10.1 INTRODUCTION [1]

At the initiation of these analyses in 1960, it seemed convincing that (1) a purely analytic orbit calculation would be worth trying, for reasons of insight and economy; (2) to obtain the geophysically interesting tesseral harmonics, the sparseness of the data required formulation of partial derivatives with respect to the observations, rather than analysis of variations in the Kepler elements; (3) effects of tracking station location error, drag, radiation pressure, and luni-solar attraction would be comparable to tesseral harmonic effects; and (4) the optimum solution would combine satellite and terrestrial data. These ideas were the main themes of all the work described here. Most of the techniques are fully described in Kaula (1966b); more details on some other aspects are given in Kaula (1965) and Kaula (1971a).

The satellite orbit analyses described herein can be divided into four phases, which coincide with different data blocks, but which also entailed some differences of technique: (I) MINITRACK interferometry, 1960–1961; (II) early Baker-Nunn camera directions (i.e., rather active Sun, 1959–1961), 1961–1963; (III) late Baker-Nunn camera directions (i.e., quiet Sun, 1962–1963), 1963–1966; (IV) combined Baker-Nunn camera and TRANET Doppler data, 1966–1967.

[1] This work was originally undertaken in the Theoretical Division at NASA/GSFC in response to exhortations from R. D. Jastrow and J. A. O'Keefe to conduct work in parallel with SAO. Lloyd Carpenter helped greatly in learning how to use the computer. The setting up of the programs used in phases II–IV was done mainly in the summer of 1961 at SAO, where the advice of Imre G. Izsak was much appreciated. Later work at GSFC was assisted by Ed Monasterski, Susan Werner, and W. D. Putney. Subsequent to 1963, work at UCLA was done under NASA grant NSR 05-007-060, with the help of E. J. Bryan; much work was also done at Aerospace Corporation, El Segundo, California, assisted by D. H. Adams, and at USAF Aero Chart and Information Center, St. Louis, assisted by C. F. Martin and H. White.

10.2 ORBITAL DYNAMICS

In accord with premise 1 of the introduction, the theory of Brouwer (1959) was used throughout for the oblateness to order J_2^2 in long-period and secular effects. Linear analytic theories were developed for the effects of gravitational field spherical harmonics (Kaula, 1961a) and the Sun and Moon (Kaula, 1962). These theories were completely general as to harmonic degree and order and enabled considerably more compact computer programming than earlier developments. The analytic spirit was extended as far as possible by using numerical harmonic analysis for radiation pressure (Kaula, 1962, 1963a) and drag (Kaula, 1963a). The atmospheric models used for the drag effects were by Jacchia (1960) and Harris and Priester (1962).

Occasional examination was made of possible errors introduced by inadequacies in the Brouwer (1959) theory, using the higher-order theory of Kozai (1962b). However, the effects were always found to be less than 10 meters, and programming of a more accurate replacement never rose to high priority. If the effort had been continued, a more accurate and efficient theory, such as that of Aksnes (1970), would have to be programmed.

The physics of orbits will always make spherical harmonic coefficients the most effective means of representing the Earth's gravitational field in their analysis (Kaula, 1971b). For expansion of the inclination functions, the half-angle formulas (Izsak, 1964; Jeffreys, 1965) would probably be more efficient than the formulas of Kaula (1961a), but not so much so as to warrant a reprogramming.

The drag models were found to significantly improve the fit to orbital arcs which were of more than 10 days' duration, pre-1962, and at perigee below 1000 km. However, for orbits more suitable to satellite geodesy, the improvement over arbitrary accelerations for the mean anomaly and partial derivatives with respect thereto for the other elements was negligible, and hence the drag routines lapsed into desuetude.

A formulation of tidal effects on orbits similar in form to the luni-solar perturbation theory was developed (Kaula, 1969), but never applied extensively in data analysis.

10.3 SATELLITE DATA ANALYSES

The phase I analyses of MINITRACK (Kaula, 1961b,c) data were rather crude. The phase II analyses of early Baker-Nunn camera data (Kaula, 1963a,b) involved an awesome variety of modeling and statistical complications in an attempt to overcome the inadequate distribution of orbital specifications and tracking stations and the excessive drag effects. The phase III analyses (Kaula, 1966c) were somewhat simpler because of the much better data. There were also significant improvements through adoption of the technique of partitioned normals (Anderle and Smith, 1967; Guier and Newton, 1965; Kaula, 1966b, pp. 104–106) and correction of a programming error which had caused previous solutions for coefficients S_{lm}, l-m odd, to have the wrong sign.

Since phase I–III analyses are fully described in Kaula (1961b,c, 1963a,b, 1966c), the discussion here concentrates on the phase IV analyses of Baker-Nunn directions combined with TRANET (Doppler) range rate, previously described only in a report of limited distribution (Kaula, 1968).

