

The correspondence between the Poincare sphere and the complex p plane is established using a stereographic projection as shown in Fig. 5.

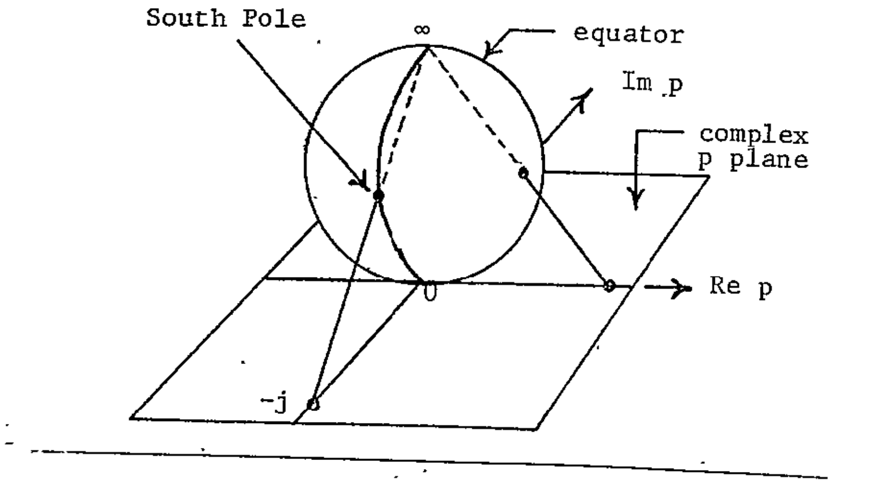


Figure 5. Relationship between points on Poincare sphere and complex p-plane [2].

The normalized Stokes parameters are deriveable from the complex polarization factor as follows

$$s_1 = \frac{1 - |p|^2}{1 + |p|^2}$$

$$s_2 = - \frac{2 \operatorname{Re}(p)}{1 + |p|^2} \tag{47}$$

$$s_3 = \frac{2 \operatorname{Im}(p)}{1 + |p|^2}$$

The complex polarization factor is obtained from the normalized Stokes parameters using

$$|p| = \sqrt{\frac{1 - s_1}{1 + s_1}} \tag{48}$$

$$\delta = \tan^{-1}\left(\frac{s_3}{s_2}\right) \tag{49}$$

Also the complex polarization factor is related very simply to the γ, δ representation

$$|p| = \frac{E_2}{E_1} = \tan\gamma \tag{50}$$

$$\text{phase }(p) = \delta$$

The complex polarization factor is also conveniently related to the complex vector form. From (6)

$$E = E_x\,\hat{x} + E_y\,\hat{y} = E_x(\hat{x} + \frac{E_y}{E_x}\,\hat{y})$$

$$= E_x(\hat{x} + p\,\hat{y}) \tag{51}$$

Now,

$$\hat{x} + p\,\hat{y} = \hat{x} + \tan\gamma\, e^{j\delta}\,\hat{y}$$

$$= \frac{1}{\cos\gamma}\,(\cos\gamma\,\hat{x} + \sin\gamma\, e^{j\delta}\,\hat{y})$$

$$= \frac{1}{\cos\gamma}\,\vec{e}$$

from (32). Or

$$\vec{e} = \cos\gamma(\hat{x} + p\,\hat{y}) \tag{52}$$

H. Other Representations

Many other representations of the polarization state of a wave can be used. For example, a set of axes at 45° to the x and y axes can be used to decompose the wave. Also, a wave can be represented as a sum of right and left hand circularly polarized components. See [2] and [3] for a discussion of these. The[illegible]s and others can be handled very nicely using the complex vector representation and will be discussed later.

The usefulness of a representation depends upon the specific application. If one is merely trying visualize various polarization states the polarization ellipse or Poincare sphere are well suited. If the behavior of a wave as it passes through a depolarizing medium is to be studied the complex polarization factor is appropriate. For antenna-wave interaction the Stokes parameters and complex vector representations are easy to use.

To illustrate the representations above a few examples will be given.

Example 1. Horizontal linear.

A. $\varepsilon = 0,\ \tau = 0$

C. $\delta = \tan^{-1}\left(\frac{\tan 2\varepsilon}{\sin 2\tau}\right) = \tan^{-1}\left(\frac{0}{0}\right) = 45°$ or anything

$\gamma = \frac{1}{2}\cos^{-1}(\cos 2\varepsilon \cos 2\tau) = \frac{1}{2}\cos^{-1}(1) = 0$

B. $e_x = \cos\gamma = \cos 0 = 1$

$e_y = \sin\gamma\, e^{j\delta} = \sin 0\, e^{j0} = 0$

D. $s_1 = \cos 2\varepsilon \cos 2\tau = \cos 0 \cos 0 = 1$

$s_2 = \cos 2\varepsilon \sin 2\tau = \cos 0 \sin 0 = 0$

$s_3 = \sin 2\varepsilon = \sin 0 = 0$

$$[s_i] = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

G. $p = \frac{e_y}{e_x} = \frac{0}{1} = 0$

or

$$|p| = \sqrt{\frac{1 - s_1}{1 + s_1}} = \sqrt{\frac{1 - 1}{1 + 1}} = 0$$

$$\delta = \tan^{-1}\left(\frac{s_3}{s_2}\right) = \tan^{-1}(0) = 0$$

or

$$|p| = \tan\gamma = \tan 0 = 0$$

Example 2. Right Hand Circular Polarization

A. $\varepsilon = -45°$, τ = anything, say not 0

C. $\delta = \tan^{-1}\left(\frac{\tan 2\varepsilon}{\sin 2\tau}\right) = \tan^{-1}(-\infty) = -90°$

$$\gamma = \frac{1}{2}\cos^{-1}(\cos 2\varepsilon \cos 2\tau) = \frac{1}{2}\cos^{-1}(0) = 45°$$

B. $e_x = \cos\gamma = \cos 45° = 1/\sqrt{2}$

$$e_y = \sin\gamma\, e^{j\delta} = \sin 45°\, e^{-j\,90°} = -j\,\frac{1}{\sqrt{2}}$$

D. $s_1 = \cos 2\varepsilon \cos 2\tau = \cos(-90°)\cos 2\tau = 0$

$$s_2 = \cos 2\varepsilon \sin 2\tau = \cos(-90°)\sin 2\tau = 0$$

$$s_3 = \sin 2\varepsilon = \sin(-90°) = -1$$

$$[s_i] = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

G. $p = \frac{e_y}{e_x} = \frac{-j/\sqrt{2}}{1/\sqrt{2}} = -j$

or

$$|p| = \sqrt{\frac{1 - s_1}{1 + s_1}} = \frac{1 - 0}{1 - 0} = 1$$

$$\delta = \tan^{-1}\left(\frac{s_3}{s_2}\right) = \tan^{-1}(-\infty) = -90°$$

or

$$|p| = \tan\gamma = \tan 45° = 1$$

2. Measured Quantities Which Completely Specify the Polarization State of a Wave [4,5,6]

There are several ways to determine the polarization ellipse and sense of rotation. They follow:

A. Polarization-pattern Method

In this method a linear antenna is rotated about an axis along the direction of propagation of the wave. The amplitude response of the antenna has a maximum and minimum which coincide with the major and minor axis of the polarization ellipse. Thus the axial ratio is easily computed. By noting the response of the receiving antenna as a function of its angle of rotation the tilt angle (τ) between a reference direction (x) and maximum response position can be determined. This method does not give the sense of the wave. Some other measurement must be employed, such as comparing the output of two opposite sense circularly polarized receiving antennas.

B. Linear-component Method

Measurement of the magnitudes of two orthogonal linearly polarized components, E_1 and E_2, (or their ratio), and their relative phase δ will completely characterize a wave state. This gives the complex polarization factor directly.

C. Circular-component Method

This involves measuring the magnitudes of right and left hand circularly polarized antenna outputs (or the ratio of the magnitudes) and the relative phase of the outputs.

D. Multiple -component Method

A set of four electric field magnitudes are measured. They consist of one set and one component from the remaining two sets of the following: two orthogonal linear polarizations; two orthogonal linear polarizations at 45° with respect to the first set; left and right circular polarizations. No phase measurements are required. Gains of antennas and receivers must be known.

Note that only two independent parameters need to be determined to uniquely specify the polarization state of a wave. See B and C. Three independent parameters are necessary if the power density of the wave is to be included. Four independent parameters are necessary if the wave is partially polarized.

3. Determination of the Orthogonal Wave State Representations

In polarization studies it is frequently desired to know the wave state which is orthogonal to a given state. In this section we will show how to obtain the orthogonal state for each of the representations discussed in Section 1.

The simplest representation for visualizing orthogonal wave states is the Poincare sphere (F of Section 1). Directly opposite points on the surface correspond to orthogonal wave states. For example, at the origin ($\varepsilon = 0$, $\tau = 0$) is horizontal LP and 180° around on the equator ($\varepsilon = 0$, $\tau = 90°$) is vertical LP. See Fig. 1.3. Also the north pole is LHCP and the south pole is RHCP, which are orthogonal states.

The Poincare sphere also aids in determining the orthogonal states of other representations. In the polarization ellipse ε and τ representation (A of Section 1) if ε_w and τ_w are the wave state parameters then

$$\begin{aligned} \varepsilon_{wo} &= -\varepsilon_w \\ \tau_{wo} &= \tau_w \pm 90° \qquad \text{such that } 0 < \tau_{wo} < 180° \end{aligned} \tag{1}$$

are the parameters of the orthogonal state. For example, if $\varepsilon_w = 20°$ and $\tau_w = 45°$ then $\varepsilon_{wo} = -20°$ and $\tau_{wo} = 135°$.