Tracking by the U.S. Navy TRANET network was received in the form of Doppler frequencies, scaled to a reference frequency of about 107 MHz, at intervals of 16 seconds. To utilize these data and the camera data in the same computer programs and to economize computer time, the following conversion and compression were applied to the Doppler data: (1) The form was converted to range rate in "canonical" units: Earth radii/(806.8137 sec.); (2) the time was converted from WWV emitted to Al; (3) observations within 15 degrees of the horizon were omitted, and tropospheric refraction corrections were applied; (4) three or four observations at equal intervals over each pass were selected; (5) for one day at a time, an orbit was fitted to these observations by iterated least squares, taking into account variations of the gravitational field up to l, m=4,4; (6) from this orbit, the range rate was calculated for each of the original 16-second interval observations; (7) for each pass, a combination of a polynomial in time and a station position shift was fitted to the residuals of the observed with respect to the computed range rates; (8) at three times within each pass, a range rate was calculated as the sum of the range rate from the orbit fitted for the day plus the polynomial and station shift fitted to the pass. The final information written on a binary tape for use in the subsequent analysis included as one record for each pass: a type number identifying the data as range rate, the tracking station number, the number of observations in the pass, the GST and A1 time (in modified Julian days) of the start of the pass, the three aggregated range rates formed by the process described above, and the time after pass-start for each of these range rates.

The zonal harmonics were held fixed at the values given in table 2 of Kaula (1966c). The tesseral harmonics selected for solution were all those for which a normalized coefficient of magnitude $8\times10^{-6}/l^2$ caused a perturbation of at least 10-meter amplitude in one satellite or at least 5-meter amplitude in two satellites, as listed in table 3 of Kaula (1966c)—all coefficients through 6,6; 7,1 through 7,5; 8,1 through 8,6; 9,1 and 9,2; 10,1 and 10,2; 11,1; and 12,1; plus the small-divisor, or near-resonant, harmonics: 9,9; 12,12; 13,12; 14,12; 15,12 through 15,14; and 17,14.

Thus there were a total of 88 unknowns common to all orbits. With seven unknowns represented by the Keplerian elements plus an acceleration parameter for each arc, the computer storage capacity for the normal equations as dimensioned was equalled. An increase of capacity to at least 145 unknowns could have been accomplished with very little difficulty. In the solutions described herein, the positions of 16 Baker-Nunn camera and 33 TRANET Doppler tracking stations were held fixed at the values obtained by Gaposchkin (1966c) and Anderle and Smith (1967), respectively. It was intended to modify the programs to increase the capacity for unknowns and to solve for station position shifts when warranted by the accuracy of the solution for gravitational coefficients, but this stage was not reached.

The satellites used are summarized in table 10.1. For the five satellites which also were used in the Kaula (1966c) solution, the data are essentially the same (except for 5 more months of TRANSIT 4A), because 1963 was the year of minimum disturbances of atmospheric density by solar activity. There are minor modifications in the arcs actually used; however, because of changes in acceptance criteria for arcs, as well as number of iterations and number of observations (32 for TRANSIT 4A, 40 for Vanguard 2, 60 for the others), a chi-square test was applied.

The significant additions to the data are the tracking of Courier 1B (28.2 degrees), GEOS–1 (59.5 degrees), and Beacon Explorer B (79.7 degrees). It was found that adding a satellite of different orbital inclination made much more difference in the solution than did adding Doppler tracking. Considerable testing was done using different weights of the Doppler tracking, relative to the camera tracking of GEOS–1, in particular, with very little variation in the results. While this situation added to our confidence that the Doppler portions of the program were correct and accurate, it meant that the major benefit of adding the capability to analyze Doppler data would not come until it enabled analysis of orbits of appreciably different inclination than the set in table 10.1: in particular, a polar orbiter.

In addition to Doppler tracking of a polar satellite, it would have been desirable that the amount of tracking of Beacon Explorer B be increased appreciably and that tracking of all satellites from more overseas stations be added in order to give a better distribution of observations than that indicated by table 10.2. The poor distribution apparently arose in part from the unavailability, for administrative reasons, of tracking from some overseas stations. This maldistribution was more severe than that tested by Anderle (1966).

Because the station positions were held fixed, of the three types of supplemental equations used in the earlier analyses only the 24-hour orbit accelerations were applied (see table 4 of Kaula (1966c)). If these equations are carried at unit weight, they have a mild influence on the solutions for the 2,2; 3,1; and 3,3 coefficients.

The method of partitioned normals was utilized, so that there was no limit on the number of orbital arcs which could be analyzed. In addition, one reference-frequency correction per pass was included as an additional, optional unknown to be separated out of the normals in the same manner as the orbital elements. Exercise of this option, however, appeared to make little difference in the results for the gravitational coefficients.

The normal-equation blocks generated from the Doppler data were kept separate from the blocks generated from the camera data, in order to facilitate the testing of different relative weights of Doppler versus camera tracking. However, as was mentioned previously, variety of tracking type seems to make much less difference than variety of orbital specifications.

The best solution (by the criterion of minimum discrepancy from terrestrial gravimetry (Kaula, 1966a)) is given in table 10.3. This solution utilized a priori standard deviations of $\pm 10^{-5}/l^2$ for nonresonating coefficients of degree $l \geq 7$. This limitation was disappointing; the variety of inclinations was such that more than a threefold ambiguity in periodicity of perturbations by tesseral harmonics should have been resolvable.