For the polarization ellipse γ and δ representation (C of Section 1) a wave with γ_w and δ_w has orthogonal state parameters

$$\begin{aligned} \gamma_{wo} &= 90° - \gamma_w \qquad \text{such that } 0 < \gamma_{wo} < 90° \\ \delta_{wo} &= \delta_w \pm 180° \qquad \text{such that } -180° < \delta_{wo} < 180° \end{aligned} \tag{2}$$

We will use the previous example again. Now

$$\gamma_w = \frac{1}{2}\cos^{-1}(\cos 2\varepsilon_w \cos 2\tau_w) = \frac{1}{2}\cos^{-1}(\cos 40° \cos 90°) = 45°$$

$$\delta_w = \tan^{-1}\left(\frac{\tan 2\varepsilon}{\sin 2\tau}\right) = \tan^{-1}\left(\frac{\tan 40°}{\sin 90°}\right) = 40°$$

Then

$$\gamma_{wo} = 45° \qquad \text{and} \qquad \delta_{wo} = -140° \ .$$

The above two orthogonal state representations are consistent because from (1-34)

$$\cos 2\gamma_{wo} = \cos 2\varepsilon_{wo} \cos 2\tau_{wo}$$

$$\cos 2(90° - \gamma_w) = \cos 2(-\varepsilon_w) \cos 2(\tau_w \pm 90°)$$

$$-\cos 2\gamma_w = \cos 2\varepsilon_w(-\cos 2\tau_w)$$

$$\cos 2\gamma_w = \cos 2\varepsilon_w \cos 2\tau_w$$

which is true for wave state w. Also (1-33) must be satisfied, and

$$\tan \delta_{wo} = \frac{\tan 2\varepsilon_{wo}}{\sin 2\tau_{wo}}$$

$$\tan(\delta_w \pm 180°) = \frac{\tan 2(-\varepsilon_w)}{\sin 2(\tau_w \pm 90°)}$$

$$\tan \delta_w = \frac{-\tan 2\varepsilon_w}{-\sin 2\tau_w}$$

$$= \frac{\tan 2\varepsilon_w}{\sin 2\tau_w}$$

which is true for wave state w. For the previous two examples we have for (1-34).

$$\cos 2(45°) = \cos 2(-20°) \cos 2(135°)$$

$$\cos 90° = \cos 40° \cos 270°$$

$$0 = 0$$

For (1-33)

$$\tan -140° = \frac{\tan 2(-20°)}{\sin 2(135°)}$$

$$0.839 = \frac{-0.839}{-1}$$

The complex vector representation has been studied by Kales [7]. A wave with normalized complex vector form $\vec{e}_{wo}$ is orthogonal to one with $\vec{e}_w$ if

$$\vec{e}_w \cdot \vec{e}_{wo}{}^* = 0 \tag{3}$$

We would like to solve this equation and determine an explicit form of $\vec{e}_{wo}$ in terms of $\vec{e}_w$. To do this we use

$$\vec{e}_w = e_{xw}\,\hat{x} + e_{yw}\,\hat{y} \tag{4}$$

$$\vec{e}_{wo} = e_{xwo}\,\hat{x} + e_{ywo}\,\hat{y} \tag{5}$$

in (3) giving

$$0 = e_{xw}\,e^*_{xwo} + e_{yw}\,e^*_{ywo} \tag{6}$$

This would be true if

$$e^*_{xwo} = e_{yw} \qquad \text{and} \qquad e^*_{ywo} = -e_{xw} \tag{7}$$

Complex conjugating these equations gives

$$e_{xwo} = e^*_{yw} \qquad \text{and} \qquad e_{ywo} = -e^*_{xw} \tag{8}$$

Thus

$$\vec{e}_{wo} = e^*_{yw}\,\hat{x} - e^*_{xw}\,\hat{y} \tag{9}$$

In the Stokes parameter formulation we can derive the orthogonal representation from the ε, τ parameters using the relationships to Stokes parameters in (1-41). So for the wave w we have

$$[s_{iw}] = \begin{bmatrix} 1 \\ s_{1w} \\ s_{2w} \\ s_{3w} \end{bmatrix} = \begin{bmatrix} 1 \\ \cos 2\varepsilon_w \cos 2\tau_w \\ \cos 2\varepsilon_w \sin 2\tau_w \\ \sin 2\varepsilon_w \end{bmatrix} \tag{10}$$

Then using the ε, τ parameters for the orthogonal state

$$[s_{iwo}] = \begin{bmatrix} 1 \\ s_{1wo} \\ s_{2wo} \\ s_{3wo} \end{bmatrix} = \begin{bmatrix} 1 \\ \cos 2\varepsilon_{wo} \cos 2\tau_{wo} \\ \cos 2\varepsilon_{wo} \sin 2\tau_{wo} \\ \sin 2\varepsilon_{wo} \end{bmatrix} \tag{11}$$

and from (1), (11) yields

$$\begin{aligned} s_{1wo} &= \cos 2(-\varepsilon_w) \cos 2(\tau_w \pm 90°) \\ s_{2wo} &= \cos 2(-\varepsilon_w) \sin 2(\tau_w \pm 90°) \\ s_{3wo} &= \sin 2(-\varepsilon_w) \end{aligned}$$

or

$$
\begin{aligned}
s_{1wo} &= -\cos 2\varepsilon_w \cos 2\tau_w = -s_{1w} \\
s_{2wo} &= -\cos 2\varepsilon_w \sin 2\tau_w = -s_{2w} \qquad (12) \\
s_{3wo} &= -\sin 2\varepsilon_w \qquad\qquad\quad = -s_{3w}
\end{aligned}
$$

For example, the Stokes parameters for RHCP and LHCP are, respectively,

$$
\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (13)
$$

The complex polarization factor orthogonal state representation can be derived from the γ, δ parameters using (1-50). For the wave

$$
\begin{aligned}
|p_w| &= \tan \gamma_w \\
\text{phase } (p_w) &= \delta_w
\end{aligned} \qquad (14)
$$

Then using (2) we have for the orthogonal state

$$
\begin{aligned}
|p_{wo}| &= \tan \gamma_{wo} = \tan (90° - \gamma_w) \\
&= \cot \gamma_w \\
\text{phase } (p_{wo}) &= \delta_{wo} = \delta_w \pm 180°
\end{aligned} \qquad (15)
$$

So

$$
\begin{aligned}
p_{wo} &= |p_{wo}|\, e^{j\delta_{wo}} \\
&= \cot \gamma_w \, e^{j(\delta_w \pm 180°)} \\
&= -\frac{1}{\tan \gamma_w \, e^{-j\delta_w}} \\
&= -\frac{1}{p_w{}^*}
\end{aligned} \qquad (16)
$$

4. Antenna-Wave Interaction

An extremely important problem is the interaction of an electromagnetic wave and a receiving antenna. Suppose the wave incident on the antenna has an average flux density S_{av}. From (1-20)

$$S_{av} = \frac{1}{2\eta} (E_1^2 + E_2^2) \tag{1}$$

In the ideal case the wave and antenna polarizations are identical and the antenna is oriented such that its maximum response is in the direction of arrival of the incident plane wave. Then the power received is

$$P_R = S_{av} A_e \tag{2}$$

where A_e is the effective aperture given by

$$A_e = \frac{\lambda^2}{4\pi} G \tag{3}$$

and G is the antenna gain. The power P_R is that power available from the antenna, i.e. will be delivered to a matched load.

If the antenna and wave polarizations are not matched (2) becomes

$$P_R = S_{av} A_e m_p \tag{4}$$

where m_p is the <u>polarization mismatch factor</u>, also called the polarization efficiency. The mismatch factor can vary from zero (complete wave-antenna mismatch) to unity (perfect wave-antenna match). In this section we will discuss how m_p is calculated for several representations.

The Poincare sphere representation provides a simple formulation for the polarization mismatch factor. It is given by

$$m_p = \frac{1}{2}(1 + \cos M\,M_a) = \cos^2 \frac{M\,M_a}{2} \tag{5}$$

where $M\,M_a$ is the angular separation on the Poincare sphere between the point of the wave state M and the antenna state M_a. For example, if the wave and antenna polarization states are identical then M and M_a are the same and $M\,M_a = 0$; then (5) gives $m_p = 1$. If the points $M\,M_a$ are directly opposite on the Poincare sphere then $M\,M_a = 180°$ and $m_p = 0$ from (5).

The Stokes parameter formulation is particularly well suited for antenna-wave interaction calculations. Consider a wave with Stokes parameters $[s_i]$ incident on an antenna with Stokes parameters $[a_i]$ where

$$[a_i] = \begin{bmatrix} 1 \\ \cos 2\varepsilon_a \cos 2\tau_a \\ \cos 2\varepsilon_a \sin 2\tau_a \\ \sin 2\varepsilon_a \end{bmatrix}, \quad [s_i] = \begin{bmatrix} 1 \\ \cos 2\varepsilon_w \cos 2\tau_w \\ \cos 2\varepsilon_w \sin 2\tau_w \\ \sin 2\varepsilon_w \end{bmatrix} \tag{6}$$

The polarization mismatch factor is [4]

$$m_p = \frac{1}{2}[a_i]^T\,[s_i] \tag{7}$$

where the superscript T indicates matrix transpose. Substituting (6) into (7) gives

$$\begin{aligned} m_p = \frac{1}{2}\{1 &+ \cos 2\varepsilon_a \cos 2\varepsilon_w \cos 2\tau_a \cos 2\tau_w \\ &+ \cos 2\varepsilon_a \cos 2\varepsilon_w \sin 2\tau_a \sin 2\tau_w \\ &+ \sin 2\varepsilon_a \sin 2\varepsilon_w\} \end{aligned} \tag{8}$$

or

$$m_p = \frac{1}{2}[1 + \sin 2\varepsilon_w \sin 2\varepsilon_a$$
$$+ \cos 2\varepsilon_w \cos 2\varepsilon_a \cos 2\Delta\tau] \tag{9}$$

where $\Delta\tau$ is the relative tilt angle

$$\Delta\tau = \tau_a - \tau_w \tag{10}$$

and τ_a and τ_w are the tilt angles of the antenna and wave polarization ellipses. If the ε, τ parameters (or ε_a, ε_w, and $\Delta\tau$) for the antenna and the wave are known, the polarization mismatch factor is easily evaluated using (9). The ε angles can be found from the axial ratios using (1-3) as

$$\varepsilon_w = \cot^{-1}(AR_w) \qquad \varepsilon_a = \cot^{-1}(AR_a) \tag{11}$$

Note that the axial ratios AR_a and AR_w carry a sign, being positive for left sense and negative for right sense.

The mismatch factor can be expressed directly in terms of axial ratios and relative tilt angle $\Delta\tau$ of the polarization ellipses as

$$m_p = [4(AR_w^{\ 2} + 1)(AR_a^{\ 2} + 1)]^{-1} \cdot [(AR_w + 1)^2(AR_a + 1)^2$$
$$+ (AR_w - 1)^2(AR_a - 1)^2 + 2(AR_w^{\ 2} - 1)(AR_a^{\ 2} - 1)\cos 2\Delta\tau] \tag{12}$$

This can be derived directly using circularly polarized components [9], with the aid of the Poincare' sphere [6], or from (9) after several manipulations. A somewhat simplier form follows from (12)

$$m_p = \frac{1}{2} + \frac{4\ AR_w\ AR_a + (AR_w^{\ 2} - 1)(AR_a^{\ 2} - 1)\cos 2\Delta\tau}{2(AR_w^{\ 2} + 1)(AR_a^{\ 2} + 1)} \tag{13}$$

Example: A right hand circularly polarized (RHCP) wave is incident on a left hand circularly polarized (LHCP) antenna. Then $AR_w = -1$, $AR_a = 1$, and from (13)

$$m_p = \frac{1}{2} + \frac{4(-1)(1) + 0}{2(2)(2)} = \frac{1}{2} - \frac{1}{2} = 0$$

Also, from (11)

$$\varepsilon_w = \cot^{-1}(-1) = -45°, \ \varepsilon_a = \cot^{-1}(1) = 45°$$

and (9) yields

$$m_p = \frac{1}{2}[1 + (-1)(1) + 0] = 0$$

Example: A nearly LHCP wave with $AR_w = 1.122$ (1.0 dB) is incident upon a nearly LHCP antenna of $AR_a = 1.03514$ (0.3 dB). From (13)

$$m_p = 0.99641 + 0.0019783 \cos 2\Delta\tau \tag{14}$$

From (11)

$$\varepsilon_w = \cot^{-1}(1.122) = 41.7095°$$

$$\varepsilon_a = \cot^{-1}(1.03514) = 44.0108°$$

Using these values in (9) a result identical to (14) is obtained. The mismatch factor is very close to unity for this case. It is maximum when the major axes of the polarization ellipses are aligned ($\Delta\tau = 0$) giving $m_p = 0.998388$ and minimum when they are perpendicular ($\Delta\tau = 90°$) and (14) then gives $m_p = 0.994432$.

Equation (9) is easy to use for any polarization. We have presented examples for circular and near circular polarization cases. Suppose the antenna is linearly polarized so that $AR_a = \infty$ and $\varepsilon_a = 0°$. Then (9) becomes

$$m_p = \frac{1}{2}[1 + \cos 2\varepsilon_w \cos 2\Delta\tau] \tag{15}$$

If the wave were linear and the antenna arbitrary (15) would be the same except ε_w is replaced by ε_a. In the special case where both the wave and the antenna are linearly polarized (15), for $\varepsilon_w = 0°$, reduces to

$$m_p = \frac{1}{2}[1 + \cos 2\Delta\tau] = \cos^2 \Delta\tau \tag{16}$$

which varies from 1 for $\Delta\tau = 0°$ to 0 for $\Delta\tau = 90°$.

Equation (13) may be used with a linear antenna also by first dividing the numerator and denominator of the second term by AR_a^2 and letting it then approach infinity, yielding

$$m_p = \frac{1}{2} + \frac{(AR_w^2 - 1)\cos 2\Delta\tau}{2(AR_w^2 + 1)} \tag{17}$$

If further, the wave is linearly polarized the process is repeated and (17) reduces to (16), as expected.

If the polarization states of the wave and the antenna are represented by complex polarization factors p_w and p_a, then [2, p. 186]

$$m_p = \frac{|1 + p_w\, p_a{*}|^2}{(1 + |p_w|^2)(1 + |p_a|^2)} \tag{18}$$

where * indicates complex conjugate.

<u>Example</u>: A RHCP wave and LHCP antenna. Then $p_w = -j$ and $p_a = j$ and

$$m_p = \frac{|1 - 1|}{(2)(2)} = 0$$

<u>Example</u>: A RHCP wave and linear horizontal antenna. Then $p_w = -j$ and $p_a = 0$ and

$$m_p = \frac{1}{2(1)} = \frac{1}{2}$$

<u>Example</u>: A RHCP wave and linear vertical antenna. Then $p_w = -j$ and $p_a = \infty$ and

$$m_p = \lim_{p_a \to \infty} \frac{\left| \frac{1}{|p_a|} + \frac{p_w p_a^*}{|p_a|} \right|^2}{(1 + |p_w|^2)\left(\frac{1}{|p_a|^2} + 1\right)} = \frac{1}{2}$$

The complex-vector representation can be used to calculate polarization mismatch factor also. The polarization mismatch factor is simply

$$m_p = |\vec{e}_w \cdot \vec{e}_a^*|^2 \tag{19}$$

where $\vec{e}_w$ and $\vec{e}_a$ are the normalized complex vector representations for the wave and antenna. This result is not in common useage and thus will be proved. [7] From (1-52) we can write

$$\begin{aligned} \vec{e}_w &= \cos \gamma_w (\hat{x} + p_w \hat{y}) \\ \vec{e}_a &= \cos \gamma_a (\hat{x} + p_a \hat{y}) \end{aligned} \tag{20}$$

Then (19) gives

$$m_p = \cos^2 \gamma_w \cos^2 \gamma_a |(\hat{x} + p_w \hat{y}) \cdot (\hat{x} + p_a^* \hat{y})|^2$$

$$= \frac{|1 + p_w p_a^*|^2}{\sec^2 \gamma_w \sec^2 \gamma_a} \tag{21}$$

From (1-50) $|p| = \tan\gamma$ so

$$1 + |p_w|^2 = 1 + \tan^2 \gamma_w \equiv \sec^2 \gamma_w \tag{22}$$

Similarly

$$1 + |p_a|^2 = \sec^2 \gamma_a \tag{23}$$

Using (22) and (23) in (21)

$$m_p = \frac{|1 + p_w\, p_a{}^*|^2}{(1 + |p_w|^2)(1 + |p_a|^2)}$$

which is (18) thus proving (19).

Example: A horizontal LP wave and a RHCP antenna. From the examples at the end of Section 1

$$\vec{e}_w = \hat{x}$$

$$\vec{e}_a = \frac{1}{\sqrt{2}}\,\hat{x} - j\,\frac{1}{\sqrt{2}}\,\hat{y}$$

Then

$$m_p = |\vec{e}_w \cdot \vec{e}_a{}^*|^2 = \left|\frac{1}{\sqrt{2}} + 0\right|^2 = \frac{1}{2}$$

which is the correct result.

The polarization mismatch factor is quite useful in calculating power transfer from an incident wave to an antenna. When performing power budget calculations for communication links the relationships developed so far in this section are sufficient. However, in many situations it is frequently desired to know the output voltage from an antenna. This is useful for calculating the antenna output voltage phase changes in response to incoming wave changes.

We will define a complex voltage V(w,a) which describes the interaction of wave polarization state w and an antenna of polarization state a as

$$V(w,a) = \vec{E}_w \cdot \vec{e}_a^* \tag{24}$$

This definition is similar to that employed with the commonly used antenna vector effective height $\vec{h}$ where $V = \vec{E}_w \cdot \vec{h}$. [10,11] The conjugate is not present in this definition because $\vec{h}$ is related to the transmitting properties of the antenna, whereas $\vec{e}_a$ relates to the polarization state relative to an xy coordinate system for reception.

It is important to establish the relationship between V(w,a) and the power available from the antenna. From (1-21)

$$\vec{E}_w = |E_w| \, \vec{e}_w \tag{25}$$

Then

$$\begin{aligned} |V(w,a)|^2 &= V(w,a) \, V^*(w,a) \\ &= |E_w| \, \vec{e}_w \cdot \vec{e}_a^* \, |E_w| \, \vec{e}_w \cdot \vec{e}_a^* \\ &= |E_w|^2 \, |\vec{e}_w \cdot \vec{e}_a^*|^2 \end{aligned} \tag{26}$$

Using (19) we have

$$|V(w,a)|^2 = |E_w|^2 \, m_p \tag{27}$$

But the average flux density of the wave is

$$S_{av} = \frac{1}{2\eta} |E_w|^2 \tag{28}$$

From (4) the available power is

$$P_R = S_{av} \cdot m_p A_e \tag{29}$$

Combining the above three results gives

$$P_R = \frac{|V(w,a)|^2}{2\eta} A_e \tag{30}$$

If V_{rms} is the true rms voltage at the output terminals of a matched antenna, the power available is

$$P_R = \frac{V_{rms}^2}{R_a} \tag{31}$$

where R_a is the antenna resistance which is also the load resistance under matched conditions. Comparing this with (30) we find that

$$V_{rms} = \sqrt{\frac{R_a A_e}{2\eta}} \; |V(w,a)| \tag{32}$$

We have now related V(w,a) to the power available and to the actual rms voltage available from the antenna. For a fixed antenna configuration (A_e and R_a fixed) $|V(w,a)|$ gives the relative variation in output voltage (output power if squared) as the incident wave characteristics (flux or polarization) change.

5. Cross Polarization Ratio

Any wave or antenna polarization state can be decomposed into component orthogonally polarized states. When studying the depolarizing effects of devices or media it is often necessary to make such decompositions. In this section we will discuss the terminology and techniques for analyzing the polarization components of a wave.

Let w' be the polarization state we wish to decompose. It could be that of an antenna or a wave leaving a device or medium. Also let w be a wave state and w_o its orthogonal state; these are the states into which we would like to decompose the wave. At this point we will arbitrarily let w be the "co-polarized" state and w_o be the "cross-polarized" state. State w_o is orthogonal or cross-polarized to state w. Let $P(w',w)$ be the power of wave w' in the component wave state w and $P(w',w_o)$ be the power of wave w' in the component wave state w_o. Then the <u>cross polarization ratio</u> (CPR) of wave w' relative to the w-w_o decomposition is defined to be

$$CPR = \frac{P(w',w_o)}{P(w',w)} \tag{1}$$

CPR can be defined in dB from the above ratio of powers as

$$CPR_{dB} = 10 \log CPR \tag{2}$$

Suppose states w and w_o represent orthogonally polarized output states of an ideal receiving antenna with a wave w' incident on it. Then from (4-4)

$$P(w',w) = S_{w'} A_e m_p(w',w) \tag{3}$$

and

$$P(w',w_o) = S_{w'}\ A_e\ m_p(w',w_o) \tag{4}$$

where $S_{w'}$ is the average flux density of wave w'. Substituting (3) and (4) into (1) gives

$$CPR = \frac{m_p(w',w_o)}{m_p(w',w)} \tag{5}$$

The mismatch factors in this equation can be calculated by any of the methods described in Section 4. In particular, the complex-vector formulation may be used. From (4-30)

$$P(w',w) = \frac{1}{2\eta}\,|V(w',w)|^2\ A_e \tag{6}$$

$$P(w',w_o) = \frac{1}{2\eta}\,|V(w',w_o)|^2\ A_e \tag{7}$$

Using these in (1) yields

$$CPR = \frac{|V(w',w_o)|^2}{|V(w',w)|^2} \tag{8}$$

or

$$CPR_{dB} = 20\ \log \frac{|V(w',w_o)|}{|V(w',w)|} \tag{9}$$

We have framed the definition of CPR in terms of the response of an ideal antenna which has two outputs proportional to two orthogonal components of the incident wave. The antenna is ideal in the sense that it introduces no depolarization. If the input wave state was a perfect match to the co-polarized state w the output of the cross-polarized port would be zero.