10.4 USE OF TERRESTRIAL DATA

In phases I–II the relative positions of tracking stations connected to the same triangulation systems were held fixed, and the stations were assumed to translate together in the solution. For the 12 Baker-Nunn cameras, six geodetic datums were required. Starting with phase III, station coordinates found from previous satellite orbit analyses were used as starting values, sometimes with a priori sigmas.

Terrestrial gravimetry was also used to give a priori sigmas for tesseral harmonic coefficients, to help overcome ill-conditioning. In phase II, these a priori values were based on the auto-covariance analysis of Kaula (1959b) and were extremely close to what later became familiar as the "$10^{-5}/l^2$ rule of thumb" (note the "preassigned σ" column in table 2 of Kaula (1963a)).

In 1966 a comprehensive comparison of satellite solutions with terrestrial gravimetry was undertaken (Kaula, 1966a). The principal conclusions were that the satellite analyses were indeed determining the real gravitational field, and that for the better solutions the errors of commission in the harmonic coefficients were very small in comparison with the errors of omission arising from the necessary truncation of the set of harmonics. A weighted combined solution was also made.

10.5 CONCLUSION

The four premises stated in the introduction still appear to stand. It would, though, be satisfying to see a good analytic theory used more extensively in geodetic orbit analysis. The work at UCLA was terminated in 1967 mainly because there was a shift to other interests, but also because the analyses had attained a complexity requiring attention from full-time professionals more appropriate to a government facility than a university. It was felt that our ideas of analyzing orbital data and their combination with terrestrial data were not sufficiently different from those of Gaposchkin and Lambeck (1971) to warrant continuation.

APPENDIX

TABLE 10.1.—*Satellite Specifications*

Satellite	a Earth radii	e	I Deg.	Days/ Arc	No. arcs	Total obs.	Starting date	Ending date	Type tracking
COURIER 1B	1.171	0.02	28.2	17	3	193	'65 Jun 11	'65 Oct 9	Camera
Vanguard 2	1.302	0.16	32.9	18	12	696	'62 Dec 31	'63 Dec 25	Camera
TRANSIT 4B	1.163	0.01	32.4	9	2	1350	'62 Apr 21	'62 Jun 23	Doppler
ECHO 1 ROCKET	1.250	0.01	47.2	18	14	1380	'63 Jan 1	'63 Dec 26	Camera
ANNA 2	1.177	0.01	50.1	18	15	1322	'62 Dec 31	'63 Oct 22	Camera
				9	2	3930	'63 May 16	'63 Jun 4	Doppler
GEOS-1	1.266	0.07	59.5	18	7	1126	'65 Nov 4	'66 Jun 10	Camera
				10	6	4768	'66 Jul 1	'67 Feb 9	Doppler
TRANSIT 4A	1.147	0.01	66.8	18	14	536	'62 Apr 6	'63 Dec 26	Camera
				9	2	2556	'62 Jul 19	'62 Aug 7	Doppler
Beacon Expl. B	1.154	0.01	79.7	9	2	2496	'65 Jan 30	'65 May 9	Doppler
MIDAS 4	1.568	0.01	95.9	30	12	3021	'62 Aug 3	'63 Dec 25	Camera

TABLE 10.2.—*Geographic Distribution of Doppler Tracking: Number of Passes Observed From Stations Within Each Octant*

Longitude E:		25–115	115–205	205–295	295–25
Latitude N	90 to 0	0	1109	3724	651
	0 to −90	333	352	0	315

TABLE 10.3.—*Potential Fully Normalized Spherical Harmonic Coefficients* $\times 10^8$

Degree l	Order m	$\overline{C}_{l_m}$	$\overline{S}_{l_m}$
2	2	2.45	−1.37
3	1	1.99	0.13
3	2	0.80	−0.71
3	3	0.47	1.27
4	1	−0.58	−0.39
4	2	0.40	0.68
4	3	1.02	0.08
4	4	−0.36	−0.32
5	1	−0.09	0.02
5	2	0.84	−0.14
5	3	−0.50	−0.06
5	4	0.36	0.28
5	5	−0.22	−0.14
6	1	−0.13	0.05
6	2	0.10	−0.40
6	3	0.14	0.23
6	4	−0.16	−0.84
6	5	−0.24	−0.54
6	6	−0.30	−0.80
7	1	0.17	0.05
7	2	0.34	0.04
7	3	−0.01	−0.09
7	4	−0.11	0.06
7	5	0.05	−0.03
8	1	−0.02	0.12
8	2	0.10	−0.10
8	3	0.08	0.11
8	4	−0.05	0.02
8	5	−0.02	−0.01
8	6	−0.03	0.02
9	1	0.07	−0.06
9	2	0.01	0.02
9	9	−0.18	−0.14
10	1	0.00	0.00
10	2	−0.03	0.05
11	1	−0.03	−0.04
12	1	−0.05	−0.03
12	12	−0.11	−0.01
13	12	−0.08	0.08
14	12	−0.05	−0.04
15	12	−0.08	0.01
15	13	−0.03	−0.07
15	14	−0.00	0.02
17	14	−0.05	0.12