It is perhaps more conventional to define CPR in terms of a mathematical decomposition of the wave. If E_{co} is the electric field intensity associated with the co-polarized component (state w) and E_{cross} is that associated with the cross-polarized component (state w_o), the CPR definition is

$$CPR = \frac{|E_{cross}|^2}{|E_{co}|^2} \tag{10}$$

This is, of course, equivalent to the above definitions.

There are two important special cases of CPR, linear and circular decompositions. We will present some data for these cases. First consider a linearly polarized wave with tilt angle $\tau_w{}'$ which is to be decomposed into orthogonal linear components with tilt angles τ_w and τ_{wo} which are orthogonal so

$$\tau_{wo} = \tau_w + 90° \tag{11}$$

Let $\Delta\tau$ be the angle of the wave line of polarization with respect to that of the w component, so

$$\Delta\tau = \tau_w - \tau_{w'} \tag{12}$$

Then

$$\tau_{wo} - \tau_{w'} = \tau_w + 90° - \tau_{w'} = \Delta\tau + 90° \tag{13}$$

Now from (4-16)

$$m_p(w',w) = \cos^2 (\tau_w - \tau_{w'}) = \cos^2 \Delta\tau \tag{14}$$

$$m_p(w',w_o) = \cos^2 (\tau_{wo} - \tau_{w'}) = \cos^2 (\Delta\tau + 90°) = \sin^2 \Delta\tau \tag{15}$$

The CPR follows from these and (5)

$$CPR = \frac{\sin^2 \Delta\tau}{\cos^2 \Delta\tau} = \tan^2 \Delta\tau \qquad (16)$$

This has a simple geometric interpretation as shown in Fig. 1. It can be seen

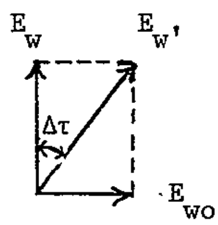


Figure 1. Linear decomposition of a linearly polarized wave.

from Figure 1 that

$$\frac{|E_{cross}|}{|E_{co}|} = \frac{|E_{wo}|}{|E_w|} = \tan \Delta\tau \qquad (17)$$

which yields (16). Table 1 gives some values of CPR as a function of $\Delta\tau$.

Table 1

CPR values of a linear wave decomposed into orthogonal linear components.

$\Delta\tau$ (degrees)	CPR_{dB}
0	$-\infty$
0.5	-41.2
1.0	-35.2

Table 1 (cont.)

$\Delta\tau$ (degrees)	CPR_{dB}
2.0	-29.1
3.0	-25.6
4.0	-23.1
5.0	-21.2
10	-15.1
20	-8.8
30	-4.8
40	-1.5
45°	0
50°	1.5
90°	$+\infty$

In the circular polarization case we wish to decompose an elliptically polarized wave w' into opposite sense circularly polarized components w and w_o where w is the same sense as w'. So we have

$$AR_w = \pm 1 \qquad \text{and} \qquad AR_{wo} = \mp 1 \tag{18}$$

Then if $AR_{w'}$ is the axial ratio of the wave w' (4-13) yields

$$m_p(w',w) = \frac{1}{2} + \frac{4|AR_{w'}|}{4(AR_{w'}{}^2 + 1)} = \frac{1}{2}\,\frac{[|AR_{w'}| + 1]^2}{|AR_{w'}|^2 + 1} \tag{19}$$

since $AR_{w'}$ $AR_w = |AR_{w'}|$, $AR_{w'}$ and AR_w are of the same sign, and $|AR_w| = 1$. Similarly

$$m_p(w',w_o) = \frac{1}{2} + \frac{-4|AR_{w'}|}{4(AR_{w'}{}^2 + 1)} = \frac{1}{2}\,\frac{[|AR_{w'}| - 1]^2}{|AR_{w'}|^2 + 1} \tag{20}$$

Using (19) and (20) in (5) yields

$$CPR = \left(\frac{|AR_{w'}| - 1}{|AR_{w'}| + 1}\right)^2 \tag{21}$$

A more direct derivation of this relationship follows from the axial ratio definition in (1-4). If E_L and E_R are the field intensities associated with the left and right hand sense circular component of the wave, the maximum and minimum (major and minor semi-axes) are the sum and difference of these, i.e.

$$|AR_{w'}| = \left|\frac{E_L + E_R}{E_L - E_R}\right| = \left|\frac{1 + E_R/E_L}{1 - E_R/E_L}\right| \tag{22}$$

$$= \frac{1 + \sqrt{CPR}}{1 - \sqrt{CPR}} \tag{23}$$

where $\sqrt{CPR}$ is E_R/E_L or E_L/E_R according as the wave is left or right hand sensed. Solving (23) gives

$$CPR = \left(\frac{|AR_{w'}| - 1}{|AR_{w'}| + 1}\right)^2 \tag{24}$$

which is (21). For example, a good quality circularly polarized antenna has an axial ratio of 0.3 dB. The CPR (dB) is then computed as

$$|AR| = 10^{\frac{AR_{dB}}{20}} = 10^{0.015} = 1.03514$$

$$CPR_{dB} = 20 \log \frac{|AR| - 1}{|AR| + 1} = 20 \log 0.01727$$

$$= -35.26 \text{ dB}$$

Table II gives CPR as a function of the wave axial ratio.

Table II

Cross Polarization Ratio and Axial Ratio
For Nearly Circularly Polarized Antennas.

AR_{dB}	$\|AR\|$	CPR (dB)
0 dB	1.00000	$-\infty$
0.1	1.01158	-44.80
0.2	1.02329	-38.78
0.3	1.03514	-35.26
0.4	1.04713	-32.76
0.5	1.05925	-30.82
0.6	1.07152	-29.24
0.7	1.08393	-27.90
0.8	1.09648	-26.74
0.9	1.10917	-25.72
1.0	1.12202	-24.81
1.5	1.18850	-21.30
2.0	1.25892	-18.81
2.5	1.33352	-16.90
3.0	1.41254	-15.34
4.0	1.58489	-12.91
5.0	1.77828	-11.05
10.0	3.13228	- 5.81
∞	∞	0.0

6. Isolation

Many devices have two output ports which are required to be completely separate. In practice, however, there is mutual coupling between the ports of the device. If an ideal signal is used as input then the power out of the desired port P_d and the power out of the undesired port P_u lead to the definition of isolation

$$I = \frac{P_d}{P_u} \tag{1}$$

In this case the isolation is residual isolation since an ideal wave was used as input. The coupling between the output ports is due entirely to the internal coupling of the device and not due to the properties of the input signal.

For devices which have two channels of output for two different polarization states we may define isolation more specifically as

$$I = \frac{\text{available output power in co-polarized channel}}{\text{available output power in cross-polarized channel}} \tag{2}$$

where "co" and "cross" are used to denote the desired and undesired output ports. The isolation is a function of the input wave state CPR. For an input wave perfectly matched to the co-polarized channel polarization state, the definition (2) is then the isolation inherent in the device itself.

Consider a receiving antenna with polarization states a_c (co-polarized) and a_x (cross-polarized). The output power from each antenna port is proportional to the input wave polarization component in that state. In general, the states a_c and a_x are not orthogonal and thus there will be internal coupling. Fig. 1 illustrates a wave of state w incident on the antenna and its associated microwave hardware. According to (2) the isolation is

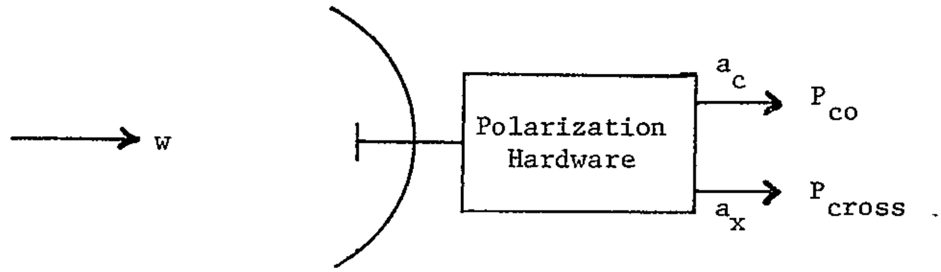


Figure 1. Dual polarized receiving antenna system.

$$I = \frac{P_{co}}{P_{cross}} \tag{3}$$

If the states a_c and a_x are orthogonal (ideal antenna) and the input wave is perfectly matched to the co-polarized state (w and a_c are the same) then the isolation is infinite since $P_{cross} = 0$. If the antenna is ideal the isolation then equals the inverse of CPR of the incoming wave if it is decomposed into a_c and a_x components.

<u>Example</u>: <u>Ideal dual linearly polarized antenna</u>. Let the co-polarized polarization state of the antenna be linear vertical ($\varepsilon_c = 0$, $\tau_c = 90°$) and the cross polarized state be horizontal linear ($\varepsilon_x = 0$, $\tau_x = 0$). Consider a general wave w incident on the antenna, We can decompose it into a_c and a_x components. From (4-17)

$$m_p(w,a_c) = \frac{1}{2} + \frac{(AR_w^{\,2} - 1)\cos 2\,\Delta\tau_c}{2(AR_w^{\,2} + 1)} \tag{4}$$

$$m_p(w,a_x) = \frac{1}{2} + \frac{(AR_w^2 - 1)\cos 2\,\Delta\tau_x}{2(AR_w^2 + 1)} \tag{5}$$

where $\Delta\tau_c = \tau_c - \tau_w$ and $\Delta\tau_x = \tau_x - \tau_w$. The isolation which equals the inverse of CPR, in this case, is from (5-5)

$$I = \frac{1}{CPR} = \frac{m_p(w,a_c)}{m_p(w,a_x)} \tag{6}$$

Using (4) and (5)

$$I = \frac{AR_w^2 + 1 + (AR_w^2 - 1)\cos 2\,\Delta\tau_c}{AR_w^2 + 1 + (AR_w^2 - 1)\cos 2\,\Delta\tau_x} \tag{7}$$

If the wave is linearly polarized ($AR_w = \infty$) with tilt angle τ_w, (7) becomes

$$I = \frac{1 + \cos 2\,\Delta\tau_c}{1 + \cos 2\,\Delta\tau_x} = \frac{1 + \cos 2(90° - \tau_w)}{1 + \cos 2(0° - \tau_w)}$$

$$= \frac{1 - \cos 2\tau_w}{1 + \cos 2\tau_w} \tag{8}$$

If $\tau_w = 90°$, $I = \infty$ which is correct since the wave is perfectly matched to the co-polarized state. If $\tau_w = 0°$, $I = 0$ and the wave is completely cross polarized to the co-polarized state.

Let us now turn to the more practical case of non-ideal antennas where a_c and a_x are not necessary orthogonal and further the incident wave may not be perfect circular, linear, etc. Figure 1 illustrates this situation and (3) is used to calculate the isolation. From (4-4)

$$P_{co} = S_w \, A_e \, m_p(w,a_c) \tag{9}$$

and

$$P_{cross} = S_w \, A_e \, m_p(w,a_x) \tag{10}$$

as we did in Section 5 except now the states w, a_c, and a_x can be any polarization states. Then (3) yields

$$I = \frac{m_p(w,a_c)}{m_p(w,a_x)} \tag{11}$$

Isolation can also be derived from the complex-vector formulation in a fashion similar to that for CPR in Section 5; see (5-6) to (5-9). Then

$$I = \frac{|V(w,a_c)|^2}{|V(w,a_x)|^2} \tag{12}$$

or

$$I_{dB} = 20 \log \frac{|V(w,a_c)|}{|V(w,a_x)|} \tag{13}$$

The general polarization mismatch factors are from (4-13)

$$m_p(w,a_c) = \frac{1}{2} + \frac{4 \, AR_w \, AR_c + (AR_w^{\,2} - 1)(AR_c^{\,2} - 1) \cos 2 \, \Delta\tau_c}{2(AR_w^{\,2} + 1)(AR_c^{\,2} + 1)} \tag{14}$$

and

$$m_p(w,a_x) = \frac{1}{2} + \frac{4 \, AR_w \, AR_x + (AR_w^{\,2} - 1)(AR_x^{\,2} - 1) \cos 2 \, \Delta\tau_x}{2(AR_w^{\,2} + 1)(AR_x^{\,2} + 1)} \tag{15}$$

Thus, the isolation is found from (14) and (15) in (11), giving

$$I = \frac{\left[\dfrac{(AR_w^{\ 2}+1)(AR_c^{\ 2}+1) + 4\ AR_w\ AR_c + (AR_w^{\ 2}-1)(AR_c^{\ 2}-1)\ \cos 2\ \Delta\tau_c}{2(AR_w^{\ 2}+1)(AR_c^{\ 2}+1)}\right]}{\left[\dfrac{(AR_w^{\ 2}+1)(AR_x^{\ 2}+1) + 4\ AR_w\ AR_x + (AR_w^{\ 2}-1)(AR_x^{\ 2}-1)\ \cos 2\ \Delta\tau_x}{2(AR_w^{\ 2}+1)(AR_x^{\ 2}+1)}\right]} \tag{16}$$

This is a completely general expression for all situations. For example, if the antennas are linearly polarized then AR_w, AR_x, and AR_c are all infinite and (16) reduces to

$$I = \frac{1 + \cos 2\ \Delta\tau_c}{1 + \cos 2\ \Delta\tau_x} \tag{17}$$

which we found in (8).
For an ideal dual linear polarized receiving system with a linearly polarized incident wave the isolation is a function of the relative tilt angles only. If the co-polarized antenna is lined up with the wave then $\Delta\tau_c = 0°$. If the cross-polarized antenna is perfectly orthogonal then $\Delta\tau_x = 90°$ and (17) becomes infinite indicating perfect isolation.

If the wave and antennas are perfectly circularly polarized then $|AR_w|$, $|AR_x|$, and $|AR_c|$ are all unity and (16) reduces to

$$I = \frac{1 + AR_w\ AR_c}{1 + AR_w\ AR_x} = \infty \tag{18}$$

This follows because the co-polarized antenna is identical to the wave so either AR_w and AR_c are both +1 or both −1 and $AR_w\ AR_c$ is always +1. If $AR_c = +1$ (LHCP) then $AR_x = -1$ (RHCP) and if $AR_c = -1$ then $AR_x = +1$; so $AR_w\ AR_x$ is always −1 and the denominator (18) is zero. Then (18) goes to infinity.

In most circularly polarized systems all axial ratios are nearly unity in magnitude. Under these conditions the co-polarized output varies only slightly as $\Delta\tau_c$ is changed. This can be seen by examining the numerator of (16). Since AR_w and AR_c are of the same sign and near unity in magnitude, the last term is negligible. Then (16) is approximately given by

$$I \simeq \frac{(AR_w{}^2 + 1)(AR_c{}^2 + 1) + 4\ AR_w\ AR_c}{(AR_w{}^2 + 1)(AR_x{}^2 + 1) + 4\ AR_w\ AR_x + (AR_w{}^2 - 1)(AR_x{}^2 - 1)\ \cos\ 2\ \Delta\tau_x} \cdot \left[\frac{AR_x{}^2 + 1}{AR_c{}^2 + 1}\right] \tag{19}$$

Maximum isolation occurs when the wave ellipse major axis is perpendicular to the cross-polarized antenna major axis, i.e. when $\Delta\tau_x = 90°$. Then (19) gives

$$I_{max} = I(\Delta\tau_x = 90°)$$

$$\simeq \frac{(AR_w{}^2 + 1)(AR_c{}^2 + 1) + 4\ AR_w\ AR_c}{2(AR_w + AR_x)^2} \left[\frac{AR_x{}^2 + 1}{AR_c{}^2 + 1}\right] \tag{20}$$

Minimum isolation occurs for $\Delta\tau_x = 0°$, and (19) yields

$$I_{min} = I(\Delta\tau_x = 0°)$$

$$\simeq \frac{(AR_w{}^2 + 1)(AR_c{}^2 + 1) + 4\ AR_w\ AR_c}{2(AR_w\ AR_x + 1)^2} \left[\frac{AR_x{}^2 + 1}{AR_c{}^2 + 1}\right] \tag{21}$$

<u>Example:</u> <u>VPI&SU CTS receiving system</u>. The receiving antenna has the

following parameters.

$$AR_c(\text{dB}) = 0.3 \text{ dB}$$

$$AR_c = -1.03514 \qquad \text{(RH)}$$

$$AR_x(\text{dB}) = 0.27 \text{ dB}$$

$$AR_x = 1.03157 \qquad \text{(LH)}$$

Then

$$\frac{AR_x^{\,2} + 1}{AR_c^{\,2} + 1} = 0.996438$$

The maximum and minimum isolations (which occur for $\Delta\tau_x = 90°$ and $0°$) are tabulated below for various incoming wave axial ratios as calculated from (20) and (21).

AR_w(dB)	AR_w	I_{max}(dB)	I_{min}(dB)
0	-1	36.2	36.2
0.3	-1.03514	58.3	29.7
0.5	-1.05925	37.6	27.1
0.7	-1.08393	32.1	25.1
1.0	-1.12202	27.5	22.7

If it is assumed that

$$AR_x = -AR_c \text{ and } \Delta\tau_c = \Delta\tau_x = \Delta\tau \qquad (22)$$

then (16) reduces to

$$I = \frac{(AR_w^2 + 1)(AR_c^2 + 1) + 4\,AR_w\,AR_c + (AR_w^2 - 1)(AR_c^2 - 1)\cos 2\,\Delta\tau}{(AR_w^2 + 1)(AR_c^2 + 1) - 4\,AR_w\,AR_c + (AR_w^2 - 1)(AR_c^2 - 1)\cos 2\,\Delta\tau} \tag{23}$$

If the wave is perfectly circularly polarized, $|AR_w| = 1$, then (23) reduces to

$$I = \left(\frac{|AR_c| + 1}{|AR_c| - 1}\right)^2 \tag{24}$$

This relationship is plotted in Fig. 2. Obviously as $|AR_c| \to 1$ (0 dB) the isolation goes to infinity. If $|AR_c| = 1.122$ (1 dB) then (24) gives I = 24.8 dB which is a point easily found on Fig. 2. If the wave is not perfectly circular the relative tilt angle $\Delta\tau$ affects the isolation. In Figures 3 through 6 the isolation is plotted as a function of antenna axial ratio with wave axial ratio and relative tilt angle as parameters. [12] For example, if the wave axial ratio is 0.5 dB the isolation for an antenna of 0.2 dB axial ratio from Fig. 4 varies between 28 and 35 dB as $\Delta\tau$ varies from 0 to 90°. We can check this example with (23). Let $AR_w = 1.05925$ (0.5 dB), $AR_c = 1.02329$ (0.3 dB), and $\Delta\tau = 0°$. Then (23) yields 27.90 dB, which agrees with the curve.

These curves can be used for examining the effect of a variable wave axial ratio also. For this application the abscissa is the wave axial ratio and the curves are for fixed antenna axial ratios of opposite sense to the wave. Then $|AR_c|$ and $|AR_w|$ interchange roles.

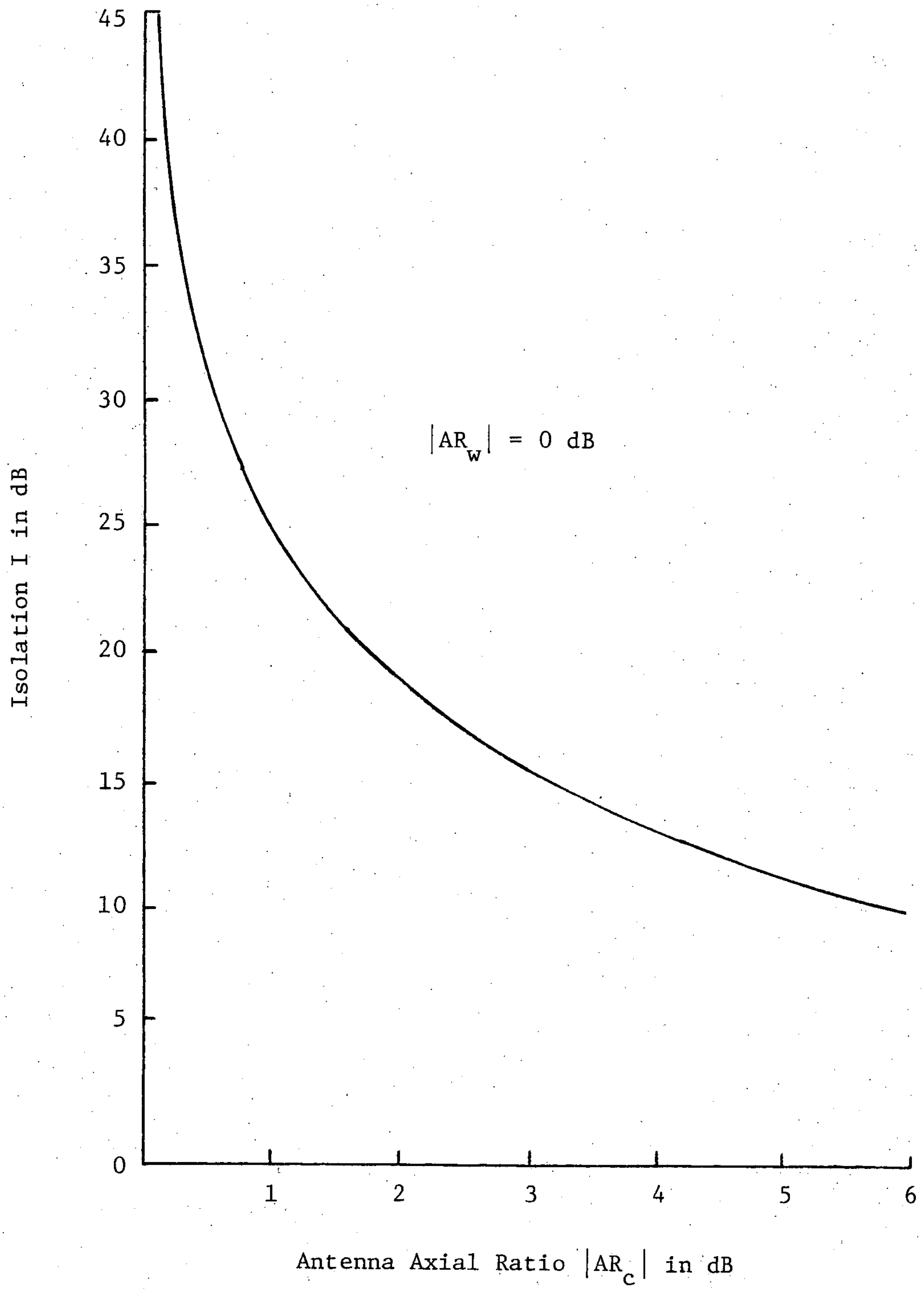


Figure 2. Isolation for perfectly circular wave polarization as a function of antenna axial ratio.

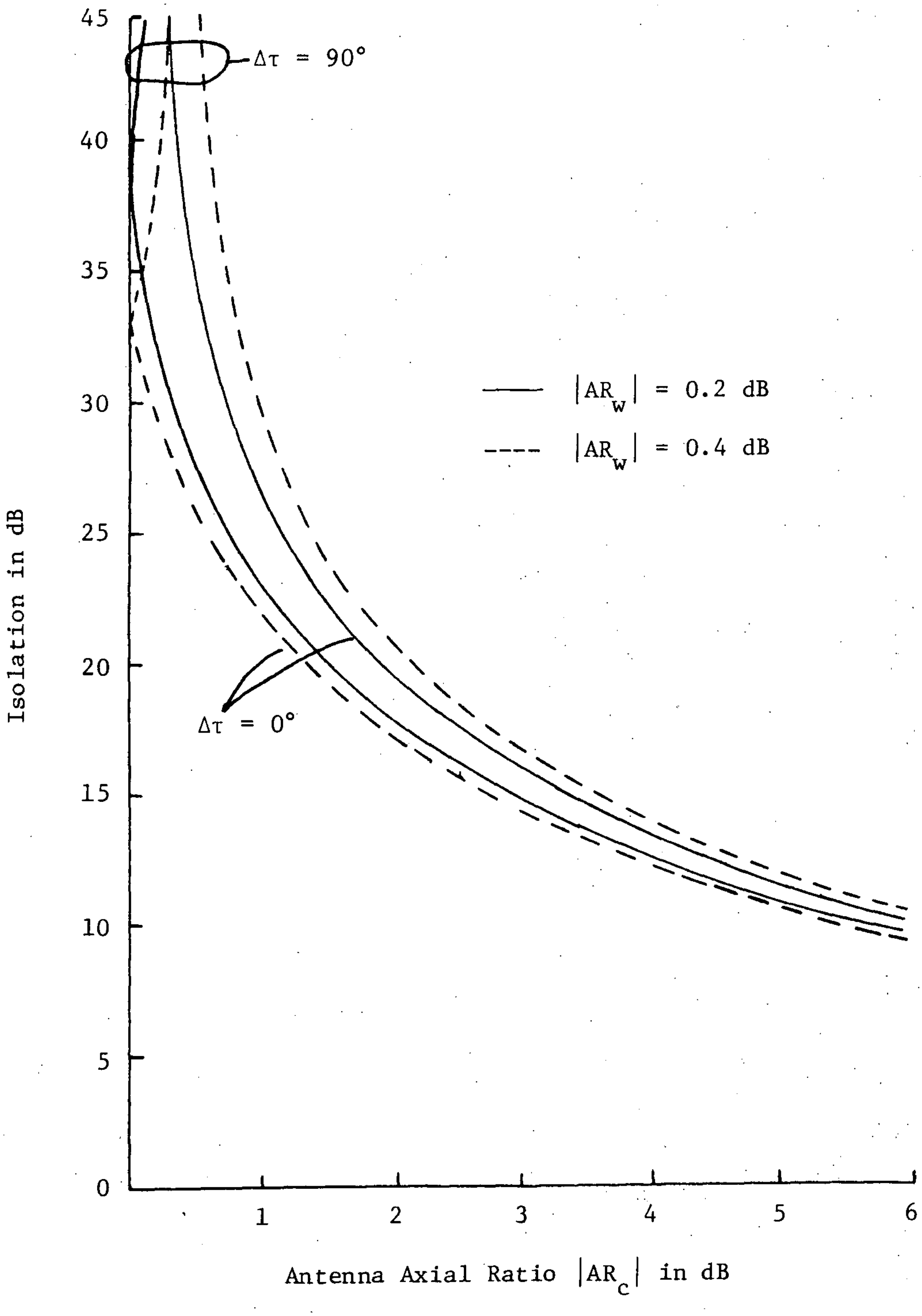


Figure 3. Isolation for waves with axial ratios of 0.2 and 0.4 dB as a function of antenna axial ratio.

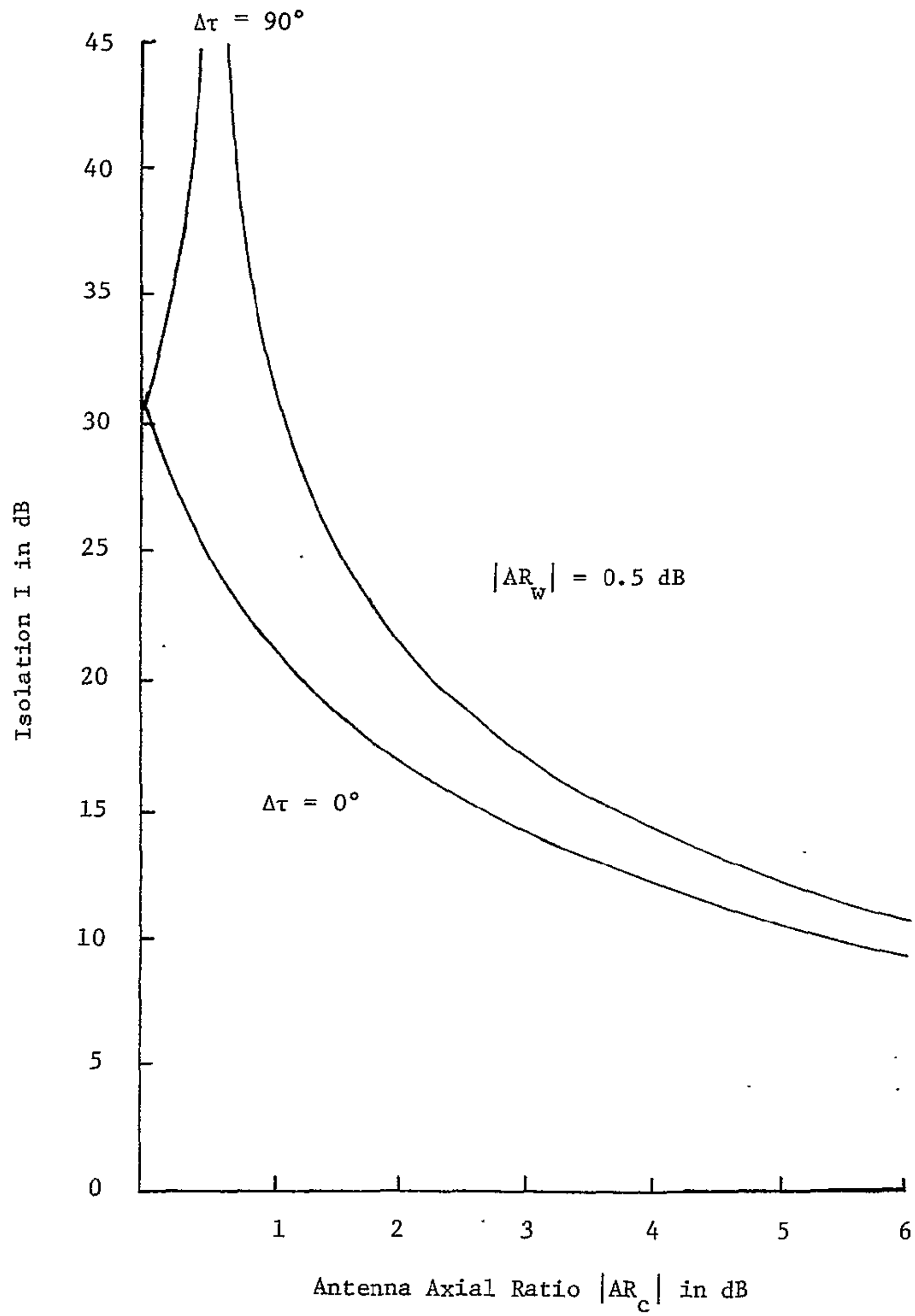


Figure 4. Isolation for a wave with an axial ratio of 0.5 dB as a function of antenna axial ratio.

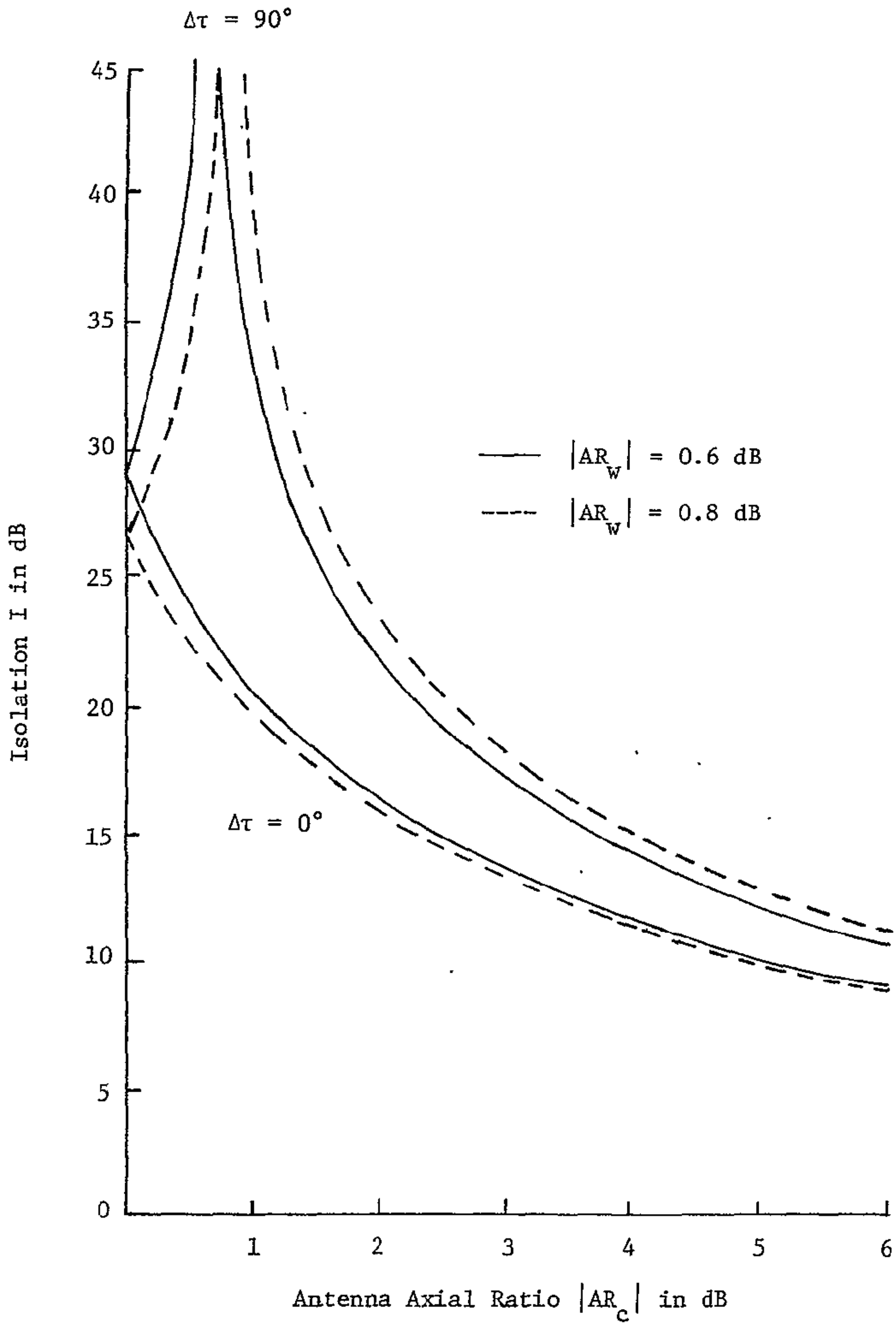


Figure 5. Isolation for waves with axial ratios of 0.6 and 0.8 dB as a function of antenna axial ratio.

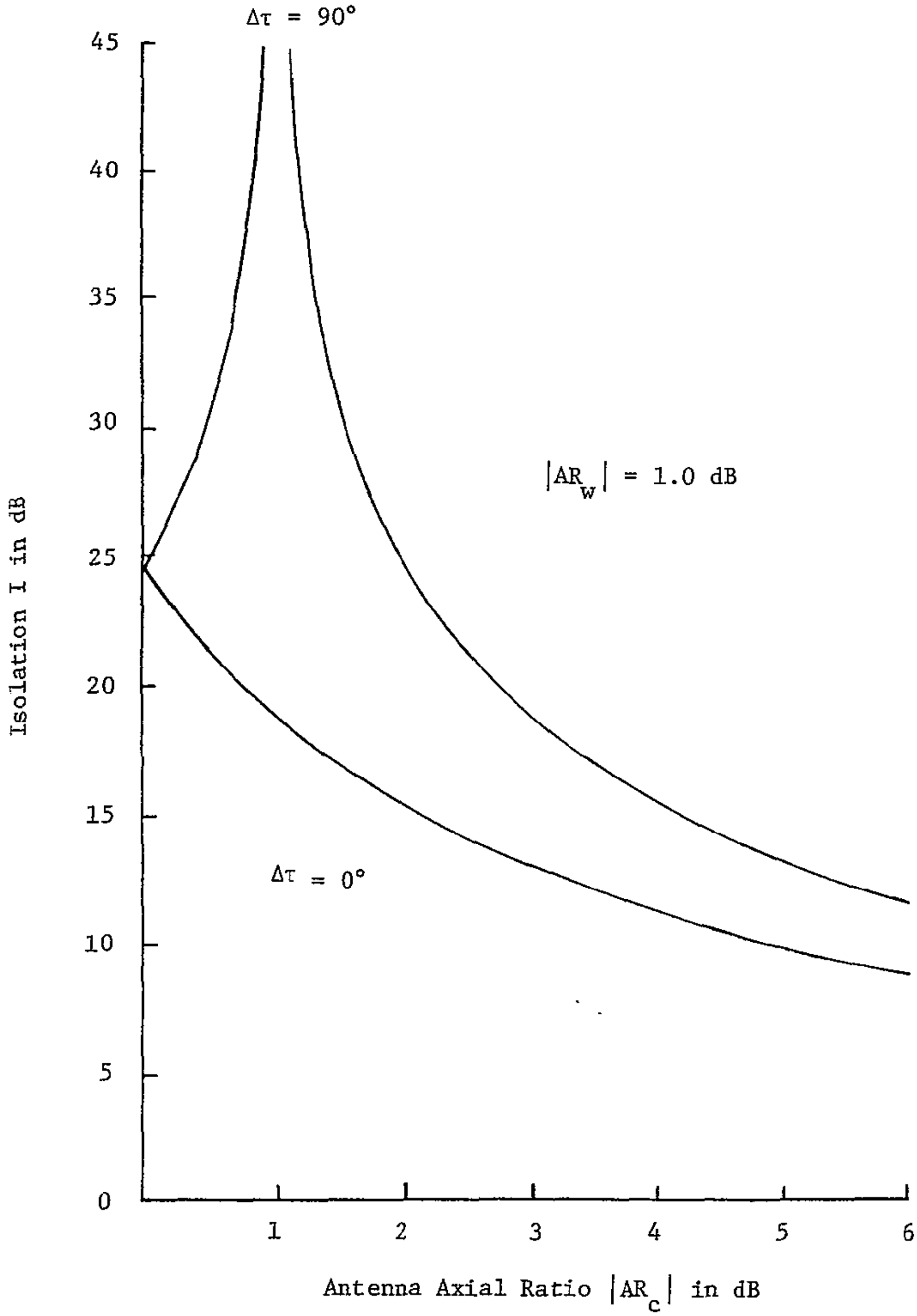


Figure 6. Isolation for a wave with an axial ratio of 1.0 dB as a function of antenna axial ratio.

7. Fade, Isolation and Phase Shift on a Dual Polarized Receiving System

A dual polarized receiving antenna has an incident wave w which has passed through a clear, stable atmosphere. This is illustrated in Fig. 1. In Fig. 2 the propagation medium is not clear air but a medium which may change the polarization state, power level, and/or the phase of the wave arriving at the antenna. The new incident wave is denoted w'. The medium may include rain, snow, and a freezing layer. To study such propagation events we can measure the signal levels out of each port and the relative phase between each port. In this section we will formulate these important observable parameters and other parameters which are useful in a theoretical model of the medium. This sets up a procedure by which mathematical models of a depolarizing medium can be compared to experimental data.

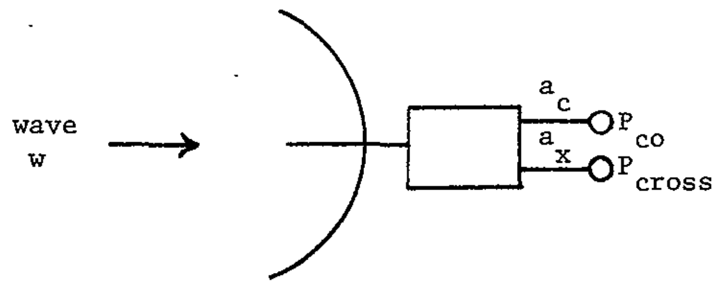


Figure 1. Clear-air dual polarized reception.

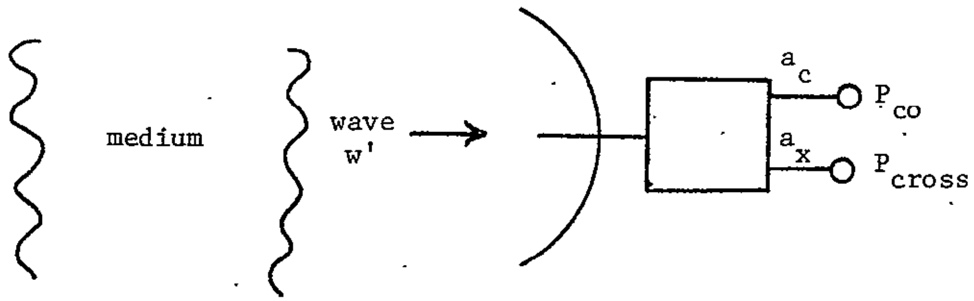


Figure 2. Dual polarized reception of a wave passing through a depolarizing medium.

It is very important to establish a fixed reference coordinate system to which all angles may be referenced. We chose the xy-coordinate system to be fixed in space. It may be helpful to think of the x-axis as the local horizontal at the receiving antenna location. Or, sometimes it is convenient to orient x and y such that $\tau_w = 0°$. The polarization ellipse for the wave w arriving at the antenna under clear weather conditions is shown in Fig. 3. The same transmitted signal now passing through a depolarizing medium arrives in wave state w' as shown in Fig. 4.

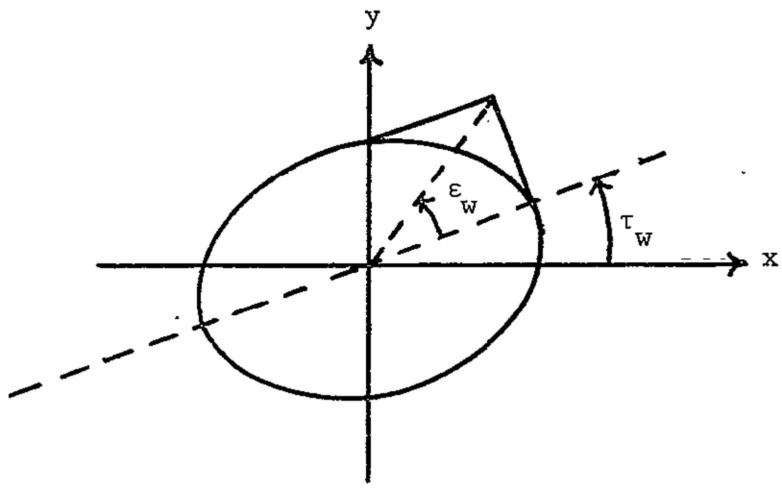


Figure 3. Incoming wave polarization ellipse during clear weather.

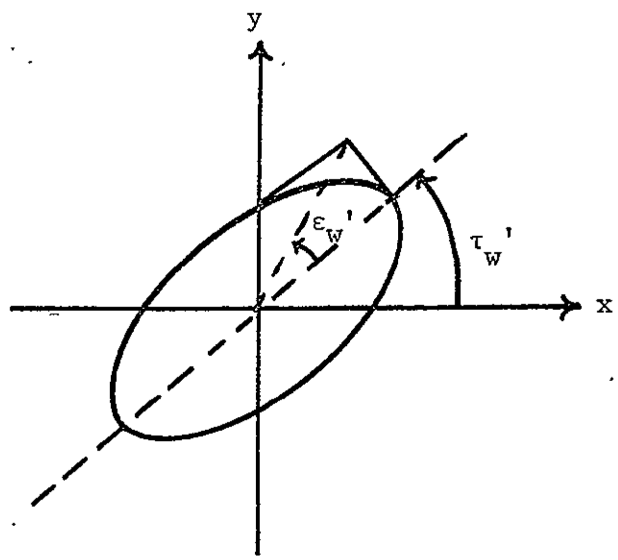


Figure 4. Incoming wave polarization ellipse after passing through a depolarizing medium.

The dual polarized receiving antenna has polarization parameters ε_c, τ_c and ε_x, τ_x for the co- and cross-polarized states. The corresponding axial ratios are

$$AR_c = \cot \varepsilon_c \qquad \text{and} \qquad AR_x = \cot \varepsilon_x \tag{1}$$

The tilt angles of the antenna polarization ellipses relative to the incoming wave states w and w' are

$$\Delta\tau_c = \tau_c - \tau_w \qquad \Delta\tau_x = \tau_x - \tau_w \tag{2}$$

and

$$\Delta\tau_c{}' = \tau_c - \tau_w{}' \qquad \Delta\tau_x{}' = \tau_x - \tau_w{}' \tag{3}$$

We wish to examine the output from the two antenna ports. In the clear weather case from (4-4)

$$P_{co} = S_w\, A_e\, m_p(w,a_c) \tag{4}$$

$$P_{cross} = S_w\, A_e\, m_p(w,a_x) \tag{5}$$

It is assumed here that the effective aperture is the same for both antenna polarization states. For dual polarized reception usually a single aperture is fed with a dual polarized feed structure and thus the effective aperture is identical in both receive states. With weather effects on the signal the received powers are

$$P'_{co} = S_{w'}\, A_e\, m_p(w',a_c) \tag{6}$$

$$P'_{cross} = S_{w'}\, A_e\, m_p(w',a_x) \tag{7}$$

With the power outputs clearly established we are prepared to formulate attenuation and isolation.

Signal loss can be defined for the medium only or for the total communication channel loss. Under most conditions they are nearly the same. The incoming (clear weather) wave w is usually very close in polarization to the co-polarized antenna state a_c. In a communications setting this is the desired channel and power out of the cross-polarized receive port from wave w is unwanted, frequently called "cross talk". We now define fade F to be the signal reduction in the co-channel due to the medium, so

$$F = \frac{P_{co}}{P_{co}'} \tag{8}$$

Using (4) and (5)

$$F = \frac{S_w}{S_w'} \cdot \frac{m_p(w,a_c)}{m_p(w',a_c)} \tag{9}$$

The first factor is medium loss and the second factor is antenna induced loss. The medium loss is usually called attenuation

$$A_t = \frac{S_w}{S_w'} \tag{10}$$

Note that this definition uses the total power density in the waves. Thus

$$F = A_t \frac{m_p(w,a_c)}{m_p(w',a_c)} \tag{11}$$

or

$$F_{dB} = A_{t,dB} + 10 \log \frac{m_p(w,a_c)}{m_p(w',a_c)} \tag{12}$$

Note that in these definitions F and A are normally greater than one. For example if $S_{w'} = \frac{1}{2} S_w$ the signal passing through the medium is one-half of the clear-air signal level. The attenuation is then a factor of 2. Similarly F_{dB} and A_{dB} are normally positive numbers.

In most communication situations w is well matched to a_c and $m_p(w,a_c)$ is nearly one. During a weather event w' is usually still close to a_c and $m_p(w',a_c)$ is also nearly one. Thus from (11)

$$F \simeq A_t \tag{12}$$

Now we turn our attention to the isolation between the co and cross polarized output ports. Under clear air conditions the discussions of the previous section apply. From (6-11)

$$I = \frac{m_p(w,a_c)}{m_p(w,a_x)} \tag{13}$$

When the propagation medium contains depolarizing components we have

$$I' = \frac{P'_{co}}{P'_{cross}} \tag{14}$$

Using (6) and (7) in the above equation

$$I' = \frac{m_p(w',a_c)}{m_p(w',a_x)} \tag{15}$$

There are, of course, many ways to calculate the polarization mismatch factors and thus the isolation I'. A general equation for isolation in terms of the wave and antenna axial ratios and relative tilt angles was given in (6-16). This can be used to calculate I' by merely replacing the wave w parameters (AR_w and τ_w) by the wave w' parameters ($AR_{w'}$ and $\tau_{w'}$).

The complex-vector formulation is particularly useful in making wave-antenna interaction calculations. In fact, it permits calculation of the phase, whereas the polarization mismatch factor developments presented above can only be used for power calculations such as for fade and isolation. So we will now discuss fade, isolation, and phase shift using the complex-vector ideas.

From (4-30) the power output from each port is

$$P_{co} = \frac{A_e}{2\eta} |V(w,a_c)|^2 \tag{16}$$

$$P_{cross} = \frac{A_e}{2\eta} |V(w,a_x)|^2 \tag{17}$$

during clear weather and

$$P'_{co} = \frac{A_e}{2\eta} |V(w',a_c)|^2 \tag{18}$$

$$P'_{cross} = \frac{A_e}{2\eta} |V(w',a_x)|^2 \tag{19}$$

during a medium disturbance. The fade is then

$$F = \frac{P_{co}}{P_{co}'} = \frac{|V(w,a_c)|^2}{|V(w',a_c)|^2} \tag{20}$$

Attenuation of the co-polarized component (defined to be state w) is

$$A = \frac{|V(w,w)|^2}{|V(w',w)|^2} \tag{21}$$

But

$$|V(w,w)|^2 = |E_w|^2 \, |\vec{e}_w \cdot \vec{e}_w{}^*|^2 = |E_w|^2 \tag{22}$$

and

$$|V(w',w)|^2 = |E_{w'}|^2 \, |\vec{e}_{w'} \cdot \vec{e}_w{}^*|^2 \tag{23}$$

Using these in (21)

$$A = \frac{|E_w^2|}{|E_{w'}|^2 \, |\vec{e}_{w'} \cdot \vec{e}_w{}^*|^2}$$

$$= \frac{S_w}{S_{w'}} \frac{1}{|\vec{e}_{w'} \cdot \vec{e}_w{}^*|^2}$$

$$= A_t \frac{1}{|\vec{e}_{w'} \cdot \vec{e}_w{}^*|^2} \qquad (24)$$

If $|\vec{e}_{w'} \cdot \vec{e}_w{}^*|^2 \simeq 1$, then $A \simeq A_t$. This is usually the case. The difference in the definitions of attenuation are due to the power which is scattered into the component w_o which is orthogonal to w. A_t includes this power since $S_{w'}$ is the total power density in wave w' summed over all components and A does not. This can be seen from (23) by noting that $|\vec{e}_{w'} \cdot \vec{e}_w{}^*|^2$ is less than 1. Then $A > A_t$ which also follows because A deals only with the change in the signal of state w and some of the power has been scattered into the orthogonal state. A is usually preferred over A_t. A_t could be expressed in terms of complex voltage as

$$A_t = \frac{|V(w,w)|^2}{|V(w',w')|^2} \qquad (25)$$

and

$$|V(w',w')|^2 = |V(w',w)|^2 + |V(w',w_o)|^2 \qquad (26)$$

$$= |E_{w'}|^2 \, |\vec{e}_{w'} \cdot \vec{e}_w{}^*|^2 + |E_{w'}|^2 \, |\vec{e}_{w'} \cdot \vec{e}_{w_o}{}^*|^2 \qquad (27)$$

The last term of the above expression represents the power which was scattered into the orthogonal state. This is included in the definition of A_t whereas it is not in A.

The isolations are

$$I = \frac{P_{co}}{P_{cross}} = \frac{|V(w,a_c)|^2}{|V(w,a_x)|^2} \tag{28}$$

and

$$I' = \frac{P'_{co}}{P'_{cross}} = \frac{|V(w',a_c)|^2}{|V(w',a_x)|^2} \tag{29}$$

The phase shifts introduced due to the medium are given by

$$\Delta\Phi_w = \text{Phase}[V(w',w)] - \text{Phase}[V(w,w)] \tag{30}$$

$$\Delta\Phi_{wo} = \text{Phase}[V(w',w_o)] - \text{Phase}[V(w,w_o)] \tag{31}$$

But Phase $(V(w,w))$ = Phase $(|E_w|^2)$ = 0 and Phase $(V(w,w_o))$ = Phase (0) = 0. So

$$\Delta\Phi_w = \text{Phase}[V(w',w)] \tag{32}$$

$$\Delta\Phi_{wo} = \text{Phase}[V(w',w_o)] \tag{33}$$

Including the antenna effects the phase at each port in clear weather is

$$\Phi_c = \text{Phase}[V(w,a_c)] \tag{34}$$

$$\Phi_x = \text{Phase}[V(w,a_x)] \tag{35}$$

During rain

$$\Phi_c' = \text{Phase}[V(w',a_c)] \tag{36}$$

$$\Phi_x' = \text{Phase}[V(w',a_x)] \tag{37}$$

Thus the phase shift at each port introduced by the medium is

$$\Delta\Phi_c = \Phi_c' - \Phi_c \tag{38}$$

$$\Delta\Phi_x = \Phi_x' - \Phi_x \tag{39}$$

The relative phase shift between output ports in clear weather is

$$\Phi_{c-x} = \Phi_c - \Phi_x \tag{40}$$

and during a medium disturbance is

$$\Phi_{c-x}' = \Phi_c' - \Phi_x' \tag{41}$$

The relative phase shift between output ports introduced by the medium is

$$\Delta\Phi_{c-x} = \Phi_{c-x}' - \Phi_{c-x} \tag{42}$$

$$= \Phi_c' - \Phi_c - (\Phi_x' - \Phi_x)$$

$$= \Delta\Phi_c - \Delta\Phi_x \tag{43}$$

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